

Holonomic Robotics

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December 25, 2011

Abstract

This paper describes the development of a holonomic robot using omni directional wheels. Both a development of the kinematic and dynamical equations of motion are derived and used as a foundation for gaining further insight into the capabilities of the robot. A discussion of the vision system used to detect objects and obstacles during navigation.

1 Variables

Parameter	Units	Description
R	m	robot radius
M	kg	mass of robot
I	$kg \cdot m^2$	inertia of robot
I_w	$kg \cdot m^2$	inertia of wheel
a_i	m/sec^2	acceleration in the i^{th} direction
v_i	m/sec	velocity in the i^{th} direction
ω_i	rad/sec	i^{th} wheel speed
r_w	m	wheel radius
τ_i	N·m	wheel torque
F_i	N	traction force (vector)
f_i	N	magnitude of traction force ($\ F_i\ $)

2 Introduction

Robots come in a variety of types and configurations: wheeled, tracked, legs, flying, etc. Common wheeled robots typically have two wheels (directly driven) with a caster wheel to make the robot stable. There are some without the caster wheel and employ a control system to keep them upright (inverted pendulum problem) and resemble a Segway scooter. All of these two wheeled robot are non-holonomic systems.

Definition 1 *A non-holonomic system in physics and mathematics is a system whose state depends on the path taken to achieve it. An automobile is an example of a non-holonomic vehicle. The vehicle has three degrees of freedom its position in two axes, and its orientation relative to a fixed heading. Yet it has only two*

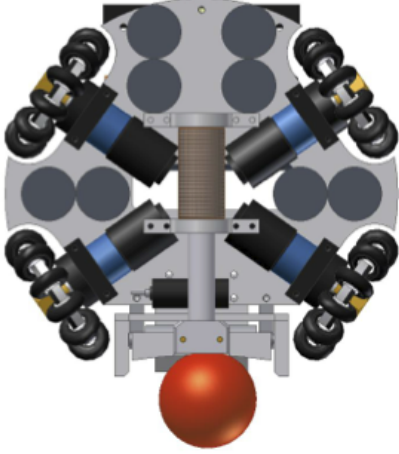


Figure 1: Holonomic soccer robot using 4 omni directional wheels and a kicking motor used to hit the red ball into a goal. [1].



Figure 2: Omni directional wheel used on the soccer robot which allows movement in any direction.

controllable degrees of freedom acceleration/braking and the angle of the steering wheel with which to control its position and orientation.

Due to these constraints, a holonomic robot (**Figure 4**) which could travel in any direction and immediately change its position and orientation is much more desirable. There are a variety of different wheels which make this type of robot possible such as mecanum or omni wheels (**Figure 2**).

Omni wheels operate like standard wheels in that the force is produced normal to the motor's axis of rotation and as a result of friction (no slip assumption). However, there are a series of smaller wheels which ring the main wheel and allow the wheel to slip in the direction of the motor rotational axis. Note that no force is produced parallel to the motor axis, just slippage.

3 Holonomic Dynamics

The dynamics for a holonomic robot, such as **Figure 4**, with 4 omni directional wheels (see **Figure 2**, can be represented as:

$$a = \sum_{i=0}^4 \frac{F_i}{M_i} = \frac{1}{M} (F_1 + F_2 + F_3 + F_4) \quad (1)$$

$$\dot{\omega} = \sum_{i=0}^4 \frac{\tau_i}{J_R} = \frac{R}{J_R} (f_1 + f_2 + f_3 + f_4) \text{ where } \tau_i = R f_i \quad (2)$$

Now looking at figure 3, and breaking equation 1 up into its x and y components

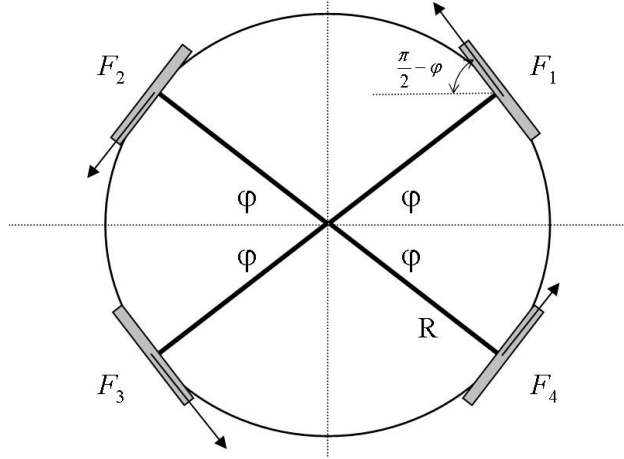


Figure 3: Coordinate system tied to the body of the robot with the origin located at the center of mass. Note that the x-axis points straight up and the y-axis points to the right. Also, the motor angle ϕ is defined as the angle measured from the y-axis. The forces (F) are the results of the motors spinning in the positive direction according to the right hand rule. Note also that no force is produced parallel to the motor's axis of rotation.

$$a_x = \frac{1}{M}(-f_1 \sin(\phi) - f_2 \sin(\phi) + f_3 \sin(\phi) + f_4 \sin(\phi)) \quad (3)$$

$$a_y = \frac{1}{M}(f_1 \cos(\phi) - f_2 \cos(\phi) - f_3 \cos(\phi) + f_4 \cos(\phi)) \quad (4)$$

$$\dot{\omega} = \frac{R}{J_R}(f_1 + f_2 + f_3 + f_4) \quad (5)$$

Now combining these into a matrix.

$$\begin{bmatrix} a_x \\ a_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{M} & 0 & 0 \\ 0 & \frac{1}{M} & 0 \\ 0 & 0 & \frac{1}{J_R} \end{bmatrix} \begin{bmatrix} \sin(\phi) & 0 & 0 \\ 0 & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} a_x \\ a_y \\ \dot{\omega} \end{bmatrix} = \beta[\phi] \gamma[f_i] \quad (7)$$

where ϕ is the angle of the motors as defined in **Figure 3**, f_i is the magnitude of the force produced by the motors, and R is the radius of the robot. This can be simplified into the following:

$$P = \frac{1}{M} A(\phi) C F \quad (8)$$

where P is the vector of accelerations, $A(\phi)$ is the matrix which defines the orientation of the motors, C defines the coupling of the motor forces, and F is the vector of forces produced by the motors. The individual motor forces can further be expressed as:

$$f_i = \frac{\tau_i}{r_w} \quad (9)$$

$$F = \frac{1}{r_w} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = \frac{1}{r_w} T \quad (10)$$

where τ_i is the torque produced by the i^{th} motor and r_w is the radius of the wheel. Here we are assuming that all wheels have the same radius r_w .

$$P = \frac{1}{Mr_w} A(\phi) CT \quad (11)$$

For a given motor orientation (e.g., $\phi = 45^\circ$) this equations provides the acceleration states as a function of motor velocity. However, the reverse would be more useful. Meaning, given a desired set of states, what are the motor velocities required?

$$T = Mr_w pinv(C) A(\phi)^{-1} P \quad (12)$$

where

$$A(\phi)^{-1} = \begin{bmatrix} \frac{1}{\sin(\phi)} & 0 & 0 \\ 0 & \frac{1}{\cos(\phi)} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \quad (13)$$

$$pinv(C) = \frac{1}{4} \begin{bmatrix} -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (14)$$

where $pinv()$ ¹ is defined as the pseudoinverse since $A(\phi)$ is not a square matrix. Finally, substituting these into the original equation, we can calculate the torques given the desired accelerations.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = \frac{Mr_w}{4} \begin{bmatrix} -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sin(\phi)} & 0 & 0 \\ 0 & \frac{1}{\cos(\phi)} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ R\dot{\omega} \end{bmatrix} \quad (15)$$

Now looking at this equation, we notice that ϕ can not be equal to 0, 90, 180, 270, or 360 otherwise we get a singularity in the $A(\phi)$ matrix. This however is not an issue in the real world, since the motors would occupy the same physical space and the robot would essentially only have 2 and not 4 motors.

3.1 State Space

Now rearranging equations (3), (4), and (5) into a state space representation where the state vector is $X = [x \ y \ R\theta \ v_x \ v_y \ R\omega]^T$ and its derivative is $\dot{X} = [v_x \ v_y \ R\omega \ a_x \ a_y \ R\dot{\omega}]^T$, the standard formulation is:

¹Pseudoinverse: for $m > n$: $A_{left}^{-1} = (A^T A)^{-1} A^T$ or $m < n$: $A_{right}^{-1} = A^T (A A^T)^{-1}$ such that $A A^{-1} = I$ or $A^{-1} A = I$

$$\dot{X} = AX + Bu \quad (16)$$

$$Y = CX \quad (17)$$

where A is the state transition matrix, B is the input matrix, u is the control input to the system, C is the output matrix which determines which states are visible, and Y is the output vector which is the observable states. Again, the nice thing about formulating the state vector X this way is that all of the units for position are in meters and all of the velocity units are in m/sec.

$$A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \quad (18)$$

$$B = \frac{1}{Mr_w} \begin{bmatrix} 0_{3 \times 4} \\ -\sin(\phi) & -\sin(\phi) & \sin(\phi) & \sin(\phi) \\ \cos(\phi) & -\cos(\phi) & -\cos(\phi) & \cos(\phi) \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad (19)$$

$$u = [\tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4]^T \quad (20)$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

The only directly measurable states come from the compass (θ) and the gyro (ω) on the IMU. Position and velocity are not directly measured. Acceleration is also measurable from the IMU and can be used to estimate the velocity and position.

$$\dot{X} = AX + Bu \quad (22)$$

$$\begin{bmatrix} v_x \\ v_y \\ R\omega \\ a_x \\ a_y \\ R\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \begin{bmatrix} x \\ y \\ R\theta \\ v_x \\ v_y \\ R\omega \end{bmatrix} + \frac{1}{Mr_w} \begin{bmatrix} 0_{3 \times 4} \\ -\sin(\phi) & -\sin(\phi) & \sin(\phi) & \sin(\phi) \\ \cos(\phi) & -\cos(\phi) & -\cos(\phi) & \cos(\phi) \\ 2 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} \quad (23)$$

Now using this we can show that this system is controllable but not fully observable.

$$\text{rank}([B \quad AB \quad A^2B \quad \dots \quad A^5B]) = 6 \quad (24)$$

$$\text{rank} \left(\begin{bmatrix} C \\ CA \\ \vdots \\ CA^5 \end{bmatrix} \right) = 2 \quad (25)$$

An estimator must be developed in order to properly control the system and keep it stable.

4 Holonomic Robot Kinematics

Now performing a similar exercise for what was done with the dynamics, looking at **Figure 3**, the velocity of motor 1 is given by $v_1 = -\sin(\phi)v_x + \cos(\phi)v_y + R\omega$. Performing this for each wheel gives:

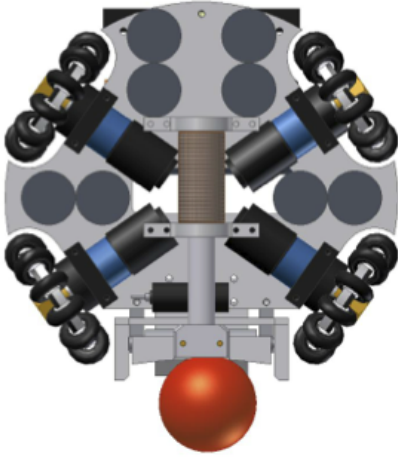


Figure 4: Configuration of three groups of motors where ϕ is 30, 45, and 60 degrees.

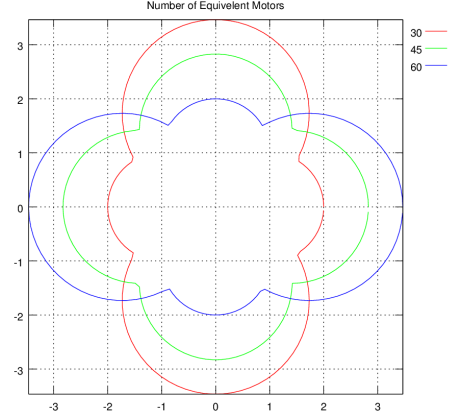


Figure 5: Number of equivalent motors for any direction under linear movement only, no rotational movement allowed.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -\sin(\phi) & \cos(\phi) & 1 \\ -\sin(\phi) & -\cos(\phi) & 1 \\ \sin(\phi) & -\cos(\phi) & 1 \\ \sin(\phi) & \cos(\phi) & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ R\omega \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sin(\phi) & 0 & 0 \\ 0 & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ R\omega \end{bmatrix} \quad (26)$$

Now setting ω to zero and calculating only linear movement, we can determine the number of equivalent motors as shown in **Figure 5**. For example, setting ϕ to 30° (the red line in **Figure 5**) and traveling in the x direction only ($[v_x \ v_y \ R\omega]^T = [1 \ 0 \ 0]^T$), the above equation simplifies to $4\sin(30)$ or 2 equivalent motors. Repeating for the y direction results in $4\cos(30)$ or 3.46 equivalent motors.

Now it is interesting to note that when ϕ is set to 30° , the robot has more equivalent motors when going forward or backwards, while a ϕ of 60° provides more equivalent motors moving left or right. When the motors are angled at 45° , movement is clearly equally optimized for both forward/backwards and left/right ($2\sin(45)$ is 2.83 motors) movement.

Figure 5 tells us that no matter how the 4 motors are oriented in a realistic configuration, the robot will never have the equivalent use of all 4 motors. Movement in one direction or another can be optimized, but then a sacrifice is made in another direction. This fact is intuitively obvious.

Another issue is these results are also ideal. This logic assumes that the wheels will not slip and have good traction in any orientation. Unfortunately real world results do not mimic this situation and the robot's performance will be reduced.

5 Control

Looking at the state space equations, the system is controllable but it is not observable. Using an IMU (accelerometer, gyro, and magnetometer), the heading (θ) can be determined from the magnetometer and the angular rate (ω) can be determined from the gyro. An observer must be used to estimate the position and velocity of the robot.

Typically encoders attached to the wheels (under the assumption of no slip) would be used to estimate velocity and position. However, with omni wheels, this is not possible since they rely on slippage in order to

Figure 6: The power system of the robot.

Figure 7: Schematic of the electronics and sensors of the robot.

achieve holonomic motion. Wheel encoders can be useful for detecting excessive amounts of wheel slippage [2] in order to optimize movement or detect failed motors.

6 Guidance and Navigation

In order to have the robot go from one location to another, the position and velocity must be estimated. A Kalman filter using the dynamic equations above will provide this solution. The general form of the Kalman filter can be found in any text book on estimation [?] and have the form:

$$\dot{x} = ax + bu \quad (27)$$

$$\tilde{x} = \hat{x} - x \quad (28)$$

where the error (\tilde{x}) is the difference between the estimated state (\hat{x}) and the true state (x).

7 Hardware Setup

7.1 Chasis

7.2 Electronic Power System

7.3 Electronics and Sensors

The robot is teleoperated from a laptop via a bluetooth link to an Arduino microcontroller which runs the motor drivers and sensors.

MiniIMU

Camera

7.4 Software

The robot utilizes the open source Robotic Operating System (ROS) [5]. The core software is run on a laptop and commands and telemetry is passed back and forth via serial bluetooth.

Figure 8: The software architecture composed of ROS nodes.

8 Results

9 References

References

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