Robust Spatio-Temporal Bayesian On-line Changepoint Detection and Model Selection

Jeremias Knoblauch[1], Theodoros Damoulas[2], Jack Jewson[1]

July 6, 2018

[1,2]University of Warwick, Department of Statistics
[2]University of Warwick, Department of Computer Science
[2]The Alan Turing Institute for Data Science and AI

Outline

(1) **Motivation** for BOCPD

(2) **Standard** BOCPD: Overview, Limitations

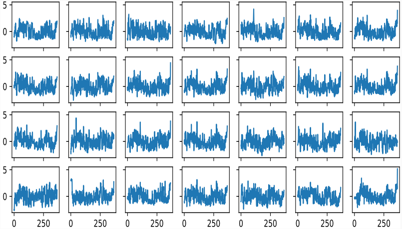
(3) **Extension I:** Multivariate/Spatio -Temporal Models

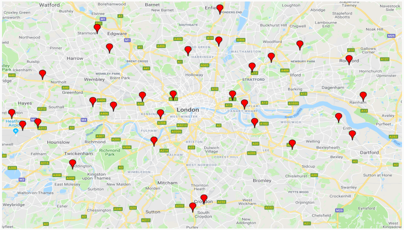
(4) **Extension II:** Model Selection

(5) **Extension III:** Robustification

[**Standard** BOCPD presented as in Adams and MacKay (2007);
Extensions I and **II** in Knoblauch and Damoulas (2018);
Extension III in Knoblauch et al. (2018)]

Motivation: Features of Air Pollution in London





(1) **Spatio-temporal** data stream

(2) Want to do principled probabilistic (= **Bayesian**) forecasting

(3) Observation frequency high $\Longrightarrow$ **on-line** treatment essential

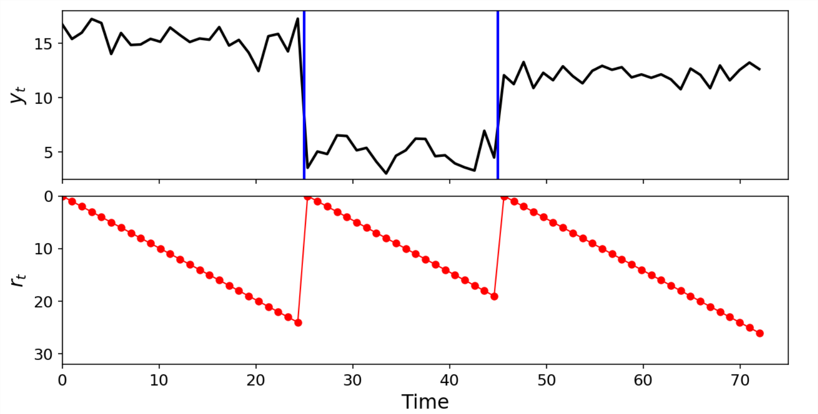
(4) Abrupt changes (congestion charge) $\Longrightarrow$ **Changepoint Detection**

$$\Longrightarrow \underbrace{\textbf{Spatio-temporal}}_{\textbf{Extension I}}\ \underbrace{\textbf{Bayesian On-line Changepoint Detection}}_{\textbf{Adams and MacKay (2007)}}$$

Standard Bayesian On-line Changepoint (CP) Detection

Idea due to Adams and MacKay (2007) and Fearnhead and Liu (2007):

(1) Define **Run-length at** $t = r_t \iff$ there was a CP at time $t - r_t$.
(2) **Inference on last CP** via $p(r_t|y_{1:t})$ rather than on *all* CPs
(3) Resulting complexity: $\mathcal{O}(t)$ **rather than** $\mathcal{O}(\prod_{i=1}^{t} i)$.



Standard BOCPD: Probabilistic model & Inference

$$r_t | r_{t-1} \sim H(r_t, r_{t-1}) \qquad \text{[conditional CP prior]} \tag{1a}$$

$$\boldsymbol{\theta} \sim \pi(\boldsymbol{\theta}) \qquad \text{[parameter prior]} \tag{1b}$$

$$\boldsymbol{y}_t | \boldsymbol{\theta} \sim f(\boldsymbol{y}_t | \boldsymbol{\theta}) \qquad \text{[observation density prior]} \tag{1c}$$

implicitly, a requirement of (f, π) is that the posterior predictives

$$f(\boldsymbol{y}_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) = \int_{\Theta} f(\boldsymbol{y}_t | \boldsymbol{\theta}) \pi(\boldsymbol{\theta} | \boldsymbol{y}_{(t-r_{t-1}):(t-1)}) d\boldsymbol{\theta} \tag{2}$$

are efficiently computable. Inference then proceeds via the recursion

$$p(\boldsymbol{y}_1, r_1 = 0) = \int_{\Theta} f(\boldsymbol{y}_1 | \boldsymbol{\theta}) \pi(\boldsymbol{\theta}) d\boldsymbol{\theta} = f(\boldsymbol{y}_1 | \boldsymbol{y}_0), \tag{3}$$

$$p(\boldsymbol{y}_{1:t}, r_t) = \sum_{r_{t-1}} \left\{ f(\boldsymbol{y}_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}) \right\}. \tag{4}$$

Observations:

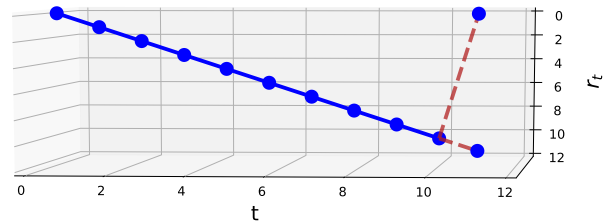
(1) Assumes that the same model (f, π) holds in each segment

(2) CPs are shifts in the parameterization $\boldsymbol{\theta}$ of that model

Standard BOCPD: Inference

$$p(\boldsymbol{y}_1, r_1 = 0) = \int_\Theta f(\boldsymbol{y}_1|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d\boldsymbol{\theta} = f(\boldsymbol{y}_1|\boldsymbol{y}_0),$$

$$p(\boldsymbol{y}_{1:t}, r_t) = \sum_{r_{t-1}} \Big\{ f(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}) H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}) \Big\}.$$



Inference:

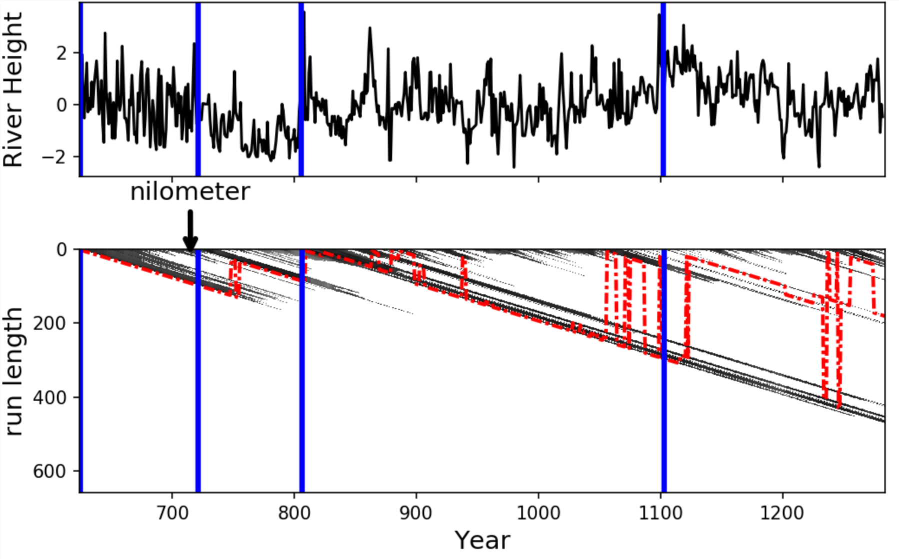
(1) **Evidence:** $p(\boldsymbol{y}_{1:t}) = \sum_{r_t} p(y_{1:t}, r_t)$

(2) **CP (run-length) posterior:** $p(r_t|\boldsymbol{y}_{1:t}) = p(y_{1:t}, r_t)/p(\boldsymbol{y}_{1:t})$.

(3) **Prediction:** $p(\boldsymbol{y}_{t+1}|\boldsymbol{y}_{1:t}) = \sum_{r_t} f(\boldsymbol{y}_{t+1}|\boldsymbol{y}_{1:t}, r_t)p(r_t|\boldsymbol{y}_{1:t})$

(4) **MAP segmentation:** $MAP_t = \max_r \{MAP_{t-r-1} p(r_t = r|\boldsymbol{y}_{1:t})\}$

Standard BOCPD: Illustration using AR(1) on Nile data



Standard BOCPD: Limitations

Limitations:

- So far: inherently **univariate** method
- Each segment described by the **same model** $m = (f, \pi)$
- **non-robust** to outliers and misspecification

Contributions:

- Construct inherently **multivariate** models m
- Allow **multiple models** $\{(f_1, \pi_1), \ldots, (f_K, \pi_K)\}$ for the segments
- Update your posteriors with **robust** divergences

Extension I: Multivariate/Spatio-Temporal models

Ingredient 1: Standard Bayesian VAR model

$$\sigma^2 \sim \mathrm{IG}(a, b) \qquad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{\Omega}) \tag{5a}$$

$$\mathrm{vec}([\boldsymbol{\alpha}, \boldsymbol{B}, \boldsymbol{A}_{1:L}]) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{V}_c) \qquad \boldsymbol{Y}_t = \boldsymbol{\alpha} + \boldsymbol{B}\boldsymbol{Z}_t + \sum_{l=1}^{L} \boldsymbol{A}_l \boldsymbol{Y}_{t-l} + \boldsymbol{\varepsilon}_t \tag{5b}$$

Ingredient 2: Neighbourhoods – Let $\mathcal{S}$ be all locations and $N(s) \subseteq \mathcal{S}$ the neighbourhood of s. (E.g., stations $>$ 10km & $>$ 20km away)

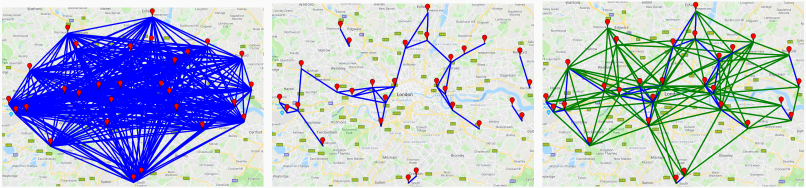
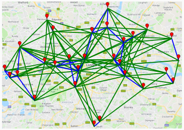
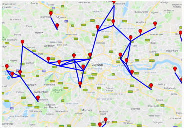


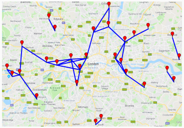
Figure 1 – Neighbourhood graphs using road distances. **Left:** Complete graph. **Center:** stations $<$ 10km apart. **Right:** Stations $<$ 10km and $<$ 20km apart.

Extension I: Multivariate/Spatio-Temporal models

$$\boldsymbol{Y}_t = \alpha + \boldsymbol{B}\boldsymbol{Z}_t \quad + \boldsymbol{A}_1\boldsymbol{Y}_{t-1} \quad + \boldsymbol{A}_2\boldsymbol{Y}_{t-2} \quad + \boldsymbol{A}_3\boldsymbol{Y}_{t-3} \quad + \varepsilon_t$$







Advantages:

(1) **Sparse** and **interpretable** coefficient matrices $\boldsymbol{A}_l$

(2) Theoretical guarantees for large class of stationary processes (Inoue et al., 2018, e.g.)

(3) Allows fast on-line computation & **incremental updates** $[\mathcal{O}(\min\{p^3, S^3\})$ for p regressors and spatial dimension $S]$

Neighbourhoods set a priori $\Longrightarrow$ different nbhs via $\{m_1, \dots, m_K\}$

Extension II: Model Selection

Idea: We allow a change of models at CP locations

New Random Variable: m_t, the model at time t

$$
\begin{aligned}
r_t | r_{t-1} &\sim H(r_t, r_{t-1}) && \text{[conditional CP prior]} && (6a)\\
m_t | m_{t-1}, r_t &\sim q(m_t | m_{t-1}, r_t) && \text{[conditional model prior]} && (6b)\\
\boldsymbol{\theta}_m | m_t &\sim \pi_{m_t}(\boldsymbol{\theta}_{m_t}) && \text{[parameter prior]} && (6c)\\
\boldsymbol{y}_t | m_t, \boldsymbol{\theta}_{m_t} &\sim f_{m_t}(\boldsymbol{y}_t | \boldsymbol{\theta}_{m_t}) && \text{[observation density prior]} && (6d)
\end{aligned}
$$

where $q(m_t | m_{t-1}, r_t) = \mathbb{1}_{\{r_t > 0\}} \delta(m_{t-1}) + \mathbb{1}_{\{r_t = 0\}} q(m_t)$.

New Recursion:

$$p(\boldsymbol{y}_1, r_1 = 0, m_1) = q(m_1) \int_{\Theta_{m_1}} f_{m_1}(\boldsymbol{y}_1 | \boldsymbol{\theta}_{m_1}) \pi_{m_1}(\boldsymbol{\theta}_{m_1}) d\boldsymbol{\theta}_{m_1} = q(m_1) f_{m_1}(\boldsymbol{y}_1 | \boldsymbol{y}_0)$$

$$p(\boldsymbol{y}_{1:t}, r_t, m_t) = \sum_{m_{t-1}, r_{t-1}} \Big\{ f_{m_t}(\boldsymbol{y}_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) q(m_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \Big\}$$

Extension II: Model Selection

$$p(\boldsymbol{y}_{1:t}, r_t, m_t) = \sum_{m_{t-1}, r_{t-1}} \Big\{ f_{m_t}(\boldsymbol{y}_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) q(m_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \\ H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \Big\}$$

For the new term involving m_t, we have that

$$q(m_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) = \begin{cases} q(m_t) & \text{if } r_{t-1} = 0, \\ q(m_{t-1} | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) & \text{if } r_{t-1} > 0. \end{cases} \tag{8}$$

with

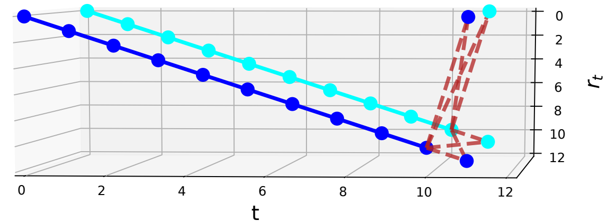
$$q(m_{t-1} | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) = \frac{p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1})}{\sum_{m_{t-1}} p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1})}. \tag{9}$$

Interpretation: model posterior at time $t-1$ becomes prior at t!

Extension II: Model Selection

$$p(\boldsymbol{y}_1, r_1 = 0, m_1) = q(m_1) \int_{\Theta_{m_1}} f_{m_1}(\boldsymbol{y}_1|\boldsymbol{\theta}_{m_1})\pi_{m_1}(\boldsymbol{\theta}_{m_1})d\boldsymbol{\theta}_{m_1} = q(m_1)f_{m_1}(\boldsymbol{y}_1|\boldsymbol{y}_0)$$

$$p(\boldsymbol{y}_{1:t}, r_t, m_t) = \sum_{m_{t-1}, r_{t-1}} \Big\{ f_{m_t}(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}) q(m_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \\ H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \Big\}$$



Observations:

(1) Each segment can now be described by a different model (f_m, π_m)

(2) CPs are shifts in models m and/or their parameterizations $\boldsymbol{\theta}_m$

(3) model prior at t = posterior at $t-1$ = $q(m_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1})$

Extension II: Model Selection

$$p(\boldsymbol{y}_{1:t}, r_t, m_t) = \sum_{m_{t-1}, r_{t-1}} \Big\{ f_{m_t}(\boldsymbol{y}_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}) q(m_t | \boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) H(r_t, r_{t-1}) p(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \Big\}$$

Inference:

(1) Evidence: $p(\boldsymbol{y}_{1:t}) = \sum_{r_t, m_t} p(\boldsymbol{y}_{1:t}, r_t, m_t)$

(2) run-length & model posterior: $p(r_t, m_t | \boldsymbol{y}_{1:t}) = p(\boldsymbol{y}_{1:t}, r_t, m_t) / p(\boldsymbol{y}_{1:t})$

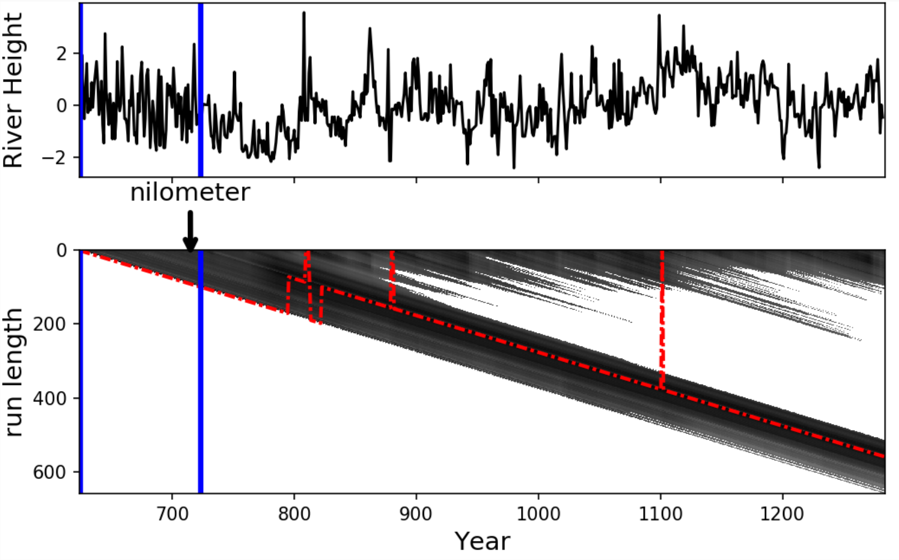
(3) Prediction: $p(\boldsymbol{y}_{t+1} | \boldsymbol{y}_{1:t}) = \sum_{r_t, m_t} f_{m_t}(\boldsymbol{y}_{t+1} | \boldsymbol{y}_{1:t}, r_t) p(r_t, m_t | \boldsymbol{y}_{1:t})$

(4) Run-length marginal posterior: $p(r_t | \boldsymbol{y}_{1:t}) = \sum_{m_t} p(r_t, m_t | \boldsymbol{y}_{1:t})$

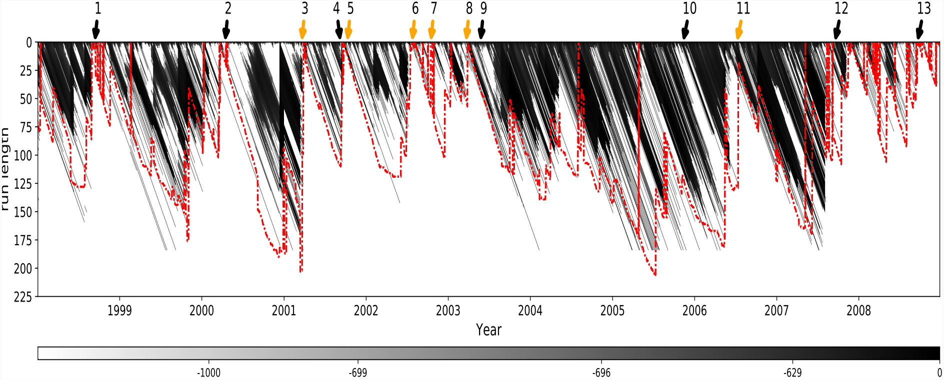
(5) Model marginal posterior: $p(m_t | \boldsymbol{y}_{1:t}) = \sum_{r_t} p(r_t, m_t | \boldsymbol{y}_{1:t})$.

(6) MAP segmentation:
$MAP_t = \max_{r,t} \{ MAP_{t-r-1} \cdot p(r_t = r, m_t = m | \boldsymbol{y}_{1:t}) \}$

Improved CP detection [on Nile data]



Improved CP detection [on 30 Portfolio return data]

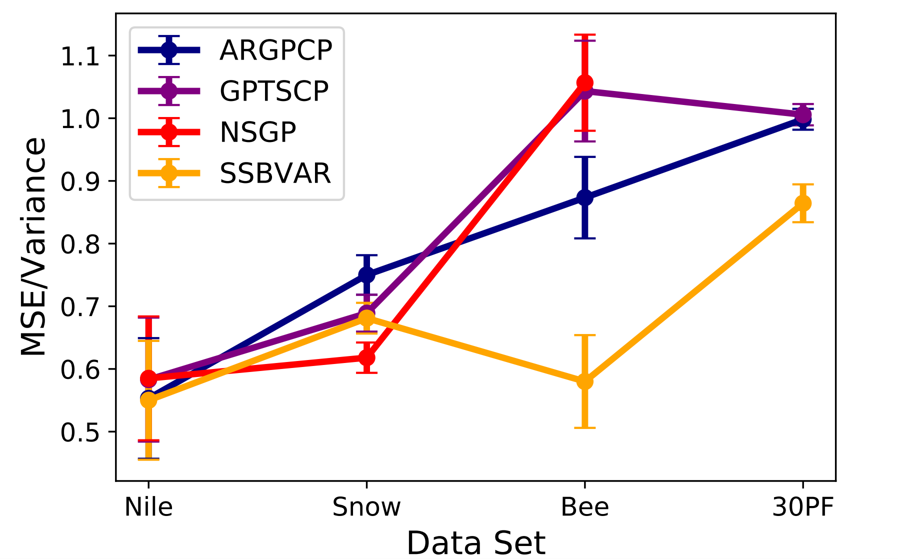


30 *Portfolio return data, 01/01/1998–31/12/2008*. CPs found with GPs by Saatçi et al. (2010) in **black**, some **new CPs** found by BOCPDMS are:

- (3) OPEC cuts output by 4%,
- (8) Iraq war,
- (11) Iran announces successful enrichment of Uranium,

Improved Prediction of multiple VARs vs single GP models

[MSE values for GP changepoint models as in Saatçi et al. (2010)]

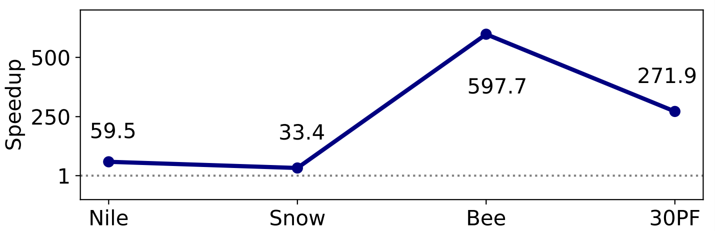


Fast computation: E.g., much faster than GP CP-models

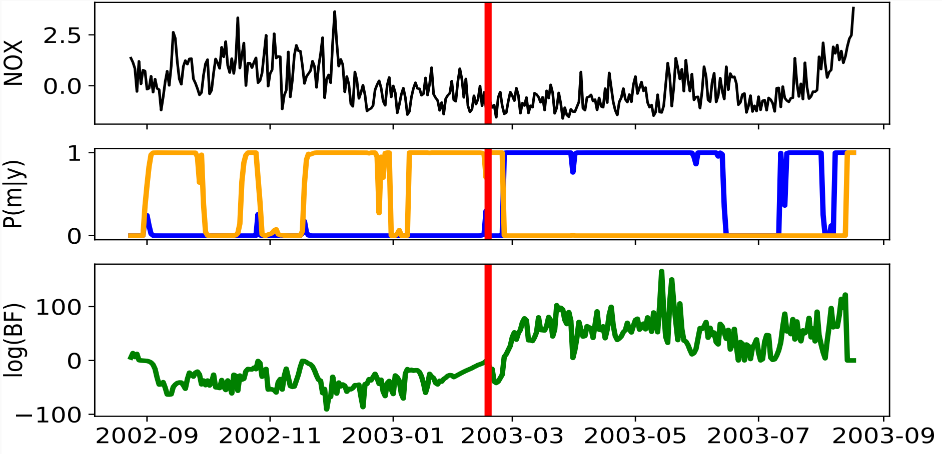
Theory:

Processing $y_{1:T} \in \mathcal{R}^{T \times d}$ using a VAR, tracking K most likely (r_t, m_t):
BOCPDMS: $\mathcal{O}(\sum_{t=1}^{T} K \cdot C_{p,d}) = \mathcal{O}(TKC_{p,d}) = \mathcal{O}(TK)$
Previous GP CP models: $\mathcal{O}(TK^3)$
$\Longrightarrow$ BOCPDMS using VARs is faster by a factor of K^2.

Practice:



✔ New capability: Model Selection on shifting multivariate dynamics



Panel 1: NOX levels in London with **congestion charge introduction**

Panel 2: Model posteriors for the two VAR models

Panel 3: Corresponding log Bayes Factors

Summary: Novelty & Implications so far

Novel Features added to BOCPD:

- Multivariate modelling of dependencies between data streams
- Generalizing BOCPD to model selection

Practical Implications:

- Improved CP detection
- Improved Prediction, especially in multivariate data
- New capability: Inference on shifting multivariate dynamics

Unresolved Limitation:

- **non-robust** to outliers
- **non-robust** to model misspecification

Limitation: BOCPD is not robust to outliers/misspecification

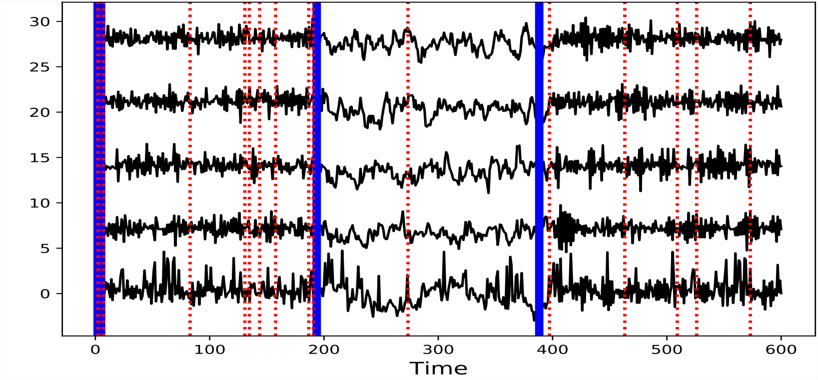


Figure 2 – E.g., 5-dimensional AR(1) with outliers in bottom-most series.
Segmentation we want and **segmentation we get** using BOCPD with AR(1).

Limitation: BOCPD is not robust to outliers/misspecification

How is it non-robust? (Superficial reasons)

- ⚠ On-line processing
- ⚠ Moderate/high dimensions for $\boldsymbol{y}_t$

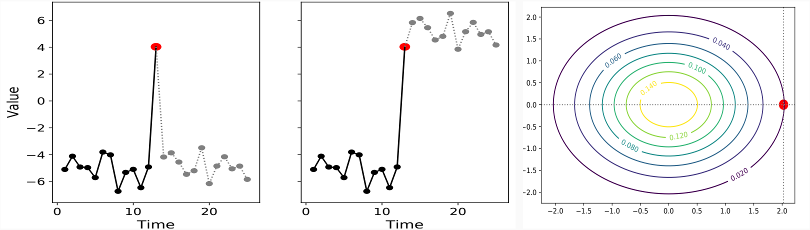


Figure 3 – Left, Center: Price for on-line processing is that outliers are confused with changepoings. **Right:** Multivariate densities become very small even if outliers occur only in a single dimension.

Limitation: BOCPD is not robust to outliers/misspecification

Why is it non-robust? (Fundamental reason for ⚠)

- **Influence function** associated with (standard) Bayesian inference!
- **Generalized Bayesian Inference** with robust divergences!

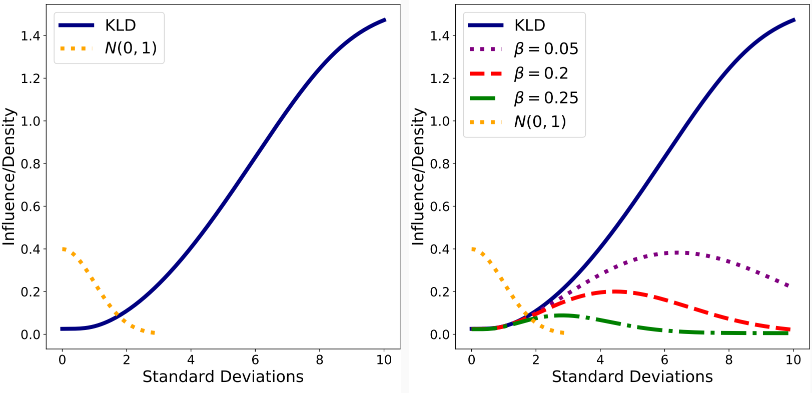


Figure 4 – Left: standard Bayes influence (Kullback-Leibler Divergence (KLD)) and standard normal density. **Right**: Robust β-Divergence (β-D) family.

Extension III: Robustification

We propose Generalized Bayesian Inference (GBI) Bissiri et al. (2016) to make BOCPD **doubly robust** for inference on $\boldsymbol{\theta}_m$ and on (r_t, m_t).

Idea of GBI: Bayesian belief updates about the parameter $\boldsymbol{\theta}_m$ driven by arbitrary loss functions $\ell(\boldsymbol{\theta}_m|\boldsymbol{y}_i)$:

$$\pi_m^{\mathrm{GBI}}(\boldsymbol{\theta}_m|\boldsymbol{y}_{(t-r_t):t}) \propto \pi_m(\theta)\exp\left\{-\sum_{i=t-r_t}^{t}\ell(\boldsymbol{\theta}_m|\boldsymbol{y}_i)\right\} \tag{11}$$

Problem: This is great if you want to do something like inference on the median (choose $\ell(\boldsymbol{\theta}_m|\boldsymbol{y}_i) = \|\boldsymbol{\theta}_m - \boldsymbol{y}_i\|_1$), but can you use this to do inference for dynamic probabilistic models ?

Solution: Work of Jewson et al. (2018) uses 1:1 correspondence between divergences and losses to propose generalized divergence-based GBI for a given probabilistic model. $\Longrightarrow$ Allows you to do robust Bayesian inference on arbitrary models

Extension III: Robustification

We propose β-D-based Generalized Bayesian Inference (GBI) to make BOCPD **doubly robust**: For the inference on $\boldsymbol{\theta}_m$ and on (r_t, m_t).

Parameter Layer robustified via β_p:

$$\pi_m^\beta(\boldsymbol{\theta}_m|\boldsymbol{y}_{(t-r_t):t}) \propto \pi_m(\theta)\exp\left\{-\textstyle\sum_{i=t-r_t}^{t}\ell^\beta(\boldsymbol{\theta}_m|\boldsymbol{y}_i)\right\}, \tag{12}$$

$$\ell^\beta(\boldsymbol{\theta}_m|\boldsymbol{y}_t) = -\left(\frac{1}{\beta_\mathsf{p}}f_m(\boldsymbol{y}_t|\boldsymbol{\theta}_m)^{\beta_\mathsf{p}} - \frac{1}{1+\beta_\mathsf{p}}\int_{\mathcal{Y}} f_m(\boldsymbol{z}|\boldsymbol{\theta}_m)^{1+\beta_\mathsf{p}}d\boldsymbol{z}\right). \tag{13}$$

We then ensure that

$$f_m(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}) = \int_{\Theta_m} f_m(\boldsymbol{y}_t|\boldsymbol{\theta}_m)\pi_m^\beta(\boldsymbol{\theta}_m|\boldsymbol{y}_{(t-r_{t-1}):(t-1)})d\boldsymbol{\theta}_m$$

is again available in closed form by forcing $\pi_m^\beta(\boldsymbol{\theta}_m|\boldsymbol{y}_{(t-r_t):t})$ into conjugate form with $f_m(\boldsymbol{y}_t|\boldsymbol{\theta}_m)$ via Structural Variational Inference.

Extension III: Robustification

We propose β-D-based Generalized Bayesian Inference (GBI) to make BOCPD **doubly robust**: For the inference on $\boldsymbol{\theta}_m$ and on (r_t, m_t).

Run-length and Model Layer robustified via β_{rlm}:

$$\widetilde{f}_m(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}) = \underbrace{e^{-\left(\frac{1}{\beta_{\text{rlm}}} f_m(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1})^{\beta_{\text{rlm}}} - \frac{1}{1+\beta_{\text{rlm}}} \int_{\mathcal{Y}} f_m(\boldsymbol{z}|\boldsymbol{y}_{1:(t-1)}, r_{t-1})^{1+\beta_{\text{rlm}}} d\boldsymbol{z}\right)}}_{=\text{Same loss function } \ell^{\beta}\text{, but this time for } (r_t,\, m_t)}$$

New Inference/Recursion:

$$p^{\beta}(\boldsymbol{y}_{1:t}, r_t, m_t) \propto \sum_{m_{t-1}, r_{t-1}} \Big\{ \widetilde{f}_{m_t}(\boldsymbol{y}_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}) q(m_t|\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \\ H(r_t, r_{t-1}) p^{\beta}(\boldsymbol{y}_{1:(t-1)}, r_{t-1}, m_{t-1}) \Big\}$$

Note: Nothing further needed to use this for inference exactly as before!

Extension III: Robustification

β-D posterior not scalable $\Longrightarrow$ Structural Variational Inference

Observation I: As $\beta \to 0$, β-D $\to$ KLD! $\Longrightarrow \pi_m^{\mathrm{KLD}} \approx \pi_m^{\beta}$ for small β!

Observation II: In fact, we prove that for most conjugate exponential family models, we get a closed-form ELBO objective approximating

$$\widehat{\pi}_m^{\beta_p}(\boldsymbol{\theta}_m) = \underset{\pi_m^{\mathrm{KLD}}(\boldsymbol{\theta}_m)}{\operatorname{argmin}} \left\{ \mathrm{KL}\left(\pi_m^{\mathrm{KLD}}(\boldsymbol{\theta}_m) \middle\| \pi_m^{\beta_p}(\boldsymbol{\theta}_m | \boldsymbol{y}_{(t-r_t):t}) \right) \right\}. \quad (14)$$

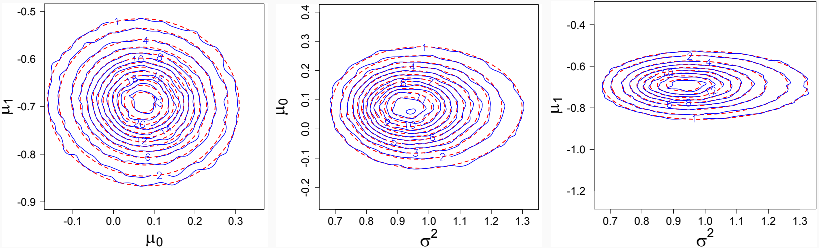


Figure 5 – Contour plots of bivariate marginals of approximation $\widehat{\pi}_m^{\beta_p}(\boldsymbol{\theta}_m)$ (dashed) and the target $\pi_m^{\beta_p}(\boldsymbol{\theta}_m | \boldsymbol{y}_{(t-r_t):t})$ (solid) estimated from 95,000 Hamiltonian Monte Carlo samples for BLR ($d = 1$, two regressors, $\beta_p = 0.25$).

Choice of $\beta \Longrightarrow$ Initialization.

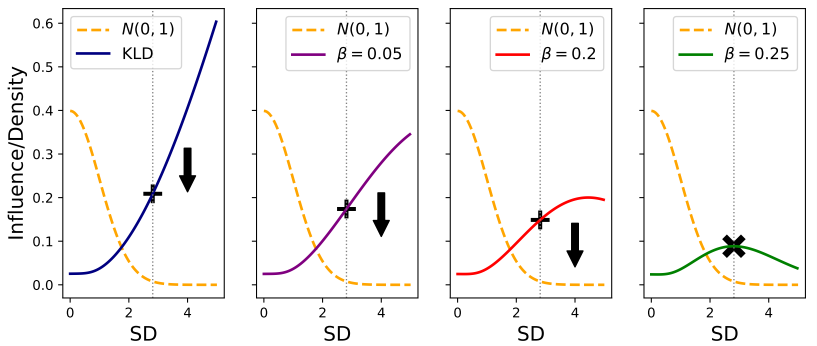


Figure 6 – Initialization procedure visualized for a (standard) normal prior on $\boldsymbol{y}_t \in \mathbb{R}$ and if we want to have maximum influence of an observation 2.75 standard deviation units from the expected value under the current belief.

Extension III: Robustification

Bad initialization of $\beta \Longrightarrow$ On-line optimization using SGD

Idea: For a predictive loss function L, and prediction $\widehat{\boldsymbol{y}}_t(\beta)$, apply SGD to $L(\boldsymbol{y}_t - \widehat{\boldsymbol{y}}_t(\beta)$ w.r.t. β

[For $p^{\beta_{\text{rlm}}}(\boldsymbol{y}_{1:t}, r_t, m_t)$: closed form gradients available; For $\widehat{\pi}_m^{\beta_{\text{p}}}(\boldsymbol{\theta}_m)$: Numerical gradient approximations used]

One can then minimize L on-line via

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} - \eta \cdot \begin{bmatrix} \nabla_{\beta_{\text{rlm},t}} L\left(\varepsilon_t(\boldsymbol{\beta}_{1:(t-1)})\right) \\ \nabla_{\beta_{\text{p},t}} L\left(\varepsilon_t(\boldsymbol{\beta}_{1:(t-1)})\right)) \end{bmatrix} \tag{15}$$

New capability: CP detection in outlier-prone data streams

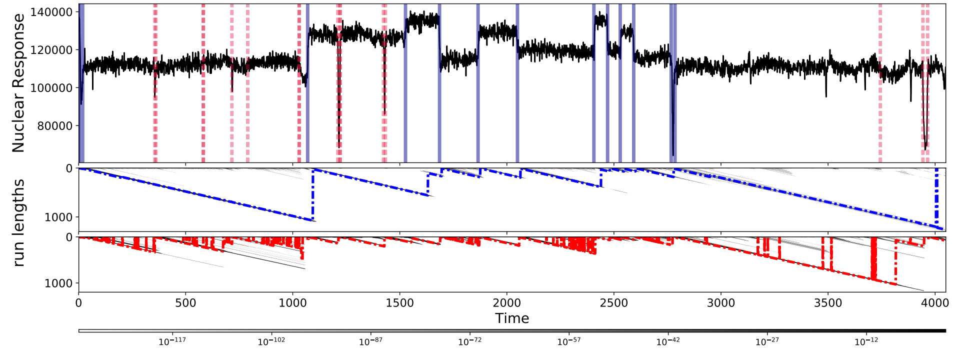


Figure 7 – **Robust segmentation and run-length distribution** and **additionally found CPs with non-robust run-length distribution**

[FDR: $> 99\% \Longrightarrow 8\%$ and reduction in MSE (MAE) by 10% (6%)]

Better Model Selection on shifting multivariate dynamics

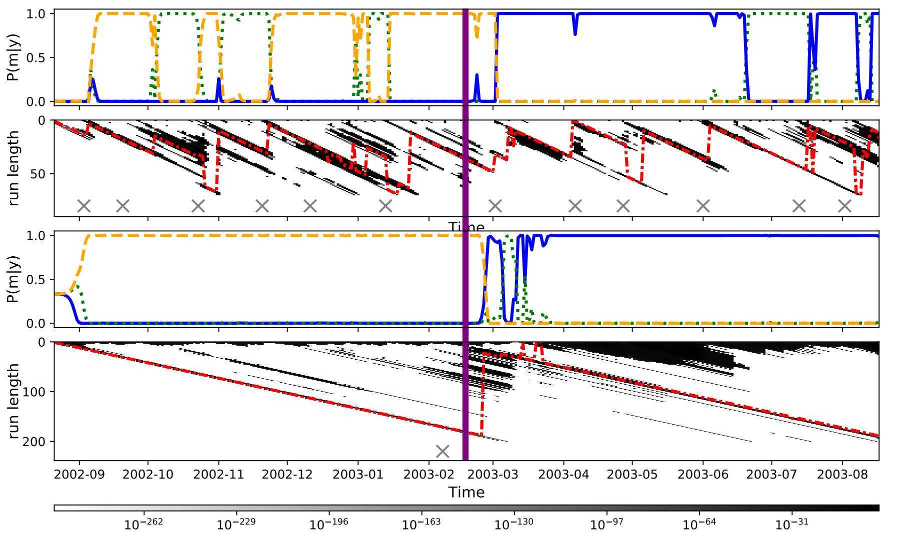


Figure 8 – Top & bottom two panels: standard & robust BOCPD.

Main References

Adams, R. P. and MacKay, D. J. (2007). Bayesian online changepoint detection. *arXiv preprint arXiv:0710.3742*.

Bissiri, P. G., Holmes, C. C., and Walker, S. G. (2016). A general framework for updating belief distributions. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 78(5):1103–1130.

Fearnhead, P. and Liu, Z. (2007). On-line inference for multiple changepoint problems. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 69(4):589–605.

Inoue, A., Kasahara, Y., Pourahmadi, M., et al. (2018). Baxter's inequality for finite predictor coefficients of multivariate long-memory stationary processes. *Bernoulli*, 24(2):1202–1232.

Jewson, J., Smith, J. Q., and Holmes, C. (2018). Principled bayesian minimum divergence inference. *arXiv preprint arXiv:1802.09411*.

Knoblauch, J. and Damoulas, T. (2018). Spatio-temporal Bayesian on-line changepoint detection with model selection. In *Proceedings of the 27th International Conference on Machine Learning (ICML-18)*. to appear.

Knoblauch, J., Jewson, J., and Damoulas, T. (2018). Doubly robust bayesian inference for non-stationary streaming data using β-divergences. In *Advances in Neural Information Processing Systems (NIPS)*. submitted to.

Saatçi, Y., Turner, R. D., and Rasmussen, C. E. (2010). Gaussian process change point models. In *Proceedings of the 27th International Conference on Machine Learning (ICML-10)*, pages 927–934.