Homework 10 Problem 2

We are given:

$$S = \exp(X)$$
$$V(X,t) = C(S,t)$$

So,

$$\frac{\partial S}{\partial X} = S$$

Now,

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial}{\partial S}\left(\frac{\partial V}{S\partial X}\right) + rS\frac{\partial V}{S\partial X} - rV = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial V}{\partial X}\frac{\partial X}{\partial S}S - \frac{\partial S}{\partial S}\frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial V}{\partial X}\frac{1}{S\partial X}S - \frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial^2 V}{\partial X^2} - \frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV = 0$$
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \left(\frac{\partial^2 V}{\partial X^2} - \frac{\partial V}{\partial X}\right) + r\frac{\partial V}{\partial X} - rV = 0$$