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Homework 10 Problem 2

We are given:

$$S = \exp(X)$$
$$V(X, t) = C(S, t)$$

So,

$$\frac{\partial S}{\partial X} = S$$

Now,

$$\begin{split} \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS \frac{\partial C}{\partial S} - rC &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial}{\partial S} \left(\frac{\partial V}{S\partial X}\right) + rS \frac{\partial V}{S\partial X} - rV &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial^2 V}{\partial S}S - \frac{\partial S}{\partial S} \frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial^2 V}{\partial S}S - \frac{\partial V}{\partial S} \frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\frac{\partial^2 V}{\partial X}S - \frac{\partial V}{\partial X}}{S^2} + r\frac{\partial V}{\partial X} - rV &= 0 \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \left(\frac{\partial^2 V}{\partial X^2} - \frac{\partial V}{\partial X}\right) + r\frac{\partial V}{\partial X} - rV &= 0 \end{split}$$