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Homework 7 Problem 4

Theorem: *It is never optimal to exercise an American call option on a non-dividend-paying stock before expiration.*

Proof

Portfolio A: American call option and $Ke^{-r(T-t)}$ in cash
Portfolio B: One share

Let, S_t be share price at time t.

For **Portfolio A**: assuming the option is exercised at some time $t < T$

$$\begin{aligned}\text{value of } \mathbf{A} &= (S_t - K) + Ke^{-r(T-t)} < S_t \\ \text{value of } \mathbf{B} &= S_t\end{aligned}$$

Now, assuming the option is exercised at T

$$\begin{aligned}\text{value of } \mathbf{A} &= max(S_T - K, 0) + K \\ &= max(S_T, K) \geq S_T \\ \text{value of } \mathbf{B} &= S_T\end{aligned}$$

So, exercising the option before maturity gives a portfolio with value less than that of **Portfolio B**. If exercised at time of maturity - the value is greater than or equal to the value of **Portfolio B**. So in the given case, an American call option should not be exercised early. □