COL 202 Homework 1

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1 Answer 1

Since there are n teams, ${}^nC_2 = \frac{n*(n-1)}{2}$ matches will be played. Therefore, there will be $\frac{n*(n-1)}{2}$ wins. By PHP, we can say that there exists at least one team whose wins are $\geq [\frac{n}{2}]$.
Because if all teams have $< [\frac{n}{2}]$ wins then maximum possible total wins will be $\leq ([\frac{n}{2}] - 1).n < \frac{n.(n-1)}{2}$.
So at least one team has won $\geq [\frac{n}{2}]$ wins.

Similarly for those $[\frac{n}{2}]$ we can say one team has $\geq [\frac{[\frac{n}{2}]}{2}]$ wins. similarly this goes on until $[\frac{[\frac{[\frac{n}{2}]}{2}]}{..}] < 1$ or we can say $[\frac{n}{2^k}] < 1([\frac{[\frac{[\frac{n}{2}]}{2}]}{ktimes}] = [\frac{n}{2^k}])$ as then no team is left and thus the sequence ends. Clearly, k is the length of the subset here . This implies $k > log_2 n$,$\Rightarrow$ $|S| > log_2 n$.

2 Answer 2

Claim: for every nice set S there exists an integer x such that $S = \{ax \mid a \in Z\}$, where if, for every $x, y \in S$ and every two integers a, b, we have $ax + by \in S$, then we call it as nice set.
Proof:
$x \in S, y \in S$ and also $ax + by \in S$.
If either $a = 0$ or $b = 0$ then $S = \{ax \mid a \in Z\}$ or $S = \{by \mid b \in Z\}$.
For $a = 0$ and $b = 0$, $S = a * 0 \mid a \in Z$ Else consider the gcd(greatest common divisor). $gcd(a, b) = g$;
We can consider $ax + by$ as a multiple of g by the result of the 8th problem of the tutorial. So we can write $ax + by$ as kg where k can be any integer.
Therefore every nice set can shown to be of the form of $\{gx \mid x \in Z\}$

3 Answer 3

We are given $b \in B$.

$B' = \{b \in B|$ there exists $b^* \in B \backslash Im(f)$ there exists $k \in NU\{0\} : (fog)^k(b^*) = b\}$.

$$A' = \{g(b)|b \in B'\}].$$

Checking statement equivalent :
a) $b \in B'$

b) If f^{-1} (b) exists then it is in A'.

c) g(b) $\in$ A'.

1) Checking a $\implies b$
given $b \in B'$

Claim: Statement b
$b \in B'$. So, $b = (fog)^k(b^*) = b$ where $k \in NU\{0\}$
$f(g(fog^{k-1}(b^*) = b$.
If f^{-1} exists, $g((fog)^{k-1}(b^*)) = f^{-1}(b)$.
As $(fog)^{k-1}(b^*) \in B'$.
So, $b_2 = (fog)^{k-1}(b^*)$

So, $g(b_2) \in A'$. (as $A' = \{g(b)|b \in B'\}$.
Checking c $\implies a$:
Given $A' = \{g(b)|b \in B'\}$.

From statement c, $g(b) \in A'$.
By definition of A', $b \in B'$.
Hence, proved.
Checking b $\implies c$
Let $f^{-1}(b) exist$. So, $f^{-1}(b) \in A'$.

$f^{-1}(b) = g(b')$ where $b \in B'$. By definition of A', Taking f both sides, $f(f^{-1}(b) = f(g(b')$.

b = fog(b').

$b' = (fog)^k(b*)$ where b* is defined above.

$b = (fog)^{k+1}(b^*) \in B'$.

$g(b) \in A'$ (By definition). Hence proved.

4 Answer 4

If A is a finite set, then let $|A| = n$. As A is finite, every subset of A will be a finite length string over A. Hence, A^* can be taken as the power set of A. Therefore, as $|A| = n$,$|A^*| = 2^n$. Hence as n is finite, 2^n is also finite, i. e. A^* is countable.
When A is countably infinite, let the elements of A be ordered in any way i. e. $A = (a_1, a_2, a_3...)$. A^* is the set of all finite subsets of A.
Let $A_i = \{A_i \in A \mid \forall\, a_m \in A_i,\ m < i\}$
$A^* = \cup_{\forall_{A_i}} A_i$
$Ai = 2^{\{a_1, a_2, ..a_{i-1}\}}$
$\Rightarrow A_i is finite.$
We know that the countable union of countable sets is countable. We are given that A is countably infinite, which means that A^* is a countable union of countable sets. Hence A^* is countable.

5 Answer 5

Proposition p(k) :- If graph has k nodes, then at least 2 nodes will have same degree(same no. of branches)
Using Principle Of Mathematical Induction :-
Base Case:
k = 2
If there are two nodes:

1. Connected
2. Not Connected

For both subcases their degree is same.
Induction Assumption: We assume for k = 3 to n-1, we have atleast 2 nodes with equal degree. Let there be h such nodes where $h \geq 2$.
Induction Step for n:
Case 1:
For $h \geq 3$, we will always have two nodes equal.
Subcase: If n^{th} node is connected to all or none such h nodes then we can observe that all h such nodes will still have equal degree.
Subcase: If n^{th} node is connected to only one such node then also there are still h-1 nodes with equal degree where $h \geq 3$.
Subcase: If n^{th} node is connected to at least 2 such nodes then we have 2 such nodes with equal degree.
Case 2:
For $h = 2$:
Subcase: When n^{th} node is connected to both such h nodes then also we have 2 nodes with equal degree.
Subcase: When n^{th} node is connected to none of such nodes then also we have 2 nodes with equal degree.
Subcase: When n^{th} node is connected to only one such node then using pigeon hole principle if we take n nodes then possible degrees are 0 to n-2 or 1 to n-1, because if there exist one such possible node which is not connected to any node then we can not have n-1 degree option. So we have n-1 options and n nodes, So at least 2 nodes should have same degree.