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## GNU Aris

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Edition 1.3, for aris version 2.2  
28 August 2013

By Ian Dunn

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This manual is for GNU Aris, the logical proof program.

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# 1 Introduction

This manual is for GNU Aris, a sequential proof program, designed to assist anyone interested in solving logical proofs. Aris supports both propositional and predicate logic, as well as Boolean algebra and arithmetical logic in the form of abstract sequences. It uses a predefined set of both inference and equivalence rules, however gives the user options to use older proofs as lemmas, including Isabelle's Isar proofs.

## 2 Terms

### Biconditional

A biconditional is a connective that connects two sentences, denoted by ' $\leftrightarrow$ '. A biconditional claims that '**sentence a if and only if sentence b**' is a new sentence. A biconditional can be inserted in Aris using the key combination *CTRL+5* (see [Section 5.4 \[Other Key Shortcuts\]](#), page 12).

### Conclusion

A conclusion is a sentence that is derived from a combination of other sentences and a rule. A focused conclusion will be highlighted in gray. A conclusion has a set of references associated with it, which are highlighted in red. Both of these colors can be changed using customization (see [Chapter 7 \[Customization\]](#), page 21).

### Conditional

A conditional is a connective that connects two sentences, denoted by ' $\rightarrow$ '. A conditional claims that '**if sentence a, then sentence b**' is a new sentence. A conditional can be inserted in Aris using the key combination *CTRL+4* (see [Section 5.4 \[Other Key Shortcuts\]](#), page 12).

### Conjunction

A conjunction is a connective that connects two or more sentences, denoted by ' $\wedge$ '. A conjunction claims that '**sentence a and sentence b**' is a new sentence. A conjunction can be inserted in Aris using the key combination *CTRL+7* (see [Section 5.4 \[Other Key Shortcuts\]](#), page 12).

### Connective

A connective is a logical symbol that connects one or more sentences. The connectives used in system PSI are conjunction (' $\wedge$ '), disjunction (' $\vee$ '), negation, (' $\neg$ '), conditional (' $\rightarrow$ '), and biconditional (' $\leftrightarrow$ '). In addition, system PSI recognizes the one-place connectives of the tautology ('T') and the contradiction ('!').

### Contradiction

A contradiction is a zero-place connective that stands on its own, denoted by '!'. A contradiction represents something that is always false. A contradiction is only used with the boolean rules (see [Section 6.4 \[Boolean Rules\]](#), page 19), and can be inserted using the key combination *CTRL+1*.

### Disjunction

A disjunction is a connective that connects two or more sentences, denoted by ' $\vee$ '. A disjunction claims that '**sentence a or sentence b**' is a new sentence. A disjunction can be inserted in Aris using the key combination *CTRL+\* (see [Section 5.4 \[Other Key Shortcuts\]](#), page 12).

### Evaluate

To evaluate a sentence means different things depending on the type of sentence. At the very least, evaluation checks the sentence for text errors, i.e. mismatched parenthesis, etc. Evaluating a conclusion checks that the sentence's text logically follows from the given references and its rule. Evaluating a goal means checking the corresponding proof for a sentence with this exact same

text. To evaluate a sentence use the key combination *CTRL+F* (see [Section 5.1 \[Proof Windows\]](#), page 9).

#### Evaluation Value

An evaluation value is the value that appears to the right of a sentence's text entry. It can either be a square, a light green check, an X, an X polygon, a pointer box, or a dark green check icon. The square means that the sentence is awaiting evaluation. The light green check means that either the conclusion logically follows from its references and rule, or that the goal has been found in the corresponding proof. In the case of premises, this means that the premise has no syntactic errors. The red X means that either the conclusion does not logically follow from its references and rule, or that the goal has not been found in the corresponding proof. The X polygon means that there is a text error with this sentence. The green check means that one of the conclusion's references has a text error. The pointer box means that the conclusion is missing a rule.

#### Existential

An existential is a quantifier that precedes the rest of sentence, denoted by '3'. An existential claims that 'there exists at least on item that has property property P' is a new sentence, assuming that 'P' is a valid predicate. An existential quantifier can be inserted into Aris using the key combination *CTRL+3*.

#### Function Symbol

A function symbol maps one object to another object. These are always lower case. Examples of a function symbol is seqlog's 'z' and 's' symbols (see [Chapter 9 \[Sequence Logic\]](#), page 24).

#### Goal

A goal is a sentence that the user is looking to meet in a certain proof. The goal window contains all of these sentences, and can be toggled by the key combination *CTRL+G* (see [Section 5.1 \[Proof Windows\]](#), page 9). When a sentence in the proof matches a goal, the proof sentence's line number is highlighted in red, while the goal's line number is changed to match the proof sentence's line number.

#### Negation

A negation is a connective that is inserted in front of a sentence, denoted by '~'. A negation claims that the opposite of the negation is true. A negation can be inserted into Aris using the key combination *CTRL+ '* (see [Section 5.4 \[Other Key Shortcuts\]](#), page 12).

#### Null Object

A null object is an object, denoted by 'nil'. In Aris it resembles a null byte, '\0', and represents an undefined object in a sequence. A null object can be inserted by using the key combination *CTRL+.*

#### Predicate

A predicate is a type of logical symbol that denotes a property of or relation between one or more objects. These always begin with capital letters, and generally use prefix notation. Exceptions of this are the identity predicate ('='), the less than predicate ('<'), and the element of predicate.

Premise	A premise is a sentence that is given. A premise has no rule associated with it, nor does it have an evaluation value, unless there is an error in it. Any variables introduced in a premise are not considered arbitrary.
Proof	A proof is a set of sentences, beginning with a set of premises and ending with a set of conclusions, that the user is trying to derive something from. The proof window is the main window that appears when the user opens up a proof.
Quantifier	A quantifier is a type of logical symbol that claims something about the amount, or quantity, of an object that holds a specific property. The quantifiers used in system PSI are the universal (' $\forall$ '), and the existential (' $\exists$ ').
Reference	A reference is a sentence that is being used to derive a conclusion. A reference is highlighted in red, and can be added or removed from the current sentence by holding down <i>CTRL</i> , and left-clicking on the desired reference.
Rules	The rules in Aris are a combination of inference rules, equivalence rules, and predicate rules that the user can use to derive sentences. The rules window (also referred to as the rules tablet) is shared amongst all of the proofs in Aris. It can be toggled by the key combination <i>CTRL+R</i> (see <a href="#">Section 5.1 [Proof Windows]</a> , <a href="#">page 9</a> ) from any proof window. For the list of rules, <a href="#">Chapter 6 [Rules Index]</a> , <a href="#">page 13</a> .
Sentence	A sentence is a line in Aris. A sentence always consists of a text entry, a line number, and an evaluation value.
Subproof	A proof within a proof. To begin a subproof in Aris, use the key combination <i>CTRL+SHIFT+A</i> , and to exit one, use the key combination <i>CTRL+E</i> .
Tautology	A tautology is a zero-place connective that stands on its own, denoted by 'T'. A tautology represents something that is always true. A tautology is only used with the boolean rules (see <a href="#">Section 6.4 [Boolean Rules]</a> , <a href="#">page 19</a> ), and can be inserted using the key combination <i>CTRL+6</i> .
Universal	A universal is a quantifier that precedes the rest of sentence, denoted by ' $\forall$ '. A universal claims that ' <b>for all items, they have property P</b> ' is a new sentence, assuming that 'P' is a valid predicate. A universal quantifier can be inserted into Aris using the key combination <i>CTRL+2</i> .
Variable	A variable represents an object within a proof. A variable is introduced when it is first used. If the line that it is introduced in is a premise, the start of a subproof, or a line using existential elimination (see <a href="#">Section 6.3.4 [ee]</a> , <a href="#">page 18</a> ), then it is <b>not</b> considered arbitrary. Otherwise, it is. Only the variables from lines that can be selected from the current line are taken into account when processing it. This means that after a subproof is ended, then lines after it don't worry about variables introduced within it.

### 3 Options

`'-a VARIABLE'`

`'--variable=VARIABLE'`

Use VARIABLE as a known variable in evaluation mode. Prepend an '\*' to specify that the variable is arbitrary.

`'-b'`

`'--boolean'`

Start Aris in boolean mode.

`'-c CONCLUSION'`

`'--conclusion=CONCLUSION'`

Use CONCLUSION as a conclusion in evaluation mode. This flag can only be specified once.

`'-e'`

`'--evaluate'`

Run Aris in evaluation mode. This means that no GUI will be loaded.

`'-f FILE'`

`'--file=FILE'`

Evaluate FILE if running Aris in evaluation mode, otherwise load FILE in Aris. This flag can be specified multiple times.

`'-g'`

`'--grade'` Grades a file specified by the file flag. This flag is ignored if used more than once.

`'-l'`

`'--list'` List the rules available in Aris, and exit.

`'-p PREMISE'`

`'--premise=PREMISE'`

Use PREMISE as a premise in evaluation mode. This flag can be specified multiple times.

`'-r RULE'`

`'--rule=RULE'`

Use RULE as a rule in evaluation mode. This flag can only be specified once.

`'-t TEXT'`

`'--text=TEXT'`

Simply check the correctness of TEXT in evaluation mode.

`'-v'`

`'--verbose'`

Run Aris verbosely, printing status and error messages.

`'-x'`

`'--latex=FILE'`

Convert FILE to a LaTeX proof file in evaluation mode.



`--version`  
Print the version of Aris and exit.

`-h`  
`--help` Print a help message and exit.

## 4 Basic Usage

This chapter describes the basic usage of GNU Aris.

### 4.1 Startup

When Aris is loaded up, you will see a few things. You will see the rules window and a proof window. The rules window contains the different rules. A rule will not be selected if a premise is in focus.

The initial layout of Aris is a single sentence. From left to right, the items of a sentence are: its line number, its text entry, its evaluation value, and its rule. The rule will not be initially visible, since no rule has been selected. In addition, premises do not have rules, and thus the rule will not appear.

### 4.2 Connectives

Aris has several connectives (see [Chapter 2 \[Terms\], page 2](#)). When the keyboard command for the desired connective is activated, the desired connective will be inserted at the current cursor point, overwriting selected text.

For example, pressing **Ctrl+7** inserts a conjunction. This is the character that looks like an upside-down ‘v’. This is also called a ‘logical and’.

### 4.3 Adding Sentences

Hitting **Ctrl+A** adds a conclusion to the proof. A conclusion is always added after the current line, or, if the line is a premise, then the conclusion is added after the last premise. If the current line is a conclusion, then it is highlighted in gray.

Hitting **Ctrl+P** adds a premise to the proof. A premise will always be added after the last premise.

Pressing **Ctrl+Shift+A** adds a subproof to the proof. When in a subproof, a conclusion will always be added within the subproof. The first line of a subproof does not require a rule, but instead acts as a premise. Pressing **Ctrl+E** ends the current subproof. This creates a new conclusion just after the subproof.

To undo a command, simply press **Ctrl+Z**. This will undo the last text modification, insertion, or deletion. In the case of several insertions or several deletions, undo will undo all of them. To undo an undo, press **Ctrl+Y**, or redo.

### 4.4 Selecting Sentences

Holding **CTRL**, and left-clicking on a sentence will select a sentence as a reference sentence. The current line’s reference sentences are highlighted in red. Only a line before the current line can be selected as a reference. In addition, if the sentence is in a different subproof than the current line, then the sentence cannot be selected as a reference. Each rule requires a specific amount of references (see [Chapter 6 \[Rules Index\], page 13](#)).

Holding **SHIFT** and left-clicking on a sentence will select the sentence. The sentence will be highlighted in violet. Multiple sentences can be selected this way, however when another action is taken, all of them will be de-selected. Pressing **CTRL+SHIFT+X** will cut the

selected lines, and *CTRL+SHIFT+C* will copy the selected lines. If no lines are selected, then the current line will be used.

## 4.5 Aris Syntax

Aris expects a certain form of syntax. Most of the connectives are infix, which means that they are placed in between their arguments. The negation is one exception to this.

Aris expects that all predicates start with an uppercase character. Aris also expects that all function symbols begin with a lower case character. After this, Aris will accept any combination of upper case letters, lower case letters, numbers, or ‘\_’.

To assist in understanding the proofs, Aris also allows for comments in sentences. To make a comment, simply insert a ‘;’. Aris will ignore everything after a ‘;’ when evaluating a sentence. This way, plaintext can be written into sentences.

## 5 Menu Options

The main GUIs for Aris all have menu bars. Each of the three types of GUIs have different menu bars, and the options for each of these are described in this section.

The keyboard shortcuts here (with the exception of the connectives) can be changed using customization. See [Chapter 7 \[Customization\], page 21](#). The shortcuts listed here are the default shortcuts for Aris.

### 5.1 Proof Windows

These are the menu options for the main proof windows. Each one can be assigned a key command.

<b>‘New’</b>	
<b>‘Ctrl+N’</b>	Starts a new proof. A new window is opened for this proof.
<b>‘Open’</b>	
<b>CTRL+O</b>	Opens an existing proof in a new window.
<b>‘Save’</b>	
<b>CTRL+S</b>	Saves the current proof.
<b>‘Save As’</b>	
<b>CTRL+SHIFT+S</b>	Saves the current proof under a different name.
<b>‘Export to LaTeX...’</b>	
	Exports the current proof to a LaTeX file.
<b>‘Close’</b>	
<b>CTRL+W</b>	Closes the current proof.
<b>‘Quit’</b>	
<b>CTRL+Q</b>	Exits Aris. But since logic is so much fun, I doubt you’ll ever want to use this one.
<b>‘Add Premise’</b>	
<b>CTRL+P</b>	Inserts a new premise at the end of the other premises.
<b>‘Add Conclusion’</b>	
<b>CTRL+A</b>	Inserts a new conclusion after the current line if it is a conclusion, or at the start of the conclusions if it is a premise.
<b>‘Add Subproof’</b>	
<b>CTRL+SHIFT+A</b>	Begins a new subproof after the current line if it is a conclusion, or at the start of the conclusions if it is a premise. This is unavailable in boolean mode, since subproofs can’t be used.
<b>‘End Subproof’</b>	
<b>CTRL+E</b>	Ends the current subproof, if there is one. Otherwise, this doesn’t do anything. This is unavailable in boolean mode, since subproofs can’t be used.

'Undo'	
<i>CTRL+Z</i>	Undos the last modification to the current proof. On a new file, this does nothing.
'Redo'	
<i>CTRL+Y</i>	Undos an undo operation. If no undo has been made, then this does nothing.
'Copy Line'	
<i>CTRL+SHIFT+C</i>	Copy the current line.
'Cut Line'	
<i>CTRL+SHIFT+X</i>	Cuts the current line. This removes the line from the proof.
'Paste Line'	
<i>CTRL+SHIFT+V</i>	Inserts the copied/cut line.
'Delete Line'	
<i>CTRL+D</i>	Deletes the current line from the proof. Can also be evoked by pressing (CTRL+DELETE).
'Evaluate Line'	
<i>CTRL+F</i>	Evaluates the logical validity of the current line.
'Evaluate Proof'	
<i>CTRL+SHIFT+F</i>	Evaluates the logical validity of the current proof. This evaluates each line of the proof.
'Toggle Goals...'	
<i>CTRL+G</i>	Toggles the goal window for the current proof.
'Toggle Boolean Mode'	
<i>CTRL+B</i>	Toggles boolean mode for the current proof See <a href="#">Section 6.4 [Boolean Rules]</a> , <a href="#">page 19</a> .
'Import Proof'	
	Imports another proof into this one. This will merge the premises of the other proof into the current one, and insert the goals of the other proof as conclusions. In addition, it sets the conclusions' references as the premises, and sets them all to use the lemma rule see <a href="#">Section 6.5.1 [lm]</a> , <a href="#">page 20</a> .
'Toggle Rules'	
<i>CTRL+R</i>	Toggles the rules window.
'Small'	
<i>CTRL+-</i>	Changes the font size to small (8pt).
'Medium'	
<i>CTRL+0</i>	Changes the font size to medium (12pt).
'Large'	
<i>CTRL+=</i>	Changes the font size to large (16pt).

- 'Custom'      Changes the font size to a custom size. This menu option opens a dialog box with a numerical entry.
- 'Contents'
- F1*            Displays Aris help. This is the only key command that cannot be modified.
- 'About GNU Aris'
- Displays information about GNU Aris.

## 5.2 Rules Table

These are the commands for the rules window. Many of them are the same as for the main proof window.

- 'New'
- 'Ctrl+N'      Starts a new proof. A new window is opened for this proof.
- 'Open'
- CTRL+O*      Opens an existing proof in a new window.
- 'Submit Proofs...'
- Submits all open proofs for grading. There is more on this in the Submission session in this manual See [Chapter 8 \[Submission\]](#), page 23.
- 'Quit'
- CTRL+Q*      Exits Aris.
- 'Small'
- CTRL+-*      Changes the font size to small (8pt).
- 'Medium'
- CTRL+O*      Changes the font size to medium (12pt).
- 'Large'
- CTRL+=*      Changes the font size to large (16pt).
- 'Custom'      Changes the font size to a custom size. This menu option opens a dialog box with a numerical entry.
- 'Contents'
- F1*            Displays Aris help. This is the only key command that cannot be modified.
- 'Customize...'
- Opens the customization dialog. For more information on this, see See [Chapter 7 \[Customization\]](#), page 21.
- 'About GNU Aris'
- Displays information about GNU Aris.

## 5.3 Goal Window

These are the commands for the goal window. Many of them are the same as for the main proof window.

- 'Add Goal'
- 'Ctrl+A'      Inserts a new goal at the end of the other goals.

‘Remove goal’

**CTRL+D** Deletes the current line from the goals. Can also be evoked by pressing (CTRL+DELETE).

‘Check Line’

**CTRL+F** Checks whether the current goal has been met, if the goal is logically valid. Otherwise, this doesn’t do anything.

‘Check All’

**CTRL+SHIFT+F** Checks whether the current goal has been met, if the goal is logically valid. Otherwise, this doesn’t do anything. This checks all the goals.

‘Hide Goals’

**CTRL+G** Hides the goal window.

## 5.4 Other Key Shortcuts

These are the keyboard shortcuts for each of the connectives. Unlike most of the other keyboard shortcuts, these cannot be modified.

**CTRL+7** Inserts a conjunction (‘^’) into Aris.

**CTRL+\** Inserts a disjunction (‘v’) into Aris.

**CTRL+ ‘** Inserts a negation (‘-’) into Aris.

**CTRL+4** Inserts a conditional (‘→’) into Aris.

**CTRL+5** Inserts a biconditional (‘<->’) into Aris.

**CTRL+2** Inserts a universal (‘V’) into Aris.

**CTRL+3** Inserts an existential (‘3’) into Aris.

**CTRL+6** Inserts a tautology (‘T’) into Aris.

**CTRL+1** Inserts a contradiction (‘!’) into Aris.

**CTRL+;** Inserts an ‘element of’ predicate into Aris.

**CTRL+.** Inserts a null object (‘nil’) into Aris.

## 6 Rules Index

The rules are divided into five categories: Inference, Equivalence, Predicate, Boolean, and Miscellaneous.

### 6.1 Inference Rules

The premises of any of these rules can be in any order.

#### 6.1.1 Conjunction

$$\begin{array}{l} P \\ Q \\ R \\ \hline P \wedge Q \wedge R \end{array}$$

What conjunction is saying is the exact opposite of simplification. If P is known, and Q is known, and R is known, etc. then P and Q and R, etc. is also known.

Take for example, that it is known that ‘I don’t like green eggs and ham’, and ‘I would not eat them in a house’, and ‘I would not eat them with a mouse’. Conjunction allows us to infer that ‘I don’t like green eggs and ham, and I would not eat them in a house, and I would not eat them with a mouse.’.

Conjunction requires at least two references.

#### 6.1.2 Simplification

$$\begin{array}{l} P \wedge Q \wedge R \wedge \dots \\ \hline P \text{ (or } Q, \text{ or } R, \text{ or } \dots) \end{array}$$

Simplification says that if it is known that P and Q and R, etc. is known to be true, then P is true.

For instance, if it is known that ‘It is cloudy, and it is raining’, then simplification allows the inference of ‘It is cloudy’ and ‘It is raining’.

Simplification requires exactly one reference.

#### 6.1.3 Addition

$$\begin{array}{l} P \\ \hline P \vee Q \vee R \vee \dots \end{array}$$

What addition says is that something is already known, so it must be true that that something or something else, or something else, etc. must also be true.



For example, if it is known that ‘The sky is blue’, then addition says that it can be inferred that ‘The sky is blue, or the sky is yellow, or the sky is pink’, since only one of those statements has to be true.

Addition requires exactly one reference.

#### 6.1.4 Disjunctive Syllogism

$$\begin{array}{l} \sim P \\ P \vee Q \vee R \\ \sim R \\ \hline Q \end{array}$$

Disjunctive syllogism is commonly used when disjunctions are present. It claims that if one knows that ‘P or Q or R’, and ‘P is false’, and ‘R is false’, then Q must be true. This works with any number of disjuncts.

#### 6.1.5 Modus Ponens

$$\begin{array}{l} P \rightarrow Q \\ P \\ \hline Q \end{array}$$

One of the basic rules of logic, modus ponens say that ‘if P happens, then Q must happen. P happened, so Q must happen’.

For example, if it is known that ‘If the dog begins to bark, then someone is at the door’, and it is also known that ‘the dog has begun to bark’, then modus ponens says that ‘someone must be at the door’.

Modus Ponens requires exactly two references.

#### 6.1.6 Hypothetical Syllogism

$$\begin{array}{l} P \rightarrow Q \\ R \rightarrow S \\ Q \rightarrow R \\ \hline P \rightarrow S \end{array}$$

Also referred to as the chain rule, hypothetical syllogism states that if one knows that ‘if P then Q’, and ‘if R then S’, then one can infer ‘if P then S’. For example, if it is known ‘if it is raining, then it is cloudy’, and ‘if it is cloudy, then it is not sunny’, and ‘if it is not sunny, then it is cold’, then hypothetical syllogism allows us to infer that ‘if it is raining, then it is cold’. This works with any number of conditional statements, as long as they all follow this pattern.

Hypothetical Syllogism requires at least two references.

### 6.1.7 Excluded Middle

$$\text{—————}$$

$$P \vee \sim P$$

A law of logic, excluded middle asserts that something is either true, or it is not true.

Excluded middle requires zero references.

### 6.1.8 Constructive Dilemma

$$P \rightarrow R$$

$$P \vee Q$$

$$Q \rightarrow S$$

$$\text{—————}$$

$$R \vee S$$

Constructive Dilemma requires at least three references.

## 6.2 Equivalence Rules

Equivalence rules operate on any valid part of the sentence, and work both ways. Each equivalence rule requires one reference.

### 6.2.1 Association

$$P \wedge (Q \wedge R) \iff P \wedge Q \wedge R$$

$$P \vee (Q \vee R) \iff P \vee Q \vee R$$

A note to users, typically association is used as  $P \wedge (Q \wedge R) \iff (P \wedge Q) \wedge R$ . While Aris will allow you to prove that this is equivalent, association allows the removal of one pair of parentheses at a time.  $(P \wedge Q) \wedge (R \wedge S) \iff P \wedge Q \wedge R \wedge S$  is also valid in Aris, because association allows recursion, but only when removing several sets of parentheses or adding several sets of parentheses.

### 6.2.2 Commutativity

$$P \wedge Q \wedge R \iff Q \wedge R \wedge P$$

$$P \vee Q \vee R \iff Q \vee R \vee P$$

Just like addition and multiplication, conjunctions and disjunctions are commutative. This of course means that ‘I would like some pie and I would like some cake’ is the same as saying ‘I would like some cake and I would like some pie’.

### 6.2.3 DeMorgan

$$\sim(P \wedge Q) \iff \sim P \vee \sim Q$$

$$\sim(P \vee Q) \iff \sim P \wedge \sim Q$$

$$\sim \exists x(P(x)) \iff \forall x(\sim P(x))$$

$$\sim \forall x(P(x)) \iff \exists x(\sim P(x))$$

DeMorgan’s Laws.

### 6.2.4 Double Negation

$$\sim\sim P \iff P$$

You probably learned in English class that saying ‘I would not like to disagree’ is the same thing as saying ‘I would like to agree’. That’s what double negation claims.

### 6.2.5 Distribution

$$P \wedge (Q_0 \vee Q_1 \vee \dots \vee Q_n) \iff (P \wedge Q_0) \vee (P \wedge Q_1) \vee (P \wedge Q_2) \vee \dots \vee (P \wedge Q_n)$$

$$P \vee (Q_0 \wedge Q_1 \wedge \dots \wedge Q_n) \iff (P \vee Q_0) \wedge (P \vee Q_1) \wedge (P \vee Q_2) \wedge \dots \wedge (P \vee Q_n)$$

$$\exists x(P(x) \vee Q(x)) \iff \exists x(P(x)) \vee \exists x(Q(x))$$

$$\forall x(P(x) \wedge Q(x)) \iff \forall x(P(x)) \wedge \forall x(Q(x))$$

### 6.2.6 Subsumption

$$P \wedge (P \vee Q) \iff P$$

$$P \vee (P \wedge Q) \iff P$$

Also called absorption. This rule can be used in Boolean mode.

### 6.2.7 Idempotence

$$P \wedge P \wedge Q \wedge R \wedge R \wedge R \iff P \wedge Q \wedge R$$

$$P \vee P \vee Q \vee R \vee R \vee R \iff P \vee Q \vee R$$

Idempotence claims that ‘I like blue and I like blue’ is the same as saying ‘I like blue’.

### 6.2.8 Implication

$$P \rightarrow Q \iff \sim P \vee Q$$

Implication uses the definition of the conditional. It is also valid to claim something such as  $\sim(\sim P \vee Q) \vee (\sim R \vee S) \iff (P \rightarrow Q) \rightarrow (R \rightarrow S)$ , because implication is recursive.

### 6.2.9 Equivalence

$$P \leftrightarrow Q \iff (P \rightarrow Q) \wedge (Q \rightarrow P)$$

Equivalence uses the definition of the biconditional. Claiming that ‘P if and only if Q’ is exactly the same as claiming ‘if P then Q’ and ‘if Q then P’. Equivalence is the only rule that works with biconditionals explicitly, and is thus used any time a biconditional is seen.

### 6.2.10 Exportation

$$(P \wedge Q) \rightarrow R \iff P \rightarrow (Q \rightarrow R)$$

This is one of the few equivalence rules that deals with conditionals.

### 6.2.11 Recursion in the Equivalence Rules

For the convenience of the user, the equivalence rules work recursively. For example

$$\sim(\sim P \vee Q) \vee (\sim R \vee S)$$

---


$$(P \rightarrow Q) \rightarrow (R \rightarrow S)$$

This is an example of using implication recursively. Recursion only works if the rule is being used the same way. For example, removing multiple parentheses with association is fine, however adding and removing parentheses with association is not.

Commutativity and idempotence work differently than the others when it comes to recursion. If commutativity is applied to a connective, then no parts of that connective, or parts of those parts, and so on, can be used in commutativity. However, other parts from the sentence can be rearranged. The same goes for idempotence.

## 6.3 Predicate Rules

The predicate rules are the rules that work specifically with predicate logic.

### 6.3.1 Universal Introduction

$$P(a) ; a \text{ is arbitrary}$$


---

$$\forall x(P(x))$$

Universal Introduction claims that if a property ‘P’ is true for some arbitrary object, then it is true for all objects. A symbol is arbitrary if nothing is known about, or rather if it was not introduced through a premise or using existential Elimination. [Chapter 2 \[Terms\]](#), [page 2](#)

Universal Introduction uses exactly one reference.

### 6.3.2 Universal Elimination

$$\forall x(P(x))$$


---

$$P(a)$$

Universal Elimination claims that if a property ‘P’ is true for all objects, then it must be true for an object ‘a’.

Universal Elimination uses exactly one reference.

### 6.3.3 Existential Introduction

$$P(a)$$


---

$$\exists x(P(x))$$

Existential Introduction claims that if ‘P’ is true for some object, then there exists an object for which ‘P’ is true.

Existential Introduction uses exactly one reference.

### 6.3.4 Existential Elimination.

$$\exists x(P(x))$$


---

$P(a)$  ;  $a$  must not have been used before

Existential Elimination claims that if there exists an object for which property ‘ $P$ ’ is true, then it can be claimed that some unused object has this property. In this case, ‘ $a$ ’ becomes a placeholder for the object. [Chapter 2 \[Terms\], page 2](#)

Existential Elimination uses exactly one reference.

### 6.3.5 Bound Variable

$$\forall x(P(x)) \iff \forall y(P(y))$$

$$\exists x(P(x)) \iff \exists y(P(y))$$

Bound Variable allows the user to substitute any bound variable for another bound variable, given that the second bound variable does not appear anywhere in the scope of the quantifier of the first bound variable. For example, if it is known that  $\forall x(\forall y(P(x) \wedge P(y)))$ , an invalid use of bound variable would be to state that  $\forall x(\forall x(P(x) \wedge P(x)))$ .

As an equivalence rule, bound variable uses only one reference, and can work on any part of the sentence.

### 6.3.6 Free Variable

$$a = b$$

$$P(a)$$


---

$$P(b)$$

Free Variable allows the user to substitute a free variable for another free variable, given that the two are identical.

Free Variable uses exactly two references.

### 6.3.7 Null Quantifier

$$\forall x(P(a)) \iff P(a)$$

If a quantifier’s bound variable does not appear in its scope, then the quantifier is said to be null, and can be removed using Null Quantifier.

As an equivalence rule, null quantifier can be used on any part of the sentence, and only uses one reference.

### 6.3.8 Prenex

$$\exists x(P(x) \wedge Q(a)) \iff \exists x(P(x)) \wedge Q(a)$$

$$\forall x(P(x) \wedge Q(a)) \iff \forall x(P(x)) \wedge Q(a)$$

$$\exists x(P(x) \vee Q(a)) \iff \exists x(P(x)) \vee Q(a)$$

$$\forall x(P(x) \vee Q(a)) \iff \forall x(P(x)) \vee Q(a)$$

The Prenex Laws are used to move quantifiers to the start of the sentence.

Prenex uses only one reference, and, being an equivalence rule, can be used on any part of the sentence.

### 6.3.9 Identity

---


$$a = a$$

Identity asserts that any variable is identical to itself.

Identity does not use any references.

## 6.4 Boolean Rules

Aris can be set to use 'boolean mode', a mode used for boolean algebra. In boolean mode, only equivalence rules that handle negations, conjunctions, or disjunctions and boolean rules can be used. In standard mode, the boolean rules can still be used, however.

### 6.4.1 Boolean Identity

$$A \wedge T \iff A$$

$$A \vee ! \iff A$$

Boolean Identity claims that the conjunction of a sentence with a tautology is logically equivalent to the sentence. It also claims that the disjunction of a sentence and a contradiction is logically equivalent to the sentence.

### 6.4.2 Boolean Negation

$$A \wedge \sim A \iff !$$

$$A \vee \sim A \iff T$$

Boolean Negation claims that the conjunction of a sentence and its contradiction is a contradiction, and the disjunction of a sentence and its negation is a tautology.

### 6.4.3 Boolean Dominance

$$A \wedge ! \iff !$$

$$A \vee T \iff T$$

Boolean Dominance claims that the conjunction of a sentence and a contradiction is logically equivalent to a contradiction. It also claims that the disjunction of a sentence and a tautology is logically equivalent to a tautology.

### 6.4.4 Symbol Negation

$$\sim T \iff !$$

$$\sim ! \iff T$$

Symbol Negation claims that a tautology is the opposite of a contradiction.

## 6.5 Miscellaneous Rules

### 6.5.1 Lemma

This handy little rule allows one to use proofs one has already done. The premises don't have to match exactly, but they must be of the same form. Aris will check for each symbol it recognizes (connectives, quantifiers, parentheses, comma, and identity). These symbols must match exactly. Aris will then check that the sentences match the correct form, or rather that they appear in the correct order.

For example, if you already did a proof of the form:

$$\begin{array}{l} A \leftrightarrow B \\ A \\ \hline B \end{array}$$

And want to reuse it, then your reference sentences must be in the form 'A'  $\leftrightarrow$  'B', and 'A'. In general, they do not have to be in that order, however. Then, your conclusion must be the second half of the biconditional.

This is where Isar interoperability comes in. Instead of selecting a previous Aris proof, a .thy file can be used. Aris will attempt to translate it into a form that it can use, using most of the keywords as references, and the lemmas and theorems as goals. These are the sentences that can be proved. For more information, see [Section 10.1 \[Isabelle/Isar\]](#), page 25.

### 6.5.2 Subproof

Given a subproof with premise 'P' and conclusion (the LAST sentence) 'Q', one can infer from subproof 'P  $\rightarrow$  Q'. In some circles, this is called conditional introduction.

### 6.5.3 Sequence

This introduces a new sequence given a function. The sequence introduced this way must not have been used, and the final argument of the given function must be the bound variable of the sentence.

### 6.5.4 Induction

$$\begin{array}{l} P(z(a)) \\ P(x) \rightarrow P(s(x)) \\ \hline \forall x(P(x)) \end{array}$$

This rule is how Aris implements mathematical induction. 'P(z(x))' is the base case, and the inductive step is 'P(x)'  $\rightarrow$  'P(s(x))'.

## 7 Customization

GNU Aris can be customized using the customization dialog, or manually through the customization file. The customization dialog can be accessed through the rules table, under 'Help'.

### 7.1 Customization Dialog

When the dialog appears, there are several tabs. These are explained as follows:

#### Main Keys

This tab allows the user to customize the keyboard shortcuts that Aris uses with the proof and rules table menus. Each entry corresponds to a menu item. The only option that cannot be edited is the 'Contents' menu keyboard shortcut.

**Goal Keys** This tab allows the user to customize the keyboard shortcuts that Aris uses with the goal menus.

**Display** This tab allows the user to customize the display settings. Included in here are the font size presets, which will be set to the indicated size when activated; the default font size, which Aris will be in when loaded; and the color preferences, which will change the colors that Aris highlights different objects in.

#### Grade Server

This tab allows the user to set preferences specific to the grade server. The two options here are for the IP address of the grading server, and the password used to authenticate into the server. (see [Chapter 8 \[Submission\]](#), page 23)

For a description of the format of key commands, see [Section 7.2 \[Config File\]](#), page 21.

### 7.2 Customization File

The config file uses s-expressions to store the customization file. It is stored under the home directory, and called '.aris'. There are several key words that it recognizes:

`'key-cmd'`

`'(key-cmd 'cmd' 'key)'`

Assigns menu 'cmd' to keyboard shortcut 'key'. The keyboard shortcuts are all in the same format, which is either *s* or *c*, a plus sign, then a letter. A *c* before the letter means 'Hold control, and press the key', and *s* means the same except with the shift key.

`'font-size'`

`'(font-size 'type' size)'`

Assigns 'size' to font type 'type'. The 'type' key word can be either 'Small', which means set the small font preset, 'Medium', which means set the medium font preset, 'Large', which means set the large font preset, or 'Default', which means set the font size that Aris loads up with initially.

`'color-pref'`

`'(color-pref 'type' color)'`

Assigns 'color' to color preference 'type'. The 'color' key word is in hexadecimal.



`'grade'`

`'(grade 'key' 'value)'`

This allows customization of the grade server's information (see [Chapter 8 \[Submission\]](#), [page 23](#)). The two options for `'key'` are `'ip'` and `'pass'`. The `'ip'` key sets the grade server's IP address, and the `'pass'` sets up the password GNU Aris will use for authentication.

## 8 Proof Submission

GNU Aris allows users to submit their proofs to a grading server, which allows instructors to use Aris in their classes. Aris submits proofs through FTP, and allows users to indicate an email for themselves and an optional email for their instructors.

Submission allows for all open proofs to be submitted. This is done by specifying a problem designation (`'11.10'`, `'nats'`, etc.). Only those that have designations are submitted. The designation is changed by editing the text box next to each file name in the submission dialog box.

Grading runs by checking the correctness of the entire proof. It is the responsibility of the grading server to confirm that the proof is the correct proof.

When the files are submitted, Aris submits them to the server, along with a directive file. The directive file will be named `'USER.directive'`, where `'USER'` is the base name of the email address specified. The files submitted will be renamed to be `'BASE-USER.tle'`, where `'BASE'` is the original basename of the file. This prevents filename conflicts on the FTP server. The grading server will then run Aris in grade mode (see [Chapter 3 \[Options\]](#), [page 5](#)), and use the email provided to email the results back to the user. If the user specified an instructor's email address, then Aris will CC the instructor.

Sample scripts are included with GNU Aris. These are the files `'doc/collect.sh'` and `'doc/collect.el'`. The grade server would run `'collect.sh'`, which will call GNU Emacs in batch mode while loading `'collect.el'`. It is `'collect.el'` that handles the emails.

## 9 Sequence Logic

Sequence Logic, often abbreviated 'seqlog', is an alternative arithmetical representation system from the standard Peano Axioms in First-Order Logic. Seqlog's original purpose was allowing more natural definition of recursive functions in FOL.

Seqlog uses the symbols 's' (the successor function), 'z' (the zero function), 'v' (the value function), and '\0' (null object).

### 9.1 Axioms

Sequence Logic, often abbreviated 'seqlog', uses the following six axioms:

- $\forall x \forall y (\sim s(x) = z(y))$
- $\forall x \forall y (s(x) = s(y) \rightarrow x = y)$
- $\forall x (v(S, x) = f.S(x))$
- $\forall x (v(\backslash 0, x) = \backslash 0)$

The first axiom states that no successor is the zero object, or, to put it differently, that the zero object is the first object. The second axiom states that no two different objects have the same successor. Using these two axioms, a 'Universal Sequence' can be defined, in a way similar to how the Peano Axioms define the natural numbers. The third axiom is the definition of a sequence, stating that the value under a given sequence 'S' of every object 'x' can be determined by a function. The rule 'sq' introduces such a sequence (see [Section 6.5.3 \[sq\], page 20](#)). The fourth axiom defines the nil object. This is a lot like 'NULL' in C, or 'nil' in lisp.

The natural numbers are defined as a sequence. For example,  $\forall x (x = \text{nat} \rightarrow \forall y \forall z (v(\text{nat}, y) = v(\text{nat}, z) \rightarrow y = z) \wedge \forall y (\sim v(\text{nat}, y) = \backslash 0))$ . This will define an infinite sequence (2nd part), that is one-to-one (1st part). Then, to define zero, one simply states  $\forall x (v(\text{nat}, z(x)) = 0)$ . This means that the zero'th element of the 'nat' sequence is the object '0'.

### 9.2 Induction

Mathematical induction requires a base case, and an inductive step. In Aris, this is used in conjunction with seqlog. For seqlog, the induction scheme is:

$$\forall x (P(z(x)) \wedge (P(x) \rightarrow P(s(x)))) \rightarrow \forall x (P(x))$$

## 10 Interoperability

In addition to everything else Aris can do, Aris can also use other proofs from other systems with the lemma rule (see [Section 6.5.1 \[lm\], page 20](#)).

### 10.1 Isar Interoperability

Aris will scan an Isar proof, which is a proof done using Isabelle, and look for certain keywords. This is still being tested, and doesn't work fully yet. This section will be updated as more of this is implemented.

#### 10.1.1 fun keyword.

Standard definition of a function in seqlog.

#### 10.1.2 type\_synonym keyword

#### 10.1.3 lemma and theorem keywords

Lemmas and theorems are treated the same. Lemmas end up as the goals of the proofs that Aris creates, and are the actual sentences that can be deduced. It takes the 'if-then' form of each lemma.

#### 10.1.4 case keyword

#### 10.1.5 primrec keyword

#### 10.1.6 definition keyword

#### 10.1.7 datatype keyword

#### 10.1.8 class keyword

#### 10.1.9 instance keyword

#### 10.1.10 everything else