

# Knights and Knaves Puzzle Generation

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# Determining Constraints

- Decided to use truth tables in order to search for solutions
- Would be bad for large groups of people, as computation gets too intense
- Decided that a person would only ever say one thing about at most two people
- What are the things that a person can say?

# A Person $\phi$ and a Person $\psi$

- $\phi$
- $\psi$
- $\sim\phi$
- $\sim\psi$
- $\phi \wedge \psi$
- $\sim\phi \wedge \psi$
- $\phi \wedge \sim\psi$
- $\sim\phi \wedge \sim\psi$

- $\phi \vee \psi$
- $\sim\phi \vee \psi$
- $\sim\phi \vee \sim\psi$
- $\phi \leftrightarrow \psi$
- $\sim\phi \leftrightarrow \psi$

# The Truth Table

$\phi$	$\psi$	$\neg\phi$	$\neg\psi$	$\phi\wedge\psi$	...	$\neg\phi\leftrightarrow\neg\psi$
T	T	F	F	T		F
T	F	F	T	F		T
F	T	T	F	F		T
F	F	T	T	F		F

# Finding Solutions

- Have a truth table now, but how do we find a solution?
- Still need to choose what people are saying. This involves two steps
  1. Generate combinations of what people can say from the truth table, e.  
g.  $\Phi$  and  $\Phi \vee \Psi$ ,  $\sim\Phi \vee \sim\Psi$  and  $\Phi \leftrightarrow \Psi$ , etc.
  2. Generate permutations of how people will say these combinations, e.  
g.  $\Phi$  says  $\Phi$  and  $\Psi$  says  $\Phi \vee \Psi$ .

# Finding a Solution Extended Example

$\phi$	$\psi$	...	$\phi$	...	$\phi \wedge \neg \psi$	...
T	T		T		F	
T	F		T		T	
F	T		F		F	
F	F		F		F	

# Similar Solutions

- Tried to make sure that problems did not have duplicates, e.g.  $\Phi$  says  $\Phi \vee \Psi$  vs  $\Phi$  says  $\Psi \vee \Phi$
- This happened for the most part, except ...
- When two people have the same solution assignment (T and T or F and F), what they say doesn't matter, since both of the statements have the same truth assignment.
- Once people are given names, though logically very similar, the puzzles do seem different.

## Example of Similar Solution

$\Phi$	$\Psi$	...	$\Phi$	...	$\Phi \wedge \neg \Psi$	...
T	T		T		F	
T	F		T		T	
F	T		F		F	
F	F		F		F	

The diagram shows a truth table with columns for variables and logical expressions. The first row is the header:  $\Phi$ ,  $\Psi$ , ...,  $\Phi$ , ...,  $\Phi \wedge \neg \Psi$ , ... . The subsequent rows show truth values (T for True, F for False). A red arrow points from the first column to the sixth column, and a green arrow points from the second column to the fourth column. The last row, where all values are False, is circled.



# Some Results

- The generation only ran up to 4 people.
- 74 solutions involving 2 people.
- 15,540 solutions involving 3 people.
- 6,562,536 solutions involving 4 people.
- To go to higher numbers of people, it is probably necessary to look at the problem differently.
- Maybe to generate a random solution and see if it is a valid puzzle.