MATH3001 Project Report
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UNIVERSITY OF LEEDS

Flood-Excess Volume Analysis of Recent River Ouse Floods: Quantification and Mitigation Assessment

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1 Introduction

1.1 Background

The city of York is located on the confluence of the River Ouse and the River Foss. The River Ouse, which is the main river flowing through York, collects water from several other rivers (The Ure, Swale and Nidd) coming down from higher ground in Yorkshire (York Stories, 2012). During periods of heavy rain, the water runs off all the higher ground and down the tributaries towards York. Also, York is situated on low lying land which means the city is unfortunately prone to flooding (York FAS, 2018).

Furthermore, due to the increase in populated areas, more buildings and roads are being built (York Stories, 2012), meaning that there are more concrete and impermeable surfaces where water cannot infiltrate. These surfaces cause water to run off quickly and flow straight towards the river. By reducing the time for rainfall to reach the river, this means that the river levels rise quickly and are more likely to flood.

There are many existing flood defenses in York that have been built over many years, but these are no longer enough to prevent flooding (York FAS, 2018). One reason could be due to climate change causing increased volumes of precipitation which the existing flood defenses were not built to manage. A well established principle in climate science is that for every 1°C increase in temperature, the atmosphere can hold about 7% more water (Copernicus Climate Change Service, 2025). It follows that more moisture is available for extreme rainfall events to occur. York has one of the longest continual records in the country and peak annual maximum levels show a steady rise in river levels in the city with the vast majority of major events occurring in the last 30 years (Local Government Association, 2022).

Two of the biggest floods that York has experienced have been the flood in November 2000 and the flood in December 2015. In November 2000, the River Ouse rose to a record high 5.40m and flooded 540 properties which caused an estimated £2 million in damage (York Civic Trust, no date). In 2015, the Ouse rose to 5.20m and around 650 homes and businesses were directly affected by the flooding. During the flood of 2015, 250 people were evacuated from their homes and over 10,000 sandbags were used, as well as the clean up alone costing the council over half a million pounds (York Press, 2016).

The Foss Barrier

It will be worth to mention the Foss Barrier as it is one of the most important flood defences in York. The Foss Barrier was built following the floods of 1982 as part of a plan to reduce flooding (Environment Agency, 2016).

The Foss Barrier's main purpose is to prevent high water levels from the River Ouse from backing up into the River Foss. It was built because historically the high levels of the River Ouse would force water back up the River Foss and hence flood the surrounding properties. This is what caused the severity of the floods of 1947, 1978 and 1982 (Environment Agency, 2016). The barrier weighs 16.5 tonnes and it is located just before the River Foss joins the River Ouse. When it is lowered, it forms a seal with the river bed, preventing water from the Ouse backing up into the Foss. When the barrier is down, the water from the Foss must be pumped around the barrier and into the River Ouse to stop the Foss from backing up (Environment Agency, 2016). However, this project will focus on the defences that prevent the river Ouse from flooding not the Foss.

1.2 Project Outline

This project aims to quantify the severity of recent flooding events in York and evaluate mitigation strategies by looking into cost effectiveness. To begin, a method for quantifying floods will be introduced followed by a brief description on how cost effective analysis will be performed. With the foundation in place, the method introduced for quantifying a flood will be used on the data from the York floods of 2000 & 2015. Then, by researching completed and proposed mitigation strategies, we can determine the volume of water that they help to mitigate. As well as this, we can estimate the cost of each of the mitigation strategies which then provides a basic cost effective analysis.

Several challenges were encountered during this project such as missing data provided by the Environmental Agency (EA). This required me to use a previous students data and make large assumptions and estimates to come up with a useable dataset, which will be explained more clearly later in Section 4. Because of this, the final values and outcomes found are highly likely to be very inaccurate. However, this project focuses on finding a way to quantify floods relatively simply and hence large uncertainties are expected. It is encouraged to refer to Subsection 7.2, where you will find how to access all the code and data that has been used in this report.

2 Flood-Excess Volume (FEV)

In this section, I will introduce the concept of Flood-Excess Volume (FEV) and explore one of the methods for approximating its value. This entire section will briefly follow the methodology written by Bokhove in Section 2 ("Tool:Flood-Excess Volume (FEV)") (Bokhove et al., 2020).

2.1 Notation

This subsection provides an overview of the notation that will be used throughout the rest of the report.

- The in-situ **river level** (stage) is denoted as $\bar{h}$ and is estimated across the entire river width at a specific location. It is measured in metres and note that $\bar{h} = 0$ is the lowest point on the river bed.
- The **threshold level**, which is the maximum river level $\bar{h}$ at which the river does not flood, is denoted h_T.
- The **discharge** is denoted as Q and is defined as the volume of water flowing through a cross section of the river at a specific point in time. This is measured in volume per unit time, usually m^3/s.
- The **flood duration** is denoted at T_f and is defined as the time difference between the river level $\bar{h}$ increasing above the threshold h_T and then dropping below it.

2.2 What is FEV?

Flood-Excess Volume (FEV), denoted V_e, can be used to quantify the size of a flood event. More specifically, it is the volume of the total river discharge above a predefined threshold river level (h_T) which is causing the flooding .

To calculate FEV, we will need to know the discharge Q. If the direct discharge is not known, one way of estimating Q is by using a rating curve which uses the in-situ river level $\bar{h}$. Usually, the Environmental Agency (EA) provides a rating curve as well as the river level $\bar{h}$ at 15 minute intervals. This is done by taking velocity measurements across the river at various depths and integrating it to give a value for the in situ discharge Q. By doing this for a range of water levels, a rating curve $Q = Q(\bar{h})$ is established. Therefore, you are given this equation for discharge Q as a function of $\bar{h}$:

$$Q = c_k(\bar{h} - a_k)^{b_k} \quad , \quad k = 1, \dots, N. \tag{2.1}$$

The coefficients a_k, b_k and c_k are given by the Environmental Agency (EA) to fit the depth data in each interval, also know as stages or limbs. The intervals are given by

$$h_{k-1} < \bar{h} < h_k \quad , \quad k = 1, \dots, N,$$

where h_N is the recorded limit and the upper bound of the interval $h_{N-1} < \bar{h} < h_N$. However, during a flood event it is very common for $\bar{h} > h_N$, hence the discharge is estimated by extrapolating the final interval to the maximum level of $\bar{h}$. There are often no velocity measurements for $\bar{h} > h_N$ so uncertainties can be quite high. Especially during extreme flooding where the banks overflow, the rating curves can then become highly approximated. Therefore, rating curves are an estimate and during extreme flooding they might include unverified extrapolations.

However, in this project, the data received only included river level and no rating curve. Section 4 explains how this was overcome and how a rating curve was produced. It also explains how we have data for the discharge at 15 minute intervals just like the river level. Hence, for this project we will not use the rating curve to get discharge as function of river level ($Q(\bar{h})$), we will just use the rating curve to estimate the threshold discharge $Q(h_T)$. The threshold discharge, $Q(h_T)$, is needed because, to calculate the FEV, we determine the total volume of water that exceeds $Q(h_T)$. In Subsection 2.3, we will introduce how to calculate the Flood-Excess Volume (FEV).

2.3 Calculating FEV

FEV can be approximated differently depending on what data is available and each method has varying accuracy. During this section, we will make reference to Figure 1 which can be found at the end of this section. Before looking at the approximation, the exact definition of FEV is calculated by

$$V_e = \int_{t_f}^{t_f+T_f} (Q(t) - Q(h_T)\, dt, \tag{2.2}$$

where $Q(t)$ is the flow rate (discharge) as a function of time t, h_T is the threshold river level as defined in Subsection 2.1, t_f is the first time at which $Q(t_f) = Q(h_T)$ (threshold discharge) for the flood event and T_f is the duration of the flood. To summarise, this is just calculating the volume of water where the discharge $Q(t)$ is larger than the threshold discharge $Q(h_T)$ for the duration of the flood T_f. However, we usually only have river level $\bar{h}$ measurements at 15 minute intervals, provided by the Environment Agency, so we need to adjust this equation slightly.

2.3.1 Method

The best approximation for FEV, denoted V_{e_1}, is used when one has river level measurements $\bar{h}$ at frequent points in time over the duration of the flood T_f, as well as the rating curve $Q = Q(\bar{h})$. Let us start with calculating the total volume V of a river. As mentioned before, for this project we have data for discharge at 15 minute intervals, so we can denote the discharge as $Q(t_k)$ where k is a discrete time index. So we can approximate the total volume of the river over the duration of the flood T_f as

$$V \approx \sum_{k=1}^{n} (Q(t_k))\Delta t, \tag{2.3}$$

where n is the number of values for $Q(t_k)$ over the flood duration T_f. The time between each value of $Q(t_k)$ is denoted Δt where $\Delta t = t_k - t_{k-1}$, in this project Δt is 15 minutes . FEV concerns the volume of water above the threshold level h_T, hence Equation 2.3 can be easily adjusted to give the first approximation for FEV

$$V_e \approx V_{e_1} = \sum_{k=1}^{n} (Q(t_k) - Q(h_T))\Delta t, \tag{2.4}$$

where $Q(h_T)$ is found using the rating curve and is a constant as h_T is a fixed value. As we can see, this is very similar to Equation 2.2 for exact FEV but instead using time intervals Δt. Since n is the number of $Q(t_k)$ values over the flood duration T_f, we can write $\Delta t = T_f/n$. This means, as $n \to \infty$, $\Delta t \to 0$ and hence this approximation becomes the exact area between the curve $Q(t_k)$ and the constant value $Q(h_T)$. This can be visualised in Figure 1 as the light blue shaded area in the top right quadrant between the curve $Q(\bar{h})$ and the constant $Q(h_T)$. Notice that in Figure 1, the discharge is a function of river level as it uses the rating curve, this is because there is no discharge data as a function of time for the River Aire.

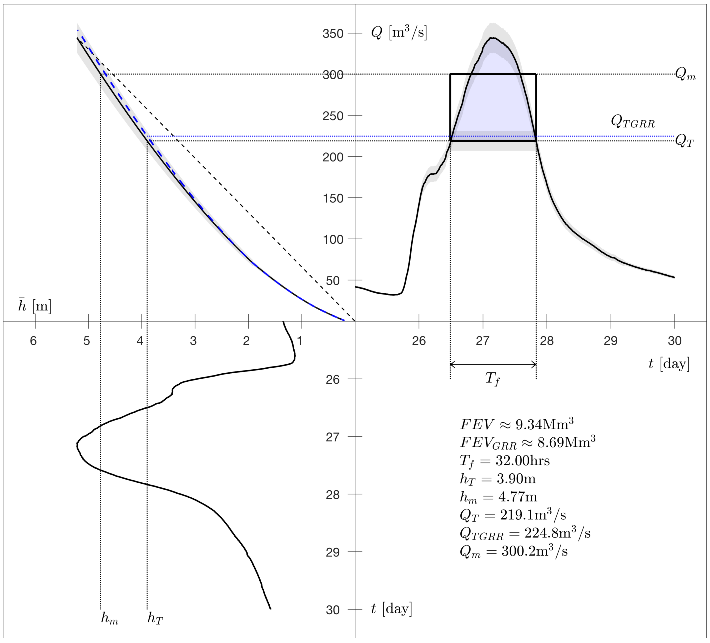


Figure 1: The rating curve (top-left panel) is displayed as well as river levels (lower-left panel) and discharge rates (top-right panel) of the River Aire at Armley, Leeds, around Boxing Day 2015. Dotted lines indicate a chosen threshold h_T, a mean river level h_m and the corresponding discharge Q_T and Q_m. The FEV is found by determining the light blue shaded area, as in 2.4, under the discharge curve $Q(\bar{h})$. Screenshot taken from (Bokhove et al., 2020).

3 Using FEV for Cost-Effective Analysis

Now that the method for calculating FEV have been established, we can introduce how FEV will be used to analyse the different mitigation strategies.

We need to show a way of communicating FEV in a more understandable way for the reader because FEV can be millions of cubic metres. Hence, the idea is to express the FEV as a 2 metre deep square 'flood-excess lake' where the volume of the lake is equal to the FEV. This can quickly help to visualise the amount of water that caused the flooding and hence needed to mitigate. If we look at this 'flood-excess lake' from above we can start to split up this lake into different strategies that will mitigate that amount of volume. The cost of each measure can be estimated as well, which leads to a simple cost effective assessment. This is done by finding the ratio between the FEV and the cost of each strategy which means cost effectiveness can be measured as a cost per percentage of FEV mitigated (Bokhove et al., 2019).

To help illustrate this, let us look at the River Aire floods of boxing day 2015 written in Bokhove et al. (2020). Bokhove uses the same method stated in Section 2.3.1 but in this case discharge is a function of river level, $Q(\bar{h})$, not time . By doing this, an estimate for the FEV was found to be $9,340,000$m^3 which can be seen in Figure 1 (Bokhove et al., 2020). Hence, by dividing this value by two (since the 'flood-excess lake' is of depth 2 metres) and then square rooting it, we can get a value for the side length of the 'flood-excess lake'.

$$\frac{9340000}{2} = 4670000 \quad \Longrightarrow \quad \sqrt{4670000} = 2161.018278$$

Therefore, the 'flood-excess lake' can be visualised as seen below in Figure 2.

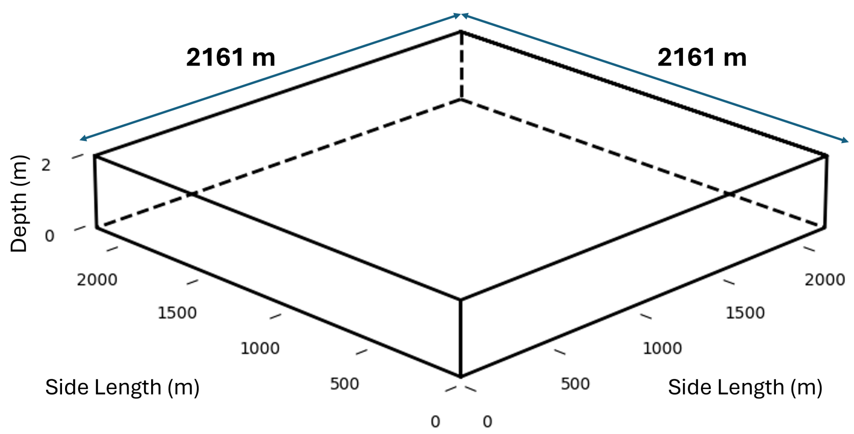


Figure 2: FEV of the River Armley 2015 boxing day floods expressed as a flood-excess Lake.

This is then a really clear visual of how much water flooded and hence needs to be mitigated. After researching different mitigation strategies, information of the cost will be found for each strategy as well as the amount of FEV that it will help to mitigate.

Hence, by looking at Figure 2 from above, you could then start to partition this square up into sections, each one representing a different flood mitigation measure and the size of it depending on the volume of FEV that it will mitigate. This can then be used to find the cost effectiveness of each mitigation strategy. For example, if the strategy costs a lot of money but only helps to mitigate a small proportion of the FEV, we would say that this plan is not cost effective.

4 Data

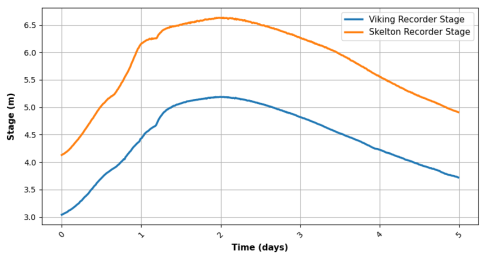
The main aim of this project was to calculate the FEV for the Viking Recorder in York for both the 2000 and 2015 floods. This is because the Viking Recorder is located in the city centre, so I decided this would be the best monitoring station to request data from. The following was requested from the Environment Agency (EA):

- Stage (river level) data at 15 minute intervals over the duration of the floods of 2000 and 2015.
- The most recent Rating Curve at this location.

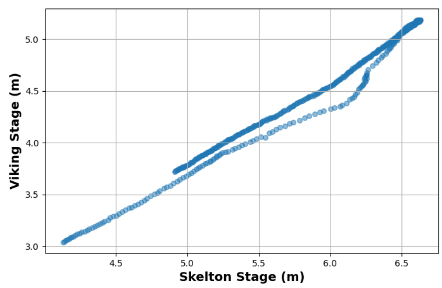
However, the EA only had stage data for the flood of 2015 and no rating curve. This immediately created a challenge as there was not sufficient data to carry out FEV calculations. Fortunately, due to previous groups, I had access to the data from the Skelton Recorder at the same time periods. The Skelton recorder, which is 5.5km upstream of the River Ouse from the Viking recorder, had stage and discharge data at 15 minute intervals. Hence, the 2015 river level data from the Viking recorder provided by the EA will be used as well as the Skelton recorder data from the previous student to find a correlation between the two monitoring stations. Following on from this, by using simple linear regression, we can create an equation that links the Viking recorder data with the Skelton data. Hence, we can use this equation to estimate the river level at the Viking Recorder in 2000.

4.1 Comparing 2015 Data

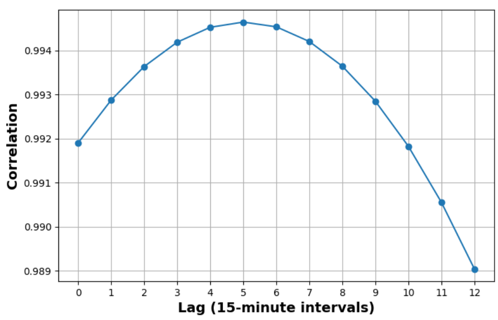
Firstly, let us plot the Stage data in 2015 for both recorders against time to visualise if there is a correlation.



(a) A Graph to show the difference in river level between the Skelton and Viking recorders during the flood of 2015.



(b) Correlation between Viking Stage vs Skelton Stage



(c) How correlation changes when shifting Skelton Data forward

Figure 3: The Python Code used to plot (b) and (c) can be found in Appendix A.1.

By looking at Figure 3a, we can clearly see a trend in the river level at the two recorders. Hence, we will use a scatter plot of Viking Stage against Skelton Stage to verify this trend, which we can see by Figure 3b, that there is clearly a positive correlation between the two recorders. However, I wanted to find if there was a better correlation if we shifted the Skelton data forward. This is because the Viking Recorder is around 5.5km downstream from the Skelton Recorder so the water will take time to travel down the river. Hence, I plotted how correlation changes when shifting the Skelton Stage data forward by 15 minute intervals. Figure 3c shows that the maximum correlation occurs when there is a $5 \times 15 = 75$ minute time lag between the two recorders. This means that the Viking Stage at time t will be estimated using the Skelton Stage at time $t - 75$ minutes. We can now use this new lagged Skelton Data together with Linear Regression to form an equation for the Viking river level.

4.2 Approximating Viking Recorder River Level

The method we will be using is a straightforward and reliable approach when you expect a mostly linear relationship as we have here. Let us introduce the simple linear regression equation that we will be using which is adapted from the methods in (Benoit, 2010):

$$V = (a \times S) + b \tag{4.1}$$

where:

- V is the Viking Stage,
- S is the Skelton Stage (with the lag of 75 minutes),
- a and b are regression coefficients.

For this equation, a tells you how much Viking Stage changes for every 1 metre change of Skelton Stage and b represents the Viking Stage when Skelton Stage is zero. Now let us show how a and b are calculated.

- The slope a:

$$a = \frac{\sum(S_i - \bar{S})(V_i - \bar{V})}{\sum(S_i - \bar{S})^2}$$

- The intercept b:

$$b = \bar{V} - a \cdot \bar{S}$$

  where: $\bar{V}$ is the mean of Viking Stage and $\bar{S}$ is the mean of the Skelton Stage. Here, S_i and V_i denote each data point for Skelton and Viking stage respectively.

Using Python, code shown in Appendix A.1, this was easily calculated and we are left with the following equation,

$$\text{Viking Stage} = 0.8412 \times \text{Skelton Stage} - 0.4557, \tag{4.2}$$

which we can then apply to the Skelton data from 2000 to get estimated river level data for the Viking Recorder in 2000 (See Figure 4).

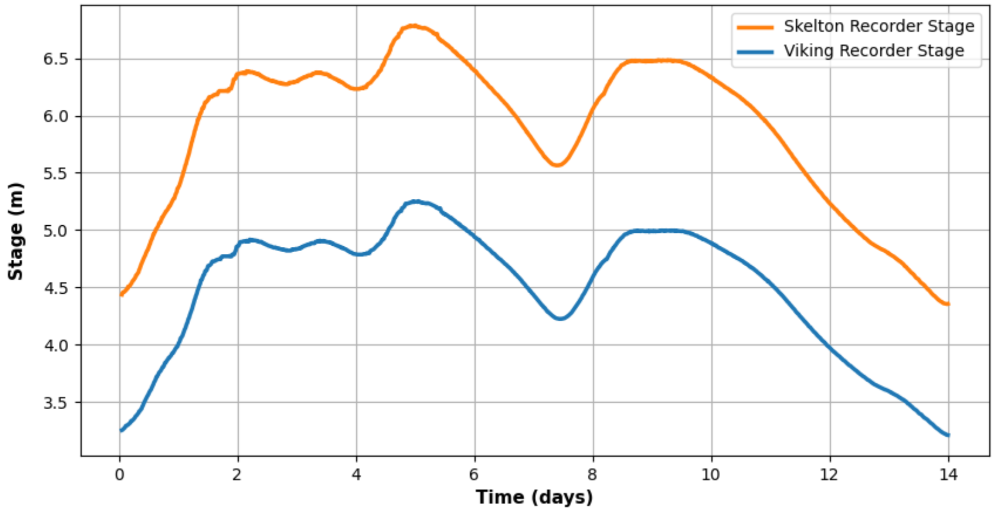


Figure 4: A Graph to show the estimated river level at the Viking Recorder during the flood in 2000.

It is worth mentioning that this data has extreme inaccuracy as it is all estimated by assuming that the relationship between Skelton Stage and Viking Stage in 2015 is identical to that in 2000. However, river behaviour changes over time due to factors like erosion, sediment deposition, and infrastructure changes. Also, the 2000 and 2015 floods could have had very different characteristics, for example, larger or faster rising floods could affect water travel time and the linearity of the relationship. However, since this data was not available by the Environment Agency, I have use what was available to approximate this data.

Due to this high inaccuracy of the 2000 data, this project will mainly focus on the data from 2015. However, a quadrant plot (same form as Figure 1) for the 2000 data was still calculated and can be found in Appendix A.6. The next step is to determine a rating curve for the Viking recorder, this will enable us to find the threshold discharge $Q(h_T)$ and hence the FEV.

4.3 Rating Curve at The Viking Recorder

Since no rating curve was provided by the EA, a rating curve will be found using the data available.

Using the flow (discharge) data from the Skelton Recorder and the adjusted time lag that was calculated in Subsection 4.1 (75 minutes), we can assume the flow at the Viking Recorder is the same just delayed slightly due to travel time. This is usually valid because this river has no significant additional inflows or outflows between the two recorders so water volume is conserved.

Therefore, by applying this to the Skelton data from 2015, we get Viking Recorder discharge in 2015 at 15 minute intervals. Then, by using the stage data for the Viking recorder in 2015 that was provided by the EA, we can plot stage against discharge and find a fit. Figure 5, shows the results of this, where the red line is the fitted rating curve.

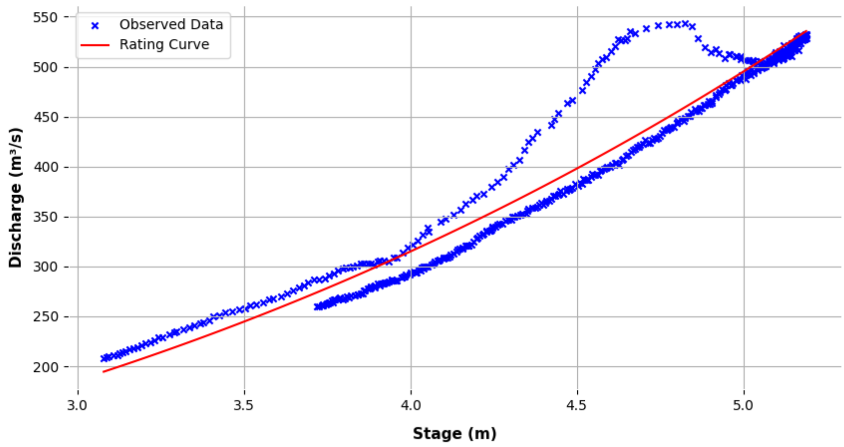


Figure 5: A graph to show Stage plotted against Discharge for the Viking Recorder in 2015. The red line shows the fitted rating curve.

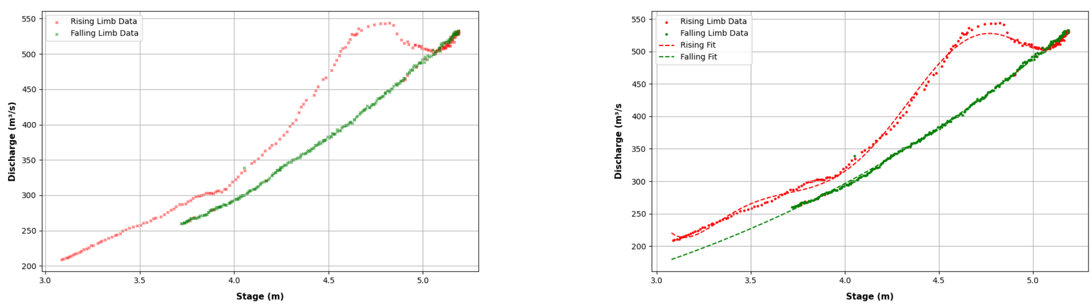
The rating curve was fitted using the form of Equation 2.1 but with just one set of coefficients a, b and c. This was done by minimising the sum of squared errors (SSE) (Appendix A.2 to see the Python code used). Hence, we get the rating curve as follows.

$$Q = 0.73\,(\bar{h} + 2.61)^{3.21} \tag{4.3}$$

The root mean squared error (RMSE) was also calculated to find how well the rating curve explains the observed data.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} = 26.1341 \tag{4.4}$$

Lower RMSE values indicate that the fit is more accurate to the real data. A value of 26 means that on average the predictions made by the fit are off by 26 units (26m^3/s) and hence suggests that the fit is not actually very accurate. By looking at Figure 5, it is quite clear there are two separate curves created. Therefore, the data was plotted again but this time, it was made clear which points the river level is rising and which points it is sinking.



(a) (b)

Figure 6: (a) A plot to show the stage-discharge relationship for the Viking Recorder in 2015. (b) A plot to show the rising and sinking limb rating curves, where the rising limb fit is a polynomial of degree eight. In these two plots, the data points from when the river level is rising has been marked in red and when the river level is sinking, it has been marked in green.

From Figure 6a, it is quite clear that we have some sort of hysteresis. Hysteresis, in this case, refers to the phenomenon where the discharge differs between the rising and falling stages of a flood event for the same water level (Perret et al., 2022). This is because, on the rising limb water moves rapidly downstream, while on the falling limb water drains more slowly. This results in a looped relationship rather than a single valued curve. Due to this, the different rating curves for the rising and sinking limbs have been calculated which can be seen in Figure 6b.

For the rising fit, a polynomial of degree 8 was chosen as it gave the best fit to the data. Due to its unusual shape, the normal form of a rating curve given by the EA (Equation 2.1), which was used for the falling fit, could not be used for the rising fit. Hence, we get Q_{up} which is the rating curve for the rising limb and Q_{down} which is the rating curve for the falling limb, where $\bar{h}$ is the river level.

Rising limb Fit,

$$\begin{aligned} Q_{up} = &- 381.71514\bar{h}^8 + 12824.94386\bar{h}^7 - 187271.92939\bar{h}^6 \\ &+ 1552352.23483\bar{h}^5 - 7990127.78084\bar{h}^4 + 26151800.13352\bar{h}^3 \\ &- 53159626.22415\bar{h}^2 + 61366729.01755\bar{h} - 30805740.47068 \end{aligned}$$

Falling limb Fit,

$$Q_{down} = 0.00035\,(\bar{h} + 6.93586)^{5.70405}$$

These equations were calculated using python functions that calculate the best fit by minimising sum of squared errors (SSE). The code used to get this plot and the equations can be found in Appendix A.3. Let us now look at the root mean square error (RMSE) of each curve to see if it is an improvement to the original rating curve seen in Equation 4.3. Using the same method as defined in Equation 4.4, we get the following.

RMSE for Rising Limb (Polynomial Fit): 9.1733
RMSE for Falling Limb (Power-Law Fit): 3.2064

For the rising limb, different degrees of polynomials were tested against there respective RMSE values. This helped decide which degree polynomial to use as our rising limb fit, see Table 1.

Polynomial Degree	2	3	4	5	6	7	8	9	10
RMSE (m^3/s)	28.04	19.25	18.61	14.72	9.87	9.86	9.17	8.68	8.57

Table 1: RMSE for different polynomial degrees

From Table 1, it is quite clear that as the degree of polynomial increases, the RMSE decreases. However, you can see that once the degree reaches six, the RMSE value starts to decrease very slowly. This means that there is no need to use a very high degree polynomial as it does not improve the fit significantly. A degree of eight was chosen for the fit as this felt reasonable and by looking at Figure 6b, we can see it is clearly a good approximation.

We can see that these values of RMSE are a large improvement to the 26.1341 found using the original rating curve in Equation 4.4. Hence, we will now use these two rating curves to calculate FEV for the Ouse flood of 2015.

5 FEV Analysis of the 2015 River Ouse Flood

5.1 Calculating the FEV

Let us look at the quadrant plot for the 2015 floods using the original rating curve found in Equation 4.3, giving us an approximate for the FEV. The threshold level h_T is the maximum river level at which the river does not flood, which was found to be $h_T = 4.57$m for the Viking recorder (GOV.UK, 2025).

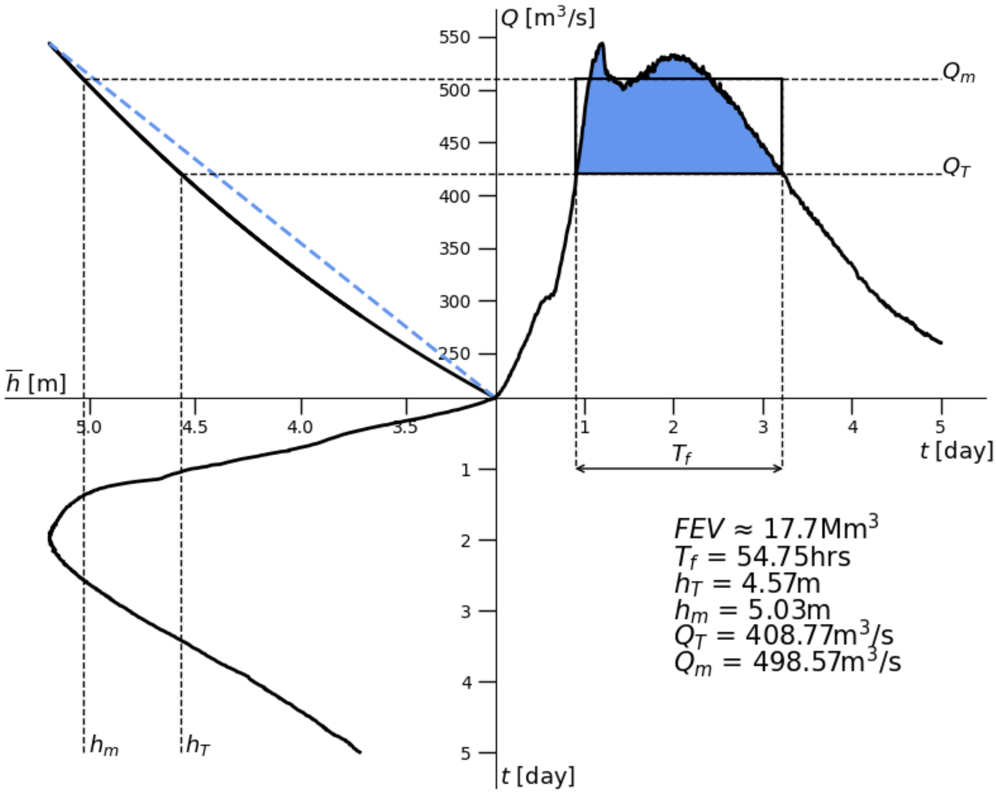


Figure 7: The rating curve (top-left panel) and its linear approximation (dashed line) are displayed as well as river levels (lower-left panel) and flow rates (top-right panel) of the River Ouse in 2015. Here the rating curve is not used to determine the flow rate as we have it as a function of time. Hence, the rating curve is used to find $Q_T = Q(h_T)$ and therefore the FEV (Blue area).

Hence, the corresponding FEV using the original rating curve is 17.7Mm^3. But, as mentioned in Section 4.3, the stage-discharge relationship showed evidence of hysteresis, we need to look into calculating the FEV using the two different rating curves for the rising and falling limbs.

For the general case we have Q_T, which is the threshold discharge found using the original rating curve. We will denote the two other thresholds for discharge as $Q_{T_{up}}$ and $Q_{T_{down}}$ where $Q_{T_{up}}$ is the threshold discharge on the rising limb and $Q_{T_{down}}$ is for the falling limb. These values were found by making a quadrant plot as in Figure 7 for both the eight degree polynomial fit and one using the falling limb rating curve, these can be found in Appendix A.4.

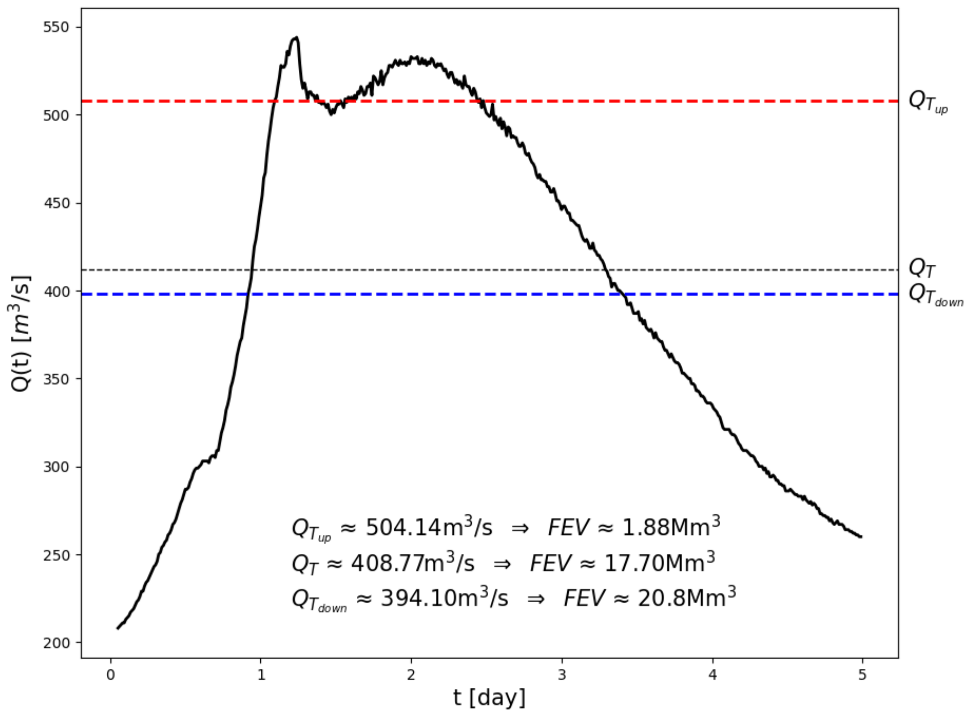


Figure 8: Plot to show the discharge of the flood and how the different threshold discharges impact the value of FEV. Each dashed line $Q_{T_{up}}$, Q_T and $Q_{T_{down}}$ define a different FEV by integrating the (positive part of the) difference $(Q(t) - Q_{T_{up}})$, $(Q(t) - Q_T)$ and $(Q(t) - Q_{T_{dwon}})$ respectively.

We can clearly see that the different thresholds change the value of FEV significantly. Hence, by linking $Q_{T_{up}}$ and $Q_{T_{down}}$ in a linear way, we can define a new threshold discharge as a function of time, $Q_T(t)$. This yields another definition of FEV, as follows

$$V_e = \int_{Q(t) \geq Q_T(t)} (Q(t) - Q_T(t))\, dt. \tag{5.1}$$

Therefore, an equation for $Q_T(t)$ needs to be found and then the corresponding FEV.

A equation for $Q_T(t)$ is found relatively simply by finding the intersections of $Q_{T_{up}}$ and $Q_{T_{down}}$ with the discharge $Q(t)$. Using these intersections you can find the equation for the straight line that connects them, shown by the green line in Figure 9. The result is:

$$Q_T(t) = -46.2490t + 552.1974.$$

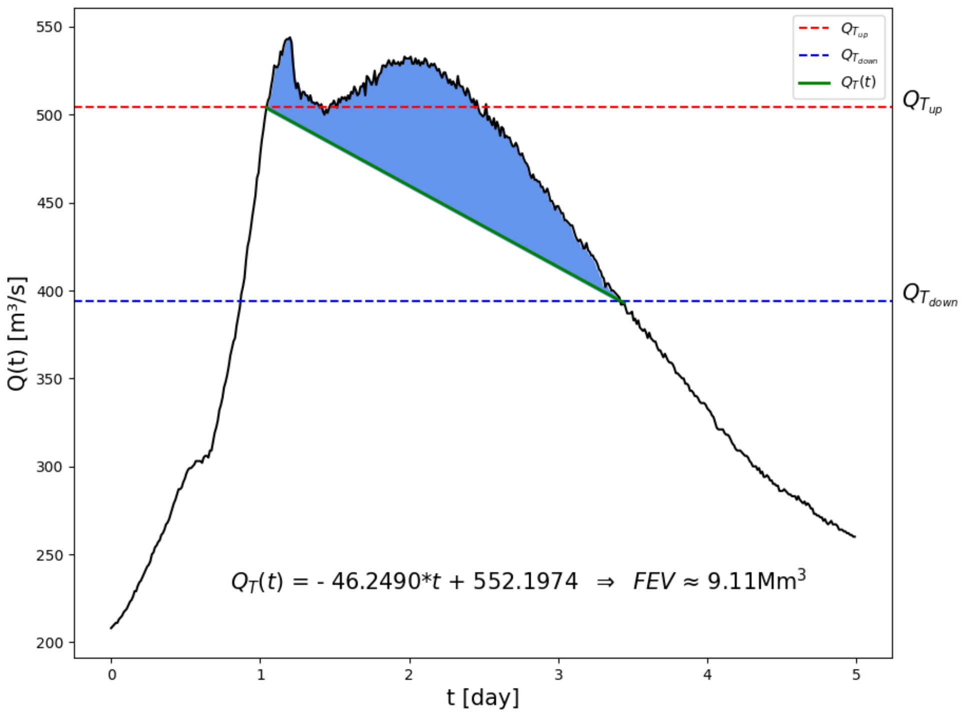


Figure 9: Plot to show the discharge $Q(t)$ and the new found threshold discharge $Q_T(t)$ (green line). Using Equation 5.1, we can find the value of FEV as 9.11 Mm^3 (See Appendix A.5).

Therefore, by using this new threshold discharge $Q_T(t)$, we have found a value for the FEV of the 2015 Ouse flood (9.11 Mm^3).

In Figure 8, we saw how much the FEV changes depending on the rating curve used. We have assumed a linear relationship when connecting the rising limb and the sinking limb to get $Q_T(t)$. In practice, this could not be the case, maybe the rising limb connects to the sinking limb earlier and hence the $Q_T(t)$ we used is incorrect. This could be improved by looking at other cases with a different maximum height and maximum discharge. And checking whether a flood with a lower max height and a smaller max discharge still has a similar sinking limb. And does the current rising limb get connected earlier to the sinking limb? This would tell you whether the simple linear link we made is reasonable or not.

5.2 Flood Excess Lake

We now have a value for the FEV of the River Ouse flood in 2015. Let us quickly produce a 'flood-excess lake' to help visualise this value. Firstly let us calculate the side length of the lake,

$$\frac{9110000}{2} = 4555000 \quad \Longrightarrow \quad \sqrt{4555000} = 2134.244597.$$

Hence, the side length of our 2 metre deep lake will be 2134 metres, Figure 10 shows the results below.

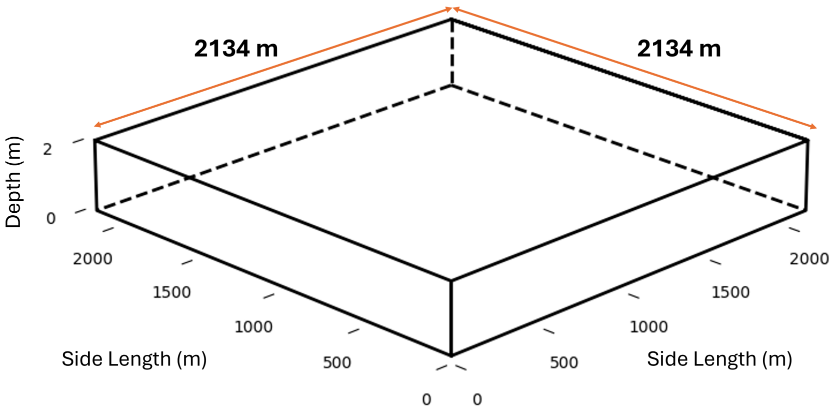


Figure 10: The FEV of the 2015 River Ouse flood expressed as a 2 metre deep 'flood-excess lake'.

6 Mitigation Strategies

6.1 Completed Mitigation Strategies in York

Following the floods of December 2015, the Environmental Agency (EA) were given an additional £45 million to better protect properties in York from flooding. Since receiving the additional funding, the EA has evaluated potential improvements to the city's existing flood defences and what new defences could be built. In November 2016, the EA published the York Five Year Plan which split up York into 10 communities (see Figure 11) and outlined potential flood mitigations for each of these. The EA has identified these communities based on historical flooding data and an understanding of existing flood defences. They are naturally divided by land features such as roads, rivers, and high ground (Environmental Agency, 2016).

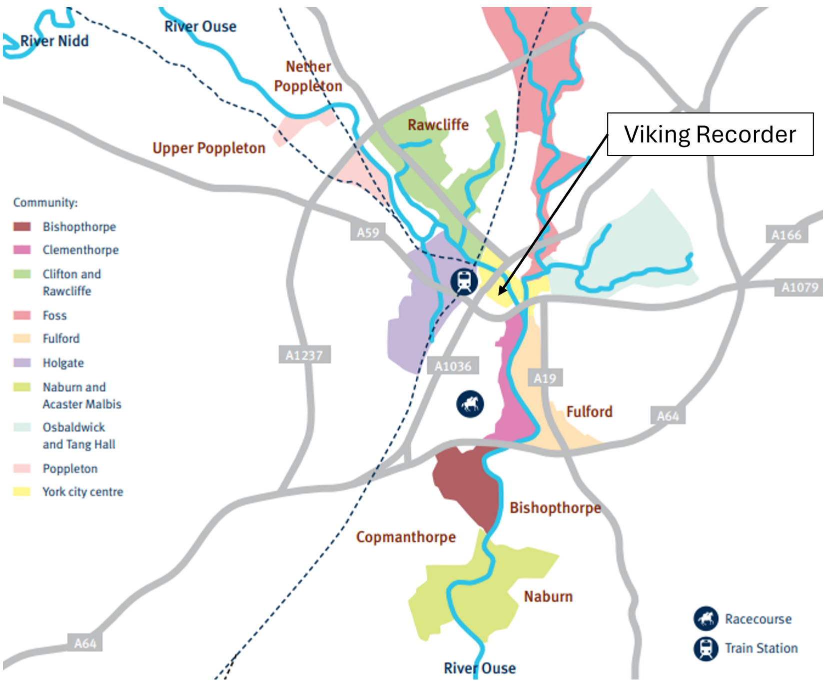


Figure 11: A map to show York and the ten different communities that it has been split up into by the EA (Environmental Agency, 2016).

An additional £17 million was allocated to improving the Foss Barrier. As part of the improvement work, they have installed new pumps which have a higher pumping capacity. The pumps would now be able to deal with the amount of water that came down the River Foss on Boxing Day 2015. This means that if that amount of water occurred again, the Foss Barrier would be able to deal with it and hence the Foss community would be less likely to encounter flooding.

However, for this project we are focussing on the flooding of the Ouse river so we will look at the mitigation plans that improve the flood protection of the Ouse not the Foss. By looking at Figure 11, we can see that the Viking Recorder is situated in the York city centre community, hence, we will be mainly evaluating potential flood mitigation in this area. As well as this, the planned mitigation for the Ouse in the Poppleton community (upstream of York city centre) is building a new 2.5m flood wall. This will force the water towards the city centre instead of flooding upstream, another reason why we will focus on evaluating the mitigation strategies near the Viking recorder (Environmental Agency, 2016).

The map shown in Figure 11 has then be split up into multiple flood cells. The York city centre section has been split into four different cells: B4, B7, B12 and B15. These are shown below in Figure 12.

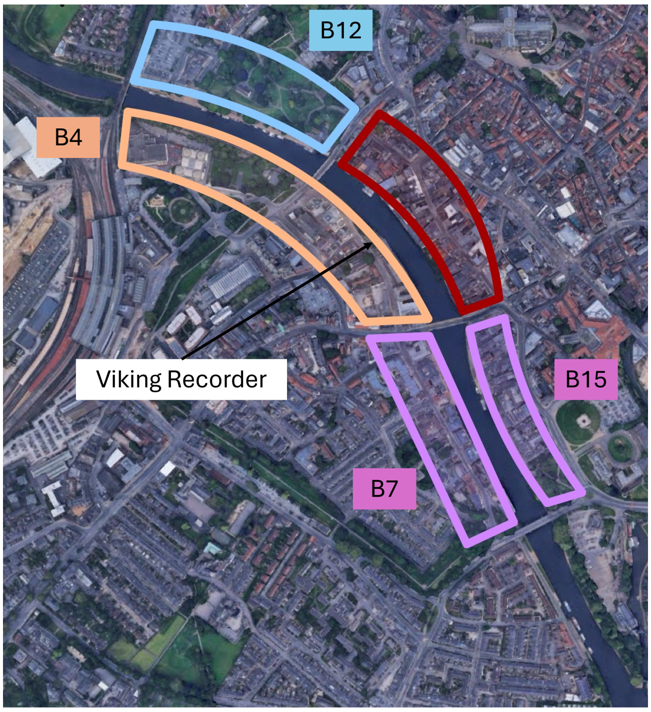


Figure 12: Map to show the four different cells in the York city centre, including the location of the Viking recorder. The red cell marks an area where no further flood mitigation was needed.

We will now look at each cell separately and explain the completed mitigation strategies as well as their cost.

Cell B4

Cell B4 follows the right bank of the Ouse from Scarborough bridge to Ouse bridge. The mitigation strategies for cell B4 were completed by February 2022 in hope to better protect 39 properties (businesses or homes) (Environmental Agency, 2021a).

One of the completed flood defences was a new 0.70m flood wall along Leeman Road from Westgate Apartments to the grounds of York City Rowing Club. This also included demountable barriers on the entrances to Memorial Gardens that will be put up in the event of a flood. A new embankment was also made in the grounds of the York City Rowing club.

As well as this, the Lendal Bridge flood gate was increased by 0.30m in height with an additional 0.15m demountable panel for extreme flooding events. The existing flood wall along North Street was raised by 0.30m and new floodgates were installed.

All this work in cell B4 claims to have better protected 39 properties and in total came to a cost of £2,555,000 (Environmental Agency, 2021a).

Cell B12

Cell B12 follows the left bank of the Ouse from Scarborough bridge to Lendal bridge and the work was completed in July 2022 (Environmental Agency, 2020).

Existing defences on Earlsborough Terrace were raised by 0.40m by installing glass panels on top of the existing brick walls and replacing the 10 existing flood gates. The existing flood wall from Earlsborough Terrace to Marygate was also raised by 0.45m.

The embankment in Museum Gardens which is already 2.50m high and 130m long was raised by 0.30m - 0.60m. The design of the embankment has been developed to not only act as a viable flood defence, but to also to fit in with the much used and well loved public space.

The work completed in cell B12 has better protected 42 homes and 15 businesses (57 properties) and in total came to a cost of £2,950,000 (Environmental Agency, 2020).

Cell B7 & B15

Cell B7 & B15 cover the area between Ouse bridge and Skeldergate bridge. There are no formal flood defences and no plans for these cells. This is for a variety of complicated reasons and hence 'hard' defences (flood walls) have not been built in this location. Many of the properties here are designed to cope with flooding on the ground floor or have already installed Property Flood Resilience (PFR) measures. This includes either preventing water from entering the property or minimising damage if water does enter. Common installations are door and window seals, waterproof walls or one way valves on toilets/drainage pipes to prevent sewage backup (York FAS, 2018).

6.1.1 Cost Efficacy

This section will look into the effectiveness of the mitigation strategies that have already been completed in York city centre.

From the completed works, we can see that within cells B4 and B12 the flood walls, embankments and flood gates have all been increased by various heights. However, we can see that they have all been increased in height by a minimum of 0.30m. This increases the flow capacity of the river, which effectively increases the threshold river level value h_T. Hence, if we take the minimum of 0.30m and add that onto the original threshold level of h_T = 4.57m, we get a new value of h_T = 4.87m. By applying this to the calculations done in section 5.1, we get that the FEV reduces from 9.11 Mm^3 down to 3.48 Mm^3 (See Figure 13a). This means that it has reduced the FEV by,

$$\frac{9.11 - 3.48}{9.11} \times 100 = 61.8\%.$$

These combined works for cells B4 and B12 cost,

$$2,555,000 + 2,950,000 = 5,505,000,$$

which means that 61.8% of the FEV was mitigated for a cost of £5,505,000. This is equivalent to £89,077 per 1% of FEV mitigated.

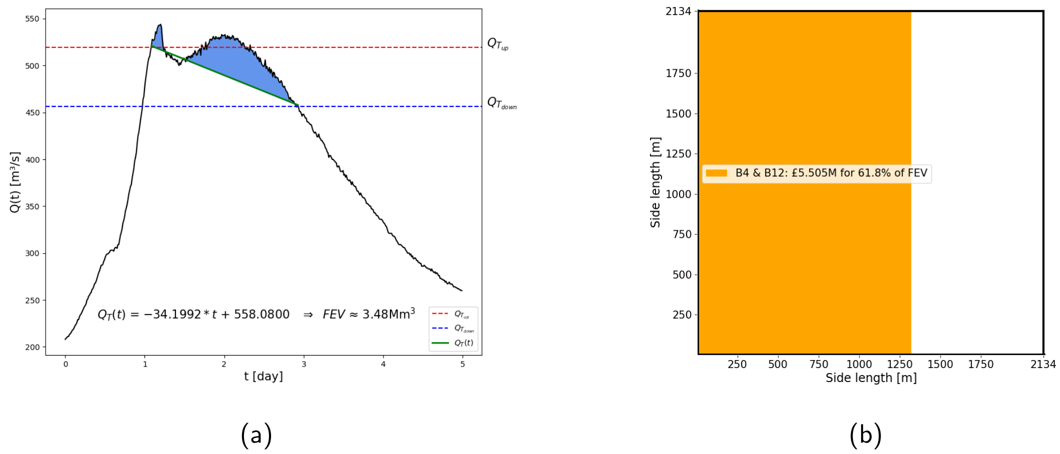


Figure 13: (a) Graph to show how both the FEV changes when h_T is risen by 0.3m. (b) Plot to show the flood excess lake (Figure 10) as seen from above. The orange area represents the volume of water mitigated by the works done in cells B4 and B12.

Therefore, the work completed in York city centre has already helped to reduce the FEV by 61.8%. However, if a flood like this were to happen again, 38.2% of the FEV in 2015 would still flood. This is still 3.48Mm^3 and hence, more mitigation is needed. The following section will look into potential further mitigation plans to reduced that remaining 38.2%.

6.2 Proposed Mitigation Strategies for York

In this subsection, different mitigation strategies that have not been planned by the EA will be introduced. After completing cost effectiveness analysis, we can see if they would be worth building in York.

The first and potentially most obvious would be to just raise the flood walls higher to mitigate the remaining FEV. This would involve raising the walls by another 0.3m to reduce the FEV to 0 (This was worked out by just adjusting the threshold height h_T to 5.17m and repeating the process that was done in Subsection 5.1). We can estimate the cost of doing this to be the same as before, so £5.505 million. This would then mitigate the remaining 38.2% and be equivalent to £144,110 per 1% of FEV mitigated.

6.2.1 Sustainable urban Drainage Systems (SuDS)

Sustainable urban drainage systems, or 'SuDS', aim to manage surface water and are a natural alternative to traditional drainage networks like pipes and sewers. These can help in flood mitigation as they can act as flood storage volume. There are many different types of SuDS such as: Bioretention strips, green roofs, permeable paving, rainwater harvesting and more (The Flood Hub, no date).

Permeable paving is different from traditional concrete paving as it allows surface water to infiltrate into the ground, thus managing flood risk (The Flood Hub, no date). It was found that permeable paving stores 48 litres of water per square metre of paving, that is 0.048m^3 . Let us look into the potential effects of installing 100,000m^2 of permeable paving, this is equivalent to approximately 13 football pitches in size. The cost of installing permeable paving is £40 per square metre and hence £4 million for 100,000m^2 (Environmental Agency, 2021bb). The volume of water the permeable paving could store would be 4,800m^3 and hence 0.05% of the FEV (3.48Mm^3). This is equivalent to £80 million per 1% of FEV mitigated, which is very large compared to the £89,077 per 1% for the work done in cells B4 and B12.

Another way of reducing flood risk is simple rainwater harvesting which involves collecting and storing rainwater that would usually flow into the river. Rainwater that has been collected can be re-used by the home owners for gardening and more. The installation of a 300 litre (0.3m^3) water butt costs between £100 - £243 per property (Environmental Agency, 2021bb) . Let us look into installing one of these water butts into 10,000 properties in York. This would help store 3000m^3 of water for a cost of £1 million, and hence 0.03% of the FEV. This is then £33.33 million per 1% of FEV mitigated. If we include both the permeable paving and the rainwater harvesting, then we mitigate 0.08% of the FEV for £5 million.

As we have seen from the above results, SuDS are a good way of managing flood risk as they have many other benefits for the community. However, in terms of a monetary view, they are not as effective as hard flood defences such as flood walls. If York were able to introduce many different SuDS, such as detention basins, swales and wetlands, it could be effective. However, this could be a long and expensive process, and maybe implementing more walls and flood gates would be more beneficial.

6.2.2 Natural Flood Management (NFM)

Natural Flood Management (NFM) describes the use of natural processes in reducing flood risk. Examples of NFM are flow-attenuation features, planting of peat and trees to absorb more rainwater and the re-meandering of brooks and rivers to slowdown the flow and to increase flood-plain storage (Bokhove et al., 2019).

Tree planting acts as flood risk management as it absorbs the rainwater from the ground, therefore extending the amount of time before the ground gets water logged. This means that there is less water run-off that flows into the river and causes flooding. A project in South Yorkshire claimed that 46.5 hectares (0.465km^2) of new woodland created 4000m^3 of extra flood storage volume at a cost of £130,000 (Environmental Agency, 2017). The River Ouse catchment area is approximately 3,300km^2. If York were to turn 100km^2 into new woodland, then there would be 860,000km^2 of extra flood storage for a cost of £27 million. This is equivalent to 9.44% of the FEV, and hence £2.86 million per 1% of FEV mitigated.

6.2.3 Flood Storage Basin

A potential mitigation strategy for York is building a new flood storage basin. An example of this is Brentingby Dam which is upstream of Melton Mowbray, near the confluence of the River Eye and the River Wreake. The dam was built to protect Melton Mowbray from future flooding and cost £6.7 million (Environmental Agency, 2021c). The dam can retain approximately 3.7Mm^3

of water (ABG Geosynthetics, no date), so during flooding, it acts as a reservoir by storing the excess water to protect the community .

Considering that there is 3.48Mm^3 left of FEV to mitigate, a similar storage basing could be developed upstream of York. Site selection would be crucial as potential sites should have adequate land availability with minimal environmental or social impacts. It may involve collaboration with local stakeholders and landowners to ensure the feasibility of the project. If a storage basin that could hold 3.48Mm^3 of water was built and cost £6.7 million then it would mitigate 38.2% of the FEV. This is equivalent to £175,393 per 1% of FEV mitigated.

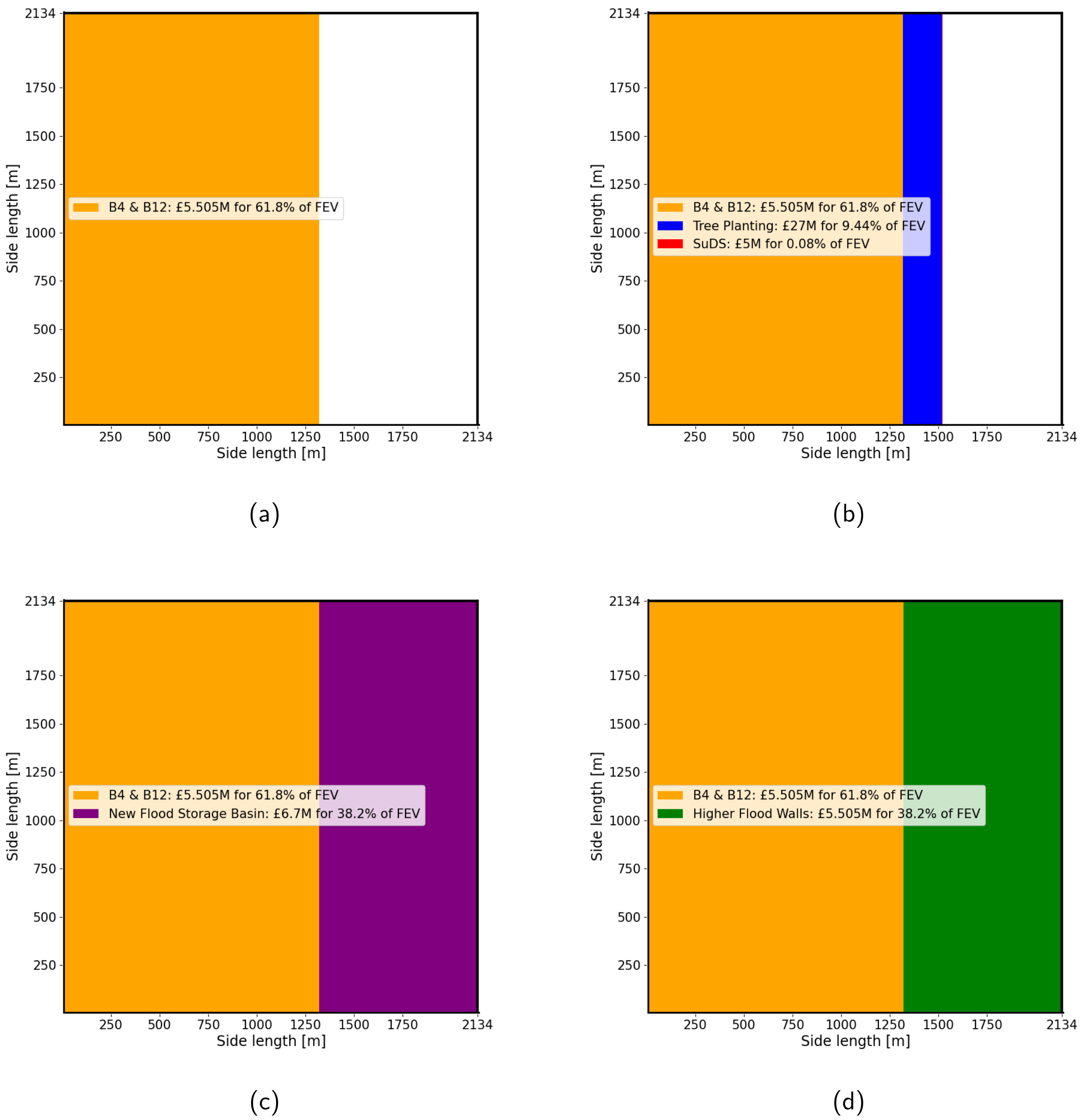


Figure 14: Four square lakes that represent completed and proposed mitigation strategies for York. Each coloured strip corresponds to a different mitigation method. The size of each strip is equivalent to the volume of the 2m deep square lake (FEV) that it mitigates.

6.3 Comparison of Mitigation Strategies

Using all the information on each mitigation strategy, Figure 14 was created to visualise the volume of FEV that each strategy mitigates. Figure 15 has also been made to show the cost per 1% of FEV mitigated. Utilising these figures, we can now start to see which mitigation plans are the most effective in York.

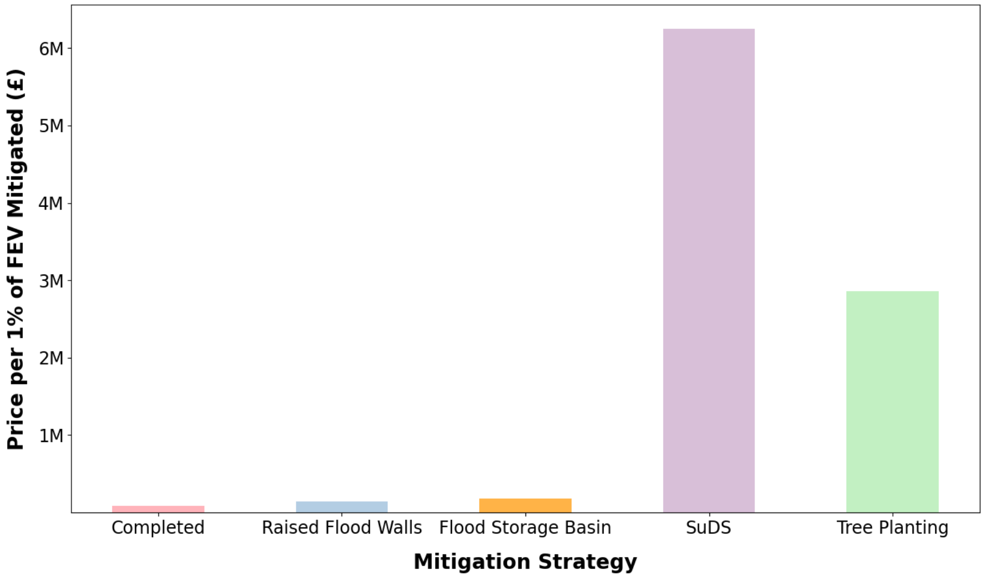


Figure 15: A Bar Graph to show the cost per 1% of FEV mitigated for each of the mitigation strategies discussed in Section 6. The price on the y-axis is listed as 1M, 2M, . . . where the M denotes a million (i.e 3M is 3 million pounds).

It is clear to see that the completed work in York city centre has already been extremely effective at mitigating FEV but also at a low cost; this work is the least costly per 1% of FEV mitigated (see Figure 15). Taking this into account, it would be reasonable to replicate this work but increase all the flood walls again by 0.3m. This would then mitigate 100% of FEV (Figure 14d) and be very cost effective. However, raising all the flood walls through York city centre is not a long term solution. Raising the walls would worsen flooding downstream as they would just force the water through the city and then the river would flood more significantly as soon as the walls stop. As well as this, hard flood defences are not the most environmentally friendly since producing concrete has high carbon emissions, 8% of global carbon emissions are caused by concrete production (Watts, 2019). Tall flood walls disconnect communities from the river and may impact York's historic character and tourism. Although flood walls typically have a long lifespan of around 50 years, they require ongoing maintenance, which incurs additional costs. Also, due to increasing flood risks the walls would most likely need to be raised again in the coming years.

Sustainable urban Drainage Systems (SuDS) are the most expensive relative to the FEV that they mitigate, which is very clearly shown in Figure 15. Given York's allocated budget of £45 million, prioritising SuDS may not be the most efficient approach, as the city must focus on high impact, cost effective solutions that provide immediate flood risk reduction. However, despite their high cost, SuDS offer long term benefits that extend beyond flood protection such as improving water quality in the area by filtering pollutants and enhancing biodiversity by creating green spaces and wetlands (The Flood Hub, no date). Therefore, while SuDS may not be the primary solution for York given budget constraints, they should still be implemented alongside more immediate flood defences which could offer both short term protection and long term sustainability.

Looking at Figure 14c and Figure 15, it would be fair to assume a new flood storage basin could be the solution for mitigating a flood of this magnitude. The cost sits at £175,393 per 1% of FEV mitigated which is approximately 35 times cheaper than SuDS per 1%, making it a highly efficient investment within the £45 million budget. Flood storage basins can enhance biodiversity by providing habitats and supporting ecosystems that thrive in fluctuating water conditions (The Flood Hub, no date). However, the success of this strategy depends heavily on land availability, a suitable upstream location must have the capacity to store this volume without affecting local ecosystems, agricultural land or communities. The site would also need to be located strategically to intercept and store peak flows of the Ouse. While additional hydrological and feasibility studies would be required, this solution offers a highly efficient flood mitigation strategy for York.

The final proposed mitigation strategy is tree planting, which could mitigate 9.44% of the FEV for £27 million, equating to £2.86 million per 1% of FEV mitigated. Although this is significantly more expensive than 'hard' flood defences, tree planting contributes to climate change mitigation by absorbing carbon dioxide through photosynthesis (Woodland Trust, no date). Given that climate change is projected to increase the severity and frequency of extreme rainfall events, reducing emissions could indirectly mitigate future flood risks. Woodlands also help to reduce the risk of flash flooding by intercepting and storing rain water, and hence reducing the time between peak rainfall and peak river flow (Woodland Trust, no date). Conversely, there is delayed effectiveness in tree planting as trees take time to grow before they can significantly impact flood resilience. Substantial land availability is also required which may compete with agricultural and urban land use and cause more tension between landowners in York. To summarise, tree planting is not an immediate flood prevention measure and is not the most cost effective but serves as a long term investment in climate resilience and environmental sustainability.

7 Summary and Discussion

Flood-Excess Volume (FEV) provides a fast and effective method for quantifying the magnitude of a flood which can then be used to evaluate flood mitigation strategies. In this project, data from the Environmental Agency (EA) was used to approximate a dataset for the York flood of 2000, however, this came with high inaccuracies so it was decided to continue just using the data from 2015. Furthermore, the data was used to generate a rating curve for the Viking Recorder in York as one was not made available by the EA. Due to hysteresis effects, separate rating curves were developed for the rising and falling limbs of the flood event which allows for more accuracy in the FEV calculations. Using the calculated rating curve, the FEV for the 2015 River Ouse flood was determined which provided a key value for assessing the flood severity. Hence, the FEV was used to evaluate various mitigation measures by comparing their cost effectiveness. Completed flood defences in York city centre have already reduced FEV by 61.8%, but further defences are needed to address the remaining flood risk. By applying FEV analysis, different strategies such as flood storage basins, SuDS, tree planting, and additional flood wall increases were assessed. This helped to show the balance between cost, effectiveness and sustainability of the flood mitigation measures. This project shows that FEV is a useful tool for flood management, providing quantitative data to support better decision making.

Flood management is a complex issue leading to ongoing debate about the most effective solutions. Natural habitats like upland bogs, woodlands, wetlands, and species rich grasslands can help absorb excess water, reducing runoff and lowering flood risks. Traditional hard engineered defences such as flood walls, embankments and barriers remain essential for flood control but are often costly and not environmentally friendly. Also, hard engineered defences offer little benefit to river ecosystems or to the aesthetic of the town. As a result, flood defence strategies are increasingly shifting towards sustainable solutions that focus on reducing runoff and enhancing natural flood management (York Civic Trust, no date).

7.1 Limitations and Further Research

While this project provides a structured approach to quantifying flood severity and evaluating mitigation strategies, there are several limitations that introduce uncertainties in the outcomes. A major source of inaccuracy is the reliance on h_T, the threshold river level at which flooding begins. Since h_T is not explicitly known in some cases, its estimation significantly affects the calculated FEV and hence the evaluation of mitigation strategies. Future research should focus on developing more accurate methods for determining h_T such as hydraulic modelling or improved gauging techniques.

Another limitation in this project lies in the rating curve generation process, which was crucial for estimating the threshold discharge Q_T and hence has a significant impact on the value of FEV. The rating curve used was generated using discharge data from the Skelton Recorder and assuming a time lagged relationship with the Viking Recorder. However, this approximation may not fully capture hydrological variations or changing riverbed conditions. Since the shape of rivers change over time, the stage discharge relationship also changes and hence rating curves need to be updated regularly to ensure accuracy.

Additionally, the availability of mitigation data poses a challenge. Much of the information on existing and proposed flood mitigation measures is not readily accessible online, particularly regarding precise water storage capacities and cost breakdowns. This makes it difficult to perform a comprehensive cost effectiveness analysis. Future research should involve collaborating with local authorities, the Environment Agency, and engineering firms to obtain detailed hydrological and financial data on mitigation strategies.

7.2 Further Reading & Acknowledgements

The main reference used for the Flood-Excess Volume analysis used in this project was provided by Bokhove in Bokhove et al. (2020). If there is a wish to gain more of an understanding, then it would be highly recommended to read this article.

Acknowledgement goes to the Environment Agency for contributing river height and discharge data for the Viking and Skelton monitoring stations.

If the reader is interested in recreating the plots and calculations found in this project or expanding on them, then all the data and code used can be found on GitHub here: `https://github.com/Bram-Mac/York-FEV-Flooding-Report-.git`

Bibliography

ABG Geosynthetics. [no date]. *Erosion Control Flood Embankment, Brentingby, UK*. [Online]. [Accessed 11 March 2025]. Available from: https://abg-geosynthetics.com/case-studies/erosion-control/erosion-control-flood-embankment-brentingby-uk/

Benoit, K. 2010. Ordinary Least Squares Regression. PO7001 Quantitative Methods 1. London School of Economics. [Online]. [Accessed 27 February 2025]. Available from: https://pdfs.semanticscholar.org/d116/17ca6854f7c61d07ea1ac1a0be2a86d7883c.pdf

Bokhove, O., Kelmanson, M.A., Kent, T., Piton, G. and Tacnet, J. 2019. Communicating (nature-based) flood-mitigation schemes using flood-excess volume. *River research and applications*. [Online]. **35**(9), pp.1402–1414. [Accessed 31 January 2025]. Available from: https://doi.org/10.1002/rra.3507

Bokhove, O., Kelmanson, M.A., Kent, T., Piton, G. and Tacnet, J.-M. 2020. A Cost-Effectiveness Protocol for Flood-Mitigation Plans Based on Leeds' Boxing Day 2015 Floods. *Water*. [Online]. **12**(3), pp.6-7. [Accessed 31 January 2025]. Available from: https://doi.org/10.3390/w12030652

Copernicus Climate Change Service. 2025. *Global Climate Highlights 2024*. [Online]. [Accessed 2 February 2025]. Available from: https://climate.copernicus.eu/global-climate-highlights-2024

Environment Agency. 2016. *The Foss Barrier: reducing the risk of flooding*. [Online]. [Accessed 31 January 2025]. Available from: https://www.gov.uk/government/publications/the-foss-barrier-reducing-the-risk-of-flooding/the-foss-barrier#:~:text=When%20the%20River%20Ouse%20is,to%20prevent%20it%20backing%20up.

Environment Agency. 2016. *York Flood Action Plan*. [Online]. [Accessed 06 March 2025]. Available from: https://assets.publishing.service.gov.uk/media/5a7f0ea0e5274a2e8ab49d73/EA_York_Flood_Action_Plan__FINAL.pdf

Environmental Agency. 2017. *Working with natural processes: evidence directory*. [Online]. [Accessed 10 March 2025]. Available from: https://assets.publishing.service.gov.uk/media/6036c5468fa8f5480a5386e9/Working_with_natural_processes_evidence_directory.pdf

Environment Agency. 2020. *York flood defence plans for Museum Gardens and Marygate*. [Online]. [Accessed 07 March 2025]. Available from: http://www.gov.uk/government/news/york-flood-defence-plans-for-museum-gardens-and-marygate

Environment Agency. 2021a. *Major milestone for £45 million York Flood Alleviation Scheme*. [Online]. [Accessed 07 March 2025]. Available from: https://www.gov.uk/government/news/major-milestone-for-45-million-york-flood-alleviation-scheme

Environmental Agency. 2021b. *Cost estimation for SUDS*. [Online]. [Accessed 10 March 2025]. Available from: https://assets.publishing.service.gov.uk/media/6034ee6c8fa8f54334a5a6a9/Cost_estimation_for_SUDS.pdf

Environmental Agency. 2021c. *Cost estimation for flood storage*. [Online]. [Accessed 11 March 2025]. Available from: https://assets.publishing.service.gov.uk/media/6034edf1e90e076609e4c522/Cost_estimation_for_flood_storage.pdf

European Environment Agency. 2020. *Floodplains: a natural system to preserve and restore*. [Online]. [Accessed 11 March 2025]. Available from: https://damremoval.eu/wp-content/uploads/2020/03/TH-AL-20-004-EN-N-Floodplains-a-natural-system-to-preserve-and-restore-final-version.pdf

GOV.UK. 2025. *River Ouse level at Viking Recorder*. [Online]. [Accessed 2 March 2025]. Available from: https://check-for-flooding.service.gov.uk/station/8208

Lisa, York Stories. 2012. *Why does York flood?*. [Online]. [Accessed 31 January 2025]. Available from: http://yorkstories.co.uk/why-does-york-flood/

Local Government Association. 2022. *City of York Council: Towards long-term flood resilience*. [Online]. [Accessed 12 February 2025]. Available from: https://www.local.gov.uk/case-studies/city-york-council-towards-long-term-flood-resilience

Perret, E., Lang, M., Le Coz, J. 2022. A framework for detecting stage-discharge hysteresis due to flow unsteadiness: Application to France's national hydrometry network. *Journal of Hydrology*. [Online]. **608**, article no: 127567 [no pagination]. [Accessed 02 March 2025]. Available from: https://doi.org/10.1016/j.jhydrol.2022.127567

The Flood Hub. [no date]. *Sustainable Drainage Systems (SuDS), The Flood Hub*. [Online]. [Accessed 10 March 2025]. Available from: https://thefloodhub.co.uk/suds/

Watts, J. 2019. Concrete: the most destructive material on Earth. *The Guardian*. [Online]. 25 February. [Accessed 17 March 2025]. Available from: https://www.theguardian.com/cities/2019/feb/25/concrete-the-most-destructive-material-on-earth

Woodland Trust. [no date]. *Can woods and trees reduce flooding?*. [Online]. [Accessed 19 March 2025]. Available from: https://www.woodlandtrust.org.uk/trees-woods-and-wildlife/british-trees/flooding/

York Civic Trust. [no date]. *Flood heights on the Ouse*. [Online]. [Accessed 12 March 2025]. Available from: https://yorkcivictrust.co.uk/heritage/civic-trust-plaques/flood-heights-on-the-ouse/

York Flood Alleviation Scheme. 2018. *York Flood Alleviation Scheme (FAS) Information page*. [Online]. [Accessed 31 January 2025]. Available from: https://consult.environment-agency.gov.uk/yorkshire/yorkfas/

York Press. 2016. York floods 2015: How the devastating floods unfolded and how York rallied superbly. *The Press*. [Online]. 28 July. [Accessed 12 March 2025]. Available from: https://www.yorkpress.co.uk/news/ryedale/14168299.york-floods-2015-how-the-devastating-floods-unfolded-and-how-york-rallied-superbly/

A Appendices

A.1 Python Code for Correlation Plots and Linear Regression Calculation

```
import pandas as pd

file_path = r'C:\Users\macha\Documents\Uni\Year 3
\Project in Maths\Ouse Data
\Comparing stage at Viking and Skelton.csv'
df = pd.read_csv(file_path)

import matplotlib.pyplot as plt

df["Time"] = pd.to_datetime(df["Time"], format="%d/%m/%Y %H:%M")

# Scatter plot of Skelton vs. Viking stage levels
plt.figure(figsize=(8, 5))
plt.scatter(df["Skelton Stage"], df["Viking Stage"],
            alpha=0.5, s=30)
plt.xlabel("Skelton Stage (m)", fontsize=14, fontweight="bold")
plt.ylabel("Viking Stage (m)", fontsize=14, fontweight="bold")
plt.grid(True)
plt.show()

import numpy as np

max_lag = 12
correlations = []

for lag in range(max_lag + 1):
    shifted_skelton = df["Skelton Stage"].shift(lag)
    correlation = shifted_skelton.corr(df["Viking Stage"])
    correlations.append(correlation)

best_lag = np.argmax(correlations)
best_corr = correlations[best_lag]

# Plot correlation vs. lag
plt.figure(figsize=(8, 5))
plt.plot(range(max_lag + 1), correlations,
         marker="o", linestyle="-")
plt.xlabel("Lag (15-minute intervals)",
           fontsize=14, fontweight="bold")
plt.ylabel("Correlation", fontsize=14,
           fontweight="bold")
#plt.title("Correlation vs. Time Lag")
plt.xticks(range(max_lag + 1))
plt.grid(True)
plt.show()

best_lag, best_corr
```

```
# Shift Skelton Stage by 75 minutes (5 intervals)
df["Skelton-Stage-Lagged"] = df["Skelton-Stage"].shift(5)

df_lagged = df.dropna(subset=["Skelton-Stage-Lagged",
                              "Viking-Stage"])

S = df_lagged["Skelton-Stage-Lagged"].values
V = df_lagged["Viking-Stage"].values

S_mean = S.mean()
V_mean = V.mean()

# Compute slope (a)
a = sum((S - S_mean) * (V - V_mean)) / sum((S - S_mean) ** 2)

# Compute intercept (b)
b = V_mean - a * S_mean
```

A.2 Python Code for determining the Rating Curve

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

df = pd.read_csv(r'C:\Users\macha\Documents\Uni\Year 3
\Project in Maths\Ouse Data\2015 Stage and Flow Data.csv')
stage_vr = df["Viking Stage"].values
discharge_vr = df["Viking Flow"].values

def rating_curve(h, c, b, a):
    return c * (h - a) ** b

a_guess = min(stage_vr)
p0 = [1, 1, a_guess]

params, covariance = curve_fit(rating_curve, stage_vr,
discharge_vr, p0=p0)

c_fitted, b_fitted, a_fitted = params

stage_range = np.linspace(min(stage_vr), max(stage_vr), 100)
fitted_discharge = rating_curve(stage_range, c_fitted, b_fitted,
a_fitted)

plt.figure(figsize=(10, 5))
plt.scatter(stage_vr, discharge_vr, label="Observed Data",
color="blue", s=20, marker = 'x')
plt.plot(stage_range, fitted_discharge, label= 'Rating Curve',
color="red")
plt.xlabel("Stage (m)", fontsize=11,fontweight="bold",
labelpad = 10)
plt.ylabel("Discharge (m$^3$/s)", fontsize=11, fontweight="bold",
labelpad = 6)
plt.legend()
plt.grid()
plt.show()

print(f"Estimated Rating Curve: Q = {c_fitted:.2f}
(h - {a_fitted:.2f})^{b_fitted:.2f}")

residuals = discharge_vr - rating_curve(stage_vr, *params)
ss_res = np.sum(residuals**2)
ss_tot = np.sum((discharge_vr - np.mean(discharge_vr))**2)
r2 = 1 - (ss_res / ss_tot)
print(r2)
```

A.3 Python Code for Rising and Falling limb fits

```
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

file_path = r'C:\Users\macha\Documents\Uni\Year 3\Project in Maths
\Ouse Data\2015 Stage and Flow Data.csv'
df = pd.read_csv(file_path)

stage_vr = df["Viking Stage"].values
discharge_vr = df["Viking Flow"].values

dh_dt = np.diff(stage_vr, prepend=stage_vr[0])

rising_indices = np.where(dh_dt > 0)[0]
falling_indices = np.where(dh_dt < 0)[0]

stage_rising = stage_vr[rising_indices]
discharge_rising = discharge_vr[rising_indices]

stage_falling = stage_vr[falling_indices]
discharge_falling = discharge_vr[falling_indices]

plt.figure(figsize=(10, 5))
plt.scatter(stage_rising, discharge_rising,
label="Rising Limb Data", color="red", s=10, marker='x',
alpha=0.5)
plt.scatter(stage_falling, discharge_falling,
label="Falling Limb Data", color="green", s=10, marker='x',
alpha=0.5)
plt.xlabel("Stage (m)", fontsize=11, fontweight="bold",
labelpad=10)
plt.ylabel("Discharge (m$^3$/s)", fontsize=11, fontweight="bold",
labelpad=6)
plt.legend()
plt.grid()


def power_law(h, a, b, h0):
    return a * (h - h0) ** b


poly_degree = 8

poly_coeffs = np.polyfit(stage_rising, discharge_rising,
poly_degree)
poly_fit = np.poly1d(poly_coeffs)
```

```
print("Rising limb (degree 8 polynomial) equation:")
poly_equation = "Q = " + " + ".join([f"{coef:.5f} *
h^{poly_degree - i}" for i, coef in enumerate(poly_coeffs)])
print(poly_equation)

h0_falling = np.min(stage_falling)
p0_falling = [np.mean(discharge_falling), 2, h0_falling]
params_falling, _ = curve_fit(power_law, stage_falling,
discharge_falling, p0=p0_falling, maxfev=10000)

a_fall, b_fall, h0_fall = params_falling
print(f"Falling limb (power-law) equation: Q = {a_fall:.5f} *
(h - {h0_fall:.5f})^{b_fall:.5f}")

h_range = np.linspace(min(stage_vr), max(stage_vr), 300)

Q_rising_fit = poly_fit(h_range)
Q_falling_fit = power_law(h_range, *params_falling)

plt.figure(figsize=(10, 6))

plt.scatter(stage_rising, discharge_rising, color= 'red', s=5,
label='Rising Limb Data', marker='o')
plt.scatter(stage_falling, discharge_falling, color= 'green', s=5,
label='Falling Limb Data', marker='o')

plt.plot(h_range, Q_rising_fit, 'r--', label='Rising Fit')
plt.plot(h_range, Q_falling_fit, 'g--', label='Falling Fit')

plt.xlabel('Stage (m)')
plt.ylabel('Discharge (m$^3$/s')
plt.legend()
plt.grid(True)
```

A.4 Quadrant plots using Rising and Falling fits

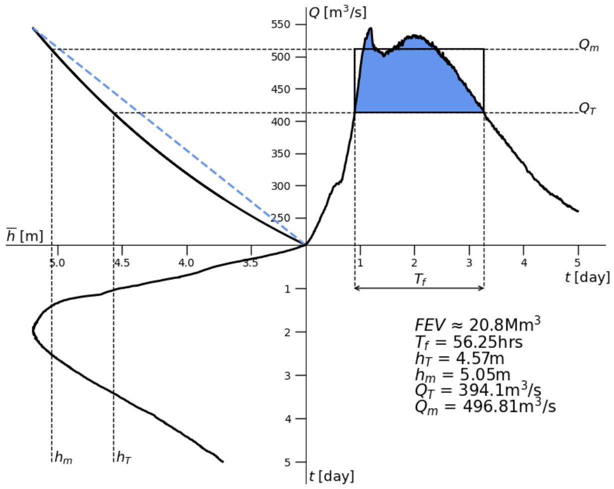


Figure 16: Quadrant plot for the 2015 flood using the falling limb fit.

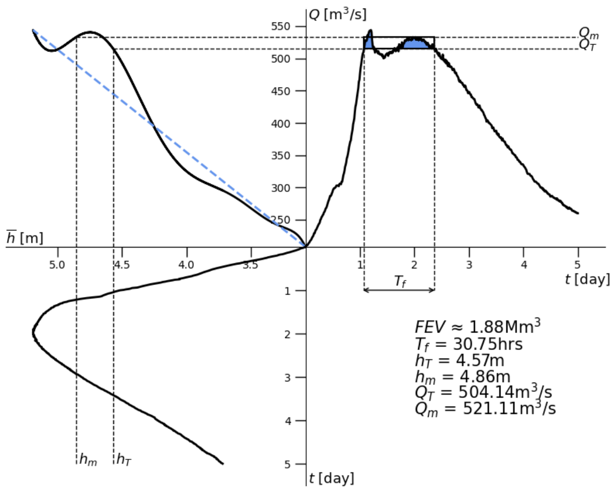


Figure 17: Quadrant plot for the 2015 flood using the eight degree polynomial rising limb fit.

A.5 FEV calculation using $Q_T(t)$

```
import pandas as pd
Data = pd.read_csv(r'C:\Users\macha\Documents\Uni\Year-3
\Project-in-Maths\Ouse-Data\2015-Stage-and-Flow-Data.csv')
time = Data['Day']
Flow = Data['Viking-Flow']
time_increment = (time[1]-time[0])*24*3600

def Q_T(t):
    return -46.2490 * t + 552.1974

fev = []
for i, t in enumerate(time):
    QT_value = Q_T(t)
    if Flow[i] >= QT_value:
        difference = (Flow[i] - QT_value) * time_increment
        fev.append(difference)

FEV=sum(fev)
```

A.6 Quadrant for the Flood of 2000

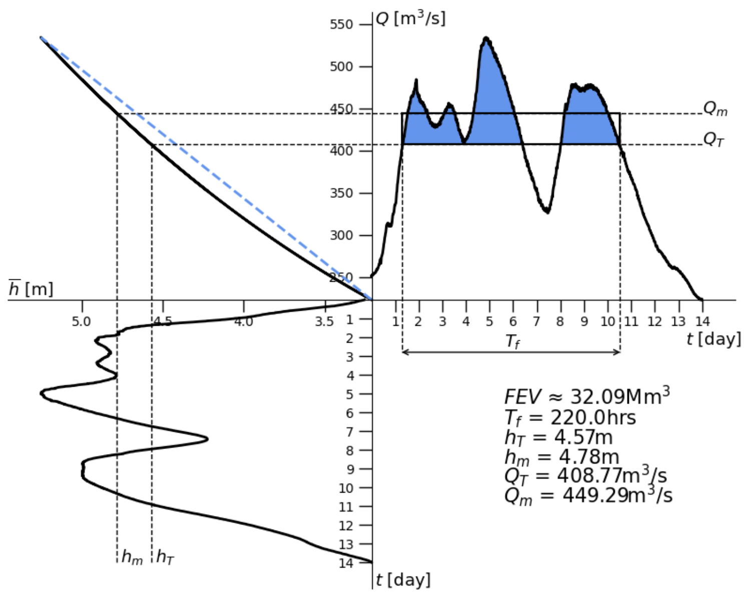


Figure 18: Quadrant plot for the York flood of 2000. This used the stage data found in Section 4.2 and the rating curve seen in Equation 4.3.

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I am aware that the University defines plagiarism as presenting someone else's work, in whole or in part, as your own. Work means any intellectual output, and typically includes text, data, images, sound or performance.

I promise that in the attached submission I have not presented anyone else's work, in whole or in part, as my own and I have not colluded with others in the preparation of this work. Where I have taken advantage of the work of others, I have given full acknowledgement. I have not resubmitted my own work or part thereof without specific written permission to do so from the University staff concerned when any of this work has been or is being submitted for marks or credits even if in a different module or for a different qualification or completed prior to entry to the University. I have read and understood the University's published rules on plagiarism and also any more detailed rules specified at School or module level. I know that if I commit plagiarism I can be expelled from the University and that it is my responsibility to be aware of the University's regulations on plagiarism and their importance.

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I confirm that I have declared all mitigating circumstances that may be relevant to the assessment of this piece of work and that I wish to have taken into account. I am aware of the University's policy on mitigation and the School's procedures for the submission of statements and evidence of mitigation. I am aware of the penalties imposed for the late submission of coursework.

Signed: Bram MacGibbon Harrold **Date:** 28 March 2025