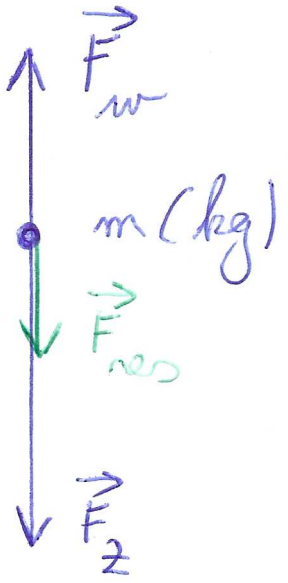
a.)



$F_z = m.g$

$F_w \sim v \Rightarrow F_w = k.v$

$F_{res} = m.a$

$s(t)$ = afgelegde valafstand op tijdstip t

$v(t) = s'(t)$ en $a(t) = s''(t)$

Wet van Newton: $F_{res} = F_z - F_w$

$m.a = m.g - k.v$

$m.s''(t) = m.g - k.s'(t)$

$m.s''(t) + k.s'(t) = m.g$

$s''(t) + \frac{k}{m}.s'(t) = g$

b.) $v(t=0) = v_0$

① $s_H(t)$

$$s''(t) + \frac{k}{m} \cdot s'(t) = 0$$

Karakteristieke vgl.: $r^2 + \frac{k}{m} \cdot r = 0$

$$r \cdot \left(r + \frac{k}{m}\right) = 0$$

$$r_1 = 0 \; ; \; r_2 = -\frac{k}{m}$$

$$s_H(t) = c_1 \cdot e^{0 \cdot t} + c_2 \cdot e^{-\frac{k}{m} \cdot t}$$

$$s_H(t) = \underbrace{c_1}_{*} + \underbrace{c_2 \cdot e^{-\frac{k}{m} \cdot t}}$$

② $s_p(t)$

Voorstel: $s_p(t) =$ veralgemening van g

$$s_p(t) = \underbrace{a}_{*}$$

overeenkomst met $s_H(t)$

$\Rightarrow$ maal t doen

$$s_p(t) = \underbrace{a \cdot t}$$

geen overeenkomst met $s_H(t)$

a bepalen door deze $s_p(t)$ in te vullen in de opgave: $s_p(t)'' + \frac{k}{m} \cdot s_p(t)' = g$

$$0 + \frac{k}{m} \cdot a = g \Rightarrow a = \frac{m \cdot g}{k}$$

$$s_p(t) = \frac{mg}{k} \cdot t$$

(3) $s(t) = s_H(t) + s_p(t)$

$$s(t) = C_1 + C_2 \cdot e^{-\frac{k}{m} \cdot t} + \frac{m \cdot g}{k} \cdot t$$

$v(t=0) = v_0$ gebruiken:

$$v(t) = s'(t) = -C_2 \cdot \frac{k}{m} \cdot e^{-\frac{k}{m} \cdot t} + \frac{mg}{k}$$

$$v_0 = -C_2 \cdot \frac{k}{m} \cdot \underbrace{e^0}_{=1} + \frac{mg}{k}$$

$$C_2 \cdot \frac{k}{m} = \frac{mg}{k} - v_0$$

$$C_2 = \frac{m}{k} \cdot \left(\frac{mg}{k} - v_0 \right)$$

$$s(t) = C_1 + \frac{m}{k}\cdot\left(\frac{m\cdot g}{k} - v_0\right)\cdot e^{-\frac{k}{m}\cdot t} + \frac{m\cdot g}{k}\cdot t$$

$s(t=0)=0$ gebruiken:

$$0 = C_1 + \frac{m}{k}\cdot\left(\frac{mg}{k} - v_0\right)\cdot \underbrace{e^{0}}_{=1} + 0$$

$$C_1 = \frac{m}{k}\left(v_0 - \frac{mg}{k}\right)$$

$$s(t) = \frac{m}{k}\left(v_0 - \frac{mg}{k}\right) + \frac{m}{k}\left(\frac{mg}{k} - v_0\right)\cdot e^{-\frac{k}{m}\cdot t} + \frac{mg}{k}t$$

$$s(t) = \frac{m}{k}\cdot\left[\left(v_0 - \frac{mg}{k}\right)\cdot\left(1 - e^{-\frac{k}{m}t}\right) + g\cdot t\right]$$