$$y'' + y' - 2y = 8.\sin(2x) + 3$$

① $\underline{\underline{y_H(x)}}$

$y'' + y' - 2y = 0 \longrightarrow$ karakteristieke vgl.:

$$r^2 + r - 2 = 0$$

csolve$(r^2+r-2=0, r)$

$$\Rightarrow r_1 = 1 \; ; \; r_2 = -2$$

$$y_H(x) = \underline{C_1 \cdot e^{1\cdot x}} + \underline{C_2 \cdot e^{-2\cdot x}}$$

② $\underline{\underline{y_p(x)}}$

rechterlid $= f(x) = \underbrace{8.\sin(2.x)}_{f_1(x)} + \underbrace{3}_{f_2(x)}$

$$f_1(x) = 8.\sin(2.x)$$

$$= e^{m.x}.\left[V_1(x).\cos(\vartheta.x) + V_2(x).\sin(\vartheta.x)\right]$$

$$= e^{0.x}.\left[0.\cos(2.x) + 8.\sin(2.x)\right]$$

$\Downarrow$

$$y_{p_1}(x) = x^?.\, e^{m.x}.\left[W_1(x).\cos(\vartheta.x) + W_2(x).\sin(\vartheta.x)\right]$$

$$= x^?.\underbrace{e^{0.x}}_{=1}.\left[a.\cos(2.x) + b.\sin(2.x)\right]$$

$$y_{p_1}(x) = x^0 \cdot [\underbrace{a \cdot \cos(2 \cdot x)} + \underbrace{b \cdot \sin(2 \cdot x)}]$$

geen overeenkomst tussen de termen van $y_{p_1}(x)$ en de termen van $y_H(x)$

$$y_{p_1}(x) = a \cdot \cos(2x) + b \cdot \sin(2x)$$

$$f_2(x) = 3$$

$$= e^{mx} \cdot [V_1(x) \cdot \cos(\theta x) + V_2(x) \cdot \sin(\theta x)]$$

$$= \underbrace{e^{0 \cdot x}}_{=1} \cdot [3 \cdot \underbrace{\cos(0 \cdot x)}_{=1} + \ldots \cdot \underbrace{\sin(0 \cdot x)}_{=0}]$$

$\Downarrow$

$$y_{p_2}(x) = x^0 \cdot e^{m \cdot x} \cdot [W_1(x) \cdot \cos(\theta \cdot x) + W_2(x) \cdot \sin(\theta \cdot x)]$$

$$= x^0 \cdot \underbrace{e^{0 \cdot x}}_{=1} \cdot [c \cdot \underbrace{\cos(0 \cdot x)}_{=1} + \ldots \cdot \underbrace{\sin(0 \cdot x)}_{=0}]$$

$$= x^0 \cdot c$$

geen overeenkomst tussen de termen van $y_{p_2}(x)$ en de termen van $y_H(x)$

$$y_{p_2}(x) = c$$

$\Downarrow$

$y_p(x) = y_{p_1}(x) + y_{p_2}(x) = a.\cos(2x) + b.\sin(2x) + c$

Nu a, b, c bepalen door $y_p(x)$ in te vullen in de opgave: $y_p(x)'' + y_p(x)' - 2.y_p(x) = 8.\sin(2.x) + 3$

rekentoestel:

$yp(x) := a.\cos(2.x) + b.\sin(2.x) + c$

$\frac{d^2}{dx^2}(yp(x)) + \frac{d}{dx}(yp(x)) - 2.yp(x)$

$\rightsquigarrow (2b - 6a).\cos(2x) + (-6b - 2a).\sin(2x) - 2.c$

$= 8.\sin(2x) + 3$

$\Downarrow$

$$\begin{cases} 2b - 6a = 0 \\ -6b - 2a = 8 \\ -2c = 3 \end{cases} \Rightarrow \begin{cases} a = -2/5 \\ b = -6/5 \\ c = -3/2 \end{cases}$$

$\Rightarrow y_p(x) = -\frac{2}{5}.\cos(2x) - \frac{6}{5}.\sin(2x) - \frac{3}{2}$

③ $y(x) = y_H(x) + y_p(x)$

$= c_1.e^{x} + c_2.e^{-2x} - \frac{2}{5}.\cos(2.x) - \frac{6}{5}.\sin(2.x) - \frac{3}{2}$