$$\frac{d^2y}{dt^2} + 3\cdot\frac{dy}{dt} + 2\cdot y = \cosh(t) \text{ en } y(0)=1; y'(0)=-1$$

① $\underline{\underline{y_H(t)}}$

$$\frac{d^2y}{dt^2} + 3\cdot\frac{dy}{dt} + 2\cdot y = 0 \longrightarrow \text{karakteristieke vgl.}$$

$$r^2 + 3\cdot r + 2 = 0$$

$$\text{csolve}(r^2+3.r+2=0, r)$$

$$\Rightarrow r_1 = -2 ; r_2 = -1$$

$$y_H(t) = c_1\cdot e^{-2.t} + c_2\cdot e^{-1.t}$$

*

② $\underline{\underline{y_p(t)}}$

$$\text{rechterlid} = f(t) = \cosh(t)$$

$$= \frac{e^t + e^{-t}}{2}$$

$$= \underbrace{\frac{1}{2}\cdot e^t}_{f_1(t)} + \underbrace{\frac{1}{2}\cdot e^{-t}}_{f_2(t)}$$

$$f_1(t) = \frac{1}{2} . e^{t}$$

$$= e^{m.t} . \left[V_1(t) . \cos(\vartheta . t) + V_2(t) . \sin(\vartheta . t) \right]$$

$$= e^{1.t} . \left[\frac{1}{2} . \underbrace{\cos(0.t)}_{=1} + \ldots . \underbrace{\sin(0.t)}_{=0} \right]$$

$$\Downarrow$$

$$y_{p1}(t) = t^{s} . e^{m.t} . \left[W_1(t) . \cos(\vartheta . t) + W_2(t) . \sin(\vartheta . t) \right]$$

$$= t^{s} . e^{1.t} . \left[a . \underbrace{\cos(0.t)}_{=1} + \ldots . \underbrace{\sin(0.t)}_{=0} \right]$$

$$= t^{s} . a . e^{t}$$

geen overeenkomst tussen de term van $y_{p1}(t)$ en de termen van $y_H(t)$

$$\Downarrow$$

$$y_{p1}(t) = a . e^{t}$$

$$f_2(t) = \frac{1}{2} . e^{-t}$$

$$= e^{m.t} . \left[V_1(t) . \cos(\vartheta . t) + V_2(t) . \sin(\vartheta . t) \right]$$

$$= e^{-1.t} . \left[\frac{1}{2} . \underbrace{\cos(0.t)}_{=1} + \ldots . \underbrace{\sin(0.t)}_{=0} \right]$$

$\Downarrow$

$$y_{p_2}(t) = t^0 . e^{m.t} . \left[W_1(t).\cos(\theta.t) + W_2(t).\sin(\theta.t) \right]$$

$$= t^0 . e^{-1.t} . \left[b . \underbrace{\cos(0.t)}_{=1} + \ldots . \underbrace{\sin(0.t)}_{=0} \right]$$

$$= t^0 . \underbrace{b . e^{-t}}_{*}$$

* overeenkomst tussen de term van $y_{p_2}(t)$ en een term van $y_H(t)$

$\times t$ doen

$$= t^0 . \underbrace{b.t.e^{-t}}$$

geen overeenkomst tussen de term van $y_{p_2}(t)$ en de termen van $y_H(t)$

$$y_{p_2}(t) = b.t.e^{-t}$$

$\Downarrow$

$$y_p(t) = y_{p1}(t) + y_{p2}(t)$$
$$= a \cdot e^{t} + b \cdot t \cdot e^{-t}$$

Nu a, b bepalen door $y_p(t)$ in te vullen

in de opgave: $\dfrac{d^2 y_p(t)}{dt^2} + 3 \cdot \dfrac{d\, y_p(t)}{dt} + 2 \cdot y_p(t)$

$$= \frac{1}{2} \cdot e^{t} + \frac{1}{2} \cdot e^{-t}$$

rekentoestel:

$yp(t) := a \cdot e^{t} + b \cdot t \cdot e^{-t}$

$$\frac{d^2}{dt^2}(yp(t)) + 3 \cdot \frac{d}{dt}(yp(t)) + 2 \cdot yp(t)$$

$$\rightsquigarrow \quad e^{-t} \cdot (6a \cdot e^{2t} + b) = \frac{1}{2} e^{t} + \frac{1}{2} e^{-t}$$

$$\Rightarrow \quad 6a \cdot e^{t} + b \cdot e^{-t} = \frac{1}{2} \cdot e^{t} + \frac{1}{2} \cdot e^{-t}$$

$$\Rightarrow \begin{cases} 6a = \frac{1}{2} \\ b = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} a = 1/12 \\ b = 1/2 \end{cases}$$

$$\Rightarrow y_p(t) = \frac{1}{12} \cdot e^{t} + \frac{1}{2} \cdot t \cdot e^{-t}$$

③ $y(t) = y_H(t) + y_p(t)$

$$= c_1 \cdot e^{-2 \cdot t} + c_2 \cdot e^{-t} + \frac{1}{12} \cdot e^{t} + \frac{1}{2} \cdot t \cdot e^{-t}$$

c_1 en c_2 bepalen met $y(t=0) = 1$ en $y'(t=0) = -1$:

rekentoestel:

$$y(t) := C1 \cdot e^{-2 \cdot t} + C2 \cdot e^{-t} + \frac{1}{12} \cdot e^{t} + \frac{1}{2} \cdot t \cdot e^{-t}$$

$$\text{solve}\left(\begin{cases} y(0) = 1 \\ \frac{d}{dt}(y(t))\Big|_{t=0} = -1 \end{cases}, C1, C2\right)$$

$\leadsto C1 = 0{,}666\ldots = \frac{2}{3}$ en $C2 = 0{,}25 = \frac{1}{4}$