Basisformules van dynamica

KINEMATICA

Rechtlijnige beweging van een puntmassa

| Variabele a | Constante $a = a_c$ |
|---------------------|---|
| $a = \frac{dv}{dt}$ | $v = v_0 + a_c t$ |
| $v = \frac{ds}{dt}$ | $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ |
| a ds = v dv | $v^2 = v_0^2 + 2a_c(s - s_0)$ |

Kromlijnige beweging van een puntmassa

| x, y, z Coördinaten | | r, θ, z Coördinaten | | |
|---------------------|-----------------|----------------------------|----------------------------|--|
| | $v_x = \dot{x}$ | $a_x = \ddot{x}$ | $v_r = \dot{r}$ | $a_r = \ddot{r} - r\dot{\theta}^2$ |
| | $v_y = \dot{y}$ | $a_y = \ddot{y}$ | $v_{	heta} = r\dot{	heta}$ | $a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ |
| | $v_z = \dot{z}$ | $a_z = \ddot{z}$ | $v_z = \dot{z}$ | $a_z = \ddot{z}$ |
| | n th Coi | irdinaten | | |

$$v = \dot{s}$$
 $a_t = \dot{v} = v \frac{dv}{ds}$ $a_n = \frac{v^2}{\rho}$ $\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relatieve beweging

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Beweging van een star onvervormbaar lichaam om

| Variabele α | Constante $\alpha = \alpha_c$ |
|-----------------------------------|---|
| $\alpha = \frac{d\omega}{dt}$ | $\omega = \omega_0 + \alpha_c t$ |
| $\omega = \frac{d\theta}{dt}$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$ |
| $\omega d\omega = \alpha d\theta$ | $\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$ |

Voor punt P geldt

$$s = \theta r$$
 $v = \omega r$ $a_t = \alpha r$ $a_n = \omega^2 r$

Relatieve algemene beweging in een plat vlak; translerende assen

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A \text{(schamier)}}$$
 $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A \text{(schamier)}}$

Relatieve algemene beweging in het platte vlak; translerende en roterende assen

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A}$$
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A})$$

KINETICA

Massatraagheidsmoment
$$I = \int r^2 dm$$

Verschuivingsstelling $I = I_G + md^2$
Gyrostraal $k = \sqrt{\frac{I}{m}}$

| Th | | 100 | 1 * |
|-------|--------|----------|--------|
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| Puntmassa | $\Sigma \mathbf{F} = m\mathbf{a}$ |
|-------------------|--|
| Star lichaam | $\Sigma F_{x} = m(a_{G})_{x}$ |
| (Vlakke beweging) | $\Sigma F_y = m(a_G)_y$ |
| | $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma (\mathcal{M}_k)_P$ |

Principe van arbeid en energie

$$T_1 + U_{1-2} = T_2$$

Kinetische energie

| Puntmassa | $T = \frac{1}{2}mv^2$ | |
|-------------------------------|--|--|
| Star onvervormbaar lichaam | $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ | |
| (Beweging in het platte vlak) | | |
| Variabele kracht | $U_F = \int F \cos \theta ds$ | |

Variabele kracht
$$U_F = \int F \cos \theta \, ds$$

Constante kracht $U_F = (F_c \cos \theta) \, \Delta s$

Gewicht $U_W = -W \, \Delta y$

Veer $U_s = -(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2)$

Koppel $U_M = M \Delta \theta$

Vermogen en rendement

$$P = rac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \quad \epsilon = rac{P_{
m out}}{P_{
m in}} = rac{U_{
m out}}{U_{
m in}}$$

Wet van behoud van energie

$$T_1 + V_1 = T_2 + V_2$$

Potentiële energie

$$V = V_g + V_e$$
, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principe van stoot en hoeveelheid van beweging

| Puntmassa | $m\mathbf{v}_1 + \sum \int \mathbf{F} dt = m\mathbf{v}_2$ |
|----------------------------------|---|
| Star onvervormbaar lichaam | $m(\mathbf{v}_G)_1 + \sum \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ |

Behoud van hoeveelheid van beweging $\Sigma(\text{stelsel } m\mathbf{v})_1 = \Sigma(\text{stelsel } m\mathbf{v})_2$

Restitutiecoëfficiënt
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Principe van stootmoment en moment van hoeveelheid van beweging

| Puntmassa | $ (\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2 $ |
|-------------------------------|--|
| | waarin $H_O = (d)(mv)$ |
| Star onvervormbaar | $(\mathbf{H}_G)_1 + \sum \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ |
| lichaam | waarin $H_G = I_G \omega$ |
| (Beweging in het platte vlak) | $(\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ |
| P ······ | waarin $H_O = I_O \omega$ |

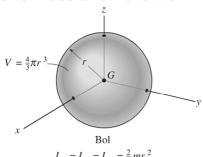
Behoud van het moment van hoeveelheid van beweging

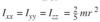
 $\Sigma(\text{stelsel }\mathbf{H})_1 = \Sigma(\text{stelsel }\mathbf{H})_2$

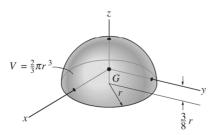
Formularium Mechanica niet op schrijven

Aangrijpingspunt van de zwaartekracht en massatraagheidsmoment van -

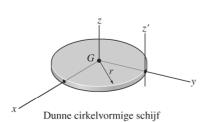
homogene massieve lichamen



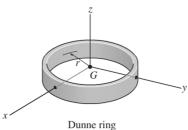




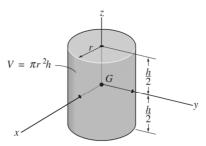
Halve bol $I_{xx} = I_{yy} = 0.259 mr^2 \ I_{zz} = \tfrac{2}{5} mr^2$



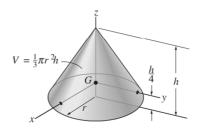
 $I_{xx} = I_{yy} = \tfrac{1}{4} \, m r^2 \quad I_{zz} = \, \tfrac{1}{2} m r^2 \quad I_{z'z'} = \tfrac{3}{2} \, m r^2$



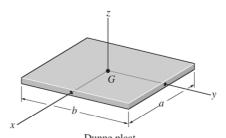
 $I_{xx} = I_{yy} = \frac{1}{2} mr^2 \qquad I_{zz} = mr^2$



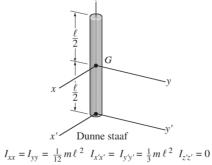
 $I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2)$ $I_{zz} = \frac{1}{2} mr^2$



 $I_{xx} = I_{yy} = \frac{3}{80} m (4r^2 + h^2) I_{zz} = \frac{3}{10} mr^2$



 $I_{xx} = \tfrac{1}{12} \ mb^2 \quad I_{yy} = \tfrac{1}{12} \ ma^2 \quad I_{zz} = \tfrac{1}{12} \ m(a^2 + b^2)$



$$I_{xx} = I_{yy} = \frac{1}{12} m \ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m \ell^2 \quad I_{z'z'} = 0$$