$z = f(x,y) = x^3 - 4 . x . y + y^3$

rechte in het xy-vlak: $r : \begin{cases} x = 3 - t \\ y = 2 - t \end{cases}$

punt $A(1,2)$

a.) richtingsvector $\vec{r}$ op rechte r bepalen:

$$\vec{r} = \frac{(-1,-1)}{\|(-1,-1)\|} = \frac{(-1,-1)}{\sqrt{(-1)^2 + (-1)^2}} = \frac{(-1,-1)}{\sqrt{2}} = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$D_{\vec{r}} f(x,y) = \frac{-9}{4}$$

$$\Leftrightarrow \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right) \bullet \vec{r} = \frac{-9}{4}$$

$$\Leftrightarrow \left(3x^2 - 4y, -4x + 3y^2\right) \bullet \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = \frac{-9}{4}$$

$$\Leftrightarrow -\frac{3x^2 - 4y}{\sqrt{2}} - \frac{-4x + 3y^2}{\sqrt{2}} = \frac{-9}{4}$$

$$\Leftrightarrow (3x^2 - 4y) + (-4x + 3y^2) = \frac{9 . \sqrt{2}}{4}$$

$$\Leftrightarrow 3x^2 - 4y - 4x + 3y^2 = \frac{9 . \sqrt{2}}{4}$$

richtingsvector $\vec{s}$ volgens de x-as:

$\vec{s} = (1,0)$

$D_{\vec{s}} f(x,y) = 2$

$\Leftrightarrow (3x^2-4y, -4x+3y^2)\cdot(1,0) = 2$

$\Leftrightarrow 3x^2-4y = 2$

Beide voorwaarden moeten samen gelden:

$$\begin{cases} 3x^2-4y-4x+3y^2 = \dfrac{9\sqrt{2}}{4} \\ 3x^2-4y = 2 \end{cases}$$

Stelsel oplossen

$\Rightarrow (-0{,}115413 ; -0{,}49001)$

of

$(1{,}68292 ; 1{,}62416)$

b.) richtingsafgeleide meest negatief

$\Leftrightarrow$ richting $= -\nabla f$

in punt $A(1,2)$ is $\nabla f(1,2) = \left(\frac{\partial f}{\partial x}(1,2), \frac{\partial f}{\partial y}(1,2)\right)$

$= (3 \cdot 1^2 - 4 \cdot 2\ , -4 \cdot 1 + 3 \cdot 2^2) = (-5\ , 8)$

Eenheids-richtingsvector maken:

$$\frac{(-5,8)}{\|(-5,8)\|} = \frac{(-5,8)}{\sqrt{(-5)^2+8^2}} = \frac{(-5,8)}{\sqrt{89}} = \left(\frac{-5}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$$

De gevraagde eenheids-richtingsvector volgens de steilste daling is dus:

$$-\frac{\nabla f(A)}{\|\nabla f(A)\|} = -\left(\frac{-5}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right) = \left(\frac{5}{\sqrt{89}}, \frac{-8}{\sqrt{89}}\right)$$