$$y'' - 2 \cdot y' - 3 \cdot y = x \cdot (1 + e^{3x})$$

① $y_H(x)$

$y'' - 2 \cdot y' - 3 \cdot y = 0 \rightarrow$ karakteristieke vgl.:

$$r^2 - 2 \cdot r - 3 = 0$$

$$\text{csolve}(r^2 - 2 \cdot r - 3 = 0, r)$$

$$\Rightarrow r_1 = -1 \; ; \; r_2 = 3$$

$$y_H(x) = C_1 \cdot e^{-1 \cdot x} + C_2 \cdot e^{3 \cdot x}$$

*

② $y_p(x)$

$$\text{rechterlid} = f(x) = \underbrace{x}_{f_1(x)} + \underbrace{x \cdot e^{3x}}_{f_2(x)}$$

$$f_1(x) = x$$

$$= e^{m \cdot x} \cdot \left[V_1(x) \cdot \cos(\alpha \cdot x) + V_2(x) \cdot \sin(\alpha \cdot x) \right]$$

$$= \underbrace{e^{0 \cdot x}}_{=1} \cdot \left[x \cdot \underbrace{\cos(0 \cdot x)}_{=1} + \ldots \cdot \underbrace{\sin(0 \cdot x)}_{=0} \right]$$

$\Downarrow$

$$y_{p1}(x) = x^? \cdot e^{m \cdot x} \cdot \left[W_1(x) \cdot \cos(\alpha \cdot x) + W_2(x) \cdot \sin(\alpha \cdot x) \right]$$

$$= x^? \cdot \underbrace{e^{0 \cdot x}}_{=1} \cdot \left[(a x + b) \cdot \underbrace{\cos(0 \cdot x)}_{=1} + \ldots \cdot \underbrace{\sin(0 \cdot x)}_{=0} \right]$$

$$y_{p_1}(x) = x^0 . (\underbrace{ax} + \underbrace{b})$$

$\downarrow$ geen overeenkomst tussen de termen van $y_{p_1}(x)$ en de termen van $y_H(x)$

$$y_{p_1}(x) = ax+b$$

$$f_2(x) = x.e^{3x}$$

$$= e^{m.x}.[V_1(x).\cos(\theta.x)+V_2(x).\sin(\theta.x)]$$

$$= e^{3.x}.[x.\underbrace{\cos(0.x)}_{=1} + \dots .\underbrace{\sin(0.x)}_{=0}]$$

$\Downarrow$

$$y_{p_2}(x) = x^?.e^{m.x}.[W_1(x).\cos(\theta.x)+W_2(x).\sin(\theta.x)]$$

$$= x^?.e^{3x}.[(c.x+d).\underbrace{\cos(0.x)}_{=1} + \dots .\underbrace{\sin(0.x)}_{=0}]$$

$$= x^?.(\underbrace{c.x.e^{3x}} + \underbrace{d.e^{3x}}_{*})$$

$\downarrow$ *: overeenkomst tussen een term van $y_{p_2}(x)$ en een term van $y_H(x)$

$\times x$ doen

$$y_{p_2}(x) = x^?.(\underbrace{c.x^2.e^{3x}} + \underbrace{d.x.e^{3x}})$$

$\downarrow$ geen overeenkomst tussen de termen van $y_{p_2}(x)$ en de termen van $y_H(x)$

$$\downarrow$$

$$y_{p_2}(x) = c.x^2.e^{3x} + d.x.e^{3x}$$

$$\Downarrow$$

$$y_p(x) = y_{p_1}(x) + y_{p_2}(x) = a.x + b + c.x^2.e^{3x} + d.x.e^{3x}$$

Nu a, b, c, d bepalen door $y_p(x)$ in te vullen in de opgave: $y_p(x)'' - 2.y_p(x)' - 3.y_p(x) = x + x.e^{3x}$

rekentoestel:

$$yp(x) := a.x + b + c.x^2.e^{3.x} + d.x.e^{3.x}$$

$$\frac{d^2}{dx^2}(yp(x)) - 2.\frac{d}{dx}(yp(x)) - 3.yp(x)$$

$$\leadsto [8.c.x + 2.(c+2.d)].e^{3x} - 3a.x - 2a - 3b$$

$$\to 8c.x.e^{3x} + (2c+4d).e^{3x} - 3a.x - 2a - 3b$$

$$\Downarrow$$

$$8c.x.e^{3x} + (2c+4d).e^{3x} - 3a.x - 2a - 3b = x + x.e^{3x}$$

$$\Downarrow$$

$$\begin{cases} 8c = 1 \\ 2c + 4d = 0 \\ -3a = 1 \\ -2a - 3b = 0 \end{cases} \Rightarrow \begin{cases} a = -1/3 \\ b = 2/9 \\ c = 1/8 \\ d = -1/16 \end{cases}$$

$\Downarrow$

$$y_p(x) = -\frac{1}{3}\cdot x + \frac{2}{9} + \frac{1}{8}\cdot x^2\cdot e^{3\cdot x} - \frac{1}{16}\cdot x\cdot e^{3\cdot x}$$

③ $$y(x) = y_H(x) + y_p(x)$$

$$= c_1\cdot e^{-x} + c_2\cdot e^{3x} - \frac{1}{3}x + \frac{2}{9} + \frac{1}{8}x^2 e^{3x} - \frac{1}{16}x e^{3x}$$