

FDTD Simulation of Lossless Transmission Lines



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Abstract

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1 The update functions

The update functions are given as:

$$\tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}} = \tilde{I}_{n+\frac{1}{2}}^{m-\frac{1}{2}} + \alpha (V_n^m - V_{n+1}^m), \quad (1)$$

$$V_n^{m+1} = V_n^m + \alpha \left(\tilde{I}_{n-\frac{1}{2}}^{m+\frac{1}{2}} - \tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}} \right), \quad (2)$$

where

$$\alpha \triangleq \frac{v \Delta t}{\Delta z}, \quad (3)$$

is the dimensionless Courant factor and

$$\tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}} = I_{n+\frac{1}{2}}^{m+\frac{1}{2}} R_c \quad (4)$$

is the rescaled current.

At the boundaries the update function for V takes another form.

- At $z = 0$

The voltage update function is given as:

$$V_0^{m+1} = V_0^m + \frac{2\Delta t}{C\Delta z} \left(I_g^{m+\frac{1}{2}} - I_{\frac{1}{2}}^{m+\frac{1}{2}} \right), \quad (5)$$

with

$$I_g^{m+\frac{1}{2}} = \frac{E_g^{E+\frac{1}{2}}}{R_g} - \frac{V_0^m + V_0^{m+1}}{2R_g}. \quad (6)$$

Substituting (6) in (5) and using (3), the two relations $v = \frac{1}{\sqrt{LC}}$ and $R_c = \sqrt{\frac{L}{C}}$ and the new defined constant $\tilde{R}_g = \frac{R_c}{R_g}$ yields, after some rearrangements:

$$V_0^{m+1} = C_1 V_0^m + C_2 \left(E_g^{m+\frac{1}{2}} \tilde{R}_g - I_{\frac{1}{2}}^{m+\frac{1}{2}} \right) \quad (7)$$

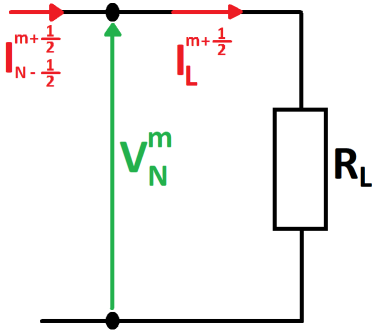
where

$$C_1 = \frac{1 - \alpha \tilde{R}_g}{1 + \alpha \tilde{R}_g}, \quad (8)$$

$$C_2 = \frac{2\alpha}{1 + \alpha \tilde{R}_g}, \quad (9)$$

are two dimensionless constants.

- At $z = l$



The voltage update function becomes:

$$V_N^{m+1} = V_N^m + \frac{2\Delta t}{C\Delta z} \left(I_{N-\frac{1}{2}}^{m+\frac{1}{2}} - I_L^{m+\frac{1}{2}} \right) \quad (10)$$

Kirchoff's voltage law in discretized form states that

$$I_L^{m+\frac{1}{2}} = \frac{V_N^{m+\frac{1}{2}}}{R_L} \quad (11)$$

$$= \frac{V_N^m + V_N^{m+1}}{2R_L} \quad (12)$$

Substituting (12) in (10) and using the same relations as for $z = 0$ and the new defined constant $\tilde{R}_L = \frac{R_C}{R_L}$ yields, after some rearrangements:

$$V_N^{m+1} = C_3 V_N^m + C_4 \tilde{I}_{N-\frac{1}{2}}^{m+\frac{1}{2}}, \quad (13)$$

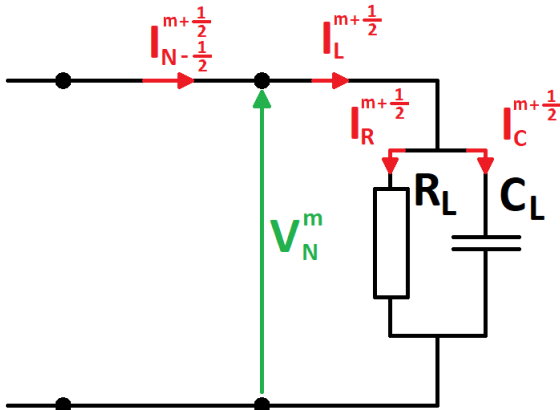
where

$$C_3 = \frac{1 - \alpha \tilde{R}_L}{1 + \alpha \tilde{R}_L}, \quad (14)$$

$$C_4 = \frac{2\alpha}{1 + \alpha \tilde{R}_L}, \quad (15)$$

are two dimensionless constants.

Now if there would be a capacitor added to the load the following situation will occur



Kirchoff's current law states that

$$I_L^{m+\frac{1}{2}} = I_R^{m+\frac{1}{2}} + I_C^{m+\frac{1}{2}}. \quad (16)$$

Using Kirchoff's voltage law at the resistor gives

$$I_R^{m+\frac{1}{2}} = \frac{V_N^{m+\frac{1}{2}}}{R_L} \quad (17)$$

$$= \frac{V_N^m + V_N^{m+1}}{2R_L} \quad (18)$$

The relation between the current and the voltage at the capacitor is given as

$$\hat{i} = -C \frac{\partial \hat{v}}{\partial t}. \quad (19)$$

For a first order FDM this turns into

$$I_R^{m+\frac{1}{2}} = -C_L \frac{V_N^{m+1} - V_N^m}{\Delta t}. \quad (20)$$

Substituting (18) and (20) into (16) gives

$$I_L^{m+\frac{1}{2}} = \left(\frac{1}{2R_L} - \frac{C_L}{\Delta t} \right) V_N^{m+1} + \left(\frac{1}{2R_L} + \frac{C_L}{\Delta t} \right) V_N^m \quad (21)$$

and can be rewritten as

$$\tilde{I}_L^{m+\frac{1}{2}} = \frac{\tilde{Z}_1}{2} V_N^{m+1} + \frac{\tilde{Z}_2}{2} V_N^m \quad (22)$$

with

$$\tilde{Z}_1 = R_C \left(\frac{1}{R_L} - 2 \frac{C_L}{\Delta t} \right) \quad (23)$$

$$\tilde{Z}_2 = R_C \left(\frac{1}{R_L} + 2 \frac{C_L}{\Delta t} \right) \quad (24)$$

Substituting (22) into the adjusted voltage update function at $z = l$ yields

$$V_N^{m+1} = C'_3 V_N^m + C'_4 \tilde{I}_{N-\frac{1}{2}}^{m+\frac{1}{2}}, \quad (25)$$

where

$$C'_3 = \frac{1 - \alpha \tilde{Z}_2}{1 + \alpha \tilde{Z}_1}, \quad (26)$$

$$C'_4 = \frac{2\alpha}{1 + \alpha \tilde{Z}_1}. \quad (27)$$

Notice when $C_L = 0$ then $C'_3 = C_3$ and $C'_4 = C_4$.