# Chapter 1

# Soft Tissue Engineering

## 1.1 Stress & strain in large deformations

### 1.1.1 Kinematics

- Mapping function

$$\vec{x} = \underline{\varphi}(\vec{X}, t) \tag{1.1}$$

- Displacement function

$$\vec{x} = \vec{X} + \underline{U}(\vec{X}, t) \tag{1.2}$$

- The deformation gradient tensor

$$d\vec{x} = \mathbf{F}(X, t)d\vec{X} \Rightarrow \mathbf{F} = \frac{\partial \vec{x}}{\partial \vec{X}} = \frac{\partial \underline{\varphi}}{\partial \vec{X}} \tag{1.3}$$

- Volume transformation

$$dv = \det(\mathbf{F})dV = JdV \tag{1.4}$$

- Surface transformation: Nanson's Formula

$$\vec{n}ds = J\mathbf{F}^{-\top}\vec{N}dS \tag{1.5}$$

- Polar decomposition

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R} \tag{1.6}$$
$$\Rightarrow \mathbf{U}^2 = \mathbf{F}^\top\mathbf{F} = \mathbf{C} \tag{1.7}$$
$$\Rightarrow \mathbf{V}^2 = \mathbf{F}\mathbf{F}^\top = \mathbf{B} \tag{1.8}$$

### 1.1.2 Strain

- Nominal strain, Biot strain or Engineering strain

$$\mathbf{e} = \mathbf{U} - \mathbf{I} \tag{1.9}$$

- Green-Lagrange strain

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^\top\mathbf{F} - \mathbf{I}) = \frac{1}{2}(\mathbf{U}^2 - \mathbf{I}) \tag{1.10}$$

- Euler-Almansi strain

$$\mathbf{A} = \frac{1}{2}(\mathbf{I} - \mathbf{V}^{-2}) \tag{1.11}$$

- True strain or logarithmic strain

$$\boldsymbol{\varepsilon} = \ln(\mathbf{U}) \tag{1.12}$$

### 1.1.3 Stress

- True stress or Cauchy stress

$$\boldsymbol{\sigma}^{\top} = \frac{d\vec{f}}{\vec{n}dS} \tag{1.13}$$

- 1PK: first Piola Kirchoff stress (Nominal stress or Engineering stress)

$$\mathbf{P} = \frac{d\vec{f}}{\vec{N}dS_0} \tag{1.14}$$

- 2PK: second Piola Kirchoff stress

$$\mathbf{S}^{\top} = \frac{\mathbf{F}^{-1}d\vec{f}}{\vec{N}dS_0} \tag{1.15}$$

- Swithcing between stresses:

| | $\boldsymbol{\sigma}$ | $\mathbf{P}$ | $\mathbf{S}$ |
|---|---|---|---|
| $\boldsymbol{\sigma}$ | $\boldsymbol{\sigma}$ | $J^{-1}\mathbf{P}\mathbf{F}^{\top}$ | $J^{-1}\mathbf{F}\mathbf{S}\mathbf{F}^{\top}$ |
| $\mathbf{P}$ | $J\boldsymbol{\sigma}\mathbf{F}^{\top}$ | $\mathbf{P}$ | $\mathbf{F}\mathbf{S}$ |
| $\mathbf{S}$ | $J\mathbf{F}^{-1}\boldsymbol{\sigma}\mathbf{F}^{-\top}$ | $\mathbf{F}^{-1}\mathbf{P}$ | $\mathbf{S}$ |

## 1.2 Constitutive modeling

## 1.3 Mechanical testing & parameter fitting

### 1.3.1 Uniaxial tensile testing

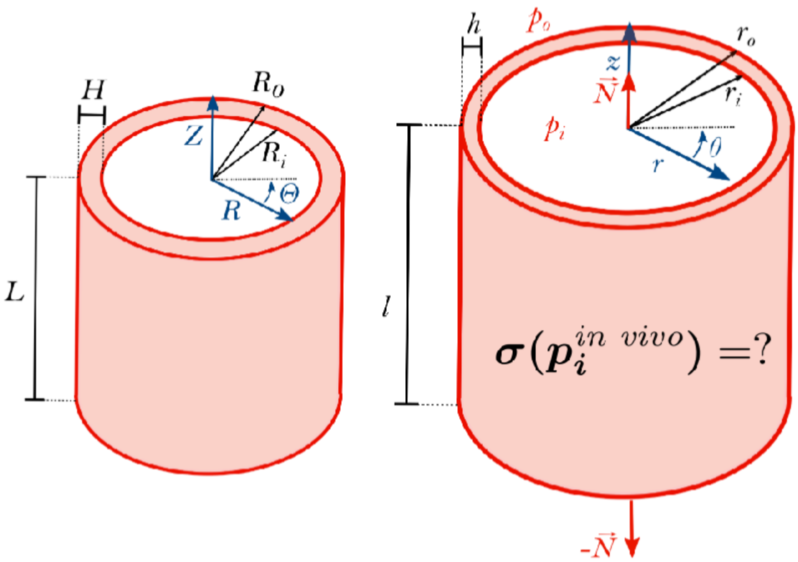
### 1.3.2 Biaxial tensile testing

### 1.3.3 Extenson-inflation testing

### 1.3.4 Strain mapping

## 1.4 In vivi wall stress estimation

### 1.4.1 Laplace's law for thin-walled tubes




The deformation gradient tensor is givan as

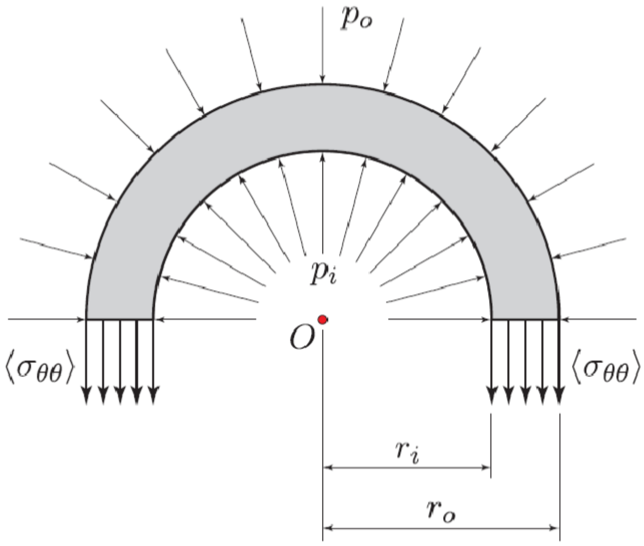
$$\mathbf{F} = \begin{bmatrix} 1/(\lambda_\theta \lambda_z) & 0 & 0 \\ 0 & \lambda_\theta & 0 \\ 0 & 0 & \lambda_z \end{bmatrix} \tag{1.16}$$

with corresponding stresses

$$\langle \sigma_{rr} \rangle = -\frac{P}{2} \tag{1.17}$$

$$\langle \sigma_{\theta\theta} \rangle = \frac{P}{h/r_i} \tag{1.18}$$

$$\langle \sigma_{zz} \rangle = \frac{||\vec{N}||}{\pi(r_0^2 - r_i^2)} \tag{1.19}$$




### 1.4.2 Analytical analysis fot thick-walled cylinders

### 1.4.3 The Finite Element Method

## 1.5 Other biomechanical applications