Report Electromagnetism I

FDTD Simulation of Lossless Transmission Lines

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Abstract
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1 The update functions

The update functions are given as:

$$\tilde{I}^{m+\frac{1}{2}}_{n+\frac{1}{2}} = \tilde{I}^{m-\frac{1}{2}}_{n+\frac{1}{2}} + \alpha\left(V^m_n - V^m_{n+1}\right), \tag{1}$$

$$V^{m+1}_n = V^m_n + \alpha\left(\tilde{I}^{m+\frac{1}{2}}_{n-\frac{1}{2}} - \tilde{I}^{m+\frac{1}{2}}_{n+\frac{1}{2}}\right), \tag{2}$$

where

$$\alpha \triangleq \frac{v\Delta t}{\Delta z}, \tag{3}$$

is the dimensionless Courant factor and

$$\tilde{I}^{m+\frac{1}{2}}_{n+\frac{1}{2}} = I^{m+\frac{1}{2}}_{n+\frac{1}{2}} R_c \tag{4}$$

is the rescaled current.
At the boundaries the update function for V takes another form.

- At $z = 0$
  The voltage update function is given as:

$$V^{m+1}_0 = V^m_0 + \frac{2\Delta t}{C\Delta z}\left(I^{m+\frac{1}{2}}_g - I^{m+\frac{1}{2}}_{\frac{1}{2}}\right), \tag{5}$$

with

$$I^{m+\frac{1}{2}}_g = \frac{E^{E+\frac{1}{2}}_g}{R_g} - \frac{V^m_0 + V^{m+1}_0}{2R_g}. \tag{6}$$

Subsituting (6) in (5) and using (3), the two relations $v = \frac{1}{\sqrt{LC}}$ and $R_c = \sqrt{\frac{L}{C}}$ and the new defined constant $\tilde{R}_g = \frac{R_c}{R_g}$ yields, after some rearrangements:

$$V^{m+1}_0 = C_1 V^m_0 + C_2\left(E^{m+\frac{1}{2}}_g \tilde{R}_g - \tilde{I}^{m+\frac{1}{2}}_{\frac{1}{2}}\right) \tag{7}$$

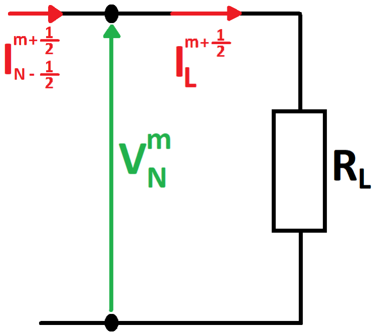
where

$$C_1 = \frac{1-\alpha\tilde{R}_g}{1+\alpha\tilde{R}_g}, \tag{8}$$

$$C_2 = \frac{2\alpha}{1+\alpha\tilde{R}_g}, \tag{9}$$

are two dimensionless constants.

- At $z = l$



The voltage update function becomes:

$$V_N^{m+1} = V_N^m + \frac{2\Delta t}{C\Delta z}\left(I_{N-\frac{1}{2}}^{m+\frac{1}{2}} - I_L^{m+\frac{1}{2}}\right) \quad (10)$$

Kirchoff's voltage law in discretized form states that

$$I_L^{m+\frac{1}{2}} = \frac{V_N^{m+\frac{1}{2}}}{R_L} \quad (11)$$

$$= \frac{V_N^m + V_N^{m+1}}{2R_L} \quad (12)$$

Subsituting (12) in (10) and using the same relations as for $z = 0$ and the new defined constant $\tilde{R}_L = \frac{R_c}{R_L}$ yields, after some rearrangements:

$$V_N^{m+1} = C_3 V_N^m + C_4 \tilde{I}_{N-\frac{1}{2}}^{m+\frac{1}{2}}, \quad (13)$$

where

$$C_3 = \frac{1-\alpha\tilde{R}_L}{1+\alpha\tilde{R}_L}, \quad (14)$$

$$C_4 = \frac{2\alpha}{1+\alpha\tilde{R}_L}, \quad (15)$$

are two dimensionless constants.