Report Electromagnetism I

FDTD Simulation of Lossless Transmission Lines

Bram Popelier, Constantijn Coppers, Vincent Belpaire
Supervisors: Prof. Dries Vande Ginste, ir. Emile Vanderstraeten
Dec. 23, 2022

University of Ghent
Faculty of Engineering and Architecture
Bachelor of Science: Biomedical Engineering

Abstract
Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.
Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

1 The update functions

The update functions are given as:

$$\tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}} = \tilde{I}_{n+\frac{1}{2}}^{m-\frac{1}{2}} + \alpha\left(V_n^m - V_{n+1}^m\right), \tag{1}$$

$$V_n^{m+1} = V_n^m + \alpha\left(\tilde{I}_{n-\frac{1}{2}}^{m+\frac{1}{2}} - \tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}}\right), \tag{2}$$

where

$$\alpha \triangleq \frac{v\Delta t}{\Delta z}, \tag{3}$$

is the dimensionless Courant factor and

$$\tilde{I}_{n+\frac{1}{2}}^{m+\frac{1}{2}} = I_{n+\frac{1}{2}}^{m+\frac{1}{2}} R_c \tag{4}$$

is the rescaled current.
At the boundaries the update function for V takes another form.

- At $z = 0$
  The voltage update function is given as:

$$V_0^{m+1} = V_0^m + \frac{2\Delta t}{C\Delta z}\left(I_g^{m+\frac{1}{2}} - I_{\frac{1}{2}}^{m+\frac{1}{2}}\right), \tag{5}$$

with

$$I_g^{m+\frac{1}{2}} = \frac{E_g^{m+\frac{1}{2}}}{R_g} - \frac{V_0^m + V_0^{m+1}}{2R_g}. \tag{6}$$

Subsituting (6) in (5) and using (3), the two relations $v = \frac{1}{\sqrt{LC}}$ and $R_c = \sqrt{\frac{L}{C}}$ and the new defined constant $\tilde{R}_g = \frac{R_c}{R_g}$ yields, after some rearrangements:

$$V_0^{m+1} = C_1 V_0^m + C_2\left(E_g^{m+\frac{1}{2}}\tilde{R}_g - \tilde{I}_{\frac{1}{2}}^{m+\frac{1}{2}}\right) \tag{7}$$

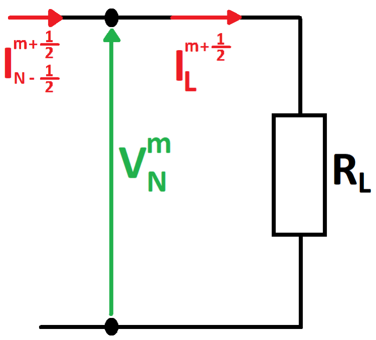
where

$$C_1 = \frac{1-\alpha\tilde{R}_g}{1+\alpha\tilde{R}_g}, \tag{8}$$

$$C_2 = \frac{2\alpha}{1+\alpha\tilde{R}_g}, \tag{9}$$

are two dimensionless constants.

- At $z = l$



The voltage update function becomes:

$$V_N^{m+1} = V_N^m + \frac{2\Delta t}{C\Delta z}\left(I_{N-\frac{1}{2}}^{m+\frac{1}{2}} - I_L^{m+\frac{1}{2}}\right) \quad (10)$$

Kirchoff's voltage law in discretized form states that

$$I_L^{m+\frac{1}{2}} = \frac{V_N^{m+\frac{1}{2}}}{R_L} \quad (11)$$

$$= \frac{V_N^m + V_N^{m+1}}{2R_L} \quad (12)$$

Subsituting (12) in (10) and using the same relations as for $z = 0$ and the new defined constant $\tilde{R}_L = \frac{R_c}{R_L}$ yields, after some rearrangements:

$$V_N^{m+1} = C_3 V_N^m + C_4 \tilde{I}_{N-\frac{1}{2}}^{m+\frac{1}{2}}, \quad (13)$$

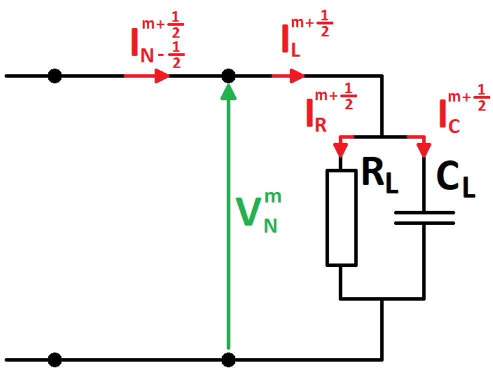
where

$$C_3 = \frac{1 - \alpha\tilde{R}_L}{1 + \alpha\tilde{R}_L}, \quad (14)$$

$$C_4 = \frac{2\alpha}{1 + \alpha\tilde{R}_L}, \quad (15)$$

are two dimensionless constants.

Now if there would be a capacitor added to the load the following situation will occur



Kirchoff's current law states that

$$I_L^{m+\frac{1}{2}} = I_R^{m+\frac{1}{2}} + I_C^{m+\frac{1}{2}}. \quad (16)$$

Using Kirchoff's voltage law at the resistor gives

$$I_R^{m+\frac{1}{2}} = \frac{V_N^{m+\frac{1}{2}}}{R_L} \quad (17)$$

$$= \frac{V_N^m + V_N^{m+1}}{2R_L} \quad (18)$$

The relation between the current and the voltage at the capitor is given as

$$\hat{i} = -C\frac{\partial \hat{v}}{\partial t}. \quad (19)$$

For a first order FDM this turns into

$$I_R^{m+\frac{1}{2}} = -C_L \frac{V_N^{m+1} - V_N^m}{\Delta t}. \quad (20)$$

Substituting (18) and (20) into (16) gives

$$I_L^{m+\frac{1}{2}} = \left(\frac{1}{2R_L} - \frac{C_L}{\Delta t}\right) V_N^{m+1} + \left(\frac{1}{2R_L} + \frac{C_L}{\Delta t}\right) V_N^m \quad (21)$$

and can be rewritten as

$$\tilde{I}_L^{m+\frac{1}{2}} = \frac{\tilde{Z}_1}{2} V_N^{m+1} + \frac{\tilde{Z}_2}{2} V_N^m \quad (22)$$

with

$$\tilde{Z}_1 = R_C\left(\frac{1}{R_L} - 2\frac{C_L}{\Delta t}\right) \quad (23)$$

$$\tilde{Z}_2 = R_C\left(\frac{1}{R_L} + 2\frac{C_L}{\Delta t}\right) \quad (24)$$

Substituting (22) into the adjusted voltage update function at $z = l$ yields

$$V_N^{m+1} = C_3' V_N^m + C_4' \tilde{I}_{N-\frac{1}{2}}^{m+\frac{1}{2}}, \quad (25)$$

where

$$C_3' = \frac{1 - \alpha\tilde{Z}_2}{1 + \alpha\tilde{Z}_1}, \quad (26)$$

$$C_4' = \frac{2\alpha}{1 + \alpha\tilde{Z}_1}. \quad (27)$$

Notice when $C_L = 0$ then $C_3' = C_3$ and $C_4' = C_4$.