Formularium - Electromagnetism I

1 Introduction

$$AB + BA = \heartsuit \tag{1}$$

2 Maxwell's Equations

3 Electrostatics

4 Magnetostatics

5 Plane Waves

5.1 Plane waves in a lossless dielectric

Assumptions Source free, homogenous, lossless and isotrpoic dielectric characterized by $\epsilon = \epsilon_0 \epsilon_r$ and $\mu = \mu_0 \mu_r$.
Plane wave propagation to an arbitrary direction $\vec{u}$:

$$\vec{e}(\vec{r}) = \vec{A} e^{-jk\vec{u}\cdot\vec{r}}, \tag{2}$$

$$\vec{h}(\vec{r}) = \frac{1}{Z_c} \vec{u} \times \vec{e}(r), \tag{3}$$

with $\vec{A} \cdot \vec{u} = 0$, $Z_c = \frac{\mu}{\epsilon}$, $k^2 = \omega^2 \epsilon \mu$ and Poynting's vector:

$$\vec{p}(\vec{r}) = \frac{|\vec{A}|^2}{2Z_c} \vec{u}. \tag{4}$$

Plane waves in a lossy dielectric

Equations 2 and 3 with $\vec{A} \cdot u = 0$ remain unchanged but k and Z_c are now complex valued (in VI). Denote $k = \beta - j\alpha$, the new Poynting vector becomes

$$\vec{p}(\vec{r}) = \frac{|\vec{A}|^2}{2Z_c} e^{-2\alpha\vec{u}\cdot\vec{r}} \vec{u}. \tag{5}$$

This shows that when a wave propagates over a distance d in a lossy medium, its power descreases by a factor $e^{-2\alpha d}$. Expressed in decibel (dB) gives

$$-10 \log_{10}(e^{-2\alpha d}) \approx 8.686 \alpha d, \tag{6}$$

with $L = 8.686\alpha$ the relative power loss in (dB/m).
Considering a material with conduction losses ($\epsilon + \frac{\sigma}{j\omega}$):

$$k = \omega \sqrt{\epsilon \mu} \sqrt{1 + \frac{\sigma}{j\omega}}, \tag{7}$$

$$Z_c = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\sqrt{1 + \frac{\sigma}{j\omega}}}, \tag{8}$$

gives rise to two interesting situations:

1. Low-loss dielectric ($\sigma \ll \omega\epsilon$):

$$k = \beta - j\alpha \approx \omega\sqrt{\epsilon\mu} - j\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}, \tag{9}$$

$$Z_c \approx \sqrt{\frac{\mu}{\epsilon}}, \tag{10}$$

2. Good conductor ($\sigma \gg \omega\epsilon$):

$$k = \beta - j\alpha \approx \frac{1-j}{\delta}, \tag{11}$$

$$Z_c \approx \frac{1+j}{\sigma\delta}, \tag{12}$$

with $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$ known as the *skin depth.*

5.2 Reflection and transmission at a plane interface

Given the incident fields in medium 1:

$$\vec{e_i} = \vec{A}e^{-jk_1\vec{u_i}\cdot\vec{r}}, \tag{13}$$

$$\vec{h_i} = \frac{1}{Z_1}\vec{u_i} \times \vec{e_i}, \tag{14}$$

with $u_i = \cos(\theta_i)\vec{u}_z + \sin(\theta_i)\vec{u}_x$ and $\vec{A} \cdot \vec{u}_i = 0$. Then:

1. The reflected wave becomes

$$\vec{e_r} = \vec{B}e^{-jk_1\vec{u_r}\cdot\vec{r}}, \tag{15}$$

$$\vec{h_r} = \frac{1}{Z_1}\vec{u_r} \times \vec{e_r}, \tag{16}$$

with $u_r = -\cos(\theta_i)\vec{u}_z + \sin(\theta_i)\vec{u}_x$ and $\vec{B} \cdot \vec{u}_r = 0$.

2. The transmitted wave becomes

$$\vec{e_t} = \vec{C}e^{-jk_2\vec{u_t}\cdot\vec{r}}, \tag{17}$$

$$\vec{h_t} = \frac{1}{Z_2}\vec{u_t} \times \vec{e_t}, \tag{18}$$

with $u_r = \cos(\theta_t)\vec{u}_z + \sin(\theta_t)\vec{u}_x$, $\vec{C} \cdot \vec{u}_t = 0$ and where Snell's law states that

$$k_2 \sin\theta_t = k_1 \sin\theta_i. \tag{19}$$