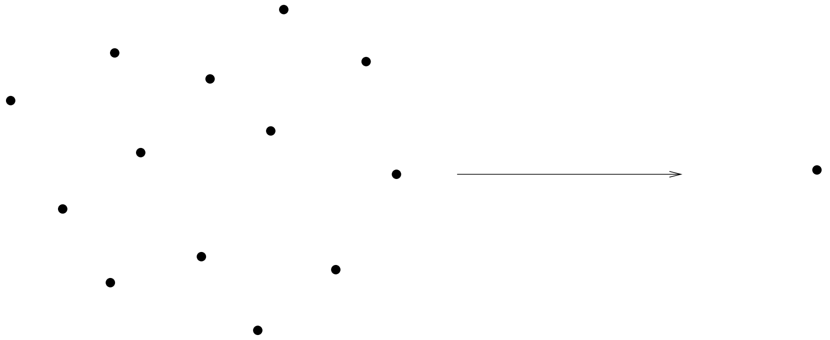
Self-stabilizing Deterministic Gathering

Yoann Dieudonné Franck Petit

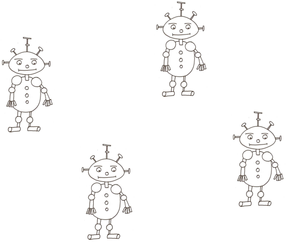
Université de Picardie Jules Verne, France

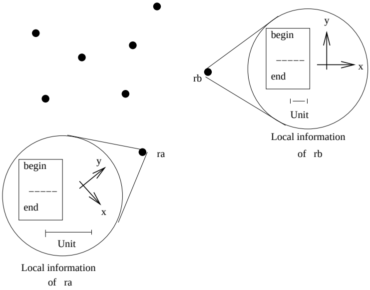
Rhodes, ALGOSENSORS 2009

Gathering Problem

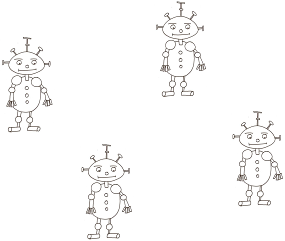


Model SSM




rb
begin
-----
end
y
x
Unit
Local information
of rb
ra
begin
-----
end
y
x
Unit
Local information
of ra

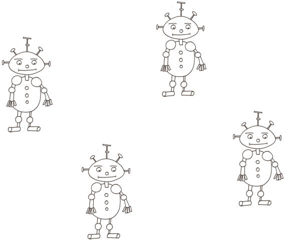
Model SSM



Model

- autonomous

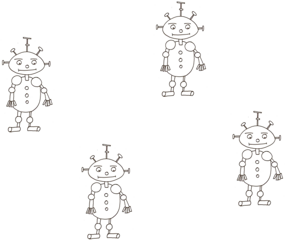
Model SSM



Model

- autonomous
- mobile

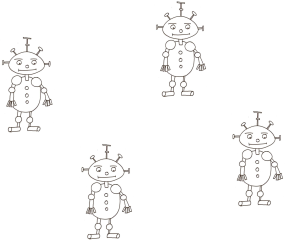
Model SSM



Model

- autonomous
- mobile
- anonymous

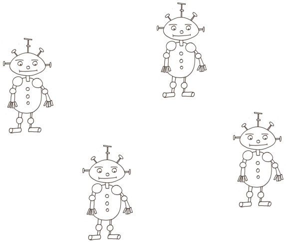
Model SSM



Model

- autonomous
- mobile
- anonymous
- oblivious

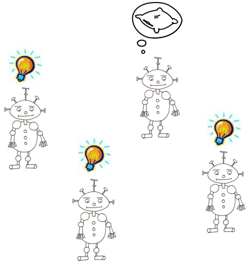
Model SSM



Model

- autonomous
- mobile
- anonymous
- oblivious
- no communication

Model SSM



T_1	T_2	T_3	T_4	T_5	------

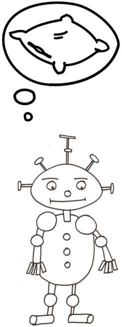
Time

- Time is represented as an infinite sequence of time instants.
- At each time instant, every robot is either active or inactive.

Fairness

Every robot is infinitely often activated.

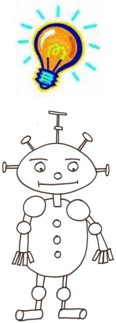
Model SSM



An inactive robot :

It remains motionless

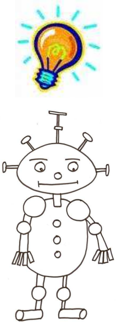
Model SSM



An active robot :

- Observes

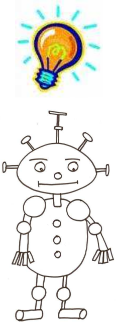
Model SSM



An active robot :

- Observes
- Computes

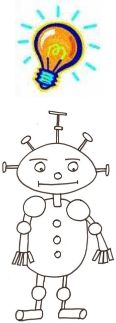
Model SSM



An active robot :

- Observes
- Computes
- Moves

Model SSM



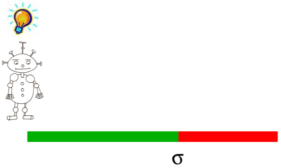
An active robot :

- Observes
- Computes
- Moves

Atomic actions

Observing, computing and moving are instantaneous.

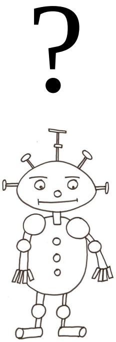
Model SSM



When a robot is active

In every single activation, the distance traveled by any robot r is bounded by σ_r.

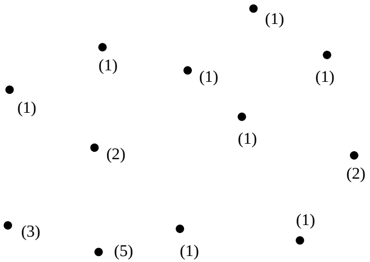
Model SSM



Remark

From the obliviousness, the robots are unable to remember any past action or observation from previous steps.

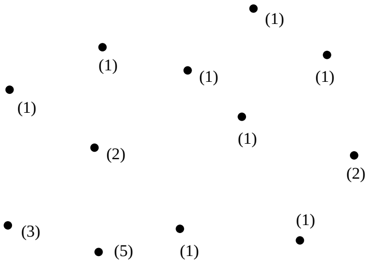
Model SSM



Assumption 1

Two or more robots can occupy the same point

Model SSM



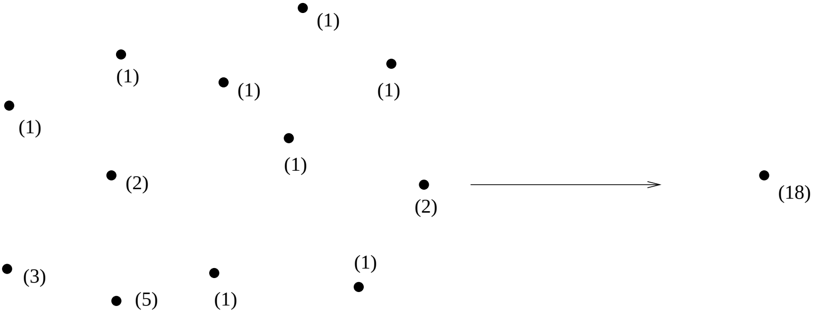
Assumption 1

Two or more robots can occupy the same point

Assumption 2

All the robots can determine the number of robots on each point

Problem Definition



In literature

In literature

Mainly studied :

- In a probabilistic way [Suzuki et al. 99, Gradinariu et al. 07,08,09]
- Starting from a configuration where all the robots are initially at distinct positions [Suzuki et al. 99, Cieliebak 03, Flocchini et al. 03,...]

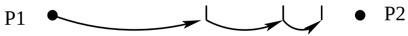
In literature

- Starting from an arbitrary configuration and in a deterministic way, only the following result

Corollary (Suzuki and yamashita 99)

Starting from an arbitrary configuration the gathering problem cannot be solved in SSM in a deterministic way if the number of robots is even.

In literature



Impossibility of gathering

only converge in an infinite number of steps

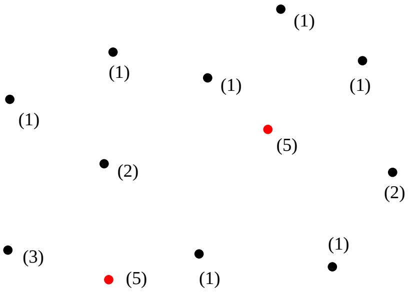
Results

- we provide a deterministic protocol for solving the gathering problem for an odd number of robots starting from an arbitrary configuration

Results

- we provide a deterministic protocol for solving the gathering problem for an odd number of robots starting from an arbitrary configuration
- The protocol is self-stabilizing because "regarless of the initial states", it is guaranteed to converge in a finite number of steps.

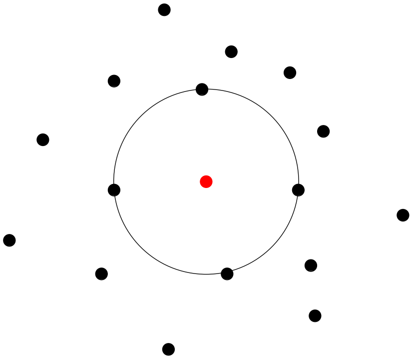
Our Protocol



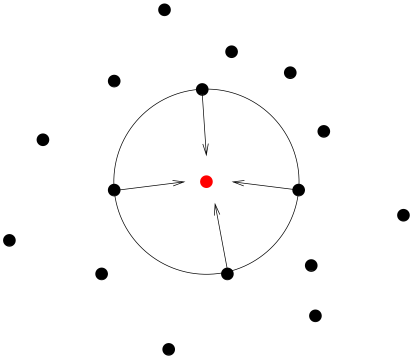
Definition (Maximal Points)

A point p is maximal $\iff \forall$ point p', $|p| \geq |p'|$

If there is only one maximal point



If there is only one maximal point



- Only the closest robots are allowed to move toward the maximal point.

If there are only one maximal point

- The unique maximal point remains unique
- The number of robots located at the maximal point increases in finite time
- Every robot becomes a close robot in finite time

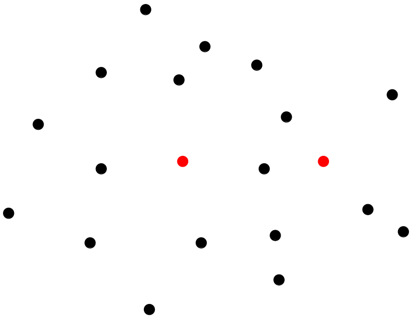
If there are only one maximal point

- The unique maximal point remains unique
- The number of robots located at the maximal point increases in finite time
- Every robot becomes a close robot in finite time

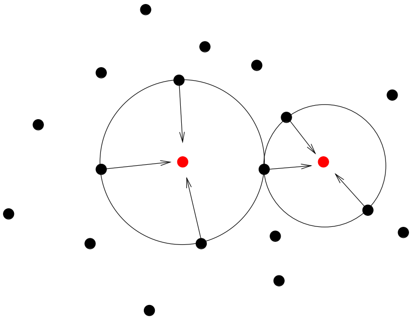
Lemma

The deterministic gathering can be solved in SSM for an odd number of robots if there exists only one maximal point.

If there are exactly 2 maximal points



If there are exactly 2 maximal points



- Only the closest robots to one of the two maximal points are allowed to move

If there are exactly 2 maximal points

- no other maximal point is created
- number of robots on the two maximal points increases
- exactly one maximal point in finite time because the number of robots is odd

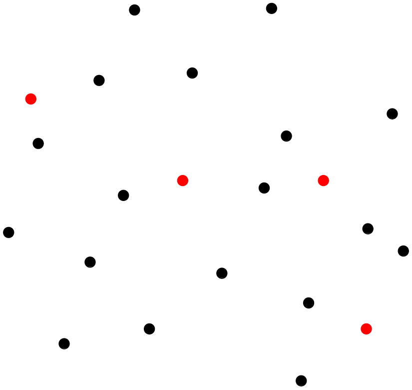
If there are exactly 2 maximal points

- no other maximal point is created
- number of robots on the two maximal points increases
- exactly one maximal point in finite time because the number of robots is odd

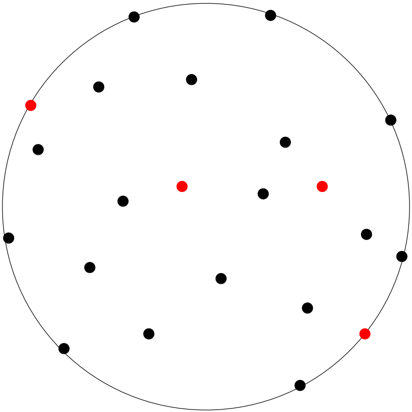
Lemma

The deterministic gathering can be solved in SSM for an odd number of robots if there exist exactly 2 maximal points.

If there are at least 3 maximal points



If there are at least 3 maximal points



- We consider the smallest enclosing circle

If there are at least 3 maximal points

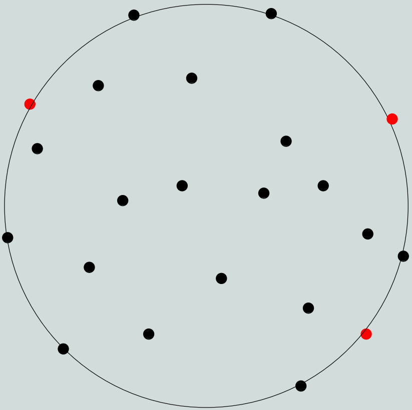
Theorem (Meggido 86)

The smallest circle enclosing a set of points is unique and can be computed in linear time

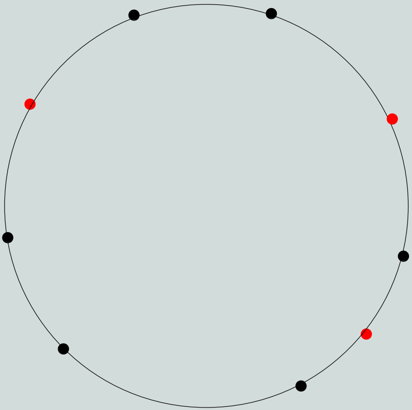
If there is exactly 3 maximal points

2 cases to consider

At least one robot inside *SEC*

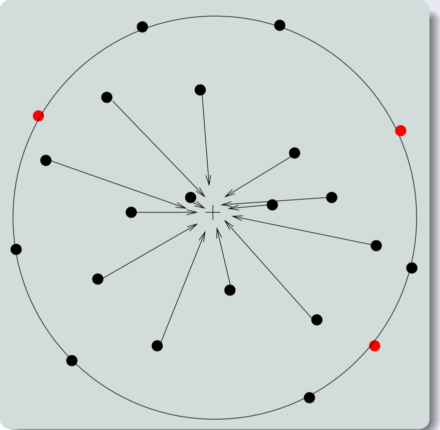


No robot inside *SEC*



If there is exactly 3 maximal points

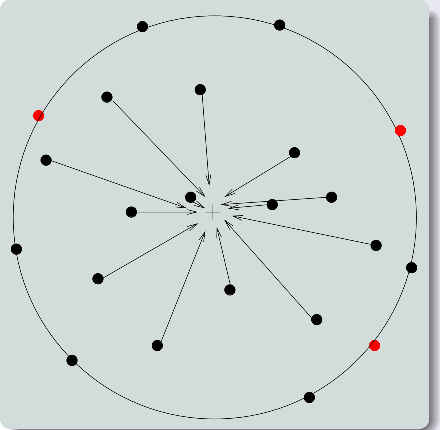
At least one robot inside *SEC*



- the robots inside *SEC*, move to the center of *SEC*

If there is exactly 3 maximal points

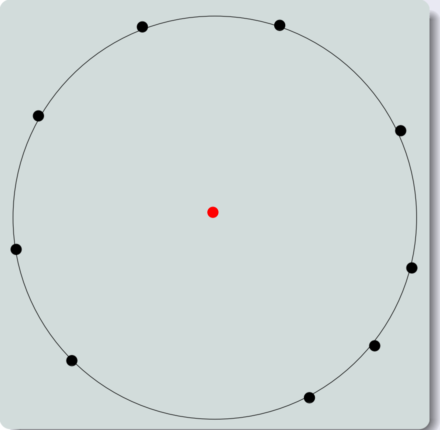
At least one robot inside *SEC*



- the robots inside *SEC*, move to the center of *SEC*
- *SEC* remains invariant

If there is exactly 3 maximal points

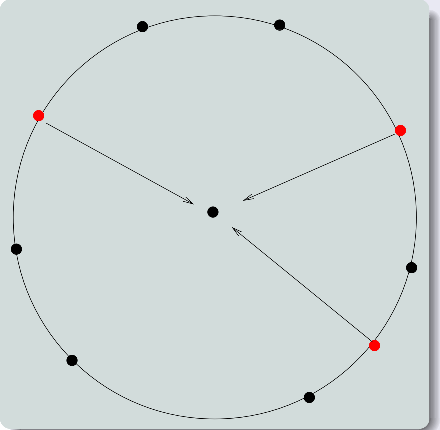
At least one robot inside *SEC*



- A unique maximal may appear at the center of *SEC*

If there is exactly 3 maximal points

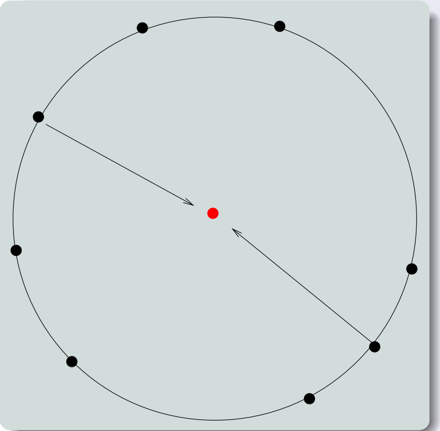
At least one robot inside *SEC*



- Only the maximal points are allowed to move towards the center

If there is exactly 3 maximal points

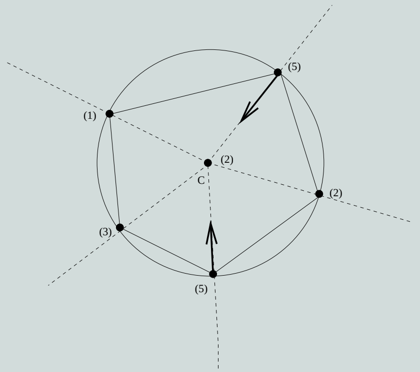
At least one robot inside *SEC*

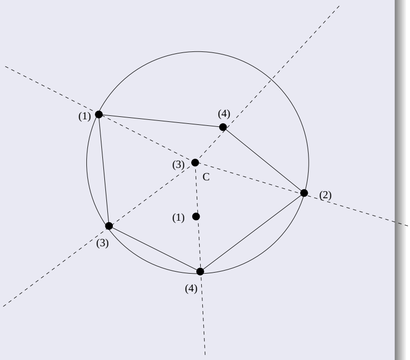


- A unique maximal may appear at the center of *SEC*

If there is exactly 3 maximal points

At least one robot inside *SEC*





If there is exactly 3 maximal points

Theorem (Chrystal 1885)

All the points inside the convex hull are inside SEC

If there is exactly 3 maximal points

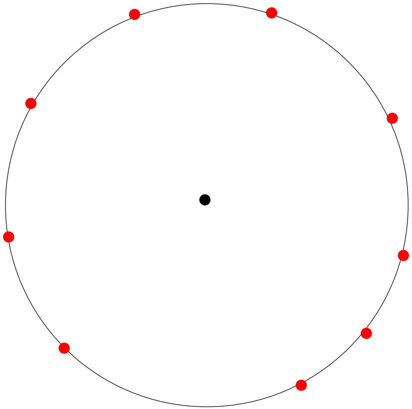
Theorem (Chrystal 1885)

All the points inside the convex hull are inside SEC

Lemma

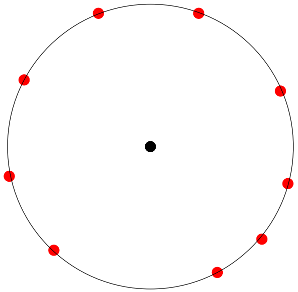
All the robots inside SEC remains inside SEC even if SEC changes while there exist at least three maximal points

If there is exactly 3 maximal points



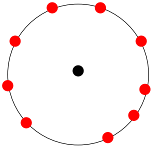
- the number of robots inside SEC never increases

If there is exactly 3 maximal points



- the number of robots inside SEC never increases

If there is exactly 3 maximal points



- the number of robots inside SEC never increases

If there is exactly 3 maximal points

- the number of robots inside SEC never increases

If there is exactly 3 maximal points

- Otherwise the number of robots inside *SEC* increases and we obtain a unique maximal point in finite time.

If there is exactly 3 maximal points

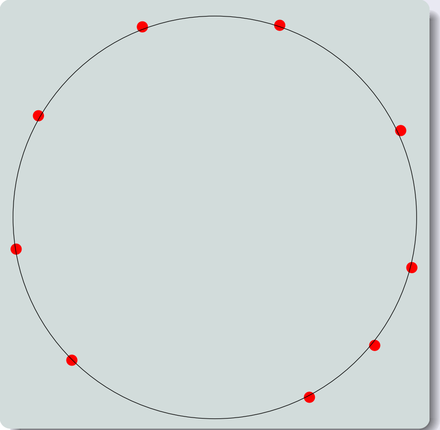
- Otherwise the number of robots inside *SEC* increases and we obtain a unique maximal point in finite time.

Lemma

The deterministic gathering can be solved in SSM for an odd number of robots if there exist at least 3 maximal points and at least one robot inside SEC.

If there is exactly 3 maximal points

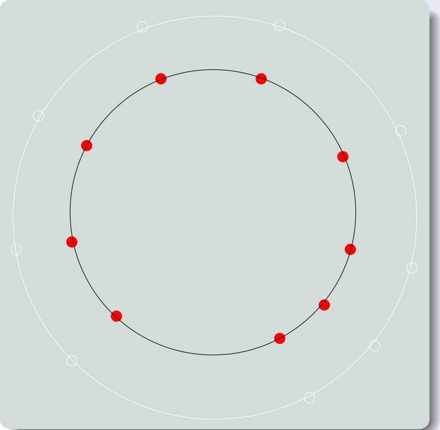
No robot inside *SEC*



- there are never some robots inside *SEC*

If there is exactly 3 maximal points

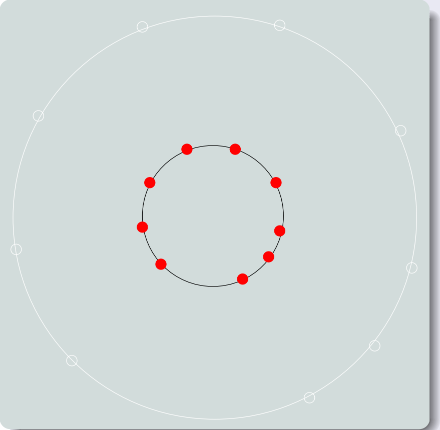
No robot inside *SEC*



- there are never some robots inside *SEC*

If there is exactly 3 maximal points

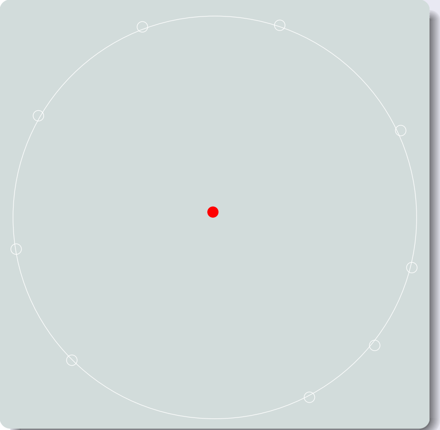
No robot inside *SEC*



- there are never some robots inside *SEC*

If there is exactly 3 maximal points

No robot inside *SEC*



- there are never some robots inside *SEC*

Final result

Theorem

Starting from an arbitrary configuration the gathering problem can be solved in SSM in a deterministic way $\Longleftrightarrow$ *the number of robots is odd.*

Future work

- a self-stabilizing deterministic gathering in a fully asynchronous model ?

Future work

- a self-stabilizing deterministic gathering in a fully asynchronous model ?
- minimal conditions for an even number of robots ?

- Thank you for your attention...

- Thank you for your attention...