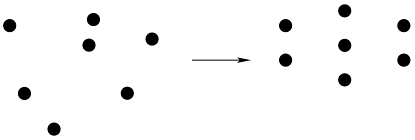
Deterministic Pattern Formation in Swarms of Robots

Franck Petit

Problem



Definition (Arbitrary Pattern Formation)

Given a swarm of n robots scattered on the plan, designing a deterministic algorithm so that, the robots eventually form a pattern $\mathcal{P}$ made of n positions and known by each of them in advance.

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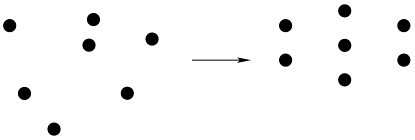

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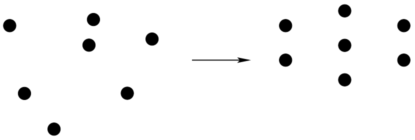

Definition (Arbitrary Pattern Formation)

Given a swarm of n robots scattered on the plan, designing a deterministic algorithm so that, the robots eventually form a pattern $\mathcal{P}$ made of n positions and known by each of them in advance.

In other words, at the end of the computation, the positions of the robots coincide, with the positions of $\mathcal{P}$, where $\mathcal{P}$ can be translated, rotated, and scaled in each local coordinate system.

Previous Works

SoD and Chirality

[Flocchini et al., 1999]

A solution exists for any n and any $\mathcal{P}$.

SoD and No Chirality

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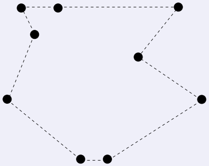

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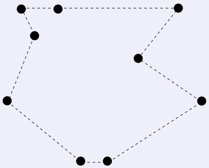

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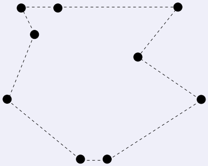

SoD and No Chirality

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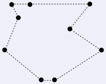
If n is odd, a solution exists for any $\mathcal{P}$.



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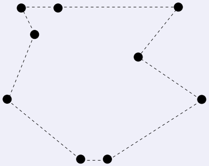

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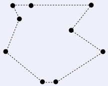
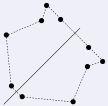


If n is even, a solution exists provided that $\mathcal{P}$ is symmetric.



Pattern Formation With No Sense of Direction

Question

Assuming no sense of direction (with or without chirality), which kind of patterns can be formed in a deterministic way?

Pattern Formation With No Sense of Direction

Theorem

[Flocchini et al., 2001]
If the robots share no Sense of Direction, then they cannot solve the Arbitrary Pattern Formation problem deterministically, even having the ability of chirality.

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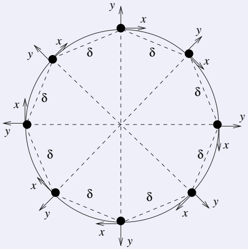

Pattern Formation With No Sense of Direction

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If the robots share no Sense of Direction, then they cannot solve the Arbitrary Pattern Formation problem deterministically, even having the ability of chirality.

Let Θ be the class of patterns that can be solved deterministically assuming robots devoid of sense of direction.

Corollary

Either Θ *is equal to the set of regular polygons (n-gons)*
or $\Theta = \emptyset$.

Circle Formation

Question

Assuming no sense of direction, is it possible to eventually form a regular n-gon (circle)?

Circle Formation

Previous work

[Défago and Konagaya, 2002] [Chatzigiannakis et al., 2004] [Défago and Souissi, 2008]

Asymptotic convergence toward the n-gon.

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Any n (in ASYNC), but the n-gon is not systematically achieved.

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Circle Formation

All previous protocols are based on the two following steps:

1. Move the robots on (the boundary of) a circle $\mathcal{C}$.
2. Without leaving $\mathcal{C}$, arrange them evenly along $\mathcal{C}$.

See also: Flocchini et al. « Distributed Computing by Mobile Robots: Uniform Circle Formation » 2016

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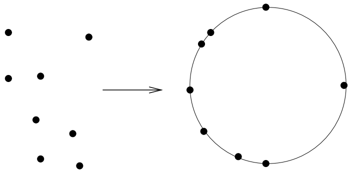

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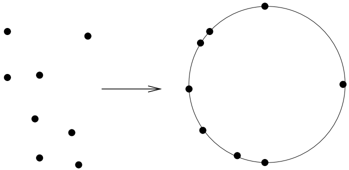

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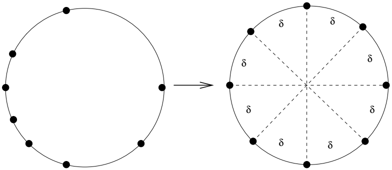

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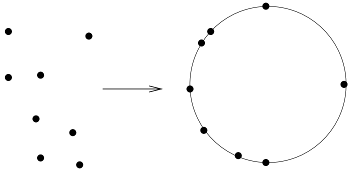

EASY PART

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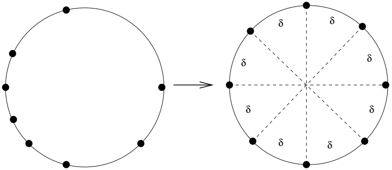

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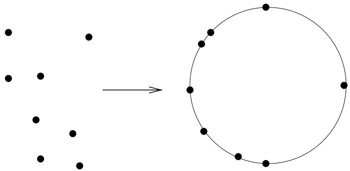

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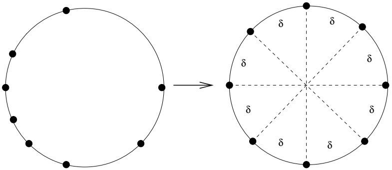

HARD PART

Circle Formation

Theorem

[Flocchini et al, 2006]
There exists no deterministic algorithm that eventually arrange n robots evenly along a circle $\mathcal{C}$ without leaving $\mathcal{C}$.

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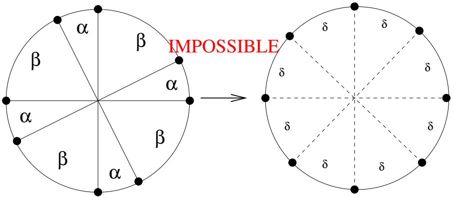

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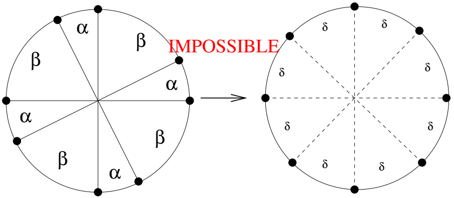

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Are we done, *i.e.*, $\Theta = \emptyset$?

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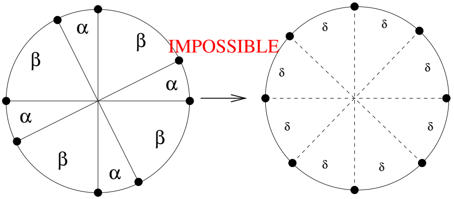

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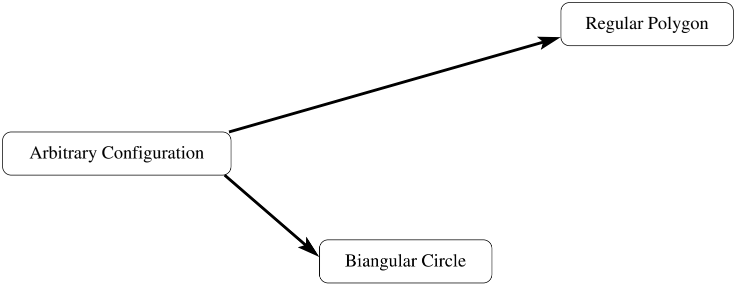
Fortunately, not!

An almost *n*-gon algorithm



[Katreniak, 2005]

Starting from a biangular circle

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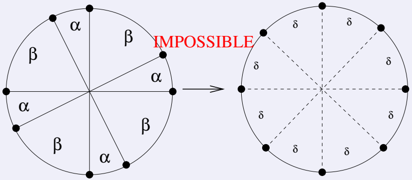

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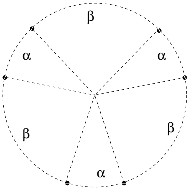

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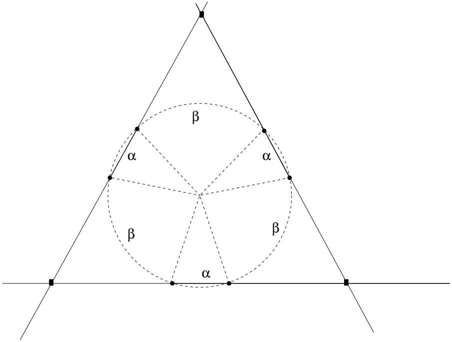

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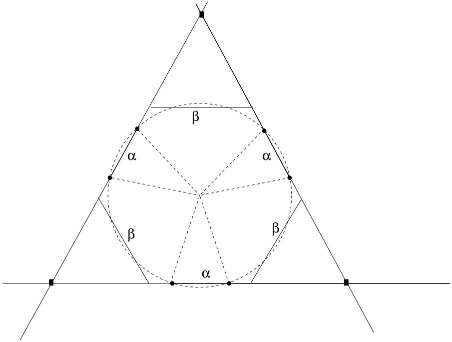

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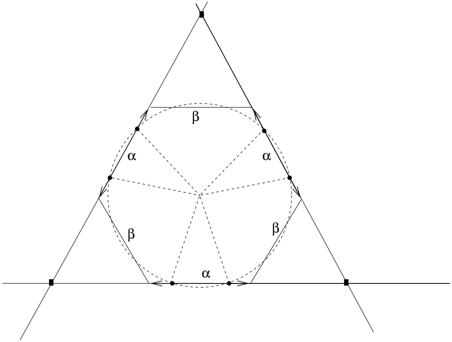

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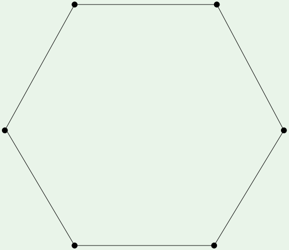
Every robot is active



Regular Polygon

Starting from a biangular circle

Let us leave $\mathcal{C}$

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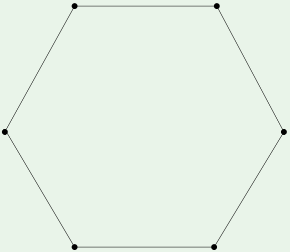

Regular Polygon

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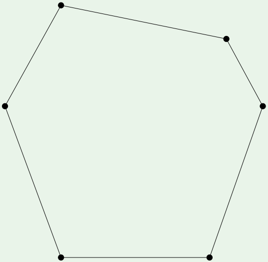

Ideal Polygon

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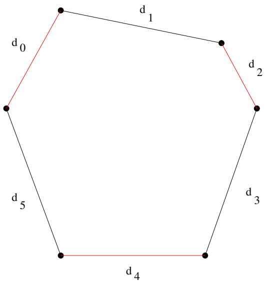

Property 1

- Either $d_0 \geq d_1 \leq \cdots \geq d_{n-1} \leq d_0$
- or $d_0 \leq d_1 \geq \cdots \leq d_{n-1} \geq d_0$

Starting from a biangular circle

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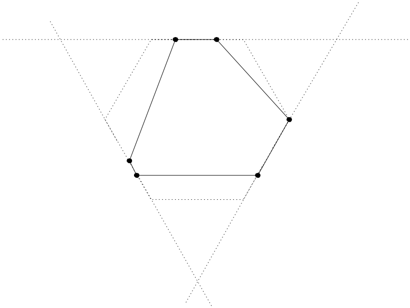

Property 2

A regular n-gon can be associated to an Ideal Polygon.

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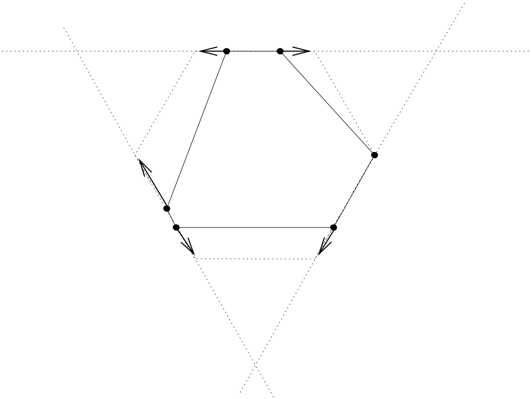
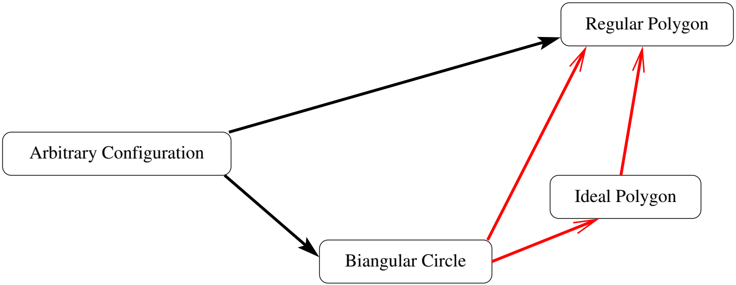


Circle Formation



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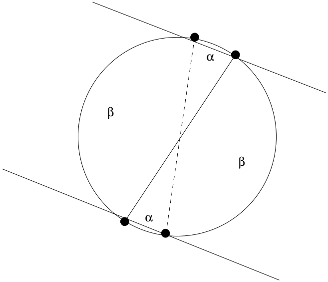
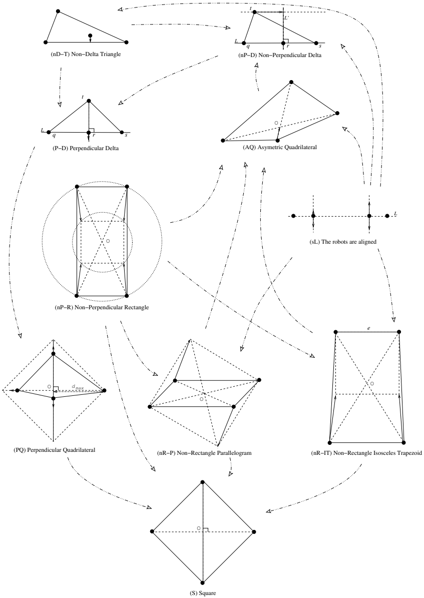


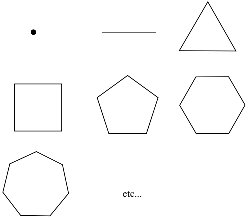
It works if $n \geq 5$ only

1. [Katreniak 2005] works if $n \geq 5$ only
2. Difficulty to find an invariant


(nD–T) Non–Delta Triangle
(nP–D) Non–Perpendicular Delta
(P–D) Perpendicular Delta
(AQ) Asymetric Quadrilateral
(sL) The robots are aligned
(nP–R) Non–Perpendicular Rectangle
(PQ) Perpendicular Quadrilateral
(nR–P) Non–Rectangle Parallelogram
(nR–IT) Non–Rectangle Isosceles Trapezoid
(S) Square

$n \leq 4$

Arbitrary Pattern Formation



Theorem

$\forall n$, the class of patterns that can be solved deterministically assuming robots devoid of sense of direction (Θ) is equal to the set of regular polygons (n-gons).