Ring Exploration by Semi-Synchronous Oblivious Robots

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Context

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- A team of *k* "*weak*" robots evolving into a ring of *n* nodes

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- A team of *k* *"weak"* robots evolving into a ring of *n* nodes
  - Autonomous : No central authority

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- A team of *k* "*weak*" robots evolving into a ring of *n* nodes
  - Autonomous
  - Anonymous : Undistinguishable

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- A team of *k* *"weak"* robots evolving into a ring of *n* nodes
  - Autonomous
  - Anonymous
  - Oblivious : No mean to know the past

Context

- A team of *k* *"weak"* robots evolving into a ring of *n* nodes
  - Autonomous
  - Anonymous
  - Oblivious
  - Disoriented : No mean to agree on a common direction or orientation

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  - Atomicity : In every configuration, each robot is located at exactly one node

Context

- A team of *k* "*weak*" robots evolving into a ring of *n* nodes
  - Atomicity
  - Multiplicity : In every configuration, each node contains zero, one, or more than one robot

    (every robot is able to detect it)

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    1. Look : Instantaneous snapshot with multiplicity detection

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  - SSM : In every configuration, k' robots are activated ($0 < k' \leq k$)
  - The k' activated robots execute the cycle:
    1. Look
    2. Compute: Based on this observation, decides to either stay idle or move to one of the neighboring nodes

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- A team of *k* "*weak*" robots evolving into a ring of *n* nodes
  - SSM : In every configuration, k' robots are activated ($0 < k' \leq k$)
  - The k' activated robots execute the cycle:
    1. Look
    2. Compute
    3. Move : Move toward its destination

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Starting from a configuration where no two robots are located at the same node:

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- Exploration:
  Each node must be visited by at least one robot

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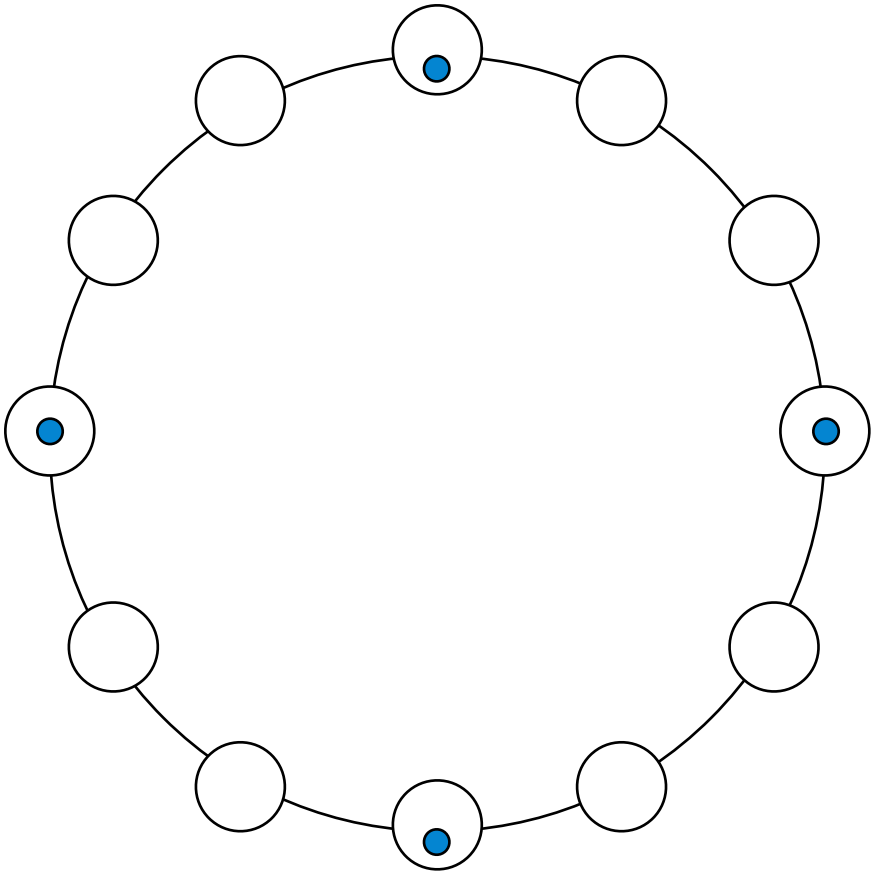
- Termination:
  Eventually, every robot stays idle
- Performance: Number of robots ($k<n$)

Lower Bound (1/2)

Deterministic Exploration impossible if $k \mid n$



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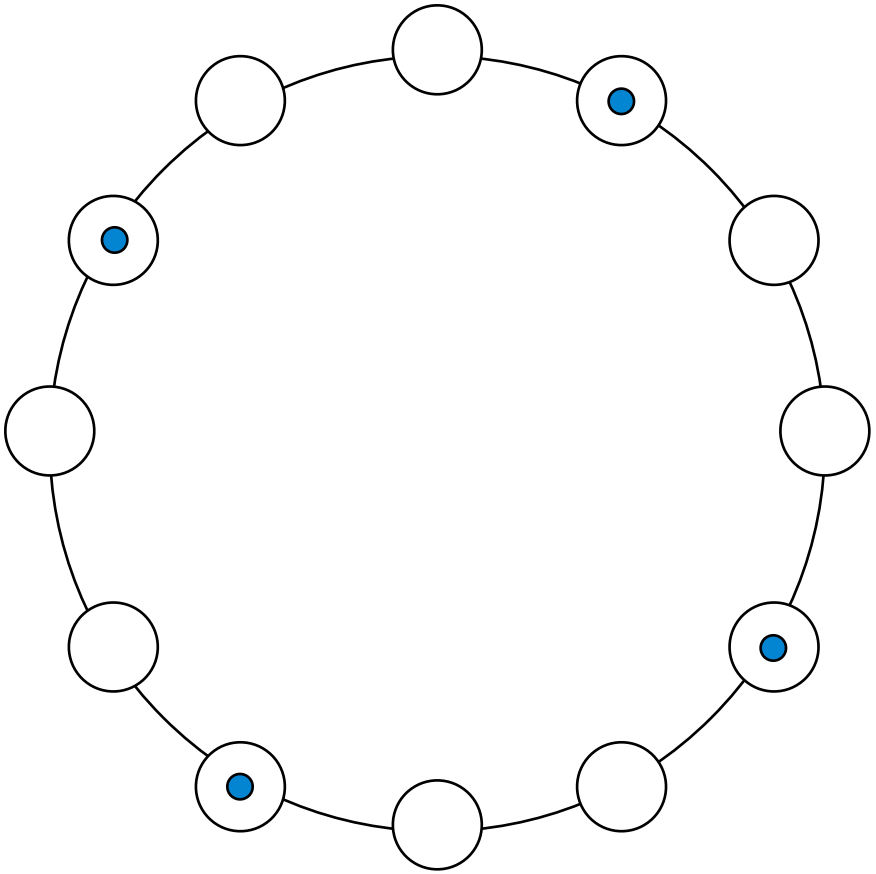

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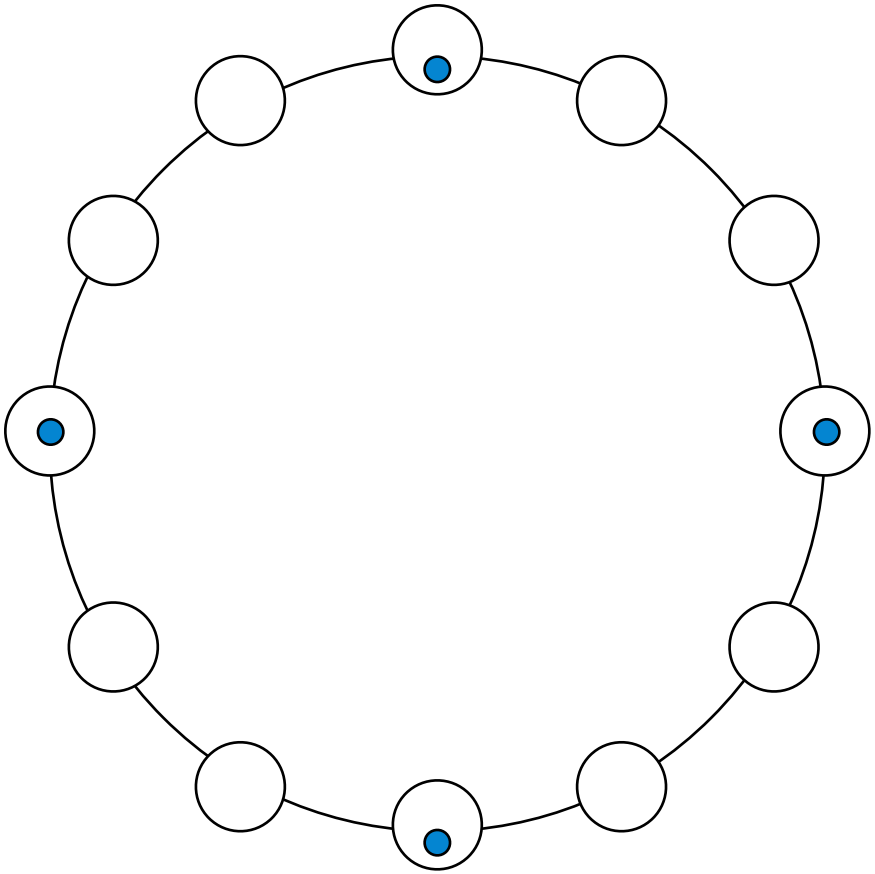
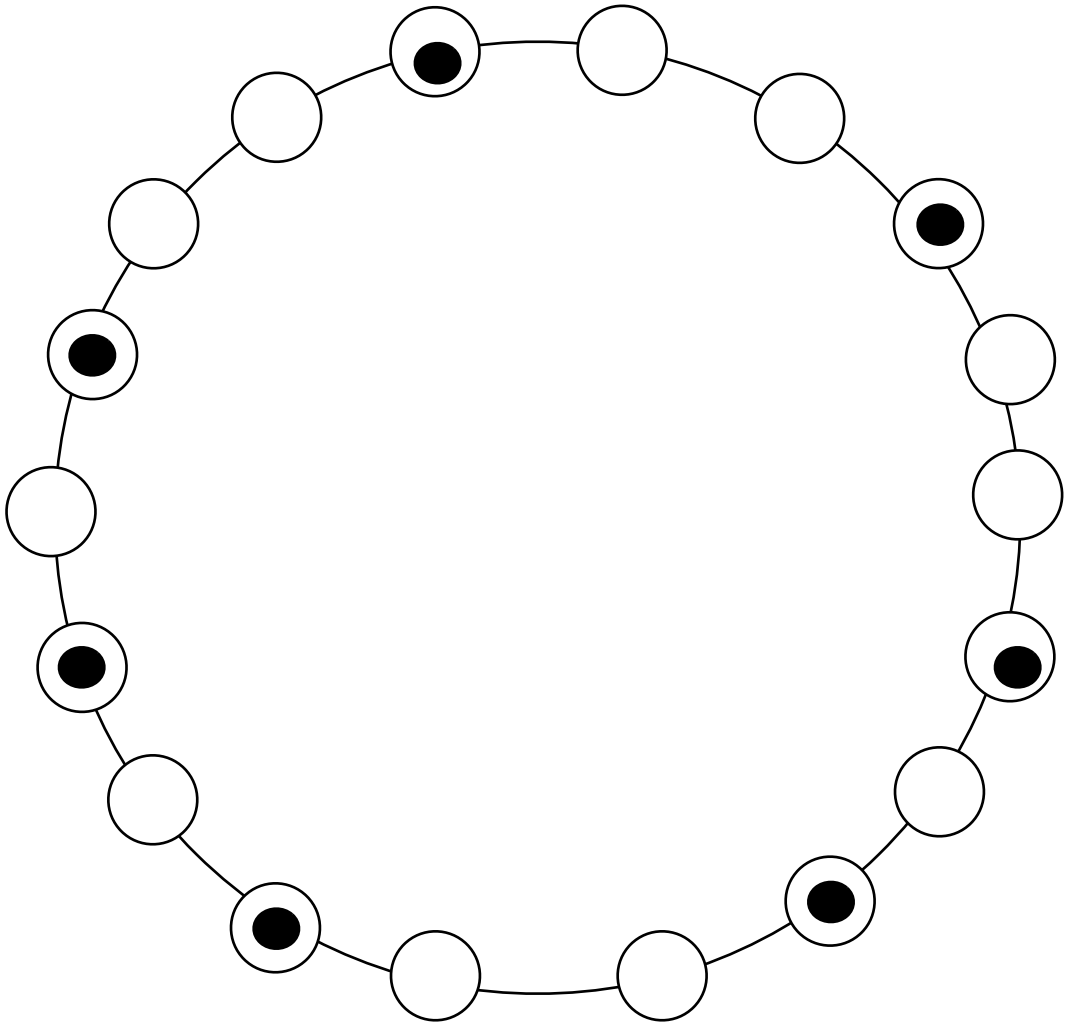


Deterministic Algorithm

- $k \nmid n$, $k > \log n$, and $n > 16$



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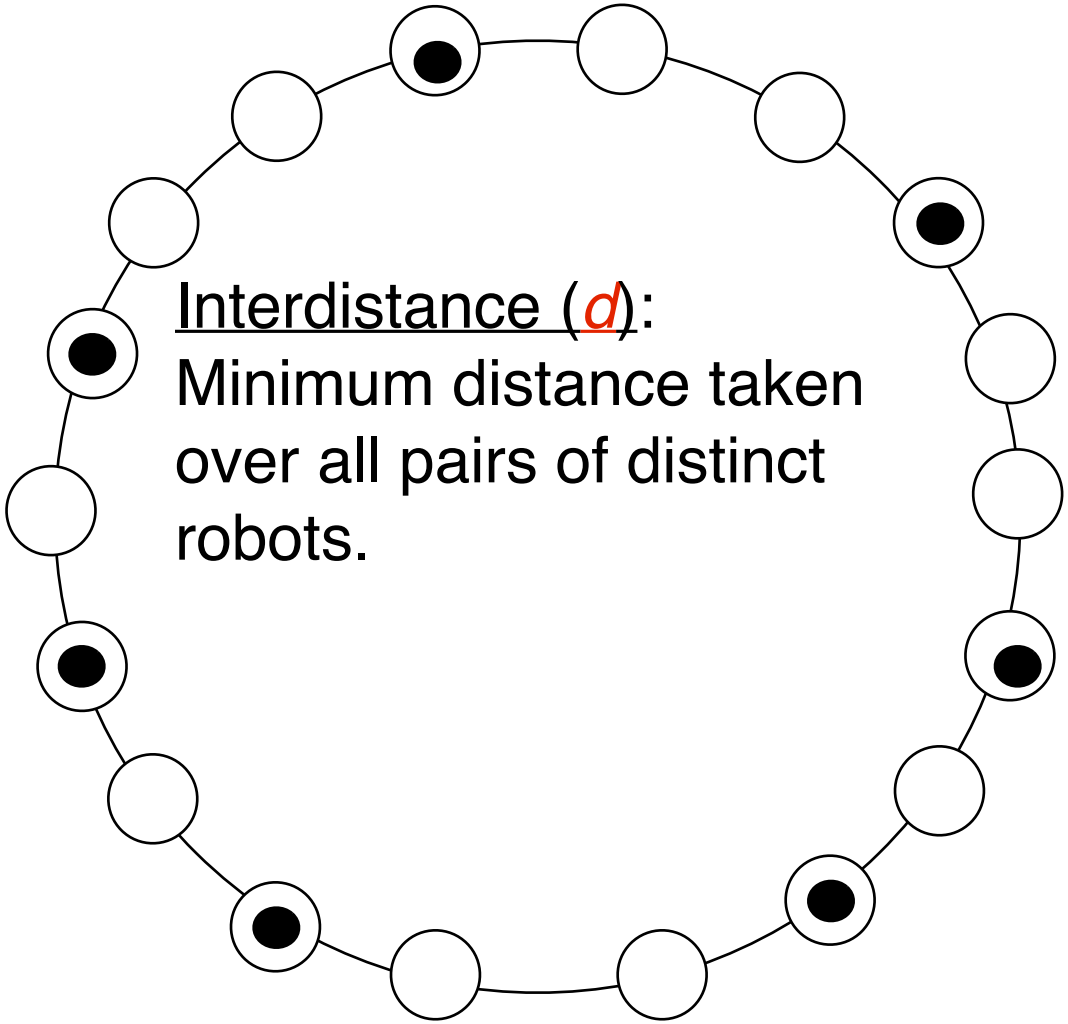

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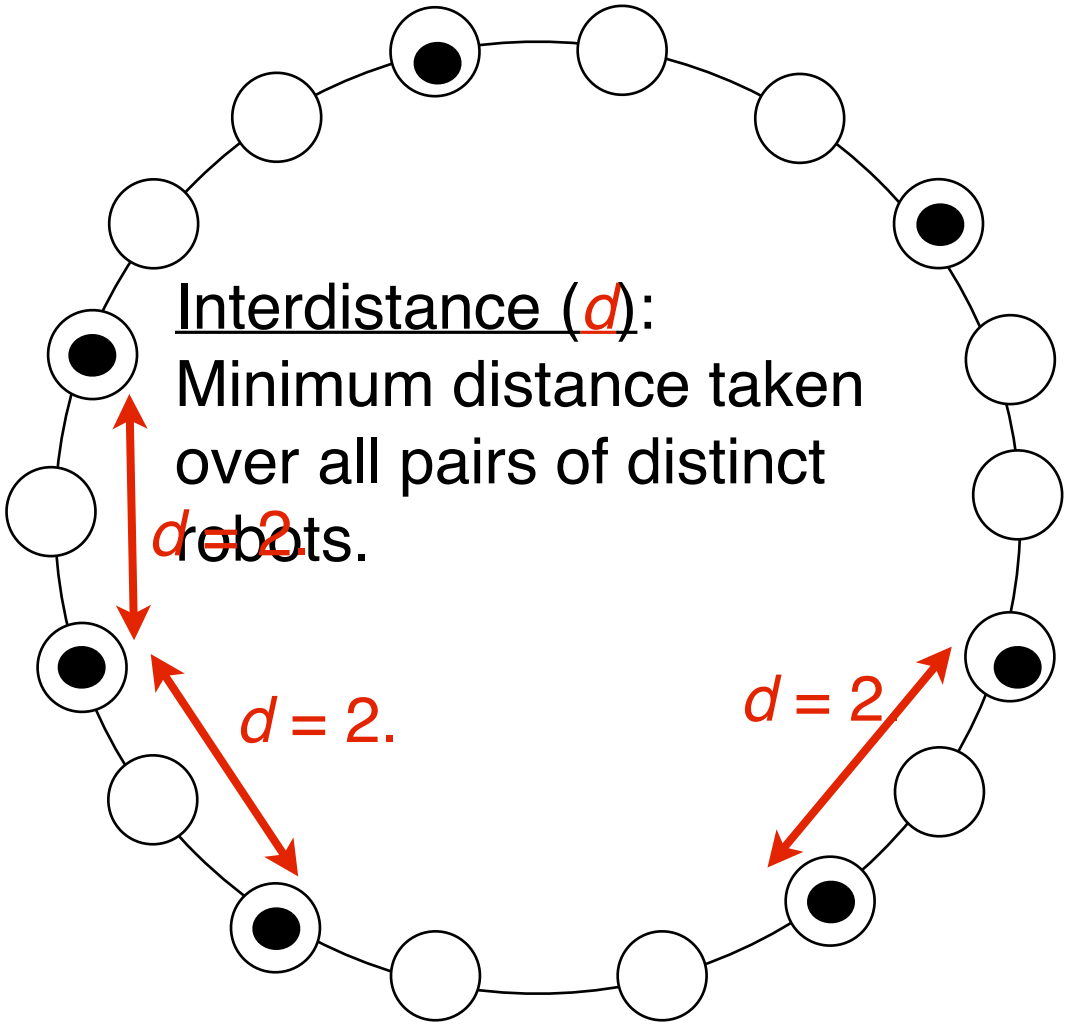

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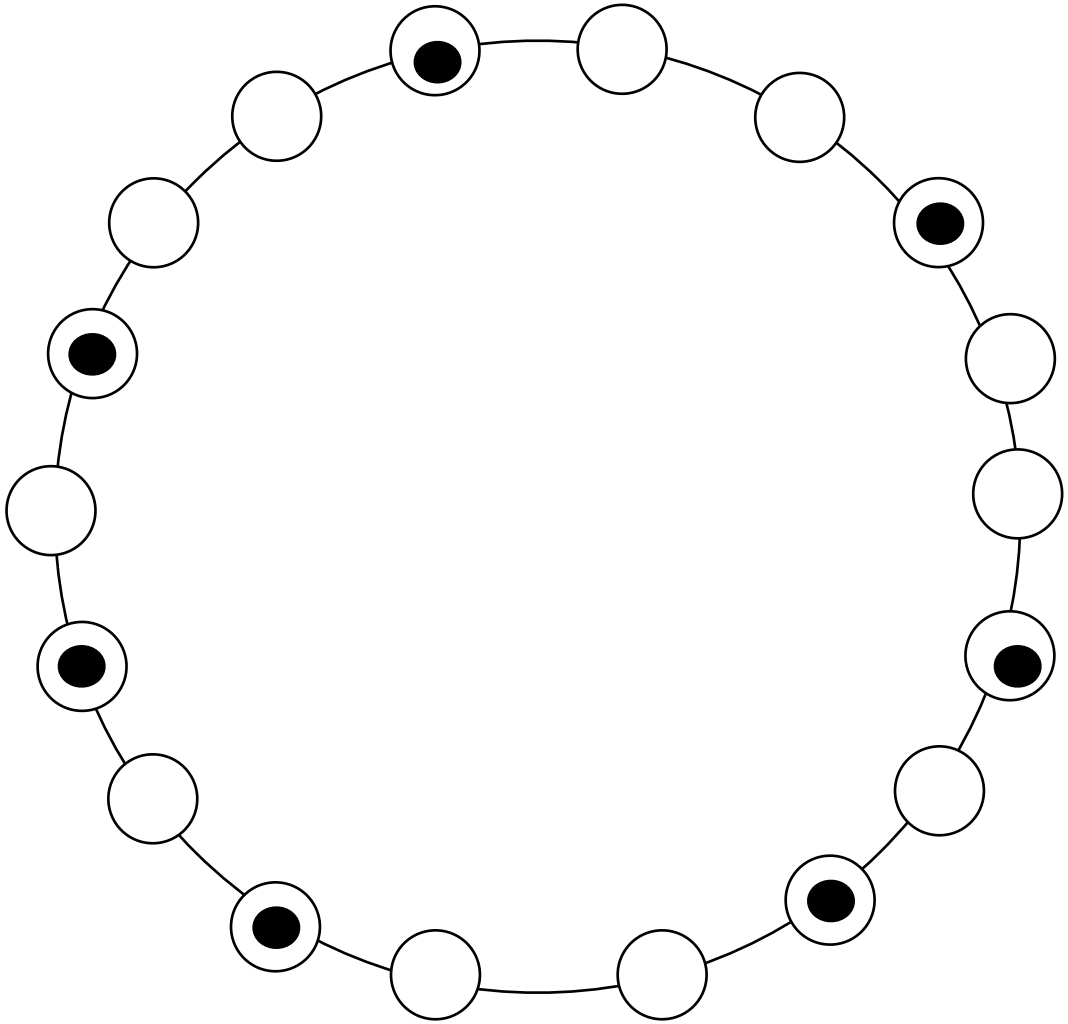

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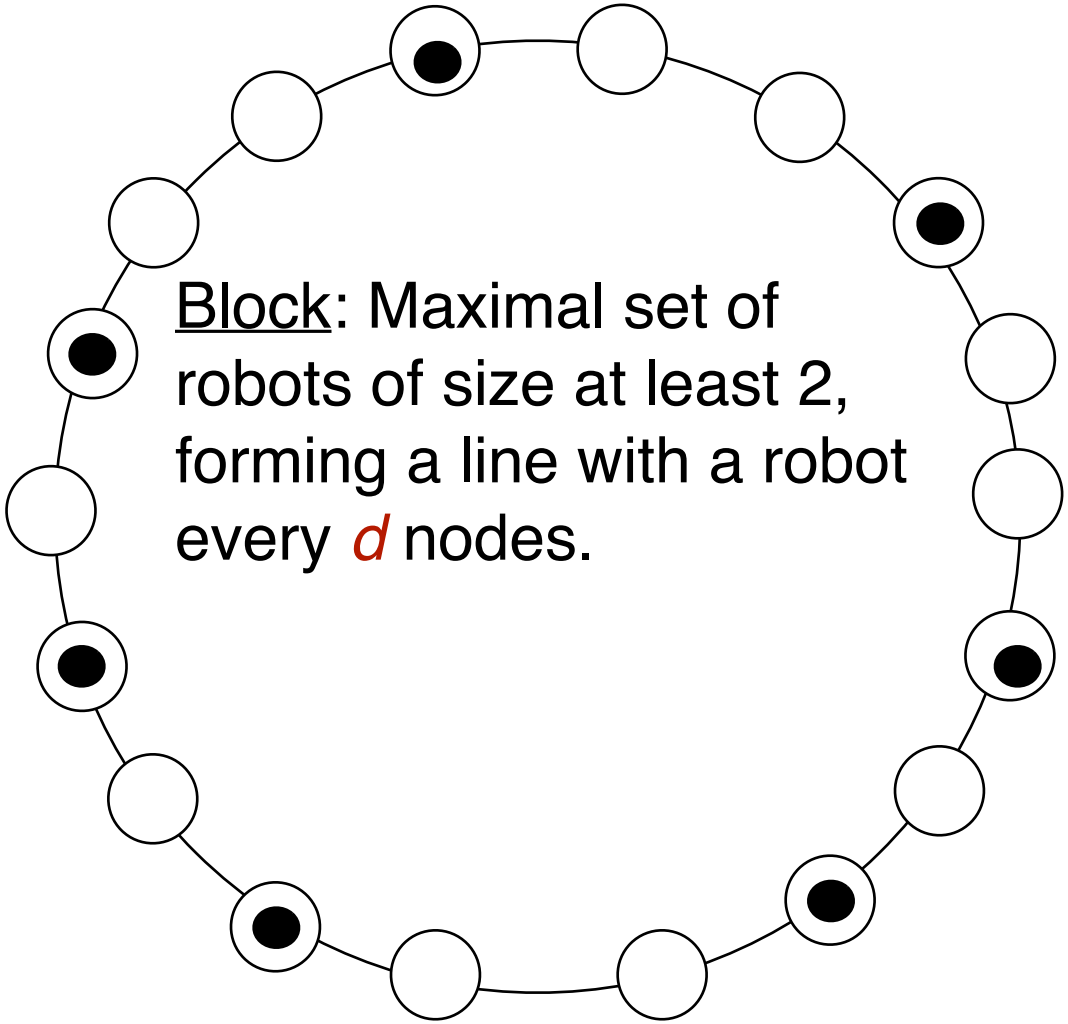

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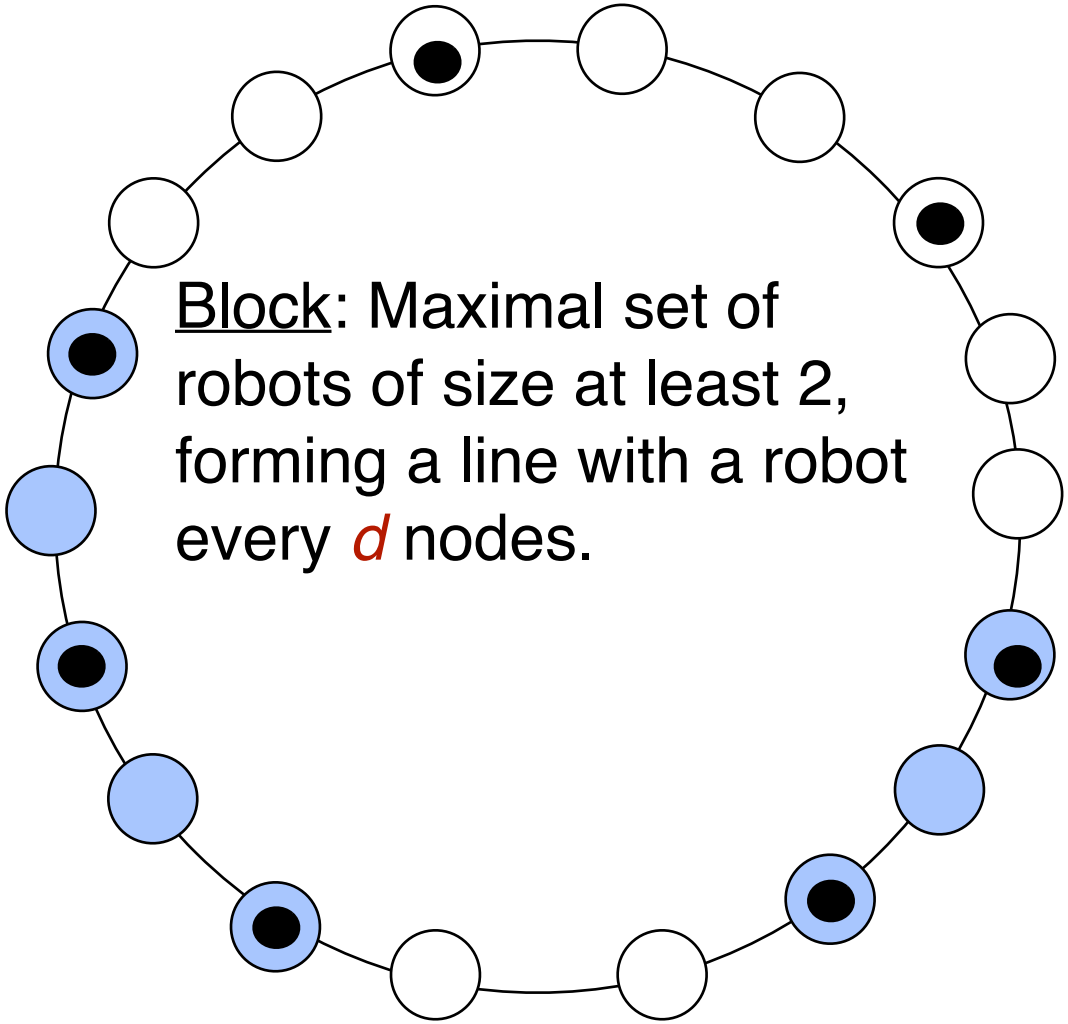

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  Method: Decrease the number of blocks whenever possible. Otherwise, decrease the interdistance.

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- Tower Phase:
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- Exploration Phase:
  Goal: Perform exploration until reaching an identified final configuration.

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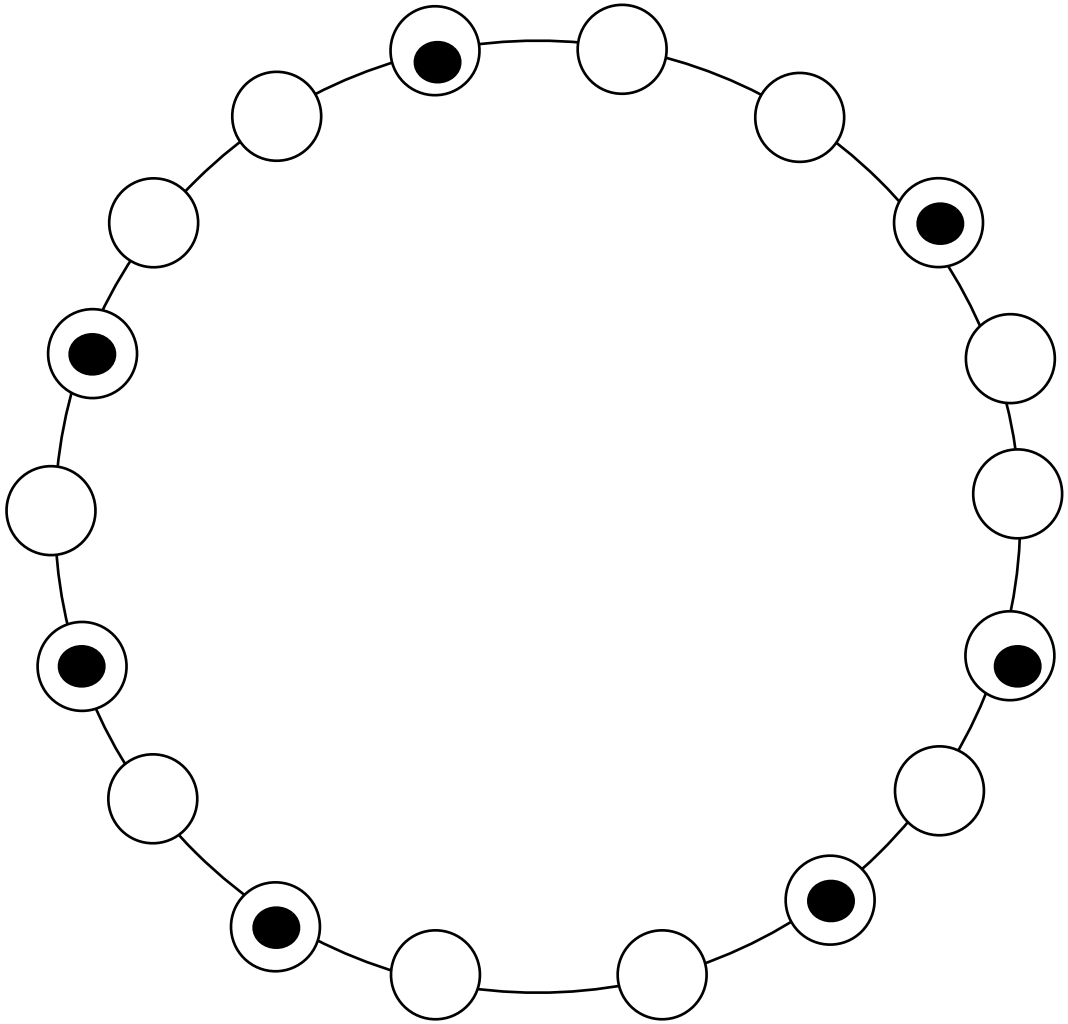

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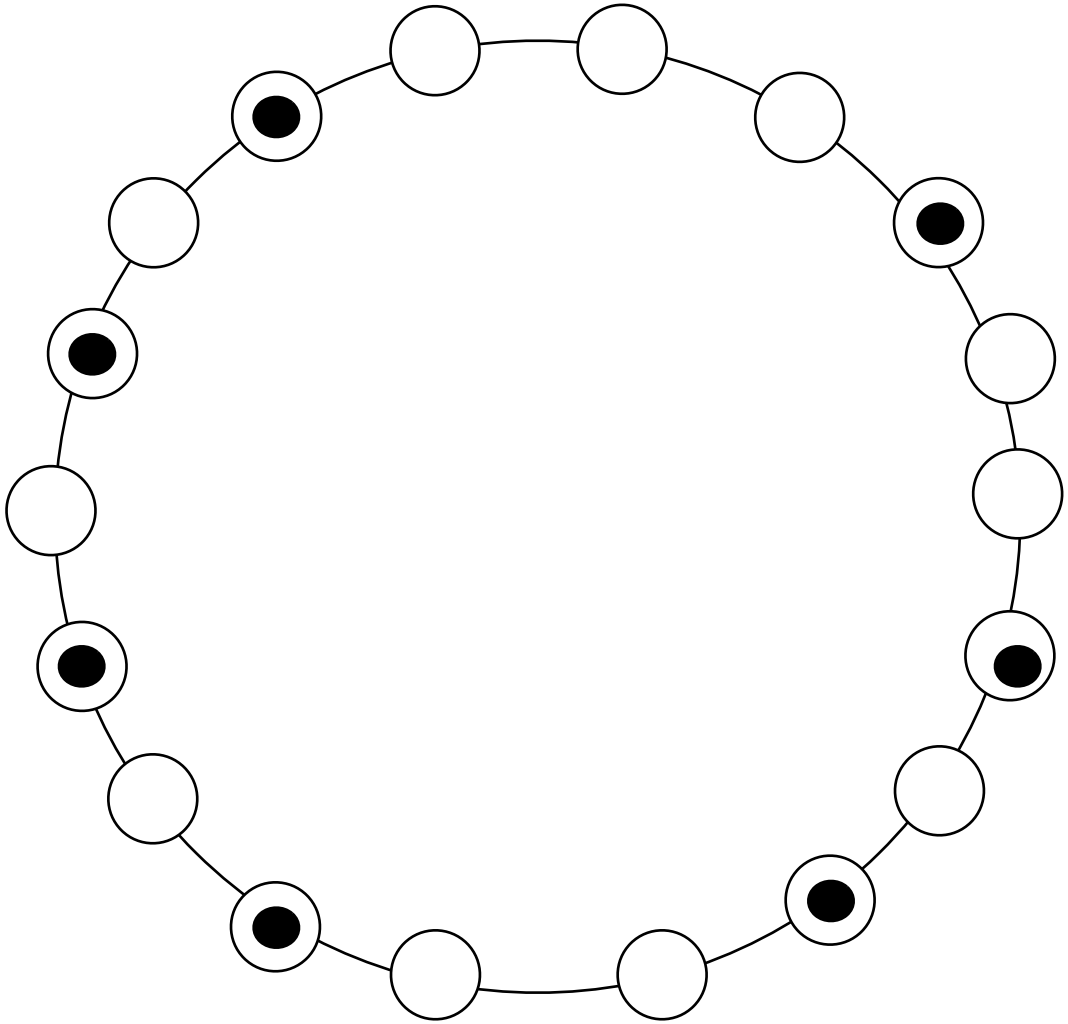

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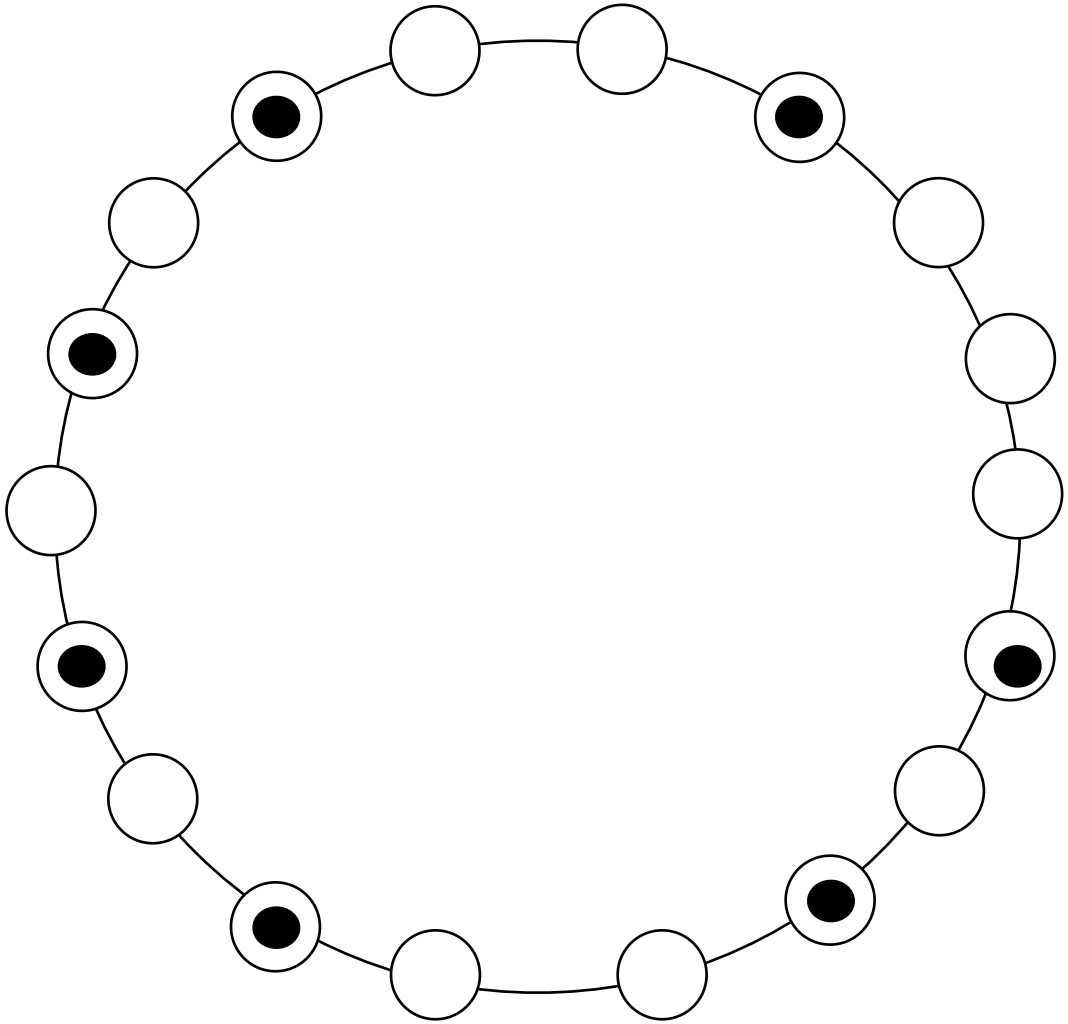

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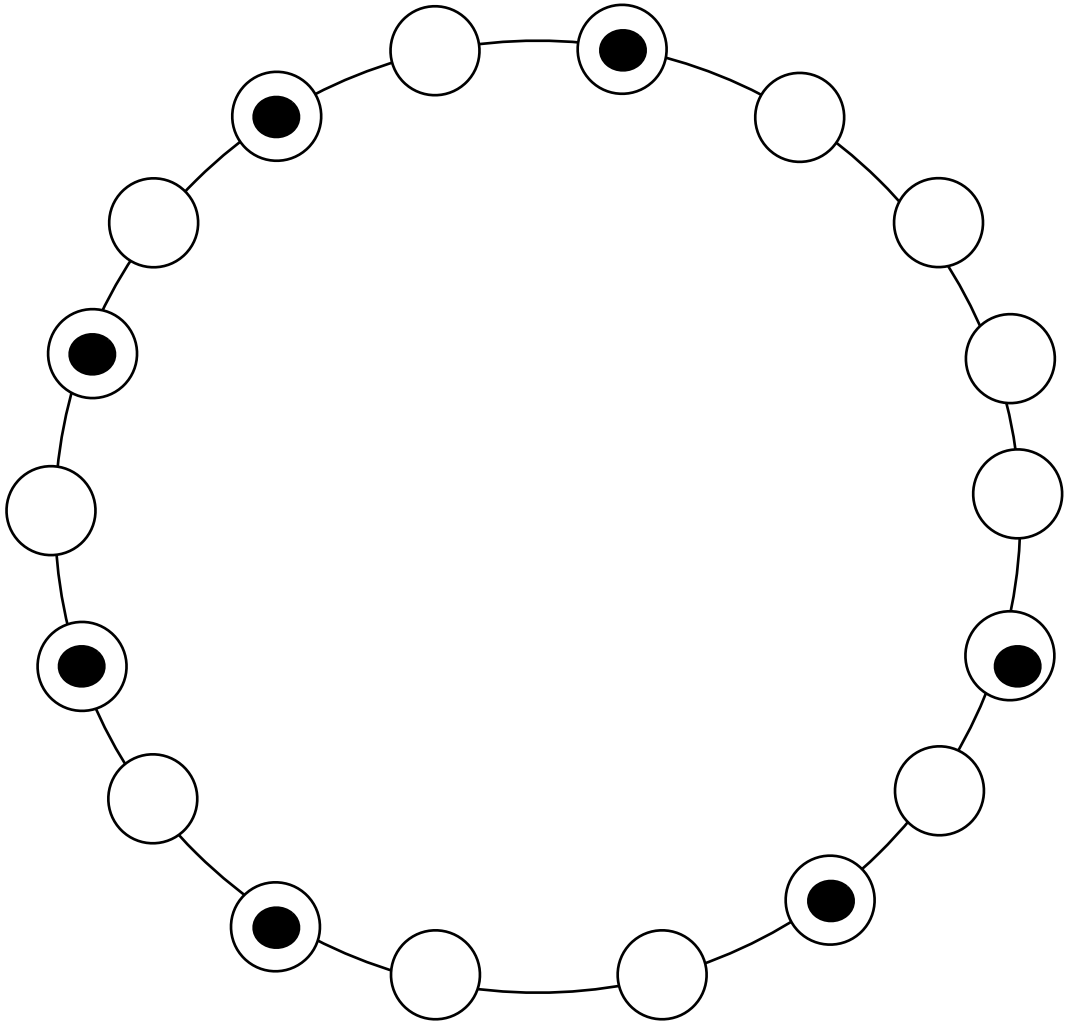

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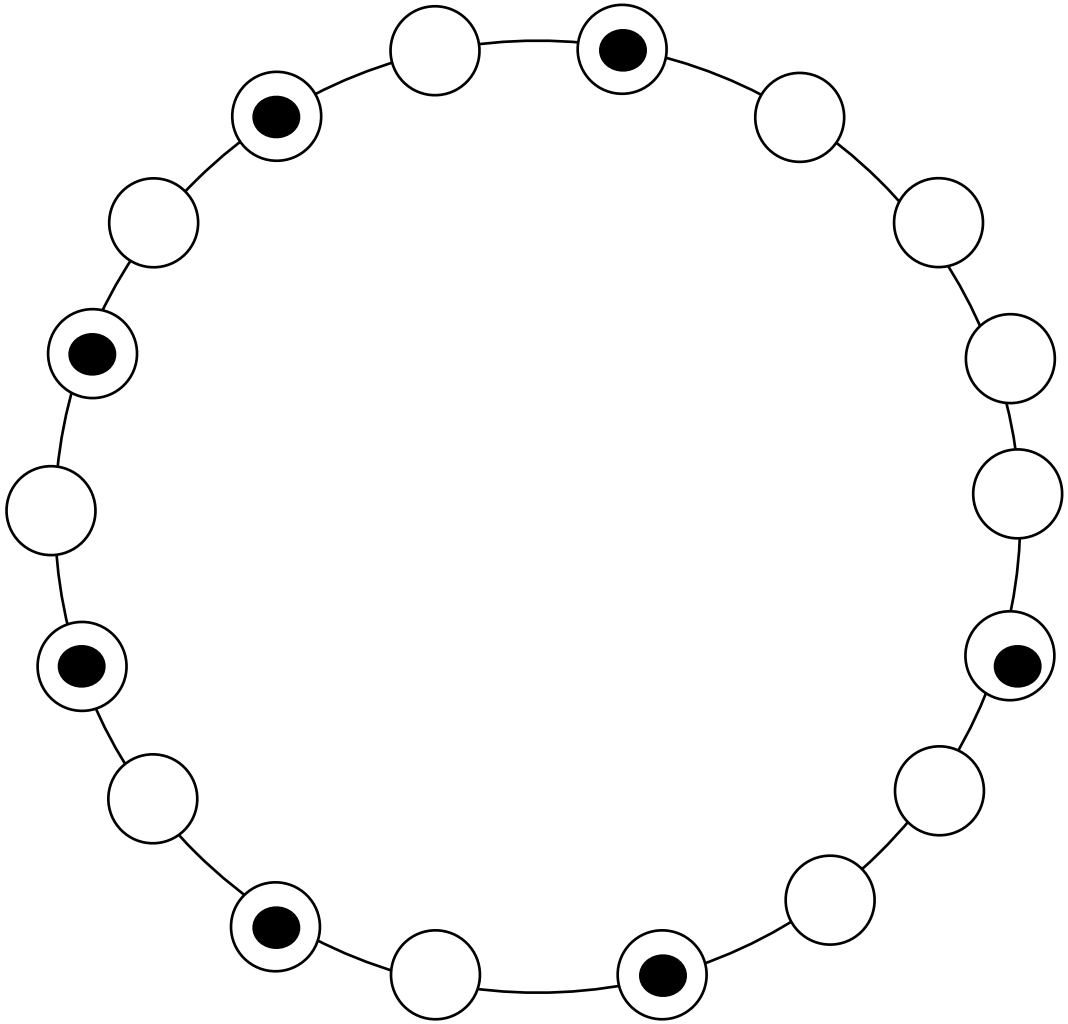

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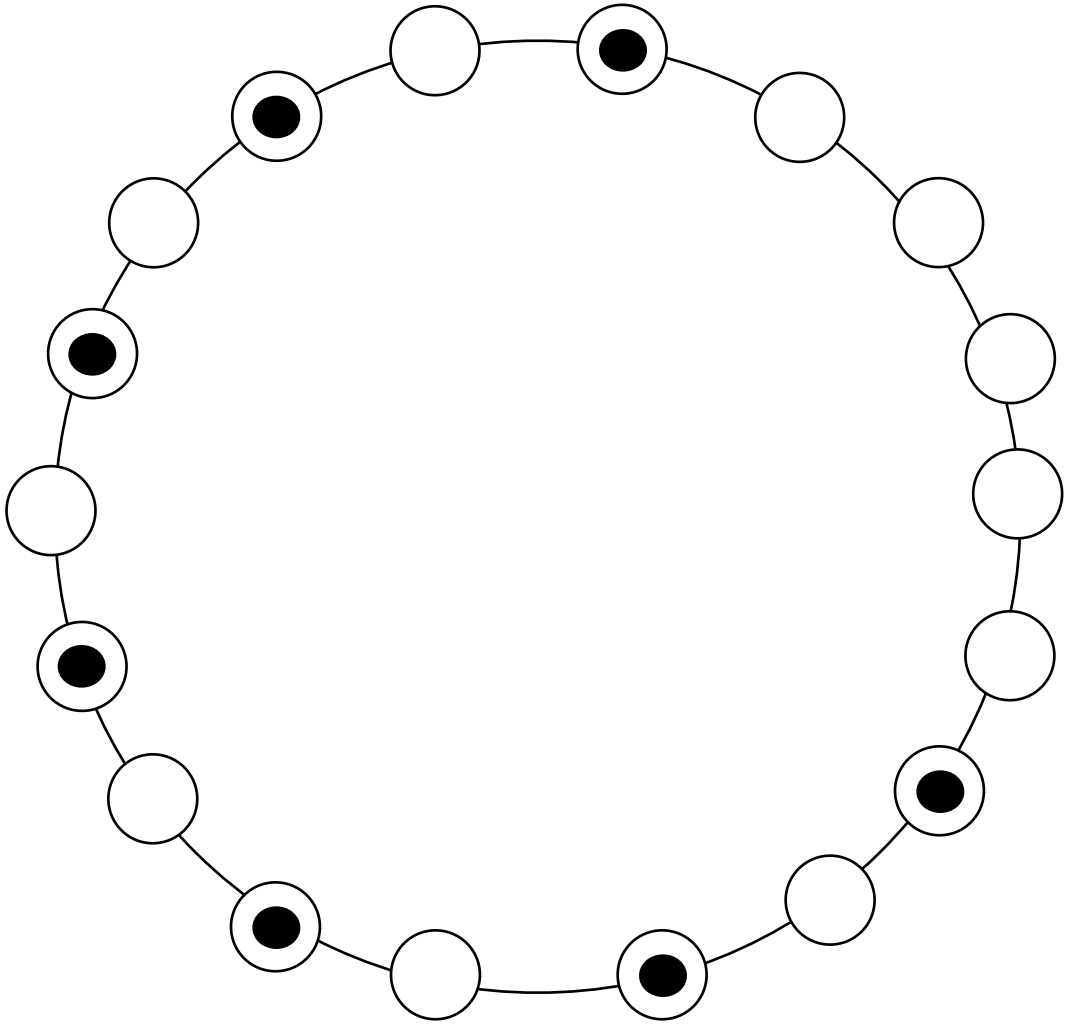

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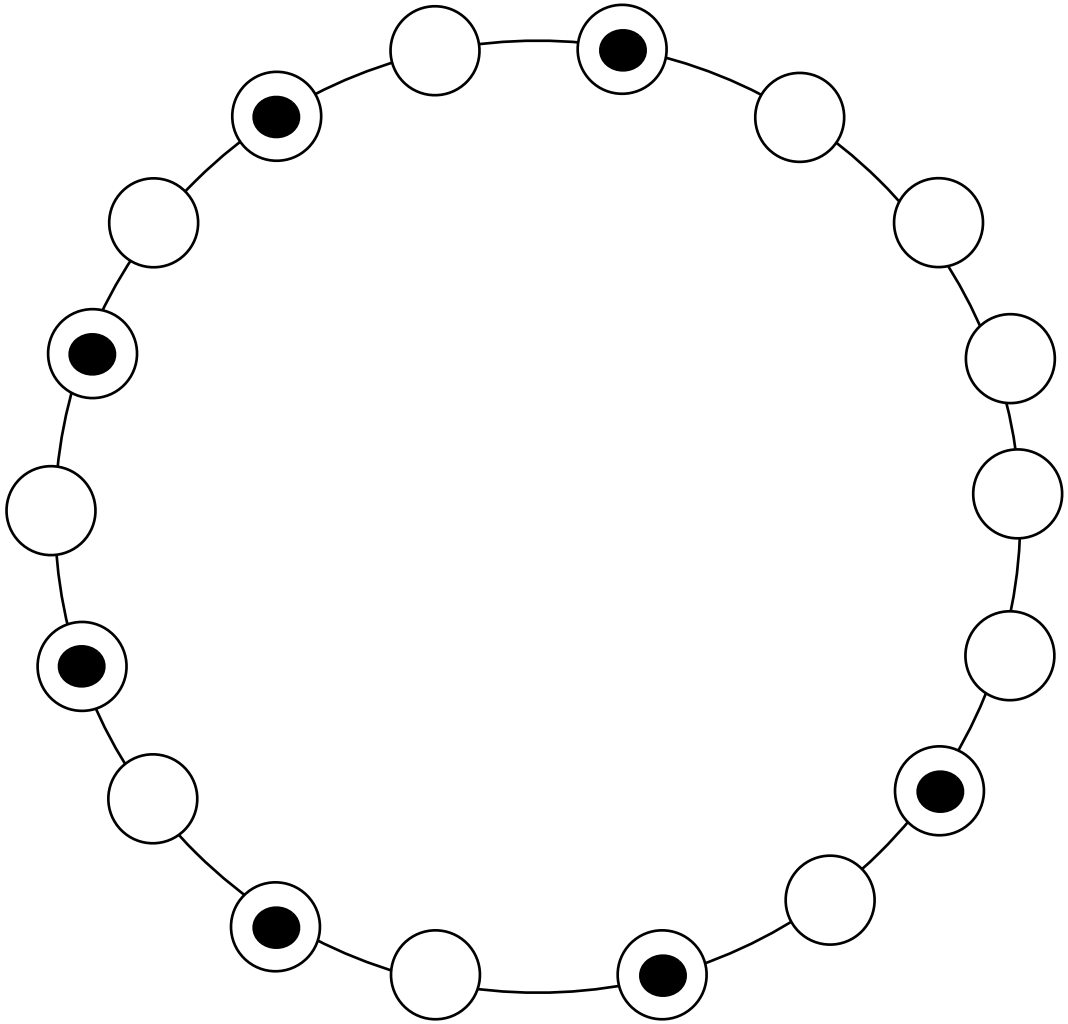

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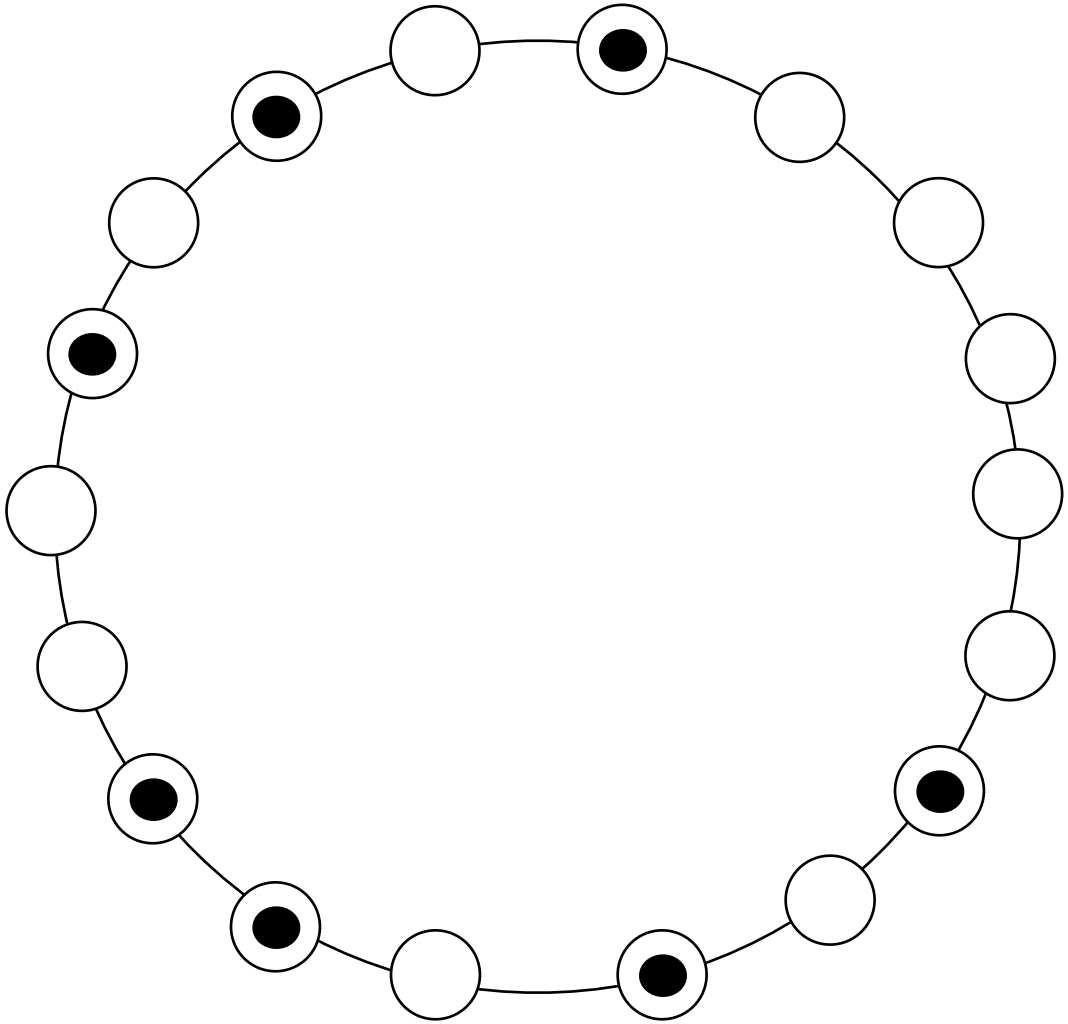

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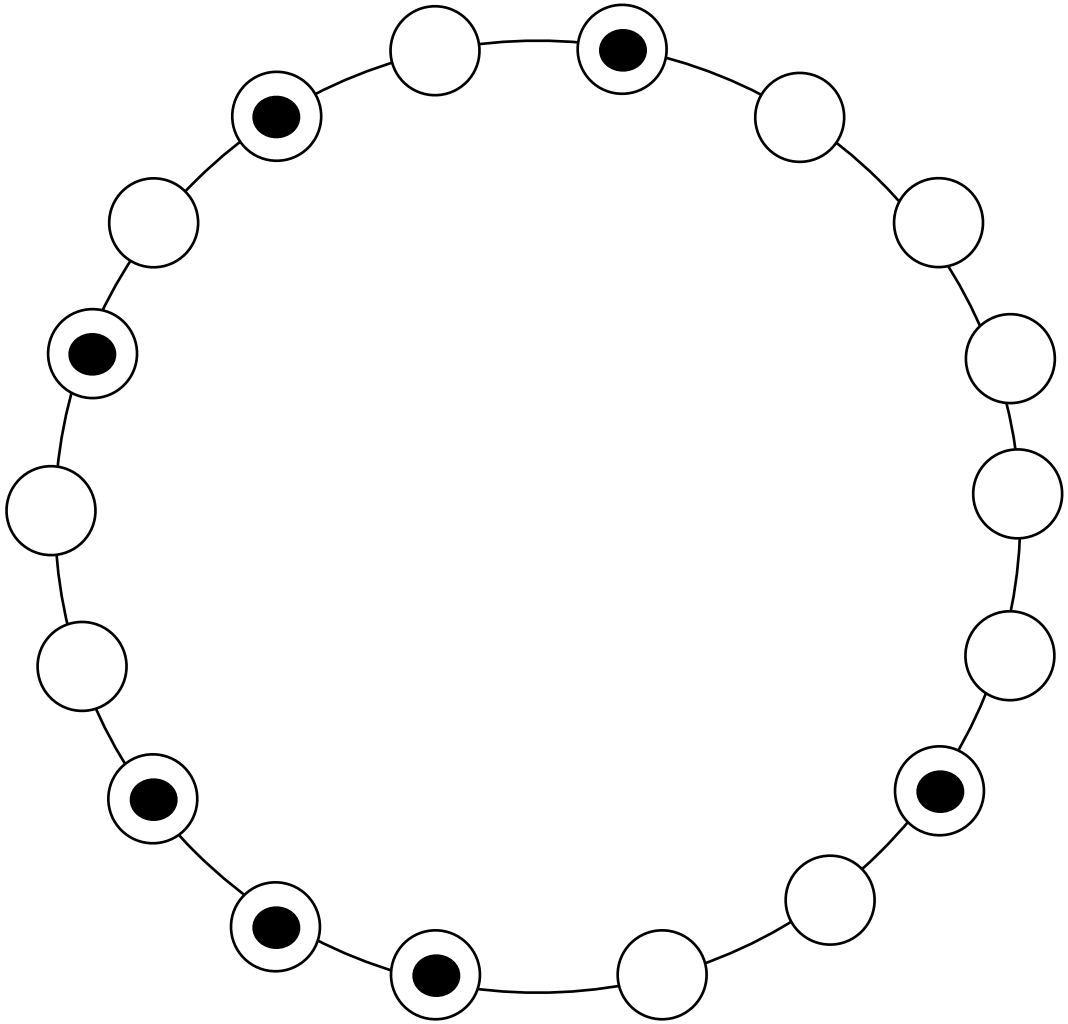

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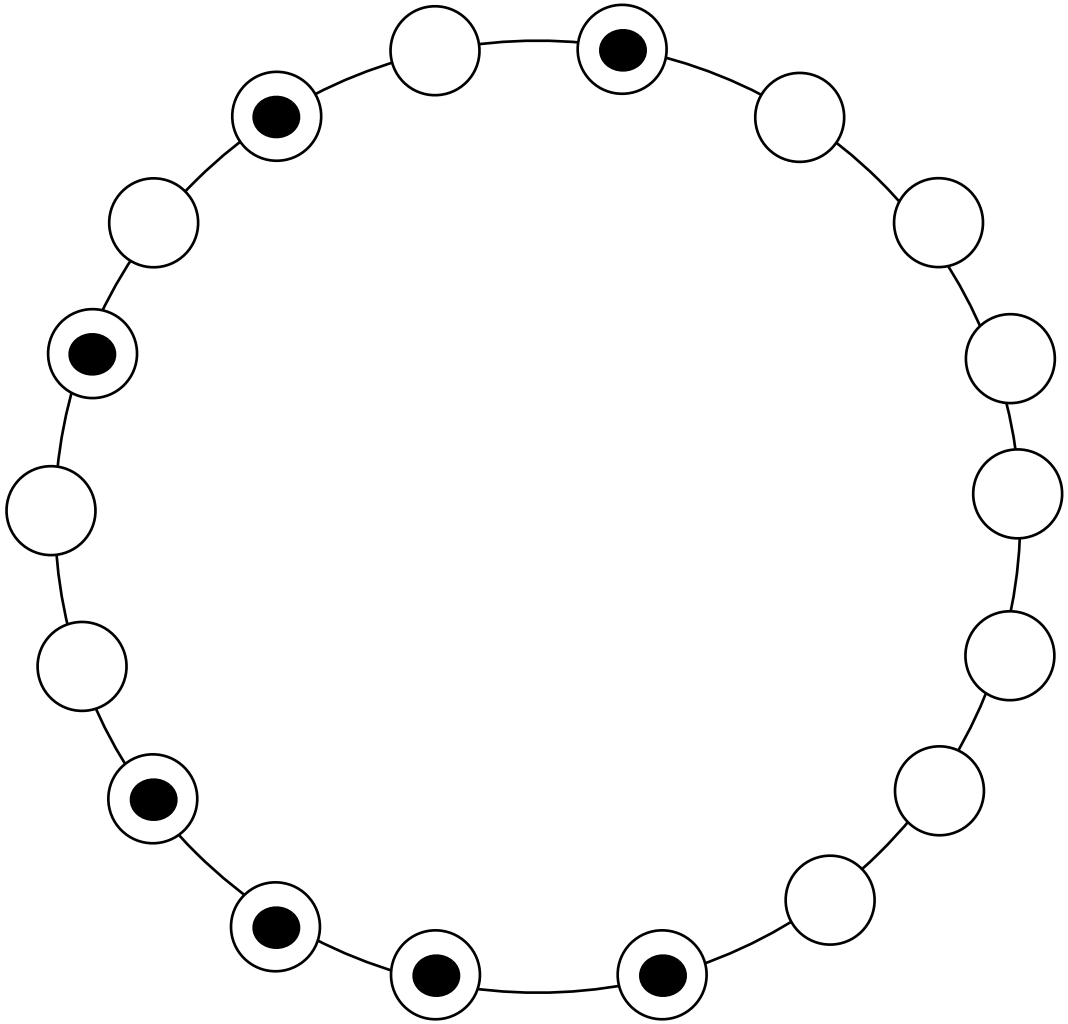

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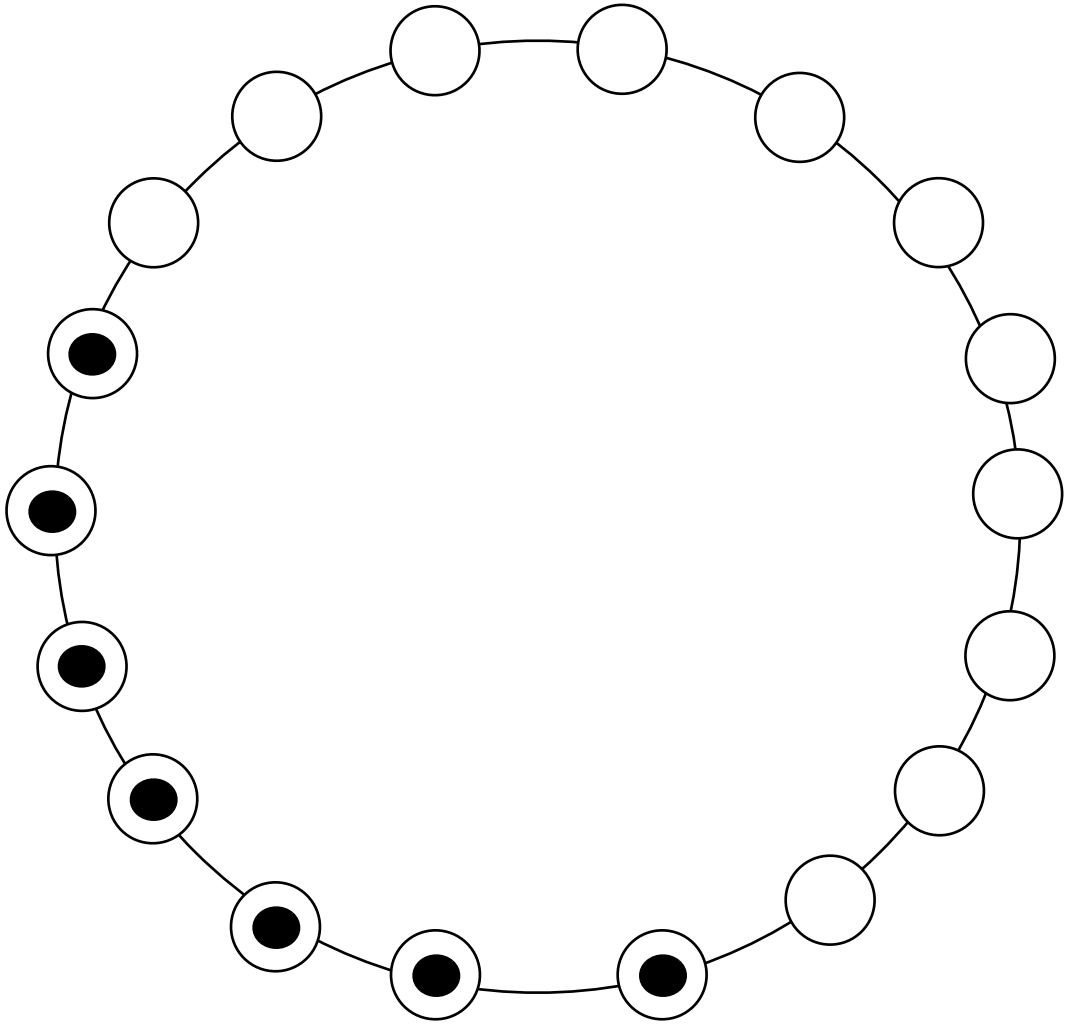

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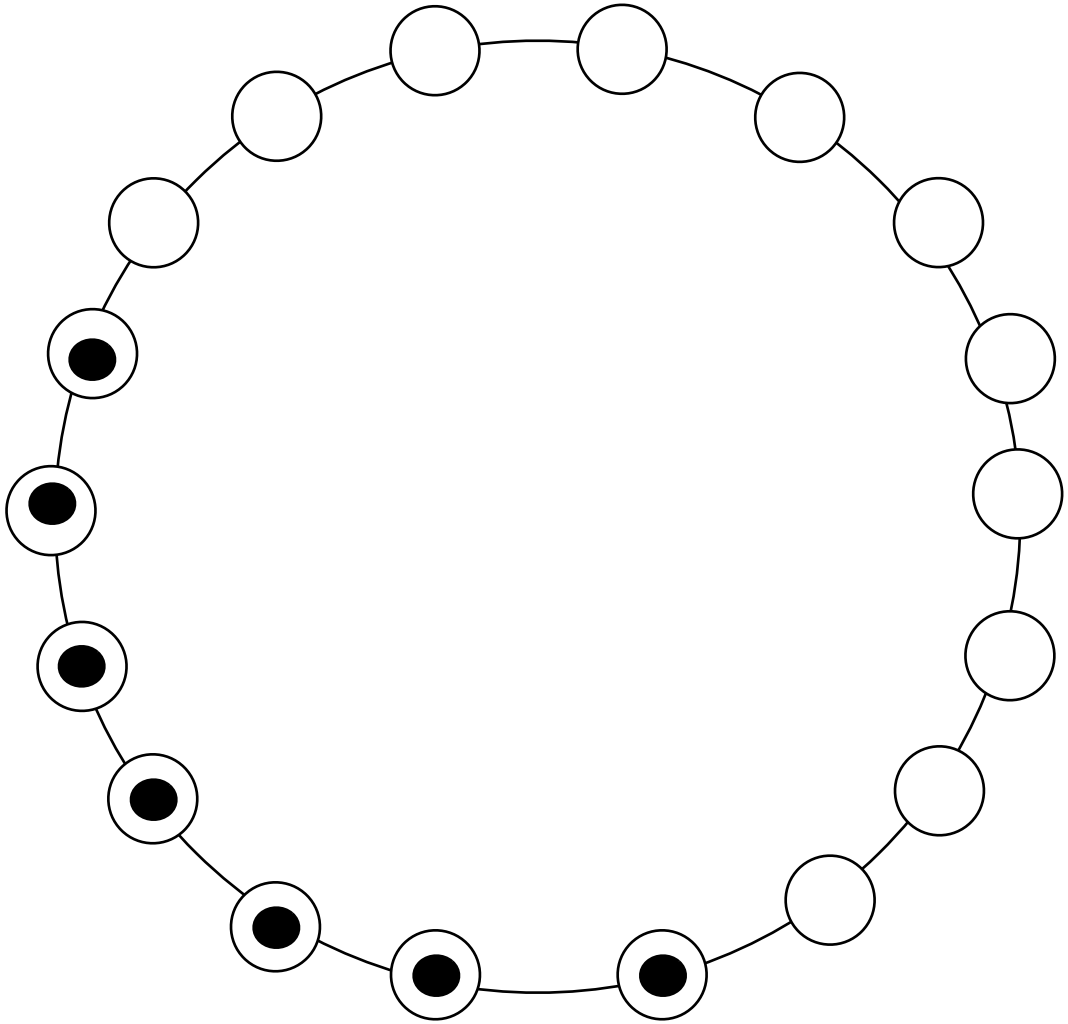

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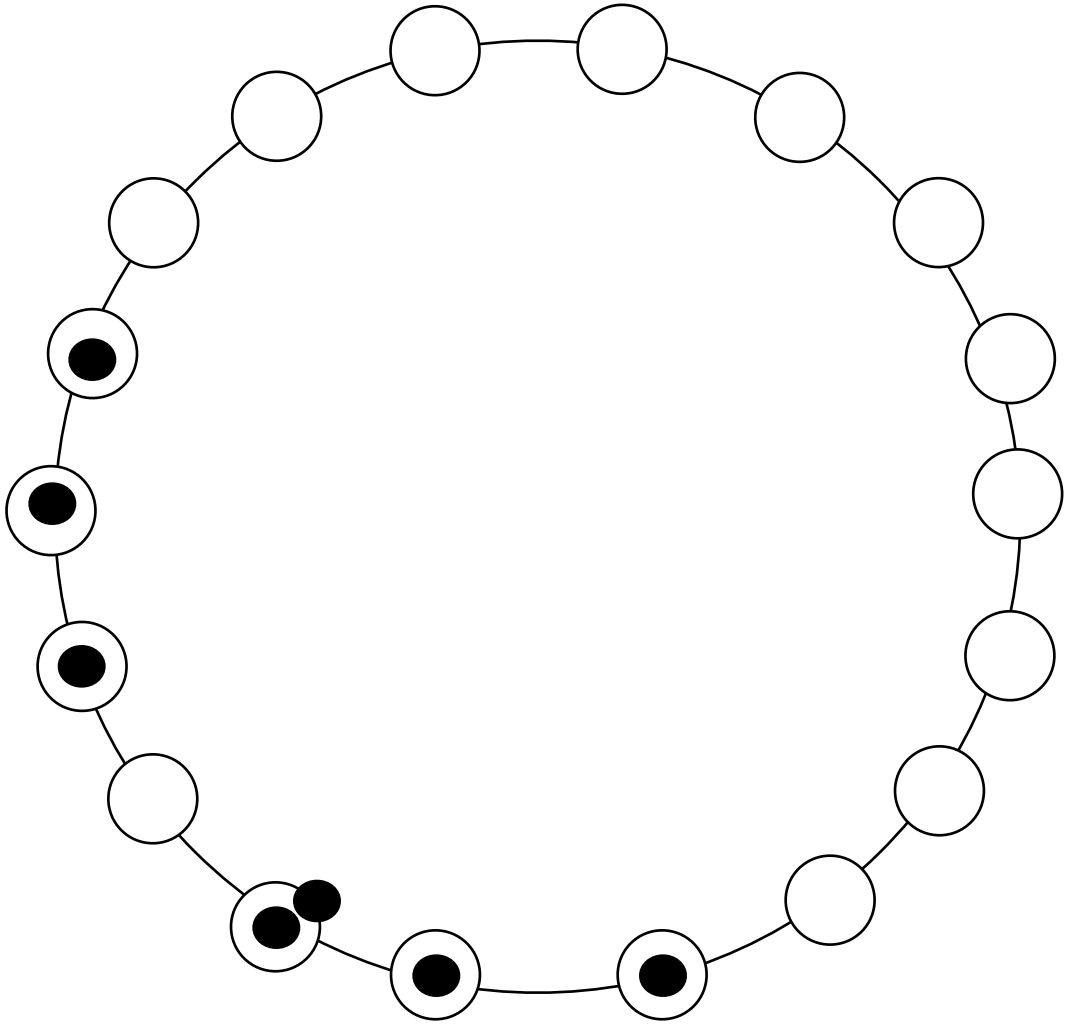

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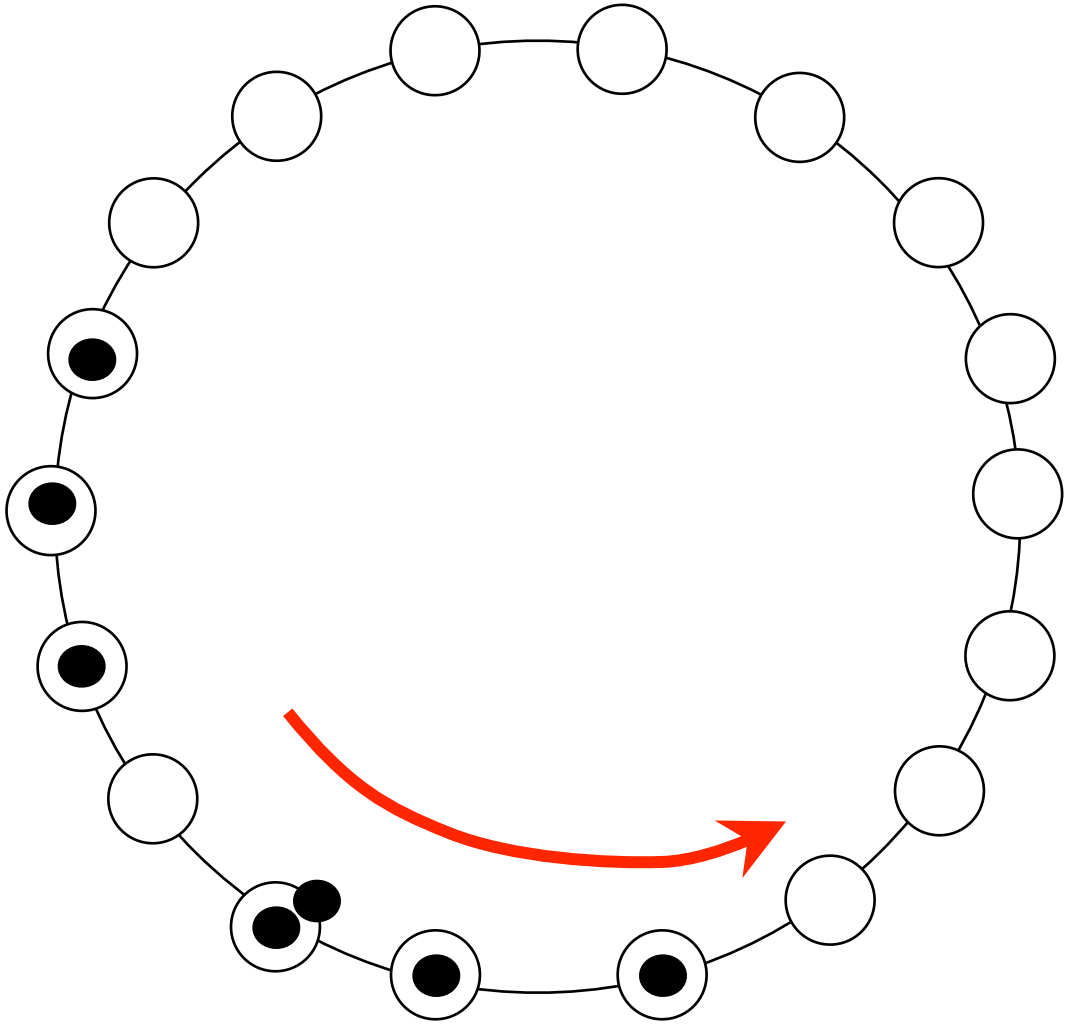

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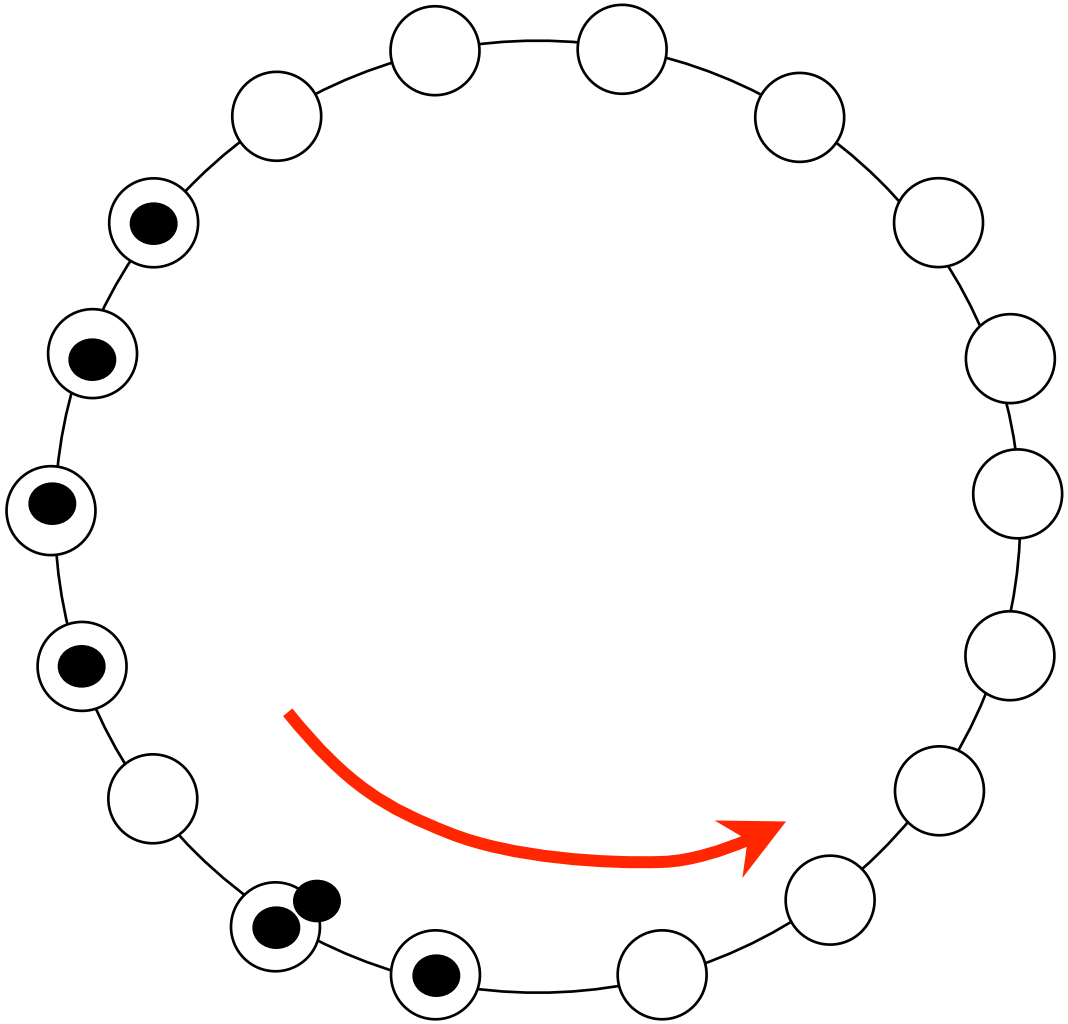

Optimality

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- Minimal Number of Robots?

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Theorem.
4 **probabilistic** robots are necessary and sufficient, provided that $n > 4$

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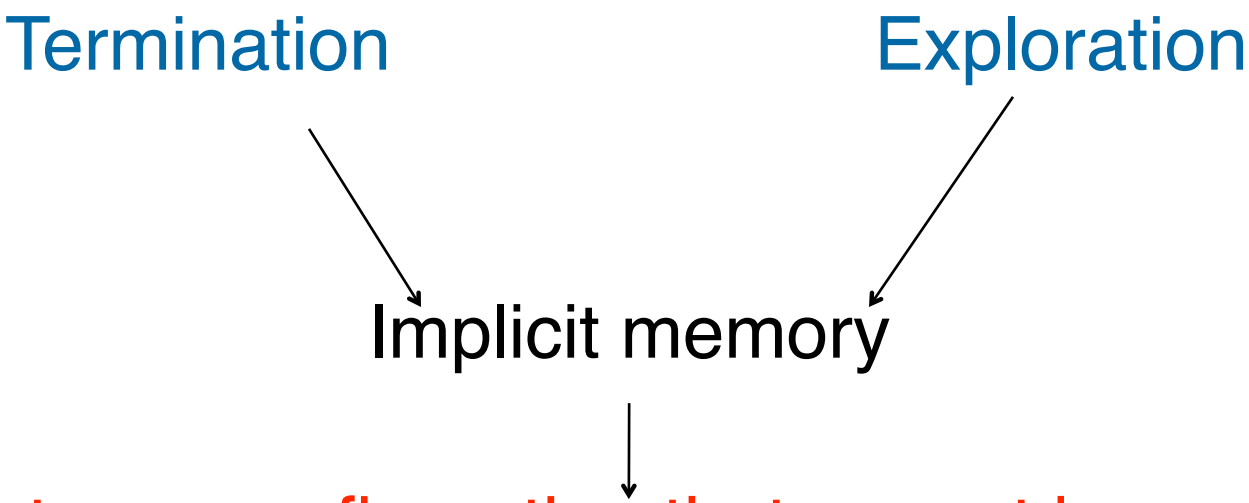
Oblivious Robots

Termination

Exploration

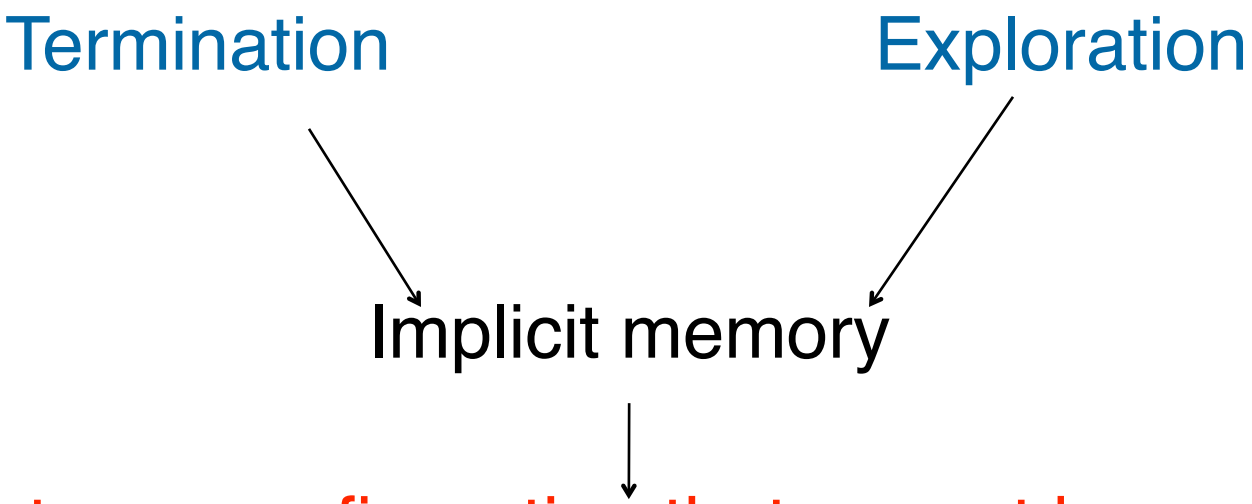
Oblivious Robots

Oblivious Robots



At least one configuration that cannot be an initial configuration

Oblivious Robots



Remark.
If $n > k$, any terminal configuration of any protocol contains at least one *tower*.

Tower

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A node with at least two robots.

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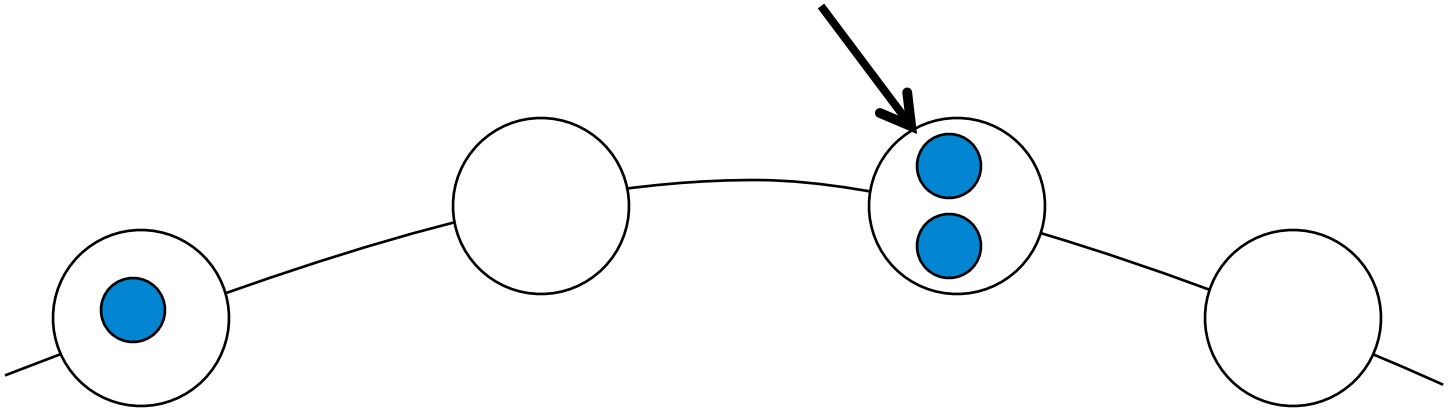

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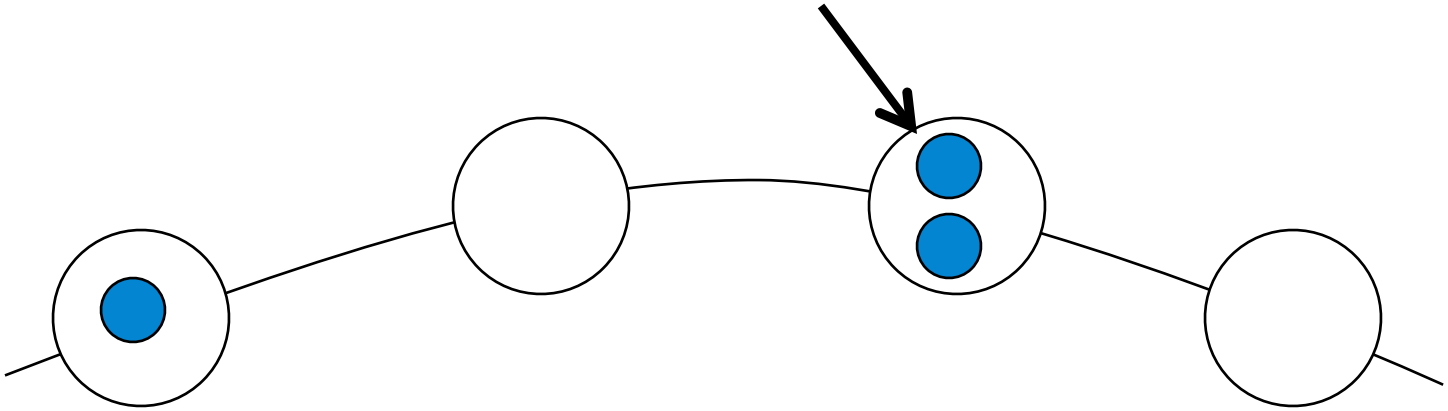

$k \geq 2$

Tower Building

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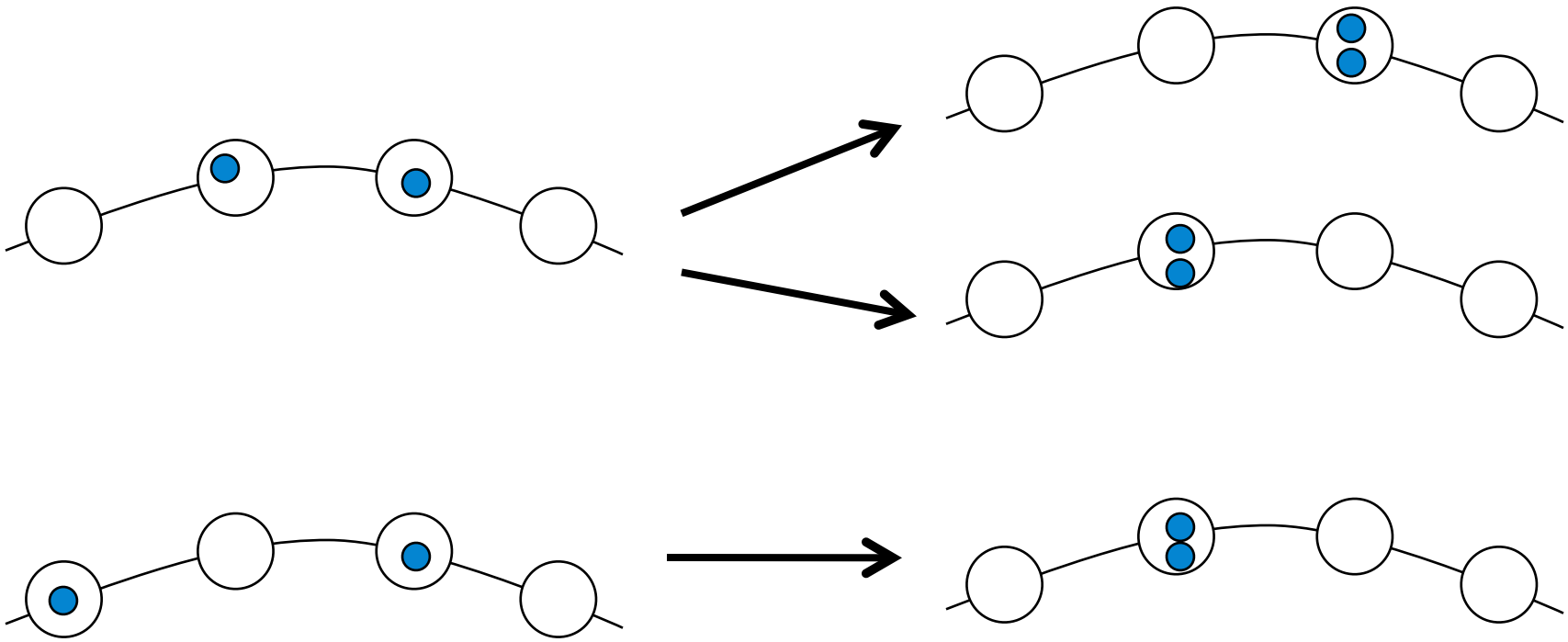

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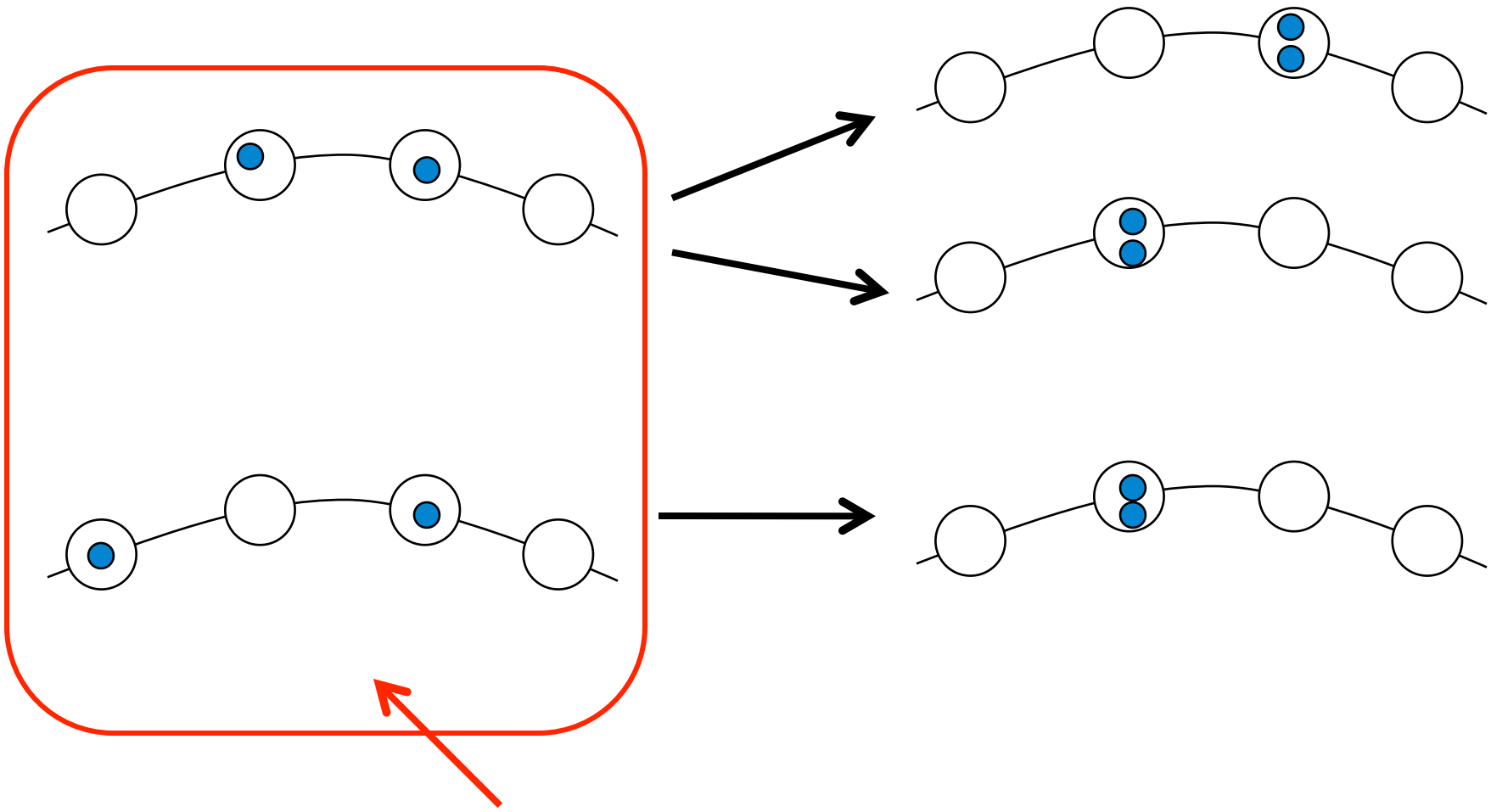
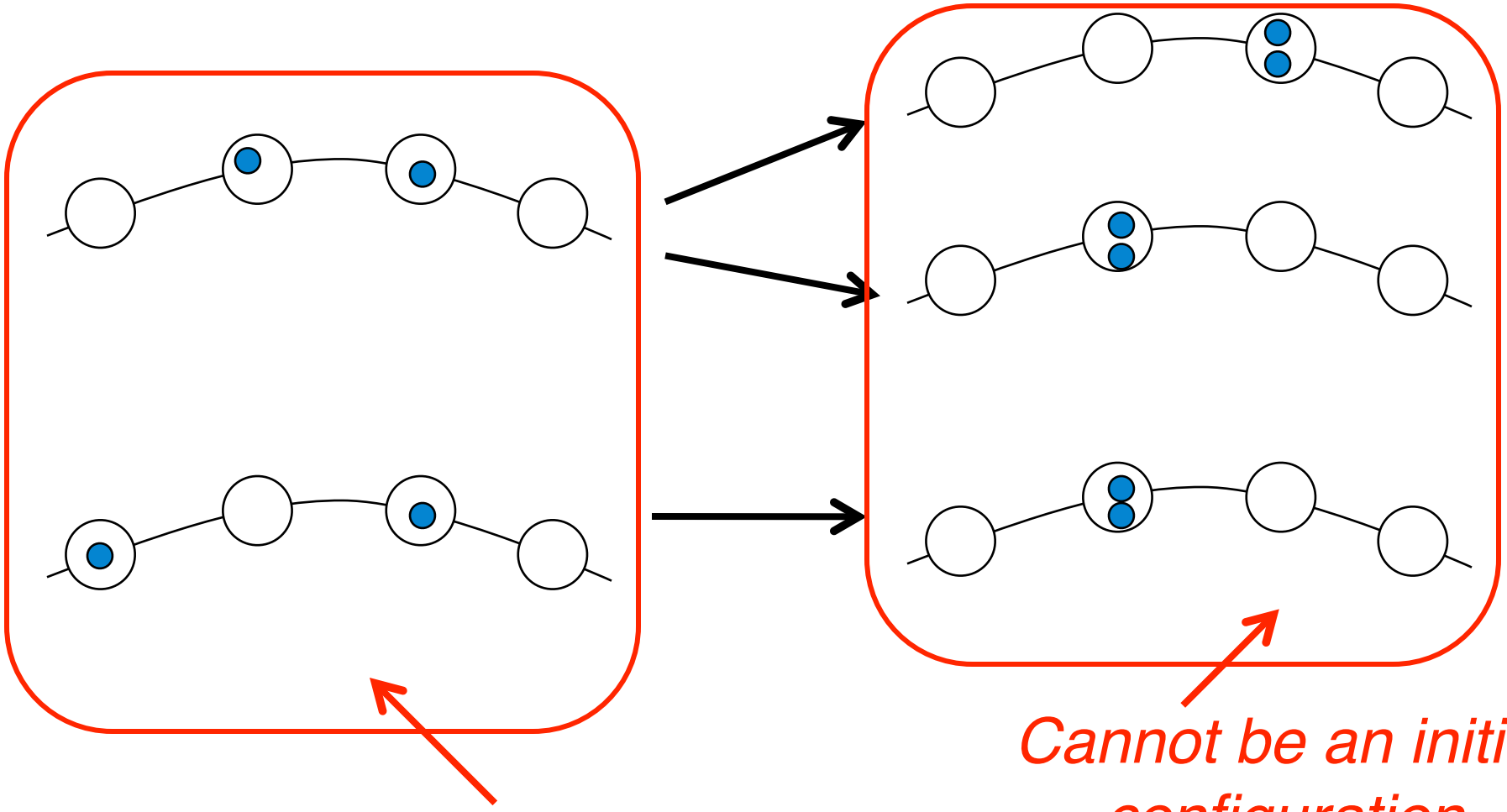
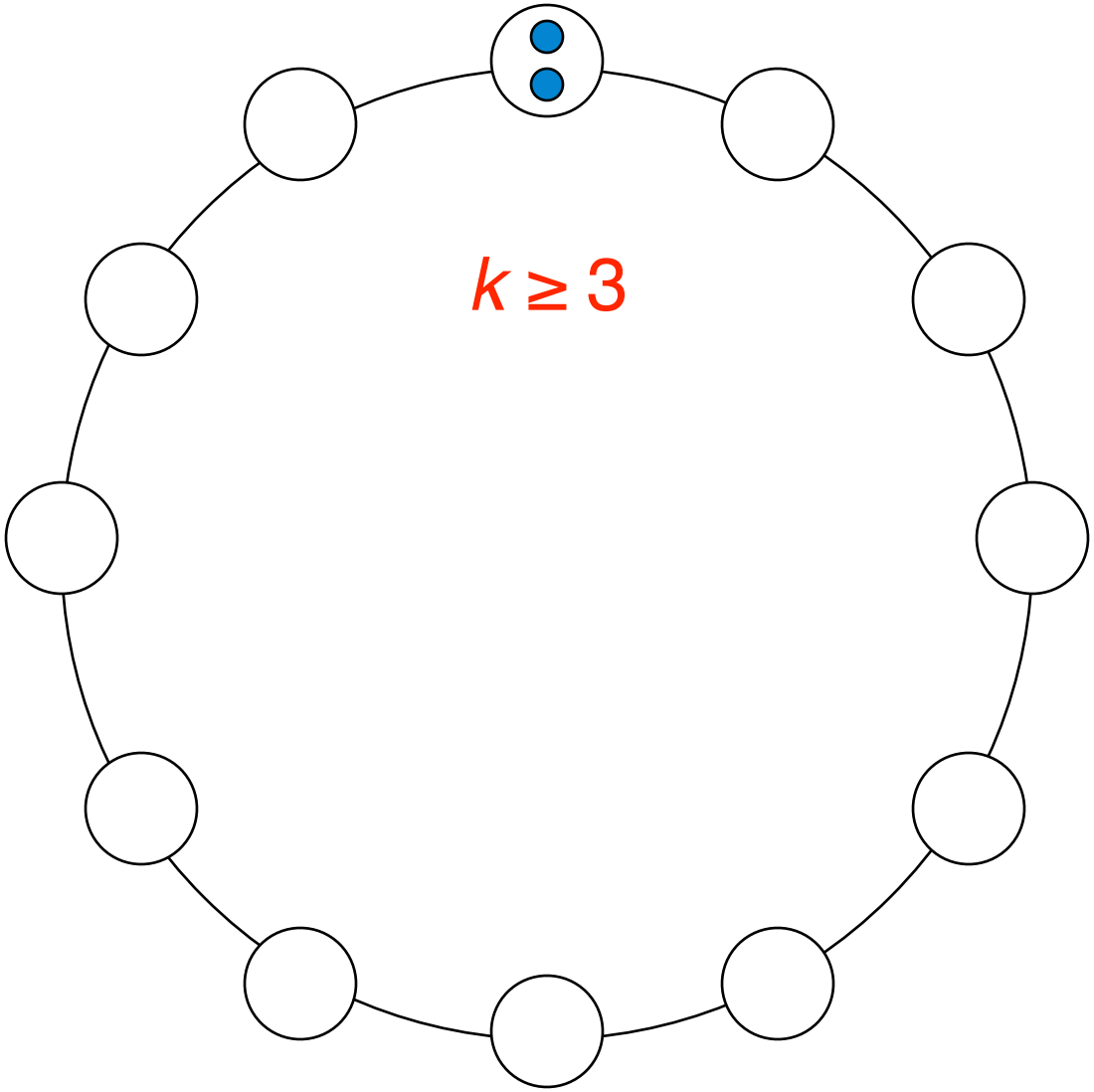


Can be an initial configuration

Tower Building



Enabling Exploration



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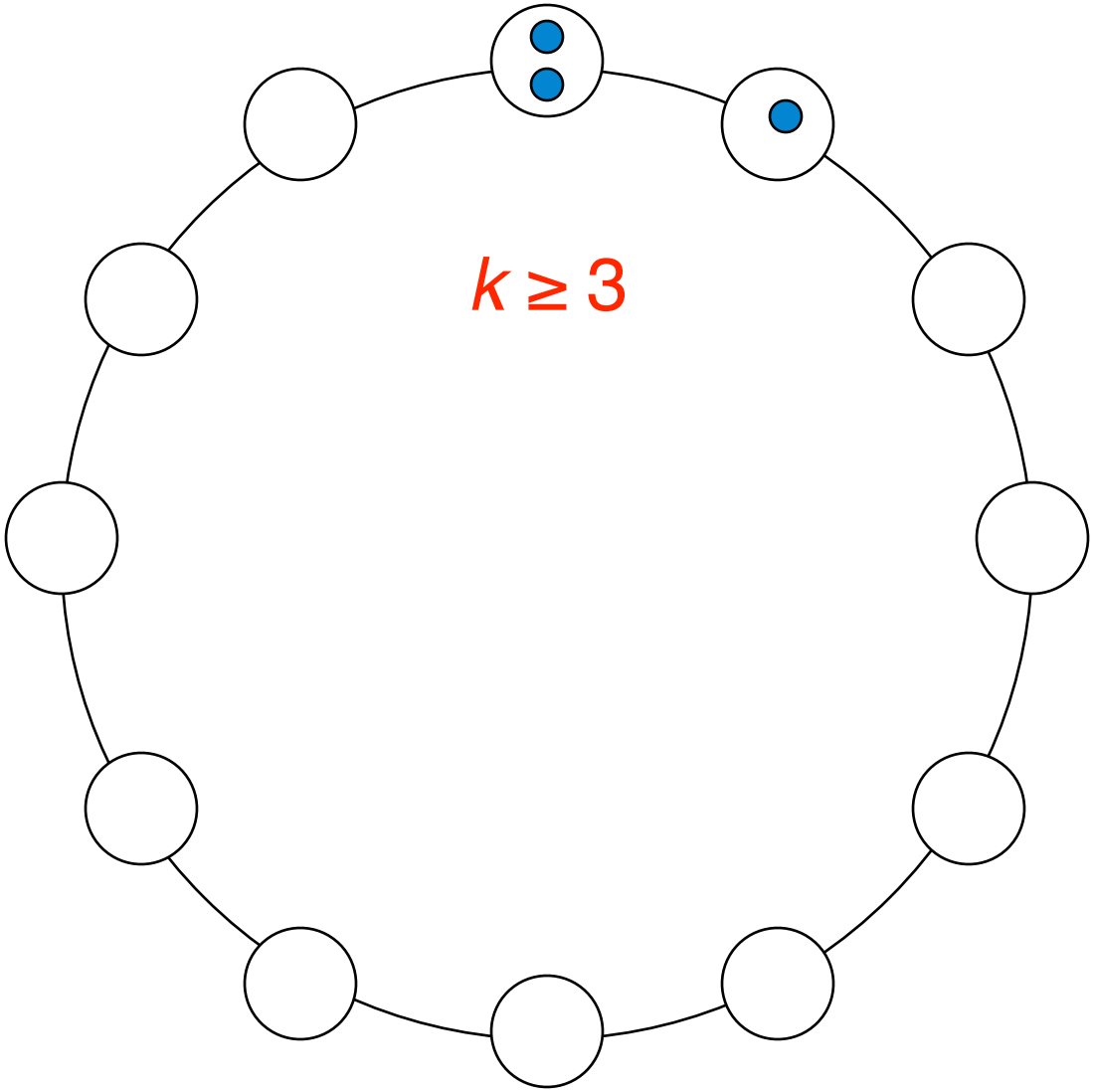

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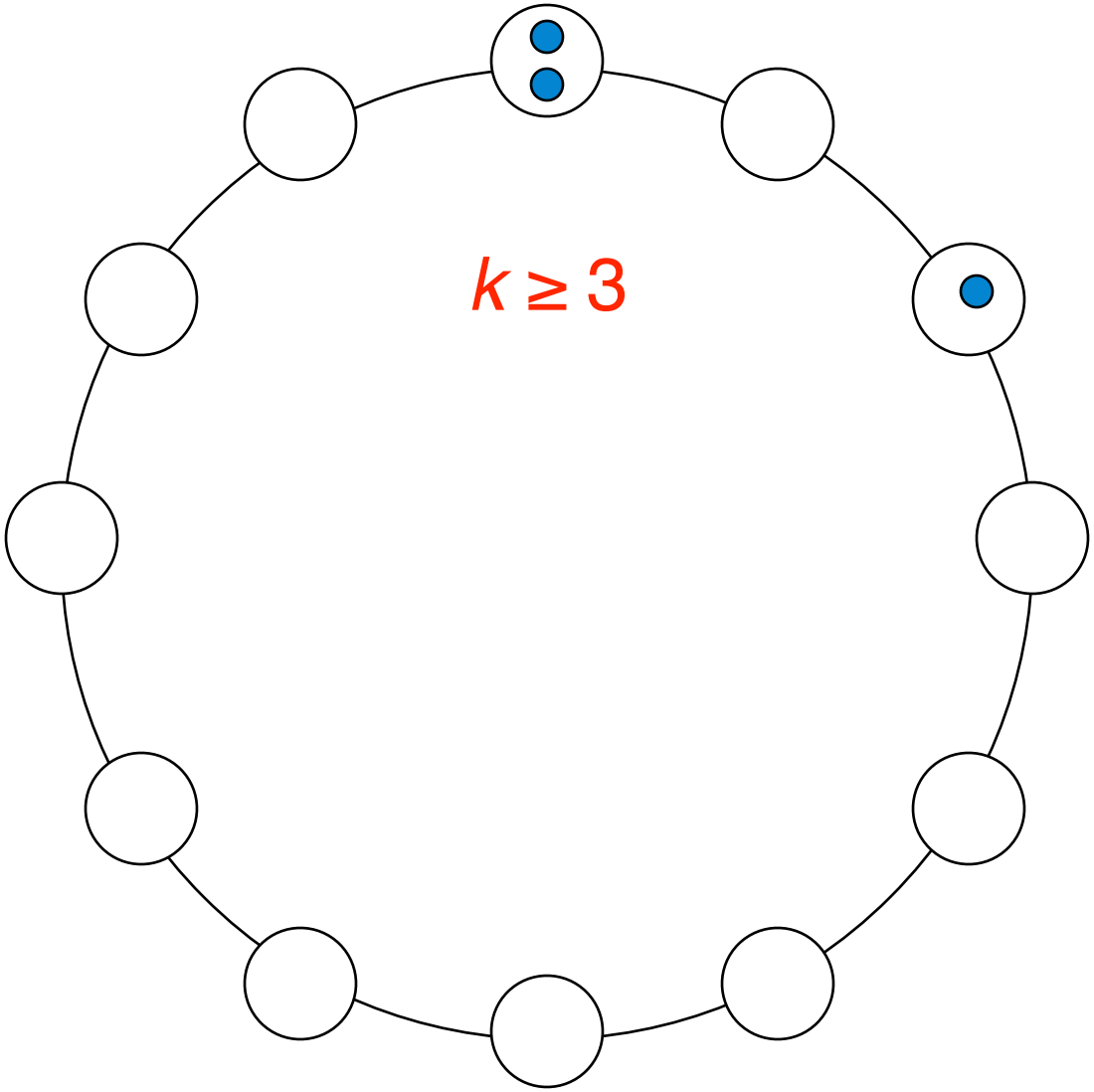

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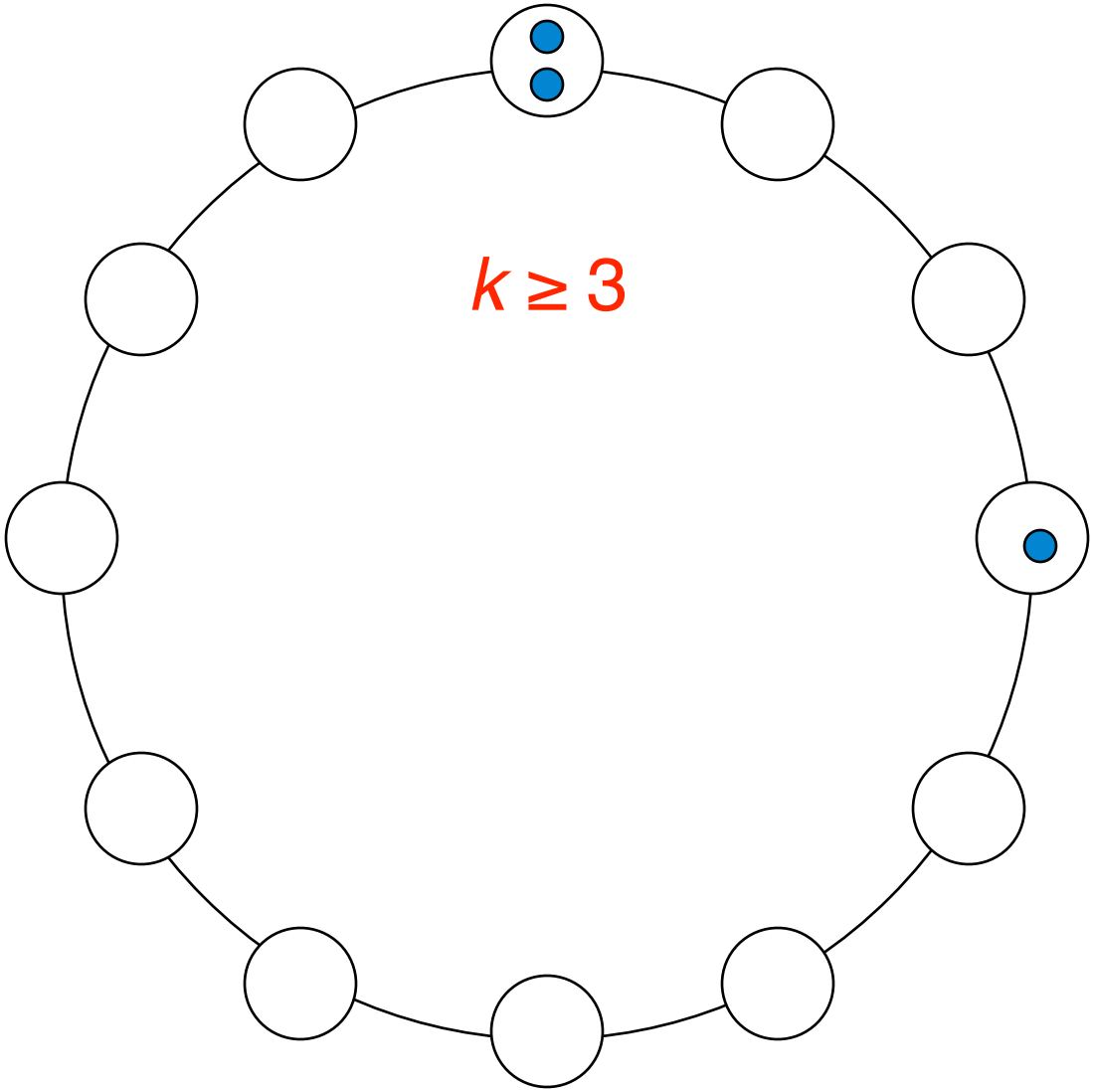

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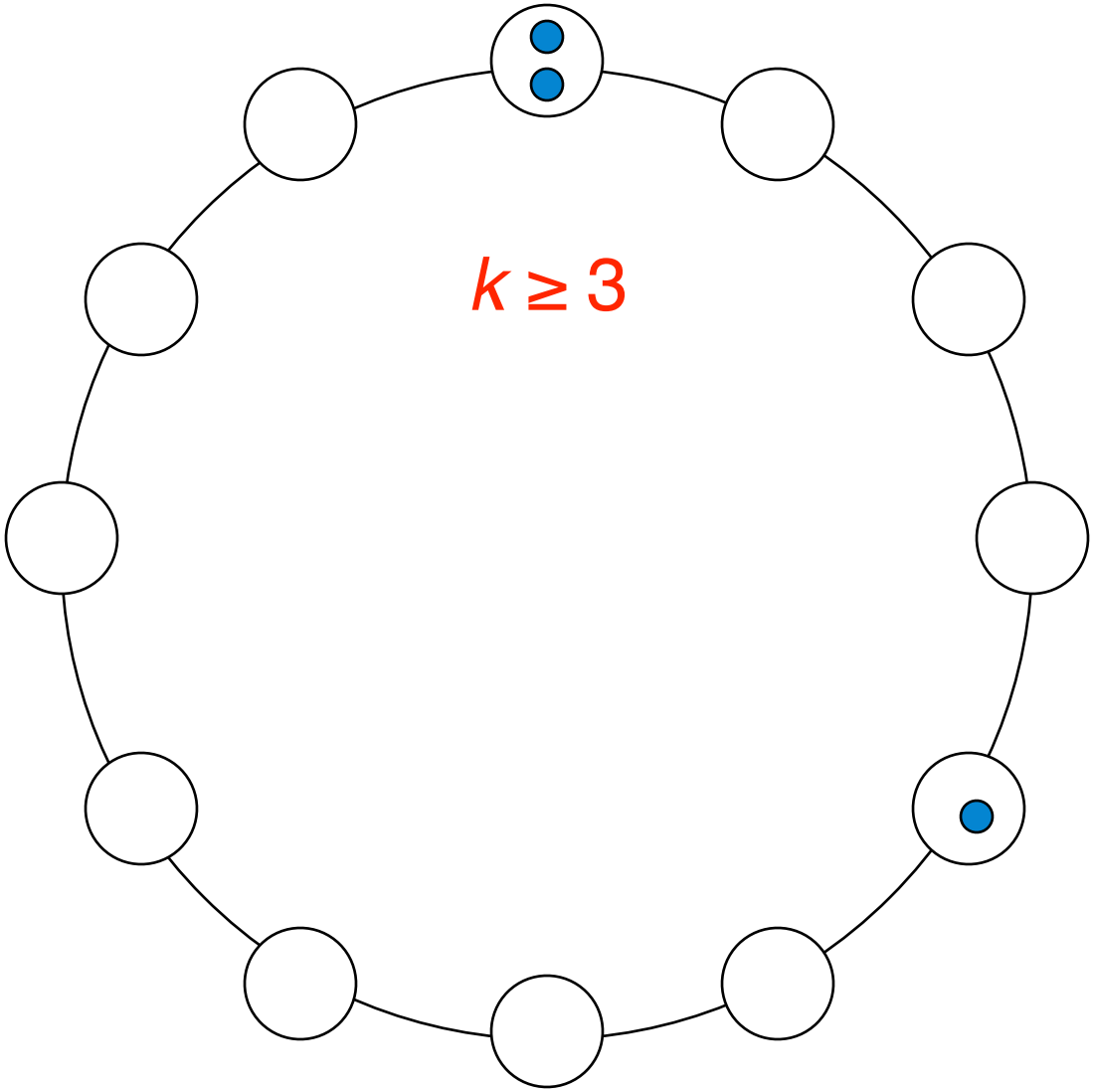

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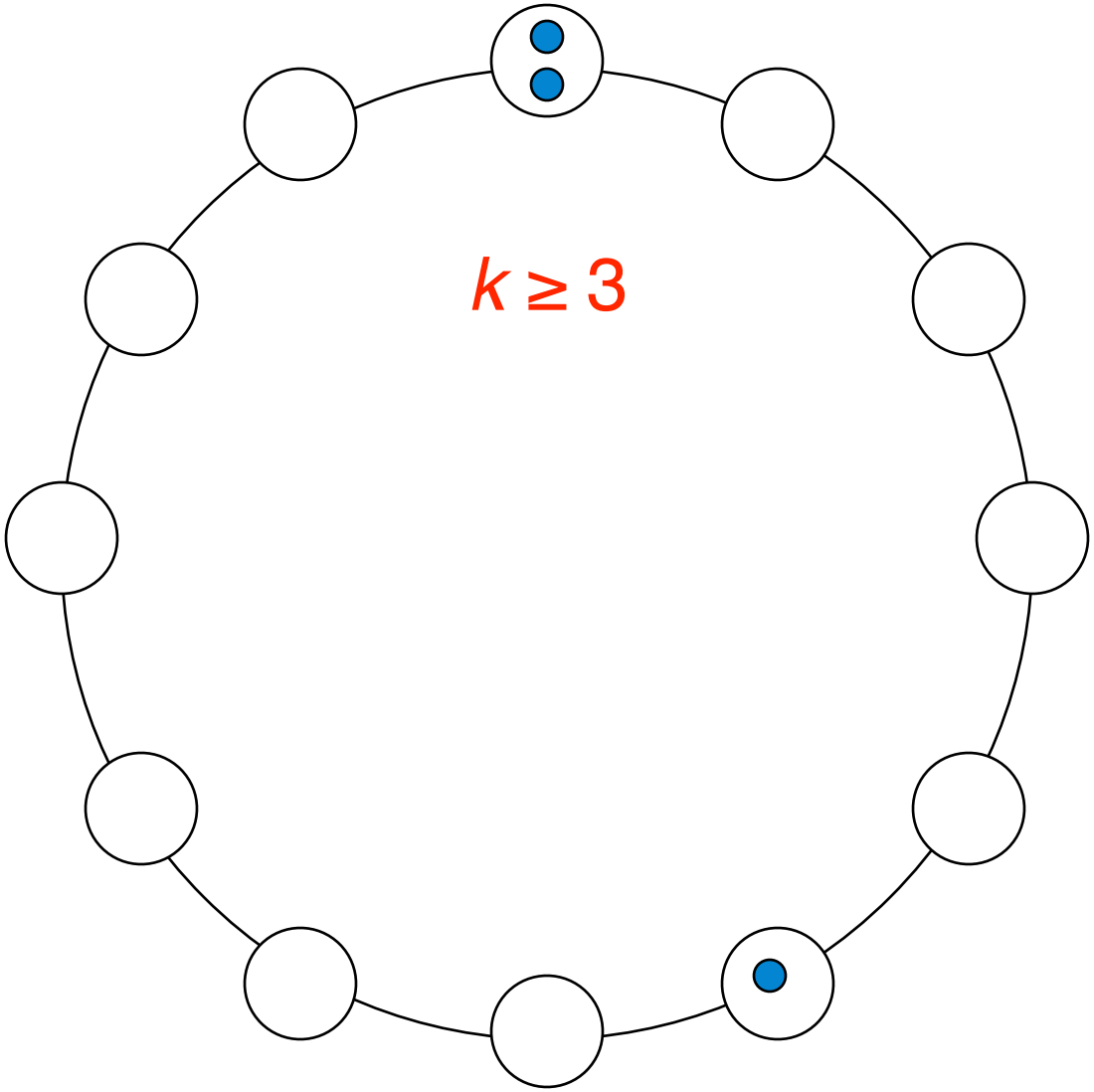

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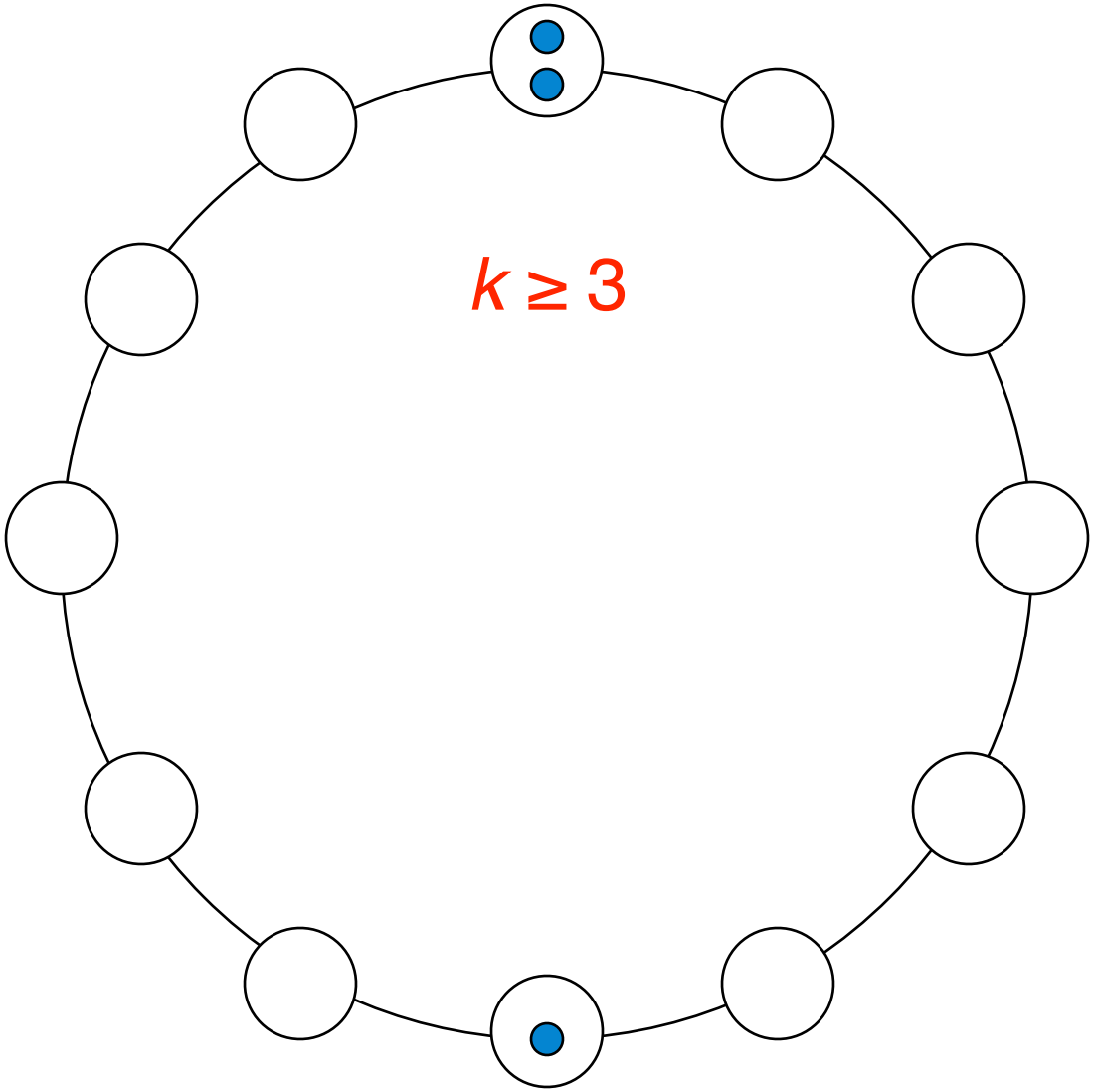

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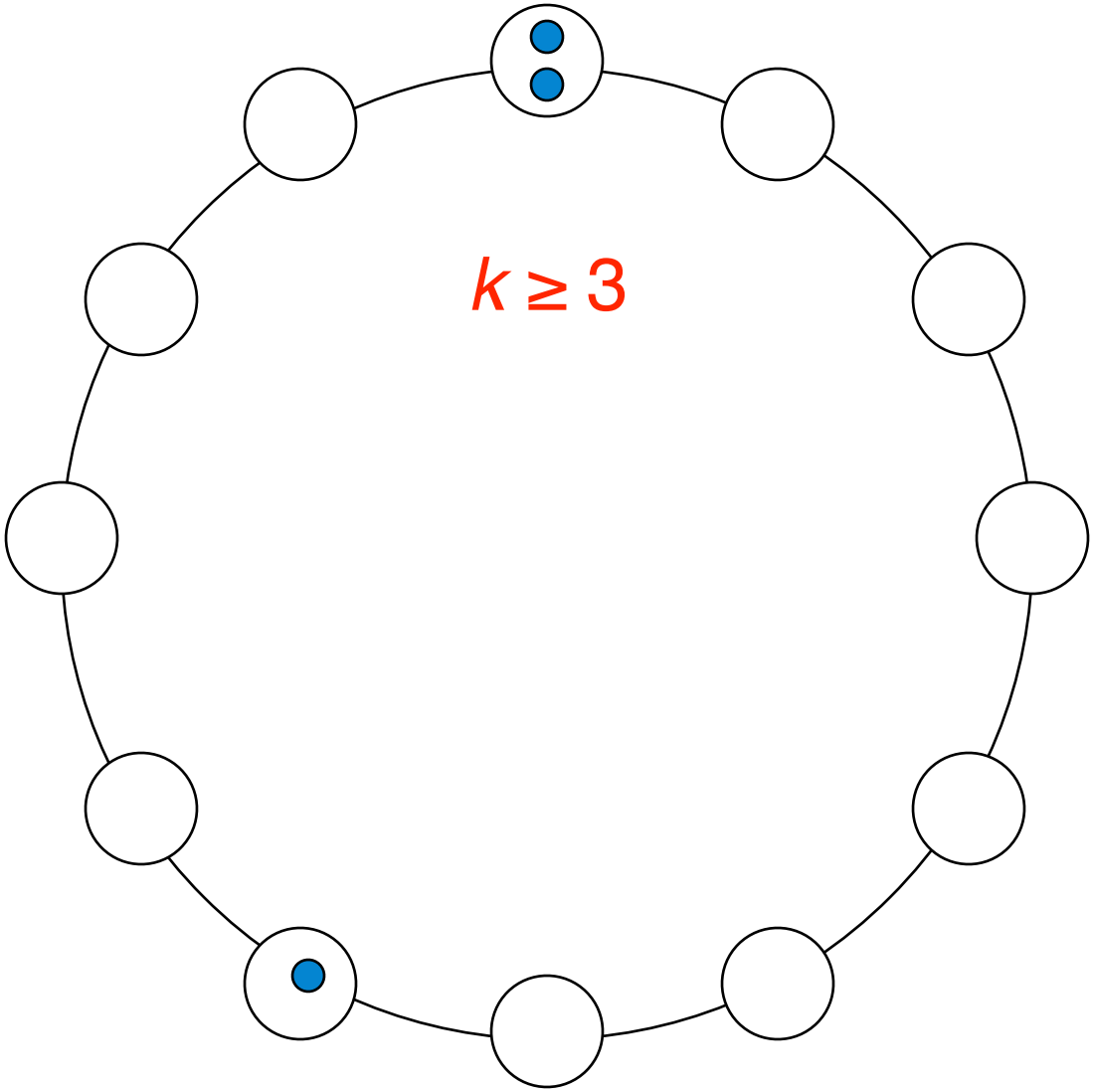

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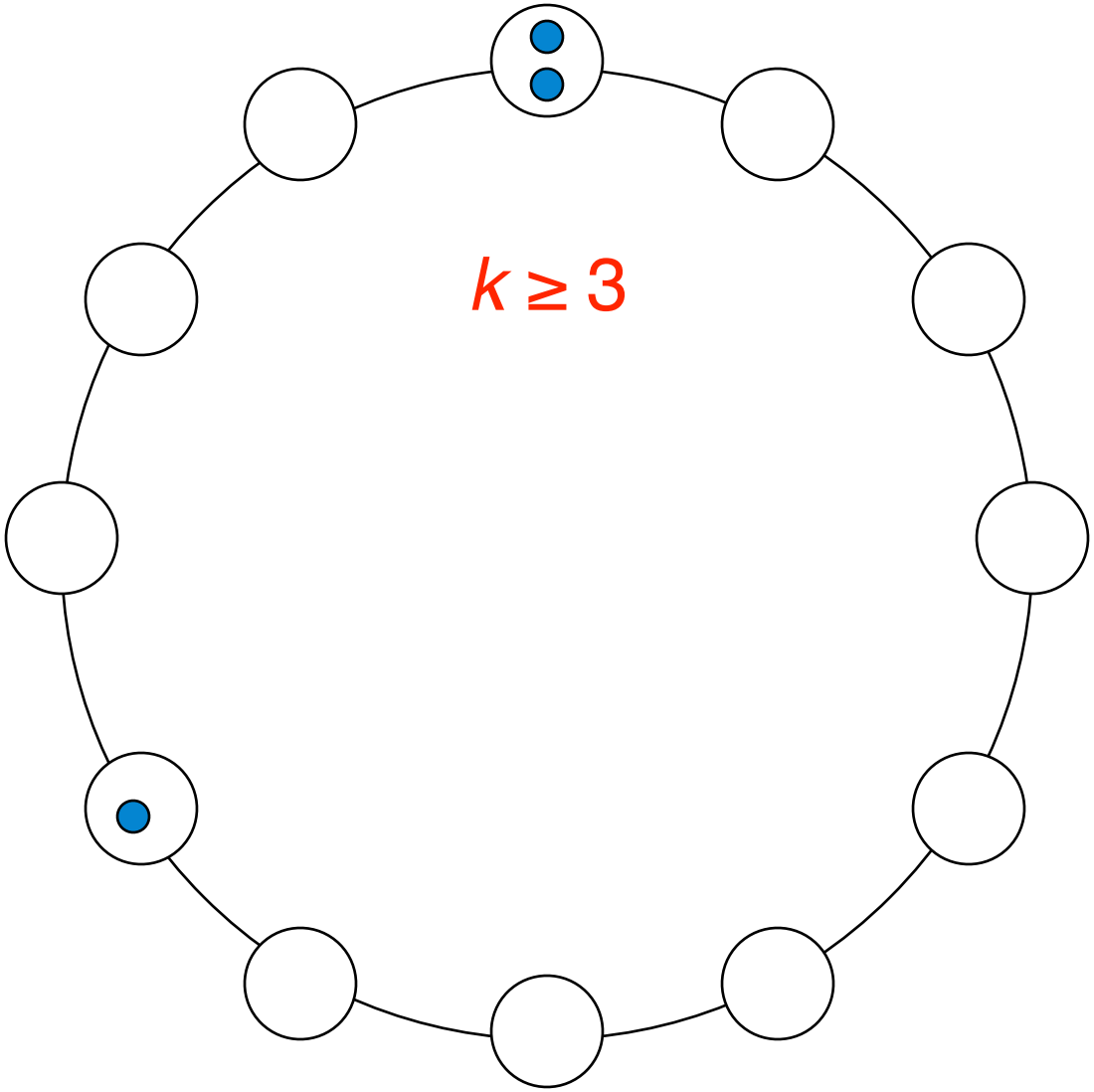

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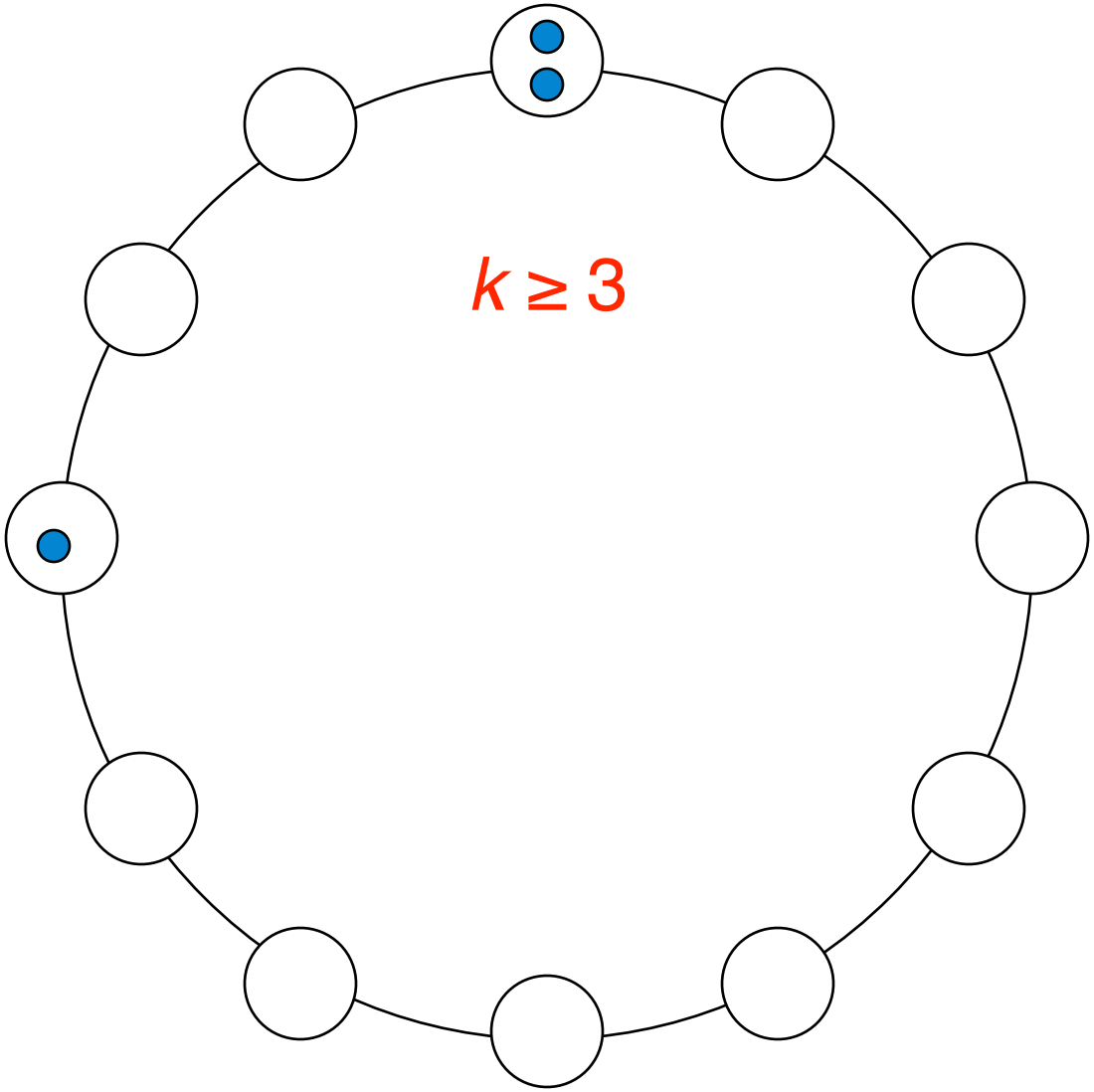

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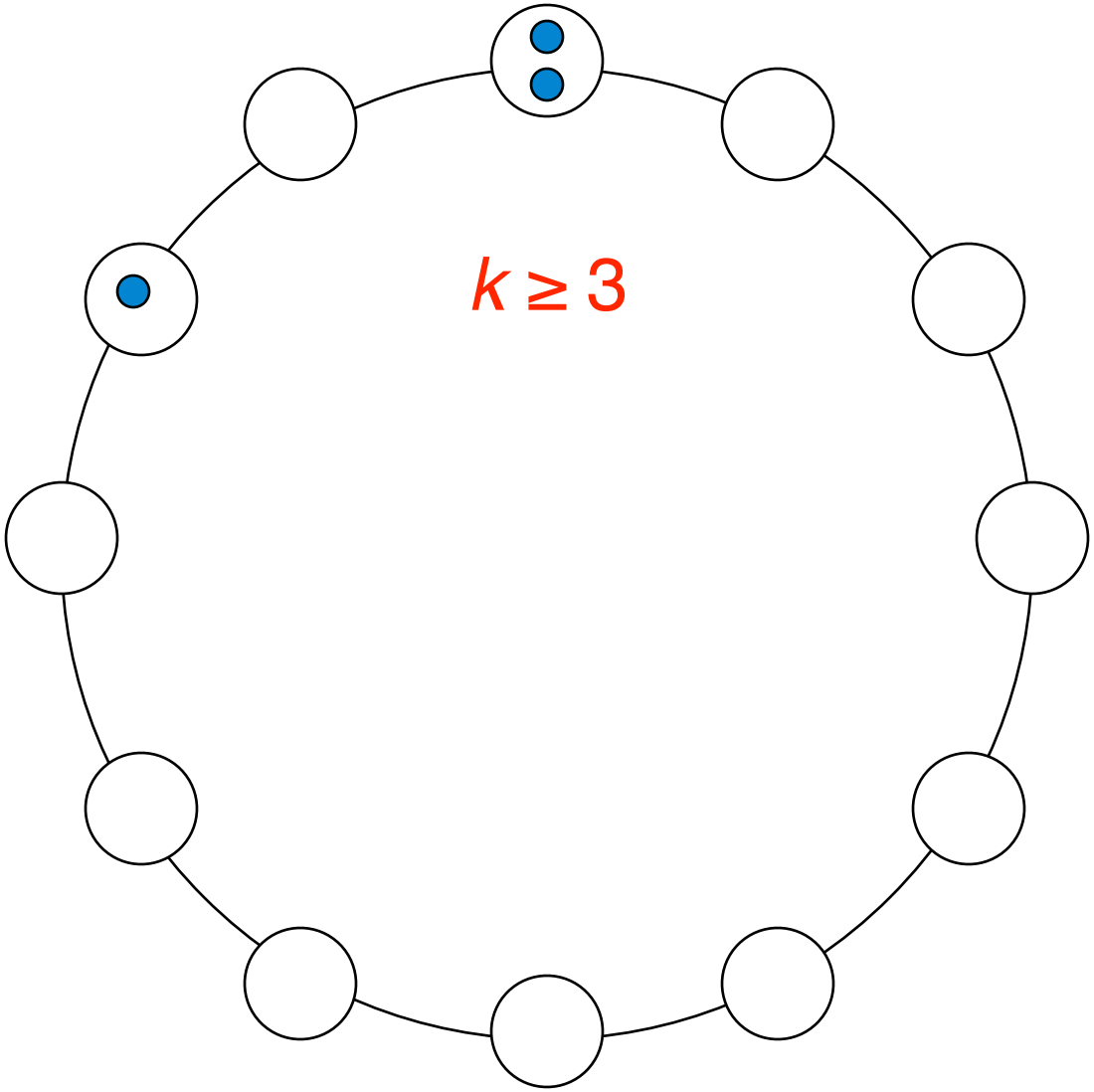

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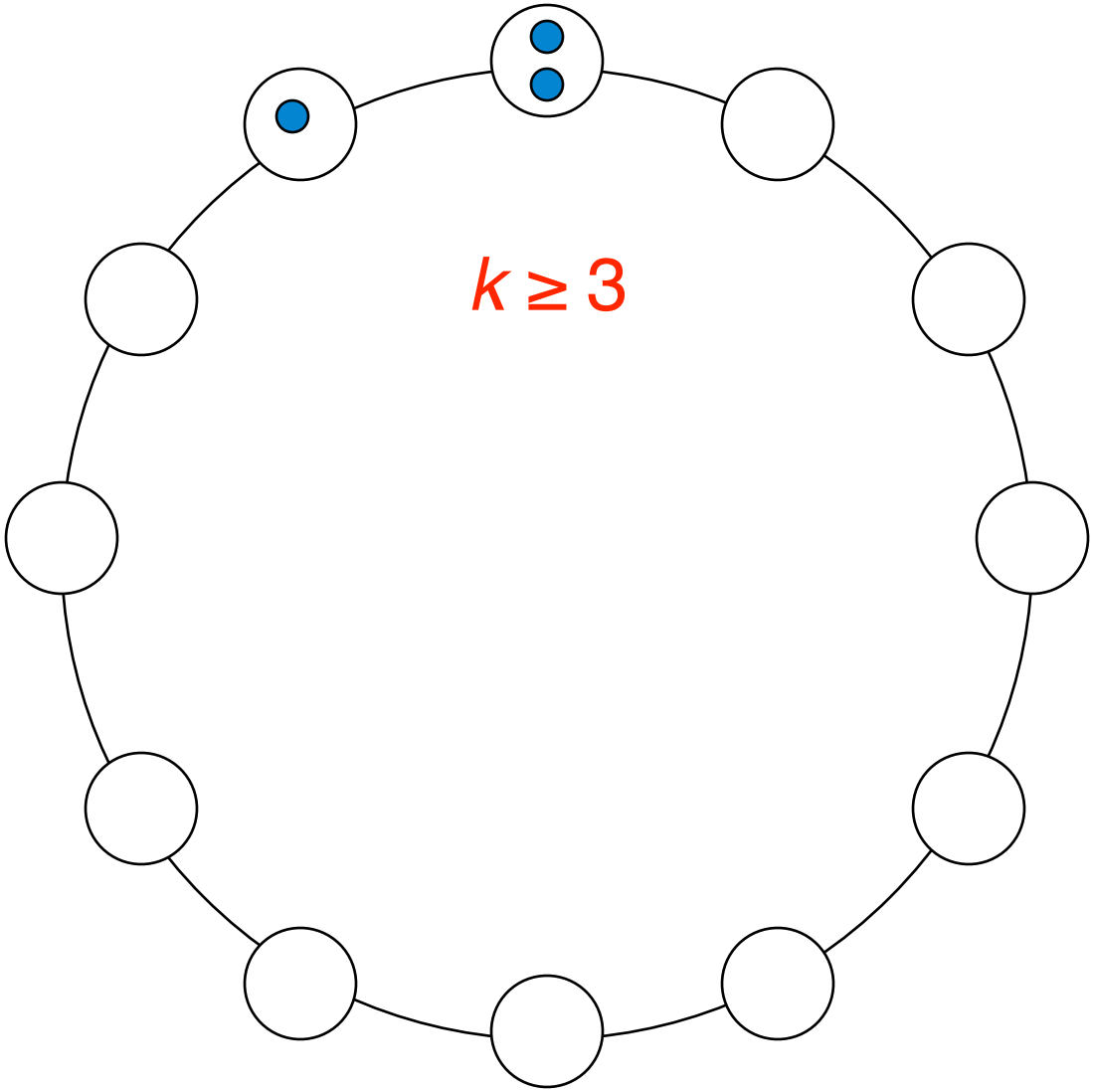

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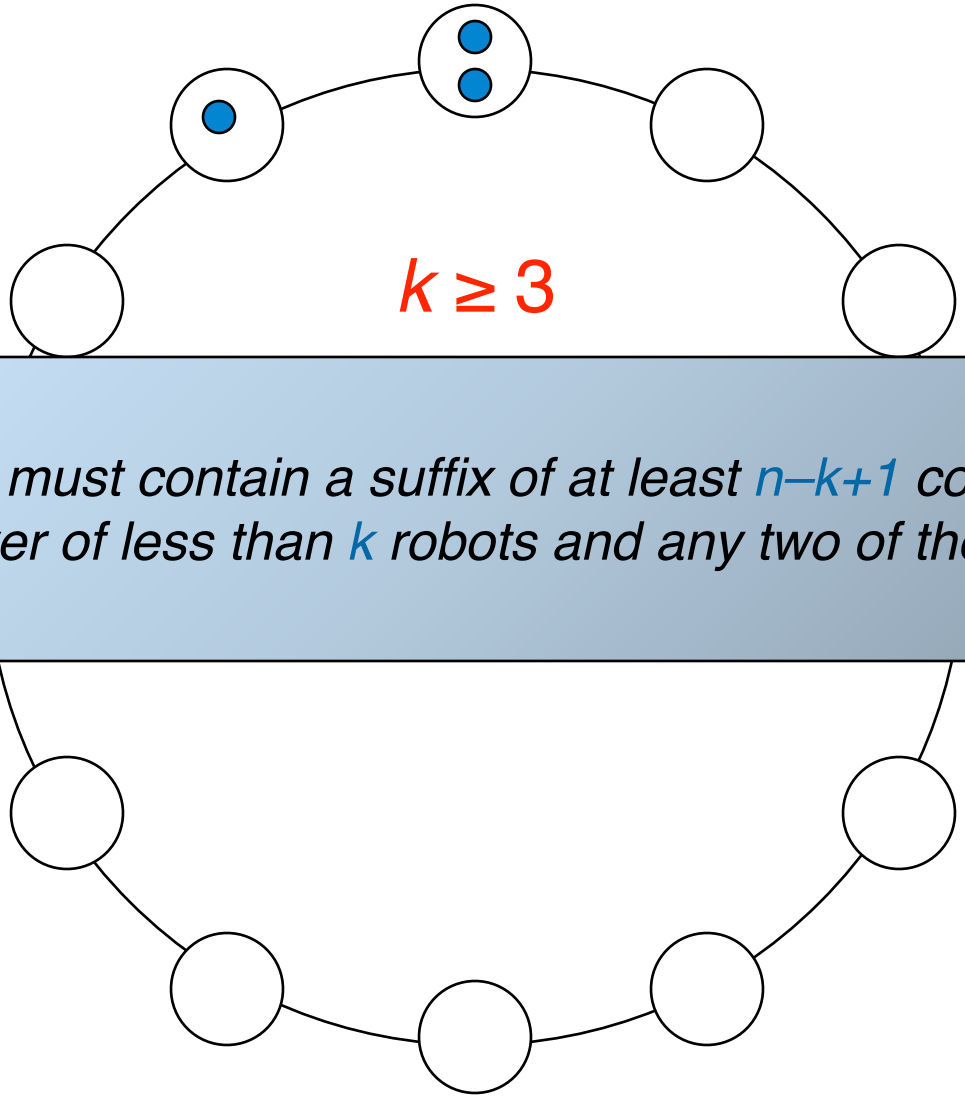
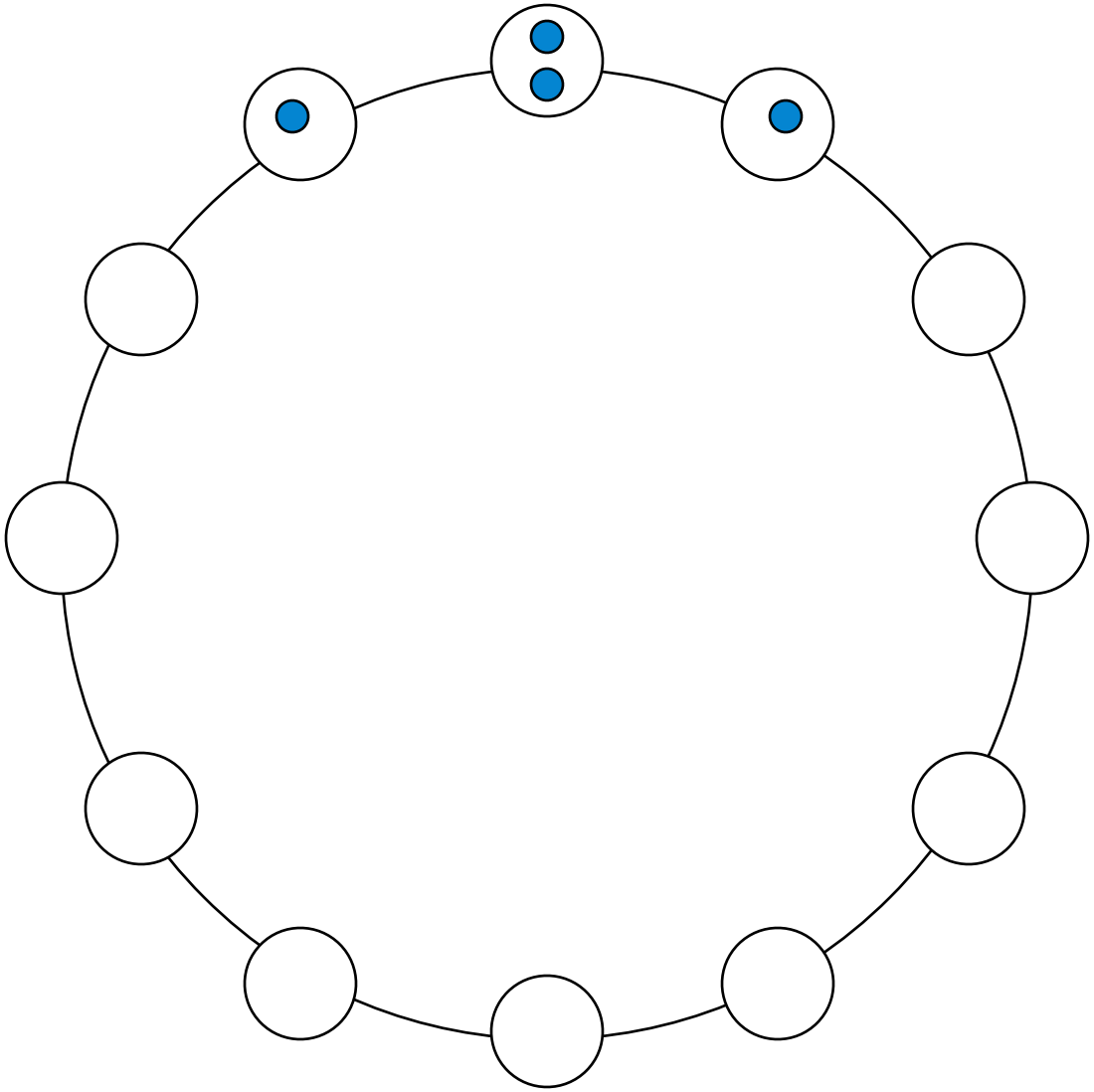


Lemma.
Every execution must contain a suffix of at least $n-k+1$ configurations containing a tower of less than k robots and any two of them are distinguishable.

Enabling Exploration



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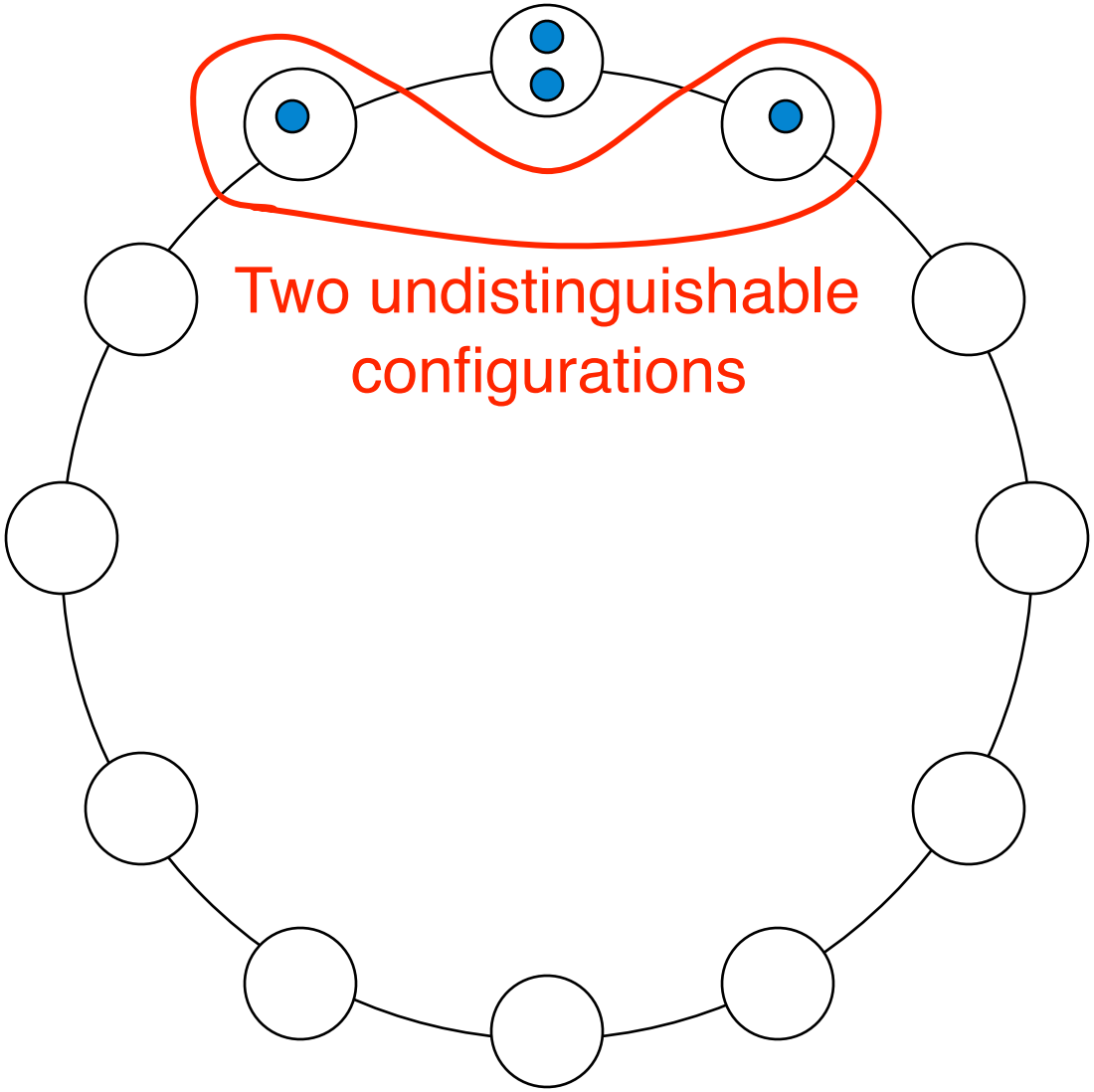

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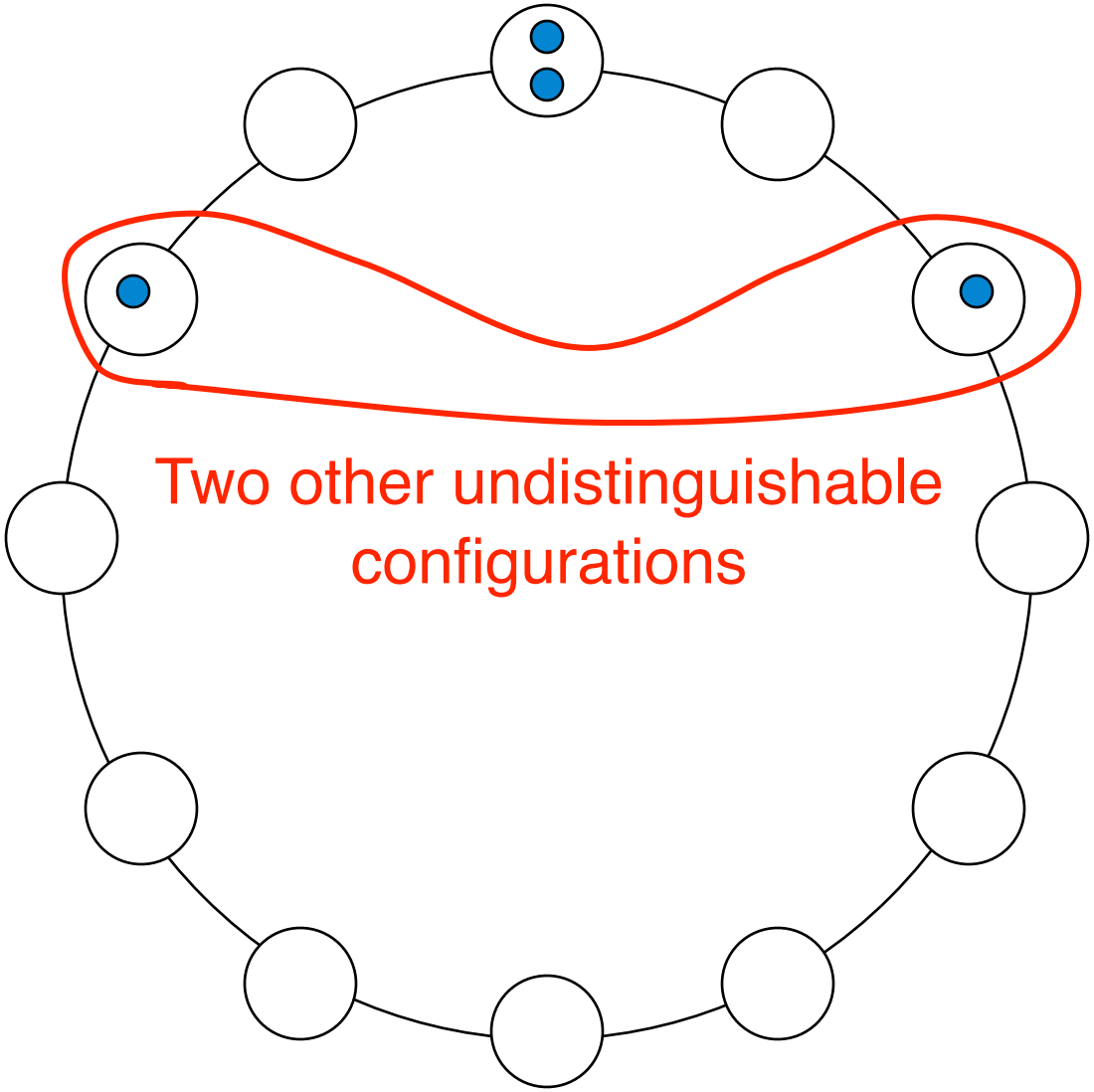

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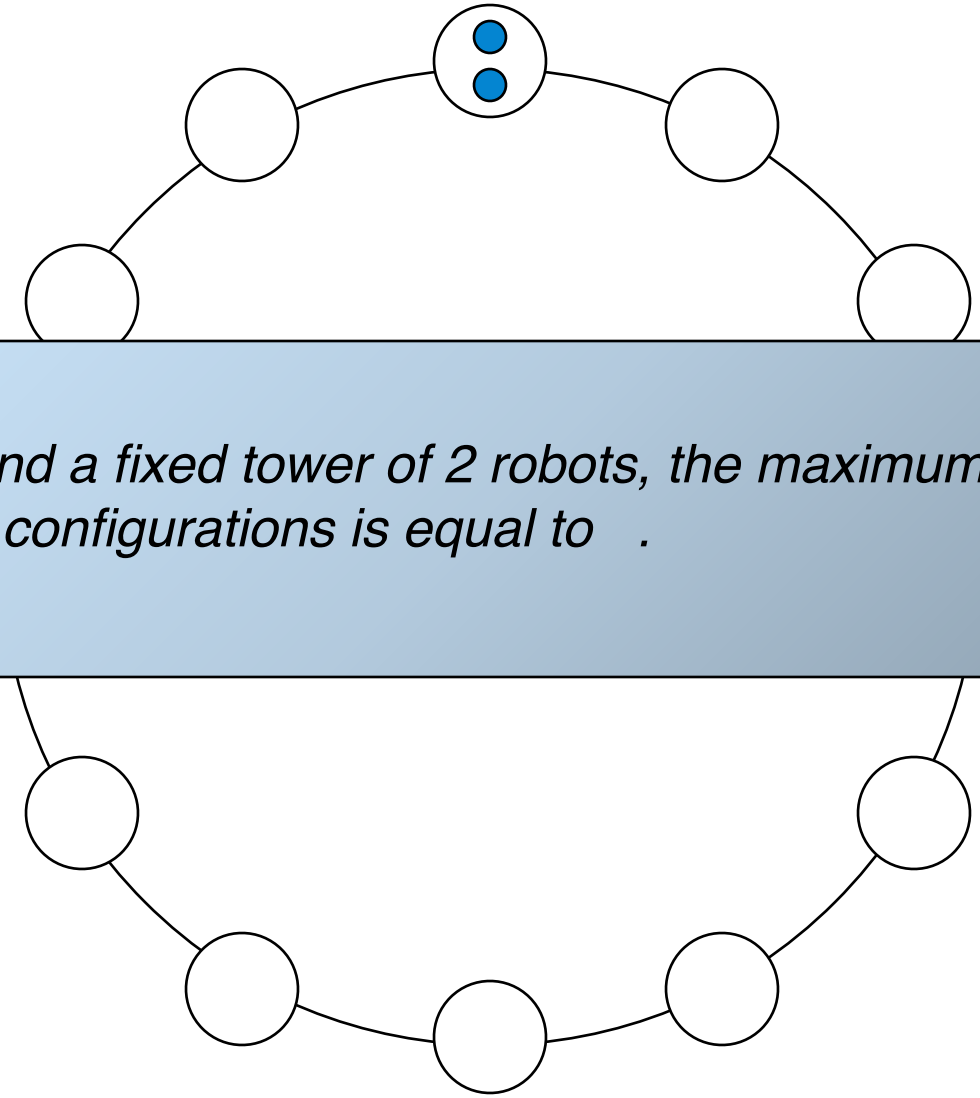

Lemma.
With 3 robots and a fixed tower of 2 robots, the maximum number of distinguishable configurations is equal to $\left\lfloor \frac{n}{2} \right\rfloor$.

Enabling Exploration

Enabling Exploration

Theorem.
For every $n > 4$, there exists no exploration protocol (even probabilistic) of an n-size ring with 3 robots.

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For every $n > 4$, there exists no exploration protocol (even probabilistic) of an n-size ring with 3 robots.

Proof :

$$\left\lfloor \frac{n}{2} \right\rfloor \geq n - k + 1 \Rightarrow n \leq 4$$

Contribution

Theorem.
4 **probabilistic** robots are necessary and sufficient, provided that $n > 4$

- The theorem holds even if k divides n.

1. Exploration impossible with less than *4* robots
2. Give an algorithm working with *4* probabilistic robots

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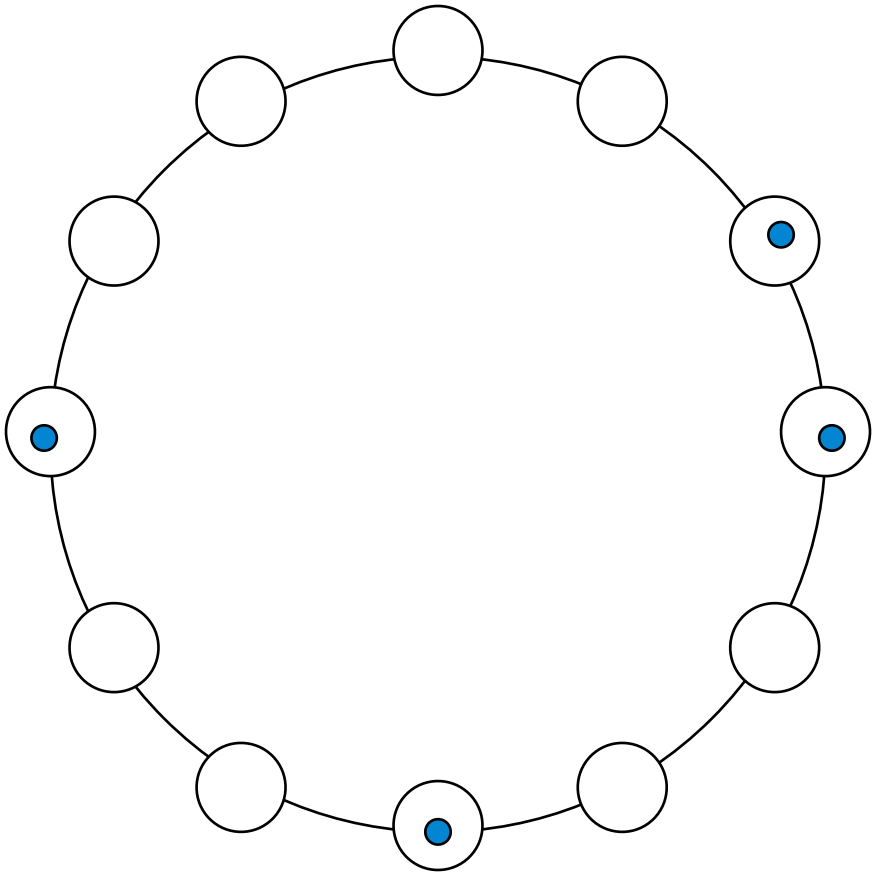

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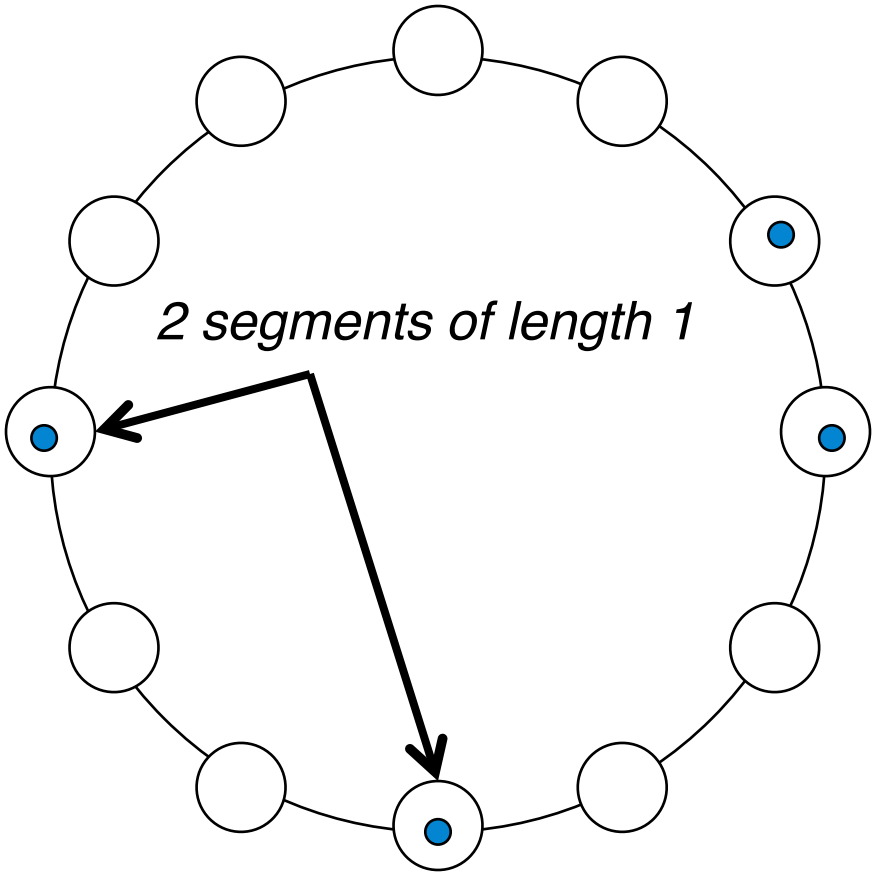

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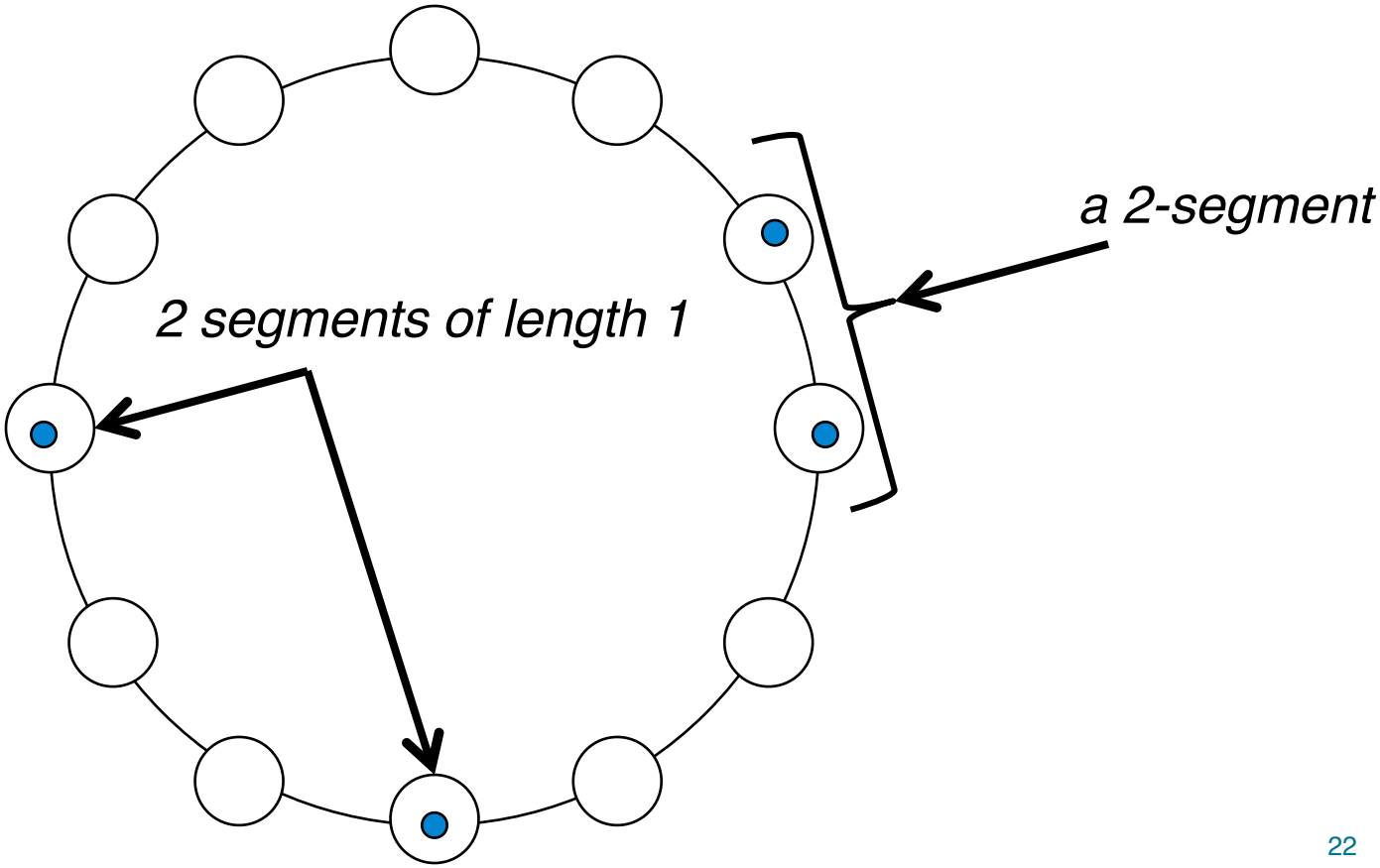

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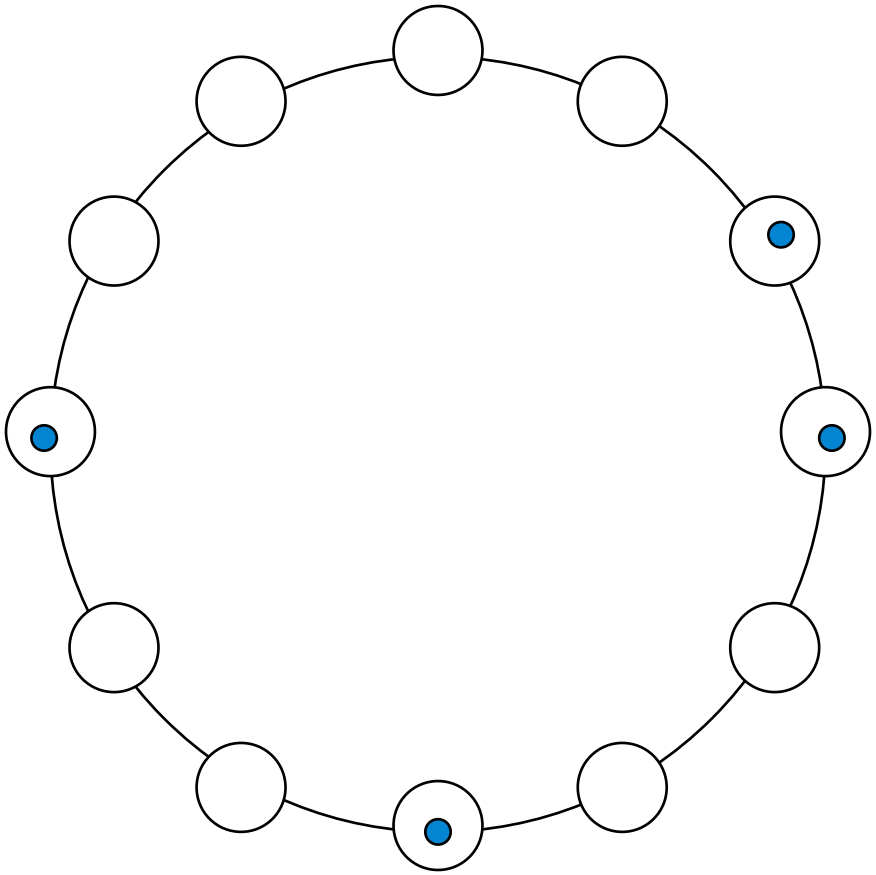

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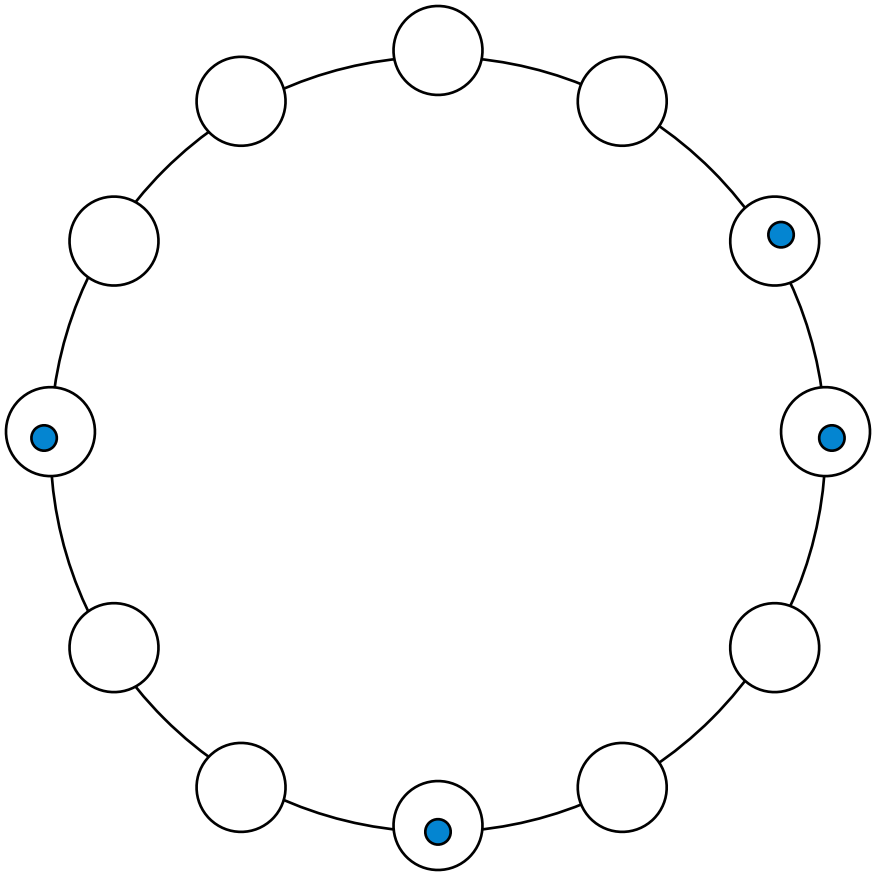

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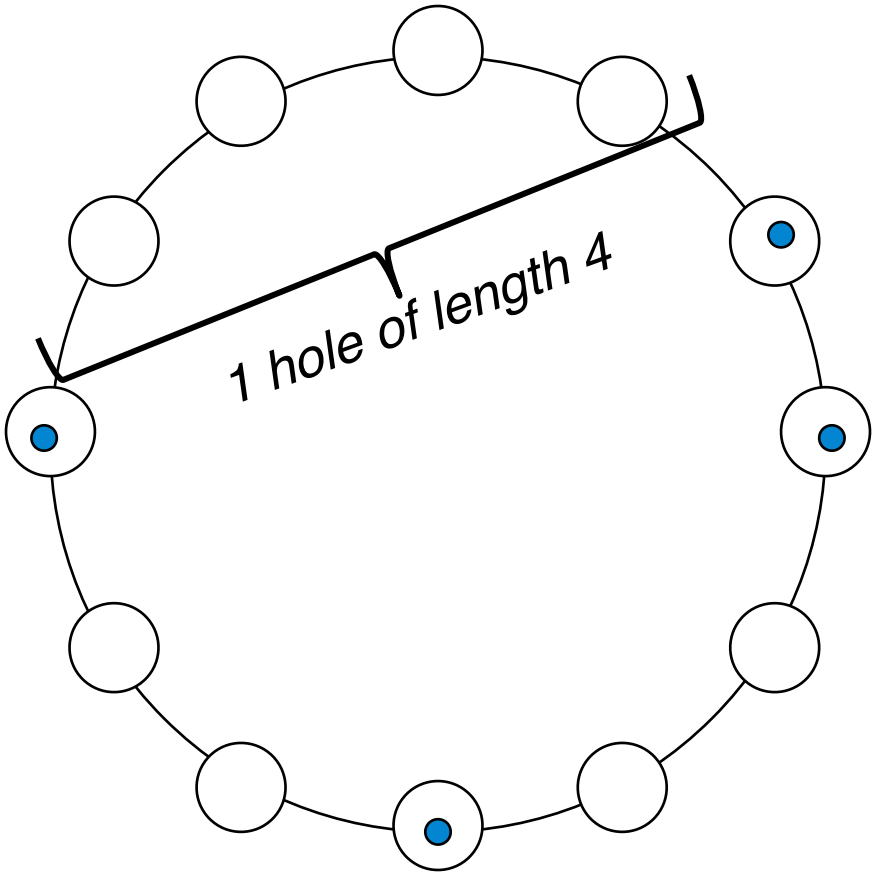

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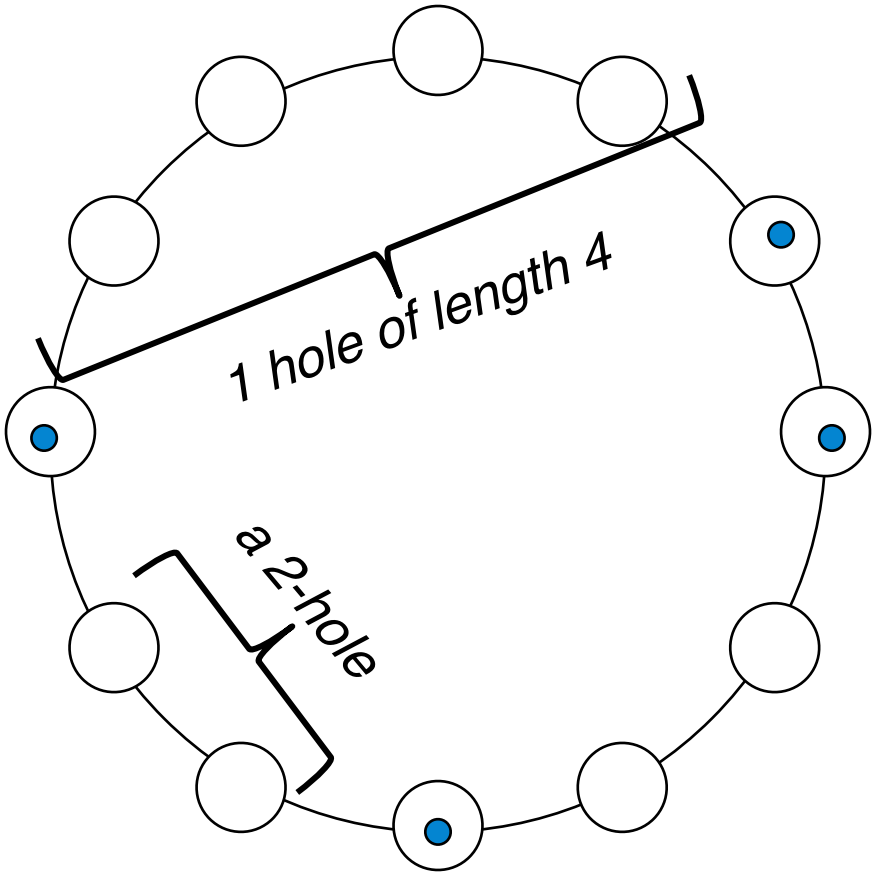

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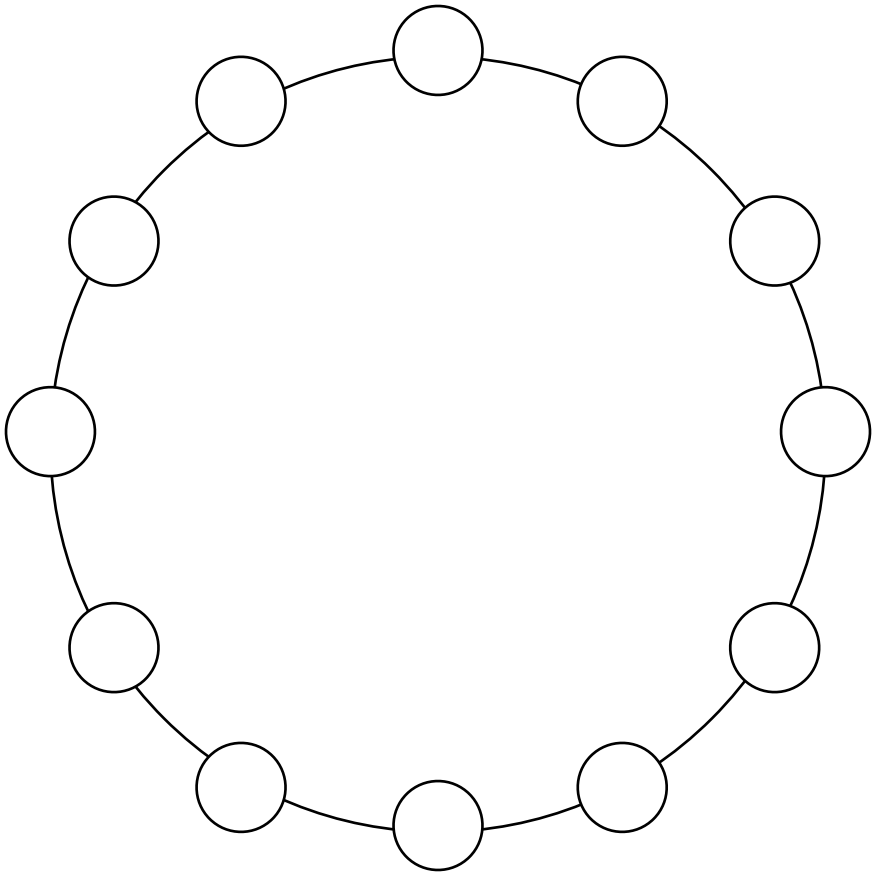

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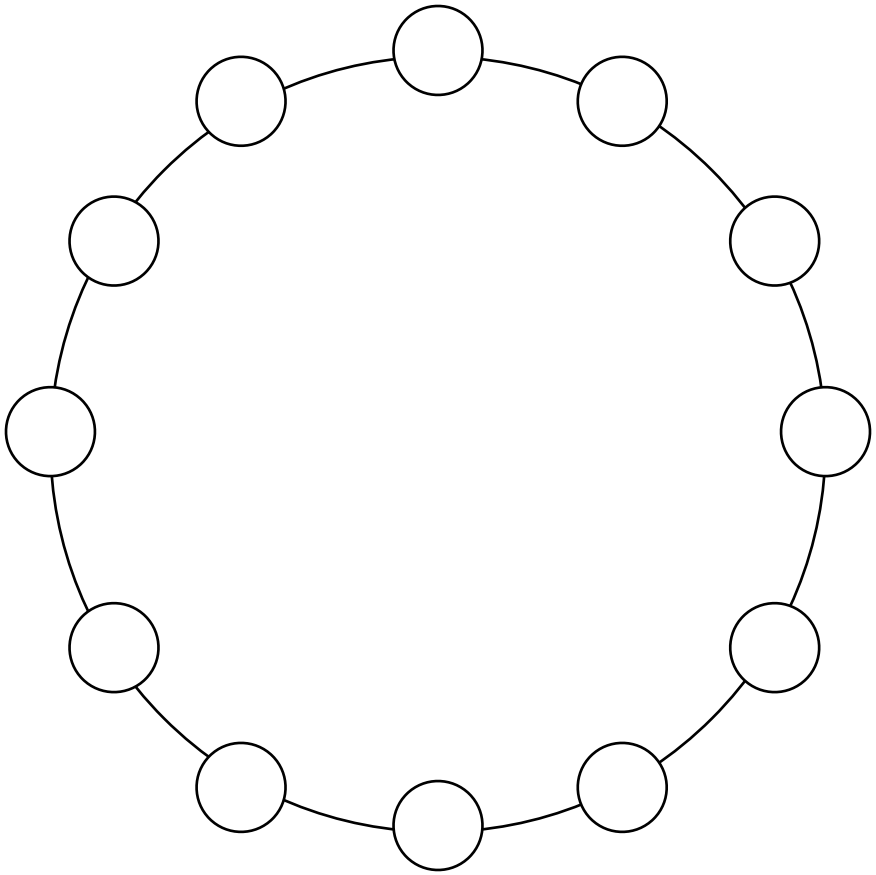

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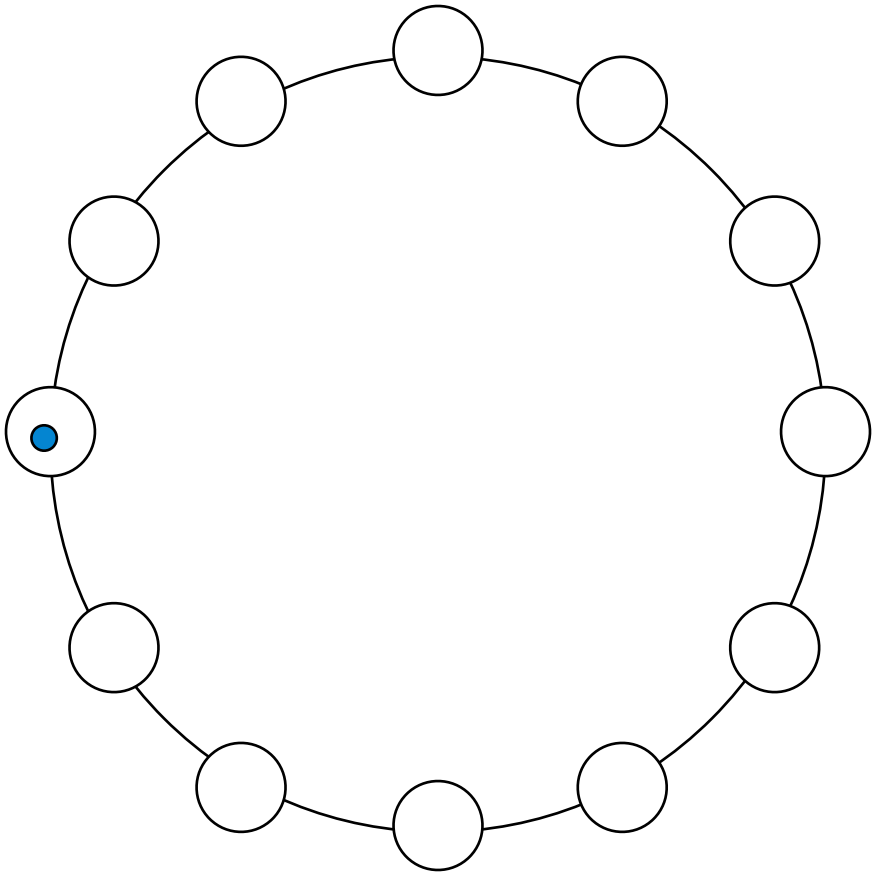

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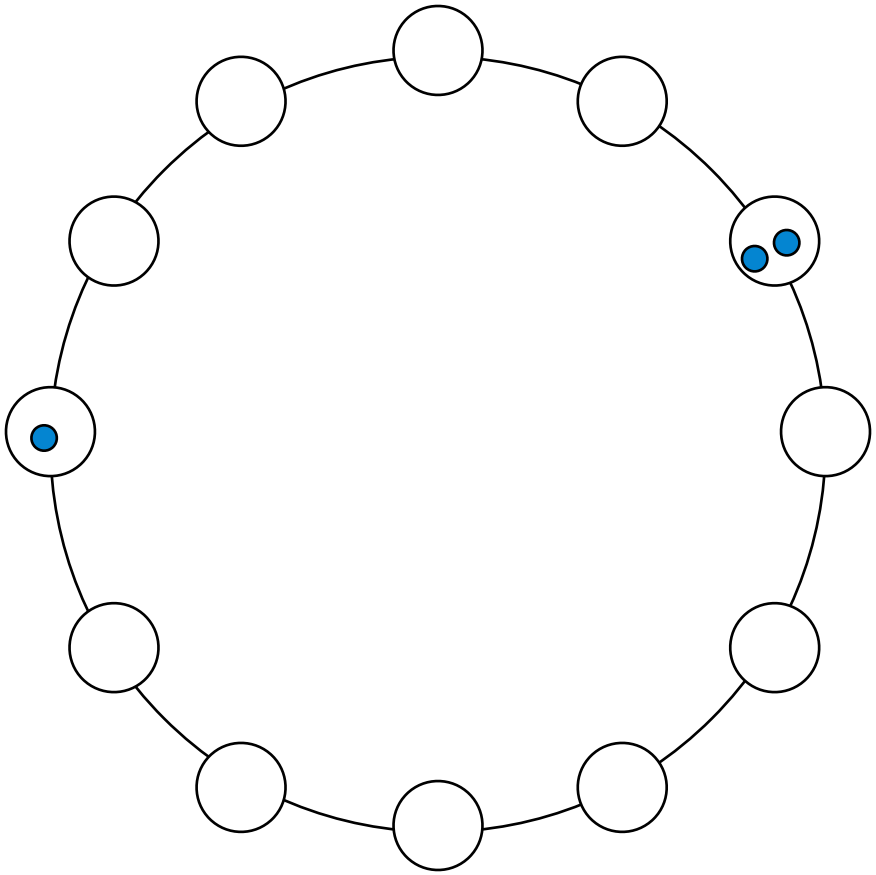

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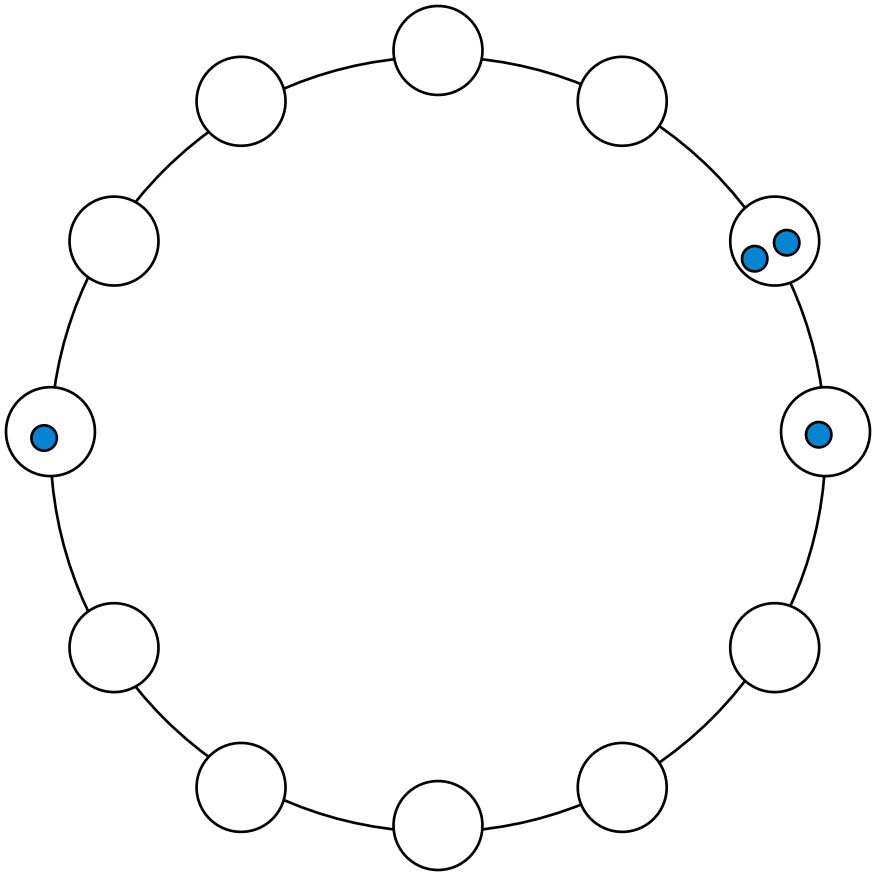

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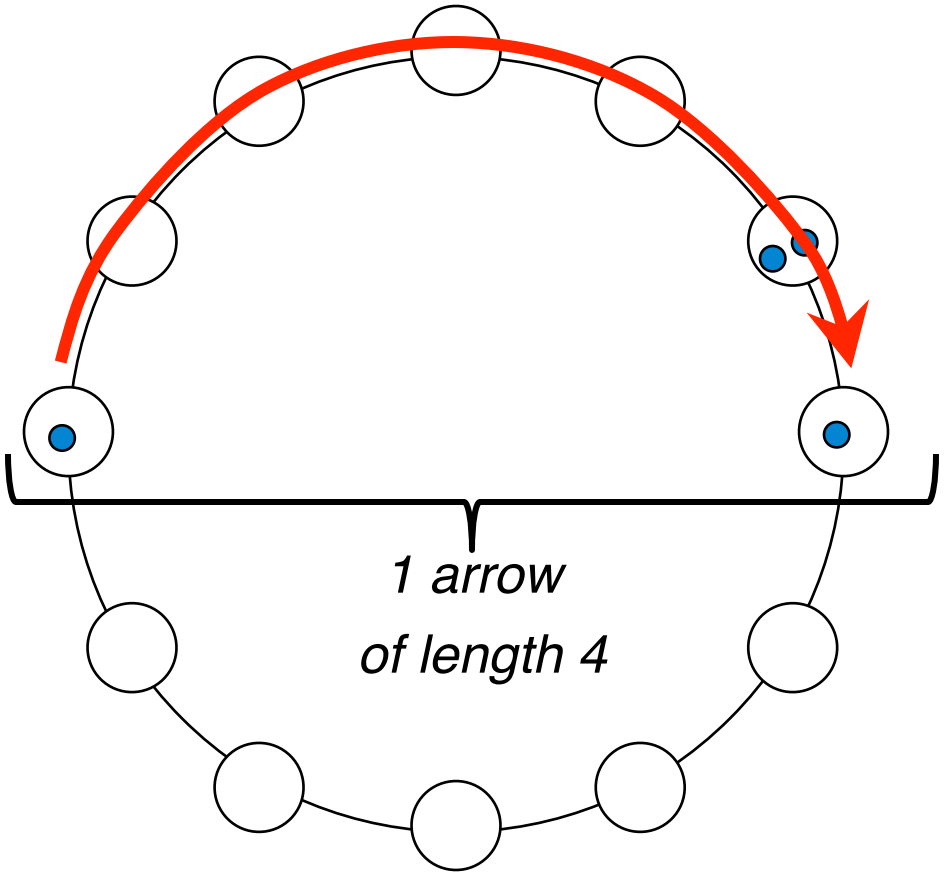

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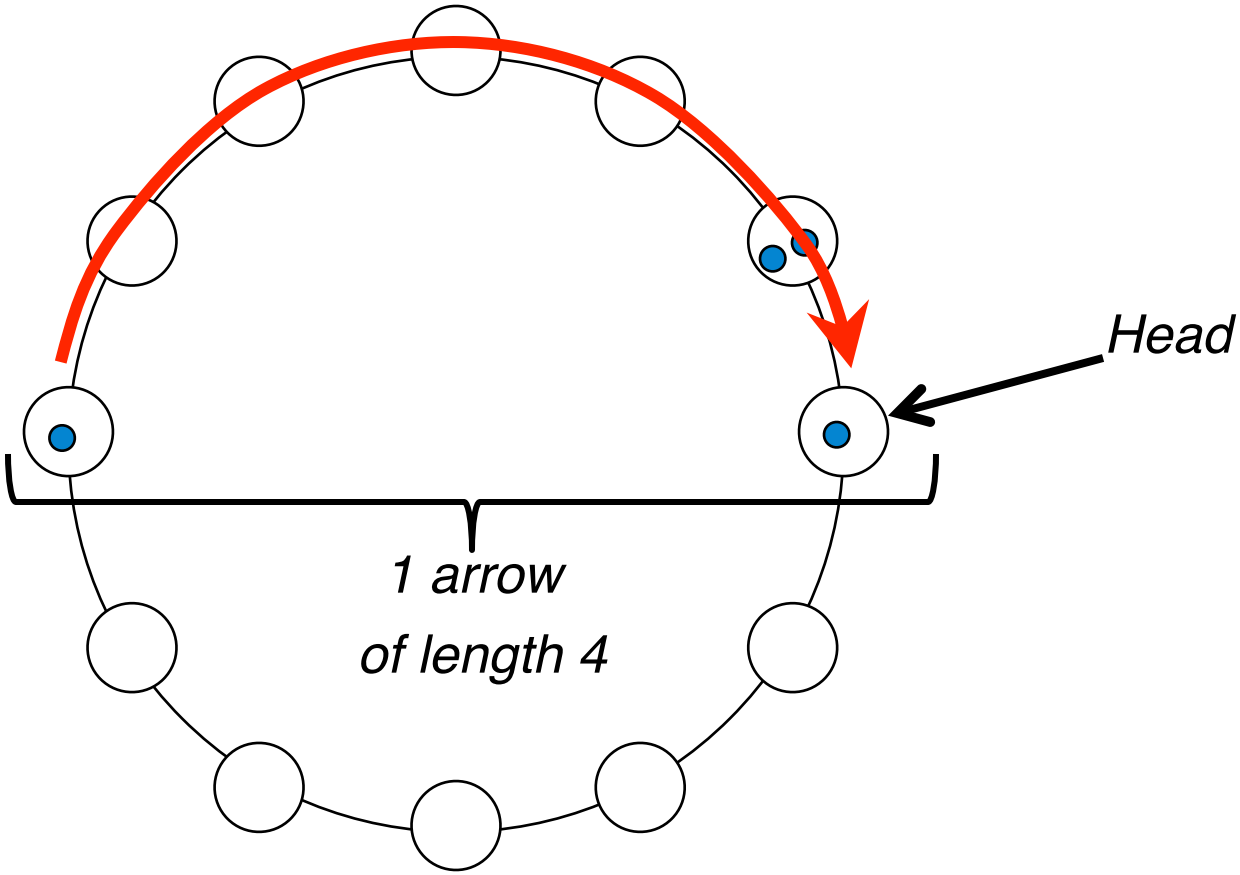

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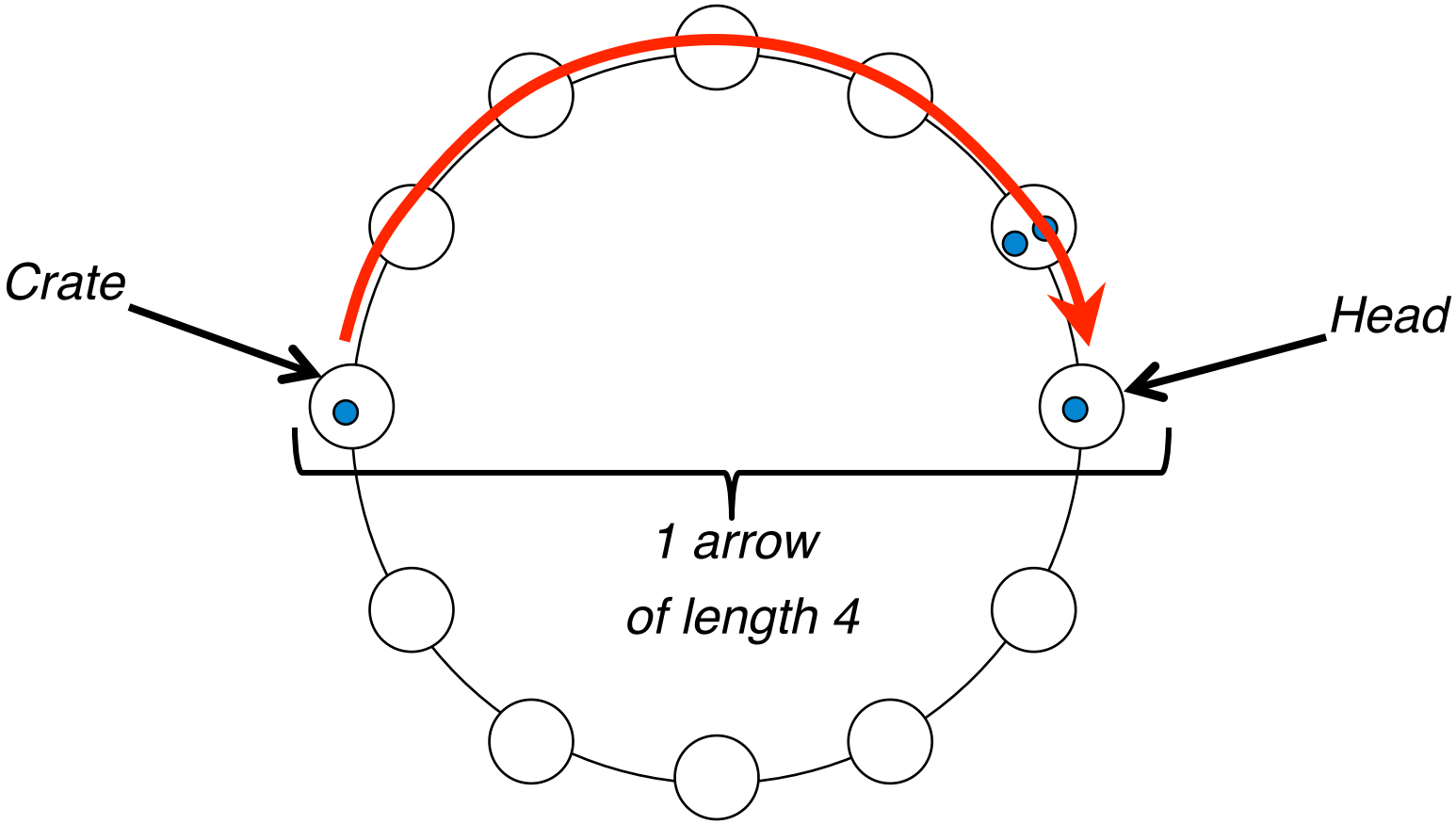

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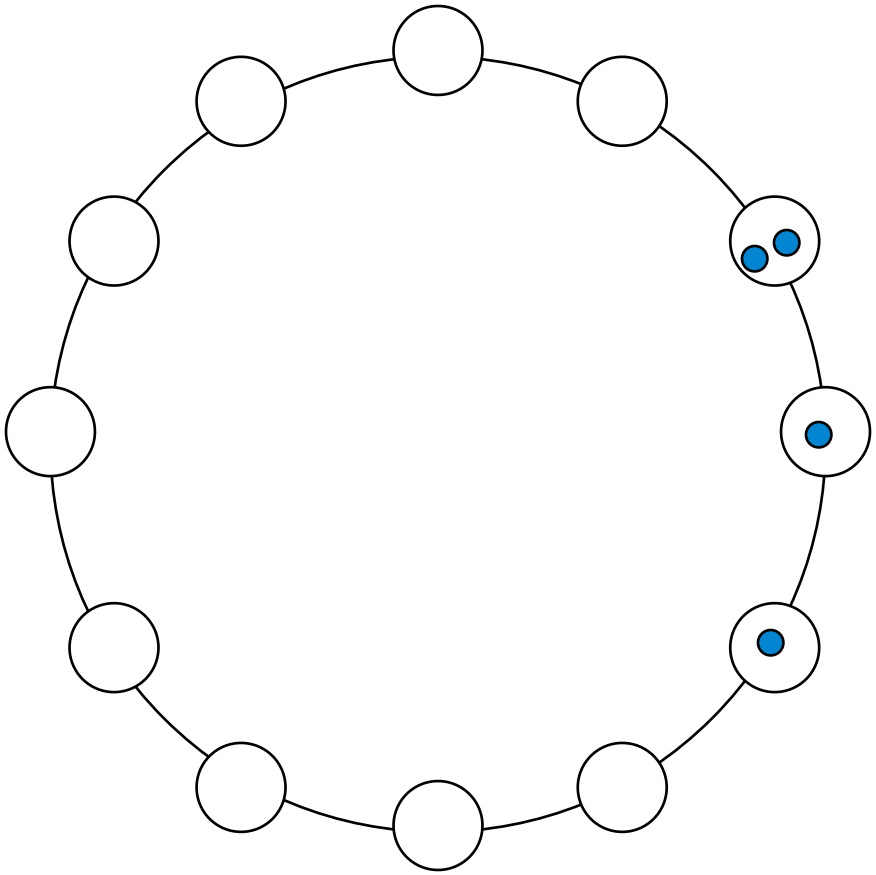

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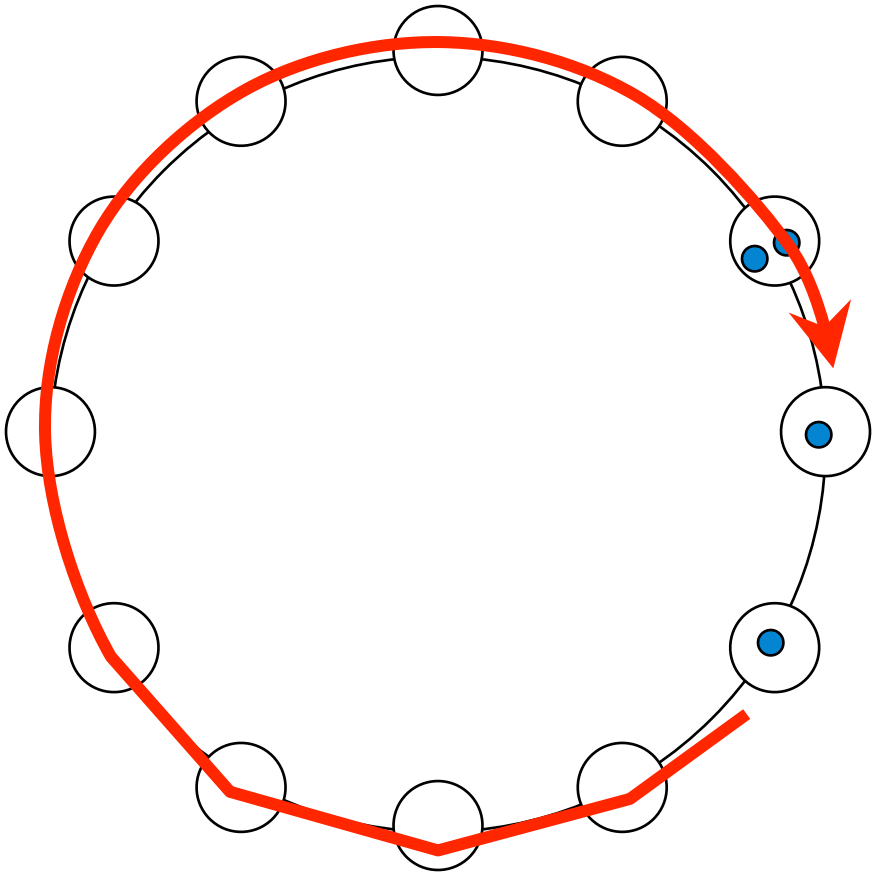

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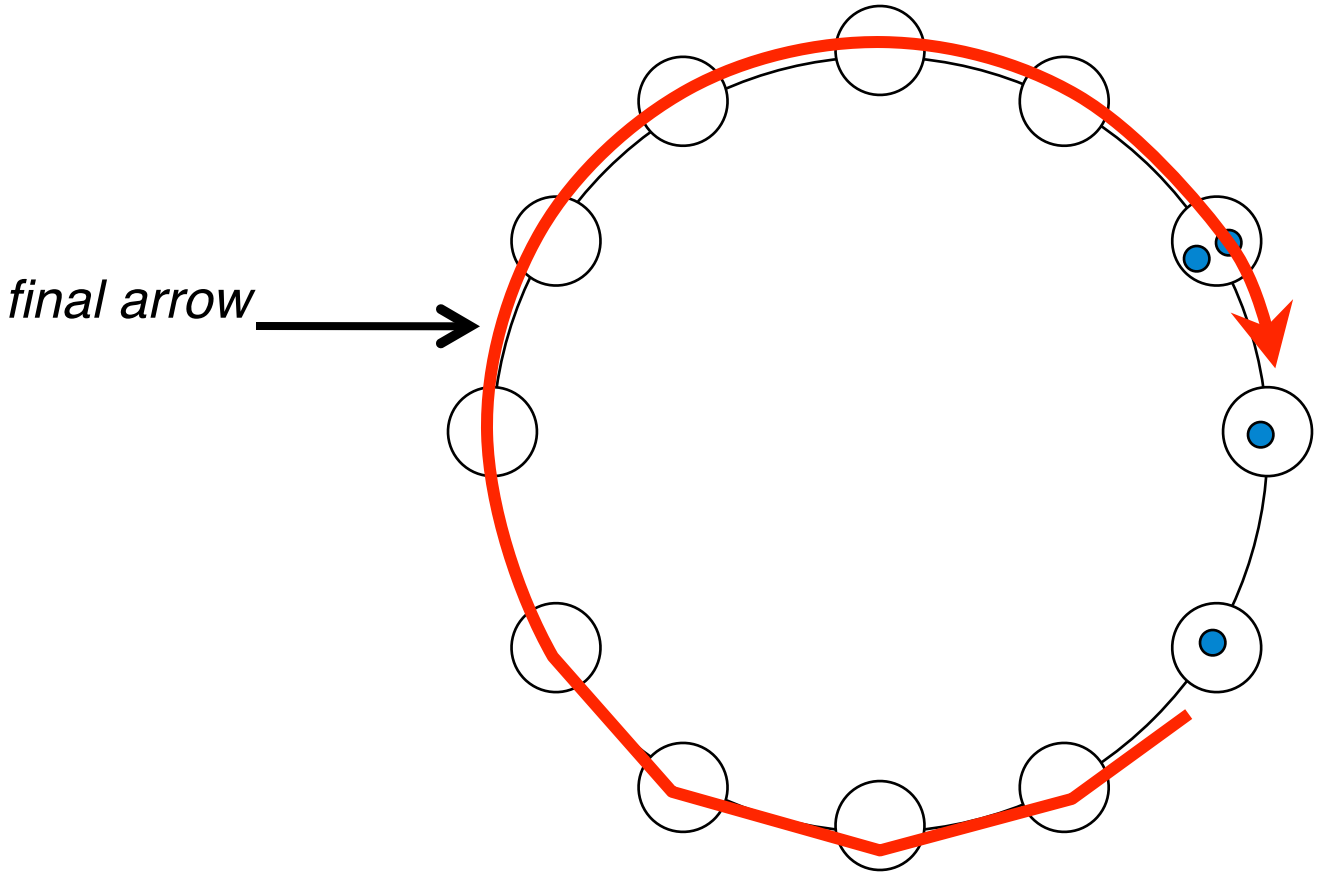

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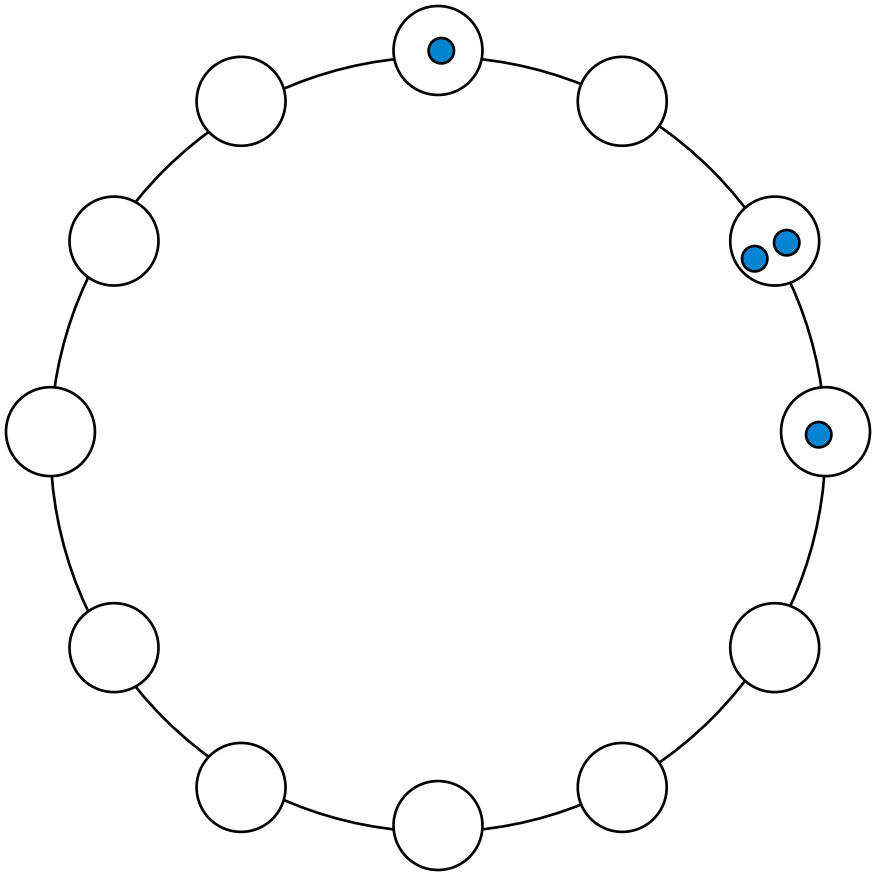

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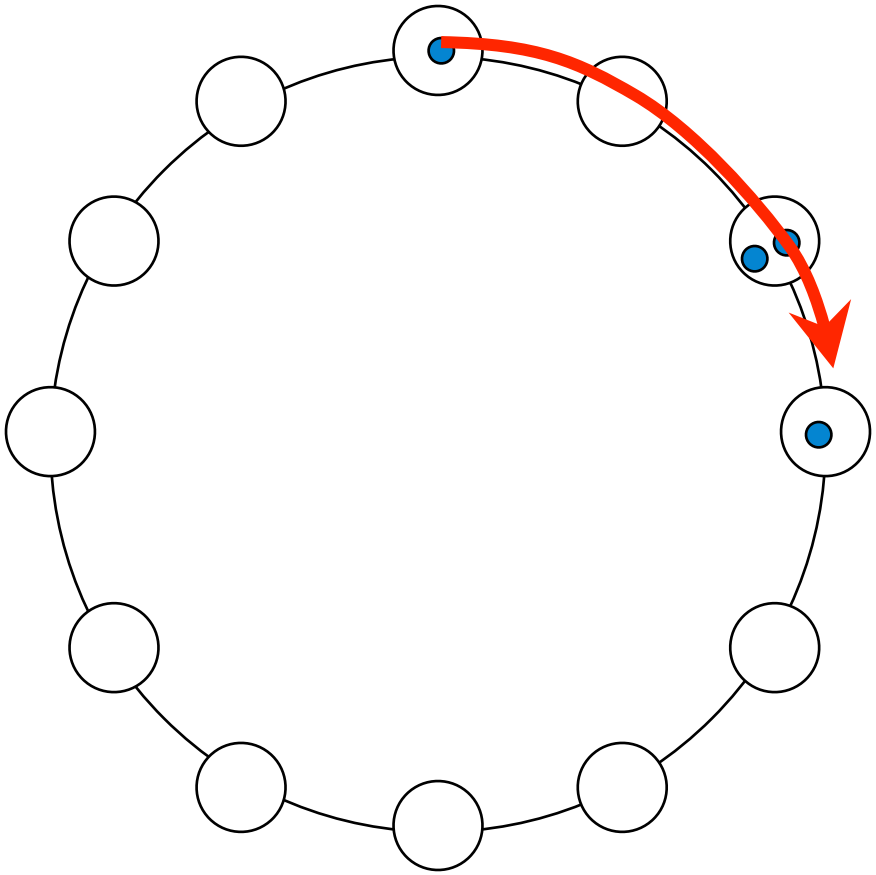

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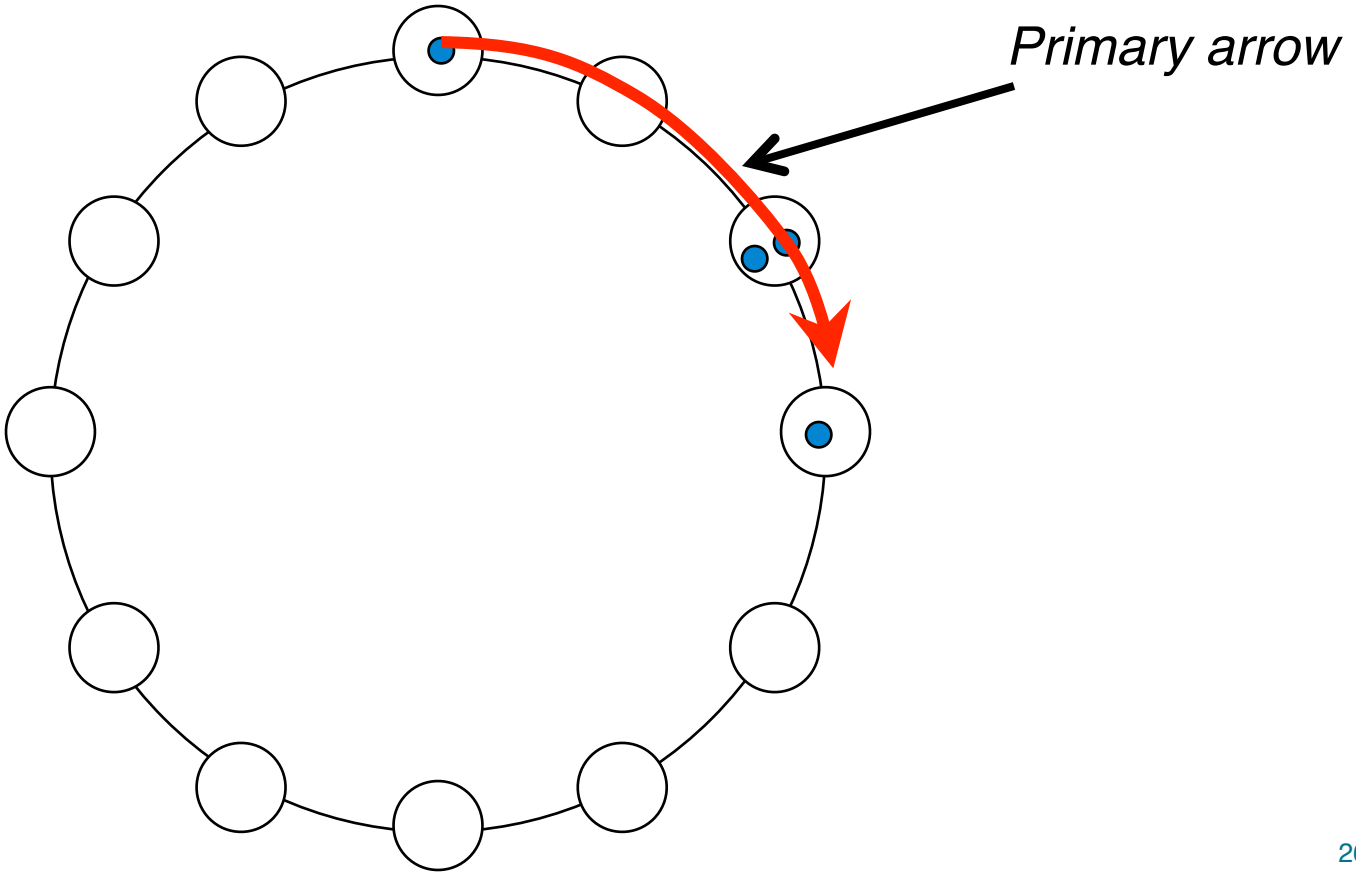

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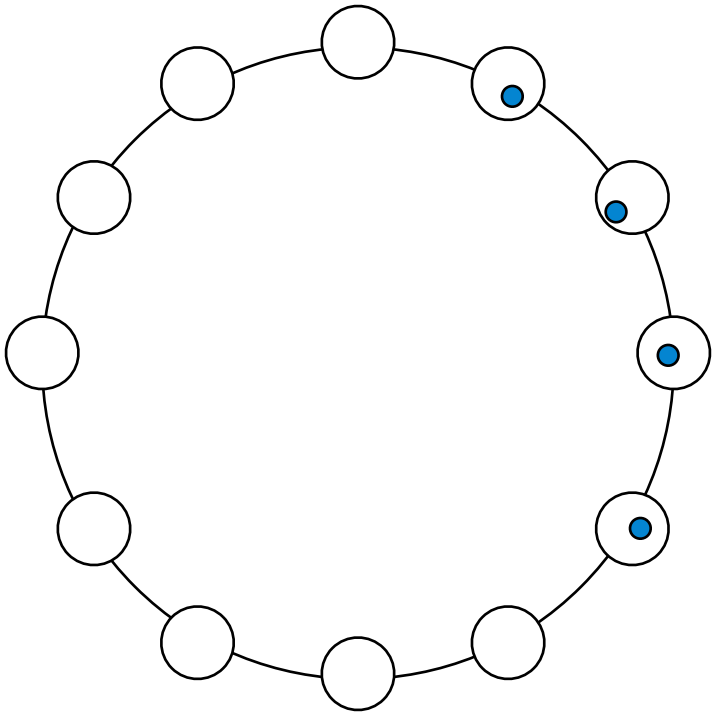

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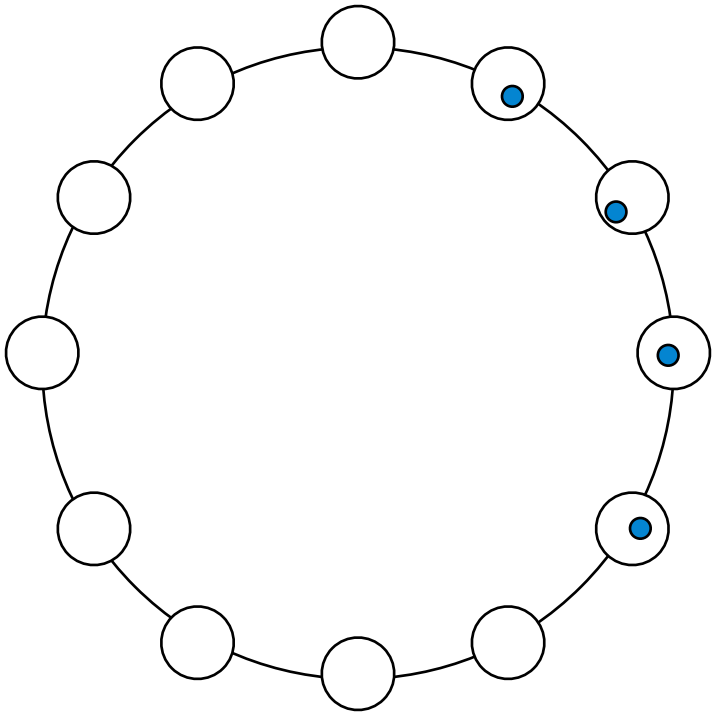

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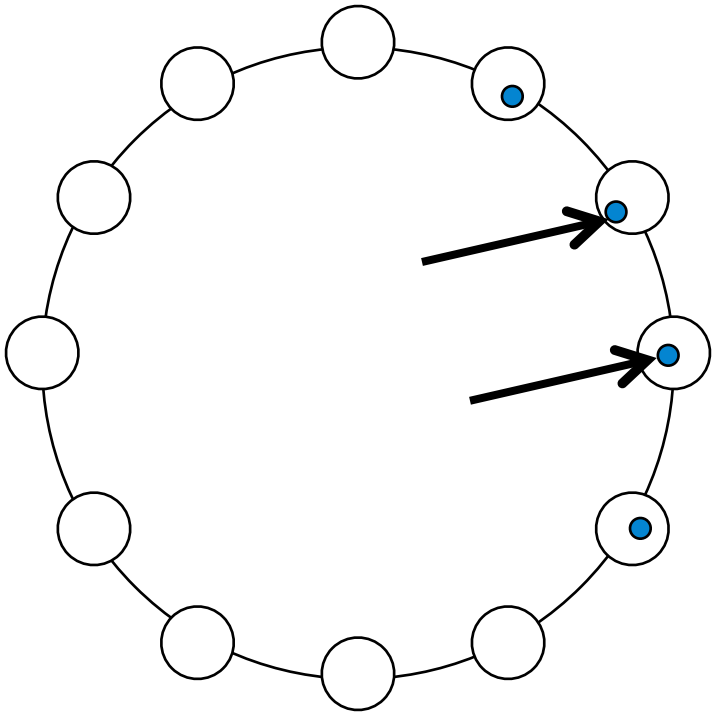

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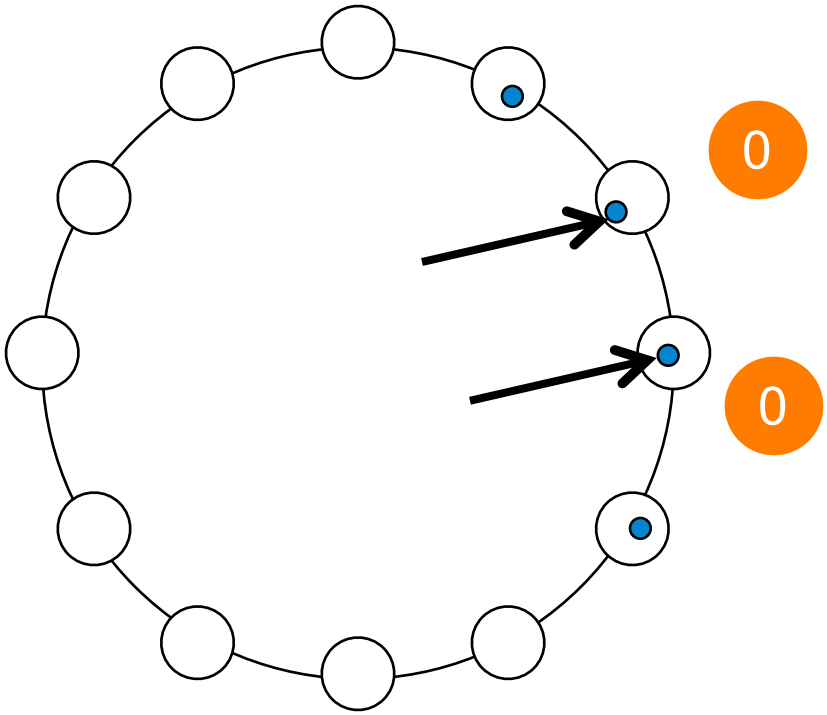

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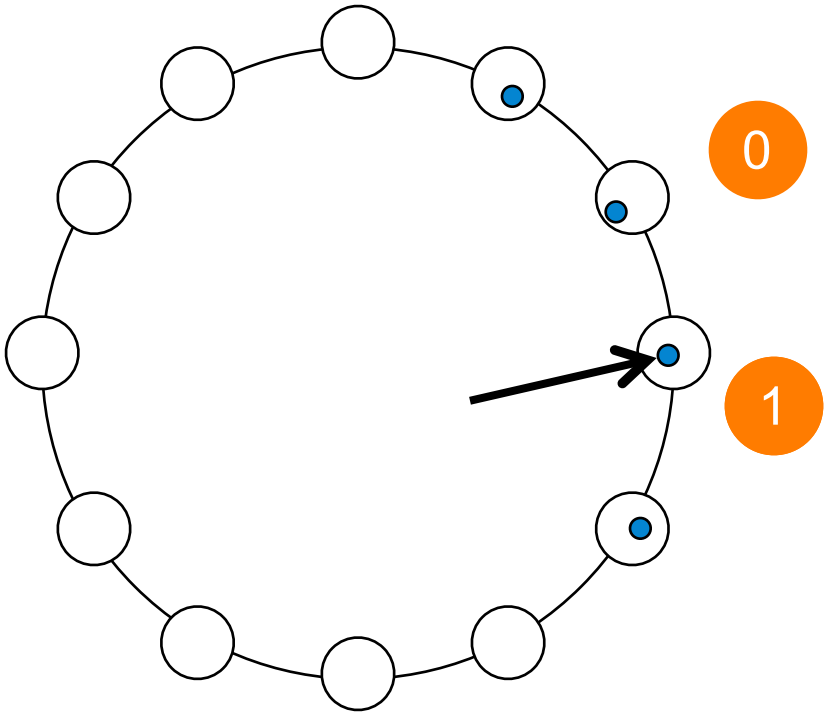

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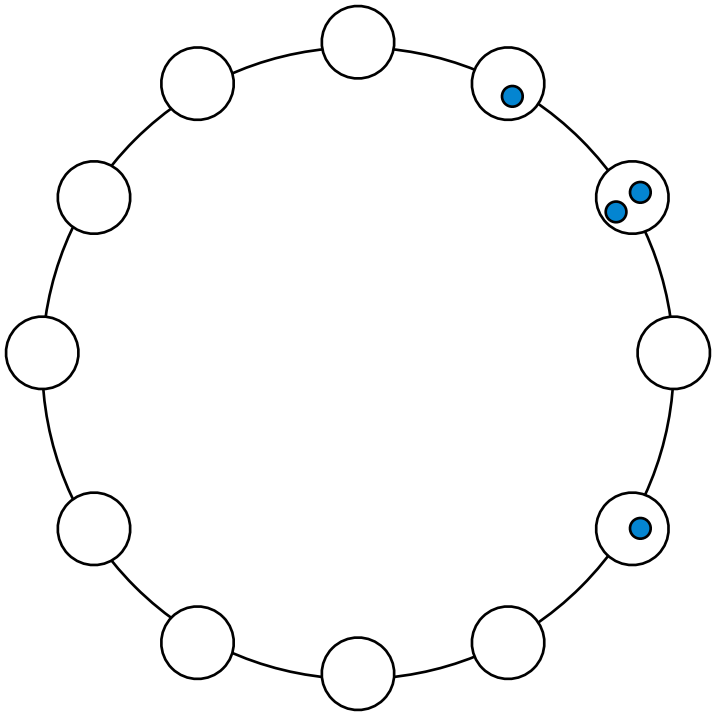

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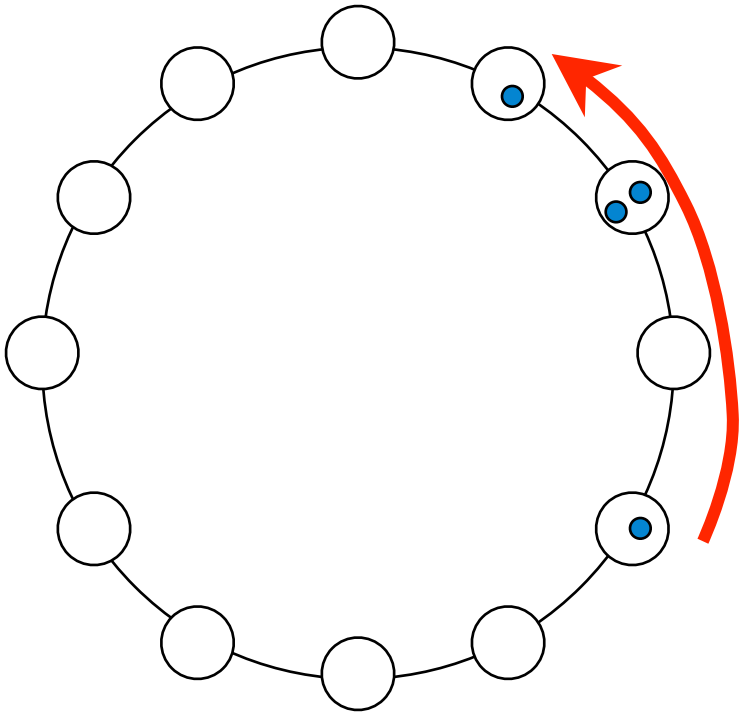

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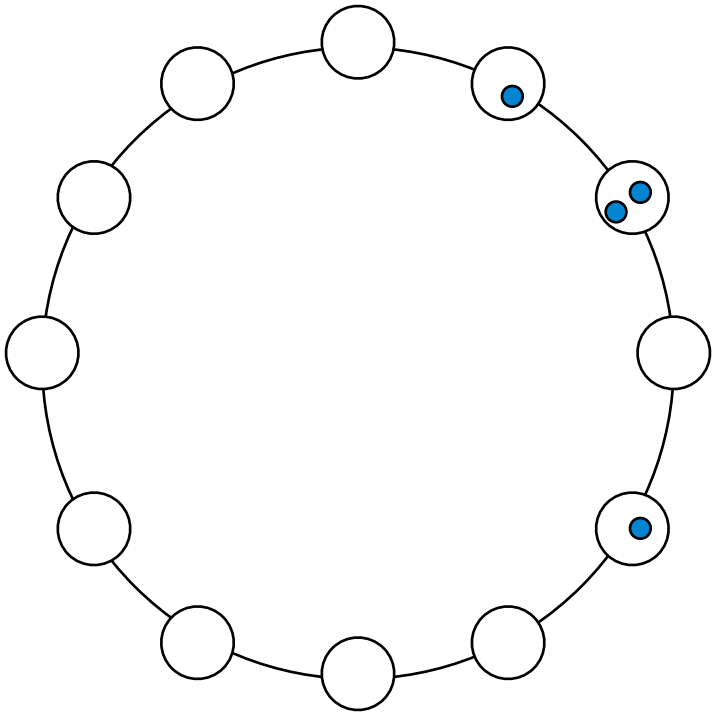

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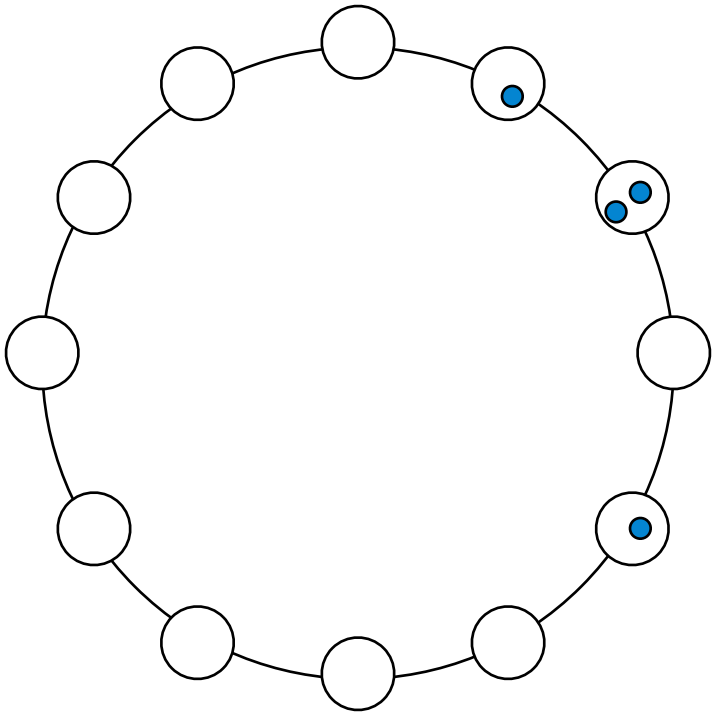

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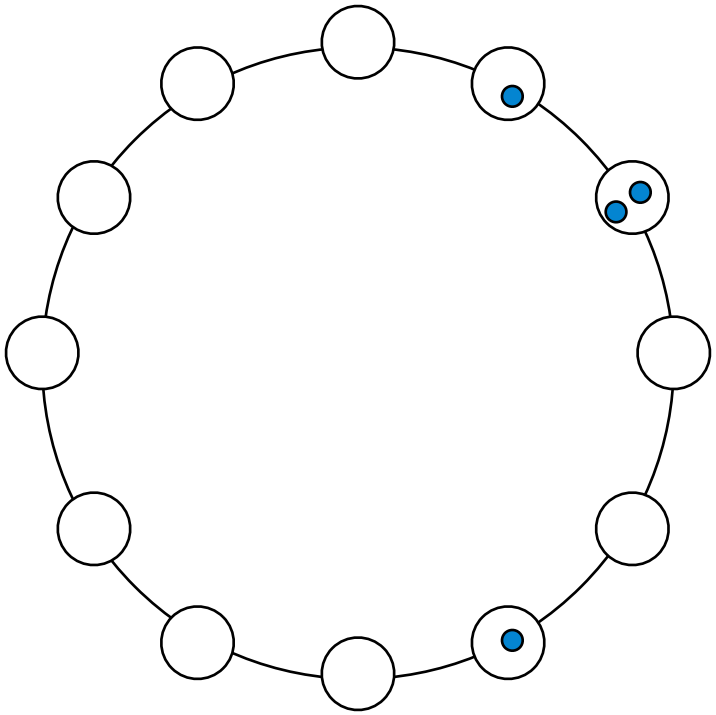

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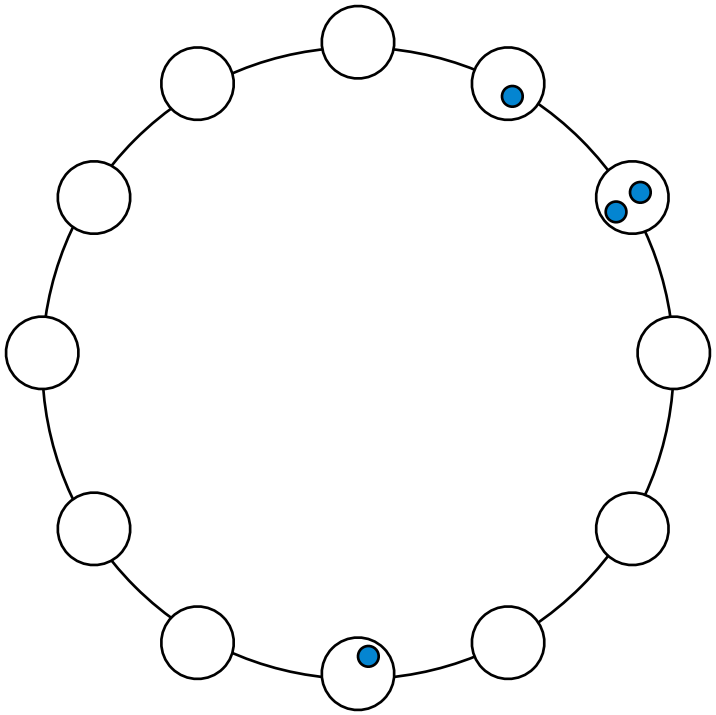

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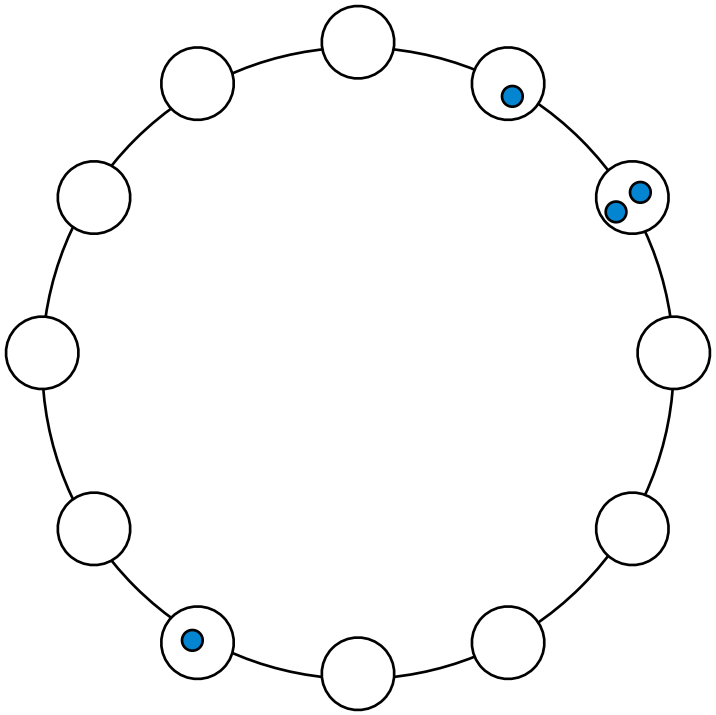

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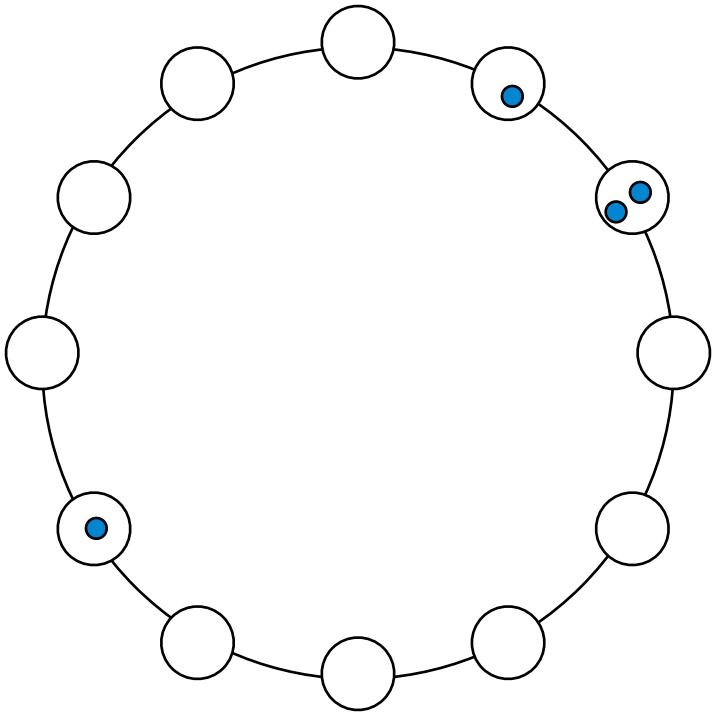

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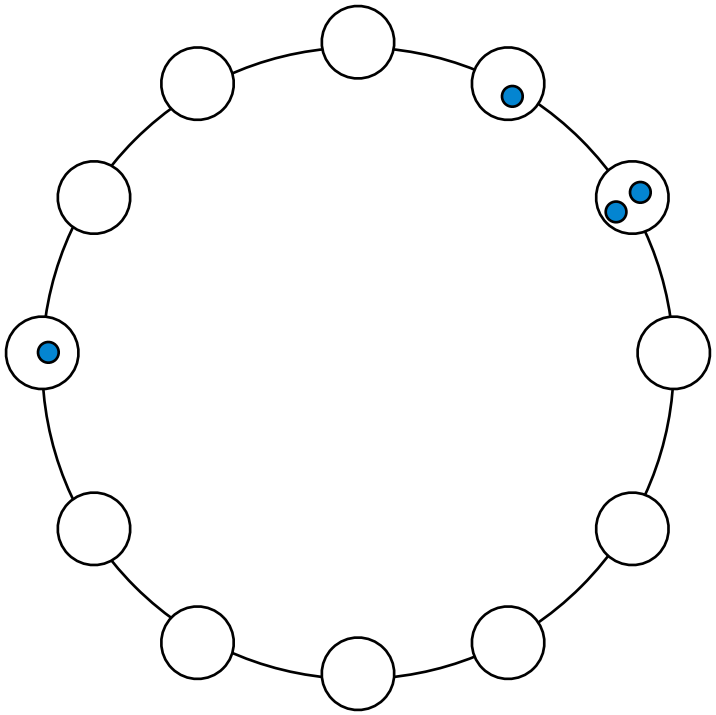

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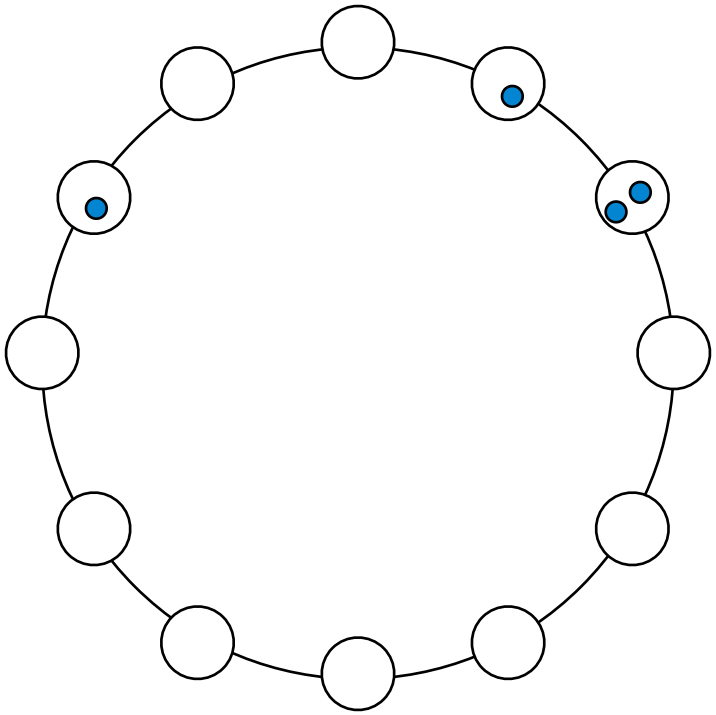

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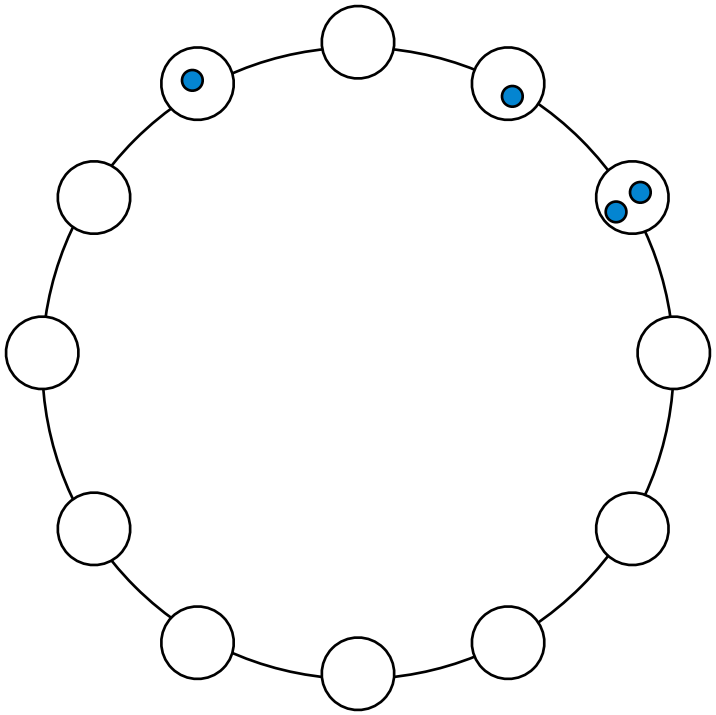

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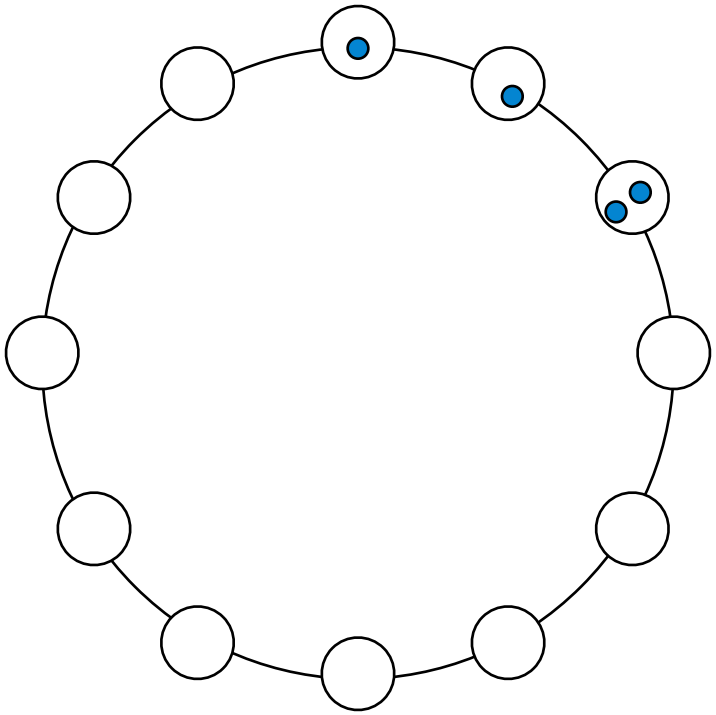

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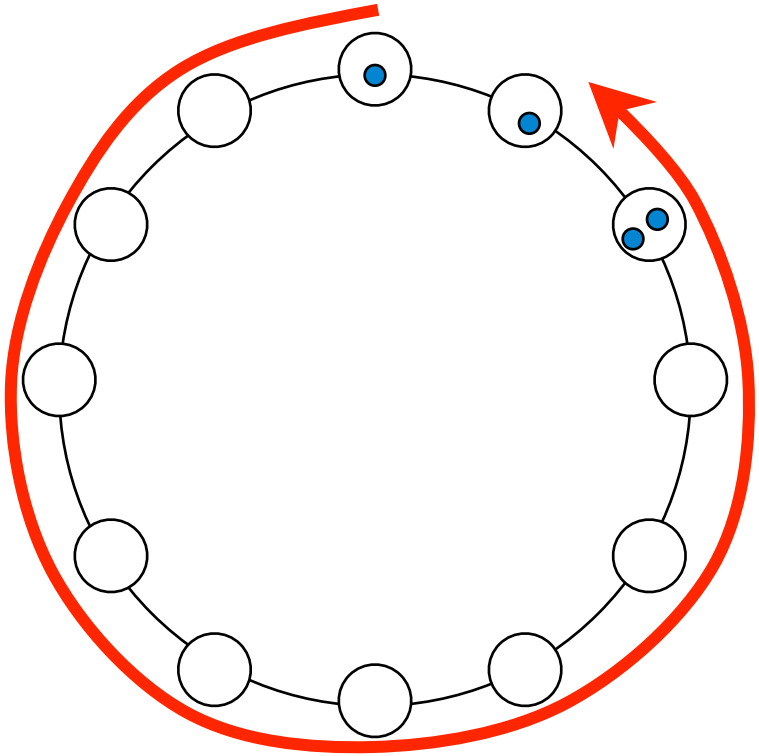

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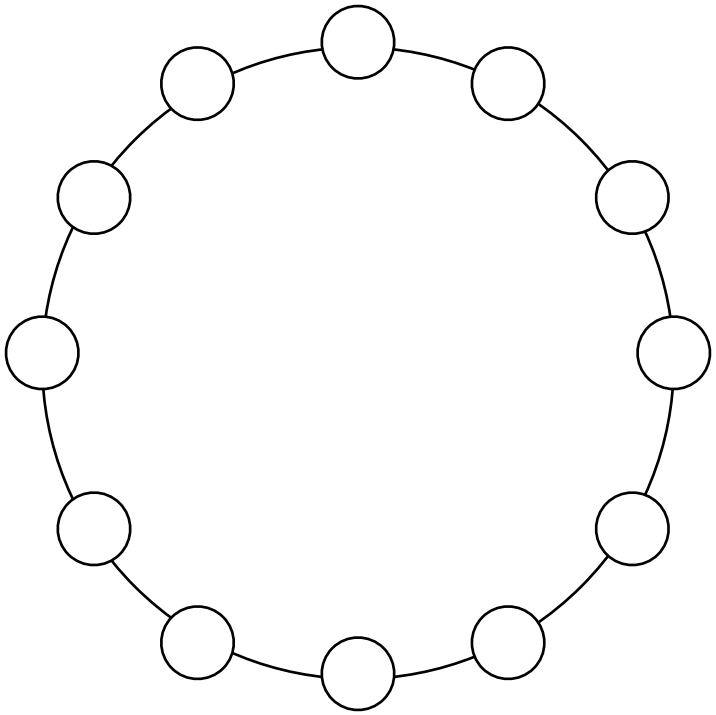

Probabilistic Algorithm

- *Initially, there is no tower*

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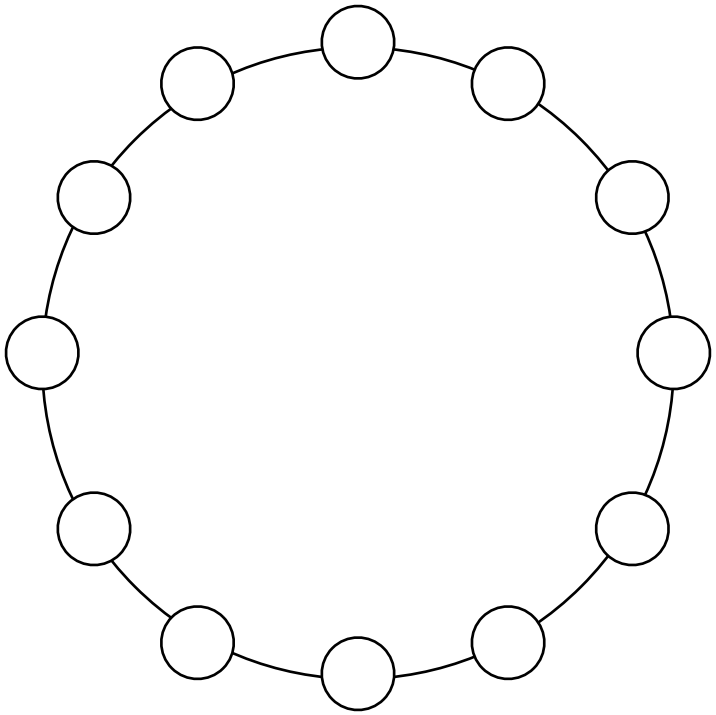
2. Build a tower ➔ Primary arrow
3. Visit the ring and terminate ➔ Final arrow



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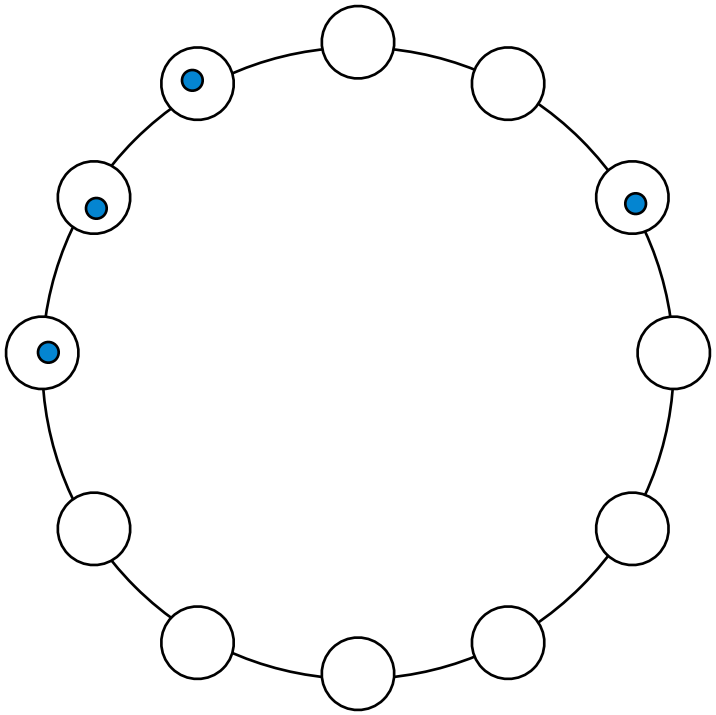
a) 3-segment



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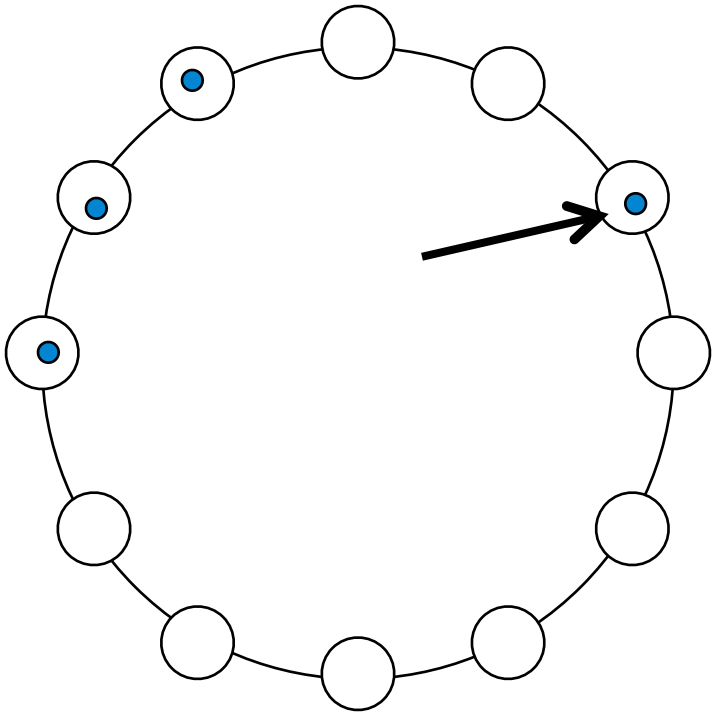

If I am the isolated node, then I move through a shortest hole.

Probabilistic Algorithm

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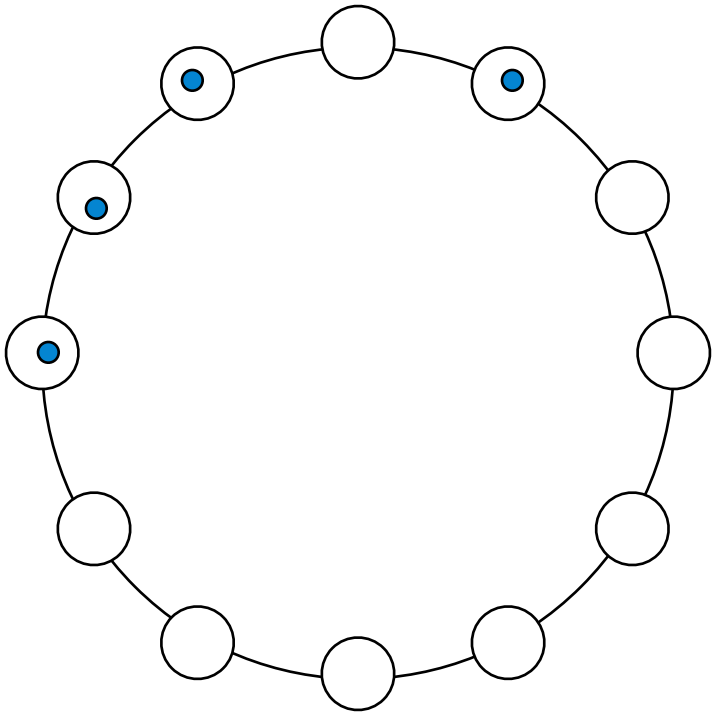

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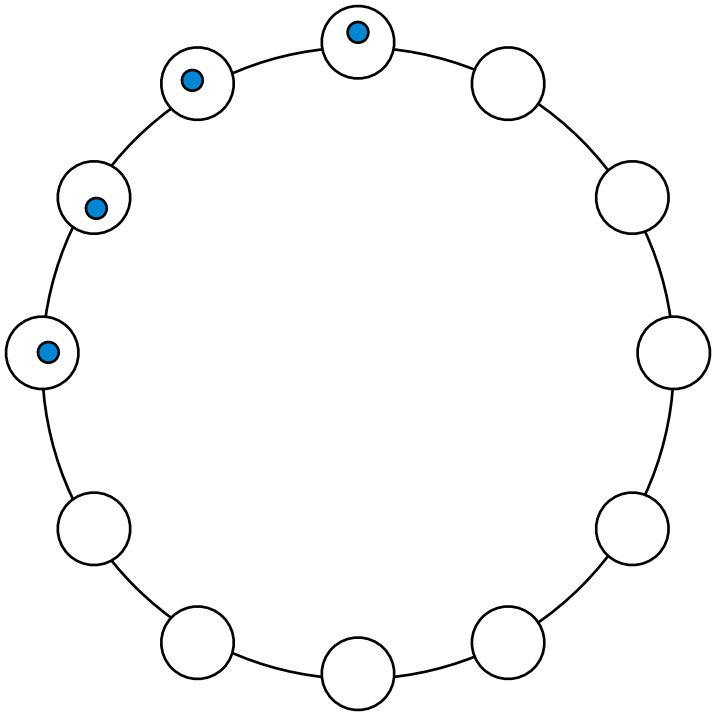

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Probabilistic Algorithm

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1. Converge toward a *4*-segment
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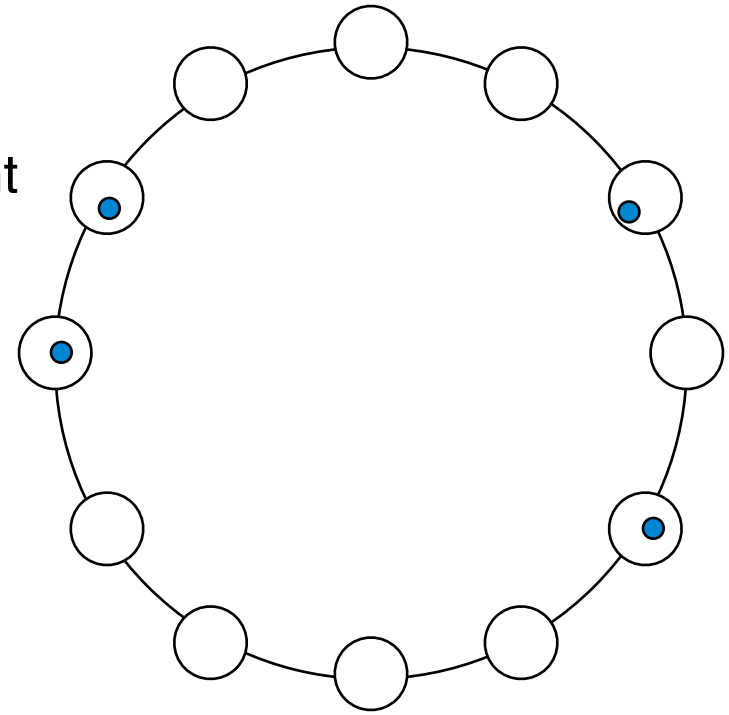
a) 3-segment
b) a unique 2-segment



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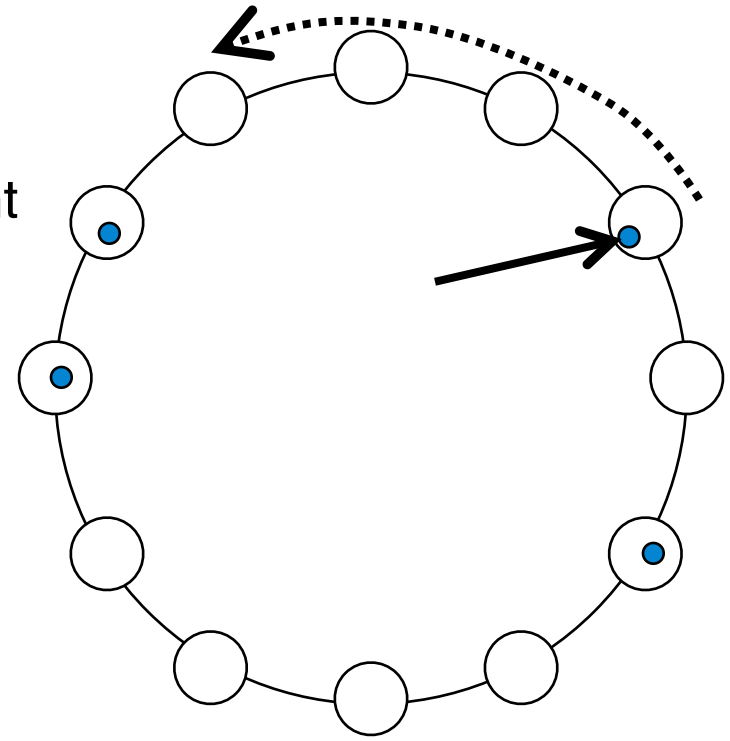
If I am at the closest distance from the 2-segment, then I move toward the closest extremity.



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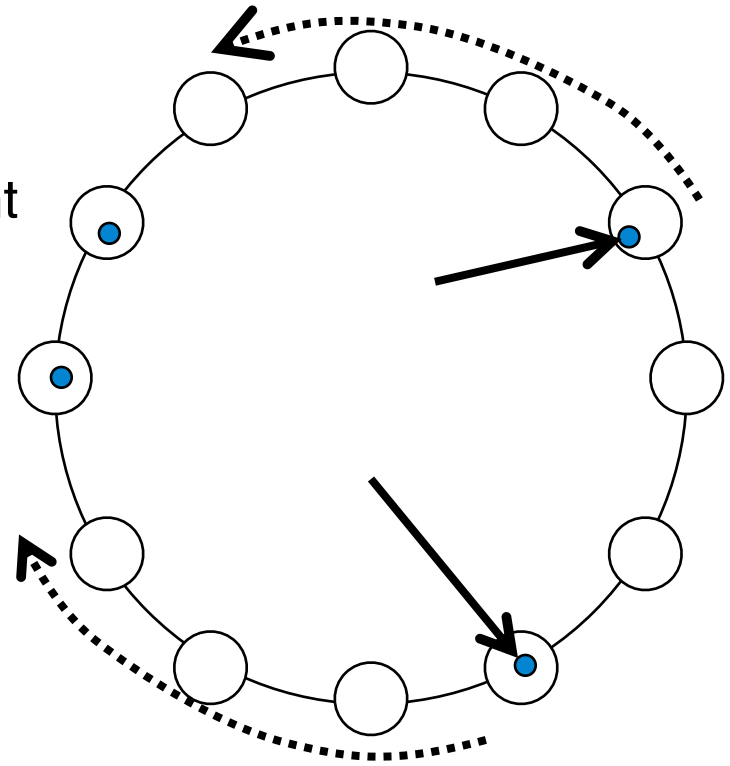

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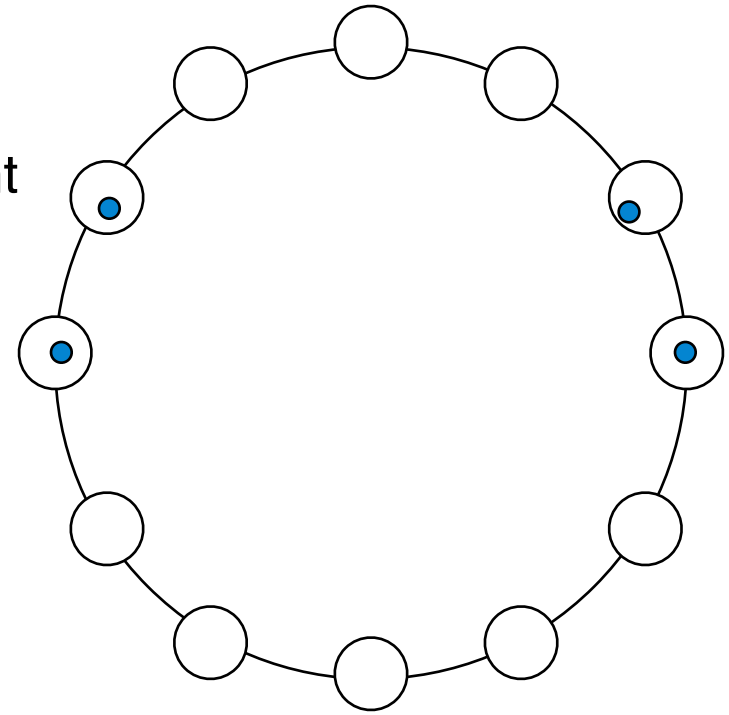

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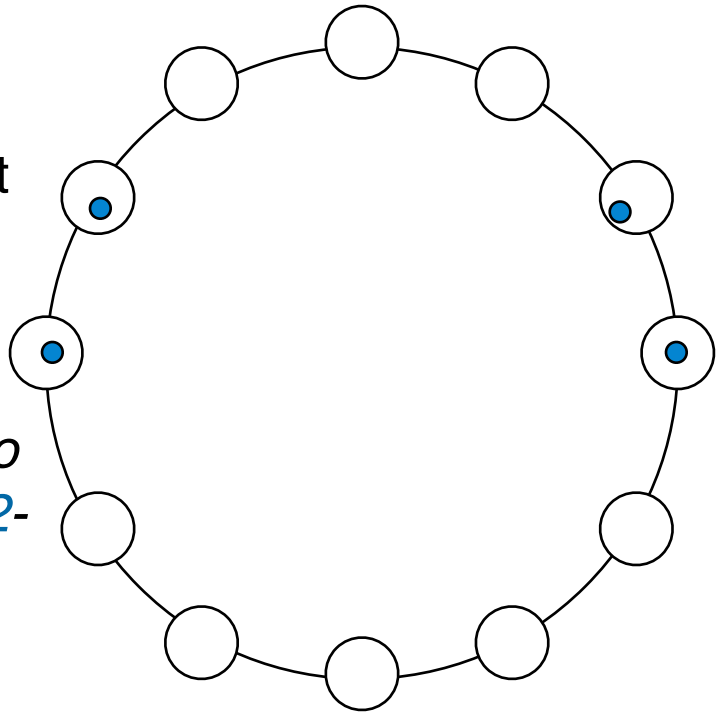
If I am a neighbor of the longest hole, then I try to move toward the other 2-segment.



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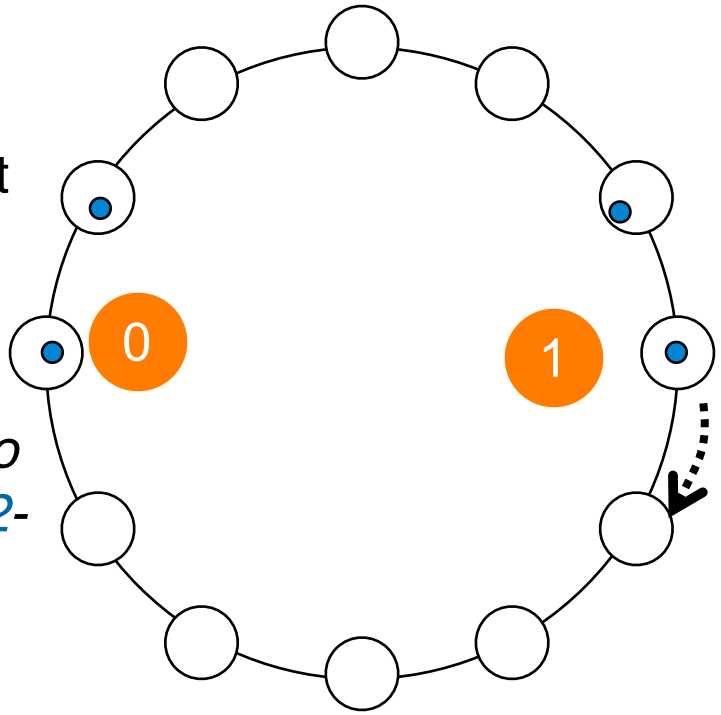

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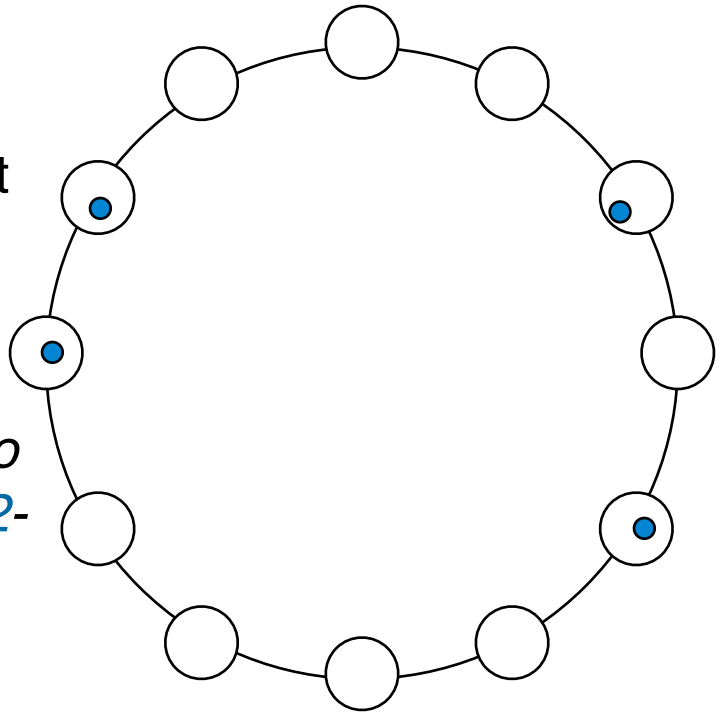

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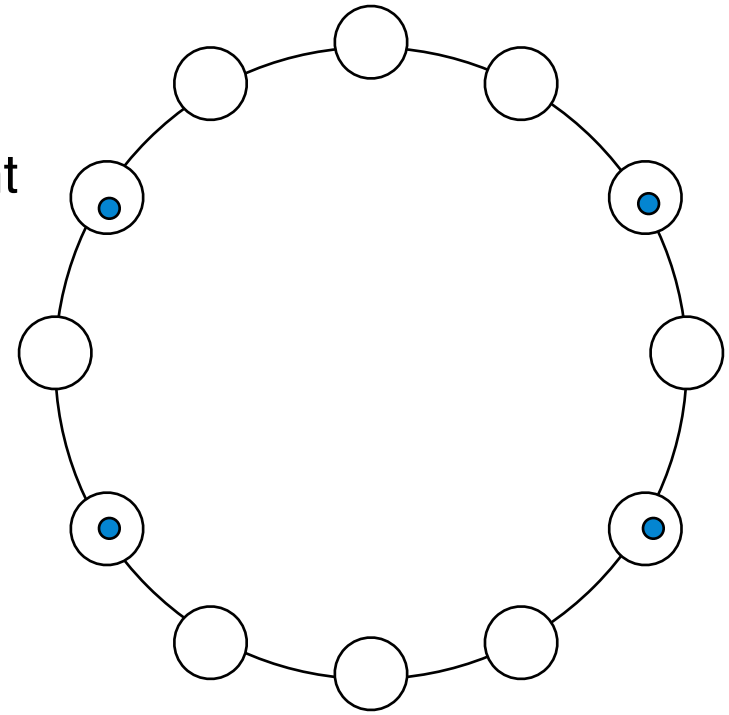

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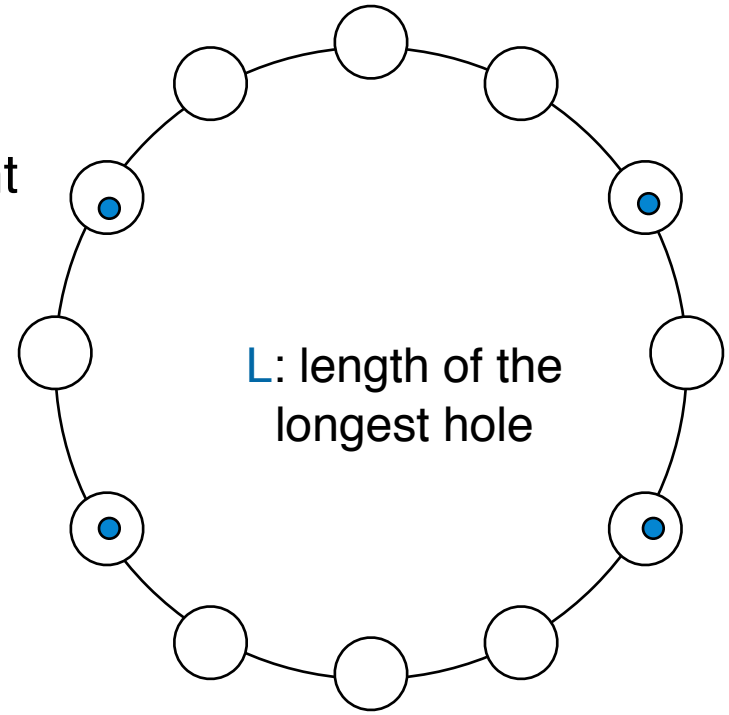

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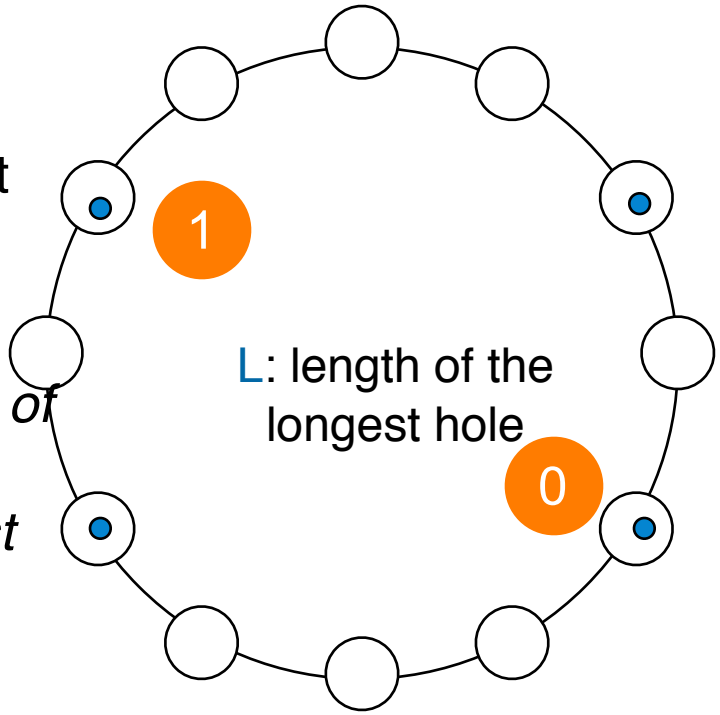
If 4 robots are neighbors of an L-hole, then I try to move through my longest neighboring hole.



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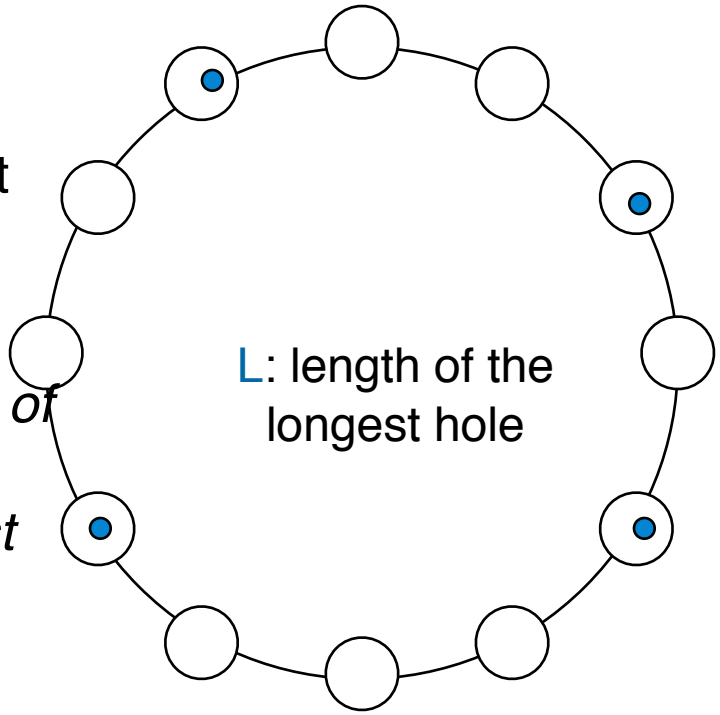

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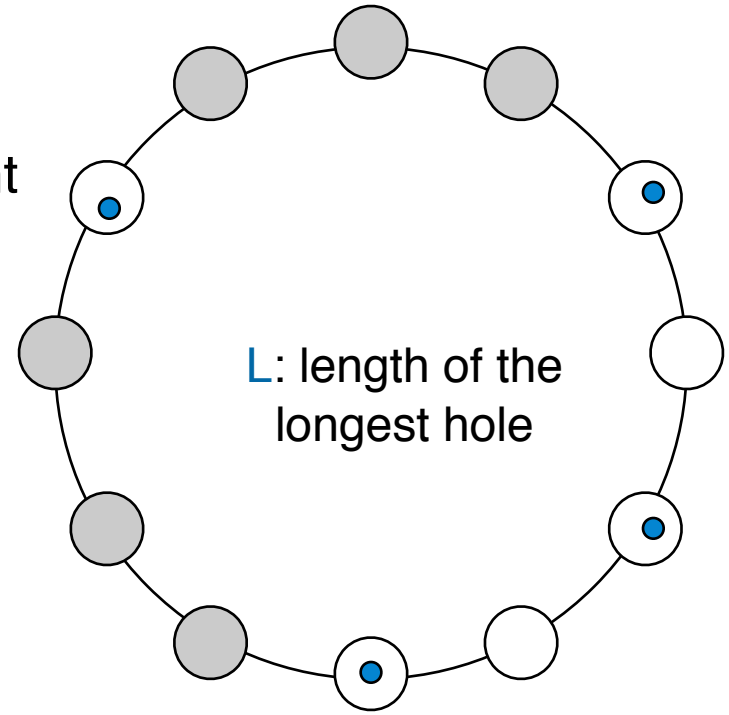

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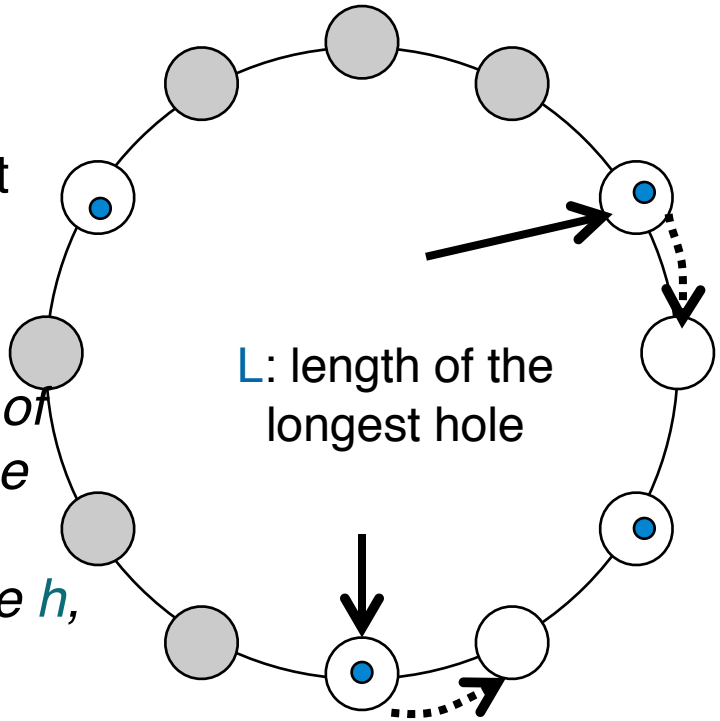
If 3 robots are neighbors of an L-hole, then if I am one of this 3 robots and a neighbor of a smaller hole h, then I move through h.



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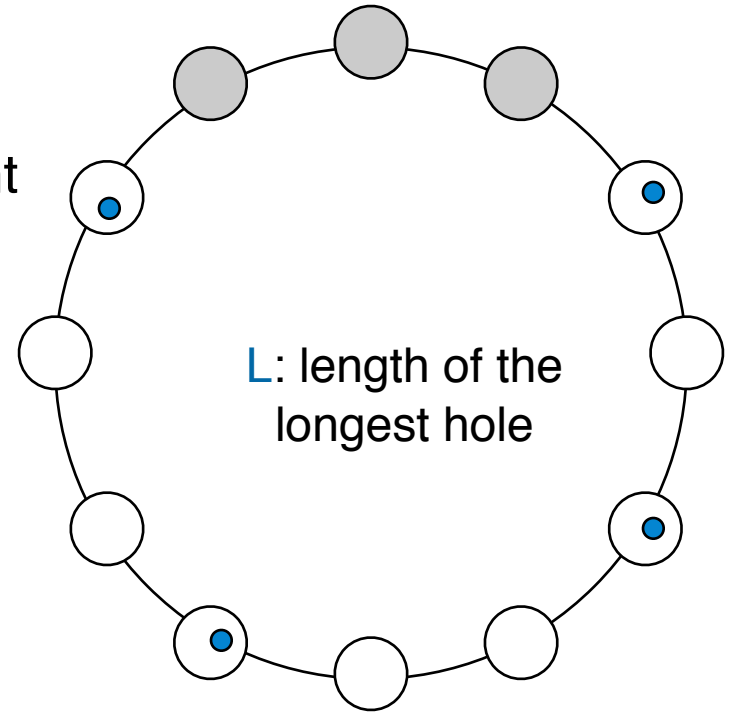

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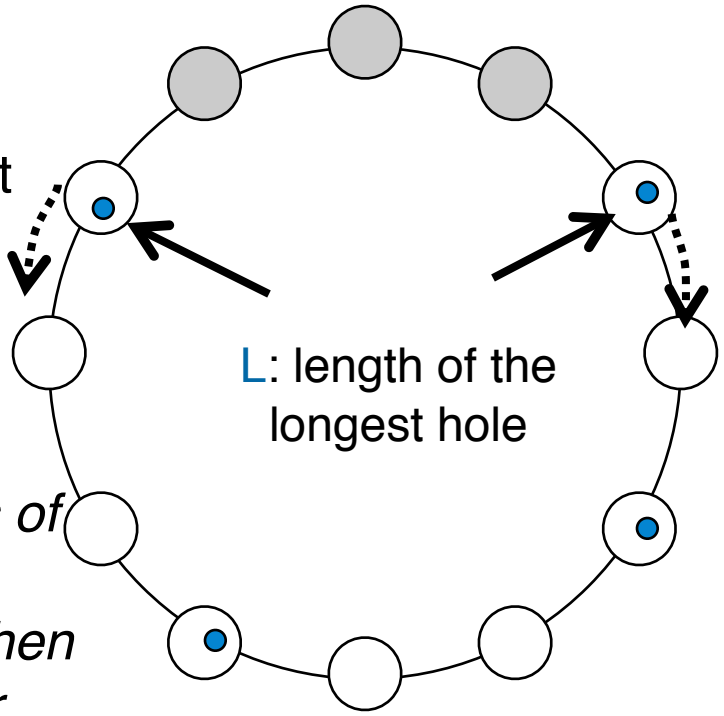
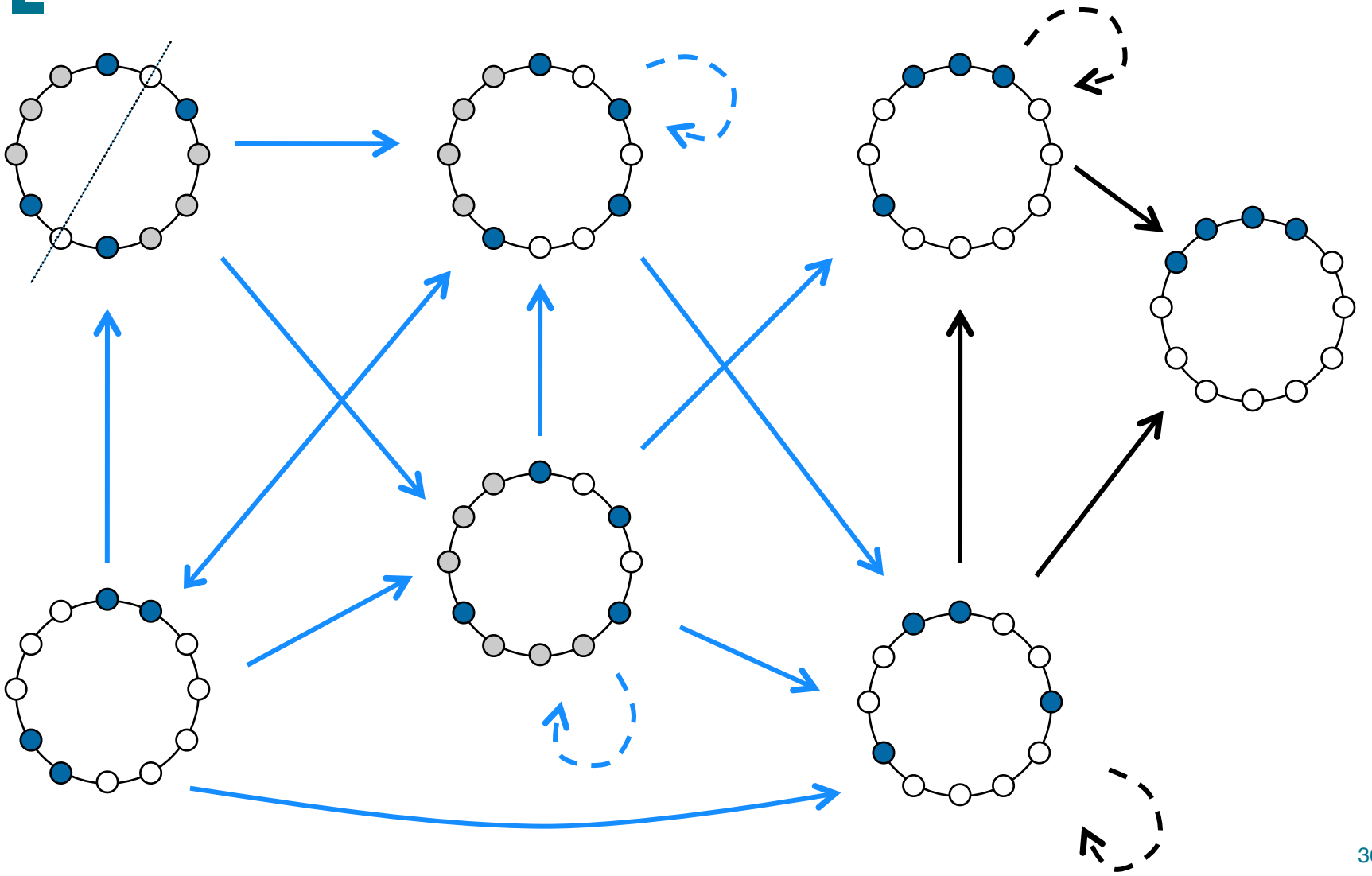


If 2 robots are neighbors of an L-hole, then if I am neighbor of the L-hole, then I move through the other

Phase 1, Summary



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Lemma.
No tower is created during Phase 1 in a n-ring with $n > 8$.

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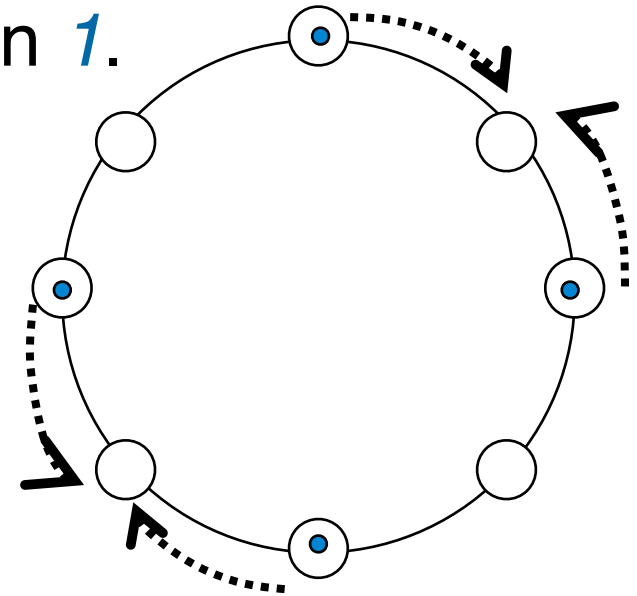
Proof Bas:

With $n > 8$ and 4 robots, there always exists a hole of length greater than 1.



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Starting from any initial configuration, the system reaches in finite expected time a configuration containing a 4-segment.

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No tower is created during Phase 1 in a n-ring with $n > 8$.

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Starting from any initial configuration, the system reaches in finite expected time a configuration containing a 4-segment.

Theorem.
The algorithm (Phases 1 to 3) is a probabilistic exploration protocol for 4 robots in a ring of $n > 8$ nodes.

cases n=5, 6, 7 are treated separately.

Conclusion

n: Number of nodes
k: Number of agents

- ✓ *Ring*

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✓ *Ring* [*Flocchini et al.*, OPODIS 2007]

- Deterministic exploration impossible if *k* divides *n* (except if *k* = *n*)
- Asynchronous deterministic algorithm with *k* >16
- Deterministic or probabilistic exploration impossible if *k* < *4*
- Probabilistic algorithm impossible in asynchronous settings
- Optimal Semi-synchronous Probabilistic Algorithm
- Deterministic exploration impossible if *k* < 5 and *n* even
- Optimal asynchronous deterministic algorithm, k = 5 and n even
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✓ *Tree* [*Flocchini et al.*, SIROCCO 2008]

- Asynchronous deterministic algorithm for trees with maximum degree equal to 3: $k \in \Theta(\log n / \log \log n)$
- Arbitrary tree: $k \in \Theta(\log n)$

✓ *Chain* [*Flocchini et al.*, IPL 2011]

- Characterization of k: $k = 3$, $k > 4$, or $k = 4$ and n odd

Conclusion

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✓ *Grid* [*Devismes et al.*, SSS 2012]

- Deterministic or probabilistic exploration impossible if $k < 2$
- Optimal Semi-synchronous Deterministic Algorithm, $k = 3$