$\boxed{\text{Measuring fitness} \quad r \text{ vs. } \lambda \text{ vs. } R_0}$

- $r = b - d$ intrinsic growth rate at low population size
- $\lambda = 1 + b - d$ ~~intrinsic~~ geometric growth rate at low population size
- $R_0 = \sum_{x=\alpha}^{\omega} l_x m_x$ basic reproductive ratio: average number of offspring to reach adulthood produced by an adult during its lifetime when population size is low.

Genotype 1

x	1	2
l_x	1	0
m_x	2	2

$\boxed{R_0 = 2}$

Genotype 2

x	1	2	3
l_x	1	1	0
m_x	0	3	3

$\boxed{R_0 = 3}$

$l_2 = 0 \Rightarrow$ no age-structure for genotype 1: all individuals are between 0 and 1.

$l_3 = 0 \Rightarrow$ two age class for genotype 2: ages 0-1 and 1-2 since no individuals survive to age 3.

Genotype 1

d: prob of dying from age 1 to 2 $\Rightarrow d = 1$

b: # offspring per individual $b = 2$.

$$N_{t+1} = b N_t - d N_t + N_t$$

$$= (2 - 1 + 1) N_t$$

$$\Rightarrow \boxed{\lambda = 2}$$

Genotype 2

$$A = \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\lambda = \sqrt{3}}$$

Genotype 1 : Lower R_0, but higher λ.

- A limitation of R_0 is that it is a lifetime measure : genotypes with long generation times may have high R_0's but actually increase, per unit time, slowly.

- Both calculating λ and R_0 in this way has ignored density dependence.

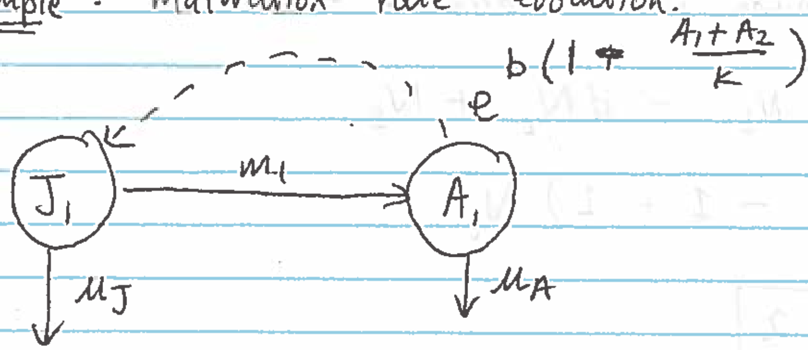
Evolutionarily stable strategies (ESS)

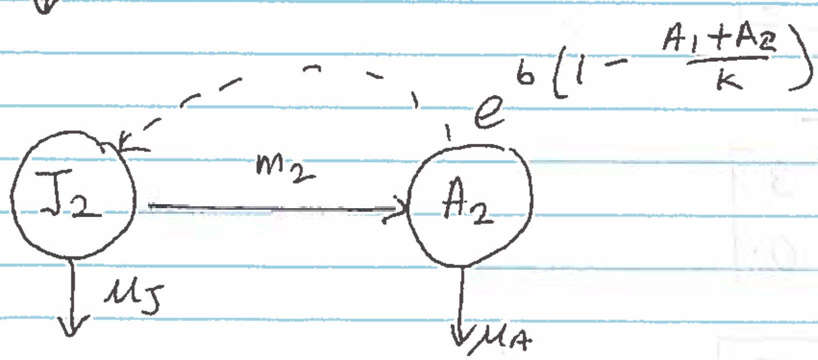
- "Strategy" here refers to a genotype/phenotype.

A strategy that if established in the population cannot be invaded by a rare mutant.

Two genotypes : mutant $\rightarrow$ rare
resident $\rightarrow$ abundant.

Example : Maturation rate evolution.





$i \in \{1, 2\}$

J_i : # of juveniles (non-reprod.) of genotype i
A_i : # of adults (reprod.) of genotype i
m_i : maturation rate of genotype i

large m_i $\Rightarrow$ low age at first reproduction.

μ_A and μ_J : natural mortality rates.

$e^{b\left(1 - \frac{A_1 + A_2}{k}\right)}$: per capita reproductive rates.

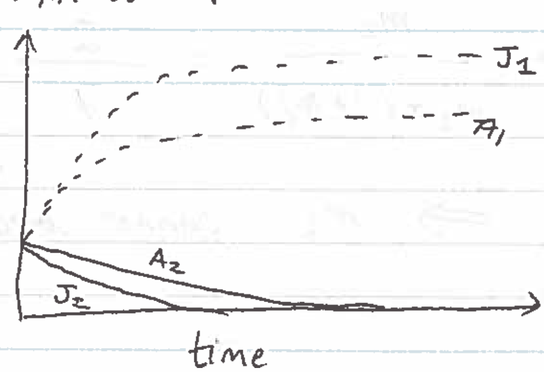
This model can be written as a system of differential equation which has an equilibrium.

$$\left.\begin{aligned} J_1^* &= \frac{\mu_A}{m_1} A_1^* \\ A_1^* &= k\left(1 - \frac{\ln\left(\mu_A\left(1 + \frac{\mu_J}{m_1}\right)\right)}{b}\right) \end{aligned}\right\} \text{resident} \rightarrow \text{abundant (given appropriate parameters)}$$

$$\left.\begin{aligned} J_2^* &= 0 \\ A_2^* &= 0 \end{aligned}\right\} \text{mutant} \rightarrow \text{rare}$$

Finding ESS $\Leftrightarrow$ finding m_1 such that this equilibrium is <u>stable</u> for all $m_1 \neq m_2$

Dynamics if MFE stable

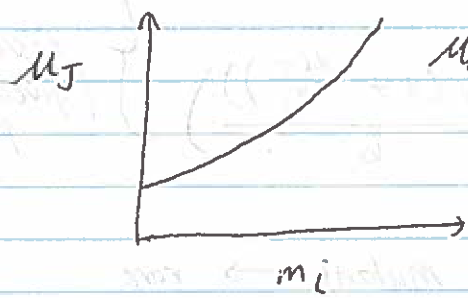


~~The~~ After some mathematical analysis, I can show that the MFE is stable if

$$\underbrace{\frac{b}{\mu_A}}_{\text{expected reproductive output as an adult}} \frac{m_1}{m_1+\mu_J} > \frac{b}{\mu_A} \underbrace{\frac{m_2}{m_2+\mu_J}}_{\text{prob becoming adult}}$$

Simplifies to $\frac{m_1}{m_1+\mu_J} > \frac{m_2}{m_2+\mu_J}$ $(*)$

Assume a trade-off where:



$\mu_J = 1 + m_i^2$

<u>Example</u> $m_1 = 1$ and $m_2 = 2$. Does m_2 invade?

$$\frac{1}{1+(1+1^2)} = \frac{m_1}{m_1+(1+m_1^2)} = \frac{1}{3} = 0.33$$

$$\frac{2}{2+(1+2^2)} = \frac{m_2}{m_2+(1+m_2^2)} = \frac{2}{7} = 0.29$$

$0.33 > 0.29 \implies m_2$ cannot invade (by $*$)

Infact, no value for m_2 will invade if $m_1 = 1$, which means that $m_1 = 1$ is the ESS.

That $m_1 = 1$ is the ESS can be show by using ~~calculatn~~ calculus to find the value of m that maximizes

$$f(m) = \frac{m}{m + (1 + m^2)}.$$