Game theory / Evolution of co-operation

- Nash equilibrium : no other strategy has higher payoff.
- Evolutionarily stable strategy : strategy / phenotype that if established in the population cannot be invaded by a rare mutant.

Let $W(x,y)$ be the payoff to an individual with a phenotype x, in a population of y-individuals (payoff or fitness)

Nash equilibrium. The strategy or phenotype, π^*, is a Nash equilibrium if

$$W(\pi, \pi^*) \leq W(\pi^*, \pi^*) \quad \text{for all} \quad \pi \neq \pi^*$$

Evolutionarily stable strategy The strategy or phenotype, π^*, is an Evolutionarily stable strategy if

$$W(\pi, \pi^*) < W(\pi^*, \pi^*)$$

or

$$W(\pi, \pi^*) = W(\pi^*, \pi^*) \quad \text{and}$$

$$W(\pi, \pi) < W(\pi^*, \pi) \quad \text{for all} \quad \pi \neq \pi^*$$

- All ESSs are also NE
- The condition for an ESS is more strict.
- In the instance were two phenotypes have equal

fitness/payoffs, then if by drift π becomes dominant, π^*, can only be an ESS if it can invade, i.e. $W(\pi,\pi) < W(\pi^*,\pi)$.

<u>Prisoners dilemma</u>

opponent

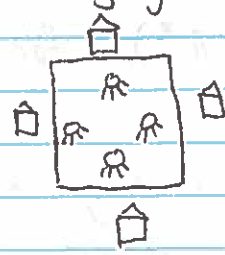
		C	D
focal	C	(2, 2)	(0, 3)
	D	($\boxed{3}$, 0)	($\boxed{1}$, 1)

□ your best response (focal)

O opponents best response.

NE is intersection of best responses : D is NE.

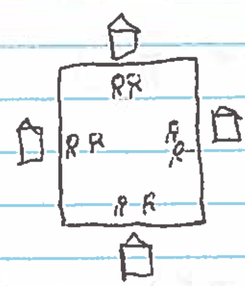
Exploitation of public goods as prisoners dilemma
— tragedy of the commons.



W(C,C) = \$100

W(C,D) = \$80

W(D,C) = \$180



W(D,D) = \$ 20

opponent

		C	D
focal	C	(100,100)	(80,180)
	D	(180,80)	(20,20)

Climate warning:

C country meets Paris agreement targets

D country pollutes.

②

		opponent C	opponent D
focal	C	modest economy stable climate	~~modest~~ modest economy climate emergency
	D	good economy ~~stable climate~~ climate emergency	good economy. climate energy

Prisoner's dilema type poblem if:

~~...~~ $W(C, D)$ lowest.
$W(D, C)$ highest.
$W(C, C) > W(D, D)$

where $F(x, y)$ is the payoff to the focal individual when the opponent plays y (or has the y phenotype).

Using the definition of a Nash equilibrium to show that ~~always ride~~ D is a Nash equilibrium.

$$W(\pi, \pi^*) \leq W(\pi^*, \pi^*) \text{ for all } \pi \neq \pi^*.$$

Let $\pi^* = D$

$$\underbrace{W(C, D)}_{0} \leq \underbrace{W(D, D)}_{1}.$$

$0 \leq 1$ and C is the only option for $\pi \neq \pi^* = D$.

Therefore, $\pi^* = D$ is a NE.

Using the definition of an ESS to show D is an ESS

$$W(\pi, \pi^*) < W(\pi^*, \pi^*) \quad \text{for all } \pi \neq \pi^*$$

$$W(C, D) < W(D, D)$$

$$0 < 1$$

There are no other strategies $\neq D$.

$\therefore$ D is an ESS.

Hawk - dove Example

opponent

focal	H	D
H	$(\frac{1}{2}, \frac{1}{2})$ Equal chance winning/injury	$(3, 0)$ Hawk wins resource
D	$(0, 3)$ Dove loses resource	$(\frac{3}{2}, \frac{3}{2})$ Doves split resource.

H: agressive behavioral strategy

D: submissive behavioral strategy.

Not prisoner's dilemma since.

$W(D, H)$ lowest ~~that~~ ✓

$W(H, D)$ highest ✓

$W(H, H) < W(D, D)$. ✗

(x, y) gives pay-offs to the focal individual $= x$, and to the opponent $= y$. The interaction gives one pay-off to each interactant.

$W(x, y)$ is the payoff to x in a y-population (a population consisting of y individuals). Therefore, x is the focal individual and the payoff to the y-individuals is irrelevant, what's relevant is that the presence of y individuals affects x's payoffs.

An example where an NE is not an ESS

		opponent	
		x	y
focal	x	$(\boxed{2}, 2)$	$(0, 2)$
	y	$(\boxed{2}, 0)$	$(\boxed{3}, 3)$

□ best response of focal.

○ best response of opponent.

Two NE are x and y, both strategies are NE!

$\pi^* = y$ $\quad W(x, y) = 0 = W(\pi, \pi^*)$

$\quad\quad\quad\quad W(y, y) = 3 = W(\pi^*, \pi^*)$

$\underbrace{W(\pi, \pi^*)}_{0} < \underbrace{W(\pi^*, \pi^*)}_{3} \quad \forall \ \pi \neq \pi^*$ so y is a NE and an ESS.

$\pi^* = x$

$W(y, x) = 2 = W(\pi, \pi^*)$

$W(x, x) = 2 = W(\pi^*, \pi^*)$

$W(\pi, \pi^*) \leq W(\pi^*, \pi^*)$ for all $\pi \neq \pi^*$ so x is a NE. But is it an ESS?

need to check :

$W(\pi, \pi) < W(\pi^*, \pi)$ for all $\pi \neq \pi^*$.

$W(y, y) = 3$

$W(x, y) = 0$

$W(\pi, \pi) > W(\pi^*, \pi)$ so $\pi^* = x$ is not an ESS.