BIOL 3295: Formula sheet

Exponents

$x^m \times x^n = x^{m+n}$

$x^0 = 1$

$x^1 = x$

Logarithms

$\ln(xy) = \ln(x) + \ln(y)$

$\ln(x^t) = t \times \ln(x)$

$\ln(e^x) = x$

$\ln(e^1) = 1$

$e = 2.71828$

Geometric growth

$N_{t+1} = N_t + bN_t - dN_t$

$N_{t+1} = \lambda N_t$

Exponential growth

$\frac{dN(t)}{dt} = (b-d)N(t)$

$\frac{dN(t)}{dt} = rN(t)$

Logistic growth - continuous time

$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right)$

Logistic growth - discrete time

$N_{t+1} = N_t + \lambda N_t \left(1 - \frac{N_t}{K}\right)$

Ricker model

$N_{t+1} = N_t e^{r\left(1-\frac{N_t}{K}\right)}$

Beverton-Holt model

$N_{t+1} = \frac{\lambda N_t}{1+\frac{\lambda-1}{K}N_t}$

Allele frequency

$\frac{dp(t)}{dt} = (r_A - r_a)p(t)(1-p(t))$
$\frac{dp(t)}{dt} = sp(t)(1-p(t))$

per capita growth rate

$\frac{1}{N(t)}\frac{dN(t)}{dt}$,
$\mathrm{Ln}\left(\frac{N_{t+1}}{N_t}\right)$ or $\frac{N_{t+1}}{N_t}$

Eigenvalues for a 2 × 2 projection matrix

$\lambda_1 = \frac{a_{11}+a_{22}+\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}$,
$\lambda_2 = \frac{a_{11}+a_{22}-\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}$

Euler-Lotka equation

$\sum_{x=\alpha}^{\omega} m_x l_x dx$

Metapopulation models

$\frac{dp}{dt} = mp(1-p) - ep$	Levins (1969)
$\frac{dp}{dt} = mp(1-p) - ep(1-p)$	Hanski (1982)
$\frac{dp}{dt} = mp - ep(1-p)$	Gotelli (1991)

Diffusion coefficient

$D = \frac{2M_D(t)^2}{\pi t}$

Asymptotic spread rate

$V_F = \sqrt{4\alpha D}$

Reaction diffusion - 1 spatial dimension

$\frac{\partial N}{\partial t} = f(N) + D\frac{\partial^2 N}{\partial x^2}$

Reaction diffusion - 2 spatial dimensions

$\frac{\partial N}{\partial t} = f(N) + D\left[\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right]$

Nash equilibrium

A strategy π^* is a Nash equilibrium if,

$$W(\pi^*, \pi^*) \geq W(\pi, \pi^*) \quad \text{for all} \quad \pi \neq \pi^*$$

where $W(x, y)$ is the fitness of a focal individual with the strategy x against an opponent with strategy y.

Evolutionarily stable strategy

A strategy π^* is an evolutionarily stable strategy if,

$$W(\pi^*, \pi^*) > W(\pi, \pi^*) \quad \text{for all} \quad \pi \neq \pi^*$$

or

$$W(\pi^*, \pi^*) = W(\pi, \pi^*) \quad \text{and}$$
$$W(\pi^*, \pi) > W(\pi, \pi) \quad \text{for all} \quad \pi \neq \pi^*$$

where $W(x, y)$ is the fitness of a focal individual with the strategy x against an opponent with strategy y.