1. Write a C/C++ program to compute the linear approximation table for this S-box

In the picture below, here is the generated output of the linear approximation table.

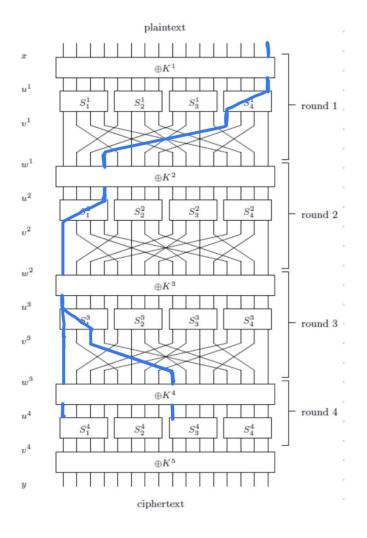
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	16	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
1	8	10	6	8	10	8	8	6	4	6	6	8	10	8	4	10
2	2 8	10	8	10	6	8	6	8	6	8	10	4	4	6	8	10
3	8	8	10	10	8	12	10	6	6	6	8	8	10	6	12	8
4	8	10	8	6	8	10	8	6	10	4	10	8	6	8	6	4
5	8	12	6	6	10	10	8	12	6	10	8	8	8	8	10	6
6	8	8	12	8	10	10	6	10	8	8	8	12	6	6	6	10
7	7 8	6	6	8	12	6	10	8	8	6	6	8	4	6	10	8
8	8	10	10	8	8	6	6	8	10	8	4	6	10	4	8	6
9	8 (8	8	12	10	10	6	10	10	6	6	6	8	12	8	8
LŒ	8 (12	10	10	6	6	12	8	8	8	6	10	6	10	8	8
11		6	12	6	8	6	8	10	4	6	8	6	8	10	8	6
12	8	8	10	10	12	8	10	6	8	12	10	6	8	8	6	6
13	8	6	8	6	6	12	10	8	8	10	4	6	6	8	6	8
4	8	6	6	12	6	8	8	10	6	8	8	10	8	6	6	4
.5	8	8	8	8	8	8	12	12	10	6	10	6	10	6	6	10

The C++ code will be provided as a different file.

2. Find a linear approximation using three active S-boxes and use the piling-up lemma (you need to research about it) to estimate the bias of the random variable

$$X_{16} \oplus U_{1}^{4} \oplus U_{4}^{9}$$

In order to find the linear approximation using three active S-Boxes, we first have to indicate what the Active S boxes are. Given the random variables, we can find the corresponding lines within the given SPN. The blue lines drawn in the figure are the paths that show the active S-Boxes.



So the 3 active S-Boxes with the bias according to the LAT are:

$$S_{4}^{1}$$
 = (The random Variable T_{1} = $U_{6}^{1} \oplus V_{13}^{1}$ has bias -1/4)

$$S_1^2 = (The random Variable T_2 = U_4^2 \oplus V_1^2 has bias -1/4)$$

$$S_1^3 = \text{(The random Variable T}_3 = U_1^3 \oplus V_1^3 \oplus V_3^3 \text{ has bias -1/4)}$$

The piling Lemma is defined as
$$2^{n-1} \prod \epsilon_i = 2^{3-1} * -\frac{1}{4} * -\frac{1}{4} * -\frac{1}{4} = -\frac{1}{16}$$

3. Implement a linear attack to find the eight sub-key bits in the last round (corresponding to the linear approximation in (b)). The plaintext-ciphertext pairs (encrypted with the same key) are in the le pairs (on each line in pairs the first is the plaintext and the second is the ciphertext; in decimal - convert to binary). You have to write a C/C++ program which reads the pairs from pairs le and outputs the biases for all candidate sub-keys. You pick then the right eight sub-key bits.