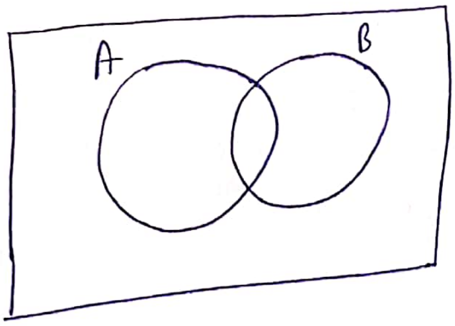
TD4.

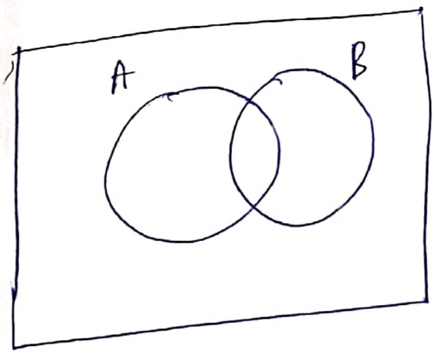
Exercice 1:

a) $A \cup B = B \Leftrightarrow A ? B$



$A \cup B = B \Leftrightarrow A \subset B$.

b) $A ? B = B \Leftrightarrow B \subset A$



$A \cap B = B \Leftrightarrow B \subset A$

c) $A \times B = \emptyset \Leftrightarrow A = \emptyset$ ou $B = \emptyset$.

d) $-3 \notin \mathbb{N}$.

e) $\mathbb{Z} \subset \mathbb{R}$

f) $-\frac{1}{2} \notin]-\frac{4}{9}, +\infty[$ car $-\frac{1}{2} < -\frac{4}{9}$.

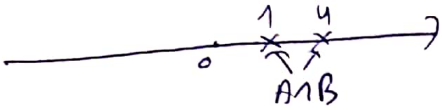
g) $\{\sqrt{2}\} \subset]1, +\infty[$ car $\sqrt{2} \in]1, +\infty[$ vu que $1 < \sqrt{2}$.

Exercice 2 :

1 - $A = \{1, 5, 4, 3, 7, 6\}$, $B = \{2, 1, 0, 4, 8\}$ et $C = \{0, 2, 8, 10\}$

On a * $A \cap B = \{1, 5, 4, 3, 7, 6\} \cap \{2, 1, 0, 4, 8\}$

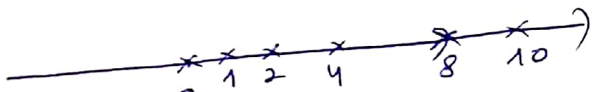
$= \{1, 4\}$



* $A \cap C = \{1, 5, 4, 3, 7, 6\} \cap \{0, 2, 8, 10\} = \emptyset$.

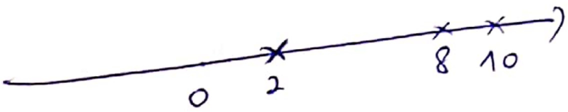
* $(A \cap B) \cup C = \{1, 4\} \cup \{0, 2, 8, 10\}$

$= \{0, 1, 2, 4, 8, 10\}$

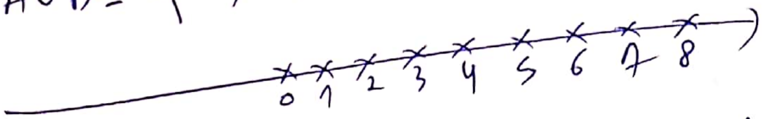


* $B \cap C = \{2, 1, 0, 4, 8\} \cap \{0, 2, 8, 10\}$

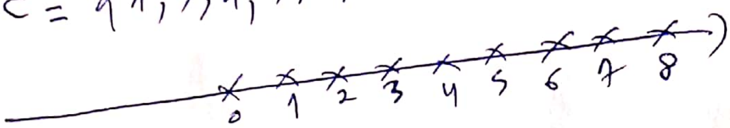
$= \{2, 8, 10\}$



* $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

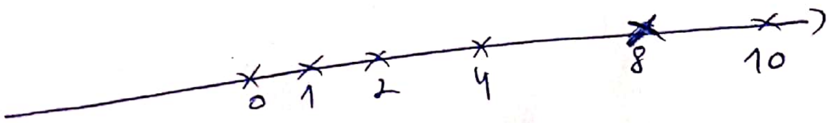


* $A \cup C = \{1, 5, 4, 3, 7, 6\} \cup \{0, 2, 8, 10\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$



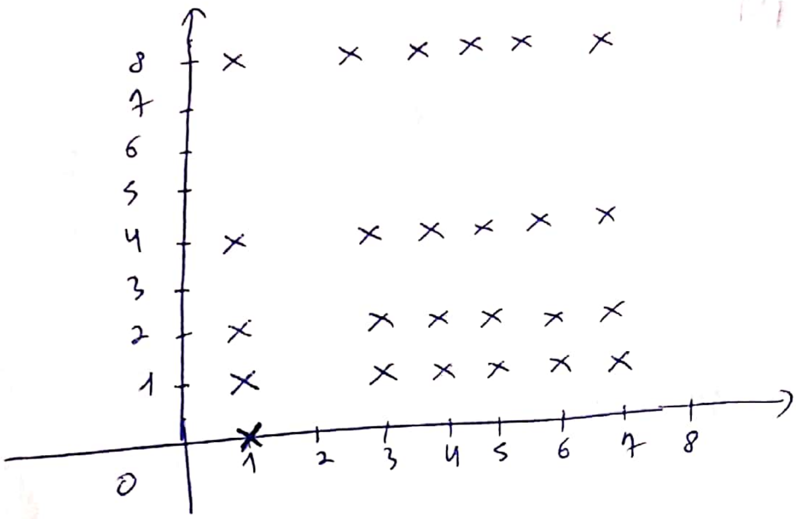
* $B \cup C = \{2, 1, 0, 4, 8\} \cup \{0, 2, 8, 10\}$

$= \{0, 1, 2, 4, 8, 10\}$



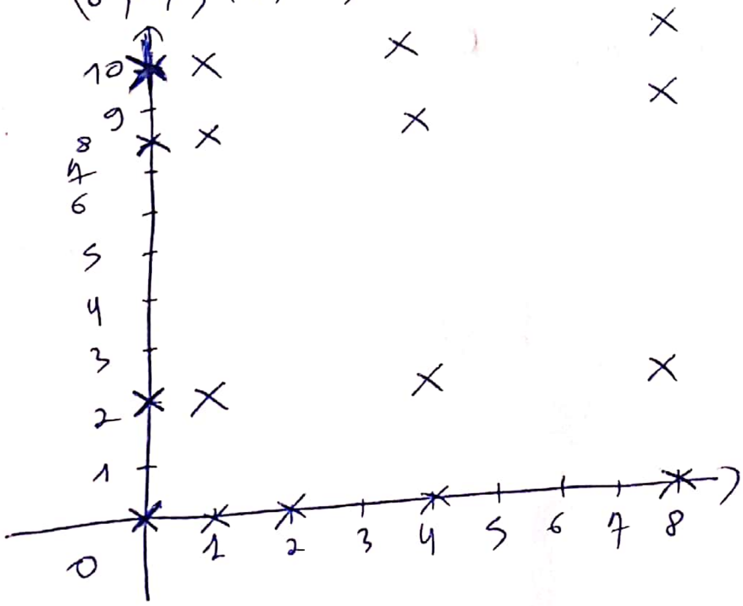
* $A \times B = \{1, 5, 4, 3, 7, 6\} \times \{2, 1, 0, 4, 8\}$

$= \{(1,0); (1,1); (1,2); (1,4); (1,8); (3,0); (3,1); (3;2); (3,4); (3;8);$
$(4,0); (4;1); (4,2); (4,4); (4,8); (5,0); (5,1); (5;2); (5,4); (5,8);$
$(6,0); (6,1); (6,2); (6,4); (6,8); (7,0); (7,1); (7;2); (7,4); (7,8)\}$

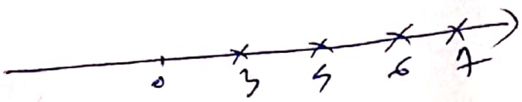


* $B \times C = \{2, 1, 0, 4, 8\} \times \{0, 2, 8, 10\}$

$= \{(0,0); (0,2); (0,8); (0,10); (1,0); (1,2); (1,8); (1,10);$
$(2,0); (2,2); (2,8); (2,10); (4,0); (4,2); (4,8); (4,10);$
$(8,0); (8,2); (8,8); (8,10)\}$

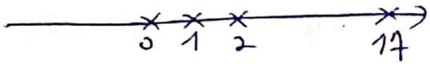


* $A \setminus B = \{1, 5, 4, 3, 7, 6\} - \{2, 1, 0, 4, 8\} = \{3, 5, 6, 7\}$

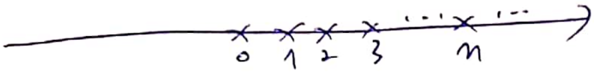


2. $A = [0, +\infty[$, $B = \{2, 1, 0, 17, -5\}$ et $C = \mathbb{Z}$.

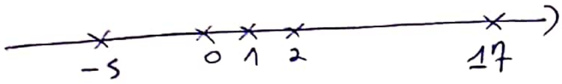
* $A \cap B = [0, +\infty[\cap \{2, 1, 0, 17, -5\} = \{0, 1, 2, 17\}$.



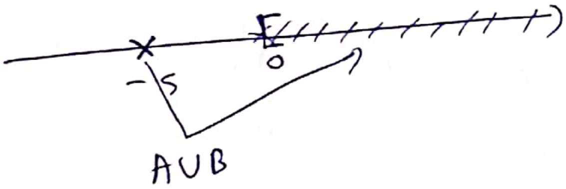
* $A \cap C = [0, +\infty[\cap \mathbb{Z} = \mathbb{N}$



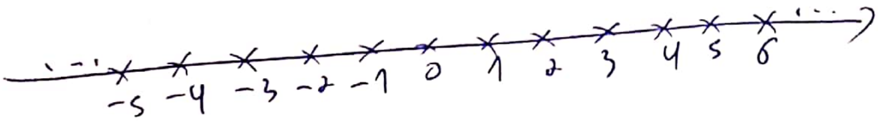
* $B \cap C = B$ car $B \subset C$.



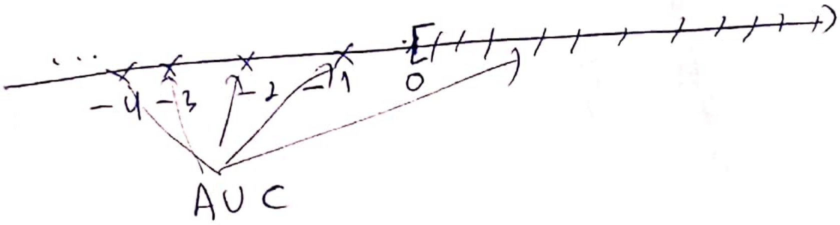
* $A \cup B = [0, +\infty[\cup \{2, 1, 0, 17, -5\}$
$= \{-5\} \cup [0, +\infty[$



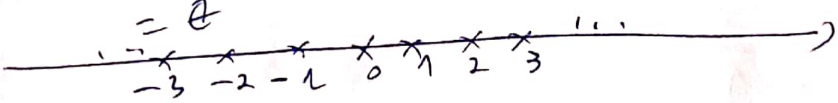
* $(A \cap B) \cup C = \{0, 1, 2, 17\} \cup C$
$= C$ car $\{0, 1, 2, 17\} \subset \mathbb{Z} = \mathbb{Z}$



* $A \cup C = [0, +\infty[\cup \mathbb{Z}$
$= [0, +\infty[\cup \mathbb{Z}^-$ où $\mathbb{Z}^- = \mathbb{Z} \setminus \mathbb{N}$.

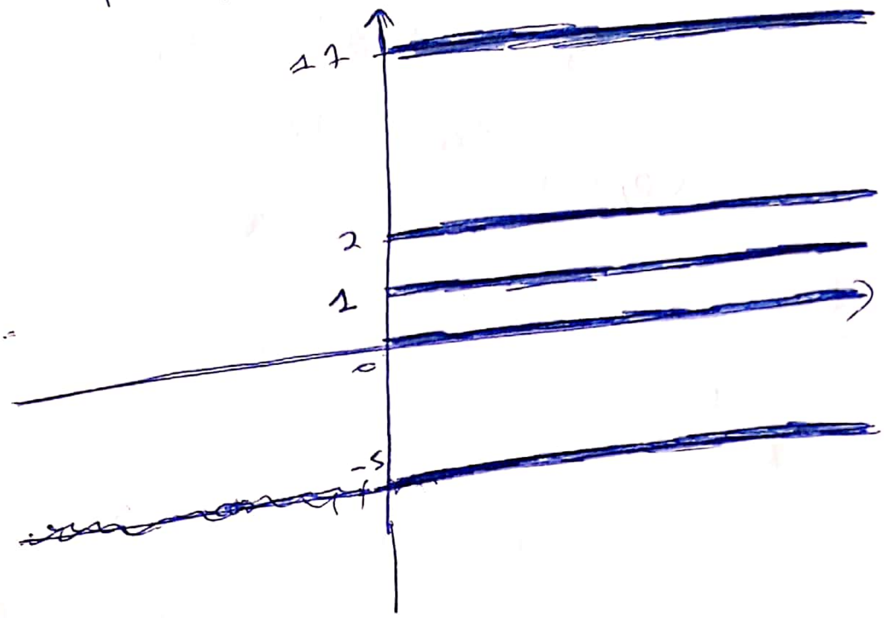


. $B \cup C = C$ car $B \subset C$
$= \mathbb{Z}$

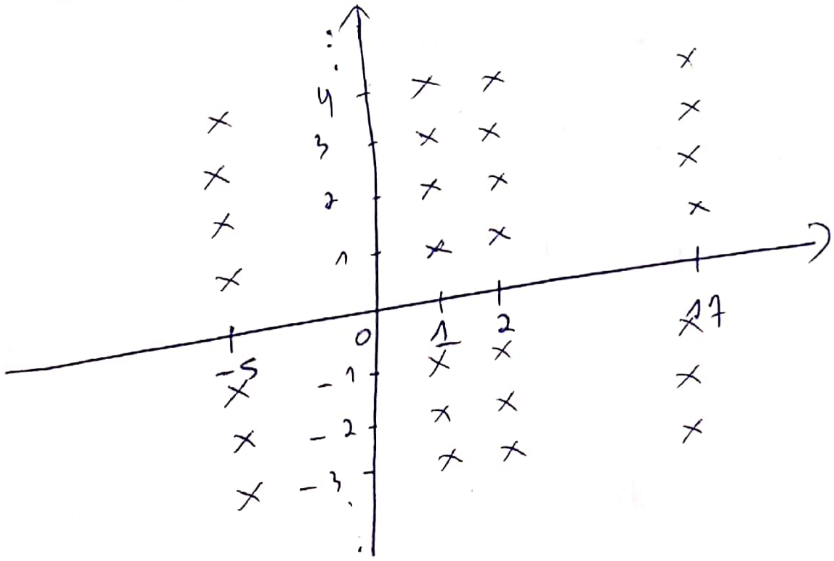


* $A \times B = [0, +\infty[\times \{2, 1, 0, 17, -5\}$

$= \{(x,0); (x,1); (x,2); (x,-5); (x,17) \mid x \geq 0\}$

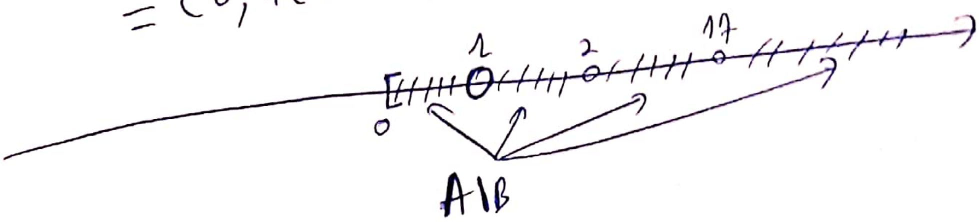


* $B \times C = \{2, 1, 0, 17, -5\} \times \mathbb{Z}$



* $A \setminus B = [0, +\infty[\setminus \{2, 1, 0, 17, -5\}$

$= [0, 1[\cup]1, 2[\cup]2, 17[\cup]17, +\infty[$



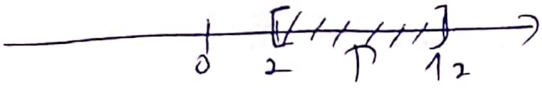
3- $A =]-\infty, 12]$, $B = [2, +\infty[$ et $C = \mathbb{N}$

* $A \cap B = [2, 12]$. En effet $A =]-\infty, 12] \cap [2, +\infty[$

$= \{x \text{ tq } x \leq 12 \text{ et } x \leq 2\}$

$= \{x \text{ tq } 2 \leq x \leq 12\}$

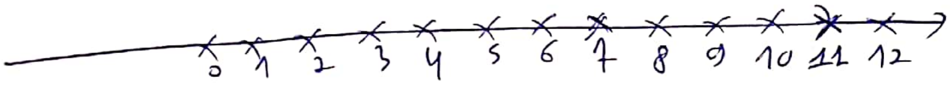
$= [2, 12]$



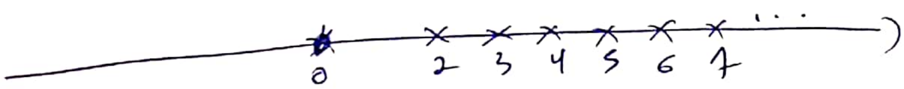
$A \cap B$

* $A \cap C =]-\infty, 12] \cap \mathbb{N}$

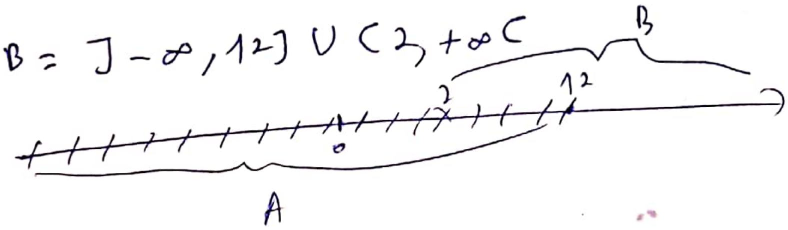
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$



* $B \cap C = [2, +\infty[\cap \mathbb{N} = \{2, 3, 4, 5, 6, 7, \dots\}$



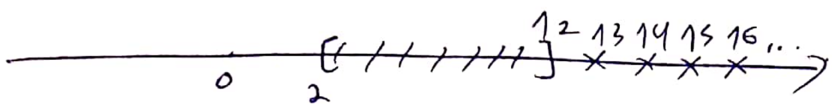
* $A \cup B =]-\infty, 12] \cup [2, +\infty[$



On a $A \cup B =]-\infty, +\infty[= \mathbb{R}$

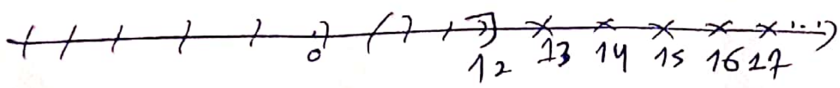
* $(A \cap B) \cup C = [2, 12] \cup \mathbb{N}$

$= [2, 12] \cup \{13, 14, 15, 16, \dots\}$



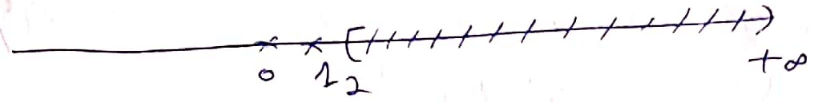
* $A \cup C =]-\infty, 12] \cup \mathbb{N}$

$=]-\infty, 12] \cup \{13, 14, 15, 16, \dots\}$

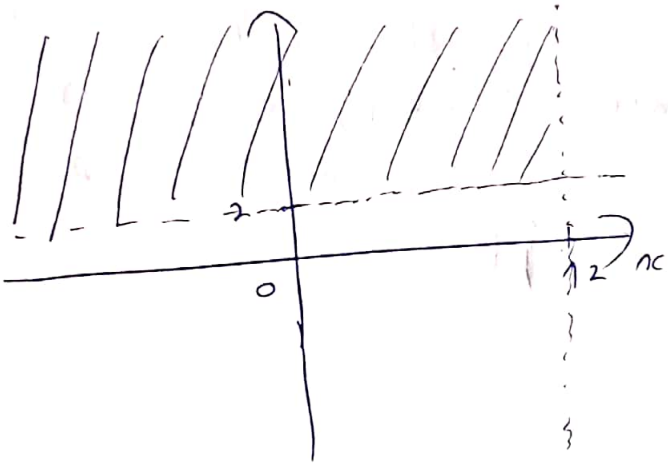


④

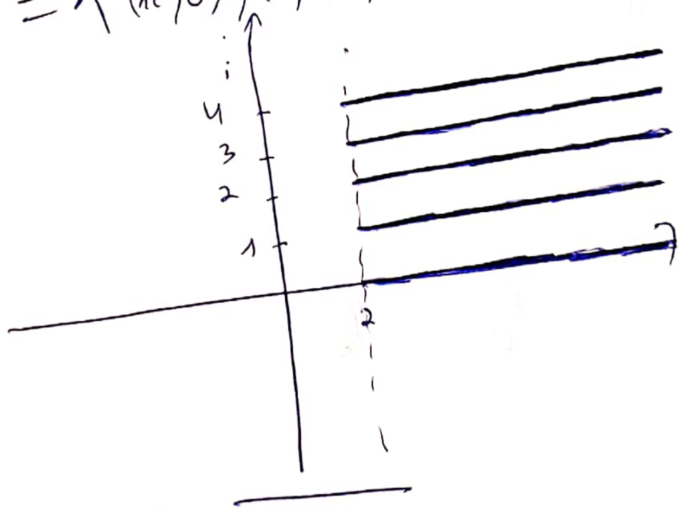
* $B \cup C = [2, +\infty[\cup \mathbb{N}$
$= \{0, 1\} \cup [2, +\infty[$



* $A \times B =]-\infty, 12] \times [2, +\infty[$
$= \{(x, y) \text{ tq } x \leq 12 \text{ et } y \geq 2\}$



* $B \times C = [2, +\infty[\times \mathbb{N}$
$= \{(x, y) \text{ tq } x \geq 2 \text{ et } y \in \mathbb{N}\}$
$= \{(x, 0), (x, 1), (x, 2), \dots, (x, 10), \dots\}$,



* $A \setminus B =]-\infty, 12] \cap \overline{[2, +\infty[}$
$= \{x \text{ tq } x \leq 12 \text{ et } x < 2\}$
$= \{x \text{ tq } x < 2\} =]-\infty, 2[$

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Exercice 3 :

- $A = \{0,1,2,3\}$

$P(A) = \{\emptyset; \{0\}; \{1\}; \{2\}; \{3\}; \{0,1\}; \{0,2\}; \{0,3\}; \{1;2\}; \{1,3\}; \{2,3\};$
$\{0,2,3\}; \{0,1,2\}; \{0,1;3\}; \{1,2,3\}; A\}$

- $B = \{0,2\} \times \{-1,0\}$

$= \{(0,-1); (0,0); (2,-1); (2,0)\}$

$P(B) = \{\emptyset; \{(0,-1)\}; \{(0,0)\}; \{(2,-1)\}; \{(2,0)\}; \{(0,-1); (0,0)\};$
$\{(0,-1); (2,-1)\}; \{(0,-1); (2,0)\}; \{(0,0); (2,-1)\}; \{(0,0); (2,0)\};$
$\{(2,-1); (2,0)\}; \{(0,-1); (0,0); (2,0)\}; \{(0,-1); (2,0); (2,-1)\};$
$\{(0,-1); (0,0); (2,-1)\}; \{(0,0); (2,-1); (2,0)\}; B\}$

Exercice 5 :

- $\text{Card } A = 4$
- $\text{Card } P(A) = 2^{\text{card } A} = 2^4 = 16$
- $\text{card}(B) = \text{card}(\{1,2,3\} \times \{-11,21,521\})$

$= \text{card}(\{1,2,3\}) \times \text{card}(\{-11,21,521\})$

$= 3 \times 3$

$= 9$

- $\text{card } C = [1,3] = +\infty$

- $\text{card } C = \text{card } \{1,12,14,15,16,28,35,50\} \times \text{card }]1,2[$

$= 8 \times +\infty = +\infty$

- $\text{card } E = 4$
- $\text{card } E \cap A = \{-5,0,2,7\} \cap \{-5,1,2,3\}$

$= \{2,-5\}$, donc $\text{card}(E \cap A) = 2$

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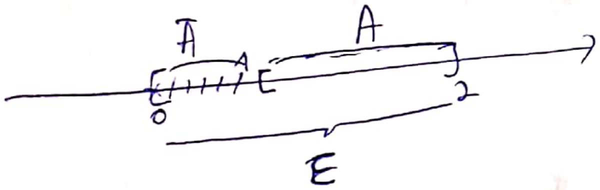
- $\text{card}(E \cup A) = \text{card}(E) + \text{card}(A) - \text{card}(E \cap A)$
$$= 4 + 4 - 2$$
$$= 6$$

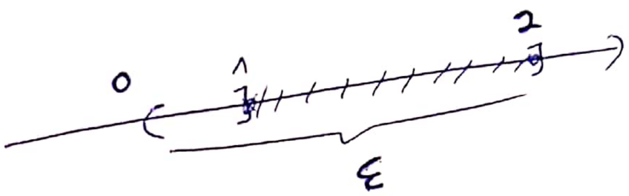
On peut aussi expliciter $E \cup A$. On a $E \cup A = \{-5, 0, 2, 7\} \cup \{-5, 1, 2, 3\}$
$$= \{-5, 0, 1, 2, 3, 7\}.$$
donc $\text{card}(E \cup A) = 6$

Exercice 8 :

1- On a $\bar{A} = \{x \in [0,2] \text{ tq } x < 1\} = [0, 1[$ | $E = [0,2]$ et $A = [1,2]$

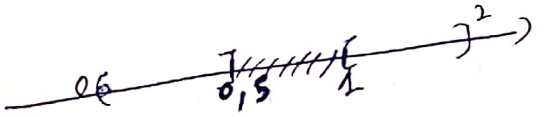


2- On a $\bar{A} = [0, 1] \cup \{2\}$ | $E = [0,2]$ et $A =]1;2[$



3- $E = [0;2]$ et $A =]0;0,5] \cup [1;2]$

On a $\bar{A} = \{0\} \cup]0,5;1[$



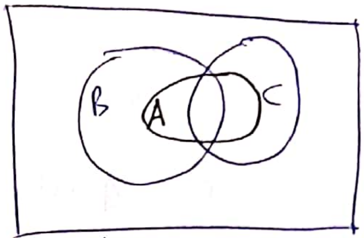
4- $E = \{0;1;2;3;4\}$

$A = \{1;2\}$

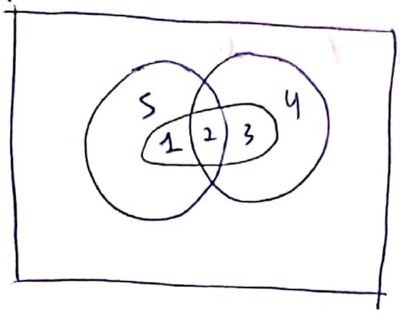
On a $\bar{A} = \{0;3;4\}$

Exercice 9 :

1. $A \subset B \cup C$ implique que $A \subset B$ ou $A \subset C$.



Non, c'est faux.



$B = \{1, 2, 5\}$ et $C = \{2, 3, 4\}$ et $A = \{1, 2, 3\}$

Alors $A \subset B \cup C = \{1, 2, 3, 4, 5\}$ mais $A \not\subset B$ et $A \not\subset C$.

2 - $P(A \cup B) = P(A) \cup P(B)$: fausse

En général pour 1) $P(A \cup B) \not\subset P(A) \cup P(B)$. Avec le même contrexemple $T = \{1, 2, 3\}$, $A = \{1, 2, 5\}$ et $B = \{2, 3, 4\}$. On a $T \in P(A \cup B)$ mais $T \notin P(A)$ et $T \notin P(B)$.

3 - $P(A \cap B) = P(A) \cap P(B)$

Pour montrer que deux ensembles E et F sont égaux, il faut et il suffit de montrer que $E \subset F$ et $F \subset E$.

- M.q $P(A \cap B) \subset P(A) \cap P(B)$; mais

Soit $E \in P(A \cap B)$. On a $E \subset A \cap B \subset A$ et $E \subset A \cap B \subset B$, donc $E \in P(A)$ et $E \in P(B)$, Ainsi $E \in P(A) \cap P(B)$.

D'où $P(A \cap B) \subset P(A) \cap P(B)$.

- M.q $P(A) \cap P(B) \subset P(A \cap B)$.

Soit $E \in P(A) \cap P(B)$, c'est-à dire $E \in P(A)$ (ou $E \subset A$) et $E \in P(B)$ (ou $E \subset B$).
Donc $E \subset A \cap B$; Ainsi $E \in P(A \cap B)$.
D'où $P(A) \cap P(B) \subset P(A \cap B)$)

Exercice 7 : Donner une partition de

1 - $A = \{-5, 1, 2, 3\}$.

$A_1 = \{-5, 1, 2\}$ et $A_2 = \{3\}$.

. $A_1 \neq \phi$, $A_2 \neq \phi$

. $A_1 \cap A_2 = \phi$.

. $A_1 \cup A_2 = A$.

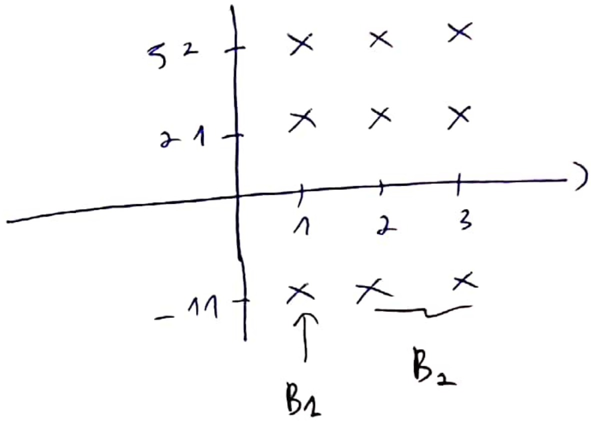
2 - $B = \{1, 2, 3\} \times \{-11, 21, 52\}$.

$B_1 = \{1\} \times \{-11, 21, 52\}$ et $B_2 = \{2, 3\} \times \{-11, 21, 52\}$

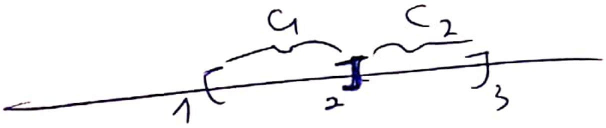
. $B_1 \neq \phi$, $B_2 \neq \phi$

. $B_1 \cap B_2 = \phi$

. $B_1 \cup B_2 = B$



3 - $C = [1, 3]$

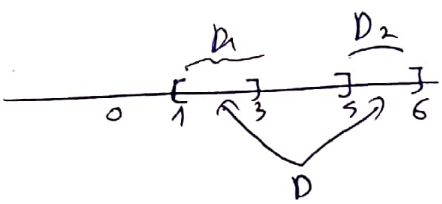


$C_1 = [1, 2]$ et $C_2 =]2, 3]$

. $C_1 \neq \phi$, $C_2 \neq \phi$

. $C_1 \cap C_2 = \phi$

. $C_1 \cup C_2 = C$



4 - $D = [1, 3] \cup]5; 6]$.

$D_1 = [1, 3]$ et $D_2 =]5, 6]$.

5 - $E = \{1, 12, 14, 15, 16, 28, 35, 50\} \times]1, 2[$.

* $E_1 = \{1, 12\} \times]1, 2[$
* $E_2 = \{14, 15, 16\} \times]1, 2[$
* $E_3 = \{28, 35, 50\} \times]1, 2[$

. On a . $E_1 \neq \phi$, $E_2 \neq \phi$, $E_3 \neq \phi$

. $E_1 \cap E_2 = \phi$, $E_2 \cap E_3 = \phi$, $E_1 \cap E_3 = \phi$

. $E_1 \cup E_2 \cup E_3 = E$