N-Body Simulation (1)

Objective is to find positions and movements of bodies in space (say planets) that are subject to gravitational forces from other bodies using Newtonian laws of physics.

The gravitational force between two bodies of masses m_a and m_b is given by

$$F = \frac{Gm_a m_b}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

Subject to forces, a body will accelerate according to Newton's second law:

$$F = ma$$

where m is the mass of the body, F is the force it experiences, and a is the resultant acceleration.

N-Body Simulation (2)

Let the time interval be Δt. Then, for a particular body of mass m, the force is given by

$$F = \frac{m(v^{t+1} - v^t)}{\Delta t}$$

and a new velocity

$$v^{t+1} = v^t + \frac{F\Delta t}{m}$$

where v^{t+1} is the velocity of body at time $t + 1$ and v^t is the velocity of body at time t.

If a body is moving at a velocity v over the time interval Δt, its position changes by

$$x^{t+1} - x^t = v\Delta t$$

where x^t is its position at time t.

Once bodies move to new positions, the forces change and the computation has to be repeated.

N-Body Simulation (3)

Three-Dimensional Space

In a three-dimensional space having a coordinate system (x, y, z), the distance between the bodies at (x_a, y_a, z_a) and (x_b, y_b, z_b) is given by

$$r = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2}$$

The forces are resolved in the three directions, using, for example,

$$F_x = \frac{Gm_a m_b}{r^2}\left(\frac{x_b - x_a}{r}\right)$$

$$F_y = \frac{Gm_a m_b}{r^2}\left(\frac{y_b - y_a}{r}\right)$$

$$F_z = \frac{Gm_a m_b}{r^2}\left(\frac{z_b - z_a}{r}\right)$$

where particles are of mass m_a and m_b and have coordinates (x_a, y_a, z_a) and (x_b, y_b, z_b).

N-Body Simulation (4)