MATH1902: SOME REMARKS ON NOTATION

Element of. The symbol $\in$ means *element of.* So if A is a set, when you write $x \in A$ you are saying that "x is an element of the set A". If $A = \{x, y, z\}$, then $x \in A$ is a true statement. If $A = \{y, z\}$, then you can still write $x \in A$, but you are just telling a lie. You should write $x \notin A$ if you want to be truthful; $\notin$ means *not an element of.*

Defined to be equal. As we know, the symbol $=$ means equals. The symbol $:=$ means *defined to be equal.* If I want to consider the whole numbers between 1 and 5 (inclusive) and I want to label this set A, then I would write $A := \{1, 2, 3, 4, 5\}$. I'm defining A to be this set. This is different from an equality that is true for other reasons. For instance, if $x := 3$ and $y := 6$, then I would write $x + y = 9$, not $x + y := 9$. This is because $x + y$ is 9 because of the particular values of x and y, not because I'm defining it to be 9. I should say that we are sometimes lazy with including the : with the = when we really should, and you will never ever lose marks in 1902 for forgetting the : or even unnecessarily adding a :.

Implication arrows. Suppose P and Q are statements; for example, P might be the statement "I eat ice cream", and Q the statement "I am happy". We write

$$P \implies Q \text{ to mean "}P\text{ implies }Q\text{";}$$

$$P \impliedby Q \text{ to mean "}P\text{ is implied by }Q\text{";}$$

and

$$P \iff Q \text{ to mean "}P\text{ is equivalent to }Q\text{", or "}P\text{ if and only if }Q\text{".}$$

In particular, $\implies$, $\impliedby$, and $\iff$ have different meanings, depending on the arrows and their directions.

Note that in our example, $P \implies Q$ is a true statement (right?!), but $P \impliedby Q$ is not because there a lots of ways to be happy.

Unions and intersections. If A and B are sets, then we write $A \cup B$ for the *union* of A and B, which is the set of elements in A *or* B; and we write $A \cap B$ for the *intersection* of A and B, which is the set of elements in A *and* B. So, for example, if $A := \{1, 2, 3\}$ and $B := \{2, 3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$ and $A \cap B = \{2, 3\}$.

Number systems. We will often refer to natural numbers, integers, rationals, real numbers, and complex numbers. We use "blackboard bold" font for the sets of these numbers:

$$\begin{aligned}
&\mathbb{N} := \{0, 1, 2, 3, \dots\};\\
&\mathbb{Z} := \{\dots, -2, -1, 0, 1, 2, \dots\};\\
&\mathbb{Q} := \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\};\\
&\mathbb{R} := \{\text{real numbers}\} = \mathbb{Q} \cup \{\text{irrational numbers}\};\ \text{and}\\
&\mathbb{C} := \{a + ib : a, b \in \mathbb{R}\}.
\end{aligned}$$

Intervals. I think this is standard. Round brackets means the number is not included, and square brackets means the number is included. So, for example, the interval $[0, 1]$ is the set of all real numbers between 0 and 1, and including 0 and 1. The interval $(2, 3]$ is the set of all real numbers between 2 and 3, including 3 but not including 2.

Set-builder notation. We use curly brackets $\{$ and $\}$ for sets. For example $\{1, 2, 3, 4, 5\}$ is a set containing the whole numbers 1 through to 5. We can also describe this set using set-builder notation:

$$\{x \in \mathbb{Z} : 1 \leq x \leq 5\}.$$

The stuff to the left of the colon tells you where your elements live, and the stuff on the right of the colon is the condition they must satisfy to live in the set. So other examples include $\{x \in \mathbb{Z} : x \geq 0\}$, which is just the set of natural numbers $\mathbb{N}$. Or $\{x \in \mathbb{R} : 2 < x \leq 3\}$, which is just the interval $(2, 3]$. Or maybe something like

$$\{f : \mathbb{R} \to \mathbb{R} : f(x) > 0 \text{ for all } x \in \mathbb{R}\},$$

which is the set of functions from $\mathbb{R}$ to $\mathbb{R}$ (see below) that are positive. In other words, their graph is always above the x-axis.

Functions. If A and B are sets, and I want to talk about a function called f that inputs values from A and outputs values from B, then I write $f\colon A \to B$. For example if we let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2$, then f is just the squaring function, satisfying, for example, $f(1) = 1$ and $f(2) = 4$.