The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH2069: Discrete Mathematics and Graph Theory — Semester 1, 2023

This assignment is worth 5% of the assessment for MATH2069. It is due **before 11:59pm on Friday 24 March**. Please upload your assignment as a single PDF file to Canvas. Poorly scanned or otherwise difficult to read assignments may not be marked.

To be awarded full marks, all working must be shown and your reasoning be explained.

All answers should be given in the final form as natural numbers.

1. In all parts of this question we consider the numbers (positive integers) which contain digits only from amongst $5, 6, 7, 8, 9$ with possible repetition.

(a) What is the number of such positive integers in which all digits are different?

(b) What is the number of 7-digit numbers in which the digits weakly increase from left to right?

(c) Now take an arbitrary 52-digit number N and let n_i be the number of times the digit $i \in \{5, 6, 7, 8, 9\}$ occurs in N. Find all possible values which the maximum of the five numbers n_5, n_6, n_7, n_8, n_9 can take.

2. Eight numbered white balls and three indistinguishable black balls are to be placed in five labelled boxes.

(a) What is the number of placements with the condition that each box contains at least one white ball?

(b) What is the number of placements where each box contains at most one black ball and none of the boxes are left empty?