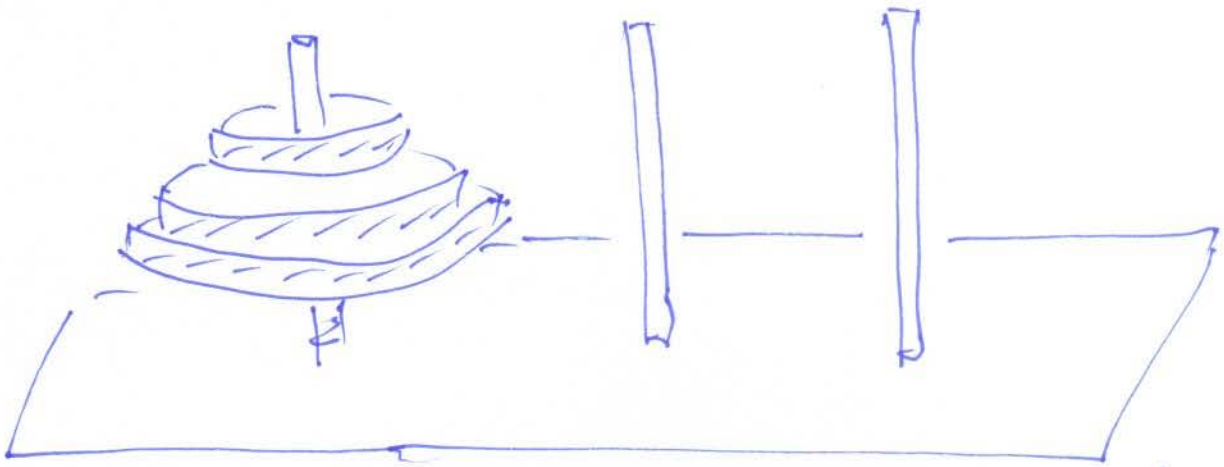
Hanoi Tower



Let h_n be the smallest number of moves required to move n disks.

$h_0 = 0, \quad h_1 = 1, \quad h_2 = 3$

Recurrence relation:

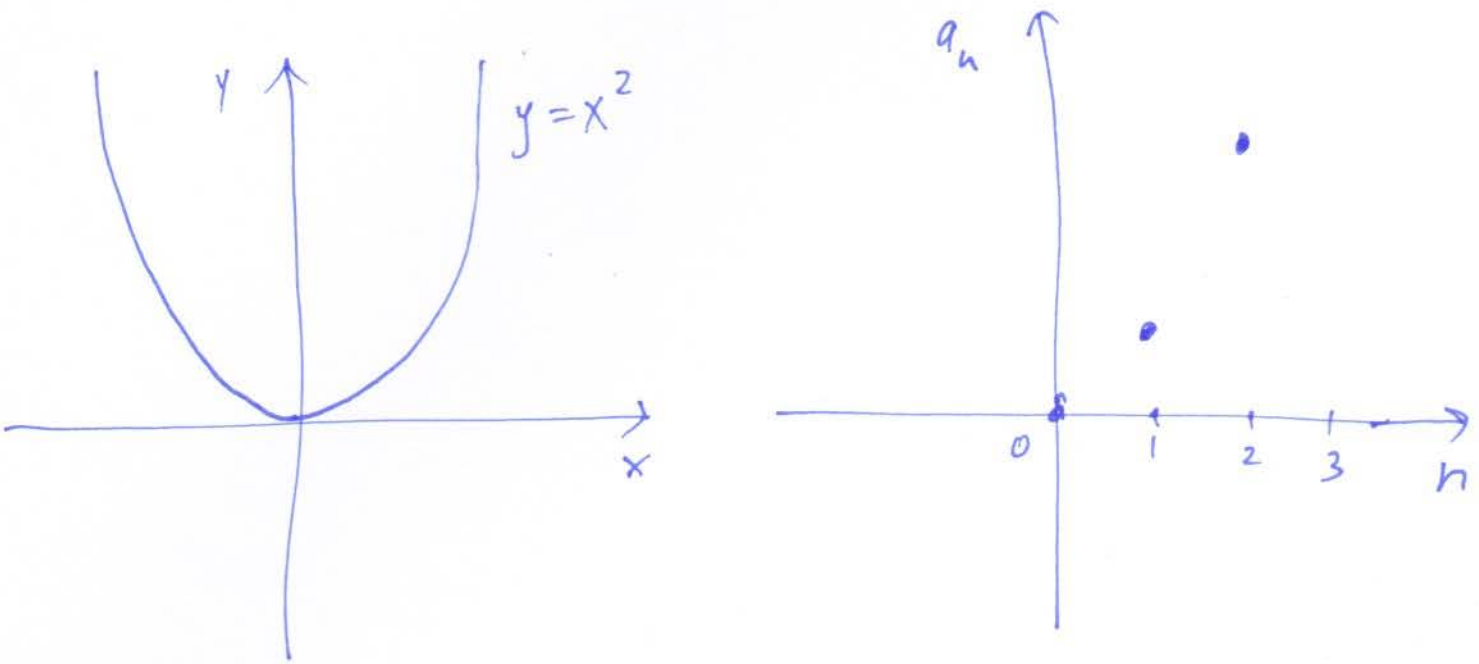
$$h_n = h_{n-1} + 1 + h_{n-1} = 2h_{n-1} + 1, \quad n \geq 1.$$

n	0	1	2	3	4	5	...
h_n	0	1	3	7	15	31	...

Conjecture: $h_n = 2^n - 1$.

Verify by substituting into the rel. n.

$h_n = 2^n - 1$ is an example of a sequence.



Sets

A set is a collection of objects.

$X = \{0, 1, 5\}$, $5 \in X$, $|X| = 3$,

the cardinality (size) of X is 3.

$\mathbb{N} = \{0, 1, 2, \ldots\}$, natural numbers.

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Q} = \{ \frac{m}{n} \mid n \in \mathbb{N},\ n \neq 0,\ m \in \mathbb{Z} \}$.

$\mathbb{R}$ = set of real numbers.

$S = \{ \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R} \}$,

$|S| = 4$.

Subsets: $A \subset X$ means A is a subset of X.

Notation:

$\emptyset$ is the empty set.

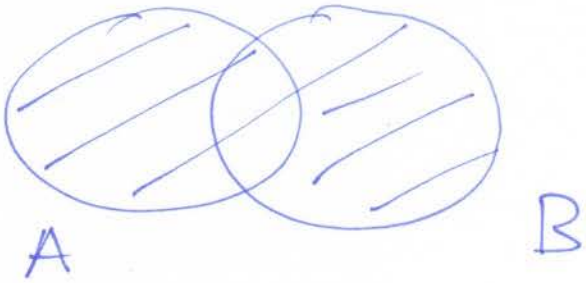
$|\emptyset| = 0$. Note: $\emptyset \subset X$ for any set X.

$\mathcal{P}(X)$ = power set of a set X, the set of all subsets of X.

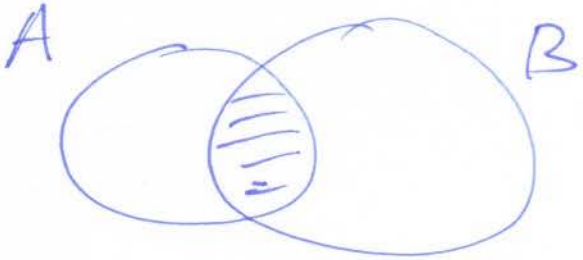
Union of sets

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Venn diagram.

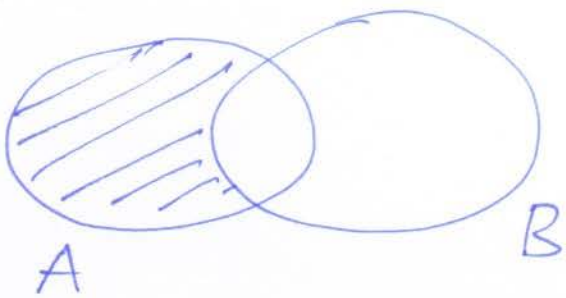


Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



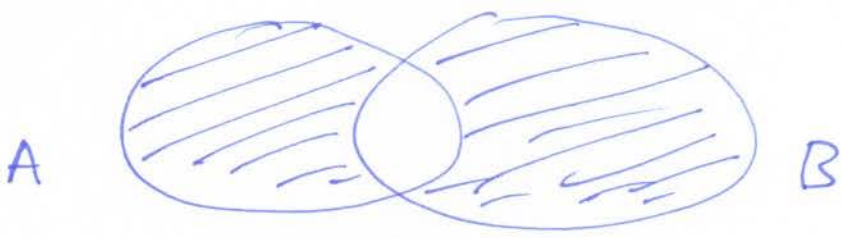
Difference:

$A \setminus B = \{x \mid x \in A \text{ but } x \notin B\}$.



Symmetric difference:

$A + B = (A \setminus B) \cup (B \setminus A)$.



Russell's Paradox

A set A is normal if it does not contain itself as an element.

Let S be the set of all normal sets.

Is S normal?

If "yes", then S does not contain S as an element, contradiction.

If "no", then S contains itself as an element, so S is normal, contradiction.