The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1902: Linear Algebra (Advanced) — Semester 1, 2023

Lecturers: Nathan Brownlowe

This **individual** assignment is due by **11:59pm on 14 March 2023**, via Canvas. A single PDF copy of your answers must be uploaded at
https://canvas.sydney.edu.au/courses/48842/assignments/441196.
Please submit only one PDF document (scan or convert other formats). It should include your SID, your tutorial time, day, room and Tutor's name. Please note: Canvas does NOT send an email digital receipt. We strongly recommend downloading your submission to check it. What you see is exactly how the marker will see your assignment.
Submissions can be overwritten until the due date. To ensure compliance with our anonymous marking obligations, please do not under any circumstances include your name in any area of your assignment; only your SID should be present. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. If you have technical difficulties with your submission, see the University of Sydney Canvas Guide, available from the Help section of Canvas.

This assignment is worth 5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall mark to your assignment using the following criteria:

Rubric

Criteria	Ratings					Pts
Correct solutions to the questions	**4 pts** **Excellent** Excellent work, answering all parts correctly. There are at most only minor or trivial errors or omissions.	**3 pts** **Very good work** Making very good progress but with one or two substantial errors, misunderstandings or omissions throughout the assignment.	**2 pts** **Good work** Making good progress, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.	**1 pts** **Fair work** A reasonable attempt, but making more than three distinct substantial errors, misunderstandings or omissions throughout the assignment.	**0 pts** **No Marks** No credit awarded.	4 pts
Clear explanations, diagrams and working shown	**1 pts** **Full Marks** Criteria met.	**0 pts** **No Marks** No clear explanations.				1 pts
						Total Points: 5

1. Let $a \in \mathbb{R}$, and let A, B, C, D be the points

$$A := (6, 2, 1),\ B := (-4, 6, 2),\ C := (-2, 5, 2) \text{ and } D = (a, a+1, 2).$$

Find the values of a such that the volume V of the tetrahedron with corners A, B, C, D satisfies $V \leq 3$. Only use formulas we have seen in the unit to get your answer.

2. Consider the interval $[-1, 1] = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$, and the set

$$C([-1, 1]) := \{\text{continuous functions } f\colon [-1, 1] \to \mathbb{R}\},$$

which is a vector space under the usual pointwise operations of addition and scalar multiplication. For $f, g \in C([-1, 1])$ we define $f \odot g \in \mathbb{R}$ by

$$f \odot g := \int_{-1}^{1} f(x)g(x)\, dx.$$

(Yes, this is just the Riemann integral you know from school, and you are free to use the basic properties of the Riemann integral to do this question.)

(a) Prove that for all $f, g, h \in C([-1, 1])$ and $a \in \mathbb{R}$ we have

i. $f \odot (g + h) = f \odot g + f \odot h$,

ii. $(af) \odot g = a(f \odot g)$,

iii. $f \odot g = g \odot f$,

iv. $f \odot f = 0 \implies f = 0$ (where $f = 0$ means $f(x) = 0$ for all $x \in [-1, 1]$).

(b) Let $f \in C([-1, 1])$ be given by $f(x) = 3x + 1$. Find the condition on $a, b \in \mathbb{R}$ such that $g \in C([-1, 1])$ given by $g(x) = ax + b$ satisfies $f \odot g = 0$.

(c) Find a formula for a function $T\colon \mathbb{R}^2 \to C([-1, 1])$ that satisfies

$$T([a, b]) \odot T([c, d]) = [a, b] \cdot [c, d] \qquad \text{for all } [a, b], [c, d] \in \mathbb{R}^2.$$

Note that the $\cdot$ on the right of the equation is just the usual dot product of vectors in $\mathbb{R}^2$.

Make sure you explain your thought process that led you to arrive at your answer, and show why your formula works.