STAT2911
Probability and Statistical Models
Tutorial 1

1. Let Ω be a sample space.

   (i) What is the smallest σ-algebra on Ω?

   (ii) Is the power set of Ω, 2^{Ω}, a σ-algebra?

   (iii) Assume $|\Omega| > 1$ (so there are at least two sample points in Ω) and construct a σ-algebra which contains exactly 4 sets.

2. Using *only the definition* of a probability measure show that

   (i) $P(\emptyset) = 0$.

   (ii) If $A_1, \ldots, A_n$ are disjoint events then $P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$.

   (iii) For any event A, $P(A) + P(A^c) = 1$.

   (iv) If $A \subset B$ then $P(A) \leq P(B)$ and conclude that for any event A, $P(A) \in [0, 1]$.

3. Prove that for any three events A, B, C

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

4. Prove that for events $A_1, \cdots, A_n$,

   $$P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i).$$

5. In five tosses of a fair coin, what is the probability of a run of three heads occurring? Note that a run of four heads in this case includes a run of three heads.

6. In a deal of 5 cards

   (i) what is the chance of a full house (3 of one value and 2 of another);

   (ii) what is the chance of a pair (2 of one value and the other three values are distinct)?