MATH 1023

E1.1

(a) (i) $f(x,y) = \cos(x^2y)$

$f_x(x,y) = -\sin(x^2y)(y)(2x) = \boxed{-2xy\sin(x^2y)}$

$f_y(x,y) = -\sin(x^2y)(x^2) = \boxed{-x^2\sin(x^2y)}$

$f_{xx}(x,y) = (-2xy)[\cos(x^2y)](2xy) + \sin(x^2y)[-2y] = -2y\sin(x^2y) - 4x^2y^2\cos(x^2y)$

$f_{yy}(x,y) = (-x^2)[\cos(x^2y)](x^2) \cancel{+ \sin(x^2y)} = -x^4\cos(x^2y)$

$f_{xy}(x,y) = \dfrac{\partial(-x^2\sin(x^2y))}{\partial x} = (-x^2)\cos(x^2y)(2xy) + (\sin(x^2y))(-2x)$

$= \cancel{-2x\sin(xy^2)}$

$= -2x\sin(x^2y) - 2x^3y\cos(x^2y)$

(ii) $df(x,y) = \dfrac{\partial f}{\partial x}dx + \dfrac{\partial f}{\partial y}dy = [-2xy\sin(x^2y)]dx + [-x^2\sin(x^2y)]dy$.

$df(a,0) = 0 - a^2\sin(0)dy = 0.$

$df(0,b) = \cancel{dx} - 0\cdot dy = 0.$

(iii) $\hat{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\sqrt{2}}{2}\hat{i} + \dfrac{\sqrt{2}}{2}\hat{j}$

$D_{\hat{u}} f(x,y) = -2xy\sin(x^2y)\cdot\dfrac{\sqrt{2}}{2} - x^2\sin(x^2y)\dfrac{\sqrt{2}}{2}$

$D_{\hat{u}} f(1,\frac{\pi}{2}) = -\cancel{2}\times\dfrac{\pi}{\cancel{2}}\times\sin(\frac{\pi}{2})\times\dfrac{\sqrt{2}}{2} - \sin(\frac{\pi}{2})\times\dfrac{\sqrt{2}}{2}$

$= -\pi\times 1\times\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}\pi.$

(b) (i) $g_x(x,y) = 2x + 2y + 2y^2$

$g_y(x,y) = 2x + 2x\cdot 2y = 2x + 4xy$

$g_x(0,0) = 0 \quad g_y(0,0) = 0.$

Thus $(0,0)$ is a critical point.

For others: since both constitutes polynomials, $g_x(x,y)$ and $g_y(x,y)$ always exist!

Set ~~both to 0~~

$\Rightarrow \begin{cases} 2x + 2y + 2y^2 = 0 & ① \\ 2x + 4xy = 0 & ② \end{cases}$

For $x \neq 0$ or $y \neq 0$:

1° Suppose $x \neq 0$, ② $\Rightarrow$ $2 + 4y = 0$ ③

$y = -\frac{1}{2}$ ④

For ① $2x + 2(-\frac{1}{2}) + 2(\frac{1}{4}) = 2x - \frac{1}{2} = 0$ ⑤

$x = \frac{1}{4}$ ⑥

Thus $(\frac{1}{4}, -\frac{1}{2})$ is another

E1.2

2° Suppose $x=0$ and $y\neq 0$

For ① $\Rightarrow$ $2y+y^2=0$ ⑦

$2+y=0$

$y=-2$ ⑧

Thus $(0,-2)$ is another one.

All in all: $(0,0)$

$(0,-2)$

$(\frac{1}{4},-\frac{1}{2})$

(ii) For $(0,0)$

$\boxed{D = f_{xx}(0,0)\, f_{yy}(0,0) - f_{xy}(0,0)^2}$

$\begin{cases} g_x(x,y) = \underline{2x+2y+2y^2} \\ g_y(x,y) = 2x+4xy \end{cases}$

For $g_{xx}(x,y) = 2$

$g_{yy}(x,y) = 4x$

$g_{xy}(x,y) = 0$

$D = 2\times(4\times 0) - 0^2 = 8 > 0$

$\because D>0$

$g_{xx}(0,0) = 2 > 0$

$\therefore$ $g(x)$ has a local minimum.

(iii) To find the global ones, we ascertain others.

$g(0,-2) = 0$

$g(\frac{1}{4},-\frac{1}{2}) = -\frac{1}{16} < 0$

$g(0,0) = 0$

$\therefore$ $g(0,0) = g(0,-2) > g(\frac{1}{4},-\frac{1}{2})$

$g(0,0)$ is a global maximum.