THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Practice Class 3

MATH2069: Discrete Mathematics and Graph Theory — Semester 1, 2023

For some questions with a numerical answer, the answer is indicated in two forms, e.g. $\frac{12}{2} = 6$*. This is to give an indication of one way to obtain the answer; in a quiz, either form on its own would suffice, e.g.* $\frac{12}{2}$ *alone or* 6 *alone.*

1. Complete the Inclusion/Exclusion formulas:

(a) $|A \cup B| =$

$$|A| + |B| - |A \cap B|$$

(b) $|A \cup B \cup C| =$

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

2. Complete the table giving the number of ways to make an unordered selection of k things from n possibilities:

repetition allowed	$\binom{n+k-1}{k}$
repetition not allowed	$\binom{n}{k}$

3. Count the subsets of the set $\{A,B,C,D,E,F,G\}$:

(a) which have 3 elements — $\binom{7}{3} = 35$

(b) which have 3 elements and contain C — $\binom{6}{2} = 15$

(c) whose intersection with $\{A,B,C,D\}$ has size 2 — $\binom{4}{2} \times 2^3 = 48$

4. How many ways are there to select, from a barrel of 45 numbered balls:

(a) six balls, and then colour one of them red — $6\binom{45}{6} = 48870360$

(b) one ball to colour red, and then five more balls — $45\binom{44}{5} = 48870360$

5. Count the four-digit numbers

(a) whose digits are two 1's and two 2's in some order — $\binom{4}{2,2} = 6$

(b) whose digits are two 1's, a 2, and a 3 in some order — $\binom{4}{2,1,1} = 12$

(c) whose digits are a 1, a 3, a 7, and a 0 in some order — $4! - 3! = 18$

(d) whose digits decrease strictly from left to right — $\binom{10}{4} = 210$

(e) whose digits increase strictly from left to right — $\binom{9}{4} = 126$

6. Suppose A, B, C are subsets of a finite set X such that

$$|A| = 30,\ |B| = 40,\ |C| = 20,\ |A \cup B| = 60,\ |A \cap C| = 5,\ |B \cap C| = 0.$$

(a) Find $|A \cup C|$. $\boxed{30 + 20 - 5 = 45}$

(b) Find $|A \cap B|$. $\boxed{30 + 40 - 60 = 10}$

(c) Find $|A \cup B \cup C|$. $\boxed{60 + 20 - 5 = 75}$

7. (It may help to know that $S(7, 3) = 301$.) How many ways can you distribute 7 packages among three people A, B, and C if:

(a) the packages are all different $\boxed{3^7 = 2187}$

(b) the packages are all different and no-one must miss out $\boxed{3!S(7, 3) = 1806}$

(c) the packages are all different and exactly one person must miss out $\boxed{3!S(7, 2) = 378}$

(d) the packages are all different and person A must get three, person B and person C two each $\boxed{\binom{7}{3,2,2} = 210}$

(e) the packages are indistinguishable $\boxed{\binom{3+7-1}{7} = 36}$

(f) the packages are indistinguishable and no-one must miss out $\boxed{\binom{3+4-1}{4} = 15}$

(g) the packages are indistinguishable and exactly one person must miss out $\boxed{3 \times 6 = 18}$

(h) the packages are indistinguishable and person A must get three, person B and person C two each $\boxed{1}$

8. For each of the following statements, write T for true or F for false.

(a) The product of any 5 consecutive integers is divisible by 120. T

(b) There are 20 ways to split 6 people into two teams of 3. F

(c) $A \cap (B \cup C) = (A \cap B) \cup C$. F

(d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. T

(e) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$. T

(f) The Stirling numbers satisfy $S(n, k) = S(n-1, k-1) + S(n-1, k)$. F

(g) For all $n \geq 1$, $S(n, n-1) = \binom{n}{2}$. T

(h) For all $n \geq 1$, $S(n, n) = \binom{n}{n}$. T

(i) There are $S(n, k)$ surjective functions $\{1, \cdots, n\} \to \{1, \cdots, k\}$. F