TECNOLÓGICO DE ESTUDIOS SUPERIORES DE ECATEPEC

DIVISIÓN DE INGENIERIA EN SISTEMAS COMPUTACIONALES

MÉTODOS NUMÉRICOS

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Grupo: 5501

“Evaluación: Ejercicios Jacobi-Gauss Seidel”

Métodos Numéricos

Fecha:

28/01/21

26/01/21

Metodos Numericos

• Reyes Mitznahuatl Brandon Jesus •

• Ejercicio •

De los siguientes sistemas de ecuaciones, obtener la solución realizando:

- 4 Iteraciones completas utilizando Jacobi
- 4 Iteraciones completas utilizando Gauss Seidel
- Realizar los programas donde se grafique el valor de cada variable y la evolución a lo largo de las iteraciones.

• Jacobi •

$$9x + 2y - z = -2$$
$$7x + 8y + 5z = 3$$
$$3x + 4y - 10z = 6$$

$$x = -2/9 - 2/9y + 1/9z = -0.222 - 0.222y + 0.11z$$
$$y = 3/8 - 7/8x - 5/8z = 0.375 - 0.875x - 0.625z$$
$$z = -6/10 + 3/10x + 4/10y = -0.6 + 0.3x + 0.4y$$

$$x = -0.222 - 0.222y + 0.11z \qquad x_0 = 0$$
$$y = 0.375 - 0.875x - 0.625z \qquad y_0 = 0$$
$$z = -0.6 + 0.3x + 0.4y \qquad z_0 = 0$$

Iteración 1

$$x_1 = -0.222 - 0.222(0) + 0.11(0) = -0.22$$
$$y_1 = 0.375 - 0.875(0) - 0.625(0) = 0.375$$
$$z_1 = -0.6 + 0.3(0) + 0.4(0) = -0.6$$

$$x_1 = -0.22$$
$$y_1 = 0.375$$
$$z_1 = -0.6$$

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Iteración 2

$x_2 = -0.22 - 0.22 y_1 + 0.11 z_1 = -0.22 - 0.22(0.375) + 0.11 (-0.6) = -0.3685$

$y_2 = 0.375 - 0.875 x_1 - 0.625 z_1 = 0.375 - 0.875 (-0.22) - 0.625 (-0.6) = 0.9423$

$z_2 = -0.6 + 0.3 x_1 + 0.4 y_1 = -0.6 + 0.3(-0.22) + 0.4(0.375) = -0.516$

$x_2 = -0.3685$

$y_2 = 0.9423$

$z_2 = -0.516$

Iteración 3

$x_3 = -0.22 - 0.22 y_2 + 0.11 z_2 = -0.22 - 0.22(0.9423) + 0.11 (-0.516) = -0.4840$

$y_3 = 0.375 - 0.875 x_2 - 0.625 z_2 = 0.375 - 0.875 (-0.3685) - 0.625 (-0.516) = 1.0199$

$z_3 = -0.6 + 0.3 x_2 + 0.4 y_2 = -0.6 + 0.3(-0.3685) + 0.4(0.9423) = 0.3336$

$x_3 = -0.4840$

$y_3 = 1.0199$

$z_3 = 0.3336$

Iteración 4

$x_4 = -0.22 - 0.22 y_3 + 0.11 z_3 = -0.22 - 0.22(1.0199) + 0.11 (0.3336) = -0.4076$

$y_4 = 0.375 - 0.875 x_3 - 0.625 z_3 = 0.375 - 0.875 (-0.4840) - 0.625 (0.3336) = 1.0082$

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$z_4 = -0.6 + 0.3x_3 + 0.4y_3 = -0.6 + 0.3(-0.4840) + 0.4(1.0199) = -0.3372$

$x_4 = -0.4076$
$y_4 = 1.0082$
$z_4 = -0.3372$

Gauss Seidel

$$9x + 2y - z = -2$$
$$7x + 8y + 5z = 3$$
$$3x + 4y - 10z = 6$$

$x = -0.22 - 0.22y + 0.11z$
$y = 0.375 - 0.875x - 0.625z$
$z = -0.6 + 0.3x + 0.4y$

$x_0 = 0$
$y_0 = 0$
$z_0 = 0$

Iteración 1

$x^1 = -0.22 - 0.22y^0 + 0.11z^0 = -0.22$
$y^1 = 0.375 - 0.875x^1 - 0.625z^0 = 0.375 - 0.875(-0.22) - 0.625(0) = 0.5675$
$z^1 = -0.6 + 0.3x^1 + 0.4y^1 = -0.6 + 0.3(-0.22) + 0.4(0.5675) = -0.439$

$x_1 = -0.22$
$y_1 = 0.5675$
$z_1 = -0.439$

Iteración 2

$x^2 = -0.22 - 0.22y^1 + 0.11z^1 = -0.22 - 0.22(0.5675) + 0.11(-0.439) = -0.3931$

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$y^2 = 0.375 - 0.875x^2 - 0.625z^1 = 0.375 - 0.875(-0.3931) - 0.625(0.439) = 0.9933$

$z_2 = -0.6 + 0.3x^2 + 0.4y^2 = -0.6 + 0.3(-0.3931) + 0.4(0.9933) = -0.0847$

$x^2 = -0.3931$

$y^2 = 0.9933$

$z^2 = -0.0847$

Iteración 3

$x^3 = -0.22 - 0.22y^2 + 0.11z^2 = -0.22 - 0.22(0.9933) + 0.11(0.0847) = -0.4292$

$y^3 = 0.375 - 0.875x^3 - 0.625z^2 = 0.375 - 0.875(-0.4292) - 0.625(0.0847) = 0.8034$

$z^3 = -0.6 + 0.3x^3 + 0.4y^3 = -0.6 + 0.3(-0.4292) + 0.4(0.8034) = -0.4074$

$x_3 = -0.4292$

$y_3 = 0.8034$

$z^3 = -0.4074$

Iteración 4

$x^4 = -0.22 - 0.22y^3 + 0.11z^3 = -0.22 - 0.22(0.8034) + 0.11(-0.4074) = -0.4415$

$y^4 = 0.375 - 0.875x^4 - 0.625z^3 = 0.375 - 0.875(-0.4415) - 0.625(-0.4074) = 1.0159$

$z^4 = -0.6 + 0.3x^4 + 0.4y^4 = -0.6 + 0.3(-0.4415) + 0.4(1.0159) = -0.3260$

$x^4 = -0.4415$

$y^4 = 1.0159$

$z^4 = -0.3260$

Ejercicio 2 ~ 26/01/21

$$5x_1 + 2x_2 = 12$$
$$-x_1 + 10x_1 = 8$$

• Jacobi •

$x_1 = {}^{12}/_5 - {}^{2}/_5 x_2 = 2.4 - 2.4x_2$
$x_2 = {}^{8}/_{10} + {}^{1}/_{10} x_1 = 0.8 + 0.1x_1$

$x_1 = 2.4 - 0.4x_2$ $\quad x_0^1 = 0$
$x_2 = 0.8 + 0.1x_1$ $\quad x_0^2 = 0$

Iteración 1

$x_1 = 2.4 - 0.4(0) = 2.4$ $\quad x_1 = 2.4$
$x_2 = 0.8 + 0.1(0) = 0.8$ $\quad x_2 = 0.8$

Iteración 2

$x_1 = 2.4 - 0.4(0.8) = 2.08$ $\quad x_2^1 = 2.08$
$x_2 = 0.8 + 0.1(2.4) = 1.04$ $\quad x_2^2 = 1.04$

Iteración 3

$x_1 = 2.4 - 0.4(1.04) = 1.984$ $\quad x_3^1 = 1.984$
$x_2 = 0.8 + 0.1(2.08) = 1.008$ $\quad x_3^2 = 1.008$

Iteración 4

$x_1 = 2.4 - 0.4(1.008) = 1.9968$ $\quad x_4^1 = 1.9968$
$x_2 = 0.8 + 0.1(1.984) = 0.9984$ $\quad x_4^2 = 0.9984$

Gauss Seidel 26/01/21

$x_1 = 2.4 - 0.4x_2$
$x_2 = 0.8 + 0.1x_1$

$x^1_0 = 0$
$x^2_0 = 0$

Iteración 1

$x_1 = 2.4 - 0.4(0) = 2.4$ $\quad x^1_1 = 2.4$
$x_2 = 0.8 + 0.1(2.4) = 1.04$ $\quad x^2_1 = 1.04$

Iteración 2

$x_1 = 2.4 - 0.4(1.04) = 1.984$ $\quad x^1_2 = 1.984$
$x_2 = 0.8 + 0.1(1.984) = 0.9984$ $\quad x^2_2 = 0.9984$

Iteración 3

$x_1 = 2.4 - 0.4(0.9984) = 2.0006$ $\quad x^1_3 = 2.0006$
$x_2 = 0.8 + 0.1(2.0006) = 1.0000$ $\quad x^2_3 = 1$

Iteración 4

$x_1 = 2.4 - 0.4(1) = 2$ $\quad x^1_4 = 2$
$x_2 = 0.8 + 0.1(2) = 1$ $\quad x^2_4 = 1$

Ejercicio 3

$8x_1 + x_2 = 4$
$2x_1 + 5x_2 = 3$
$x_1 + 4x_3 = 3$

- Jacobi -

$x_1 = 4/8 - 1/8x_2 = 0.5 - 0.125x_2$
$x_2 = 3/5 - 2/5x_1 = 0.6 - 0.4x_1$
$x_3 = 3/4 - 1/4x_1 = 0.75 - 0.25x_1$

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$x_1 = 0.5 - 0.125x_2$ $\quad x_0^1 = 0$
$x_2 = 0.6 - 0.4x_1$ $\quad x_0^2 = 0$
$x_3 = 0.75 - 0.25x_1$ $\quad x_0^3 = 0$

Iteración 1

$x_1 = 0.5 - 0.125(0) = 0.5$ $\quad x_1^1 = 0.5$
$x_2 = 0.6 - 0.4(0) = 0.6$ $\quad x_1^2 = 0.6$
$x_3 = 0.75 - 0.25(0) = 0.75$ $\quad x_1^3 = 0.75$

Iteración 2

$x_1 = 0.5 - 0.125(0.6) = 0.425$ $\quad x_2^1 = 0.425$
$x_2 = 0.6 - 0.4(0.5) = 0.4$ $\quad x_2^2 = 0.4$
$x_3 = 0.75 - 0.25(0.5) = 0.625$ $\quad x_2^3 = 0.625$

Iteración 3

$x_1 = 0.5 - 0.125(0.4) = 0.45$ $\quad x_3^1 = 0.45$
$x_2 = 0.6 - 0.4(0.425) = 0.43$ $\quad x_3^2 = 0.43$
$x_3 = 0.75 - 0.25(0.425) = 0.6437$ $\quad x_3^3 = 0.6437$

Iteración 4

$x_1 = 0.5 - 0.125(0.43) = 0.4462$ $\quad x_4^1 = 0.4462$
$x_2 = 0.6 - 0.4(0.45) = 0.42$ $\quad x_4^2 = 0.42$
$x_3 = 0.75 - 0.25(0.45) = 0.6375$ $\quad x_4^3 = 0.6375$

· Gauss Seidel ·

$x_1 = 0.5 - 0.125x_2$ $\quad x_0^1 = 0$
$x_2 = 0.6 - 0.4x_1$ $\quad x_0^2 = 0$
$x_3 = 0.75 - 0.25x_1$ $\quad x_6^3 = 0$

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Iteración 1

$x_1 = 0.5 - 0.125(0) = 0.5$ $\quad x^1_1 = 0.5$

$x_2 = 0.6 - 0.4(0.5) = 0.4$ $\quad x^2_1 = 0.4$

$x_3 = 0.75 - 0.25(0.5) = 0.625$ $\quad x^3_1 = 0.625$

Iteración 2

$x_1 = 0.5 - 0.125(0.4) = 0.45$ $\quad x^1_2 = 0.45$

$x_2 = 0.6 - 0.4(0.45) = 0.42$ $\quad x^2_2 = 0.42$

$x_3 = 0.75 - 0.25(0.45) = 0.6375$ $\quad x^3_2 = 0.6375$

Iteración 3

$x_1 = 0.5 - 0.125(0.42) = 0.4475$ $\quad x^1_3 = 0.4475$

$x_2 = 0.6 - 0.4(0.4475) = 0.421$ $\quad x^2_3 = 0.421$

$x_3 = 0.75 - 0.25(0.4475) = 0.6381$ $\quad x^3_3 = 0.6381$

Iteración 4

$x_1 = 0.5 - 0.125(0.421) = 0.4473$ $\quad x^1_4 = 0.4473$

$x_2 = 0.6 - 0.4(0.4473) = 0.4210$ $\quad x^2_4 = 0.4210$

$x_3 = 0.75 - 0.25(0.4473) = 0.6381$ $\quad x^3_4 = 0.6381$

Ejercicio 4

$$6x + 2x_2 + x_3 = 22$$
$$-x + 8x_2 + 2x_3 = 20$$
$$x_1 + x_2 + 6x_3 = 23$$

$x_1 = 22/6 - 2/6 x_2 - 1/6 x_3 = 3.66 - 0.33x_2 - 0.166x_3$

$x_2 = 20/8 + 1/8 x_1 - 2/8 x_3 = 2.5 + 0.125x_1 - 0.25x_3$

$x_3 = 23/6 - 1/6 x_1 + 1/6 x_2 = 3.833 - 0.1666x_1 - 0.1666x_2$

• Jacobi •

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$3.66 - 0.333x_2 - 0.166x_3$
$2.5 + 0.125x_1 - 0.25x_3$
$3.833 - 0.166x_1 - 0.166x_2$

$x_0^1 = 0$
$x_0^2 = 0$
$x_0^3 = 0$

Iteración 1

$x_1 = 3.66 - 0.333(0) - 0.166(0) = 3.66$
$x_2 = 2.5 + 0.125(0) - 0.25(0) = 2.5$
$x_3 = 3.833 - 0.166(0) - 0.166(0) = 3.833$

$x_1^1 = 3.66$
$x_1^2 = 2.5$
$x_1^3 = 3.833$

Iteración 2

$x_1 = 3.66 - 0.333(2.5) - 0.166(3.833) = 2.1912$
$x_2 = 2.5 + 0.125(3.66) - 0.25(3.833) = 1.9992$
$x_3 = 3.833 - 0.166(3.66) - 0.166(2.5) = 2.8104$

$x_2^1 = 2.1912$
$x_2^2 = 1.9992$
$x_2^3 = 2.8104$

Iteración 3

$x_1 = 3.66 - 0.333(1.9992) - 0.166(2.8104) = 2.5277$
$x_2 = 2.5 + 0.125(2.1912) - 0.25(2.8104) = 2.0713$
$x_3 = 3.833 - 0.166(2.1912) - 0.166(1.9992) = 3.1373$

$x_3^1 = 2.5277$
$x_3^2 = 2.0713$
$x_3^3 = 3.1373$

Iteración 4

$x_1 = 3.66 - 0.333(2.0713) - 0.166(3.1373) = 2.4494$
$x_2 = 2.5 + 0.125(2.5277) - 0.25(3.1373) = 2.03163$
$x_3 = 3.833 - 0.166(2.5277) - 0.166(2.0713) = 3.0695$

$x_4^1 = 2.4494$
$x_4^2 = 2.03163$
$x_4^3 = 3.0695$

Gauss Seidel

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$3.66 - 0.33x_2 - 0.166x_3$ $\quad x^1_0 = 0$

$2.5 + 0.125x_1 - 0.25x_3$ $\quad x^2_0 = 0$

$3.833 - 0.166x_1 - 0.166x_2$ $\quad x^3_0 = 0$

Iteración 1

$x_1 = 3.66 - 0.3(0) - 0.166(0) = 3.66$

$x_2 = 2.5 + 0.125(3.66) - 0.25(0) = 2.9575$

$x_3 = 3.833 - 0.166(3.66) - 0.166(2.9575) = 2.7344$

$x^1_1 = 3.66$

$x^2_1 = 2.9575$

$x^3_1 = 2.7344$

Iteración 2

$x_1 = 3.66 - 0.3(2.9575) - 0.166(2.7344) = 2.3188$

$x_2 = 2.5 + 0.125(2.3188) - 0.25(2.7344) = 2.1062$

$x_3 = 3.833 - 0.166(2.3188) - 0.166(2.1062) = 3.0984$

$x^1_2 = 2.3188$

$x^2_2 = 2.1062$

$x^3_2 = 3.0984$

Iteración 3

$x_1 = 3.66 - 0.3(2.1062) - 0.166(3.0984) = 2.5138$

$x_2 = 2.5 + 0.125(2.5138) - 0.25(3.0984) = 2.0396$

$x_3 = 3.833 - 0.166(2.5138) - 0.166(2.0396) = 3.0771$

$x^1_3 = 2.5138$

$x^2_3 = 2.0396$

$x^3_3 = 3.0771$

Iteración 4

$x_1 = 3.66 - 0.3(2.0396) - 0.166(3.0771) = 2.5373$

$x_2 = 2.5 + 0.125(2.5373) - 0.25(3.0771) = 2.0478$

$x_3 = 3.833 - 0.166(2.5373) - 0.166(2.0478) = 3.0718$

$x^1_4 = 2.5373$

$x^2_4 = 2.0478$

$x^3_4 = 3.0718$