Metodos Numericos 14/10/20

Tarea 4

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Metodo de biseccion y falsa posición

1. Se tiene el siguiente polinomio

$$f(x) = -0.5x^2 + 2.5x + 4.5$$

- Determinar las raíces reales utilizando la formula general de segundo grado
- Para la raiz positiva. Hacer el metodo de bisección (5 iteraciones a mano) y verificar el metodo computacionalmente
- Para la raiz positiva. Hacer el metodo de falsa posición (5 iteraciones a mano) y verificar el metodo computacionalmente, mostrar la grafica de convergencia.

1. $$f(x) = -0.5x^2 + 2.5x + 4.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

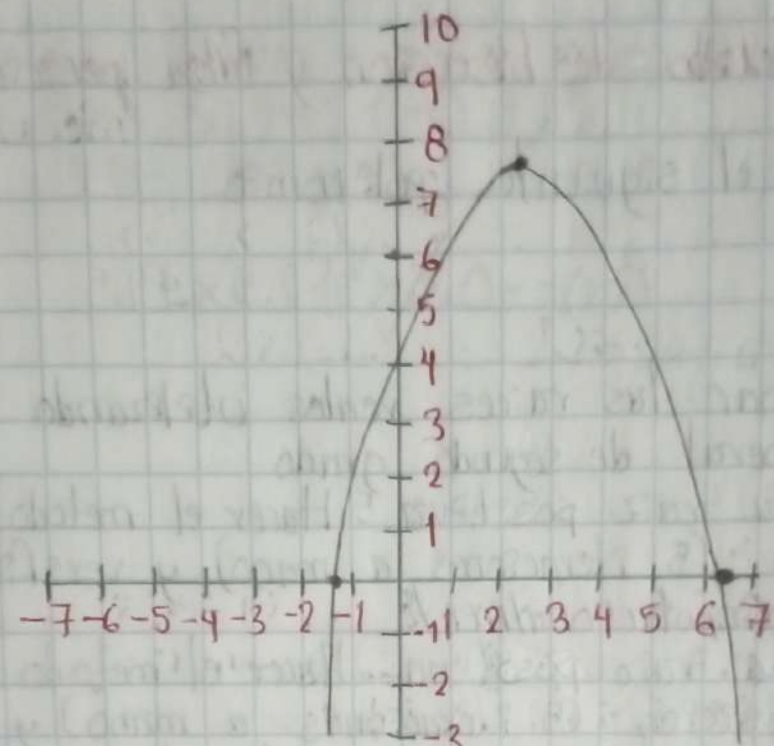
$$x = \frac{-(2.5) \pm \sqrt{(2.5)^2 - 4(-0.5)(4.5)}}{2(-0.5)} \rightarrow \frac{-2.5 \pm \sqrt{6.25 - (-9)}}{-1}$$

$$= \frac{-2.5 \pm \sqrt{6.25 + 9}}{-1} \rightarrow x = \frac{-2.5 \pm \sqrt{15.25}}{-1}$$

$$= \frac{-2.5 \pm 3.9}{-1} \rightarrow$$

$$x_1 = \frac{-2.5 + 3.9}{1} = \frac{1.4}{-1} = -1.4$$

$$x_2 = \frac{-2.5 - 3.9}{-1} = \frac{-6.4}{-1} = 6.4$$



• Raiz positiva - Bisección •

$$f(x) = -0.5x^2 + 2.5x + 4.5 \quad [6, 7]$$

- $x_u = 7$
- $x_l = 6$

$$f(x_l) = -0.5(6)^2 + 2.5(6) + 4.5$$
$$= -0.5(36) + 15 + 4.5$$
$$= -18 + 15 + 4.5$$
$$= 1.5$$

$$f(x_u) = -0.5(7)^2 + 2.5(7) + 4.5$$
$$= -0.5(49) + 17.5 + 4.5$$
$$= -24.5 + 17.5 + 4.5$$
$$= -2.5$$

$(1.5)(-2.5) = -3.75 < 0 \rightarrow$ Hay raíz en el intervalo

$$xr = xl + \frac{\Delta x}{2} = \frac{2}{2}xl + \frac{xu - xl}{2}$$

$$= \frac{2xl + xu - xl}{2} = \frac{xu + xl}{2}$$

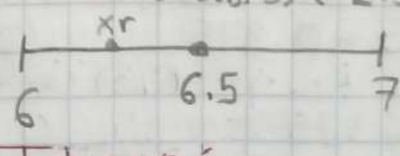
$$xr = \frac{7+6}{2} = \frac{13}{2} = 6.5$$

$$f(xr) = -0.5(6.5)^2 + 2.5(6.5) + 4.5$$
$$= -0.5(42.25) + 16.25 + 4.5$$
$$= -21.125 + 16.25 + 4.5$$
$$= \underline{-0.375}$$

$F(xl) = +1.5$
$f(xr) = -0.375$
$f(xu) = -2.5$

$f(xl)\,f(xr)$
$(1.5)(-0.375) < 0$
$f(xr)\,f(xu)$
$(-0.375)(-2.5) > 0$

- $xl = 6$
- $xu = 6.5$
- $xr = ?$



Iteración

$$x'r = \frac{x'l + x'u}{2} = \frac{6 + 6.5}{2} = \underline{6.25}$$

$$F(x'r) = -0.5(6.25)^2 + 2.5(6.25) + 4.5$$
$$= -0.5(39.0625) + 15.625 + 4.5$$
$$= 19.53125 + 15.625 + 4.5$$
$$= \underline{0.59375}$$

$F(x'l) = f(1) = 1.5$

$(x'r) = 0.59375$

$(x'u) = -0.375$

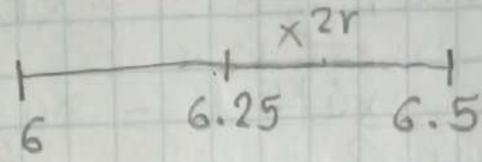
$f(x'l)\,f(xr)$

$(1.5)(0.59375) > 0$

$f(xr)\,f(xu)$

$(0.59375)(-0.375) < 0$

- $x^2l = 6.25$
- $x^2r = ?$
- $x^2u = 6.5$



Iteración

$$x^2r = \frac{x^2l + x^2u}{2} = \frac{6.25 + 6.5}{2} = 6.375$$

$$f(x^2r) = -0.5(6.375)^2 + 2.5(6.375) + 4.5$$
$$= -0.5(40.640625) + 15.9375 + 4.5$$
$$= -20.3203125 + 15.9375 + 4.5$$
$$= 0.1171875$$

- $(x^2l) = 0.59375$
- $(x^2r) = 0.1171875$
- $(x^2u) = -0.375$

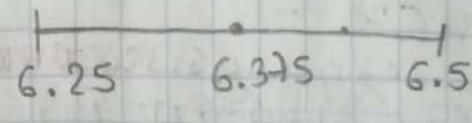
$f(x^2l)\,f(x^2r)$

$(0.59375)(0.1171875) > 0$

$f(x^2r)\,f(x^2u)$

$(0.1171875)(-0.375) < 0$

- $x^3l = 6.375$
- $x^3r = ?$
- $x^3u = 6.5$



$$x^3r = \frac{x^3l + x^3u}{2} = \frac{6.375 + 6.5}{2} = 6.4375$$

$$f(x^3r) = -0.5(6.4375)^2 + 2.5(6.4375) + 4.5$$
$$= -0.5(41.44140625) + 16.09375 + 4.5$$

$$= 20.720703 + 16.0937 + 4.5$$
$$= -0.12695313$$

$(x^3 l) = 0.1171875$

$(x^3 r) = -0.1269531$

$(x^3 u) = -0.375$

$f(x^3 l)\,f(x^3 r)$

$(0.1171875)(-0.12695) < 0$

$f(x^3 l)\,f(x^3 r)$

$(-0.126531)(-0.375) > 0$

$$Xr = \frac{6.375 + 6.437}{2} = 6.406$$

$$f(x^4 r) = -0.5(6.406)^2 + 2.5(6.406) + 4.5$$
$$= -0.5(41.036836) + 2.5(6.406) + 4.5$$
$$= -0.003418$$

- $(x^4 l) = 0.117187$
- $(x^4 r) = -0.003418$
- $(x^4 u) = -0.126953$

$f(x^4 l)\,f(x^4 r)$

$(0.117184)(-0.003418) < 0$

Iteración

$$x^5 r = \frac{6.375 + 6.406}{2} = 6.3905$$

$$f(x^5 r = -0.5(6.3905)^2 + 2.5(6.3905) + 4.5$$
$$= 0.05700487$$

$(x^5 l) = 0.117187$

$(x^5 r) = 0.05700487$

$(x^5 u) = -0.126953$

$f(x^5 r)\,f(x^5 u)$

$(0.05700487)(-0.126953) < 0$

$\varepsilon a < 0.0001 \rightarrow$ Criterio paro

$Xr^{act} = 6.3905 \quad xr^{ant} = 6.406$

$$\varepsilon a = \left| \frac{6.3905 - 6.406}{6.3905} \right| = -0.0024265$$

$$= -0.0024265 < 0.0001$$

Raíz Negativa

Falsa posición

$f(x) = -0.5x^2 + 2.5x + 4.5$

Intervalo $[-2, -1]$

- $xu = -1$
- $xl = -2$

$$(xu) = -0.5(-1)^2 + 2.5(-1) + 4.5$$
$$= -0.5(1) - 2.5 + 4.5$$
$$= 1.5$$

$$f(xl) = -0.5(-2)^2 + 2.5(-2) + 4.5$$
$$= -0.5(4) - 5 + 4.5$$
$$= -2 - 5 + 4.5$$
$$= -2.5$$

$f(xu)\, f(xl) < 0$

$(1.5)(-2.5) < 0 \rightarrow$ Si hay raíz

$$xr = xu - \frac{f(xu)(xl - xu)}{f(xl) - f(xu)}$$

$$xr = -1 - \frac{1.5(-2 - (-1))}{-2.5 - 1.5}$$

$$x_r = -1 - \frac{1.5(-2+1)}{-4} \rightarrow x_r = -1 - 0.375$$

$$x_r = -1.375$$

$$= -0.5(-1.375)^2 + 25(-1.375) + 4.5$$
$$= 0.1171875$$

- $f(x_l) = -2.5$ $\quad (-2.5)(0.11718) < 0$
  $f(x_r) = 0.11718$
  $(x_u) = 1.5$

$f(x'_u) = 0.11718$
$f(x'_l) = -2.5$

- $x_l = -2$
  $x_r = ?$
  $x_u = -1.375$

$$x'_r = -1.375 - \frac{0.11718(-2-(-1.375))}{-2.5-0.11718}$$

$$x'_r = -1.375 - \frac{0.11718(-2+1.375)}{-2.61718}$$

$$x'_r = -1.375 - \frac{0.11718(-0.625)}{-2.61718}$$

$$x'_r = -1.3470149$$

$$f(x'_r) = -0.5(-1.3470149)^2 + 2.5(-1.3470149) + 4.5$$
$$= 0.22528$$

- $(x'_l) = -2.5$ $\quad (-2.5)(0.22523) < 0$
  $(x'_r) = 0.225288$
  $(x'_u) = 0.11718$

• $x^2 l = -2$

$x^2 r = ?$

$x^2 u = -1.3470$

$$x^2 r = -1.3470 - \frac{(0.22523)(-2-(-1.3470))}{-2.5-0.22523}$$

$$x^2 r = -1.3470 - \frac{(0.22523)(-0.653)}{-2.72523}$$

• $x^2 r = -1.400969$

$$(x^2 r) = -0.5(-1.400967)^2 + 2.5(-1.400967) + 4.5$$

$$(x^2 r) = 0.016217$$

$$(-2.5)(0.016217) < 0$$

• $(x^2 l) = -2.5$

$(x^2 r) = 0.016217$

$(x^2 u) = 0.22523$

$x^3 l = -2$ $\qquad f(x^3 l) = -2.5$

$x^3 r = ?$ $\qquad f(x^3 u) = 0.016217$

$x^3 u = -1.400697$

$$x^3 r = -1400967 - \frac{0.016217(-2-(-1.40067))}{-2.5-0.016217}$$

$$x^3 r = -1.400967 - \frac{0.016217(-0.59903)}{-2.516217}$$

$$(x^3 r) = -1.4048306$$

$$(x^3 r) = -0.5(-1.40483)^2 + 2.5(-1.40483) + 4.5$$

$$= 0.0011489$$

• $(x3l) = -2.5$ $\qquad (-2.5)(0.0011489) < 0$

$(x3r) = 0.0011489$

$(x3u) = 0.0016217$

• $x^4 l = -2$ $\qquad f(x^4u) = 0.0011489$

$x^4 r = ?$ $\qquad = -2.5$

$x^4 u = -1.40483$

$$x^4 r = -1.40483 - \frac{0.0011489(-2-(1.40483))}{-2.5-0.001489}$$

$x^4 r = -1.405104$

$(x^4 r) = -0.5(-1.405104)^2 + 2.5(-1.405104) + 4.5$

$= 0.000081319$

• $(x^4 l) = -2.5$ $\qquad (-2.5)(0.000081319) < 0$

$(x^4 r) = 0.000081319$

$(x^4 u) = 0.0011489$

• $x^5 l = -2$ $\qquad f(x^5 u) = 0.000081319$

$x^5 r = ?$ $\qquad (x^5 l) = -2.5$

$x^5 u = -1.405104$

$$x^5 r = -1.405104 - \frac{0.00001319(-2-(-1.405104))}{-2.5-0.000081319}$$

$$x^5 r = -1.405104 - \frac{0.000081319(0.59849)}{-2.500081319}$$

$x^5 r = -1.4051233$

$(x^5 r) = -0.5(-1.4051233)^2 + 2.5(-1.4051233) + 4.5$

$= 0.00005755$

- $(x^5{}_l) = -2.5$ $\qquad (-2.5)(0.00000575S) < 0$

$(x^5{}_r) = 0.000005755$

$(x^5{}_b) = 0.00008131 9$

$\varepsilon_a < 0.0001 \longrightarrow$ Criterio paro

$x_r^{act} = 1.4051233 \quad x_r^{ant} = 1.405104$

$$\varepsilon_a = \left| \frac{-1.4051233 - (-1.405104)}{1.4051233} \right|$$

$\varepsilon_a = -0.000013771 < 0$ //

14/10/20

3. Sea la Función

$$f(x) = \ln(x^2) = 0.7$$

Determinar analíticamente la raíz

$\ln(x^2) = 0.7$

$x^2 = e^{0.7}$

$x_1 = \sqrt{e^{0.7}}$ $\quad$ $x_2 = -\sqrt{e^{0.7}}$

$x_1 = 1.419067$ $\quad$ $x_2 = -1.414067$

• Determinar las primeras tres iteraciones (a mano) utilizando Bisección en en intervalo $x \in [0.5, 2]$

$f(x) = \ln(x^2) = 0.7$ $\quad x_l = 0.5$ $\quad x_u = 2$

$f(x_l) = \ln((0.5)^2) - 0.7 = -2.08629436$

$f(x_u) = \ln((2)^2) - 0.7 = 0.68629436$

$f(x_u) f(x_l) = (-2.0862943)(0.6862943)$

$= 1.431812055 < 0 \rightarrow$ Negativo hay raíz en el intervalo

$$x_r = \frac{x_l + x_u}{2} = \frac{0.5 + 2}{2} = \frac{2.5}{2} = 1.25$$

$f(x_r) = \ln((1.25)^2) - 0.7 = -0.253712897$

• $(x_l) = -2.0862943$

$(x_r) = -0.2537128$ $\quad f(x_r) \cdot (x_u) < 0$

$(x_u) = 0.68629436$

Iteración

$x_l = 1.25$, $x_r = ?$ $\quad x_u = 2$

$$xr = \frac{1.25+2}{2} = \frac{3.25}{2} = 1.625$$

$$(x_l) = \ln((1.25)^2) - 0.7 = -0.25371289$$
$$(x_r) = \ln((1.625)^2) - 0.7 = 0.27101563$$
$$(x_u) = \ln((2)^2) - 0.7 = 0.6862943$$

$$f(x_l)\, f(x_r) < 0$$

$$\varepsilon a = \left| \frac{1.625 - 1.25}{1.625} \right| = 0.23076923$$

Iteración

- $x_l^2 = 1.25$
- $x_r^2 = ?$ $\quad x_r^2 = \frac{1.25 + 1.625}{2} = 1.4375$
- $x_u^2 = 1.625$

$$f(x_l^2) = \ln((1.25)^2) - 0.7 = -0.25371289$$
$$(x_r^2) = \ln((1.4375)^2) - 0.7 = 0.02581098$$
$$(x_u^2) = \ln((1.625)^2) - 0.7 = 0.27101563$$
$$f(x_r^2) - f(x_u^2) < 0$$

$$\varepsilon a = \left| \frac{1.4375 - 1.625}{1.4375} \right| = 0.13043478$$

Iteración

$x_l^3 = 1.4375 \quad x_r^3 = ? \quad x_u^3 = 1.625$

$$x_r^3 = \frac{1.4375 + 1.625}{2} = 1.53125$$

$(xr3) = \ln((1.4375)^2) - 0.7 = 0.025581098$
$(xr3) = \ln((1.53125)^2) - 0.7 = 0.1521687 9$
$(xu3) = \ln((1.625)^2) - 0.7 = 0.2710563$

Determinar las primeras tres iteraciones (a mano) utilizando falsa posición y el intervalo del punto anterior.

$f(x) = \ln(x^2) = 0.7 \quad x_l = 0.5 \quad x_u = 2$
$f(x) = 2\ln(x) - 0.7 = 0$
$f(x_l) = 2\ln(0.5) - 0.7 = -2.0862943$
$(x_u) = 2\ln(2) - 0.7 = 0.6862943$
$(x_l) \cdot f(x_u) = (-2.0862943)(0.6862943)$
$= 1.43181205 < 0$

$$x_r = \frac{f(x_l)\, x_u - f(x_u)\, x_l}{f(x_l) - f(x_u)}$$

$$x_r = \frac{(-2.0862943)(2) - (0.6862943)(0.5)}{-2.0862943 - 0.6862943}$$

$x_r = 1.628707448$
$f(x_r) = 2\ln(1.6287074) - 0.7$
$= 0.27557344$
$f(x_l) \cdot f(x_r) < 0$

Iteración

$x_l = 0.5 \quad x_r = ? \quad x_u = 1.62870744$
$f(x_l) = 2\ln(0.5) - 0.7 = -2.0862943$
$(x_u) = 2\ln(1.62870744) - 0.7 = 0.27557344$

$$x_r = \frac{(-2.08629)(1.62870) - (0.27557)(0.5)}{-2.08629 - 0.27557}$$

$xr = 1.49703$

$f(xr) = 2ln(1.49703) - 0.7 = 0.10696$

$f(xl)\ f(xr) < 0$

Iteración

$xl^2 = 0.5 \quad xr^2 = ? \quad xu^2 = 1.49703$

$f(xl^2) = 2ln(0.5) - 0.7 = -2.0862943$

$(xu^2) = 2ln(1.49703) = 0.80696$

$$xr^2 = \frac{(-2.08629)(1.49703) - (0.80696)(0.5)}{-2.08629 - 0.80696}$$

$xr^2 = 1.21894$

$f(xr^2) = 2ln(1.21894) - 0.7 = -0.30403$

$f(xr^2)\ f(xu^2) < 0$

Iteración

$xl^3 = 1.21894 \quad xr3 = ? \quad xu^3\ 1.49703$

$(xl^3) = 2ln(1.21894) - 0.7 = -0.30403$

$(xu^3) = 2ln(1.49703) - 0.7 = 0.10696$

$$x^3 = \frac{(-0.30403)(1.49703) - (0.10696)(1.21894)}{-0.30403 - 0.10696}$$

$x^3 = 1.42465$

$f(xr^3) = 2ln(1.42465) - 0.7$

$= 0.00785233403$

$f(xl^3) - f(x^3) < 0$

4. Sea la función $x^{3.5} = 80$

Determine la raiz bajo las siguientes casos.

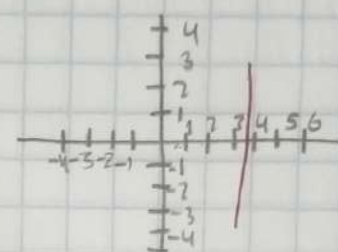
De forma analítica

$x^{3.5} = 80$

$x^{7/2} = 80$

$x = 80^{2/7}$

Forma gráfica



Utilizando bisecciones (3 iteraciones a mano) el resto por programar

$f(x) = x^{3.5} - 80 = 0 \qquad x \in [1, 3]$

$x_l = 1$

$x_u = 3$

$f(x_l) = (1)^{3.5} - 80 = -79$

$f(x_u) = (3)^{3.5} - 80 = -33.234628$

$f(x_l) \cdot f(x_u) > 0 \rightarrow$ No hay raiz

Utilizando falsa posición (3 iteraciones a mano) el resto por programa.

$f(x) = x^{3.5} - 80 = 0 \qquad x \in [1, 3]$

$x_l = 1$

$x_u = 3$

$f(x_l) = (1)^{3.5} - 80 = -79$

$f(x_u) = (3)^{3.5} - 80 = -33.2346282$

$f(x_l) \cdot f(x_u) > 0 \rightarrow$ No hay raiz.