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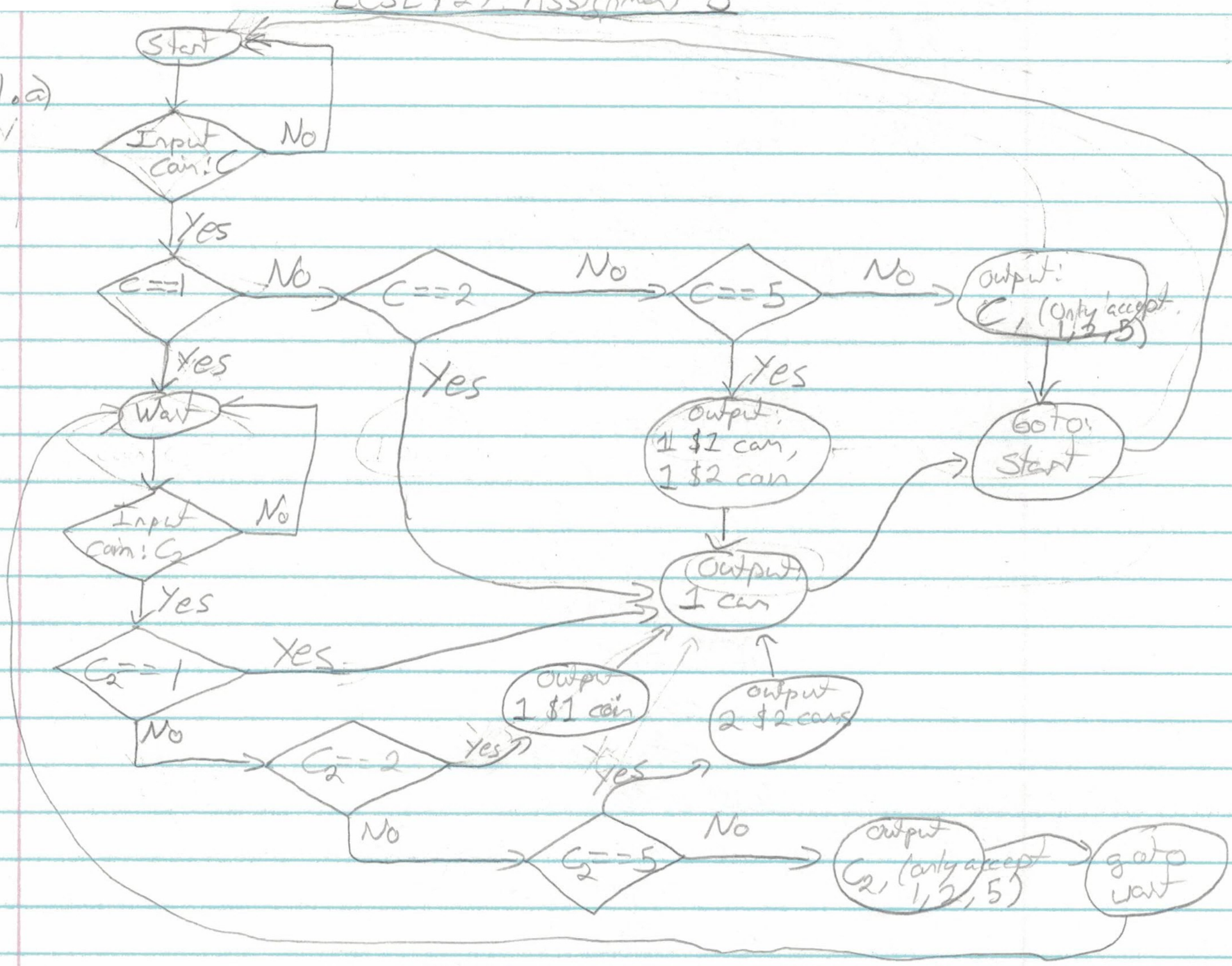
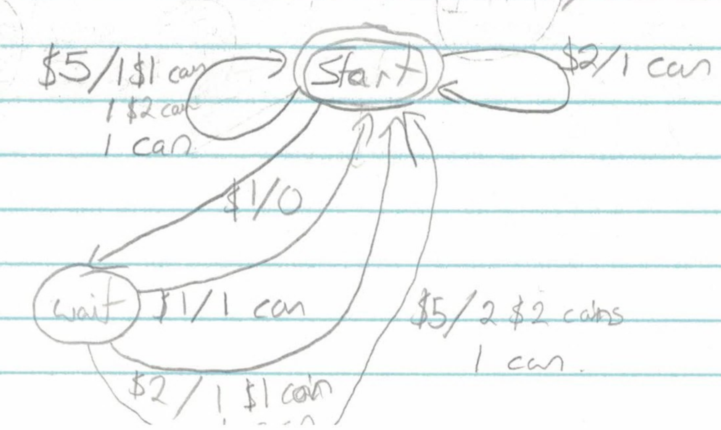
ECSE 429: Assignment 3

1.a)



b) 2 a) handles incorrect inputs, however I'm going to construct a deterministic Finite State Automata in this question so I am assuming only 1, 2, and 5 are valid inputs to the machine. Assuming as well that the machine will wait for input at any given state. The machine will be in the Mealy form. 0 out means no output.



c) Vending Machine:

```
Coin C1 ← None
Coin C2 ← None
While (True)
    While (C1 == None)
        C1 ← checkForInput()
    if (C1 == $5)
        outputChange([$2, $1])
        outputSodaCan()
    else if (C1 == $2)
        outputSodaCan()
    else if (C1 == $1)
        while (C2 == None)
            C2 ← checkForInput()
        if (C2 == $5)
            outputChange([$2, $2])
            outputSodaCan()
        else if (C2 == $2)
            outputChange([$1])
            outputSodaCan()
        else if (C2 == $1)
            outputSodaCan()
        else
            outputChange([C2])
            outputMessage("Only $1, $2, $5 coins accepted")
    else
        outputChange([C1])
        outputMessage("Only $1, $2, $5 coins accepted")
    C1 ← None
    C2 ← None
```

2. a) All of the outputs will be handled as state transitions.

2 states: 1) The previous element of w parsed was either nothing or a 0.

2) The previous element of w parsed was a 1.

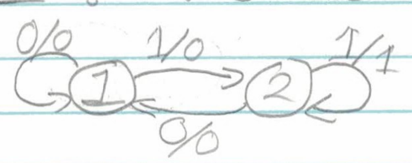
4 transitions: a) From state 1 upon parsing input 0 we output 0 and stay in state 1.

b) From state 1 upon parsing input 1 we output 0 and move to state 2

c) From state 2 upon parsing input 1 we output 1 and stay in state 2.

d) From state 2 upon parsing input 0 we output 0 and transition to state 1

b) Transition function describes input output relation as w_i/z_i. State 1 is our start state



The Mealy FSM was chosen because the output is decided by both the current state and the input at symbol i. That is to mean that if we are in state 1, the output for input 1 is different from the output for input 1 in state 2. Specifically, we output 0 for input 1 iff input 1 corresponds to the first one of substring of 1's.

3. a) Clauses: x, y

$x \vee y = x \vee (y \wedge \bar{x})$ deMorgan's $\Rightarrow x+y = x+y$ ✓

$\Rightarrow x+y = x + yx'$

eMorgan's $\Rightarrow x+y = x+(y'+x)'$

eMorgan's $\Rightarrow x+y = (x'(y'+x))'$

istributive $\Rightarrow x+y = (x'y'+x'x)'$

Complement/Identity $\Rightarrow x+y = (x'y')'$

b) Clauses: x, y

$x \wedge y = 0 \Leftrightarrow x \wedge \bar{y} = x$

$xy = 0 \Leftrightarrow x\bar{y} = x$

$xy = 0 \Leftrightarrow x\bar{y}y = xy$

$xy = 0 \Leftrightarrow x(0) = xy$

$xy = 0 \Leftrightarrow 0 = xy$ ✓

b) continued: So $xy=0$ and $x\bar{y}=x$ have the exact same truth values for all (x,y) combinations!!!

c) Clauses: x, y, z

$x \wedge (y \wedge \bar{z}) = (x \vee y) \wedge (x \wedge \bar{z})$

$\Rightarrow x(yz') = (x+y)(xz')$

Distributive $\Rightarrow x(yz') = xxz' + yxz'$

Associative/Idempotent $\Rightarrow xyz' = xz' + xyz'$

Absorption $\Rightarrow xyz' = xz'$

The equation reduced to two different boolean expressions, so we just need to find a vector that evaluates to false (i.e. $y=0$, $x\bar{z}=1$)

$(x,y,z) = (0,0,0) \Rightarrow 001 = (0+0)01$ ✓

$001 \Rightarrow 000 = (0+0)00$ ✓

$010 \Rightarrow 011 = (0+1)01$ ✓

$011 \Rightarrow 010 = (0+1)00$ ✓

$100 \Rightarrow 101 = (1+0)11$ ✗

$101 \Rightarrow 100 = (1+0)10$ ✓

$110 \Rightarrow 111 = (1+1)11$ ✓

$111 \Rightarrow 110 = (1+1)10$ ✓

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4. a) $c+(bc)'$

DeMorgan's $\Rightarrow c+(b'+c')$

Commutative $\Rightarrow c+(c'+b')$

Associative $\Rightarrow (c+c')+b'$

Complement $\Rightarrow 1+b'$

Domination $\Rightarrow 1$

b) $(ab)'(a'+b)(b'+b)$

Complement $\Rightarrow (ab)'(a'+b)(1)$

Identity $\Rightarrow (ab)'(a'+b)$

DeMorgans $\Rightarrow (ab)'(ab')'$

DeMorgans $\Rightarrow (ab+ab')'$

Distributive $\Rightarrow (a(b+b'))'$

Complement $\Rightarrow (a(1))'$

Identity $\Rightarrow a'$

c) $(a+c)(ad+ad')+ac+c$

Distributive $\Rightarrow (a+c)(a(d+d'))+ac+c$

Complement $\Rightarrow (a+c)(a(1))+ac+c$

Identity $\Rightarrow (a+c)a+ac+c$

Distributive $\Rightarrow aa+ca+ac+c$

Idempotent $\Rightarrow a+ca+ac+c$

Associative $\Rightarrow a+ac+ac+c$

Idempotent $\Rightarrow a+ac+c$

Absorption $\Rightarrow a+c$

d) $a'(a+b)+(b+aa)(a+b')$

Idempotent $\Rightarrow a'(a+b)+(b+a)(a+b')$

Distributive $\Rightarrow a'a+a'b+(b+a)a+(b+a)b'$

Distributive $\Rightarrow a'a+a'b+ba+aa+bb'+ab'$

Complement $\Rightarrow a'b+ba+aa+ab'$

Idempotent $\Rightarrow a'b+ba+a+ab'$

Absorption $\Rightarrow a'b+ba+a$

Absorption $\Rightarrow a'b+a$

DeMorgans $\Rightarrow (a+b')'+a$

DeMorgans $\Rightarrow ((a+b')a')'$

Distributive $\Rightarrow (aa'+b'a')'$

Complement $\Rightarrow (b'a')'$

DeMorgans $\Rightarrow b+a$