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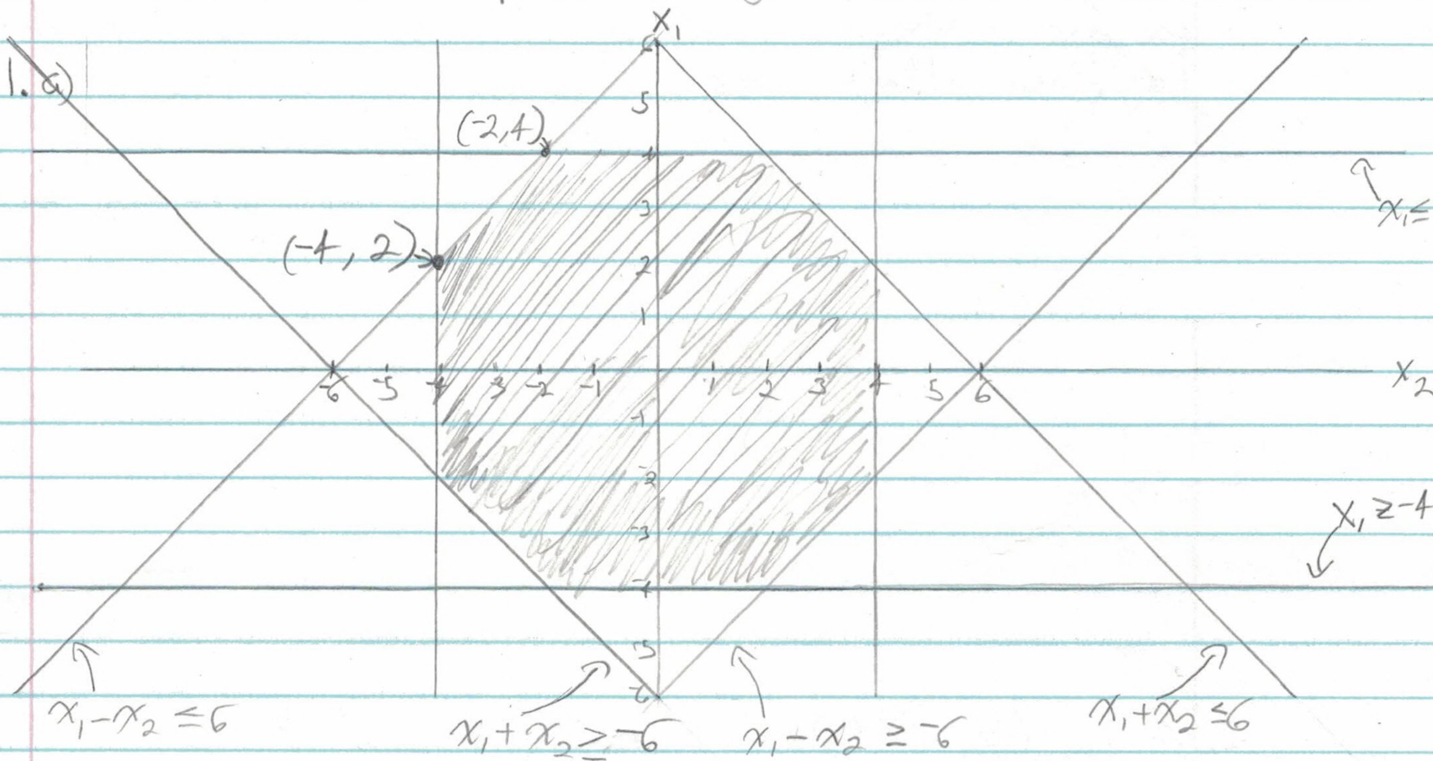
Comp 360: Assignment 3

1. a)



b) $-2x_1 + x_2 = 10 @ (-4,2),\ 6 @ (-4,-2),\ 8 @ (-2,4)$

Therefore by simplex $(-4,2)$ is optimal for Maximizing $-2x_1 + x_2$ objective fn

c) Line defined by $(-2,4)$ and $(-4,2)$ is $x_1 - x_2 = 6$

so objective function $x_1 - x_2 = 6 @ (-4,2),\ 6 @ (-2,4),\ 2 @ (2,4),$
$2 @ (-4,-2)$

Therefore by simplex the line formed by $(-4,2)$ and $(-2,4)$ will all have unique optimal solution 6 for maximizing $x_1 - x_2$ objective fn

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2. max: $c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$

subject to: $a_{11}x_1 + a_{12}x_2 + \dots + a_{15}x_5 \le b_1$
$a_{21}x_1 + \dots + a_{25}x_5 \le b_2$
$\vdots$

m 5D planes forming the boundaries of the feasible region.

* Simplex is just taking advantage of the fact that Local min/max = Global min/max for convex polytopes by picking a vertex on the polytope and then checking neighbors and always searching for incrementally better vertices, returning when it finds no better.

For most cases Simplex is polynomial time, so I'm assuming you are looking for something other than Simplex* to also account for those unlikely cases.

3. Given $(G=(V,E), s, t, \{c_e\}_{e\in E})$ and K

Variables $f(e)\ \forall e \in E$

max $\sum_{su\in E} f(su)$

Subject to: $\sum_{su\in E} K_{su} \leq K$

$f(e) \leq c_e + K_e \quad \forall e \in E$

$\sum_{vu\in E} f(vu) - \sum_{uw\in E} f(uw) = 0 \quad \forall u, v, w \in V,\ u \neq t,\ u \neq s$

$f(e) \geq 0 \quad \forall e \in E$

4. The issue is that the objective function is currently written in a non-linear form $\frac{x_1+2x_2+\dots+nx_n}{x_1+\dots+x_n}$

So ideally we'd like to convert ↑ into a linear expression.

Take $\frac{x_i}{x_1+\dots+x_n} = y_i$, then we have $y_1+2y_2+\dots+ny_n$.

This is now linear, however all our constraints are of the x variables. We want these constraints in terms of these new y variables.

$y_i = \frac{x_i}{x_1+\dots+x_n}$ where $x_i \geq 0 \; \forall i = 1, \dots, n$
and $x_1+\dots+x_n \geq 0 \; \forall$

so $\frac{\geq 0}{\geq 0} \Rightarrow \geq 0$

$y_i \geq 0 \quad \forall i = 1, \dots, n$

5. a) We are trying to maximize the sum of all x_p where x_p are positive numbers ($x_p \geq 0$) associated with given paths P.

However we then constrain the solution by an upper bound such that the sum of the x_p's passing through any edge e is less than or equal to a capacity C_e.

This constraint guarantees that in our sum over all paths' x_p's, no one edge will exceed our capacity C_e.

So this means that in maximizing the Σx_p's over all possible paths we are in turn maximizing the amount of flow SO LONG AS THE DEFINITION OF A PATH P IS SUCH THAT IT STARTS AT NODE S AND ENDS AT NODE T.

b) $y_e \times (\sum_{p: e \in P} x_p \leq C_e)$

$$\min \sum_{e \in E} y_e C_e$$

$y_e \geq 0 \quad \forall e \in E$

Primal

6. max $3.1x_1 + 10x_2 + 8x_3 - 45.2x_4 + 18x_5$

$x_1^* = x_3^* = 0.5$

$x_2^* = x_4^* = 0$

$x_5^* = 2$

s.t. $(x_1 + x_2 + x_3 - x_4 + 2x_5 \le 5) \times y_1$

$(2x_1 - 4x_2 + 1.2x_3 + 2x_4 + 7x_5 \le 16) \times y_2$

$(x_1 + x_2 - 3x_3 - x_4 - 10x_5 \le -20) \times y_3$

$(3x_1 + x_2 + 3x_3 + \frac{3}{2}x_4 + \frac{7}{3}x_5 \le 10) \times y_4$

$(x_2 + x_3 + 6x_4 + 2x_5 \le 4.5) \times y_5$

$(2x_2 - x_4 + x_5 \le 2) \times y_6$

$x_1, x_2, x_3, x_4, x_5 \ge 0$

Dual

min $5y_1 + 16y_2 - 20y_3 + 10y_4 + 4.5y_5 + 2y_6$

s.t. $y_1 + 2y_2 + y_3 + 3y_4 \ge 3.1$

$y_1 - 4y_2 + y_3 + y_4 + y_5 + 2y_6 \ge 10$

$y_1 + 1.2y_2 - 3y_3 + 3y_4 + y_5 \ge 8$

$-y_1 + 2y_2 - y_3 + \frac{3}{2}y_4 + 6y_5 - y_6 \ge -45.2$

$2y_1 + 7y_2 - 10y_3 + \frac{7}{3}y_4 + 2y_5 + y_6 \ge 18$

$y_1, y_2, y_3, y_4, y_5, y_6 \ge 0$

Complementary Slackness $\Rightarrow$

$y_1 + 2y_2 + y_3 + 3y_4 = 3.1$

$y_1 + 1.2y_2 - 3y_3 + 3y_4 + y_5 = 8$

$2y_1 + 7y_2 - 10y_3 + \frac{7}{3}y_4 + 2y_5 + y_6 = 18$

$y_1 - 4y_2 + y_3 + y_4 + y_5 + 2y_6 > 10$

$-y_1 + 2y_2 - y_3 + \frac{3}{2}y_4 + 6y_5 - y_6 > -45.2$