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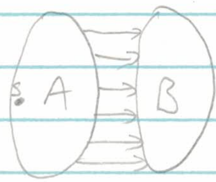
COMP 360: Assignment 2

1. A min cut is a cut(A, B) s.t. $f^{out}(A) = \sum_{\substack{uv \in E \\ u \in A \\ v \in B}} C_{uv}$.



That means that $\forall C_{uv} \in E$ s.t. $u \in A$ and $v \in B$ $f(uv) = C_{uv}$.

In order to have a different max flow f' then the flow must pass through another edge because all edges uv $u \in A, v \in B$ have $f(uv) = C_{uv}$. But as a result of $val(f) = val(f')$ then in order to modify f to get a new f' then we must redirect flow from at least 1 edge in uv to an equivalent # of edges outside uv $u \in A, v \in B$ because $C = 1$ for all $e \in E$. Therefore, we can now construct $A' = A - \{u \in A \text{ s.t. } f^{out}(u) \text{ is removed}\} + \{v \in B \text{ s.t. } f^{out}(v) \text{ is added}\}$

and $B' = B - \{v \in B \text{ s.t. } f^{out}(v) \text{ is added}\} + \{u \in A \text{ s.t. } f^{out}(u) \text{ is removed}\}$

Which exactly correlates a min-cut (A', B') for the new flow f'.

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2. $\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ & \ddots & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$

- Find the row or column with the most negative integers.
- Remove it.
- Repeat until no-such row or column found.

Naively runs around $O(n^4)$ (technically $O(2n^2 + 2(n)(n-1) + 2(n-1)^2$

- Count the number of negatives per column and per row labelling them $re_1, \dots, re_n$ and $ce_1, \dots, ce_n$. $O(n^2 + n^2) = O(n^2)$