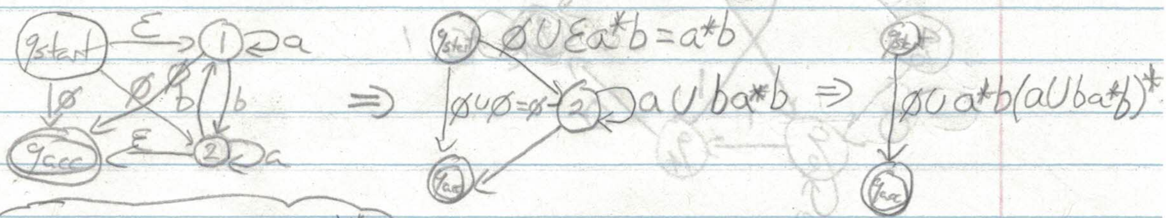
Brandon Smith Oct. 15, 2019

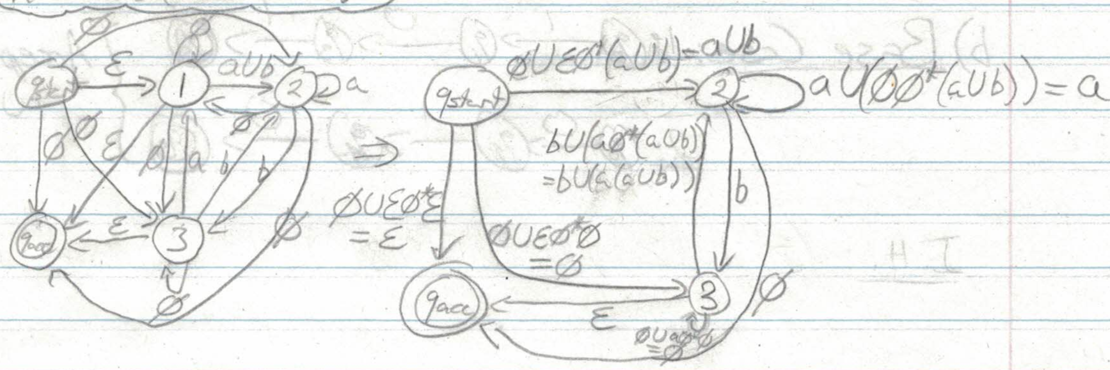
COMP330: Assignment 2

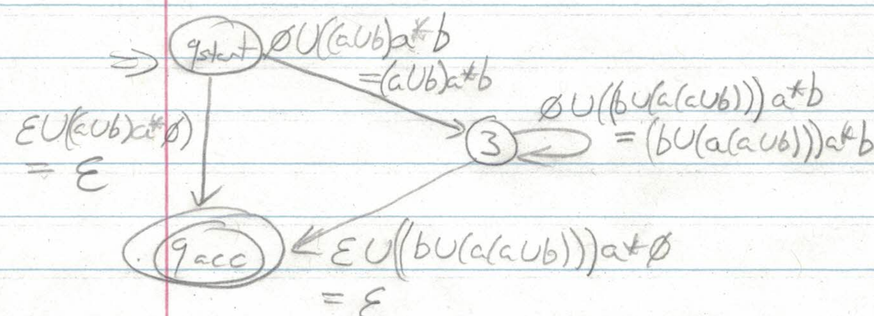
1.21 a)

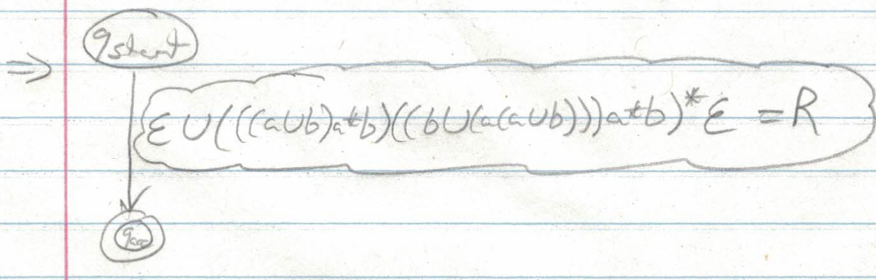


$\Rightarrow R = a^*b(a \cup ba^*b)^*$

b)







1.47 By the pumping lemma All s, $|s| \geq p$, $s = xyz$, $s \in Y$ Assume Y reg.

1. For each $i \geq 0$, $xy^iz \in Y$,
2. $|y| > 0$, and
3. $|xy| \leq p$

Choose $K = p$, $s = x_1 \# x_2 \# x_3 \ldots \# x_p$

Let $s = xyz$, in order to satisfy (1) and (2) y must be substring $y = x_i\# \ldots \#x_j\#$. We would then pump the string once to get $xyyz$,

$\Rightarrow s' = x(x_i\# \ldots \#x_j\#)(x_i\# \ldots \#x_j\#)z$

But by the definition of the language $x_i \neq x_j$ for all $i \neq j$

$\Rightarrow$ $s' \notin Y$ because x_i now occurs twice, ..., and x_j now occurs twice.

$\Rightarrow$ A contradiction has occured, and the pumping lemma condition (1) is no longer satisfied.

$\Rightarrow$ Y is not regular

1.53 Assume that the language is regular, by the Myhill Nerode theorem there exists a DFA with k states corresponding to the index of the language. Given that the language consists of $x = y + z$ where x, y, z are binary integers, then the number of symbols parsed $= \lfloor \log_2 x_{ij} + 1 \rfloor + 1$ $+ \lfloor \log_2 y_{ij} + 1 \rfloor + 1$ $+ \lfloor \log_2 z_{ij} + 1$ if X, Y, Z are being represented in decimal.

$\Rightarrow$ Number of unique expressions $> X$

$\Rightarrow$ Number of symbols in X is countably infinite because the number of integers is countably infinite.

$\Rightarrow$ Number of unique expressions is $>$ countably infinite.

$\Rightarrow$ Contradiction wherein we cannot have a DFA with finite index

$\Rightarrow$ By the Myhill Nerode theorem that the language ADD is not regular.

1.54