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Comp 330: Assignment 3

2.2 a) Example 2.36 shows that $C=\{a^n b^n c^n \mid n \geq 0\}$ is not context free so if we can show that both $A=\{a^m b^n c^n \mid m, n \geq 0\}$ and $B=\{a^n b^n c^m \mid m, n \geq 0\}$ are both context free languages then we can prove that context free languages are not closed under intersection. This is because $A \cap B = C$. By construction in A and B we have b^n in the center of the string so if $m<n$ then the intersection becomes $\{a^m b^m c^m \mid m \geq 0\}$, whereas if $m \geq n$ $A \cap B = C$. So in either case we result in C.

$$A=\left(\{S, Q, T\}, \{a, b, c\}, R_A = \begin{pmatrix} S \Rightarrow QT \\ Q \Rightarrow aQ \mid \varepsilon \\ T \Rightarrow bTc \mid \varepsilon \end{pmatrix}, S\right)$$

$$B=\left(\{S, Q, T\}, \{a, b, c\}, R_B = \begin{pmatrix} S \Rightarrow QT \\ Q \Rightarrow aQb \mid \varepsilon \\ T \Rightarrow Tc \mid \varepsilon \end{pmatrix}, S\right)$$

$\Rightarrow$ A is Context Free and B is Context Free

$A \cap B = C \Rightarrow$ Context Free languages are not closed under intersection.

b) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ Instinct is telling me to prove by contradiction

Easy to show context free languages are closed under union for any grammars G_1, G_2 As seen in class:

$$G_U = G_1 \cup G_2 = (V_1 \cup V_2 \cup \{S_U\}, \Sigma_1 \cup \Sigma_2, \{S_U \Rightarrow S_1 \mid S_2\} \cup R_A \cup R_B, S_U)$$

So $A \cup B$ is a CFL. ASSUME Context Free Languages are closed under complement.

$\Rightarrow \overline{A \cup B}$ is Context Free

DeMorgan's $\Rightarrow \overline{A} \cap \overline{B}$ is Context Free. However this is False from a) so

a contradiction has occured. Therfore => Context free languages are not closed under complement

2.15 $G = (\{S\}, \{a, b\}, R = (S \Rightarrow aSb \mid \varepsilon), S)$

I believe the CFL described by the above grammar G contradicts the construct $CFL(G)^*$ because the star by definition is a concatenation of strings that are accepted by CFL(G).

$G' = (\{S\}, \{a, b\}, R = (S \Rightarrow SS \mid aSb \mid \varepsilon), S)$

$\Rightarrow S \Rightarrow aSb \Rightarrow aSSb \Rightarrow aaSbaSbb$
$\Rightarrow aabcbb$

However aababb is not a string solely composed of concatenated substrings of CFL(G)

aaba G describes a language of $\{a^n b^n \mid n \geq 0\}$
so the grammar G' is not representing $CFL(G)^*$.

2.27a) if condition then if condition then a:=1 else a:=1

<STMT> ⇒ <IF-THEN>
⇒ if condition then <STMT>
⇒ if condition then <IF-THEN-ELSE>
⇒ if condition then if condition then <stmt> else <stmt>
⇒ if condition then if condition then a:=1 else a:=1

<STMT> ⇒ <IF-THEN-ELSE>
⇒ if condition then <stmt> else <stmt>
⇒ if condition then <IF-THEN> else a:=1
⇒ if condition then if condition the <stmt> else a:=1
⇒ if condition then if condition then a:=1 else a:=1

Therefore we have more than one leftmost derivation of a single sentence.

b) $G^1 = (\{$<STMT>, <IF-THEN>, <IF-THEN-ELSE>, <ASSIGN>$\}$,
$\{$if, condition, then, else, a:=1, '{', '}'$\}$,
R = (<STMT> => <ASSIGN> | <IF-THEN> | <IF-THEN-ELSE>
<IF-THEN> => if condition then { <STMT> }
<IF-THEN-ELSE> => if condition then {<STMT } else {<STMT>}
<ASSIGN> => a:=1)
<STMT>)

The above grammar describes an un-ambiguous language to achieve the same end-goal, however it adds two new characters to the alphabet so it is a $\boxed{\text{different language}}$. So instead we must construct a grammar that restricts the usage of <IF-THEN> such that it cannot be used within an if clause of an <IF-THEN-ELSE>

$G^2 = (\{$<STMT>, <IF-THEN>, <IF-THEN-ELSE>, <ASSIGN>, <IF-STMT>
$\{$if, condition, then, else, a:=1$\}$,
R = (<STMT> => <ASSING> | <IF-THEN> | <IF-THEN-ELSE>
<IF-THEN> => if condition then <STMT>
<IF-THEN-ELSE> => if condition then <IF-STMT> else <STMT>
<IF-STMT> => <ASSIGN> | <IF-THEN-ELSE>
<ASSIGN> => a:=1)
<STMT>)

This grammar (G^2) maintains the same language but restricts the use of <IF-THEN> so every opening if of an <IF-THEN-ELSE> remains paired with it's original else.

2.28 b) $G = (\{S, A, B\}, \{a, b, \varepsilon\}, R = \begin{pmatrix} S \Rightarrow aAS \mid bBS \mid \varepsilon \\ A \Rightarrow b \mid aAA \\ B \Rightarrow a \mid bBB \end{pmatrix}, S)$

aaababbabbba => aAS => aaAAS => aaaAAAS => aaabAAS => aaabaAAAS => aaababbaAAS => aaababbabbS
Unambiguous because S is always at the end and Starts the next set of ba's

c. $G = (\{S, A, B\}, \{a, b, \varepsilon\}, R = \begin{pmatrix} S \Rightarrow aAS \mid bBS \mid \varepsilon \\ A \Rightarrow b \mid aAA \mid \varepsilon \\ B \Rightarrow a \mid bBB \end{pmatrix}, S)$

2.30 a) $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$ Assume Context Free

choose $s = 0^p 1^p 0^p 1^p$ clearly $\in \{0^n 1^n 0^n 1^n \mid n \geq 0\}$ and $|s| \geq p$

Need to show pumping s fails ~~one of the~~ pumping lemma conditions.

$|vy| > 0$ and $|vxy| \leq p$.

V and y can either (1) contain only 0's, or 1's, or (2) v contains 0's and y contains 1's, or v or y contains 0's and 1's.

(1) If $vxy = 0^p$ or 1^p then pumping v^i and y^i to 2 will result ~~in~~ either $0^k 1^p 0^p 1^p$, $0^p 1^k 0^p 1^p$, $0^p 1^p 0^k 1^p$, or $0^p 1^p 0^p 1^k$ with $k > p$. Thus Not in the language so contradiction.

(2) If $vxy = 0^{k_1} 1^{k_2}$ or $1^{k_1} 0^{k_2}$ with $k_1, k_2 \leq p$ $k_1 = p - k_2$ then pumping v^i and y^i to 2 will result in either $0^{k_1} 1^{k_2} 0^p 1^p$, $0^p 1^{k_1} 0^{k_2} 1^p$, or $0^p 1^p 0^{k_1} 1^{k_2}$ with $k_1, k_2 > p$. Thus Not in the language so contradiction.

d) $A = \{t_1 \# t_2 \# \ldots \# t_k \mid k \geq 2$, each $t_i \in \{a, b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

Assume A is CF.

Choose $t_i = \{a, b\}^p$ $\forall i$ $0 \ldots k$. $S \in A$ and $|S| \geq p$ because $k \geq 2$

$|vy| > 0$ and $|vxy| \leq p$

Two cases: (1) $vxy = t_i$ $\forall i$ $0 \ldots k$ and (2) $vxy = t_b\#$ or $\#t_b$ $\forall i$ $0 \ldots k$ where t_b is t_i minus the first or last character

The reasoning is identical for both (1) and (2). If vxy = t or 6b# or #tb then pumping will be guaranteed to change ti so we no longer satisfy ti = tj so it is no longer in A, thus contradiction occured.

The reasoning is identical for both (1) and (2).

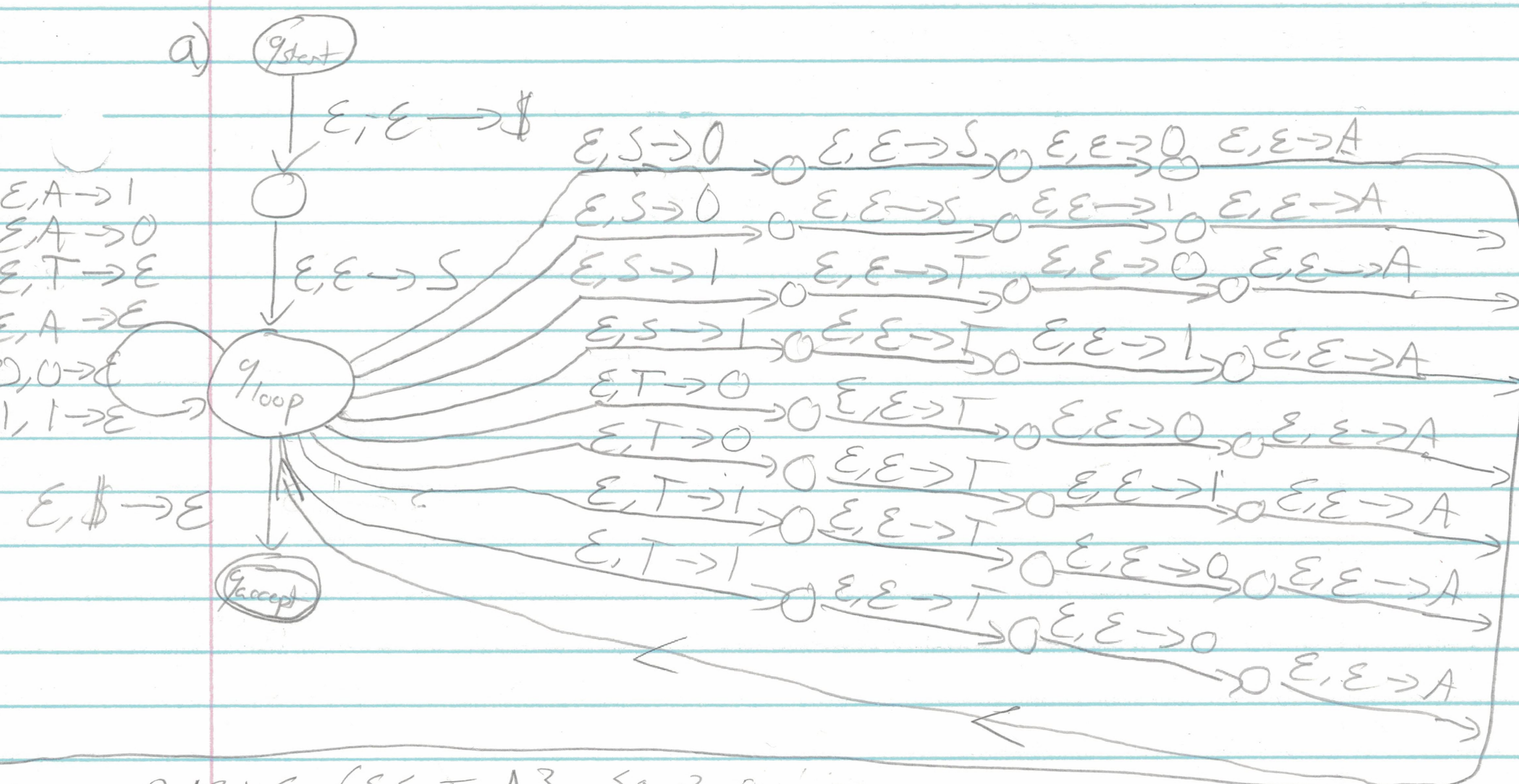
2.36 $A = \{a^i b^j c^k : i, j, k \text{ all different}\}$ is my best guess but I can't prove this is not context free at all.

Choose $a^p b^{p+1} c^{p+2}$, $|vxy| \leq p$ $|vy| > 0$

vxy can either be a's, b's, c's For all cases it can be pumped successfully.

2.47 b) $G = \left(\{S, T, A\}, \{0,1\}, R = \begin{pmatrix} S \Rightarrow A0S0 \mid A1S0 \mid A0T1 \mid A1T1 \\ T \Rightarrow A0T0 \mid A1T0 \mid A0T1 \mid A1T1 \mid \varepsilon \\ A \Rightarrow 1 \mid 0 \mid \varepsilon \end{pmatrix}, S\right)$

a)



2.48 a) $G = \left(\{S, T, A\}, \{0,1\}, R = \begin{pmatrix} S \Rightarrow ABSISBA \mid ABTITBA \\ T \Rightarrow ABTBTBA \mid \varepsilon \\ A \Rightarrow 1 \mid 0 \mid \varepsilon \\ B \Rightarrow 1 \mid 0 \end{pmatrix}, S\right)$

Assume Context free.

b) choose $s = x^p y^p z^p$, $|s| \geq p$, $s \in C_2$

$|vy| > 0$ $|vxy| \leq p$.

(1) $vxy = x^p$. pump and now $|x| \neq |z|$ so contradiction.

(2) $vxy = y^p$. pump and now $|x| = |z| \neq |y|$ so contradiction.

(3) $vxy = z^p$ pump and now $|x| \neq |z|$ so contradiction.

(4) $vxy = x^{k_1} y^{k_2}$ pump and now $|x| \neq |z|$ so contradiction

(5) $vxy = y^{k_2} z^{k_1}$ pump and now $|x| \neq |z|$ so contradiction.

So all possible decompositions result in contradiction so C_2 not context free.