Autopilot and Airframe Build Lab

November 4, 2019

1 Building your airframe

The airframe built consists of 4 major areas: assembly, installation, quantification, and configuration.

1.1 Assembly

We are using an airframe kit that includes the components you will need. Follow the instructions to assemble this according to plans.

1.2 Installation

In this section, you will install the autopilot hardware and other components as in Fig. 1.

1.2.1 Receiver and PWM converter

Install both with velcro and connect as instructed. Install the receiver in the center of the aircraft, with the label facing up and pins facing aft. Install the PWM-SBUS converter on the left side of the airframe. Caution: R/C connectors do not physically restrict you from connecting them improperly, which will result in damaged electronics. R/C connectors consist of 3 leads, ground (0V), power (+5V), and signal (pulse width modulation PWM). Pay attention to the circuit board labels to ensure each component attaches to which element.

1.2.2 Flight battery

Attach velcro to the unlabeled side of the case, then install as far forward as possible (against the aft firewall section). Warning: Lithium polymer flight batteries are flammable. Handle batteries gently, do not allow the battery to discharge below 3.5V per cell (e.g., 10.5V total), and alert a teaching staff if a battery is swollen. Never replace a green cap on a battery unless it has been freshly charged and balanced.

1.2.3 Autopilot

Assembly the Navio2 and case. Install using velcro, ensuring adequate clearance on all sides. Route the wiring underneath the case, ensuring it does not interfere with moving components.

1.2.4 915 MHz radio

Attach velcro to the radio and install it to the right side of the airframe. Use the DF13 connector to connect it to the Navio2 board.

 $^{^{1}\}mathrm{Prepared}$ by Imraan A. Faruque i.faruque@okstate.edu.

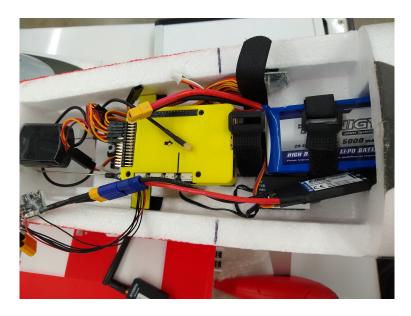


Figure 1: Autopilot and peripherals.

1.3 Quantification

In this section, you will make the measurements that are needed to properly model and simulate your flight vehicle. These consist of aerodynamic, inertial properties, and control surface deflections. Before you measure anything, choose and label a datum that is unlikely to change and also easy to measure relative to (vehicle firewall is a common choice).

Aerodynamic Measure the locations and dimensions of the vehicle's aerodynamic surfaces. You may use the attached diagram to annotate this.

Inertial We need to know the CG location, mass, and inertia tensor. We use a scale for the mass, balance for CG location, and bifilar pendulum for the moments of inertia. See the attached handout for more detail on the bifilar pendulum method. Be sure to label the CG location specified in the airframe build location and verify that your build satisfies this position.

Control surface deflections Here, we measure the deflections for a given servo command and make a simple mapping between them. Come back to this one once you have configured your onboard hardware and are able to send manual commands.

1.4 Configuration

Each of the electronic components you installed must be configured. Warning: for your safety, ensure the aircraft is securely restrained and remain clear of the propeller at any time that power is connected.

Receiver This receiver has an onboard stabilization circuit. Expect more setup instructions at the event.

915 MHz radio These are pre-paired to connect to eachother. To see the telemetry, you need a MAVlink compatible ground station such as Mission Planner or QGroundcontrol on your laptop. Be sure to label these pairs.

Transmitter The transmitter settings have already been developed for you. Use the provided transmitter SD card image to copy the model setup to the transmitter. Bind the transmitter to the receiver using the manufacturer instructions.

Autopilot hardware Using the software installation we covered last week, use your SD card to boot the Raspberry Pi computer, and download and start the Python process to run the autopilot environment in manual pass-through case. Verify that the servos move freely and do not bind at either end. Caution: binding is a common reason for servo failure. Now go back to the control surface collection and, for each control channel, move the controls and record the surface deflections.

Now that things are working, practice as if you are at a field for a flight test test and start making a checklist for flight test day. Divide this into preflight, during flight, and post-flight sections.

1.5 Wrapup

- 1. Ensure that all components are labeled
- 2. Remove the wings from the aircraft for storage.
- 3. Return the assembly kit.
- 4. Return the flight battery to the appropriate area and use a green or red cap to indicate its charge level.
- 5. Ensure your work area is clean, you have your measurements, and you have a checklist draft for flight day.

Appendix A: Kit Inventory

The kit contains the following items, all of which must be returned prior to assigning a project grade.

Airframe kit Commandor mPd 1.4m airframe kit, charged 3S 5000mAh battery with green cap

Onboard electronics Raspberry Pi, Navio2 board, power module, 3-D printed case, Spektrum PMAR636 receiver, 915 MHz radio, PWM-SBUS converter, GPS antenna

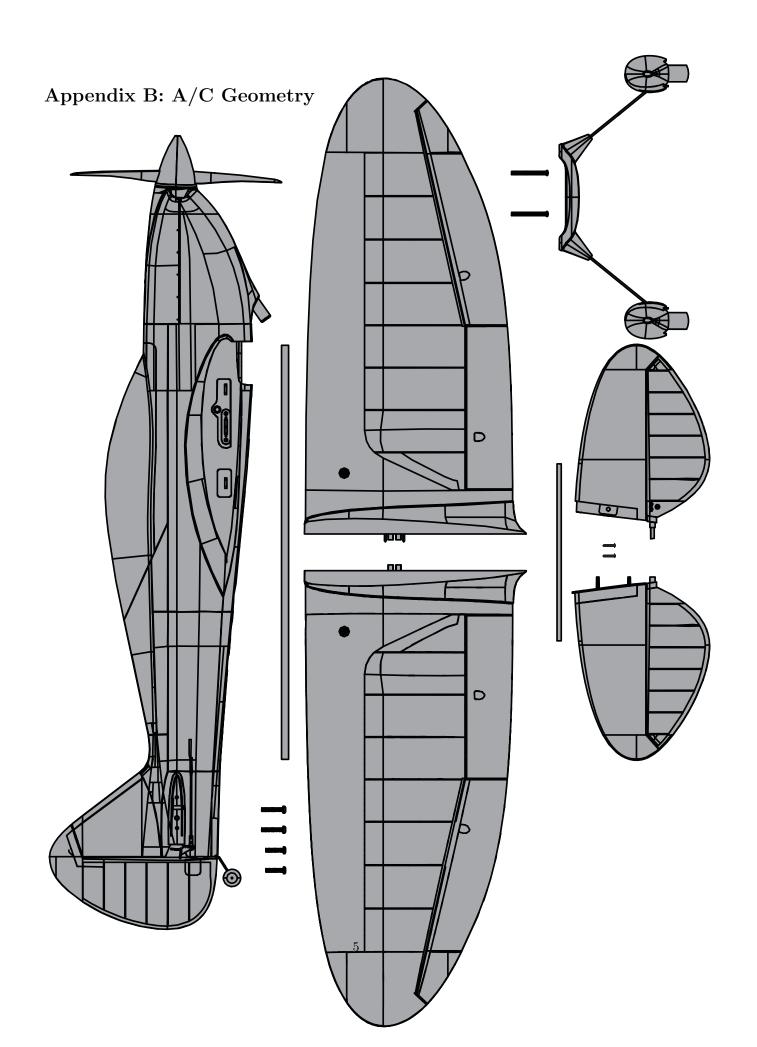
Ground components TX, RF module, battery case, AA batteries, 915MHz USB radio.

Tools Screwdriver, scissors, ruler, protractor, tape measure

Materials Velcro

Connectors Servo connectors, XT60 to EC3 converter, DF-13 radio connectors,

Other GPS mount bracket, airframe labels.



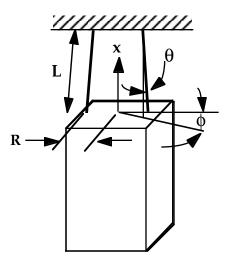


Figure 1: A simplified bifilar pendulum

1 Bifilar Pendulum

The center of mass and moment of inertia properties of the APPT were needed for the design and integration of the COMPASS Satellite. A method known as the bifilar pendulum technique was used to determine these properties. A bifilar pendulum consists of a rigid body suspended by two wires or filaments as shown in figure 1. The dynamical equations of motion in the general case are nonlinear and complicated. The equations of motion can be greatly simplified provided that certain assumptions are not violated. In this case, knowing the natural twisting period of the pendulum system about the vertical axis allows for the calculation of the moment of inertia of the body about this axis. Performing this experiment about six sufficiently different axis of the rigid body allows one to obtain the full moment of inertia tensor. Error terms are considered in section 3.2.

1.1 Equations of Motion

The bifilar pendulum equations of motion are most thoroughly treated in Ref [1]. In Ref [1] the full nonlinear equations of motion are derived without the aid of simplifying assumptions. However, if care is taken, a simplified model will accurately describe the bifilar pendulum dynamics and yields an expression for the moment of inertia about the vertical axis in terms of the period and other measurable values.

1.2 Simplified Model

Figure 1 shows the simplified bifilar pendulum configuration. This model assumes that the axis of rotation is vertical, equidistant from the two wires, and passes through the center of mass. In this case the wires have equal tension, each equal to one half of the weight (W) of the body and will provide a restoring force whenever the rigid body is rotated about the x axis. The sum of moments (M_x) incurred is, for small displacements,

$$\Sigma M_x = WRsin(\theta) \approx WR\theta.$$
 (1)

The desired moment of inertia about the x axis (I_{xx}) can be introduced using Newton's Law,

$$WR\theta = I_{xx}\ddot{\phi}. (2)$$

Using the geometric relation

$$L\theta = R\phi \tag{3}$$

an equation of motion for the θ dimension can be written:

$$\ddot{\theta} = \frac{WR^2}{LI_{xx}}\theta. \tag{4}$$

This simple harmonic oscillator system has a natural frequency (ω) given by

$$\omega = \sqrt{\frac{WR^2}{LI_{xx}}}. (5)$$

Rearranging Eqn. 5, using D = 2R, and writing τ for the natural period gives an expression for I_{xx} in terms all measurable quantities:

$$I_{xx} = \frac{WD^2\tau^2}{16\pi^2L}. (6)$$

1.3 Torsional Correction

A correction to Eqn. 6 must be made to account for the torsional effects of the wires. Modifying Eq.2 to account for this additional torque gives

$$WR\theta + 2\frac{GJ}{L}\phi = I_{xx}\ddot{\phi} \tag{7}$$

where G is the shear modulus and J is the polar moment of inertia of the wire. The expression GJ/L is the twisting spring constant k_{eff} of the wire according to strain theory. Combining Eqn. 3 and 7 and solving the harmonic system gives the correction term for I_{xx} as

$$I_{torsion} = \frac{GJ\tau^2}{2\pi^2 L} = \frac{k_{eff}\tau^2}{2\pi^2} \tag{8}$$

which agrees with Ref.[2]. The final equation used for I_{xx} is

$$I_{xx} = \frac{WD^2\tau^2}{16\pi^2L} + \frac{k_{eff}\tau^2}{2\pi^2} \tag{9}$$

1.3.1 Torsional Spring Constant Measurement

 k_{eff} can be measured experimentally by suspending an object from one of the wires and measuring the torsional period. By knowing the moment of inertia (I_{test}) of the test body, the harmonic relation

$$\frac{2\pi}{\tau_n} = \sqrt{\frac{k_{eff}}{I_{test}}} \tag{10}$$

and the natural period (τ_n) by experiment, k_{eff} can be obtained and used in Eqn. 8.

2 Moment of Inertia Matrix

Section 1.1 provides a means of determining the moment of inertia of the thruster about a vertical axis. In order to construct the full moment of inertia tensor (I), the experiment must be repeated with the thruster suspended in six different orientations. Since I is a symmetric matrix, three of the nine elements are repeated. Once six moments of inertia have been measured about six sufficiently different axis on the thruster, I can be determined for any coordinate system desired. I is constructed from the measured values through the use of a rotation transformation.

2.1 Rotation Transformation

The moment of inertia tensor transforms under a rotation according to the following relation[3]

$$\mathbf{I}^{\mathbf{a}} = (\mathbf{R}^{\mathbf{a}\mathbf{b}})^{-1} \mathbf{I}^{\mathbf{b}} \mathbf{R}^{\mathbf{a}\mathbf{b}}$$
 (11)

where R^{ab} transforms from the a frame to the b frame. This equation allows a measured moment of inertia about an arbitrary axis to be related to the moments of inertia about the desired axes. Several such measurements must be made to determine the entire moment of inertia tensor in the desired frame.

2.2 Selecting the Necessary Independent Measurements

Six orientations of the thruster need to be selected. Each experiment will provide a value for the moment

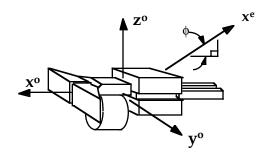


Figure 2: The LES 8/9 APPT reference frame and a sample x axis orientation.

of inertia I_{xx} about the x^e axis as labeled in figure 1 and will provide one equation of the form of Eqn. 11,

$$\begin{bmatrix} I_{xx}^{e} & - & - \\ - & - & - \\ - & - & - \end{bmatrix} = (\mathbf{R^{eo}})^{-1} \begin{bmatrix} I_{xx}^{o} & I_{xy}^{o} & I_{xz}^{o} \\ I_{yx}^{o} & I_{yy}^{o} & I_{yz}^{o} \\ I_{zx}^{o} & I_{zy}^{o} & I_{zz}^{o} \end{bmatrix} \mathbf{R^{eo}}$$
(12)

where I^o represents the moment of inertia tensor of the thruster in the desired frame. Figure 2 shows the **o** frame and an x^e axis for which the moment of inertia (I_{xx}) is measured in one experimental configuration. For example, a right handed rotation of the o frame about the y^o axis (see figure 2) of amount ϕ would align the x^o axis and the x^e axis of the experiment. Such a rotation is represented by the rotation matrix

$$R^{eo} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}. \tag{13}$$

Placing this into Eqn. 12 and writing the expression for the element I_{xx}^e gives

$$c^{2}I_{xx}^{o} - 2scI_{xz}^{o} + s^{2}I_{zz}^{o} = I_{xx}^{e}$$
 (14)

where c and s represent the cosine and sine respectively of the angle ϕ . One can see that after three different values of I^e_{xx} have been obtained from experiment I^o_{xx} , I^o_{xz} and I^o_{zz} can be solved for and another measurement in the xz plane would be redundant. In this case, I^o_{xy} and I^o_{yy} can then be obtained from two different measurements of the moment of inertia in the xy plane and lastly I^o_{yz} from a measurement in the yz plane.

3 Experimental Results

3.1 Io of the LES 8/9 APPT

The technique described above was used with the LES 8/9 APPT. In cases where simultaneous equations needed to be solved, they were solved numerically. The results are, in $kg\ m^2$,

$$\mathbf{I^{O}} = \begin{bmatrix} 3.78e - 2 & 5.45e - 4 & 5.78e - 3 \\ 5.45e - 4 & 5.63e - 2 & 5.89e - 3 \\ 7.78e - 3 & 5.89e - 3 & 4.90e - 2 \end{bmatrix}$$
(15)

3.2 Error Calculations

The errors on the above results were obtained by numerically monitoring the solutions as the inputs values were varied throughout the respective uncertainty ranges. The percent uncertainty on all moments of inertia (the diagonal elements) was found to be 1.5% and on the products of inertia 5%.

3.2.1 Center of Mass Misalignment

The center of mass was located by hanging the thruster by a wire and noting that the center of mass must align with the wire. Two such experiments locate the center of mass. Using the parallel axis theorem, the center of mass location must be known to an error of ϵ such that the bifilar pendulum is rotating about an axis a distance ϵ away from the x axis as described in section 1.2. In this case the parallel axis theorem says that the error for a rigid body mass (m) is $m\epsilon^2$ must be much smaller then I_{xx}^e as determined in the experiment.

References

- [1] T.R. Kane and Gan-Tai Tseng. Dynamics of the bifilar pendulum. *Int. J. Mech. Sci*, 1967. Vol. 9, p.83-96.
- [2] G.H. Jones. A bifilar moment of inertia facility. In 23d nat'l. conf. of the soc. of aeron. weight engr, Dallas, May 18-21 1964. N66-37974, CASI 66N37984.
- [3] E. Wiesel, W. Spaceflight dynamics. McGraw-Hill, 1989.