KALMAN FILTER EXAMPLE:

F-16 SHORT PERIOD DYNAMACICS APPROXIMATION

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + G\underline{w}_g \qquad \underline{x} = \begin{bmatrix} \alpha \\ q \end{bmatrix} \begin{matrix} \text{angle of attack} \\ \text{pitch rate} \end{matrix}$$

For $V_T = 502\ ft/s$, $CG = .35\bar{c}$

$$u = [\delta_e] \text{ elevator input}$$

$$\underline{w}_g = \text{gust velocity}$$

$$A = \begin{bmatrix} -1.01887 & .90506 \\ .82225 & -1.07741 \end{bmatrix}$$

$$B = \begin{bmatrix} -.00215 \\ -0.17555 \end{bmatrix}, \quad G = \begin{bmatrix} 0.00203 \\ -0.00164 \end{bmatrix}$$

Assume w_g IS WHITE, unit (~~on [0,1]~~)

Suppose WE HAVE NO ANGLE OF ATTACK SENSOR (OR FAILED) AND WE WANT TO ESTIMATE α. WE CAN MEASURE q, AND NORMAL ACCELERATION n_z EASILY.

FROM LINEARIZATION OF OUR SIMULATOR,

$$\underline{y} = \begin{bmatrix} n_z \\ q \end{bmatrix} = \begin{bmatrix} 15.87875 & 1.48113 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \underline{v} \equiv C\underline{x} + \underline{v}$$

where $\underline{v}$ = MEASUREMENT NOISE. WE LET OUR SENSORS SIT AT REST FOR A WHILE AND DISCOVER ITS VARIANCE IN $q = 1/60$, IN n_z IS $1/20$, THUS LET COVARIANCE R BE

$$R = \begin{bmatrix} 1/20 & 0 \\ 0 & 1/60 \end{bmatrix}$$

ASSUMED w IS UNIT INTENSITY

$$Q = [1]$$

NOW SOLVE THE ARE TO GET $P \rightarrow L = \begin{bmatrix} -.0003934 & 4.77 \times 10^{-5} \\ .002331 & 3.84 \times 10^{-3} \end{bmatrix}$

$L \quad 160]$

NOW, SOLVE THE A.R.E TO GET $P \rightarrow L = \begin{bmatrix} .002331 & 3.84 \times 10^{-5} \end{bmatrix}$

THUS OUR ESTIMATE IS

$$\dot{\hat{\underline{x}}} = (A - LC)\hat{\underline{x}} + B\underline{u} + Ly$$

WE CAN IMPLEMENT THIS IN A SUBROUTINE OR IMPLEMENT ITS

T.F. $H_\alpha(s) = \begin{bmatrix} 1 & 0 \end{bmatrix}\left[(sI - (A - LC))^{-1}\right]\begin{bmatrix} B & L \end{bmatrix}$ AND

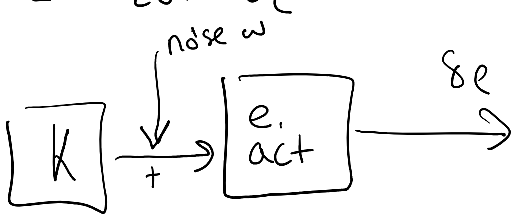
$$\hat{\alpha}(s) = H_\alpha(s)\begin{pmatrix} \bar{U}(s) \\ Y(s) \end{pmatrix}$$

ADD AN ACTUATOR W/ T.F $\dfrac{20.2}{s + 20.2} = \dfrac{\delta e}{u_e}$

$$\frac{d}{dt}\begin{bmatrix} \alpha \\ q \\ \delta e \end{bmatrix} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{A} & B \\ \hline 0 & 0 & -20.2 \end{array}\right]\begin{bmatrix} \alpha \\ q \\ \delta e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20.2 \end{bmatrix} u_e + \begin{bmatrix} G \\ 0 \end{bmatrix} w_g$$

$$\Rightarrow L = \begin{bmatrix} 0.00394 & -4.77 \times 10^{-5} \\ -0.08237 & 3.844 \times 10^{-5} \\ 0 & 0 \end{bmatrix}$$

$$\dot{\delta}_e = -20.2\,\delta_e + 20.2\,u$$



BUT WHAT IF THE DYNAMICS OR MSMNT EQN ARE NOT LINEAR? e.g., not of the form $\dot{\underline{x}} = A\underline{x} + B\underline{u}$

$\underline{y} = C\underline{x}$

SAY $\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{w})$

$\underline{y} = \underline{h}(\underline{x}, \underline{v})$

WE CAN USE A ~~SIMILAR~~ N.L. ESTIMATOR SUCH AS AN UNSCENTED K.F OR A PARTICLE FILTER OR USE

AN EXTENDED KALMAN FILTER

"USE A K.F. APPROACH, BUT WHEN CHOOSING L, YOU NEED A, B, C, SO USE A LOCAL LINEARIZATION

$$A = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}(t), \underline{u}(t)}, \qquad B = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\underline{x}(t), \underline{u}(t)}$$

$$C = \left.\frac{\partial \underline{h}}{\partial \underline{x}}\right|_{\underline{x}(t), \underline{u}(t)}$$

THEORETICAL GUARANTEES? THERE ARE NONE, BUT USUALLY WORKS.

EXAMPLE: A.H.R.S.

↑ Attitude and Heading reference system