OKLAHOMA STATE UNIVERSITY
School of Mechanical and Aerospace Engineering

MAE 5010 - Autopilot Design & Test

Homework #2 (Assigned: 9/5, Due: 9/14)
Aerodynamic Force Modeling

1. Review Stevens, Lewis, & Johnson Chapter 2.

2. Write the total aerodynamic forces in body frame coordinates for the simplified aircraft layout in Fig. 1 using the component buildup method.

   Thus, your forces should be:

$$\vec{F} = \vec{T} + \vec{L_w} + \vec{L_h} + \vec{L_v} + \vec{D_w} + \vec{D_h} + \vec{D_v} \tag{1}$$

   and

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \left[\sum \vec{L_i} + \sum \vec{D_i} + \sum \vec{S_i}\right]_B \tag{2}$$

   Hints and assumptions:

   Wing Aerodynamics Model the wing aero forces with

$$L_{\mathrm{w}} = \frac{1}{2}\rho V_T^2 S\left[C_L(\alpha) + C_{L_q}\frac{c}{2V_T}q + C_{L_{\delta_e}}\delta_e\right] \tag{3}$$

$$D_{\mathrm{w}} = \frac{1}{2}\rho V_T^2 S\left[C_D(\alpha) + C_{D_q}\frac{c}{2V_T}q + C_{D_{\delta_e}}\delta_e\right] \tag{4}$$

$$M_w = \frac{1}{2}\rho V_T^2 Sc\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{c}{2V_T}q + C_{m_{\delta_e}}\delta_e\right] \tag{5}$$

   Include stall only on the main wing with

$$C_L(\alpha) = (1-\sigma(\alpha))\left[C_{L_0} + C_{L_\alpha}\alpha\right] + \sigma(\alpha)\left[2\mathrm{sign}(\alpha)\sin^2\alpha\cos\alpha\right] \tag{6}$$

$$\sigma(\alpha) = \frac{1 + e^{-M(\alpha-\alpha_0)} + e^{M(\alpha+\alpha_0)}}{\left(1+e^{-M(\alpha-\alpha_0)}\right)\left(1+e^{M(\alpha+\alpha_0)}\right)}, M = 50, \alpha_0 = 0.471 \tag{7}$$

   Assume for this aircraft $C_{m_\alpha} = -0.38, C_{m_q} = -3.6, C_{L_q} = C_{D_q} = 0$.

   Tail surfaces You may assume the vertical tail generates only a force as a function of β (ie, not dependent on α, and and neglect β or side forces in the other aerodynamic surface models. You may assume these surfaces do not stall, and neglect the quarter chord moment generated by them as well.

Thrust modeling You may assume that $\vec{T}$ acts along $\hat{b}_x$ with magnitude $T = T_{max}\delta_T$ such that

$$X = T + \left[R_{BW}\vec{L_w}\right]_{\text{xcomponent}} + ..., \text{etc.} \tag{8}$$

Control modeling You may neglect the control surface area in your aerodynamic force expressions and account for them by $C_{l_{\delta a}} = 0.08, C_{n_{\delta a}} = 0.06, C_{L_{\delta_e}} = -0.36, C_{m_{\delta e}} = -0.5, C_{Y_{\delta_r}} = -0.17, C_{l_{\delta r}} = 0.105$

3. Estimate the stability derivatives for the Boeing 737-800 aircraft using your choice of numerical tool.

   Use a reference flight condition of $M = 0.25$ cruise at sea level, such that $\rho = 1.12$ kg/m^3 and $V_T = 85$m/s, and $\alpha = 3°$.

   Hints: You may find the provided `.avl` model helpful. To estimate the moments of inertia, you may assume $I_{xz} = 0$ and find the primary moments of inertia I_{ii} using the radii of gyration $K_x = 4.73, K_y = 9.01, K_z = 9.73$ meters using

$$I_{ii} = mK_i^2, \quad i = \{x, y, z\} \tag{9}$$

   and the properties

Param.	Value	Unit
b	34.32	m
c	4.235	m
m	79015.8	kg
S	124.862	m^2
L_{total}	38.0	m

Table 1: 737-800 parameters

4. Add a function or class method to your code that computes body frame aerodynamic forces and moments for the simplified aircraft model at a given state. You may use the following properties.

Component		Value
Wing Airfoil	C_{L0}	.28
	$C_{L\alpha}$	3.5/rad
	C_{D0}	0.015
	C_{m0}	-0.02
H. tail Airfoil		
	C_{L0}	.1
	$C_{L\alpha}$	5.79/rad
	C_{D0}	0.01
	C_{m0}	0

Component		Value
Wing	b	2.896 m
	c	0.207 m
	l_w	.3m
H. tail	b	.7m
	c	0.1m
	l_h	2.5m
V. tail	b	.3m
	c	0.1m
	l_v	2.5m

Parameter	Value
m	13.5 kg
I_x	0.824 kg-m^2
I_y	1.135 kg-m^2
I_z	1.759 kg-m^2
I_{xz}	0.1204 kg-m^2
S	0.55 m^2
T_{max}	25N
ρ	1.2682 kg/m^3

Table 2: Simplified aircraft parameters

5. Connect this aerodynamics function to your dynamics integrator system from HW1. Don't forget to include gravity. Your first runs will invariably not be at trim. By changing your initial condition and control inputs, can you find the straight and level trim position?

   Submission: Upload your submission to Canvas as a single PDF, being sure to attach a code appendix.

1 Aerodynamic Equation reference

$$F_{\text{lift}} = \frac{1}{2}\rho V_T^2 S\left[C_L(\alpha) + C_{L_q}\frac{c}{2V_T}q + C_{L_{\delta_e}}\delta_e\right] \tag{10}$$

$$F_{\text{drag}} = \frac{1}{2}\rho V_T^2 S\left[C_D(\alpha) + C_{D_q}\frac{c}{2V_T}q + C_{D_{\delta_e}}\delta_e\right] \tag{11}$$

$$M = \frac{1}{2}\rho V_T^2 Sc\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{c}{2V_T}q + C_{m_{\delta_e}}\delta_e\right] \tag{12}$$

$$C_L(\alpha) = (1-\sigma(\alpha))\left[C_{L_0} + C_{L_\alpha}\alpha\right] + \sigma(\alpha)\left[2\,\text{sign}(\alpha)\sin^2\alpha\cos\alpha\right] \tag{13}$$

$$\sigma(\alpha) = \frac{1 + e^{-M(\alpha-\alpha_0)} + e^{M(\alpha+\alpha_0)}}{\left(1 + e^{-M(\alpha-\alpha_0)}\right)\left(1 + e^{M(\alpha+\alpha_0)}\right)} \tag{14}$$

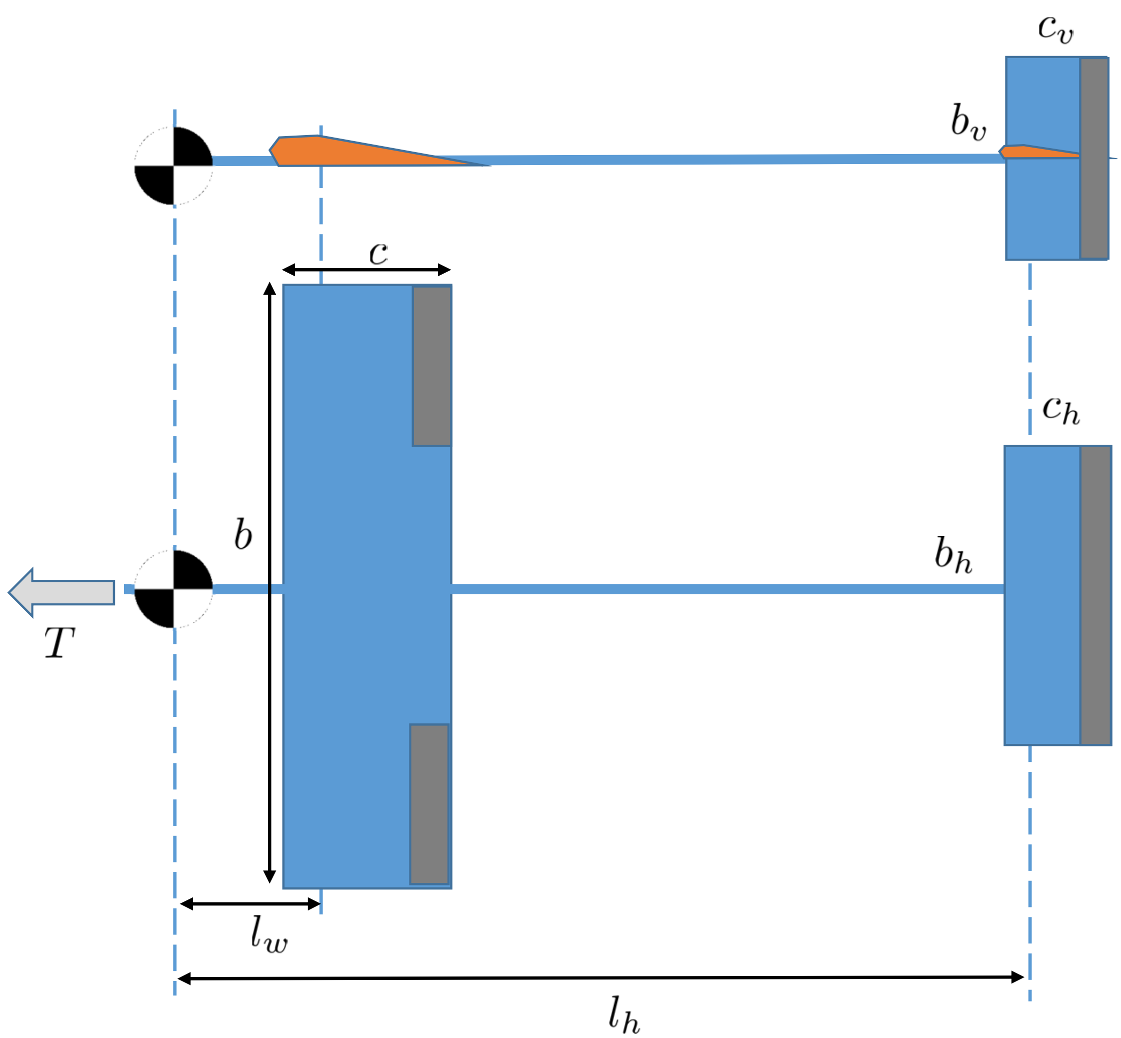

Figure 1: AC geometry