TABLE 6.4-1 The Kalman Filter

System Model

$$\dot{x} = Ax + Bu + Gw$$

$$y = Cx + v$$

$$x(0) \sim (\bar{x}_0, P_0), \quad w(t) \sim (0, Q), \quad v(t) \sim (0, R)$$

Assumptions

$w(t)$ and $v(t)$ are white noise processes orthogonal to each other and to $x(0)$.

Initialization

$$\hat{x}(0) = \bar{x}_0$$

Error Covariance ARE

$$AP + PA^{\mathrm{T}} + GQG^{\mathrm{T}} - PC^{\mathrm{T}}R^{-1}CP = 0$$

Kalman Gain

$$L = PC^{\mathrm{T}}R^{-1}$$

Estimate Dynamics (Filter Dynamics)

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$