CSC 421 Assignment3

Zimeng Ming V00844078
Ronald Liu V00838627

Question1:

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T1:

clausal form:

$(p \mid q \mid \neg r) \,\&\, ((\neg r \mid q \mid p) \rightarrow ((r \mid q) \,\&\, \neg q \,\&\, \neg p))$

I: $(p \mid q \mid \neg r) \,\&\, ((\neg r \mid q \mid p) \mid ((r \mid q) \,\&\, \neg q \,\&\, \neg p))$

N: $(p \mid q \mid \neg r) \,\&\, [(r \,\&\, \neg q \,\&\, \neg p) \mid ((r \mid q) \,\&\, \neg q \,\&\, \neg p)]$

D: $(p \mid q \mid \neg r) \,\&\, [((r \,\&\, \neg q \,\&\, \neg p) \mid (r \mid q)) \,\&\, ((r \,\&\, \neg q \,\&\, \neg p) \mid \neg q) \,\&\, ((r \,\&\, \neg q \,\&\, \neg p) \mid \neg p)]$

$= (p \mid q \mid \neg r) \,\&\, [((r \mid (r \mid q)) \,\&\, (\neg q \mid (r \mid q)) \,\&\, (\neg p \mid (r \mid q))) \,\&\, ((r \mid \neg q) \,\&\, (\neg q \mid \neg q) \,\&\, (\neg p \mid \neg q)) \,\&\, ((r \mid \neg p) \,\&\, (\neg q \mid \neg p) \,\&\, (\neg p \mid \neg p))$

$= (p \mid q \mid \neg r) \,\&\, [(r \mid q) \,\&\, (\neg q \mid r \mid q) \,\&\, (\neg p \mid r \mid q)) \,\&\, (r \mid \neg q) \,\&\, (\neg q) \,\&\, (\neg p \mid \neg q) \,\&\, (r \mid \neg p) \,\&\, (\neg q \mid \neg p) \,\&\, (\neg p)$

O: $\{p, q, \neg r\}, \{r, q\}, \{\neg q, r, q\}, \{\neg p, r, q\}, \{r, \neg q\}, \{\neg q\}$
$\{\neg p, \neg q\}, \{r, \neg p\}, \{\neg q, \neg p\}, \{\neg p\}$.

resolution step.

1. $\{p, q, -r\}$
2. $\{r, q\}$
3. $\{-q, r, q\}$
4. $\{-p, r, q\}$
5. $\{r, -q\}$
6. $\{-q\}$
7. $\{-p, -q\}$
8. $\{r, -p\}$
9. $\{-q, -p\}$
10. $\{-p\}$

resolution:

11. $\{p, q\}$ 1, 2
12. $\{q\}$ 11, 10
13. $\{\}$ 12, 6

Question2:

Q2:

$\forall x, \forall y\ (horse(x) \land dog(y) \Rightarrow faster(x, y))$

$\exists y\ (Greyhound(y) \land \forall z\ (Rabbit(z) \Rightarrow Faster(y, z)))$

$\forall y\ (Greyhound(y) \Rightarrow Dog(y))$

$\forall x\ \forall y\ \forall z\ (Faster(x, y) \land Faster(y, z) \Rightarrow Faster(x, z))$

$\neg \forall x\ \forall y\ (Horse(x) \land Rabbit(y) \Rightarrow Faster(x, y))$ negated conclusion

do casual form and do the resolution steps for each statement

① $\forall x, \forall y\ (horse(x) \land dog(y) \Rightarrow faster(x, y))$

I: $\forall x, \forall y\ (\neg horse(x) \land dog(y)) \lor faster(x, y))$

N: $\forall x, \forall y\ ((\neg horse(x) \lor \neg dog(y)) \lor faster(x, y))$

S: $\forall x, \forall y\ ((\neg horse(x) \lor \neg dog(y)) \lor faster(x, y))$

E: $\forall x, \forall y\ ((\neg horse(x) \lor \neg dog(y)) \lor faster(x, y))$

A: $(\neg horse(x) \lor \neg dog(y)) \lor faster(x, y)$

D: $\neg horse(x) \lor \neg dog(y) \lor faster(x, y)$

O: $\{ \neg horse(x),\ \neg dog(y),\ faster(x, y) \}$

② $\exists y\ (Greyhound(y) \land (\forall z\ (Rabbit(z) \Rightarrow faster(y, z))))$

I: $\exists y\ ((Greyhound(y)) \land \forall z\ (\neg Rabbit(z)) \lor faster(y, z)))$

N: $\exists y\ ((Greyhound(y)) \land \forall z\ (\neg Rabbit(z) \lor faster(y, z)))$

S: $Greyhound(a) \land \forall z\ (\neg Rabbit(z) \lor faster(a, z)))$

E: $Greyhound(a) \land \forall z\ (\neg Rabbit(z) \lor faster(a, z)))$

A: $Greyhound(a) \land (\neg Rabbit(z) \lor faster(a, z)))$

O: $\{ Greyhound(a) \}, \{ \neg Rabbit(z),\ fasten(a, z) \}$

③ $\forall y\ (Greyhound(y) \Rightarrow Dog(y))$

I: $\forall y\ (\neg Greyhound(y) \lor Dog(y))$

A: $\neg Greyhound(y) \lor Dog(y)$

O: $\{ \neg Greyhound(y),\ Dog(y) \}$

④ $\forall x \forall y \forall z\ (Faster(x,y) \wedge Faster(y,z) \Rightarrow Faster(x,z))$

I: $\forall x \forall y \forall z\ (\neg(Faster(x,y) \wedge Faster(y,z)) \vee Faster(x,z))$

N: $\forall x \forall y \forall z\ ((\neg Faster(x,y) \vee \neg Faster(y,z)) \vee Faster(x,z))$

A: $(\neg Faster(x,y) \vee \neg Faster(y,z)) \vee Faster(x,z))$

D: $\neg Faster(x,y) \vee \neg Faster(y,z) \vee Faster(x,z)$

O: $\{\neg Faster(x,y),\ \neg Faster(y,z),\ Faster(x,z)\}$

⑤ $\neg \forall x \forall y\ (Horse(x) \wedge Rabbit(y) \Rightarrow Faster(x,y))$

I: $\neg \forall x \forall y\ (\neg(Horse(x) \wedge Rabbit(y)) \vee Faster(x,y))$

N: $\exists x \exists y\ ((Horse(x) \wedge Rabbit(y)) \wedge \neg(Faster(x,y)))$

E: $(Horse(b) \wedge Rabbit(c)) \wedge \neg Faster(b,c)$

D: $Horse(b) \wedge Rabbit(c) \wedge \neg Faster(b,c)$

O: $\{Horse(b)\},\ \{Rabbit(c)\},\ \{\neg Faster(b,c)\}$

resolution steps:

① $\{\neg horse(x),\ \neg dog(y),\ faster(x,y)\}$

② $\{Greyhound(a)\}$

③ $\{\neg Rabbit(z),\ faster(a,z)\}$

④ $\{\neg Greyhound(y),\ Dog(y)\}$

⑤ $\{\neg Faster(x,y),\ \neg faster(y,z),\ faster(x,z)\}$

⑥ $\{Horse(b)\}$

⑦ $\{rabbit(c)\}$

⑧ $\{\neg Faster(b,c)\}$

$a \rightarrow y$

$b \rightarrow x$

$c \rightarrow z$

⑨ $\{\neg dog(y),\ faster(x,y)\}$ ① ⑥

⑩ $\{\neg Greyhound(y),\ faster(x,y)\}$ ④ ⑨

⑪ $\{faster(a,z)\}$ ③ ⑦

⑫ $\{faster(x,y)\}$ ② ⑩

⑬ $\{\neg faster(y,z),\ faster(x,z)\}$ ⑤ ⑫

⑭ $\{faster(x,z)\}$ ⑬ ⑪

⑮ $\{\ \}$ ⑭ ⑧

Question 3:

Assumption:
all x (hummingbird(x) -> bird(x)).
all x (hummingbird(x) -> richly_colored(x)).
all x (bird(x) & large(x) -> -live_on_honey(x)).
all x (bird(x) & -live_on_honey(x) -> -richly_colored(x)).

goal:
all x (hummingbird(x) -> -large(x)).

```
============================== prooftrans ============================
Prover9 (32) version Dec-2007, Dec 2007.
Process 79097 was started by brandonming on Brandons-MacBook-Pro.local,
Mon Nov 18 20:46:03 2019
The command was "/private/var/folders/_q/yybjd8v575d1rrgwsj7r51dr0000gn/T/
AppTranslocation/9DB74EC8-292F-4B76-8F30-33BA37F1B7DD/d/Prover9-Mace4-v05B.app/
Contents/Resources/bin-mac-intel/prover9".
============================== end of head ===========================

============================== end of input ==========================

============================== PROOF =================================

% -------- Comments from original proof --------
% Proof 1 at 0.00 (+ 0.00) seconds.
% Length of proof is 18.
% Level of proof is 5.
% Maximum clause weight is 0.
% Given clauses 0.

1 (all x (hummingbird(x) -> bird(x))) # label(non_clause).  [assumption].
2 (all x (hummingbird(x) -> richly_colored(x))) # label(non_clause).  [assumption].
3 (all x (bird(x) & large(x) -> -live_on_honey(x))) # label(non_clause).  [assumption].
4 (all x (bird(x) & -live_on_honey(x) -> -richly_colored(x))) # label(non_clause).  [assumption].
5 (all x (hummingbird(x) -> -large(x))) # label(non_clause) # label(goal).  [goal].
6 hummingbird(c1).  [deny(5)].
7 -hummingbird(x) | bird(x).  [clausify(1)].
8 -hummingbird(x) | richly_colored(x).  [clausify(2)].
9 bird(c1).  [resolve(6,a,7,a)].
10 -bird(x) | -large(x) | -live_on_honey(x).  [clausify(3)].
11 -bird(x) | live_on_honey(x) | -richly_colored(x).  [clausify(4)].
12 -large(c1) | -live_on_honey(c1).  [resolve(9,a,10,a)].
13 large(c1).  [deny(5)].
14 live_on_honey(c1) | -richly_colored(c1).  [resolve(9,a,11,a)].
15 richly_colored(c1).  [resolve(6,a,8,a)].
16 live_on_honey(c1).  [resolve(14,b,15,a)].
17 -live_on_honey(c1).  [resolve(12,a,13,a)].
18 $F.  [resolve(16,a,17,a)].

============================== end of proof ==========================
```

Question 4:

Assumption:

worth_listening_subjects(gardener).
all x (remember_battle_Waterloo(x) -> old(x)).
all x (worth_listening_subjects(x) -> remember_battle_Waterloo(x)).

Goal:

old(gardener).

============================== PROOF =================================

% -------- Comments from original proof --------
% Proof 1 at 0.00 (+ 0.00) seconds.
% Length of proof is 10.
% Level of proof is 4.
% Maximum clause weight is 0.
% Given clauses 0.

1 (all x (remember_battle_Waterloo(x) -> old(x))) # label(non_clause). [assumption].
2 (all x (worth_listening_subjects(x) -> remember_battle_Waterloo(x))) # label(non_clause). [assumption].
3 old(gardener) # label(non_clause) # label(goal). [goal].
4 -worth_listening_subjects(x) | remember_battle_Waterloo(x). [clausify(2)].
5 worth_listening_subjects(gardener). [assumption].
6 remember_battle_Waterloo(gardener). [resolve(4,a,5,a)].
7 -remember_battle_Waterloo(x) | old(x). [clausify(1)].
8 old(gardener). [resolve(6,a,7,a)].
9 -old(gardener). [deny(3)].
10 $F. [resolve(8,a,9,a)].

============================== end of proof ==========================

Question5:

Q5: $P_{3,1}$ and P_{13} has same probability since it is symmetric.

$P(P_{13} | b_{12}, b_{21}) = \alpha \sum \sum P(b_{12} | P_{13}, P_{22}) \cdot P(b_{21} | P_{31}, P_{13}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31})$

$= \alpha [P(b_{12} | P_{13}, P_{22}) \cdot P(b_{21} | P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) +$

$P(b_{12} | P_{13}, \neg P_{22}) \cdot P(b_{21} | P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) +$

$P(b_{12} | P_{13}, P_{22}) \cdot P(b_{21} | \neg P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) +$

$P(b_{12} | P_{13}, \neg P_{22}) \cdot P(b_{21} | \neg P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31})]$

$= \alpha (1 \times 1 \times 0.01 \times 0.01 \times 0.01 + 1 \times 1 \times 0.01 \times 0.99 \times 0.01 + 1 \times 1 \times 0.01 \times 0.01 \times 0.99 + 0)$

$= 0.000199\,\alpha$

$P(\neg P_{13} | b_{12}, b_{21})$

$= \alpha (1 \times 1 \times 0.99 \times 0.01 \times 0.01 + 0 + 0.99 \times 0.99 \times 0.01 \times 1 \times 1 + 0)$

$= 0.0099\,\alpha$

$\alpha = \frac{1}{0.000199 + 0.0099}$

$P(P_{13} | b_{12}, b_{21}) = \frac{0.000199}{\alpha} = 0.019705 = 1.97\%$

$P(\neg P_{13} | b_{12}, b_{21}) = \frac{0.0099}{\alpha} = 0.980295 = 98.03\%$

$P(P_{22} | b_{12}, b_{21})$

$= \alpha (1 \times 1 \times 0.01 \times 0.01 \times 0.01 + 1 \times 1 \times 0.01 \times 0.99 \times 0.01 + 1 \times 1 \times 0.01 \times 0.01 \times 0.99 + 1 \times 1 \times 0.01 \times 0.99 \times 0.99)$

$= 0.01\,\alpha$

$P(\neg P_{22} | b_{12}, b_{21})$

$= \alpha (1 \times 1 \times 0.01 \times 0.99 \times 0.01 + 0 + 0 + 0)$

$= 0.000099\,\alpha$

$\alpha = \frac{1}{0.000099 + 0.01} = \frac{1}{0.010099}$

$P(P_{22} | b_{12}, b_{21})$

$= \frac{0.01}{\alpha} \approx 0.99 \approx 99\%$

$P(\neg P_{22} | b_{12}, b_{21})$

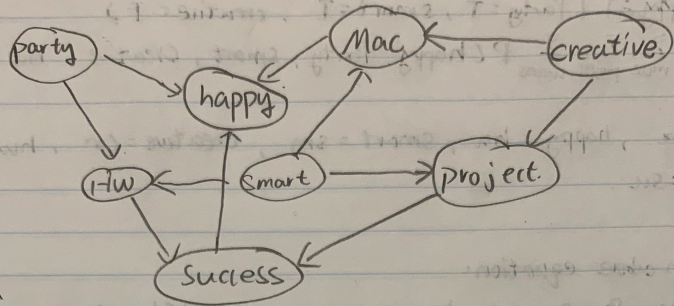
$= \frac{0.000099}{\alpha} \approx 0.9\%$

Thus, going to [2, 2] is almost certain death [3, 1] and [1, 3] is the probability step that the logical agent goes. But three box are looks have equal opportunities So 1/3 chance to die, it is same with probability = 0.2,

Question6:

Q6:

①



② $P(party = T) = 0.6022$ $P(party = F) = 0.3978$

$P(creative = T) = 0.6994$ $P(creative = F) = 0.3006$

$P(smart = T) = 0.7048$ $P(smart = F) = 0.2952$

Party	Smart	P(Hw | Party, smart)
T	T	0.802797
T	F	0.093533
F	T	0.898477
F	F	0.3049180

smart	creative	P(Mac | smart, creative)
T	T	0.685796
T	F	0.413308
F	T	0.897115
F	F	0.121560

Hw	Project	P(success | Hw, Project)
T	T	0.896629
T	F	0.306452
F	T	0.206697
F	F	0.049669

Smart	creative	P(project | Smart, creative)
T	T	0.905169
T	F	0.793814
F	T	0.402385
F	F	0.103505

Sucess	Mac	Party	P(Happy | success, Mac, Party)
T	T	T	0.959375
T	T	F	0.357992
T	F	T	0.721519
T	F	F	0.492525
F	T	T	0.306818
F	T	F	0.204969
F	F	T	0.420118
F	F	F	0.093851

③ $P(\text{Happy}=T \mid \text{Party}=T, \text{smart}=T, \text{creative}=F)$

$$= \alpha \sum_{Hw} \sum_{mac} \sum_{project} \sum_{success} P(\text{happy}, \text{Party}, \text{smart}, \text{Creative}, hw, mac, \text{project}, \text{success})$$

Party = pa, happy = ha, smart = sm, creative = cr, hw = hw, mac = mac, project = pr

Success = su.

so, from above equation:

$$= P(ha, pa, sm, cr, hw, mac, pr, su) + P(ha, pa, sm, cr, hw, mac, pr, \neg su)$$
$$+ P(ha, pa, sm, cr, hw, mac, \neg pr, su) + P(ha, pa, sm, cr, hw, mac, \neg pr, \neg su)$$
$$+ P(ha, pa, sm, cr, hw, \neg mac, pr, su) + P(ha, pa, sm, cr, hw, mac, \neg mac, pr, \neg su)$$
$$+ P(ha, pa, sm, cr, hw, \neg mac, \neg pr, su) + P(ha, pa, sm, cr, hw, \neg mac, \neg pr, \neg su)$$
$$+ P(ha, pa, sm, cr, \neg hw, mac, pr, su) + P(ha, pa, sm, cr, \neg hw, mac, pr, \neg su)$$
$$+ P(ha, pa, sm, cr, \neg hw, mac, \neg pr, su) + P(ha, pa, sm, cr, \neg hw, mac, \neg pr, \neg su)$$
$$+ P(ha, pa, sm, cr, \neg hw, \neg mac, pr, su) + P(ha, pa, sm, cr, \neg hw, \neg mac, pr, \neg su)$$
$$+ P(ha, pa, sm, cr, \neg hw, \neg mac, \neg pr, su) + P(ha, pa, sm, cr, \neg hw, \neg mac, \neg pr, \neg su)$$

$P(\text{Happy}=F \mid \text{Party}=T, \text{smart}=T, \text{creative}=F)$

$$= \alpha \sum_{hw} \sum_{mac} \sum_{su} P(\neg ha, pa, sm, cr, hw, mac, pr, su)$$

$= P(\neg ha, pa, sm, cr, hw, mac, pr, su) +$ .... same with above but using $\neg$happy ....

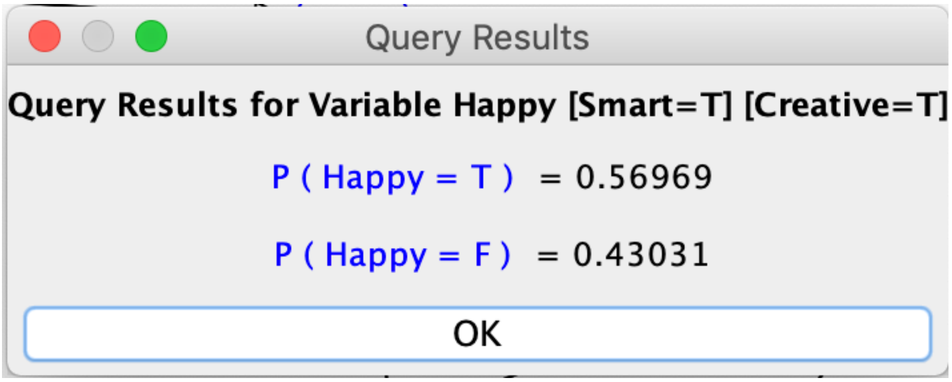
$+ P(\neg ha, pa, sm, cr, \neg hw, \neg mac, \neg pr, \neg su)$

$\alpha = 1/(P(ha \mid pa, sm, \neg c) + P(\neg ha \mid pa, sm, \neg c))$

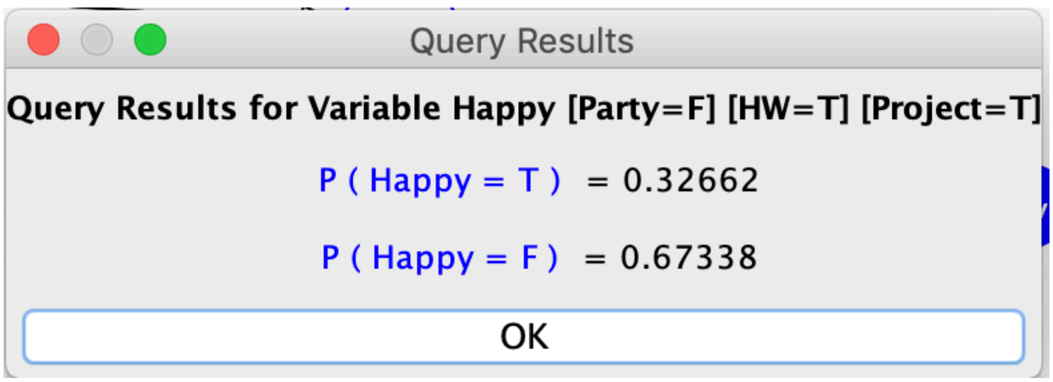
$$P(ha \mid pa, sm, \neg c) = \frac{P(ha \mid pa, sm, \neg c)}{\alpha}$$

$$P(\neg ha \mid pa, sm, \neg c) = \frac{P(\neg ha \mid pa, sm, \neg c)}{\alpha}$$

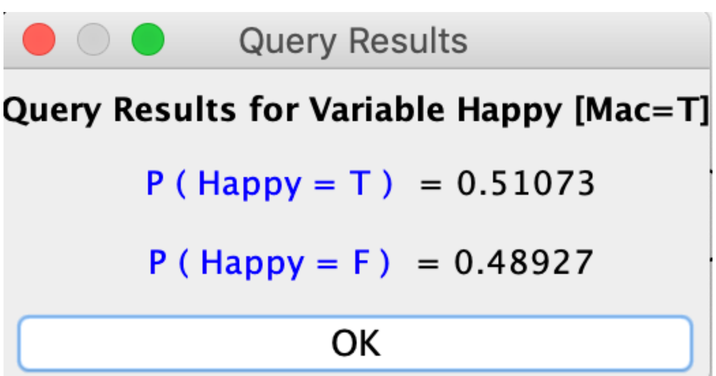
4.


Query Results
Query Results for Variable Happy [Smart=T] [Creative=T]
P (Happy = T) = 0.56969
P (Happy = F) = 0.43031
OK

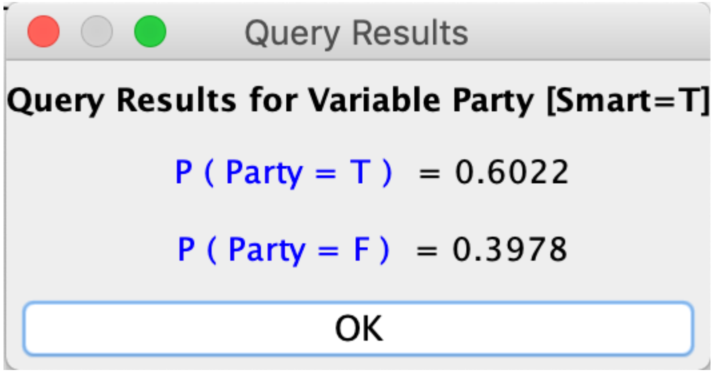
5.


Query Results
Query Results for Variable Happy [Party=F] [HW=T] [Project=T]
P (Happy = T) = 0.32662
P (Happy = F) = 0.67338
OK

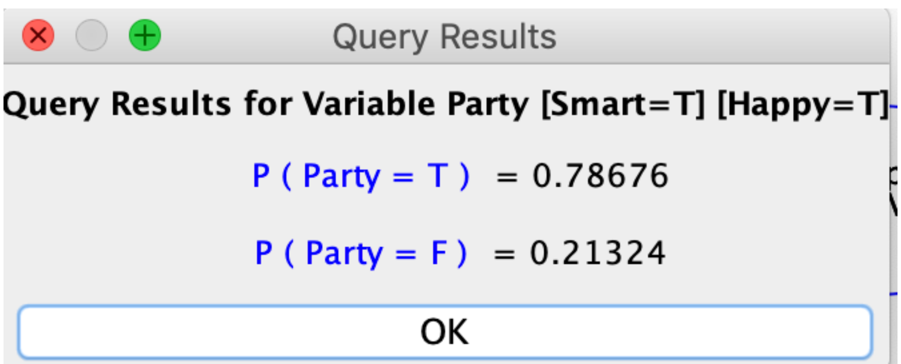
6.


Query Results
Query Results for Variable Happy [Mac=T]
P (Happy = T) = 0.51073
P (Happy = F) = 0.48927
OK

7.


Query Results
Query Results for Variable Party [Smart=T]
P (Party = T) = 0.6022
P (Party = F) = 0.3978
OK

8.


Query Results
Query Results for Variable Party [Smart=T] [Happy=T]
P (Party = T) = 0.78676
P (Party = F) = 0.21324
OK