Impedance

From AstroBaki

Short Topical Videos

Reference Material

- Horowitz & Hill, *The Art of Electronics, 2nd Ed.*, Ch. 1

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Impedance

The concept of impedance is basically an attempt to extend Ohm's Law to devices that are not resistors. Ohm's Law

$$V = IR,$$

relates voltage, V, and current, I, by a resistance, R. Impedance generalizes this by considering V and I as complex waveforms, and using the symbol Z to denote the (possibly complex, and possibly frequency-dependent) number that relates them. Just as a note, since I generally means current in electronics, the imaginary unit $\sqrt{-1}$ is generally denoted as j by electrical engineers. We'll use that notation here as well.

Capacitive Impedance

In discussing capacitors, we used capacitance C to relate current to the time-derivative of voltage:

$$\frac{dV}{dt} = \frac{I}{C}.$$

Knowing from Fourier analysis that we can decompose any waveform into a sum of sin/cos functions, it makes sense to examine how this equation works for a sinusoid waveform like

$$V = e^{j\omega t} = \cos\omega t + j\sin\omega t.$$

In this case we get:

$$\begin{aligned} j\omega e^{j\omega t} &= \frac{I}{C} \\ j\omega V &= \frac{I}{C} \\ V &= I\frac{1}{j\omega C} \end{aligned}$$

In that last line, where we've substituted V back in, we have an equation that looks very much like Ohms law, but instead of R, we have $V = IZ$, with Z, for capacitors, given by:

$$Z_c = \frac{1}{j\omega C}$$

Hence, Ohm's law can be extended to capacitors if we generalize resistance to a complex impedance, and take the imaginary part of that impedance to be a frequency-dependent quantity. Apart from changing the phase of an incoming wave by $90°$, a capacitor can pass a high-frequency wave with very little attenuation, but can completely block a DC voltage with an infinite impedance.

Inductive Impedance

Inductors, on the other hand, have a voltage that is dependent on the time derivative of current:

$$V = L\frac{dI}{dt},$$

This time, let's take the current to be a sinusoid given by:

$$I = e^{j\omega t} = \cos\omega t + j\sin\omega t.$$

In this case we get:

$$V = Lj\omega e^{j\omega t}$$
$$V = Lj\omega I$$

Again, the last line looks like Ohm's Law, if we take the impedance of an inductor to be:

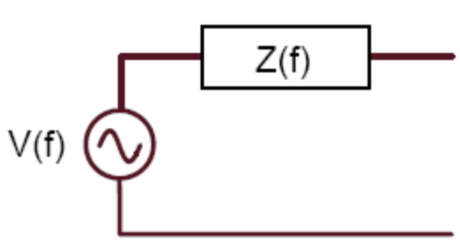
$$Z_L = j\omega L$$

So an inductor is like a resistor that changes the phase of the incoming wave by $90°$, and resists higher frequencies more strongly than lower frequencies (and behaves just as a wire to a DC voltage).

Mixed Circuits

There's actually not a lot to say here. You can pretend that impedances are just complex resistors. They add when wired in series; they add reciprocally when wired in parallel. You'll see that the rules for adding capacitors in parallel falls out naturally using the expressing for the impedance of a capacitor. Oh, and as we'll see when discussing RC and LC filters, you can use the frequency-dependence of the impedances of capacitors and inductors to construct filters that purposely attenuate waveforms based on their constituent frequencies.

Finally, the idea of the Thévenin equivalent circuit (involving a voltage source and an equivalent series resistor) generalizes naturally to impedances. Now instead of an equivalent series resistor, we instead have an equivalent (and possibly frequency-dependent) series impedance:



The Thévenin equivalent circuit, using impedances