Specific Intensity

From AstroBaki
Jump to navigationJump to search

Course Home

Short Topical Videos

- Specific Intensity: What's the Flux? (by Aaron Parsons) (https://youtu.be/7Op36as0HX4)
- Photon Buckets: How (Radio) Telescopes Receive Power (by Aaron Parsons) (http://youtu.be/YzqTpH9Y9y8)

Reference Material

- Brightness and Flux Density (Condon & Ransom, NRAO) (http://www.cv.nrao.edu/course/astr534/Brightness.html)
- Specific Intensity: The Fount of All Knowledge (Heiles, UC Berkeley) (https://github.com/AaronParsons/ugradio/blob/master/spec_intensity/fount.pdf)
- Specific Radiative Intensity (Wikipedia) (https://en.wikipedia.org/wiki/Specific_radiative_intensity)

Related Topics

- Radiative Transfer Equation
- Black-Body Radiation

Contents

Specific Intensity, I_ν, is the amount of flux ($\frac{\text{ergs}}{\text{s} \cdot \text{cm}^2}$) radiated per solid angle (sr) per bandwidth (Hz). Specific Intensity is so constructed because it is conserved along a ray. This makes it a useful quantity for propagating radiative quantities in numerical and theoretical simulations, and for forming an observable quantity that all observers can (given certain telescope specifications) agree upon.

Specific intensity may be written I_ν or I_λ, with units:

$$[I_\nu] = \frac{\text{erg}}{\text{s} \cdot \text{sr} \cdot \text{cm}^2 \cdot \text{Hz}}$$
$$[I_\lambda] = \frac{\text{erg}}{\text{s} \cdot \text{sr} \cdot \text{cm}^2 \cdot \text{cm}}$$

Note that $I_\nu \neq I_\lambda$ because $d\nu \neq d\lambda$. However, $d\nu = d\lambda \cdot c/\lambda^2$, so $d\nu/\nu = d\lambda/\lambda$. This means that

$$\nu I_\nu = \lambda I_\lambda$$

1 Units of Radiation

In order to motivate the useful properties of **specific intensity**, it's helpful to list the variety of ways we have of describing the energy transfer coming from electromagnetic waves (or photons) striking a telescope. Below are a few that are commonly used:

1.1 Voltage

Voltage (with units of Volts, V) gives the most direct view of the shape of the electromagnetic waves that are striking a (usually radio) telescope. However, astronomical signals are usually noise, so the random fluctuations in voltage aren't usually terribly illuminating, and you can't average them directly. This makes voltage a poor unit for learning about the sky.

1.2 Power

Power, P, (with units of ergs/s, or watts, or dBm), which is generally proportional to voltage squared, is a much more useful quantity. For example, it can be averaged over time. However, it encodes no information about what frequency interval (bandwidth) that the measurement was made over, and the power of a measurement is generally proportional to the bandwidth of the signals you let in.

The unit **dBm** may not be familiar to most astronomers. It refers to decibels relative to a milliwatt:

$$P_{\text{dBm}} = 10 \log_{10}\left(\frac{P}{1\text{ mW}}\right)$$

1.3 Power Density

Power density (with units of ergs/s/Hz, or dBm/Hz), divides out by the bandwidth B that the measurement is made over. However, it contains no information about how large an area this signal was collected over.

1.4 Flux

Flux, F, (with units of ergs/s/cm^2) divides power received by the area the signal was collected over, but it does **not** divide by the bandwidth. Because flux has a notion of area (i.e. the surface through which power has passed), flux has an associated direction $\hat{n}$ normal to that surface. Thus, depending which side of a surface you are measuring flux on (defined by the handedness of the perimeter of your surface), you can have a positive or negative flux, even though power is always positive.

1.5 Flux Density

Flux density, F_ν, (with units of ergs/(s cm^2 Hz), or Jy) combines the concepts of power density and flux to get a measurement that divides out both bandwidth and collecting area. Most astronomers can agree on the flux density of a source, but **if the beam of your telescope is smaller than the source on the sky**, you can get a different answer because you are not be getting all the photons from that source–you are only getting the ones illuminated by your beam.

Flux density is power per unit frequency passing through a differential area whose normal is $\hat{n}$. It relates to a plane-parallel specific intensity (I_ν) by integrating over solid angle

$$F_\nu = \int I_\nu \cos\theta d\Omega,$$

where θ is the angle between $\hat{n}$ (the normal vector of the surface) and the direction of incidence of the specific intensity.

Radio astronomers commonly use flux densities with units of Janskies. A Jansky is defined as:

$$1 \text{ Jy} \equiv 1e-23 \frac{\text{erg}}{\text{s} \cdot \text{cm}^2 \cdot \text{Hz}}$$

1.6 Specific Intensity

Specific intensity (with units of ergs/(s cm^2 Hz sr), Jy/beam) divides flux density by the angular area of the measurement (or of the source), and is intrinsic to source. Just like flux, specific intensity has a notion of area, and therefore, a notion of propagation in the direction normal to that surface, $\hat{n}$. Unlike flux density, specific intensity is conserved along a ray. Flux density (above) is related to specific intensity by integrating over solid angle:

1.7 Brightness Temperature

Brightness temperature (with units of K) uses the Rayleigh-Jeans tail of a blackbody spectrum to define an equivalent temperature corresponding to a specific intensity. It is often used as a proxy for specific intensity. (See below and Black-Body Radiation.)

$$I_\nu = \frac{2kT_b}{\lambda^2}$$

2 Specific Intensity, Specifically

Proof that Specific Intensity is conserved along a ray

The power received by the telescope is:

$$P_{rec} = I_\nu d\Omega dA$$

where $I_\nu(\alpha, \delta)$ is the intensity as a function of right-ascension (α) and declination (δ). Say that $\Sigma_\nu(\alpha, \delta)$ is the surface luminosity of a patch of sky (that is, the emitted intensity). Then power emitted by patch of sky is:

$$P_{emit} = \Sigma_\nu \frac{dA}{r^2} d\tilde{A}$$

Recognizing that $d\tilde{A} = d\Omega r^2$:

$$\Sigma_\nu = I_\nu$$

This derivation assumes that we are in a *vacuum* and that the *frequencies of photons are constant*. If frequencies change, then though specific intensity I_ν is not conserved, $\frac{I_\nu}{\nu^3}$ is. Also, for redshift z,

$$I_\nu \propto \nu^3 \propto \frac{1}{(1+z)^3}$$

so intensity decreases with redshift. Finally:

$$\frac{I_\nu}{\eta^2}$$

is conserved along a ray, where η is the index of refraction.

The Blackbody

* *See also Black-Body Radiation*.

A blackbody is the simplest source: it absorbs and reemits radiation with 100% efficiency. The frequency content of blackbody radiation is given by the *Planck Function*:

$$B_\nu = \frac{h\nu}{\lambda^2}\frac{2}{(e^{\frac{h\nu}{kT}}-1)}$$

$$B_\nu = \frac{2h\nu^3}{c^2(e^{\frac{h\nu}{kT}}-1)} \neq B_\lambda$$

(The Planck Function for Black Body Radiation)

Derivation:

The # density of photons having frequency between ν and $\nu + d\nu$ has to equal the # density of phase-space cells in that region, multiplied by the occupation # per cell. Thus:

$$n_\nu d\nu = \frac{4\pi\nu^2 d\nu}{c^3}\frac{2}{e^{\frac{h\nu}{kT}}-1}$$

However,

$$h\nu\frac{n_\nu c}{4\pi} = I_\nu = B_\nu$$

so we have it. In the limit that $h\nu \gg kT$:

$$B_\nu \approx \frac{2h\nu^3}{c^2}e^{-\frac{h\nu}{kT}}$$

Wein tail

If $h\nu \ll kT$:

$$B_\nu \approx \frac{2kT}{\lambda^2}$$

Rayleigh-Jeans tail Note that this tail peaks at $\sim \frac{3kT}{h}$. Also,

$$\nu I_\nu = \lambda I_\lambda$$

Reiteration: Conservation of specific intensity

Conservation of specific intensity told us the intensity collected by your telescope is absolutely equal to the surface intensity emitted into the angle leftrightarrowonding to your pixel on the sky. *Specific intensity is distance independent*. $1m^2$ of sky emitted into one square degree specific intensity measured by a $1m^2$ telescope pointed at a one square degree pixel of sky.

Units

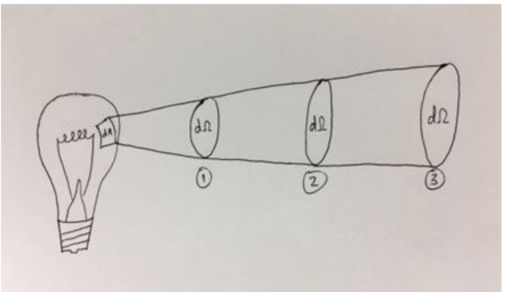
We have flux, flux density, surface brightness, and spectral energy distribution to describe luminosity. When in doubt, look at the units. Surface brightness, like specific intensity, is distance independent. It is simply specific intensity integrated over frequency.

Lenses

If you look at something through a lens, the specific intensity of a point is conserved. In the lens, the specific intensity may be greater, because $\frac{I_\nu}{\eta^2}$ is conserved (remember that η is the index of refraction of the medium), but once the light goes out again, even if the object looks bigger, the specific intensity is the same. *This does not violate conservation of energy because the light is being focused to a smaller area.*

Now if something is *inside* the lens (say, a bowl of water), then the lens bends light rays closer to each other, increasing the specific intensity. Thus, the object is emitting/scattering higher density photons, and appears more luminous, and the object also appears larger. This means that there must be some places where the object, omitting the lens, would have been visible, but now isn't.

A pictorial description of specific intensity (and proof of conservation along a ray)



In the above picture, a light bulb is emitting energy at a constant rate (i.e. Watts, or ergs / s). We have selected a small portion of the surface of the light bulb to focus on, dA. Further, we care only about the light emitted from that small area within a small frequency range $d\nu$. Lastly, we only care about light emitted from that small area into a particular solid angle $d\Omega$. So we are observing $\frac{\text{Energy}}{(\text{time})(dA)(d\Omega)(d\nu)}$, which has units of $\frac{erg}{s\ cm^2\ sr\ \nu}$, which is a specific intensity.

We can see from the picture why exactly specific intensity is conserved along a ray. At positions 1, 2, and 3, the solid angle corresponds to larger and larger surface areas. In other words, you need different sizes of detector to collect the same solid angle at different distances. And the detector size you need for a given solid angle increases with the square of the distance from the light bulb. Similarly, flux

decreases as the square of the distance, which means the two effects cancel out. Put another way, the photons that left the light bulb within a given solid angle (and surface area and frequency range) spread out with distance, but all stay within the exact same solid angle. Therefore, the flux you measure from the star within that solid angle (and surface area and frequency range) is conserved at any distance.