Transmission Lines

From AstroBaki
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Short Topical Videos

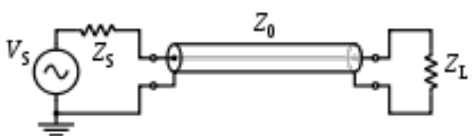
- Tektronix - Transmission Lines (http://www.youtube.com/watch?v=I9m2w4DgeVk) (amusing. bear through the "this is a transmission line" intro)

Reference Material

- Transmission Line (wikipedia) (http://en.wikipedia.org/wiki/Transmission_line)
- Characteristic Impedance (wikipedia) (http://en.wikipedia.org/wiki/Characteristic_impedance)

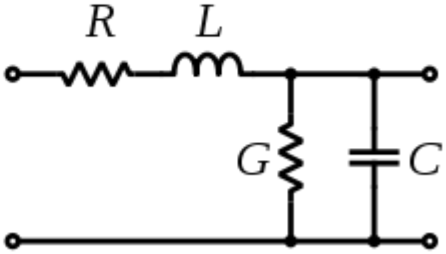
Contents

Impedance of Transmission Lines



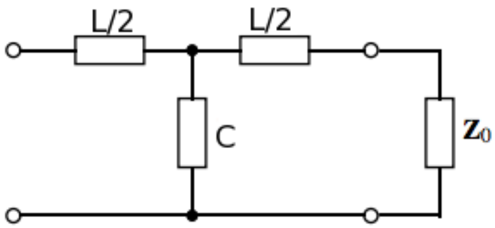
A transmission line with characteristic impedance Z_0, driven by a source with impedance Z_s, and terminated with a load impedance of Z_L

Transmission lines are a bit different than the normal wires we're used to dealing with. For example, if you measured the resistance of a 10m piece of wire, and found it to be 0.01Ω, then you might reasonably expect that you'd measure the impedance of a 20m piece of wire to be 0.02Ω. However, when we say that a coaxial cable (SMA, BNC, or otherwise) has an impedance of 50Ω, there is no mention of a length. 50Ω coaxial cable is 50Ω whether it is 1m or 100m long. How can this be?



A per-length transmission line model consisting of a (small) series resistance R, a series inductance L, a (small) parallel resistance G caused by dielectric conduction, and a parallel capacitance C.

It turns out that the impedance of a transmission line, although it is real-valued (i.e. resistive), is not caused by the resistance of the wire (which is typically quite small, and results in signal loss along the wire). Rather, for a lossless transmission line, capacitance and inductance are what give rise to the characteristic impedance. If you've ever cut a cable in half and seen the dielectric that sits between the conducting wire and the exterior sheath, you are probably not surprised that capacitance plays a role. The other key to understanding transmission lines is to recognize that they are for carrying signals. You have to launch a signal down a transmission line, so we should really be thinking about the relationship between the voltage and current of the signal that is transmitted.



Adding a differential piece of transmission line to an infinite line.

Here is a cute pedagogical derivation of how this works. Supposing a lossless transmission line, we add a differential piece of line (with two half-inductances and a capacitor, as shown above), and argue that this shouldn't change the overall impedance. In this configuration, the overall impedance of the line Z_0 is given by

$$\begin{aligned} Z_0 &= \frac{1}{2}Z_L + \frac{1}{2}Z_C \parallel (\frac{1}{2}Z_L + Z_0) \\ Z_0^2 &= \frac{1}{4}Z_L^2 + Z_L Z_C \\ Z_0 &= \sqrt{Z_L Z_C + \left(\frac{1}{2}Z_L\right)^2} \end{aligned}$$

Now if we define L to be an inductance per unit length, and C to be a capacitance per unit length, we have $Z_L = j\omega L\Delta\ell$ and $Z_C = 1/j\omega C\Delta\ell$, where $\Delta\ell$ is some small unit of length. In this case:

$$Z_0 = \sqrt{\frac{L}{C} - \left(\frac{1}{2}\omega L\Delta\ell\right)^2}.$$

Notice how the differential length and frequency dependence (and, even the definition of L and C as being per unit length) fall out of the left-hand term under the root. And, of course, as $\Delta\ell \to 0$, we are left with:

$$Z_0 = \sqrt{\frac{L}{C}}$$