CONDITIONAL OLS MINIMUM VARIANCE HEDGE RATIOS

JOËLLE MIFFRE

The paper presents a new methodology to estimate time dependent minimum variance hedge ratios. The so-called conditional OLS hedge ratio modifies the static OLS approach to incorporate conditioning information. The ability of the conditional OLS hedge ratio to minimize the risk of a hedged portfolio is compared to conventional static and dynamic approaches, such as the naïve hedge, the roll-over OLS hedge, and the bivariate GARCH(1,1) model. The paper concludes that, both in-sample and out-of-sample, the conditional OLS hedge ratio reduces the basis risk of an equity portfolio better than the alternatives conventionally used in risk management. © 2004 Wiley Periodicals, Inc. Jrl Fut Mark 24:945–964, 2004

INTRODUCTION

The predictability of returns is now a well-documented feature of risky assets. It seems to prevail across both developed and emerging markets

I would like to thank Amir Alizadeh, Bernard Bollen, Martin Martens, and Richard Priestley for helpful comments. I am also grateful to seminar participants at Cass Business School, Monash University, the University of New-South Wales, and the University of Technology, Sydney. An earlier version of the paper was presented at the 2002 meeting of the European Financial Management Association. I acknowledge financial support from the University of Technology, Sydney.
For correspondence, Cass Business School, City University, 106 Bunhill Row, London, EC1Y 8TZ, United Kingdom; e-mail: J.Miffre@city.ac.uk

Received September 2003; Accepted December 2003

■ *Joëlle Miffre is an Assistant Professor of Finance at Cass Business School, London, United Kingdom.*

Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fut.20116

and across a wide cross section of assets.[1] Most importantly, the evidence indicates that conditional asset pricing models with time-varying risk and time-varying factor risk premia capture most of the predictable variance of returns and leave little variation to be explained in terms of market inefficiency (Ferson & Harvey, 1993; Evans, 1994; Ferson & Korajczyk, 1995; Harvey, 1995). It follows that the predictability of returns is probably consistent with rational pricing in efficient markets and most likely results from variations in the return required by investors over time to compensate them for risk and deferred consumption (for an early review see Fama, 1991).

If the predictable variations in returns are rational, hedging strategies that ignore this predictability might lead to suboptimal hedging decisions. The purpose of the paper is to present a new methodology that takes the predictability of returns into account while estimating time dependent minimum variance hedge ratios. The approach modifies the traditional OLS hedge ratio to incorporate conditioning information. It relies on previous work on conditional asset pricing and on conditional portfolio performance evaluation (Ferson & Warther, 1996; Ferson & Schadt, 1996; Christopherson, Ferson, & Glassman, 1998).

This hedge ratio, referred to as the conditional OLS (ordinary least squares) hedge ratio, presents three advantages compared to the traditional naive, static OLS, and GARCH (generalized autoregressive conditional heteroscedasticity) hedge ratios typically encountered in the literature on hedging. First, unlike the naive hedge ratio, the conditional OLS hedge ratio recognizes the less than perfect correlation between spot and futures prices. Second, as opposed to the naive and the static OLS hedge ratios, the conditional OLS hedge ratio is time dependent. It takes into account the stochastic movements in hedge ratios arising from the predictability of returns. Third, the conditional OLS hedge ratio is simple to estimate and does not suffer from the problems of convergence one sometimes runs into with the family of GARCH models. Like GARCH, it readjusts hedging positions as new information becomes available. However, it does not require the sometimes tedious maximization of the GARCH log likelihood function and produces virtually instant estimates of the one-step ahead hedge ratio. For these reasons, the investment community may welcome the conditional OLS hedge ratio as an alternative tool for risk management.

[1]Among many others, the assets tested for time-varying risk premia include U.S. long-term corporate bonds (Chang & Huang, 1990), international equity indices (Ferson & Harvey, 1993; Harvey, 1995), and futures contracts (Bessembinder & Chan, 1992; Miffre, 2000).

The empirical part of the paper attempts to answer the following questions. Do information variables available at time $t - 1$ capture the variation in the optimal hedge ratio of an equity portfolio? Does conditioning the hedge ratio on past information enhance hedging effectiveness relative to the alternatives already present in the literature on risk management? The ability of the conditional OLS hedge ratio to minimize market risk is compared to conventional static and dynamic approaches, such as the naive hedge, the roll-over OLS hedge, and the bivariate GARCH(1,1) model. Special attention is given to measuring portfolio volatility in-sample and out-of-sample. The paper concludes that information variables available at time $t - 1$ capture the change in the hedge ratios of an equity portfolio and that the conditional OLS hedge ratio substantially reduces in-sample and out-of-sample volatility.

The remainder of the article is organized as follows. The second section briefly reviews the literature on hedging. The third section presents the theory that underpins the conditional OLS minimum variance hedge ratio and describes the methodologies used to estimate competing hedge ratios. The fourth section introduces the data set. The fifth section presents the empirical results. Finally the last section concludes.

A BRIEF REVIEW OF THE LITERATURE ON HEDGING

Traditionally, three approaches have been suggested as a way to minimize the risk of a cash position. The first one, called the naive or one-to-one hedge, assumes that the correlation between the spot and the futures prices is perfect and sets the hedge ratio equal to one over the period of the hedge. The problems with this hedge ratio are twofold. First, it fails to recognize that the correlation between spot and futures prices is less than perfect. Second, it fails to consider the stochastic nature of futures and spot prices and the resulting time variation in the hedge ratio.

The second approach, the static OLS hedge, accurately recognizes that the correlation between the futures and spot prices is less than perfect and estimates the hedge ratio as the OLS coefficient of a regression of spot return on futures return (Ederington, 1979; Figlewski, 1984). However it imposes the restriction of a constant joint distribution of spot and futures price changes. As such, it could lead to suboptimal hedging decisions in periods of high basis volatility. The naive and OLS approaches are static risk management strategies that involve a one-time decision about the best hedge and do not require any adjustment of the hedge ratio once this decision is taken.

More recently, quite a large literature was developed to overcome the problems inherent in the naïve and static OLS hedge ratios. The authors who focused on these issues are concerned with the dynamics in the joint distribution of returns and with the time-varying nature of optimal hedge ratios. Hedge ratios are estimated using the family of GARCH models introduced by Engle (1982) and Bollerslev (1986). These papers study a variety of commodities (Baillie & Myers, 1991; Myers, 1991), Treasury securities (Cecchetti, Cumby, & Figlewski, 1988), foreign exchange instruments (Kroner & Sultan, 1991, 1993; Lin, Najand, & Yung, 1994; Gagnon, Lypny, & McCurdy, 1998), and stock indices (Park & Switzer, 1995; Brooks, Henry, & Persand, 2002). The conclusions that emerge from these studies are that optimal hedge ratios are time dependent and that dynamic hedging reduces in-sample portfolio variance substantially better than static hedging. However, the out-of-sample advantages of the GARCH hedge ratio are much more controversial. Some argue that the GARCH hedge ratio enhances out-of-sample hedging effectiveness (Baillie & Myers, 1991; Kroner & Sultan, 1993; Park & Switzer, 1995; Brooks, Henry, & Persand, 2002). Others, however, weight the benefits of a dynamic hedge against the extra complexity of the GARCH method and the transactions costs it involves and conclude that static hedging might reduce *ex ante* volatility better (Kroner & Sultan, 1991; Myers, 1991; Lin et al., 1994). This suggests that the benefits from an active risk management strategy ought to be viewed with some caution.

METHODOLOGY

The Conditional OLS Hedge Ratio

Consider an investor who is long stocks. He wants to minimize his exposure to market risk by going short β_t dollars of futures contracts. The time t return on his hedged portfolio p_t equals

$$p_t = s_t - \beta_t f_t$$

where s_t and f_t are the changes in the futures and spot prices respectively and β_t is the time t minimum variance hedge ratio.

The variance of the returns on the hedged portfolio, conditional upon a set of L information variables Z available at time $t - 1$, equals

$$\sigma^2(p_t \mid Z_{t-1}) = \sigma^2(s_t \mid Z_{t-1}) + (\beta_t \mid Z_{t-1})^2\sigma^2(f_t \mid Z_{t-1}) - 2(\beta_t \mid Z_{t-1})\sigma(s_t, f_t \mid Z_{t-1})$$

where $\sigma^2(. \mid Z_{t-1})$ and $\sigma(.,. \mid Z_{t-1})$ denote variance and covariance conditional upon Z_{t-1}.

The minimum variance hedge ratio is traditionally defined as the value of $(\beta_t \mid Z_{t-1})$ that sets the first derivative of $\sigma^2(p_t \mid Z_{t-1})$ with respect to $(\beta_t \mid Z_{t-1})$ equal to zero (for an OLS estimation, see, for example, Figlewski, 1984; or for a GARCH estimation see Kroner & Sultan, 1991). It is the optimal proportion of the spot asset that should be hedged. It is equal to

$$(\beta_t \mid Z_{t-1}) = \frac{\sigma(s_t, f_t \mid Z_{t-1})}{\sigma^2(f_t \mid Z_{t-1})}$$

A traditional approach to estimating a minimum variance hedge ratio is to regress the return to the spot asset on the return to the futures contract, implicitly assuming β_t is constant.

$$s_t = \alpha + \beta f_t + \varepsilon_t \tag{1}$$

The slope coefficient β is the static OLS minimum variance hedge ratio. The hedger in this case makes no use of publicly available information while forming expectations of future hedge ratios. The intercept α in Equation (1) can be thought of as the mean of the change in the basis of the hedged portfolio.

Alternatively, the hedger could model the time variation in the hedge ratio by assuming a linear relationship between β_t and a set of $(L-1)$ mean zero instruments available at time $t-1, z_{t-1}$.[2] β_t then equals

$$(\beta_t \mid z_{t-1}) = \beta_0 + \beta_1 z_{t-1} \tag{2}$$

where β_0 measures the mean hedge ratio, β_1 is a $(L-1)$—vector of parameters, $z_{t-1} = Z_{t-1} - E(Z)$ is a $(L-1)$—vector of normalized deviation from Z_{t-1}. $\beta_1 z_{t-1}$ captures the time variation in the hedge ratio. It measures the departure from the mean hedge ratio β_0. In this case, the risk-minimizing hedge ratio is time dependent and changes as new information arrives to the market. The time-varying hedge ratio will reduce to the static OLS hedge ratio if no information is conveyed to the market.

[2]This assumption is in line with conventional research on conditional asset pricing models (see, among others, Harvey, 1989).

Substituting β in (1) by $(\beta_t \mid z_{t-1})$ in (2) yields

$$s_t = \alpha + \beta_0 f_t + \beta_1 f_t z_{t-1} + \varepsilon_t \tag{3}$$

Equation (3) is a regression of the spot return on a constant, the futures return, and the product of the futures return with the lagged information variables. The conditional model (3) adds a vector of $(L - 1)$ predictors $f_t z_{t-1}$ to the regression traditionally used to measure the static OLS hedge ratio. These regressors pick up the variation through time in the hedge ratio that is related to changing economic conditions. The resulting model is referred to as the conditional OLS model with a constant basis.[3]

The static OLS model can also be extended to allow for time variation in the change in the basis of the hedged portfolio. A linear relationship similar to (2) is used to model the time variation in α (Christopherson et al., 1998):

$$(\alpha_t \mid z_{t-1}) = \alpha_0 + \alpha_1 z_{t-1} \tag{4}$$

Substituting α in (3) by $(\alpha_t \mid z_{t-1})$ in (4) leads to the following regression

$$s_t = \alpha_0 + \alpha_1 z_{t-1} + \beta_0 f_t + \beta_1 f_t z_{t-1} + \varepsilon_t \tag{5}$$

This model is referred to as the conditional OLS model with a time-varying basis.

Note also that the static OLS model (1) is nested into the conditional OLS models (3) and (5). In particular, the conditional models can be assessed against the static model by testing the restrictions $\beta_1 = 0$ in (3) and $\alpha_1 = \beta_1 = 0$ in (5). Under the hypothesis that the lagged information variables are insignificant, the conditional OLS models reduce to the static OLS model.

The time t one-step ahead hedge ratios are estimated using the *ex-ante* model (3) over the in-sample period. This produces estimates of the coefficients β_0 and β_1 in (3) and hence an estimate of the one-step ahead conditional hedge ratio $(\beta_t \mid Z_{t-1})$ in (2). The sample is then rolled over to the next monthly observation and the model is re-estimated over the new sample to produce a new estimate of the one-step ahead conditional hedge ratio. Given the sample, this generates a time series of 84 rolling over forecasts of the hedge ratio over the out-of-sample period June 1996 to May 2003.[4] The assumption is, therefore, that hedgers reconsider

[3]To correct for the fact that Z_{t-1} only roughly approximates the true information set used by hedgers, the standard errors in (3) are adjusted for first order serial correlation and heteroscedasticity.
[4]The same procedure is applied to the conditional OLS model with a time-varying basis in Equation (5).

their position on a monthly basis. They update the parameter estimates at the end of each month as new information becomes available.

Alternative Specifications of the Optimal Hedge Ratio

The ability of the conditional OLS hedge ratio to minimize portfolio risk is compared to conventional static and dynamic approaches, such as the naive hedge, the static OLS hedge, and a bivariate GARCH(1,1) model. Each is defined in turn below.

The nave approach simply sets the hedge ratio equal to one over the period of the hedge. It, therefore, assumes a perfect correlation between the futures and spot prices and no time variation in the hedge ratio.

The static OLS hedge ratio is defined as β in (1). The hedger in that case does not incorporate conditioning information.

The bivariate GARCH model captures the dynamics in the second moments of the distribution of returns via a GARCH(1,1) error structure and assumes that the correlation between the spot and futures returns ρ is constant. It is as follows

$$s_t = \alpha_s + u_{s,t} \tag{6a}$$

$$f_t = \alpha_f + u_{f,t} \tag{6b}$$

$$\begin{bmatrix} u_{s,t} \\ u_{f,t} \end{bmatrix} \Bigg| \, Z_{t-1} \sim N(0, H_t) \tag{6c}$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \tag{6d}$$

$$h_{s,t}^2 = c_s + a_s h_{s,t-1}^2 + b_s u_{s,t-1}^2 \tag{6e}$$

$$h_{f,t}^2 = c_f + a_f h_{f,t-1}^2 + b_f u_{f,t-1}^2 \tag{6f}$$

where $u_{s,t}$ and $u_{f,t}$ are residuals, $h_{s,t}^2$ and $h_{f,t}^2$ are conditional variances, $h_{sf,t}$ is a conditional covariance. The parameters to estimate are α_i, a_i, b_i, c_i for $i = \{s, f\}$ and ρ. Equations (6a) and (6b) define the conditional mean returns, Equations (6c)–(6f) define the conditional variance–covariance matrix. The GARCH hedge ratio is measured as $(h_{sf,t}/h_{f,t}^2)$ in (6d) and (6f).

Two further tests are implemented. We test for the presence of time variation in the conditional variances of returns $(a_s = a_f = b_s = b_f = 0)$. Like before for the conditional OLS models, the static OLS model is nested in the GARCH model. Under the hypothesis that $(a_s = a_f = b_s = b_f = 0)$, GARCH indeed reduces to static OLS.

DATA

The data comprise end-of-month spot and futures prices on the S&P500 and the NYSE. The choice of a monthly frequency was dictated by the evidence that predictability is stronger for longer return horizons. On this ground, it did not seem justified to test the conditional OLS model with weekly or daily data as is typical in the hedging literature. To compile the time series of futures prices, the closing prices on the nearest maturity futures contract are collected, except in the maturity month when the prices on the second nearest futures are used.[5] Returns are computed as 100 times the difference in the logs of the prices at the end of the month.

The dataset spans the period June 1982 to May 2003. The sample is split into two subsamples. The in-sample period covers the period June 1982 to May 1996, approximately two-thirds of the dataset, and is used for estimation. The out-of-sample period, used for forecasting and hedging decisions, covers the remaining one-third.

Some of the information variables are specific to futures markets, while others follow from the literature on return predictability and conditional asset pricing (see, for example, Fama & French, 1989; Ferson & Harvey, 1993; Ferson & Schadt, 1996). These variables are a constant and the first lag in (1) the futures return, (2) the basis, (3) the dividend yield on the US stock index, (4) the spread between BAA and AAA corporate bond yield, and (5) the term structure of US interest rates.[6] Note also that the information variables are truly predictors, in the sense that they are constructed only with information that is already available at the time the optimal hedge ratio is calculated. For example, the information variables are demeaned using data up to the month when the hedge ratio is estimated.

Table I reports the correlation matrix in the independent variables of Equation (5) for both the S&P500 futures contracts (bottom left of the matrix) and the NYSE futures contracts (top right of the matrix). The correlations range from -0.31 to 0.37 for the S&P500 with an average of 0.02 and from -0.33 to 0.42 for the NYSE with an average of 0.06. With only a few exceptions, the correlations between the regressors are small, suggesting that multicollinearity should not be a problem.

[5]Rolling the hedge forward avoids thin trading and expiration effects. However, it introduces some uncertainty about the spread between the futures price of the contract that is closed out and the futures price of the new contract that is entered into. At these times, hedgers face roll-over risk as well as basis risk. To avoid that the former clouds inference on the latter, it is standard to delete the cross-over returns from the dataset. We nonetheless decided to include them on the ground that with monthly data and four contracts a year, deleting the roll-over returns would have substantially shortened our time series.

[6]Fama and French (1989) and Chen (1991) show that dividend yield, default spread, and the term structure predict the business cycle one period ahead. As such, they could command higher hedge ratios when a recession is anticipated and lower hedge ratios when an economic recovery is expected.

TABLE I
Correlations

	F_{t-1}	B_{t-1}	DY_{t-1}	DS_{t-1}	TS_{t-1}	f_t	$f_t F_{t-1}$	$f_t B_{t-1}$	$f_t DY_{t-1}$	$f_t DS_{t-1}$	$f_t TS_{t-1}$
F_{t-1}		0.05	0.05	−0.11	−0.08	0.01	0.05	0.21	0.01	0.04	0.05
B_{t-1}	0.01		0.25	0.01	−0.12	−0.13	0.20	0.09	−0.08	−0.03	−0.02
DY_{t-1}	0.05	0.03		0.09	0.33	0.16	0.01	−0.07	0.28	0.23	0.11
DS_{t-1}	−0.13	−0.10	0.09		0.29	−0.03	0.04	−0.03	0.24	0.20	0.04
TS_{t-1}	−0.08	−0.27	0.33	0.29		−0.05	0.04	−0.02	0.10	0.04	0.10
f_t	−0.02	−0.20	0.15	−0.06	−0.05		−0.33	0.12	0.01	0.28	−0.02
$f_t F_{t-1}$	0.02	0.23	0.00	0.03	0.03	−0.30		−0.24	−0.02	−0.16	−0.02
$f_t B_{t-1}$	0.22	0.13	−0.15	−0.10	−0.08	0.13	−0.31		0.07	−0.07	0.11
$f_t DY_{t-1}$	0.01	−0.17	0.25	0.23	0.08	−0.09	−0.06	−0.03		0.25	0.31
$f_t DS_{t-1}$	0.03	−0.12	0.23	0.18	0.04	0.30	−0.22	−0.14	0.15		0.42
$f_t TS_{t-1}$	0.04	−0.11	0.09	0.04	0.06	−0.09	−0.08	−0.07	0.33	0.37	

Note. F_{t-1}, B_{t-1}, DY_{t-1}, DS_{t-1}, and TS_{t-1} are mean zero information variables associated with futures returns, basis, dividend yield, default spread, and term structure. f_t is the futures return. f_tF_{t-1}, f_tB_{t-1}, f_tDY_{t-1}, f_tDS_{t-1}, and f_tTS_{t-1} are products of futures return and the lagged mean zero information variables. The bottom left of the correlation matrix is for the S&P500 futures, the top right is for the NYSE futures.

EMPIRICAL RESULTS

Statistical Significance of Conditioning Information: The Conditional OLS Models

Valid conclusions regarding the ability of the conditional OLS hedge to reduce portfolio volatility can only be drawn within a well-specified conditional model. To ensure that this condition is met, model (3) is estimated via OLS over the in-sample period and the hypothesis of a constant hedge ratio is tested using a χ^2 test with $(L-1)$ degrees of freedom. Table II, Panel A reports the coefficient estimates of the static OLS model (1). Table II, Panel B presents similar information for the conditional OLS model with a constant basis. Table II, Panel C reports χ^2 and probability values for the test of the marginal explanatory power of the conditioning information variables in (3). Ultimately, Table II, Panel C, tests the conditional OLS model with a constant basis against the static OLS model and tells us whether the static OLS model is misspecified.

Table II, Panel C, makes it clear that the information variables are jointly significant at 1% over the in-sample period. Similar tests

TABLE II
Conditional OLS Model With a Constant Basis

	S&P500		*NYSE*	
	Estimate	*(t-ratio) [p-value]*	*Estimate*	*(t-ratio) [p-value]*
Panel A: Static OLS model				
α	0.0176	(0.65)	0.0188	(0.79)
β	0.9735	(68.02)	0.9684	(76.53)
$\bar{R}^2$	0.9654		0.9724	
Panel B: Conditional OLS model with a constant basis				
α	0.0442	(1.99)	0.0305	(2.53)
β_0	0.9728	(55.53)	0.9660	(60.14)
$\beta_{1,F}$	0.0192	(2.07)	0.0075	(0.83)
$\beta_{1,B}$	0.1818	(2.58)	0.0893	(2.26)
$\beta_{1,DY}$	−0.0274	(−1.28)	−0.0289	(−1.35)
$\beta_{1,DS}$	0.0488	(1.27)	0.0391	(0.99)
$\beta_{1,TS}$	0.0027	(0.22)	−0.0038	(−0.28)
$\bar{R}^2$	0.9701		0.9737	
Panel C: Conditional OLS model with a constant basis versus static OLS				
$H_0 : \beta_1 = 0$	20.61	[0.00]	17.65	[0.00]

Note. The conditional OLS model with a constant basis is as follows:

$$s_t = \alpha + \beta_0 f_t + \beta_{1,F} f_t F_{t-1} + \beta_{1,B} f_t B_{t-1} + \beta_{1,DY} f_t DY_{t-1} + \beta_{1,DS} f_t DS_{t-1} + \beta_{1,TS} f_t TS_{t-1} + \varepsilon_t$$

s_t and f_t are the spot and futures returns, respectively. The information variables, measured at time $t-1$, include: F_{t-1} (futures return), B_{t-1} (basis), DY_{t-1} (dividend yield), DS_{t-1} (default spread), and TS_{t-1} (term structure). ε_t are residuals, α, β_0, and $\beta_{1,i}$ for $i = \{F, B, DY, DS, TS\}$ are the estimated parameters.

implemented over the out-of-sample period also suggest that the information variables have time and again a role to play in capturing the variation in the hedge ratio. A closer look at the *t*-ratios in Panel B indicates that the basis is particularly crucial at predicting the hedge ratio one period ahead. Clearly, according to Table II, it is important to allow the risk minimizing hedge ratio to be time dependent instead of restricting it to be constant. This suggests that the static OLS model is misspecified. Consequently, restricting the hedge ratio to be constant, instead of conditioning it on past information, might lead to poor hedging decisions. This point notwithstanding, the explanatory power of the conditional model is only marginally higher (on average, the adjusted *R*-squared of the conditional regressions is only 0.3% higher than the adjusted *R*-squared of the static models). The adjusted *R*-squareds of the regressions range from 96.5 to 97.4%. This indicates that, irrespectively of the hedge ratio considered, selling stock index futures eliminates roughly 97% of the volatility of the long position in the equity market and thus that the basis risk of the hedged portfolio is only marginal.

In Table III, the conditional OLS model with a constant basis is extended to allow for time variation in the change in the basis. Panel A, reports the estimated parameters for the change in the basis and tests the joint significance of the lagged information variables. Panel B, presents similar information for the hedge ratio. Panel C tests the joint significance of both α_1 and β_1 in Equation (5). Namely, Panel C tests the conditional OLS model with a time-varying basis against the static OLS model.

Like Table II previously, Table III documents substantial in-sample predictability of the hedge ratios (Panel B). It also reveals that the change in the basis is correlated with public information (Panel A).[7] The hypothesis of a constant basis ($\alpha_1 = 0$ in Panel A) and the hypothesis of a constant hedge ratio ($\beta_1 = 0$ in Panel B) are both rejected. The adjusted *R*-squareds of the conditional OLS model with a time-varying basis are only slightly higher than the ones reported for the static OLS model in Table II. Conditioning the change in the basis and the hedge ratio on past information only increases the explanatory power of the model by an average of 1.21%.

Table III, Panel C tests the conditional OLS model with a time-varying basis relative to the static OLS model. Namely, Panel C tests the validity of the restriction that both α_1 and β_1 are insignificant over the in-sample period. The χ^2 tests consistently reject the static OLS model in favor of the conditional specification. This reinforces the view put

[7]This result is consistent with the evidence presented in Bailey and Chan (1993), Baum and Barkoulas (1996), and Miffre (2004) on the predictability of the basis.

TABLE III
Conditional OLS Model With a Time-Varying Basis

	S&P500		*NYSE*	
	Estimate	*(t-ratio) [p-value]*	*Estimate*	*(t-ratio) [p-value]*
Panel A: Time variation in the change in the basis				
α_0	0.0148	(0.76)	0.0174	(1.06)
$\alpha_{1,F}$	0.0046	(0.25)	−0.0136	(−1.01)
$\alpha_{1,B}$	0.8853	(12.33)	0.6119	(4.94)
$\alpha_{1,DY}$	−0.0867	(−1.72)	−0.0950	(−1.58)
$\alpha_{1,DS}$	0.2337	(2.00)	0.0989	(1.50)
$\alpha_{1,TS}$	0.0251	(1.84)	−0.0052	(−0.34)
$H_{01}: \alpha_1 = 0$	289.04	[0.00]	58.43	[0.00]
Panel B: Time variation in the hedge ratio				
β_0	0.0000	(86.63)	0.9825	(79.16)
$\beta_{1,F}$	0.0010	(0.16)	−0.0019	(−0.31)
$\beta_{1,B}$	0.0842	(3.42)	0.0511	(2.02)
$\beta_{1,DY}$	−0.0248	(−1.20)	−0.0267	(−1.40)
$\beta_{1,DS}$	0.0412	(0.97)	0.0503	(1.31)
$\beta_{1,TS}$	0.0112	(1.05)	0.0021	(0.18)
$H_{02}: \beta_1 = 0$	72.39	[0.00]	130.46	[0.00]
$\overline{R}^2$	0.9817		0.9804	
Panel C: Conditional OLS model with a time-varying basis versus static OLS				
$H_{03}: \alpha_1 = \beta_1 = 0$	902.87	[0.00]	76.78	[0.00]

Note. The conditional OLS regression with a time-varying basis is as follows:

$$s_t = \alpha_0 + \alpha_{1,F}F_{t-1} + \alpha_{1,B}B_{t-1} + \alpha_{1,DY}DY_{t-1} + \alpha_{1,DS}DS_{t-1} + \alpha_{1,TS}TS_{t-1} + \cdots$$
$$\cdots + \beta_0 f_t + \beta_{1,F} f_t F_{t-1} + \beta_{1,B} f_t B_{t-1} + \beta_{1,DY} f_t DY_{t-1} + \beta_{1,DS} f_t DS_{t-1} + \beta_{1,TS} f_t TS_{t-1} + \varepsilon_t$$

s_t and f_t are the spot and futures returns respectively. The information variables, measured at time $t-1$, include: F_{t-1} (futures return), B_{t-1} (basis), DY_{t-1} (dividend yield), DS_{t-1} (default spread), and TS_{t-1} (term structure). ε_t are residuals, α_0, $\alpha_{1,i}$, β_0, and $\beta_{1,i}$ for $i = \{F, B, DY, DS, TS\}$ are the estimated parameters.

forward earlier that the static OLS model is misspecified. Further tests, not reported here, confirmed that the set of prespecified information variables also captures the time variation in the conditional OLS hedge ratio over the entire out-of-sample period.

Alternative Optimal Hedge Ratio: The GARCH Model

The paper examines the ability of different hedge ratios to minimize portfolio risk. In particular, special attention is given to estimating the minimum variance hedge ratio via a bivariate GARCH(1,1) model. The results are reported in Table IV. Panel A reports the estimated coefficients over the in-sample period. Panel B tests the GARCH model

TABLE IV
Bivariate GARCH(1,1) Model

	S&P500		*NYSE*	
	Estimate	*(t-ratio) [p-value]*	*Estimate*	*(t-ratio) [p-value]*
Panel A: Parameter estimates				
α_s	0.3655	(3.69)	0.4423	(2.87)
α_f	0.3592	(3.45)	0.4418	(2.84)
c_s	0.0847	(1.29)	1.5983	(0.60)
c_f	0.0988	(1.28)	1.6758	(0.95)
a_s	0.8491	(20.30)	0.4500	(0.50)
a_f	0.8374	(18.71)	0.4215	(0.72)
b_s	0.1286	(3.14)	0.0322	(0.55)
b_f	0.1339	(3.42)	0.0563	(0.80)
ρ	0.9839	(486.19)	0.9865	(634.85)
Panel B: GARCH versus static OLS				
$H_0 : a_s = a_f = b_s = b_f = 0$	3004.77	[0.00]	3.17	[0.53]

against the static OLS model by imposing the restrictions $a_s = a_f = b_s = b_f = 0$ on the conditional model (6a)–(6f).

The t-ratios and χ^2 tests indicate that the parameters a_s, a_f, b_s, and b_f, are jointly significant for the S&P500, suggesting that the GARCH error structure captures the dynamics in the second moments of the joint distribution of returns. Ultimately, this implies that the conditional variances ($h^2_{s,t}$ and $h^2_{f,t}$) and covariances ($h_{sf,t}$) are changing over time. This confirms that the risk-minimizing hedge ratios are indeed time-varying.

The in-sample results for the NYSE are somehow surprising given the results reported in the existing literature (Park & Switzer, 1995; Brooks, Henry, & Persand, 2002). The lagged coefficients in Equations (6e) and (6f) are insignificant, suggesting that the GARCH(1,1) specification fails to capture the time variation, if any, in the NYSE hedge ratio. This may be due to the use of monthly data (as opposed to weekly or daily data in previous studies) and thus could be the result of our short sample size. Note, however, that this is not necessarily the case since in over a third of the out-of-sample period, the hypothesis that $a_s = a_f = b_s = b_f = 0$ is rejected. This suggests that for a third of the sample, the GARCH error structure captures the time variation in the NYSE hedge ratio.

Hedging Effectiveness: In-Sample Results

The hedging effectiveness of the conditional OLS hedge ratios is compared to the ability of conventional methods, such as the naive hedge,

TABLE V
Variance of the Hedged Portfolios and Measure of Hedging Effectiveness

	S&P500			*NYSE*		
	Variance	*HE1*	*HE2*	*Variance*	*HE1*	*HE2*
Panel A: In-sample results: 30 September 1982 to 31 May 1996						
Naive hedge	0.11613	0.242	0.079	0.09007	0.143	0.066
Static OLS hedge	0.11362	0.215	0.056	0.08643	0.097	0.023
GARCH hedge	0.11721	0.253	0.089	0.08375	0.063	−0.009
Conditional OLS hedge with a constant basis	0.09352	N/A	−0.131	0.07882	N/A	−0.067
Conditional OLS hedge with a time-varying basis	0.10763	0.151	N/A	0.08448	0.072	N/A
Panel B: Out-of-sample results: 28 June 1996 to 31 May 2003						
Naive hedge	0.05469	0.054	0.129	0.05020	0.096	0.077
Roll-over OLS hedge	0.05317	0.024	0.097	0.04920	0.074	0.056
GARCH hedge	0.05601	0.079	0.156	0.05204	0.136	0.117
Conditional OLS hedge with a constant basis	0.05191	N/A	0.071	0.04579	N/A	−0.017
Conditional OLS hedge with a time-varying basis	0.04845	−0.067	N/A	0.04660	0.018	N/A

Note. The variance of the hedged portfolio is measured as $\sigma^2(s_t - h_t f_t)$, where s_t, h_t, and f_t are the time t spot return, hedge ratio, and futures return, respectively. HE1 (HE2) measures hedging effectiveness with regards to the conditional OLS hedge with a constant basis (with a time-varying basis). Hedging effectiveness is measured as the percentage increase (if positive, decrease if negative) in risk of each hedge ratio relative to the conditional OLS hedge ratios (e.g., 0.242 means that the naive hedge ratio increases basis risk by 24.2% more than the conditional OLS hedge with a constant basis).

the static OLS hedge, and the GARCH hedge, to minimize risk. For each of the respective hedge ratios, the variance of the hedged portfolio is computed each month as $\sigma^2(s_t - \beta_t f_t)$. The lower the basis risk is, the better is hedging effectiveness.

The in-sample results are summarized in Table V, Panel A. Table V reports the basis risk of the hedged portfolio under different estimates of the hedge ratio. It also reports measures of hedging effectiveness, estimated as the percentage increase (if positive, decrease if negative) in risk of each hedge ratio relative to the conditional OLS models. For example, 0.242 (0.079) in Table V, Panel A, indicates that the naive hedge ratio increases basis risk by 24.2% (7.9%) more than the conditional OLS model with a constant (time-varying) basis.

The in-sample results clearly indicate that a hedger who uses the conditional OLS hedge can substantially reduce the volatility of his hedged portfolio. For example, the basis risk of the conditional OLS hedge with a constant basis is 0.09352 for the S&P500. This compares favorably with the variance of the naive, static OLS, and GARCH hedges (0.11613, 0.11362, and 0.11721, respectively). Relative to the naive hedge, the conditional OLS hedge with a constant basis eliminates 24.2% more of the volatility of the S&P500 index. Compared to the static OLS hedge, it enhances hedging effectiveness by 21.5%. It reduces the basis risk of the hedged portfolio by a further 25.3% relative to GARCH. The conditional OLS model with a constant basis reduces basis risk better than the alternative hedging strategies for the NYSE too. Note also that allowing for time dependency in both the change in the basis and the hedge ratio does not improve in-sample hedging effectiveness relative to the conditional OLS model with a constant basis. This conclusion prevails for both the S&P500 futures and the NYSE futures.

The GARCH model fares particularly poorly for the S&P500. This may be due to the small sample size that results from the use of monthly data. Note also that the static OLS hedge reduces portfolio volatility by more than the naive hedge.

Hedging Effectiveness: Out-of-Sample Results

How much variance reduction can be achieved if one uses historical hedge ratios to determine appropriate hedging strategies for future time periods? Hedgers who use the roll-over OLS, GARCH, and conditional OLS hedges are assumed to re-estimate the risk-minimizing hedge ratio at the end of each month using the most recently available information set. This hedge ratio is then used as a basis for risk management over the following month.

The out-of-sample results are reported in Table V, Panel B. For both the S&P500 and the NYSE indices, conditioning the hedge ratio on past information reduces out-of-sample portfolio volatility more substantially than managing risk via the naive, roll-over OLS, and GARCH approaches. For the S&P500, the conditional OLS hedge with a time-varying basis reduces basis risk by 12.9% more than the naive hedge. It also fares better than the roll-over OLS and the GARCH hedges, with percentage reduction in risk that are, respectively, 9.7 and 15.6% higher. For the NYSE index, exposure to basis risk is minimized when the hedge ratio is measured via the conditional OLS model with a constant basis. Note also that the out-of-sample results for both the S&P500 and the NYSE index indicate a poor hedging performance of the GARCH hedge relative to the roll-over OLS and naive hedges. Again this might be due to our small sample.

A last test is implemented to ensure that the conditional OLS hedge ratio[8] differs from its naive, roll-over OLS, and GARCH counterparts. Figures 1 and 2 plot the estimated hedge ratios for each stock index over the whole sample. A casual look at the figures indicates that the conditional OLS hedge ratio is not a replica of the hedge ratios already present in the literature on risk management. This hypothesis is tested by calculating for each of the two indices the correlation between the conditional OLS hedge ratio and its counterparts. These correlations range from -0.23 to -0.03 for the S&P500 and from 0.05 to 0.42 for the NYSE. It seems, therefore, that the conditional OLS model does not capture the same variation in the hedge ratio as the roll-over OLS and GARCH models. As the competing hedge ratios do differ, the inferences drawn from one model will be at odds with the one advocated by a competing approach.

CONCLUSIONS

The article presents a new methodology to estimate minimum variance hedge ratios. This methodology relies on previous work on conditional asset pricing and on conditional portfolio performance evaluation (Ferson & Warther, 1996; Ferson & Schadt, 1996; Christopherson et al., 1998). It modifies the static OLS model to account for conditioning information. The advantages of this approach are that (1) it recognizes the less than perfect correlation between spot and futures prices, (2) it captures the stochastic movements in hedge ratios arising from the predictability of returns, and (3) it is easy to estimate and produces virtually instant estimates of the hedge ratio. These characteristics make the conditional OLS hedge ratio potentially superior to the traditional naive, static OLS, and GARCH hedges as a theoretical construct for risk management.

These considerations motivated the empirical analysis of the article. The hypothesis tested here is that the basis risk of the hedged portfolio is minimized when one allows via a set of lagged information variables for some stochastic movements in the hedge ratio. The ability of the conditional OLS hedge ratio to minimize portfolio risk is compared to conventional static and dynamic approaches, such as the naive hedge, the roll-over OLS hedge, and the bivariate GARCH model. Special attention is given to measuring hedging effectiveness in-sample and out-of-sample.

[8]The remainder of the paper focuses on the conditional OLS hedge ratios that perform the best out-of-sample; namely, the one with a conditional basis for the S&P500 and the one with a constant basis for the NYSE.

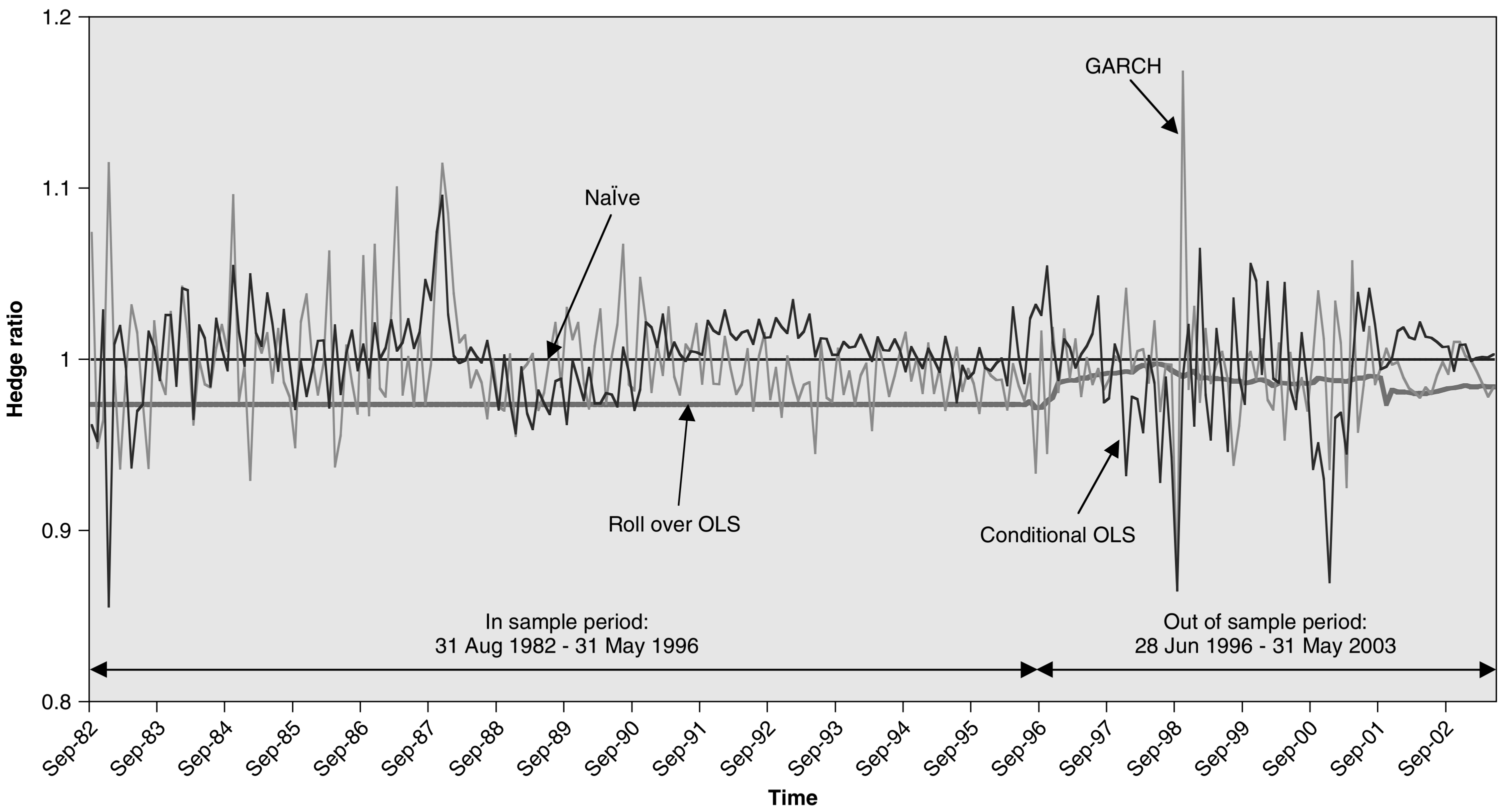

FIGURE 1
S&P500 hedge ratios.

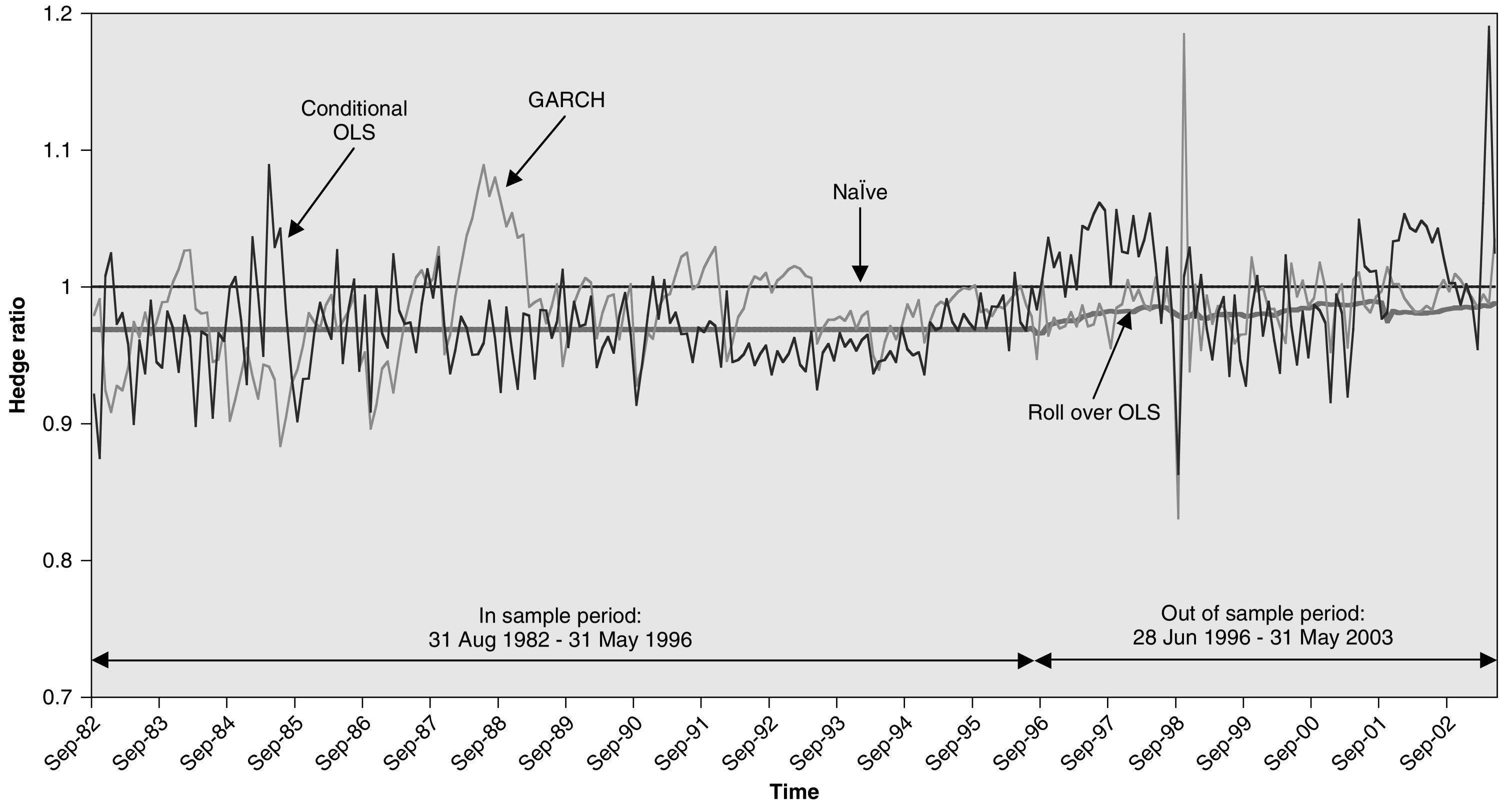

FIGURE 2
NYSE hedge ratios.

The conclusions are as follows. First, the time variation in the optimal hedge ratio is captured by a set of information variables available at time $t - 1$. Second, the conditional OLS hedge ratio reduces in-sample and out-of-sample portfolio volatility better than the naive, static OLS, and GARCH approaches. It follows that hedgers with relatively long-term horizons may welcome this approach as a new tool for risk management.

Thus far, we have not taken into consideration the impact that transaction costs may have on the desirability of our dynamic hedge. Given that the risk reductions generated through our hedge ratios are substantial, it may well be the case that the benefits of the conditional OLS hedge in terms of risk reduction more than outweigh the cost of dynamic hedging. However, we have not tested this hypothesis formally and leave it, therefore, as a possible avenue for future research in risk management.

BIBLIOGRAPHY

Bailey, W., & Chan, K. C. (1993). Macroeconomic influences and the variability of the commodity futures basis. Journal of Finance, 48, 555–573.

Baillie, R. T., & Myers, R. J. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. Journal of Applied Econometrics, 6, 109–124.

Baum, C. F., & Barkoulas, J. T. (1996). Time-varying risk premia in currency basis. Journal of Futures Markets, 16, 735–755.

Bessembinder, H., & Chan, K. (1992). Time-varying risk premia and forecastable returns in futures markets. Journal of Financial Economics, 32, 169–193.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307–327.

Brooks, C., Henry, O. T., & Persand, G. (2002). The effect of asymmetries on optimal hedge ratios. Journal of Business, 75, 333–352.

Cecchetti, S. G., Cumby, R. E., & Figlewski, S. (1988). Estimation of optimal futures hedge. Review of Economics and Statistics, 70, 623–630.

Chang, E. C., & Huang, R. D. (1990). Time-varying return and risk in the corporate bond market. Journal of Financial and Quantitative Analysis, 25, 323–340.

Chen, N. (1991). Financial investment opportunities and the macroeconomy. Journal of Finance, 46(2), 529–554.

Christopherson, J. A., Ferson, W. E., & Glassman, D. A. (1998). Conditioning managers alphas on economic information: Another look at the persistence of performance. Review of Financial Studies, 11, 111–142.

Ederington, L. H. (1979). The hedging performance of the new futures markets. Journal of Finance, 34, 157–170.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. Econometrica, 50, 987–1008.

Evans, M. D. D. (1994). Expected returns, time-varying risk and risk premia. Journal of Finance, 49, 655–679.

Fama, E. F. (1991). Efficient capital markets: II. Journal of Finance, 46, 1575–1617.

Fama, E. F., & French, K. R. (1989). Business conditions and expected returns on stocks and bonds. Journal of Financial Economics, 25, 23–49.

Ferson, W. E., & Harvey, C. R. (1993). The risk and predictability of international equity returns. Review of Financial Studies, 6, 527–566.

Ferson, W. E., & Korajczyk, R. A. (1995). Do arbitrage pricing models explain the predictability of stock returns? Journal of Business, 68, 309–349.

Ferson, W. E., & Schadt, R. W. (1996). Measuring fund strategy and performance in changing economic conditions. Journal of Finance, 51, 425–461.

Ferson, W. E., & Warther, V. A. (1996). Evaluating fund performance in a dynamic market. Financial Analysts Journal, November–December, 20–28.

Figlewski, S. (1984). Hedging performance and basis risk in stock index futures. Journal of Finance, 39, 657–669.

Gagnon, L., Lypny, G. J., & McCurdy, T. H. (1998). Hedging foreign currency portfolios. Journal of Empirical Finance, 5, 197–220.

Harvey, C. R. (1989). Time-varying conditional covariances in tests of asset pricing models. Journal of Financial Economics, 24, 289–317.

Harvey, C. R. (1995). Predictable risk and returns in emerging markets. Review of Financial Studies, 8, 773–816.

Kroner, K., & Sultan, J. (1991). Exchange rate volatility and time-varying hedge ratios. Pacific Basin Capital Markets Research, 2, 397–412.

Kroner, K., & Sultan, J. (1993). Time-varying distribution and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis, 28, 535–551.

Lin, J. W., Najand, M., & Yung, K. (1994). Hedging with currency futures: OLS versus GARCH. Journal of Multinational Financial Management, 4, 45–67.

Miffre, J. (2000). Normal backwardation is normal. Journal of Futures Markets, 20, 803–821.

Miffre, J. (in press). The conditional price of basis risk: An investigation using foreign exchange instruments. Journal of Business Finance and Accounting.

Myers, R. J. (1991). Estimating time-varying optimal hedge ratios on futures markets. Journal of Futures Markets, 11, 39–53.

Park, T. H., & Switzer, L. N. (1995). Bivariate GARCH estimation of the optimal hedge ratios for stock index futures: A note. Journal of Futures Markets, 15, 61–67.