

Random Walks

What is a random walk?



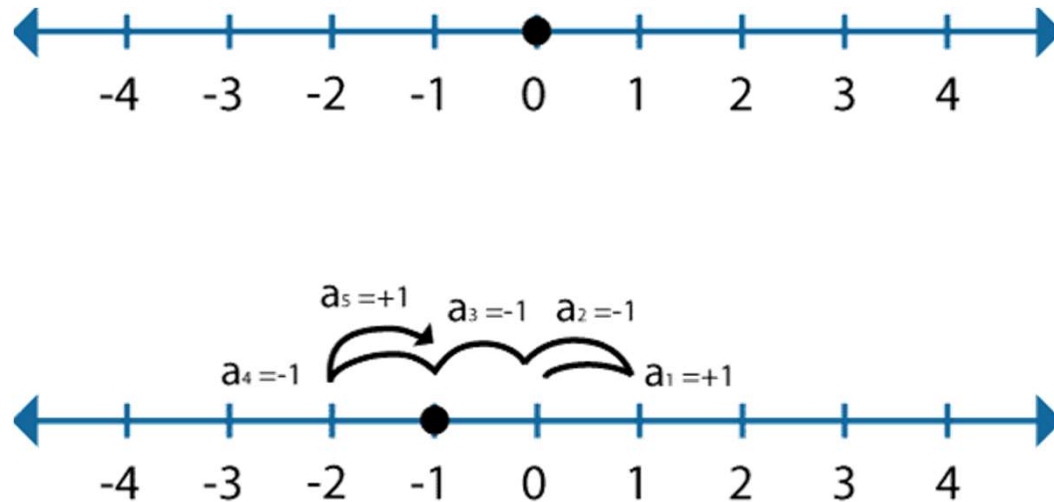
Definition:

隨機移動的物體從它們開始的地方遊走的過程

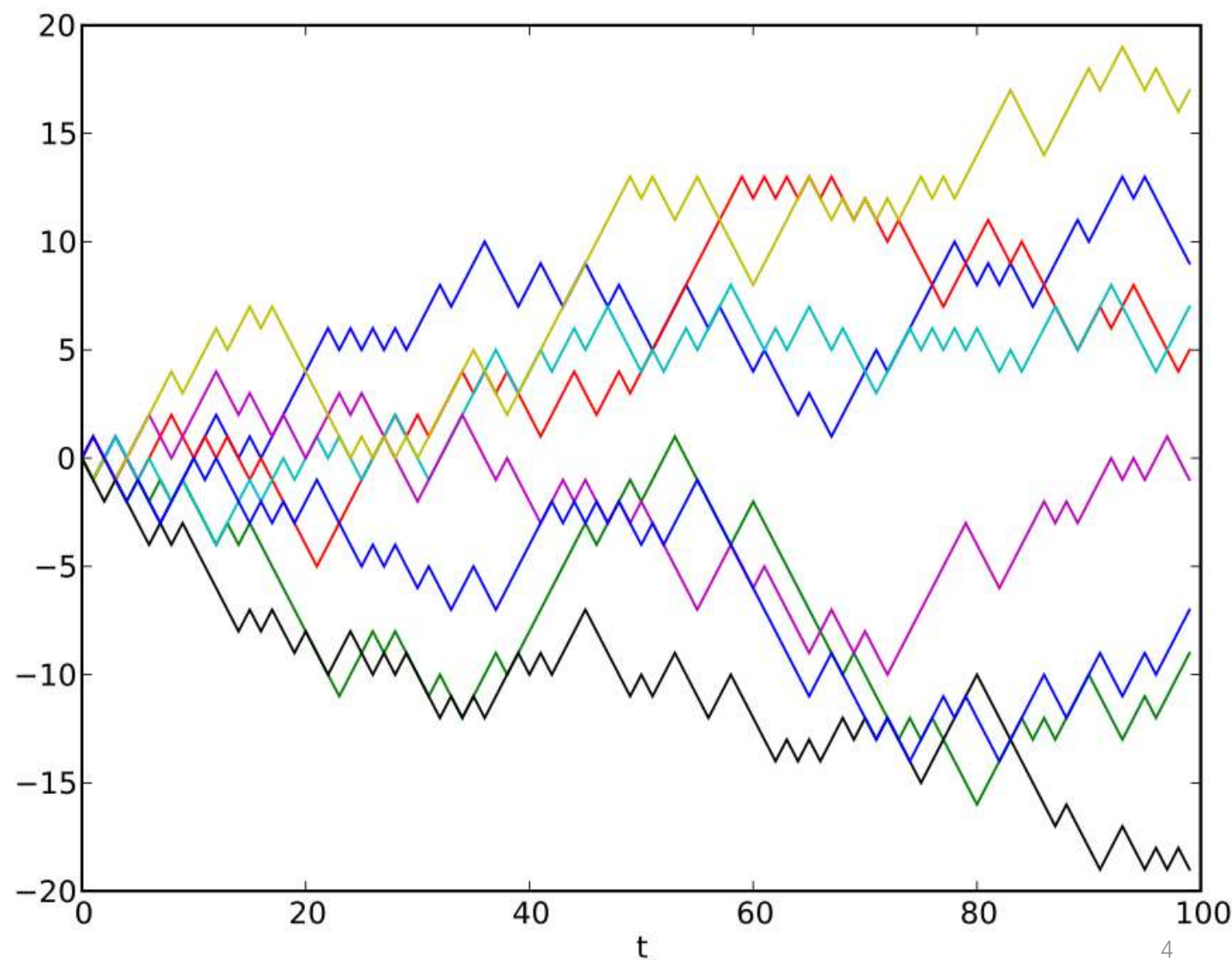
Random Walks

The Mathematics in 1 Dimension

How can we describe this mathematically?



醉漢走路：
1.多次游走后的
位置？
2.单次游走后的
位置？



Random Walk Model

Consider a random variable

$$u(i) = \{1, -1\}$$

$$y(t) = \sum_{i=1}^t u(i)$$

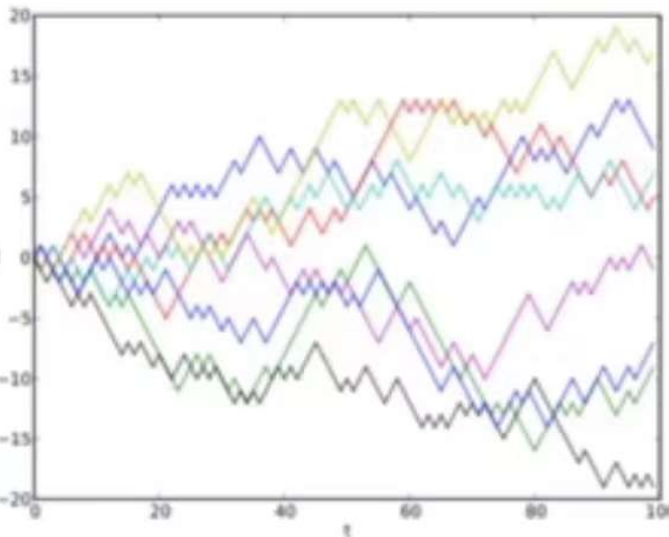
$$\langle y(t) \rangle = 0 \quad \text{Useless!}$$

$$\langle y^2(t) \rangle = \left\langle \left(\sum_{i=1}^t u(i) \right)^2 \right\rangle$$

$$= \left\langle \sum_{i=1}^t \sum_{j=1}^t (u(i) u(j)) \right\rangle = \sum_{i=1}^t \sum_{j=1}^t \langle u(i) u(j) \rangle = t$$

$$\langle u(i) u(j) \rangle = 0 \text{ for } i \neq j$$

$$\langle u(i) u(j) \rangle = 1 \text{ for } i = j$$



a_1	a_2	$a_1 a_2$
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1

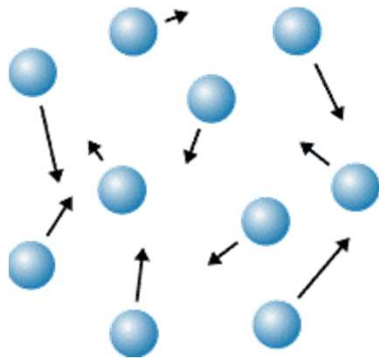
$$\langle y^2(t) \rangle = t$$

$$\sigma(t) \propto \sqrt{t}$$

More About Random Walks

Examples & Random Walks in 2D and 3D

- What does a random walk have to do with real life?



Random walks in more than one dimension

1.Keep same rule as 1-D.

2.More commonly.

Note :more memory will be changed.(baised)

Random walks with a variable step size

- $\sqrt{\langle d^2 \rangle} = \langle r \rangle \sqrt{N}$



Not all random walks are “random” (Biased random walk)

- There are several ways that a random walk can be biased.

1. Suppose that instead of an equal probability of moving left to right, there was a higher probability of moving to the right.

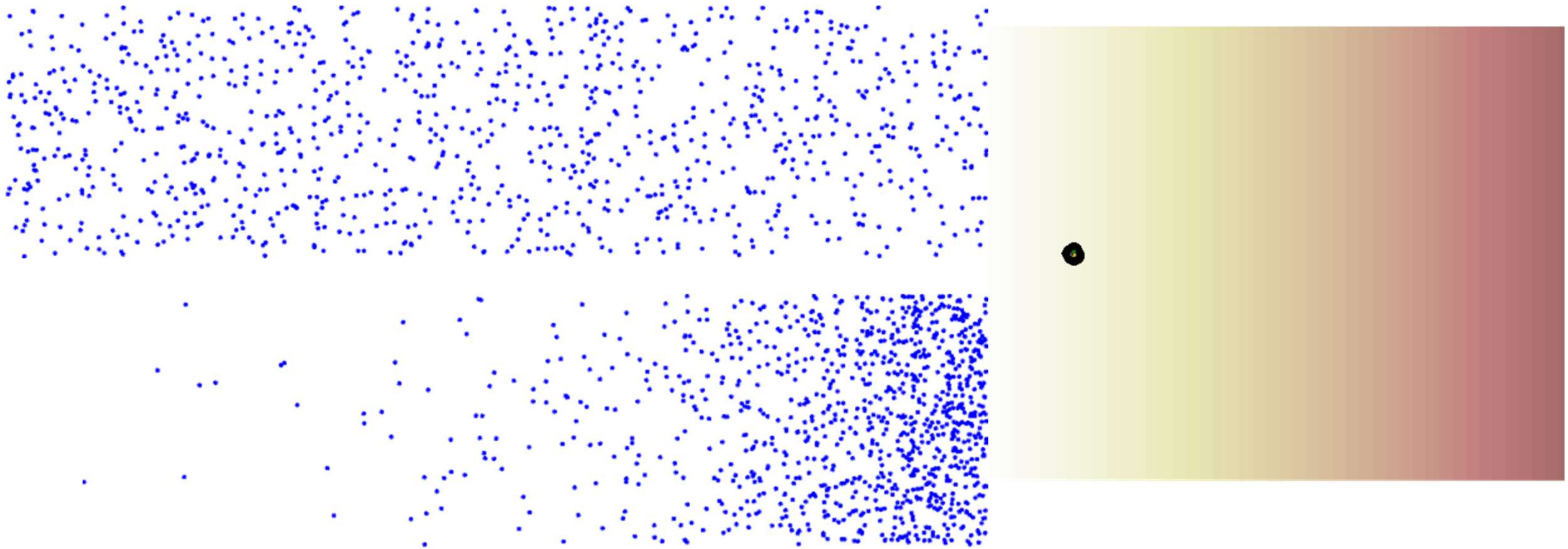
2. Suppose the probabilities of moving to the left or right remained equal, but whenever the dot moved to the right, it moved 2 spaces, and when it moved to the left, it only moved 1 space.

Not all random walks are “random” (Biased random walk)

Which situation ?



What does a concentration gradient have to do with a random walk?



Random walk in graph network

- Node embedding
- Page rank

