Assignment#6

Chapter 6

1. Compute the standard inner product on M_{22} of the given matrices.

$$U = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix} \quad , \quad V = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$$

2. Let $\mathbf{p} = x + x^3$ and $\mathbf{q} = 1 + x^2$
   Find $\|\mathbf{p}\|$ and $d(\mathbf{p}, \mathbf{q})$ relative to the evaluation inner product on P_3 at the stated sample points:

$$x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1.$$

3. Let P_2 have the evaluation inner product at the points

$$x_0 = -2, x_1 = 0, x_2 = 2$$

Show that the vectors $\mathbf{p} = x$ and $\mathbf{q} = x^2$ are orthogonal with respect to this inner product.

4. Find a basis for the orthogonal complement of the subspace of R^n spanned by the vectors.

$$\mathbf{v_1} = (1, 4, 5, 2), \mathbf{v_2} = (2, 1, 3, 0), \mathbf{v_3} = (-1, 3, 2, 2)$$

5. The vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ are **orthonormal** with respect to the Euclidean inner product on R^4. Find the orthogonal projection of $\mathbf{b} = (1, 2, 0, -1)$ onto the subspace W spanned by these vectors.

$$\mathbf{v_1} = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right), \mathbf{v_2} = \left(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\right), \mathbf{v_3} = \left(\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\right)$$

6. Find an orthogonal basis for the column space of the matrix

$$\begin{bmatrix} 6 & 1 & -5 \\ 2 & 1 & 1 \\ -2 & -2 & 5 \\ 6 & 8 & -7 \end{bmatrix}$$

7. We obtained the column vectors of Q by applying the Gram–Schmidt process to the column vectors of $\boldsymbol{A}$.
   Find a QR-decomposition of the matrix $\boldsymbol{A}$

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{3} \end{bmatrix} \quad \boldsymbol{Q} = \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

8. Find the least squares solution of the equation $\boldsymbol{A}\mathbf{x} = \mathbf{b}$.

$$\boldsymbol{A}=\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} \quad \mathbf{b}=\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$$

9. Let W be the plane with equation $5x - 3y + z = 0$.

a. Find a basis for W. (5 points)

b. Find the standard matrix for the orthogonal projection onto W.(5 points)

10. Find the least squares straight line fit

$$\boldsymbol{y} = a\boldsymbol{x} + b$$

to the data points:

$$(0, 1), (2, 0), (3, 1), (3, 2)$$

and show that the result is reasonable by graphing the fitted line and plotting the data in the same coordinate system.

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

本次作業無需每題寫心得，請選擇你認為需要的，不單獨算分，但完全不寫心得最多-20.

！！ 如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理!

截止日期：12/7 00:00(週三)