

Assignment #2

1. Evaluate $\det(A)$ by a cofactor expansion along a row or column of your choice.

$$A = \begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$$

2. By inspection, what is the relationship between the following determinants?

$$d_1 = \begin{vmatrix} a & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix} \quad \text{and} \quad d_2 = \begin{vmatrix} a + \lambda & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix}$$

3. Evaluate the determinant of the matrix by first reducing the matrix to row echelon form and then using some combination of row operations and cofactor expansion.

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

4. Confirm the identities without evaluating any of the determinants directly.

$$\begin{vmatrix} a_1 + b_1 t & a_2 + b_2 t & a_3 + b_3 t \\ a_1 t + b_1 & a_2 t + b_2 & a_3 t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

5. Find the values of k for which the matrix A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$$

6. Solve by Cramer's rule, where it applies.

$$\begin{aligned} -x_1 - 4x_2 + 2x_3 + x_4 &= -32 \\ 2x_1 - x_2 + 7x_3 + 9x_4 &= 14 \\ -x_1 + x_2 + 3x_3 + x_4 &= 11 \\ x_1 - 2x_2 + x_3 - 4x_4 &= -4 \end{aligned}$$

7. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that $\det(A) = -7$,

find a. $\det(3A)$ b. $\det(A^{-1})$ c. $\det(2A^{-1})$ d. $\det((2A)^{-1})$

$$\text{e. } \det \begin{pmatrix} a & g & d \\ b & h & e \\ c & i & f \end{pmatrix}$$

8. decide whether the matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$

9. Prove that if $\det(A) = 1$ and all the entries in A are integers, then all the entries in A^{-1} are integers.
10. Verify that $\det(AB) = \det(BA)$ and determine whether the equality $\det(A + B) = \det(A) + \det(B)$ holds.

$$A = \begin{bmatrix} -1 & 8 & 2 \\ 1 & 0 & -1 \\ -2 & 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 1 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

評分標準：

每題解答 6 分，心得 4 分。結果錯-2，過程或方法錯不給分。

繳交期限：10/11 （週二） 0:00 遲交分數*0.8

繳交格式：手寫後貼到 word 檔再轉成 PDF 格式，只接受 pdf!

檔案名稱 【LA_assignment#2_學號_version】

上傳到 Ecourse2 作業區!!!

不符合繳交格式總分-10!

如果不會請去請教同學，在檔案裡面說明你請教了誰（姓名+學號）即可，原則上不扣分！禁止抄襲！