Assignment #3

1. Consider the following 3 vectors in R^3 : $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (2, 0, 1)$, $\mathbf{w} = (3, 1, 0)$

   (a) (5 points) Project vector w orthogonally onto vector v.

   (b) (5 points) Find the vector that is the reflection of w on v

2. Let $T_1(x_1, x_2) = (x_1 - x_2, 2x_2 - x_1, 3x_1)$ and $T_2(x_1, x_2, x_3) = (4x_2, x_1 + 2x_2)$.

   (a) (4 points) Find the standard matrices for T_1 and T_2.

   (b) (3 points) Find the standard matrix for $T_1 \circ T_2$

   (c) (3 points) Use the matrix obtained in part(b) to find formular for $T_1(T_2(x_1, x_2, x_3))$

3. Consider the following 3 vectors in R^3 :

   $\mathbf{u_1} = (a, 2, -1)$, $\mathbf{u_2} = (4, 1, 0)$, $\mathbf{u_3} = (1, 5, -2)$

   Where a is an unspecified real number.

   (a) (4 points) Find the possible value(s) of a, such that the volume of the parallelepiped described by the given vectors $\mathbf{u_1}$,$\mathbf{u_2}$, and $\mathbf{u_3}$ is one

   (b) (6 points) Find the area of each face of the parallelepiped which is determined by the above given vectors $\mathbf{u}_1$,$\mathbf{u}_2$, and $\mathbf{u}_3$ for a = 0

4. Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axioms that fail.(10 points)

The set of polynomials of the form $a_0 + a_1 x$ with the operations

$$(a_0 + a_1 x) + (b_0 + b_1 x) = (a_0 + b_0) + (a_1 + b_1)x$$

and

$$k(a_0 + a_1 x) = (ka_0) + (ka_1)x$$

5. Use the Subspace Test to determine which of the sets are subspaces of R^3 .

a. (3 points)All vectors of the form (a, 0, 0).

b. (3 points)All vectors of the form (a, 1, 1).

c. (4 points)All vectors of the form (a, b, c), where b = a + c.

6. Use the Subspace Test to determine which of the sets are subspaces of M_{nn}.

a. (2.5 points)The set of all diagonal n × n matrices.

b. (2.5 points)The set of all n × n matrices A such that $\det(A) = 0$.

c. (2.5 points)The set of all n × n matrices A such that $\operatorname{tr}(A) = 0$.

d. (2.5 points)The set of all symmetric n × n matrices.

7. Express the following as linear combinations of **u** = (2, 1, 4), **v** = (1, −1, 3), and **w** = (3, 2, 5).

a. (3 points) (−9, −7, −15)

b. (3 points) (6, 11, 6)

c. (4 points) (0, 0, 0)

8.Determine whether the following polynomials span $\mathbf{P}_2$. (10 points)

$\mathbf{p}_1 = 1 - x + 2x^2$, $\mathbf{p}_2 = 3 + x$,

$\mathbf{p}_3 = 5 - x + 4x^2$, $\mathbf{p}_4 = -2 - 2x + 2x^2$

9. In each part, determine whether the vectors are linearly independent or are linearly dependent in P_2 .

a. (5 points) $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$

b. (5 points) $1 + 3x + 3x^2$, $x + 4x^2$, $5+ 6x + 3x^2$, $7 + 2x - x^2$

10.

In each part, let $T_A : R^3 \rightarrow R^3$ be multiplication by A, and let $\mathbf{u}_1 = (1,0,0)$, $\mathbf{u}_2 = (2,-1,1)$, and $\mathbf{u}_3 = (0,1,1)$. Determine whether the set $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2), T_A(\mathbf{u}_3)\}$ is linearly independent in R^3.

a. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$ **b.** $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

本次作業由于多數爲定理判斷題，所以心得不用每題都寫，寫你覺得有意義的就好，如果都沒有，也要寫一份總體的心得，心得不單獨算分，但作業內完全無心得總成績最多−20. 可以接受打字

Note:前三題原本是老師要出的期中考題, 但並未選中, 請同學認真作答, 謝謝!

繳交期限：10/25 （週二）0:00 遲交分數*0.8