Assignment #5

題目範圍: section 5.1~5.2

1. Confirm by multiplication that x is an eigenvector of A, and find the corresponding eigenvalue.

   $A=\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$;x=$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

2. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix.

   $A=\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

3. Find the eigenvalues and a basis for each eigenspace of the linear operator defined by the stated formula. [Suggestion: Work with the standard matrix for the operator.]

   $T\ (x, y, z) = (2x - y - z, x - z, -x + y + 2z)$

4. In each part of Exercises 4, find the eigenvalues and the corresponding eigenspaces of the stated matrix operator on R^2. Use geometric reasoning to find the answers. No computations are needed.

   a. Reflection about the line $y = x$. (2 points)
   b. Orthogonal projection onto the x-axis. (2 points)
   c. Rotation about the origin through a positive angle of 90°. (2 points)
   d. Contraction with factor k ($0 \leq k < 1$). (2 points)
   e. Shear in the x-direction by a factor k ($k \neq 0$). (2 points)

5. Suppose that the characteristic polynomial of some matrix A is found to be $p(\lambda) = (\lambda - 1)(\lambda - 3)^2 (\lambda - 4)^3$. In each part, answer the question and explain your reasoning.

   a. What is the size of A ? (3 points)
   b. Is A invertible ? (3 points)
   c. How many eigenspaces does A have ?(3 points)

6. Let

$$A=\begin{bmatrix}4 & 0 & 1\\ 2 & 3 & 2\\ 1 & 0 & 4\end{bmatrix}$$

a. Find the eigenvalues of A. (3 points)
b. For each eigenvalue λ, find the rank of the matrix λI-A (3 points)
c. Is A diagonalizable? Justify your conclusion.(4 points)

7. Find the geometric and algebraic multiplicity of each eigenvalue of the matrix A, and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A, and find $P^{-1}AP$.

$$A=\begin{bmatrix}-1 & 4 & -2\\ -3 & 4 & 0\\ -3 & 1 & 3\end{bmatrix}$$

8. If A , B and C are n × n matrices such that A is similar to B and B is similar to C, do you think that A must be similar to C? Justify your answer.

9. Let

$$A=\begin{bmatrix}1 & -2 & 8\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix} \text{ and } P=\begin{bmatrix}1 & -4 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

Confirm that P diagonalizes A, and then compute each of the following powers of A.

10. a. Is it possible for an n × n matrix to be similar to itself? Justify your answer. (3 points)

b. What can you say about an n × n matrix that is similar to $0_{n\times n}$? Justify your answer. (3 points)

c. Is it possible for a nonsingular matrix to be similar to a singular matrix? Justify your answer.(4 points)

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.

！！如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理！

繳交期限：11/16 （週三）00:00 遲交分數*0.8