Assignment #4

題目範圍: section 4.5~4.9

1. Show that the following polynomials form a basis for P_3 .
   $1+x$, $1-x$, $1-x^2$, $1-x^3$

   A=$\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x^0\\ x^1\\ x^2\\ x^3\end{bmatrix}$ ➔A is invertible ,so form a basis for P_3

   Or show $a(1+x)+b(1-x)+c(1-x^2)+d(1-x^3)=0$ *only a=b=c=d=0*

2. In each part, let T_A ：$R^3 \rightarrow R^3$ be multiplication by A, and let u = (1, −2, −1). Find the coordinate vector of T_A(u) relative to the basis S = {(1, 1, 0), (0, 1, 1), (1, 1, 1)} for R^3 .

   a. (5 points) A=$\begin{bmatrix}2 & -1 & 0\\ 1 & 1 & 1\\ 0 & -1 & 2\end{bmatrix}$ $\begin{bmatrix}-2\\ -6\\ 6\end{bmatrix}$

   b. (5 points) A=$\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1\\ 2\\ -3\end{bmatrix}$

   1. Au=b
   2. x=S^{-1}b

3. In each part, find a basis for the given subspace of R^3 , and state its dimension.
   a. (2.5 points) The plane $3x - 2y + 5z = 0$.
   b. (2.5 points) The plane $x - y = 0$.
   c. (2.5 points) The line $x = 2t$, $y = -t$, $z = 4t$.

   d. (2.5 points) All vectors of the form (a, b, c), where b = a + c.

   a.&b.find [3,-2,5]=0 [1,-1,0]=0nullspace to solve 只要 basis 构成与参考答

案相同 subspace 即可

   a.(2/3,1,0) (-5/3,0,1) dimension:2
   b.(1,1,0) (0,0,1) dimension:2
   c.(2,-1,4) dimension:1

   d.参数解 x=$\begin{bmatrix}x1\\ x2\\ x3\end{bmatrix}=\begin{bmatrix}a\\ a+c\\ c\end{bmatrix}=a\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}+c\begin{bmatrix}0\\ 1\\ 1\end{bmatrix}$ dimension:2

4. In each part, let T_A be multiplication by A and find the dimension of the subspace R^4 consisting of all vectors x for which T_A(x) = 0.

a. (5 points) A=$\begin{bmatrix} 1 & 0 & 2 & -1 \\ -1 & 4 & 0 & 0 \end{bmatrix}$ kernel of A is 2

b. (5 points) A=$\begin{bmatrix} 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ kernel of A is 1

5. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for R^2, where

$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\mathbf{u}'_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{u}'_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

a.(2.5 points) Find the transition matrix from B' to B. $\begin{bmatrix} 13/10 & -1/2 \\ -2/5 & 0 \end{bmatrix}$

b.(2.5 points) Find the transition matrix from B to B'. $\begin{bmatrix} 0 & -5/2 \\ -2 & -13/2 \end{bmatrix}$

c.(2.5 points) Compute the coordinate vector $[\mathbf{w}]_B$, where $\mathbf{w} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ and use (11) to compute $[\mathbf{w}]_{B'}$. $[\mathbf{w}]_B = \begin{bmatrix} \frac{-17}{10} \\ \frac{8}{5} \end{bmatrix}$ $[\mathbf{w}]_{B'} = P_{B \to B'} [\mathbf{w}]_B = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$

d.(2.5 points) Check your work by computing $[\mathbf{w}]_{B'}$ directly. $(B')^{-1}w = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$

Note: use(11) please to check the reference at the bottom of this assignment.

6. If B_1, B_2, and B_3 are bases for R^2, and if

$$P_{B_1 \to B_2} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad \text{and} \quad P_{B_2 \to B_3} = \begin{bmatrix} 7 & 2 \\ 4 & -1 \end{bmatrix},$$

then $P_{B_3 \to B_1} = \begin{bmatrix} \frac{-2}{15} & \frac{11}{15} \\ \frac{7}{15} & \frac{-31}{15} \end{bmatrix}$ $P_{B_3 \to B_1} = (P_{B_2 \to B_3} P_{B_1 \to B_2})^{-1}$

7. Suppose that $x_1 = -1$, $x_2 = 2$, $x_3 = 4$, $x_4 = -3$ is a solution of a nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ and that the solution set of the homogeneous system $A\mathbf{x} = \mathbf{0}$ is given by the formulas

$$x_1 = -3r + 4s, \quad x_2 = r - s, \quad x_3 = r, \quad x_4 = s$$

a.(5 points) Find a vector form of the general solution of $A\mathbf{x} = \mathbf{0}$.
b.(5 points) Find a vector form of the general solution of $A\mathbf{x} = \mathbf{b}$

a. A=$r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ b. x=$\begin{bmatrix} -1 \\ 2 \\ 4 \\ -3 \end{bmatrix} + r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

8. Consider the linear systems

$$\begin{bmatrix}1 & -2 & 3\\ 2 & 1 & 4\\ 1 & -7 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

and

$$\begin{bmatrix}1 & -2 & 3\\ 2 & 1 & 4\\ 1 & -7 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}2\\ 7\\ -1\end{bmatrix}$$

a. (2.5 points) Find a general solution of the homogeneous system.
   s(-11/5,2/5,1)
b. (2.5 points) Confirm that $x_1 = 1$, $x_2 = 1$, $x_3 = 1$ is a solution of the nonhomogeneous system.
   yes
c. (2.5 points) Use the results in parts (a) and (b) to find a general solution of the nonhomogeneous system.
   (1,1,1)+ s(-11/5,2/5,1)
d. (2.5 points) Check your result in part (c) by solving the nonhomogeneous system directly
   By rref

$$x=\begin{bmatrix}\frac{16}{5}\\ \frac{3}{5}\\ 0\end{bmatrix}+s\begin{bmatrix}-\frac{11}{5}\\ \frac{2}{5}\\ 1\end{bmatrix}$$,when s=1,x=(1,1,1)

9. Let $T : R^5 \to R^3$ be the linear transformation defined by the formula

$$T(x_1 , x_2 , x_3 , x_4 , x_5) = (x_1 + x_2 , x_2 + x_3 + x_4 , x_4 + x_5)$$

a. (5 points) Find the rank of the standard matrix for T. 3
b. (5 points) Find the nullity of the standard matrix for T. 5

10.a. (2.5 points) If A is a 3×5 matrix, then the rank of A is at most _3______. Why?

b. (2.5 points)If A is a 3×5 matrix, then the nullity of A is at most _5______. Why?

c. (2.5 points)If A is a 3×5 matrix, then the rank of A^T is at most _3______. Why?

d. (2.5 points)If A is a 3×5 matrix, then the nullity of A^T is at most 3___.Why?

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.

繳交期限：11/8 （週二）00:00 遲交分數*0.8

Reference:

Transforming Coordinates

Suppose now that B and B' are bases for a finite-dimensional vector space V. Since multiplication by $P_{B \to B'}$ maps coordinate vectors relative to the basis B into coordinate vectors relative to a basis B', and $P_{B' \to B}$ maps coordinate vectors relative to B' into coordinate vectors relative to B, it follows that for every vector $\mathbf{v}$ in V we have

$$\boxed{[\mathbf{v}]_{B'} = P_{B \to B'}[\mathbf{v}]_B} \tag{11}$$

$$\boxed{[\mathbf{v}]_B = P_{B' \to B}[\mathbf{v}]_{B'}} \tag{12}$$