Assignment #1

1. Use the column-row expansion of AB to express this product as a sum of matrix products.

$$A = \begin{bmatrix} 0 & 4 & 2 \\ 1 & -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 4 & 0 \\ 1 & -1 \end{bmatrix}$$

Column row expansion :

AB=C1R1+C2R2+C3R3 Ci: ith column of A, R: ith row of B

$$= \begin{bmatrix} 18 & -2 \\ -1 & -6 \end{bmatrix}$$

2. Simplify the expression assuming that A, B, C, and D are invertible .

$$(AC^{-1})^{-1}(AC^{-1})(AC^{-1})^{-1}AD^{-1}$$

CD^{-1}

3. show that the matrices A and B are row equivalent by finding a sequence of elementary row operations that produces B from A, and then use that result to find a matrix C such that CA = B

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ 3 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 9 & 4 \\ -5 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$C=E_{31}(-4)E_{31}(-2)E_{21}(-2) = \begin{bmatrix} 9 & 0 & -4 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

要按題目要求解，不能用 $C=BA^{-1}$!方法不對 -5

4. Consider the matrices

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a. Show that the equation $A\mathbf{x} = \mathbf{x}$ can be rewritten as $(A - I)\mathbf{x} = \mathbf{0}$ and use this result to solve $A\mathbf{x} = \mathbf{x}$ for $\mathbf{x}$.

b. Solve $A\mathbf{x} = 4\mathbf{x}$.

a.) $Ax=Ix \rightarrow (A-I)x=0$

$$A-I=\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix} \text{ Solve } x=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b.) $(A-4I)x=0$ $$A-4I=\begin{bmatrix} -2 & 1 & 2 \\ 2 & -2 & -2 \\ 3 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{rref}\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

let $x_3=t$, $x_2=0$, $x_1=t$

$$\rightarrow \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$