1. $p(x_0) = -2$, $p(x_1) = 0$, $p(x_2) = 2$

   $q(x_0) = 4$, $q(x_1) = 0$, $q(x_2) = 4$

   (a) $||p_1|| = \sqrt{p(x0)p(x0) + p(x1)p(x1) + p(x2)p(x2)} = \sqrt{4+4+0} = 2\sqrt{2}$

   (b) $p \cdot q = p(x_0)\, q(x_0) + p(x_1)\, q(x_1) + p(x_2)\, q(x_2)$

   $= -2\times4 + 0\times0 + 2\times4 = 0$, is orthogonal.

2. $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \end{bmatrix}$, $Ax = 0 \rightarrow x_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ \frac{7}{2} \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ -3 \\ 1 \\ \frac{7}{2} \end{bmatrix}$

3. $(\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, -\frac{1}{2})$

4. (a) $u_1 = (\frac{1}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}, \frac{7}{10})$, $u_2 = (-\frac{7}{10}, \frac{3}{10}, \frac{4}{10}, -\frac{1}{2}, \frac{1}{10})$,

   $\rightarrow v_2 - (\frac{v2 \cdot v1}{||v1||2})v_1$, $u_2 = \frac{v2'}{||v2'||} = (-7, 3, 4, -5, 1)$

   (b) $Q = \begin{bmatrix} \frac{1}{10} & -\frac{7}{10} \\ \frac{3}{10} & \frac{3}{10} \\ \frac{4}{10} & \frac{4}{10} \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{7}{10} & \frac{1}{10} \end{bmatrix}$, $R = \begin{bmatrix} 10 & 10 \\ 0 & 10 \end{bmatrix}$

   (c) $x = (A^TA)^{-1}A^Tb = R^{-1}Q^Tb = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$

5. (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -5 & 3 \end{bmatrix}$

   (b) $A(A^TA)^{-1}A^T = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & -\frac{1}{7} \\ \frac{3}{7} & \frac{26}{35} & \frac{23}{35} \\ -\frac{1}{7} & \frac{3}{35} & \frac{34}{35} \end{bmatrix}$

6. (a) $A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $P = A'(A'^TA')^{-1}A'^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$,

$Q = N(N^TN)^{-1}N^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(b) $P + Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $PQ = O_{n\times n}$, because A 的 rowspace is orthogonal to A 的 nullspace.

7. $A^TA = \begin{bmatrix} 5 & -1 & 4 \\ -1 & 11 & 10 \\ 4 & 10 & 14 \end{bmatrix}$, $A^Tb = \begin{bmatrix} -7 \\ 14 \\ 7 \end{bmatrix}$

$x = (A^TA)^{-1}A^Tb = \begin{bmatrix} -\frac{7}{6} \\ \frac{7}{6} \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} r$

8. $M = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

$V^* = (M^TM)^{-1}M^Tb = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{6} \end{bmatrix}$, $y = \frac{2}{3} + \frac{1}{6}x$

9. (a) $M = \begin{bmatrix} 1 & -5 & 25 \\ 1 & -4 & 16 \\ 1 & 0 & 0 \\ 1 & 3 & 9 \end{bmatrix}$, $b = \begin{bmatrix} -133 \\ -71 \\ -3 \\ 27 \end{bmatrix}$

$V^* = (M^TM)^{-1}M^Tb = \begin{bmatrix} \frac{3537}{781} \\ \frac{10613}{781} \\ \frac{-1821}{781} \end{bmatrix}$, $y = \frac{3537}{781} + \frac{10613}{781}x - \frac{1821}{781}x^2$

(b) $M = \begin{bmatrix} 1 & 25 \\ 1 & 16 \\ 1 & 0 \\ 1 & 9 \end{bmatrix}$, $(u^Tu)^{-1}M^Tb = \begin{bmatrix} \frac{9435}{337} \\ \frac{-1968}{337} \end{bmatrix}$, $y = \frac{9435}{337} - \frac{1968}{337}x^2$