Assignment#7

Chapter 7&9

1. Consider the rectangular $x'y'z'$ -coordinate system obtained by rotating a rectangular xyz-coordinate system counter-clock wise about the z-axis (looking down the z-axis) through the angle $\theta = \pi/4$.
   a) (5 points)Find the $x'y'z'$-coordinates of the point whose xyz coordinates are $(-1, 2, 5)$.
   b) (5 points)Find the xyz-coordinates of the point whose $x'y'z'$-coordinates are $(1, 6, -3)$.

2. Determine whether there exists a 3×3 symmetric matrix whose eigenvalues are $\lambda_1 = -1, \lambda_2 = 3, \lambda_3 = 7$ and for which the corresponding eigenvectors are as stated. If there is such a matrix, find it, and if there is none, explain why not.
   a) (5 points) $x_1=\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $x_2=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $x_3=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
   b) (5 points) $x_1=\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $x_1=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $x_1=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3. Let $T_A : R^2 \rightarrow R^2$ be multiplication by A. Find two orthogonal unit vectors $\mathbf{u_1}$ and $\mathbf{u_2}$ such that $T_A(\mathbf{u_1})$ and $T_A(\mathbf{u_2})$ are orthogonal.
   $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

4. Find an quadratic form with no cross product terms by principal Axes Theorem.
   $Q = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1 x_2 - 4x_1 x_3 - 8x_2 x_3$

5. Identify the conic section represented by the equation by rotating axes to place the conic in standard position. Find an equation of the conic in the rotated coordinates, and find the angle of rotation.
   $2x^2 - 4xy - y^2 + 8 = 0$

6. Classify the matrix as positive definite, negative definite, or indefinite.
   $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

7. .Given $A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}$,
   a) (5 points)Find a singular value decomposition of A.
   b) (5 points)Find a reduced singular value decomposition of A by part(a)

c) (5 points)Find approximation of A by **rank =1**.

8. Proof:
   a) (10 points)Prove that if $A = U\Sigma V^T$ is a singular value decomposition of A, then U orthogonally diagonalizes AA^T.
   b) (5 points) If A is an m × n matrix, then A^TA and AA^T have the same rank.

9. Use PLU-decomposition of A, and use it to solve the linear system $A\boldsymbol{x}$ = b.

$$A=\begin{bmatrix} 3 & -1 & 0 \\ 2 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix};\mathrm{b}=\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$

評分標準：

每題配分已標注，答錯即 0 分。

本次作業無需每題寫心得，請選擇你認為需要的，不單獨算分，但完全不寫心得最多-20.

！！如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理!

截止日期：12/28 00:00(週三)