Assignment#6

Chapter 6

1. (5 points)Let $\mathbf{p} = x$ and $\mathbf{q} = x^2$ and the stated sample points are given :

   $$x_0 = -2,\ x_1 = 0,\ x_2 = 2$$

   (a)Find $\|\mathbf{p}\|$ relative to the evaluation inner product on P_2.(2.5 points)

   (b)Show that the vectors **p** and **q** are orthogonal with respect to this inner product.(2.5 points)

2. (5 points)Find a basis for the orthogonal complement of the subspace of R^n spanned by the vectors.

   $$\mathbf{v_1} = (1, 4, 5, 2),\ \mathbf{v_2} = (2, 1, 3, 0),\ \mathbf{v_3} = (-1, 3, 2, 2)$$

3. (5 points)The vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ are **orthonormal** with respect to the Euclidean inner product on R^4 . Find the orthogonal projection of $\mathbf{b} = (1, 2, 0, -1)$ onto the subspace W spanned by these vectors.

   $$\mathbf{v_1} = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right), \mathbf{v_2} = \left(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\right), \mathbf{v_3} = \left(\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\right)$$

4. (15 points)Consider

   $$A=\begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix}$$

   (a) Find an orthonormal basis for the column space of **A** .(5 points)

   (b) Write A as **QR**-decomposition , where **Q** has **orthonormal** columns and **R** is upper triangular.(5 points)

   (c) Find the least square solution to $\mathbf{A}\boldsymbol{x}=\mathbf{b}$,if $\mathbf{b} = [-3, 7, 1, 0, 4]^T$.(5 points)

5. (10 points)Let W be the plane with equation $5x - 3y + z = 0$.

   a. Find a basis for W. (5 points)

   b. Find the standard matrix for the orthogonal projection onto W.(5 points)

6. (20 points)Consider

   $$A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

(a) Find the projection matrix **P** onto the row space of **A** , and the projection matrix **Q** onto the nullspace of **A.** (10 points)

(b) Find **P+Q. Explain your result.** (5 points)

(c) Find **PQ**. Explain your result.(5 points)

7. (10 points)Find parametric equations for all least squares solutions of
   $\mathbf{A}\boldsymbol{x} = \mathbf{b}$,
   Where column vectors of **A** are **not** linear independent.

$$\mathbf{A}=\begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{b}=\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$$

8. (10 points)Find the least squares straight line fit

$$\boldsymbol{y} = a\boldsymbol{x} + b$$

to the data points:

(0, 1), (2, 0), (3, 1), (3, 2)

and show that the result is reasonable by graphing the fitted line and plotting the data in the same coordinate system.

9. (20 points)Given the following set of data points :

(-5, -133), (-4, -71), (0, -3), (3, 27)

(a)Find the parabola $y=ax^2+bx+c$ which best fits these points.(10 points)
(b)Find the parabola $y=ax^2+c$ with no linear term which best fits these points. (10 points)

評分標準：

每題配分已標注，答錯即 0 分。

本次作業無需每題寫心得，請選擇你認為需要的，不單獨算分，但完全不寫心得最多-20.

！！ 如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理!

截止日期：12/8 00:00(週四)