Assignment #4

題目範圍: section 4.5~4.9

1. Show that the following polynomials form a basis for P_3 .
   $1+x, 1-x, 1-x^2, 1-x^3$

2. In each part, let $T_A : R^3 \rightarrow R^3$ be multiplication by A, and let u = (1, −2, −1). Find the coordinate vector of T_A(u) relative to the basis S = {(1, 1, 0), (0, 1, 1), (1, 1, 1)} for R^3 .
   a. (5 points) A=$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$
   b. (5 points) A=$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

3. In each part, find a basis for the given subspace of R^3 , and state its dimension.
   a. (2.5 points) The plane $3x - 2y + 5z = 0$.
   b. (2.5 points) The plane $x - y = 0$.
   c. (2.5 points) The line $x = 2t, y = -t, z = 4t$.
   d. (2.5 points) All vectors of the form (a, b, c), where b = a + c.

4. In each part, let T_A be multiplication by A and find the dimension of the subspace R^4 consisting of all vectors x for which T_A(x) = 0.
   a. (5 points) A=$\begin{bmatrix} 1 & 0 & 2 & -1 \\ -1 & 4 & 0 & 0 \end{bmatrix}$
   b. (5 points) A=$\begin{bmatrix} 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

5. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for R^2 , where
   $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\mathbf{u}'_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{u}'_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$
   a.(2.5 points) Find the transition matrix from B' to B.
   b.(2.5 points) Find the transition matrix from B to B' .
   c.(2.5 points) Compute the coordinate vector $[\mathbf{w}]_B$, where $\mathbf{w} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ and use (11) to compute $[\mathbf{w}]_{B'}$.
   d.(2.5 points) Check your work by computing $[\mathbf{w}]_{B'}$ directly.

Note: use(11) please to check the reference at the bottom of this assignment.

6. If B_1, B_2, and B_3 are bases for R^2, and if

$$P_{B_1\to B_2} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad \text{and} \quad P_{B_2\to B_3} = \begin{bmatrix} 7 & 2 \\ 4 & -1 \end{bmatrix},$$

then $P_{B_3\to B_1}$ =________.

7. Suppose that $x_1 = -1$, $x_2 = 2$, $x_3 = 4$, $x_4 = -3$ is a solution of a nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ and that the solution set of the homogeneous system $A\mathbf{x} = \mathbf{0}$ is given by the formulas

$$x_1 = -3r + 4s, \quad x_2 = r - s, \quad x_3 = r, \quad x_4 = s$$

a.(5 points) Find a vector form of the general solution of $A\boldsymbol{x} = \mathbf{0}$.
b.(5 points) Find a vector form of the general solution of $A\boldsymbol{x} = \mathbf{b}$

8. Consider the linear systems

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix}$$

a. (2.5 points) Find a general solution of the homogeneous system.
b.(2.5 points) Confirm that $x_1 = 1$, $x_2 = 1$, $x_3 = 1$ is a solution of the nonhomogeneous system.
c. (2.5 points) Use the results in parts (a) and (b) to find a general solution of the nonhomogeneous system.
d. (2.5 points) Check your result in part (c) by solving the nonhomogeneous system directly

9. Let $T : R^5 \to R^3$ be the linear transformation defined by the formula

$$T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2, x_2 + x_3 + x_4, x_4 + x_5)$$

a. (5 points) Find the rank of the standard matrix for T.
b. (5 points) Find the nullity of the standard matrix for T.

10.a. (2.5 points) If A is a 3×5 matrix, then the rank of A is at most _______.

Why?

b. (2.5 points)If A is a 3×5 matrix, then the nullity of A is at most ______. Why?

c. (2.5 points)If A is a 3×5 matrix, then the rank of A^T is at most ______. Why?

d. (2.5 points)If A is a 3×5 matrix, then the nullity of A^T is at most ______.Why?

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.

繳交期限：11/8 （週二）00:00 遲交分數*0.8

Reference:

Transforming Coordinates

Suppose now that B and B' are bases for a finite-dimensional vector space V. Since multiplication by $P_{B\to B'}$ maps coordinate vectors relative to the basis B into coordinate vectors relative to a basis B', and $P_{B'\to B}$ maps coordinate vectors relative to B' into coordinate vectors relative to B, it follows that for every vector $\mathbf{v}$ in V we have

$$\boxed{[\mathbf{v}]_{B'} = P_{B\to B'}[\mathbf{v}]_B} \tag{11}$$

$$\boxed{[\mathbf{v}]_B = P_{B'\to B}[\mathbf{v}]_{B'}} \tag{12}$$