1. $\lambda^3-12\lambda^2-16 = 0$

$\lambda_1 = -2,\ \lambda_2 = -2,\ \lambda_3 = 4$

(1) when $\lambda_1 = \lambda_2 = -2$, eigenspace base $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}$

(2) when $\lambda_3 = 4$, eigenvector $\begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\1\end{bmatrix}$

2. $\left(\begin{bmatrix}2 & 1 & -1\\-1 & 0 & 1\\-1 & -1 & 2\end{bmatrix}\right)^T = \begin{bmatrix}2 & -1 & -1\\1 & 0 & -1\\-1 & 1 & 2\end{bmatrix}$

$\lambda_1 = 1,\ \lambda_2 = 1,\ \lambda_3 = 2$

when $\lambda_1 = 1$, $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$ when $\lambda = 2$, $\begin{bmatrix}-1\\-1\\1\end{bmatrix}$

3. (a) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ $\lambda_1 = 1 \rightarrow \begin{bmatrix}1\\1\end{bmatrix}$ $\lambda_2 = -1$ $\begin{bmatrix}-1\\1\end{bmatrix}$

(b) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ $\lambda_1 = 1 \rightarrow \begin{bmatrix}1\\0\end{bmatrix}$ $\lambda_2 = 0$ $\begin{bmatrix}0\\1\end{bmatrix}$

(c) $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ and $\theta = 90^\circ \rightarrow \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$, have no eigenvalue

(d) $\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$ λ is k, the eigenspace is R^2

(e) $\begin{bmatrix}1 & k\\0 & 1\end{bmatrix}$ $\lambda = 1 \rightarrow \begin{bmatrix}1\\0\end{bmatrix}$

4. (a) size : 6×6, the degree of the characteristic polynomial is 6.

(b) A is invertible, 0 不是 $p(\lambda)$的根

(c) 3 個, 3 個不同實數根, 有 3 個 eigenspace.

5. (a) $(\lambda-3)(\lambda-3)(\lambda-5) = 0$, $\lambda = 3$ or 5

(b) $\lambda = 3$, rank = 1; $\lambda = 5$, rank = 2

(c) A is 3×3 and 24 has 3 linear independent eigenvector, so it's diagnalizable.

6. $\det(\lambda I-A) = \lambda^3-6\lambda^2+11\lambda-6 = 0 \rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

eigenvalues are $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$

$\lambda_1 = 1 \rightarrow \begin{bmatrix}1\\1\\1\end{bmatrix}$, $\lambda_2 = 2 \rightarrow \begin{bmatrix}1\\\frac{3}{2}\\\frac{3}{2}\end{bmatrix}$, $\lambda_3 = 3 \rightarrow \begin{bmatrix}1\\3\\4\end{bmatrix}$

so A is diagonaliable.

For all eigenvalues, geormetric and algebraic multiplicity is 1

$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{3}{2} & 3 \\ 1 & \frac{3}{2} & 4 \end{bmatrix}$, $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

7. $A = P^{-1}BP \rightarrow B = PAP^{-1}$

   $B = Q^{-1}CQ \rightarrow C = QBQ^{-1}$

   $\rightarrow C = Q(PAP^{-1})\, Q^{-1} = (QP)A(P^{-1}\, Q^{-1}) = (QP)A(QP)^{-1}$,

   QP is invertible, so A is similar to C

8. (a) $D = P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{1000} = I$

   (b) $A^{-1000} = I^{-1} = I$

   (c) $A^{2301} = A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

   (d) $A^{-2301} = A^{-1} = A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

9. $A = PDP^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & -1 \\ 0 & 0 & 1 \end{bmatrix}$

10. (a) $A = I_nAI_n = I_n^{-1}AI_n$,

    so any n×n matrix A is similar to itself.

    (b) zero matrix 0n×n

    (c) it is not possible, they don’t have same det