True or False : Gain 10 points for each correct answer, and detect 10 points for each incorrect answer.

(a) A square matrix that is orthogonally diagonalizable must be symmetric.
(b) If A is a symmetric matrix, then two eigenvectors with different corresponding eigenvalues are orthogonal.
(c) A symmetric matrix always has real eigenvalues.
(d) If A is an orthogonal matrix, then A***x*** • A***y*** = ***x*** • ***y***
(e) If A is an orthogonal matrix, then $A^2 = A$

(a)T(b)T (c)F (d)T (e)F

(50%) computing problem :

If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ has eigenvalues 3 and -1 with corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, respectively.

(a) What is the orthogonally diagonalization for matrix A ? (25%)
(b) What is the spectral-decomposition of matrix A ? (25%)

(a) A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ =
$\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ or
$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ = 3 $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ - 1 $\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$