Assignment #3

1. Consider the following 3 vectors in R^3 : $\mathbf{u}$ = (1, 2, 3), $\mathbf{v}$ = (2, 0, 1), $\mathbf{w}$ = (3, 1, 0)

   (a) (5 points) Project vector w orthogonally onto vector v.

   $$\mathrm{Proj}_{\mathbf{v}}\mathbf{w}=\frac{w\cdot v}{||v||^2}\,v=\left(\frac{12}{5},0,\frac{6}{5}\right)$$

   (b) (5 points) Find the vector that is the reflection of w on v

   $$\mathbf{w}_{\perp}=\mathbf{w}-\mathrm{Proj}_{\mathbf{v}}\mathbf{w}=\left(\frac{3}{5},1,\frac{-6}{5}\right)$$

   $$\textbf{reflection } \mathbf{w}'=-\mathbf{w}_{\perp}+\mathrm{Proj}_{\mathbf{v}}\mathbf{w}=\left(\frac{9}{5},-1,\frac{12}{5}\right)$$

2. Let $T_1(x_1, x_2) = (x_1 - x_2, 2x_2 - x_1, 3x_1)$ and $T_2(x_1, x_2, x_3) = (4x_2, x_1 + 2x_2)$.

   (a) (4 points) Find the standard matrices for T_1 and T_2.

   (b) (3 points) Find the standard matrix for $T_1 \circ T_2$

   (c) (3 points) Use the matrix obtained in part(b) to find formular for $T_1(T_2(x_1, x_2, x_3))$

(a) $T_1=\begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 3 & 0 \end{bmatrix}$ $T_2=\begin{bmatrix} 0 & 4 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

(b) $T_1 \circ T_2=\begin{bmatrix} -1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 12 & 0 \end{bmatrix}$

(c) $T_1(T_2(x_1, x_2, x_3))=(-x_1+2x_2,2x_1,12x_2)$

3. Consider the following 3 vectors in R^3 :

   $\mathbf{u_1}$ = (a, 2, −1), $\mathbf{u_2}$ = (4, 1, 0), $\mathbf{u_3}$ = (1, 5, −2)

Where a is an unspecified real number.

(a) (4 points) Find the possible value(s) of a, such that the volume of the parallelepiped described by the given vectors $\mathbf{u_1}$,$\mathbf{u_2}$, and $\mathbf{u_3}$ is one

V=| $\begin{vmatrix} a & 4 & 1 \\ 2 & 1 & 5 \\ -1 & 0 & -2 \end{vmatrix}$ $| = 1$,a=-2or-1

(b) (6 points) Find the area of each face of the parallelepiped which is determined by the above given vectors $\mathbf{u_1}$,$\mathbf{u_2}$, and $\mathbf{u_3}$ for a = 0

$||u_1 \times u_2||=||(1,-4,-8)||=9$

$||u_2 \times u_3||=||(-2,8,19)||=\sqrt{429}$

$||u_1 \times u_3||=||(1,-1,-2)||=\sqrt{6}$

4. Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axioms that fail.(10 points)

The set of polynomials of the form $a_0 + a_1 x$ with the operations

$$(a_0 + a_1 x) + (b_0 + b_1 x) = (a_0 + b_0) + (a_1 + b_1)x$$

and

$$k(a_0 + a_1 x) = (ka_0) + (ka_1)x$$

Yes,It's a vector space !

Step1：identify the set V of objects that will become vectors

Step2:identify the addition and scalar multipulication operations on V.

Step3: verify axiom1,6(1.close addition and 6.close multipulity)

Step4:verify axiom2,3,4,5,7,8,9 and 10 hold!

5. Use the Subspace Test to determine which of the sets are subspaces of R^3 .

a. (3 points)All vectors of the form (a, 0, 0).

b. (3 points)All vectors of the form (a, 1, 1).

c. (4 points)All vectors of the form (a, b, c), where b = a + c.

(a)Yes

(b)No

(c)Yes

6. Use the Subspace Test to determine which of the sets are subspaces of M_{nn}.

a. (2.5 points)The set of all diagonal n × n matrices.

b. (2.5 points)The set of all n × n matrices A such that det(A) = 0.

c. (2.5 points)The set of all n × n matrices A such that tr(A) = 0.

d. (2.5 points)The set of all symmetric n × n matrices.

(a)Yes

(b)No

(c)Yes

(d)Yes

7. Express the following as linear combinations of **u** = (2, 1, 4), **v** = (1, −1, 3), and **w** = (3, 2, 5).

a. (3 points) (−9, −7, −15)

b. (3 points) (6, 11, 6)

c. (4 points) (0, 0, 0)

(a)$(-9,-7,-15) = \alpha_1\mathbf{u} + \alpha_2\mathbf{v} + \alpha_3\boldsymbol{w}$

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix}\boldsymbol{\alpha} = \begin{bmatrix} -9 \\ -7 \\ -15 \end{bmatrix} \Rightarrow \boldsymbol{\alpha} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix} \quad (-9,-7,-15) = -2\mathbf{u} + \mathbf{v} - 2\mathbf{w}$$

(b) $\boldsymbol{\alpha} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$

(c) $\boldsymbol{\alpha} = \begin{bmatrix} 4 \\ -5 \\ 1 \end{bmatrix}$

(d) $\boldsymbol{\alpha} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

8.Determine whether the following polynomials span $\mathbf{P}_2$. (10 points)

$\mathbf{p_1} = 1 - x + 2x^2$, $\mathbf{p_2} = 3 + x$,

$\mathbf{p_3} = 5 - x + 4x^2$, $\mathbf{p_4} = -2 - 2x + 2x^2$

$P_2 = a_0 + a_1\boldsymbol{x} + a_2\boldsymbol{x}^2$ so, $\dim(P_2) = 3$

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ -1 & 1 & -1 & -2 \\ 2 & 0 & 4 & 2 \end{bmatrix} \text{ Guassion elimination } \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, $P_3 = 2P_1 + P_2$;$P_4 = P_1 - P_2$ $\dim(\text{span}\{P_1, P_2\}) = 2 < \dim(P_2)$

Can't span P_2!

9. In each part, determine whether the vectors are linearly independent

or are linearly dependent in P_2 .

a. (5 points) $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$

b. (5 points) $1 + 3x + 3x^2$, $x + 4x^2$, $5 + 6x + 3x^2$, $7 + 2x - x^2$

(a) $\begin{bmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{bmatrix}$ Guassion elimination $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Linear independent ! det(A)=-32! Or check det(A), if det(A)≠0, linear independent.

(b) det(A)=0, linear dependent!

10.

In each part, let $T_A : R^3 \to R^3$ be multiplication by A, and let $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (2, -1, 1)$, and $\mathbf{u}_3 = (0, 1, 1)$. Determine whether the set $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2), T_A(\mathbf{u}_3)\}$ is linearly independent in R^3.

a. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$ **b.** $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

(a)$T_A(u_1,u_2,u_3)=\begin{bmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{bmatrix}$ det($T_A(u_1,u_2,u_3)$)=-8 linear independent!

(b) $T_A(u_1,u_2,u_3)=\begin{bmatrix} 1 & 2 & 2 \\ 1 & -2 & -2 \\ 2 & 2 & 2 \end{bmatrix}$ det($T_A(u_1,u_2,u_3)$)=0 linear dependent!

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

本次作業由于多數爲定理判斷題，所以心得不用每題都寫，寫你覺得有意義的就好，如果都沒有，也要寫一份總體的心得，心得

不單獨算分，但作業內完全無心得總成績最多-20. 可以接受打字

Note:前三題原本是老師要出的期中考題, 但並未選中, 請同學認真作答, 謝謝!

繳交期限：10/25 （週二）0:00 遲交分數*0.8