Assignment #2

1. Evaluate $\det(A)$ by a cofactor expansion along a row or column of your choice.

$$A = \begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$$

det(A)=0

2. By inspection, what is the relationship between the following determinants?

$$d_1 = \begin{vmatrix} a & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix} \quad \text{and} \quad d_2 = \begin{vmatrix} a+\lambda & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix}$$

d1+λ=d2

3. Evaluate the determinant of the matrix by first reducing the matrix to row echelon form and then using some combination of row operations and cofactor expansion.

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

$$\text{ref}(A)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -1 & 0 & -3 & 0 \\ 2 & 12 & 108 & -13 \end{bmatrix}$$ det(A)=1*1*(-3)*(-13)=39

4. Confirm the identities without evaluating any of the determinants directly.

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Answer:

We have

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1-t^2b_1 & b_2-t^2b_2 & b_3-t^2b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$[-t$ times the first row was added to the second row$]$

$$= \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ (1-t^2)b_1 & (1-t^2)b_2 & (1-t^2)b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1-t^2)\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$[$A common factor of $(1-t^2)$ from the second row was factored$]$

$$= (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$[-t$ times the second row was added to the first row$]$

Thus we get the required identity.

5. Find the values of k for which the matrix A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$$

If A is invertible ,det(A) $\neq 0$

So det(A)=1(1-2k)-2(k-0)+0(2k-0)=1-4k$\neq$0

$k \neq 1/4$

6. Solve by Cramer's rule, where it applies.

$$\begin{aligned} -x_1 - 4x_2 + 2x_3 + x_4 &= -32 \\ 2x_1 - x_2 + 7x_3 + 9x_4 &= 14 \\ -x_1 + x_2 + 3x_3 + x_4 &= 11 \\ x_1 - 2x_2 + x_3 - 4x_4 &= -4 \end{aligned}$$

$x_1=5$ $x_2=8$ $x_3=3$ $x_4=-1$

7. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that $\det(A) = -7$,

find a. $\det(3A)$ b. $\det(A^{-1})$ c. $\det(2A^{-1})$ d. $\det((2A)^{-1})$

e. $det(\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix})$

a. -189 b. $-\frac{1}{7}$ c. $-\frac{8}{7}$ d. $-\frac{1}{56}$ e. 7

8. decide whether the matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$

$\det(A)=-6 \neq 0$, so A is invertible .

$A^{-1}=\frac{1}{det(A)}\text{adj}A$

adjA=$\begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix}$ A^{-1}=$\begin{bmatrix} 2 & 0 & 3/2 \\ 2/3 & 1/3 & 2/3 \\ -1 & 0 & -1 \end{bmatrix}$

9. Prove that if det(A) = 1 and all the entries in A are integers, then all the entries in A^{-1} are integers.

det(A)=1 ➔$A^{-1}=\frac{1}{det(A)}$adjA=adjA

In (adjA)^T, all Confactors are integers, because of all the entries in A are integers.

Then all the entries in A^{-1} are integers.

10. Verify that det(AB) = det(BA) and determine whether the equality det(A + B) = det(A) + det(B) holds.

$$A = \begin{bmatrix} -1 & 8 & 2 \\ 1 & 0 & -1 \\ -2 & 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 1 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

AB= $\begin{bmatrix} 6 & 15 & 26 \\ 2 & -4 & -3 \\ -2 & 10 & 12 \end{bmatrix}$, BA= $\begin{bmatrix} 5 & 8 & -3 \\ -6 & 14 & 7 \\ 5 & -2 & -5 \end{bmatrix}$

A+B=$\begin{bmatrix} 1 & 7 & -2 \\ 2 & 1 & 2 \\ -2 & 5 & 1 \end{bmatrix}$

det(A)=2 det(B)=-33 det(A+B)=-75 det(A)+det(B)=-31

det(AB)=-66 det(BA)=-66

So. det(A+B) $\neq$det(A)+det(B) det(AB)=det(BA)

評分標準：

每題解答 6 分，心得 4 分。結果錯-2，過程或方法錯不給分.

繳交期限：10/11 （週二）0:00 遲交分數*0.8