

Assignment#6

Chapter 6

1. Let $\mathbf{p} = x$ and $\mathbf{q} = x^2$ and the stated sample points are given :

$$x_0 = -2, x_1 = 0, x_2 = 2$$

(a) Find $\|\mathbf{p}\|$ relative to the evaluation inner product on P_2

(b) Show that the vectors \mathbf{p} and \mathbf{q} are orthogonal with respect to this inner product.

2. Find a basis for the orthogonal complement of the subspace of \mathbb{R}^n spanned by the vectors.

$$\mathbf{v}_1 = (1, 4, 5, 2), \mathbf{v}_2 = (2, 1, 3, 0), \mathbf{v}_3 = (-1, 3, 2, 2)$$

3. The vectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 are **orthonormal** with respect to the Euclidean inner product on \mathbb{R}^4 . Find the orthogonal projection of $\mathbf{b} = (1, 2, 0, -1)$ onto the subspace W spanned by these vectors.

$$\mathbf{v}_1 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right), \mathbf{v}_2 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right), \mathbf{v}_3 = \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right)$$

4. Find a least square solution of $\mathbf{Ax}=\mathbf{b}$ by QR -decomposition of the matrix \mathbf{A}

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 9 \\ 3 \end{bmatrix}$$

5. Let W be the plane with equation $5x - 3y + z = 0$.

a. Find a basis for W . (5 points)

b. Find the standard matrix for the orthogonal projection onto W . (5 points)

6. find parametric equations for all least squares solutions of $\mathbf{Ax} = \mathbf{b}$,

$$\mathbf{A} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$$

7. Find the least squares straight line fit

$$\mathbf{y} = \mathbf{ax} + b$$

to the data points:

$$(0, 1), (2, 0), (3, 1), (3, 2)$$

and show that the result is reasonable by graphing the fitted line and

plotting the data in the same coordinate system.

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

本次作業無需每題寫心得，請選擇你認為需要的，不單獨算分，但完全不寫心得最多-20.

！！如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理！

截止日期： 12/7 00:00(週三)