

Assignment #5

題目範圍: section 5.1~5.2

1. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix.

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

2. Find the eigenvalues and a basis for each eigenspace of the linear operator defined by the stated formula. [Suggestion: Work with the standard matrix for the operator.]

$$T(x, y, z) = (2x - y - z, x - z, -x + y + 2z)$$

3. In each part of Exercises 4, find the eigenvalues and the corresponding eigenspaces of the stated matrix operator on R^2 . Use geometric reasoning to find the answers. No computations are needed.

- a. Reflection about the line $y = x$. (2 points)
- b. Orthogonal projection onto the x -axis. (2 points)
- c. Rotation about the origin through a positive angle of 90° . (2 points)
- d. Contraction with factor k ($0 \leq k < 1$). (2 points)
- e. Shear in the x -direction by a factor k ($k \neq 0$). (2 points)

4. Suppose that the characteristic polynomial of some matrix A is found to be $p(\lambda) = (\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3$. In each part, answer the question and explain your reasoning.

- a. What is the size of A ? (3 points)
- b. Is A invertible? (3 points)
- c. How many eigenspaces does A have? (3 points)

5. Let

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

- a. Find the eigenvalues of A . (3 points)

- b. For each eigenvalue λ , find the rank of the matrix $\lambda I - A$ (3 points)
 c. Is A diagonalizable? Justify your conclusion. (4 points)

6. Find the geometric and algebraic multiplicity of each eigenvalue of the matrix A , and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A , and find $P^{-1} A P$.

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

7. If A , B and C are $n \times n$ matrices such that A is similar to B and B is similar to C , do you think that A must be similar to C ? Justify your answer.

8. Let

$$A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Confirm that P diagonalizes A , and then compute each of the following powers of A .

- a. A^{1000} b. A^{-1000} c. A^{2301} d. A^{-2301}

9. Find a 3×3 matrix A that has eigenvalues 1, -1 , and 0, and for which

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

are their corresponding eigenvectors.

10. a. Is it possible for an $n \times n$ matrix to be similar to itself? Justify your answer. (3 points)
 b. What can you say about an $n \times n$ matrix that is similar to $0_{n \times n}$? Justify your answer. (3 points)
 c. Is it possible for a nonsingular matrix to be similar to a singular matrix? Justify your answer. (4 points)

評分標準：

每題 10 分，每小題配分已標注，答錯即 0 分。

每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.

！！ 如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明

且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理！

繳交期限：11/16（週三）00:00 遲交分數*0.8