Assignment #4

題目範圍: section 4.5~4.9

1. Show that the following polynomials form a basis for 𝑃3 .

1+ *x*, 1 − *x*, 1 – *x2* , 1 − *x3*

A= 🡺A is invertible ,so form a basis for P3

Or show a(1+ *x)*+b( 1 − *x*)+c(1 – *x2)* +d( 1 − *x3)=0 only a=b=c=d=0*

1. In each part, let 𝑇𝐴 ∶𝑅3 →𝑅3 be multiplication by 𝐴, and let u = (1, −2, −1). Find the coordinate vector of 𝑇𝐴(u) relative to the basis 𝑆 = {(1, 1, 0), (0, 1, 1), (1, 1, 1)} for 𝑅3 .
2. (5 points) A=
3. (5 points) A=
4. Au=b
5. x=S-1b
6. In each part, find a basis for the given subspace of 𝑅3 , and state its dimension.

a. (2.5 points) The plane 3*x* − 2*y* + 5*z* = 0.

b. (2.5 points) The plane *x − y =* 0.

c. (2.5 points) The line *x =* 2*t, y = −t, z =* 4*t.*

d. (2.5 points) All vectors of the form (a, b, c), where b = a + c.

a.&b.find [3,-2,5]=0 [1,-1,0]=0nullspace to solve 只要basis构成与参考答案相同subspace即可

a.(2/3,1,0) (-5/3,0,1) dimension:2

b.(1,1,0) (0,0,1) dimension:2

c.(2,-1,4) dimension:1

d.参数解x== dimension:2

1. In each part, let 𝑇𝐴 be multiplication by 𝐴 and find the dimension of the subspace 𝑅4 consisting of all vectors x for which 𝑇𝐴(x) = 0.
2. (5 points) A= kernel of A is 2
3. (5 points) A= kernel of A is 1
4. Consider the bases 𝐵 = {**u1 , u2** } and 𝐵′ = {**u′1 , u′ 2 }** for 𝑅2 , where

**u1** = , **u2** = , **u′1** = , **u′2** =

a.(2.5 points) Find the transition matrix from 𝐵 ′ to 𝐵.

b.(2.5 points) Find the transition matrix from 𝐵 to 𝐵 ′ .

c.(2.5 points) Compute the coordinate vector [**w**]𝐵, where **w** = and use (11) to compute [**w**]𝐵′ . [**w**]𝐵= [**w**]𝐵′=PB🡺B’ [**w**]𝐵 =

d.(2.5 points) Check your work by computing [**w**]𝐵′ directly. (B′)-1w =

Note: use(11) please to check the reference at the bottom of this assignment.

1. If 𝐵1 , 𝐵2 , and 𝐵3 are bases for 𝑅2 , and if

𝑃𝐵1→𝐵2 = and 𝑃𝐵2→𝐵3 = ,

then 𝑃𝐵3→𝐵1 = 𝑃𝐵3→𝐵1= ( 𝑃𝐵2→𝐵3 𝑃𝐵1→𝐵2 )-1

1. Suppose that *x1*=−1, *x2* =2, *x3*=4, *x4*=−3 is a solution of a nonhomogeneous linear system **𝐴x = b** and that the solution set of the homogeneous system **𝐴x = 0** is given by the formulas

*x1* = −3*r* + 4*s* , *x2* = *r – s* , *x3* = *r* , *x4* = *s*

a.(5 points) Find a vector form of the general solution of **𝐴*x* = 0.**

b.(5 points) Find a vector form of the general solution of **𝐴*x* = b**

**a. A=**+s b. x=++s

1. Consider the linear systems

=

and

*=*

1. (2.5 points) Find a general solution of the homogeneous system.

s(-11/5,2/5,1)

1. (2.5 points) Confirm that *x1* = 1*, x2* = 1, *x3* = 1 is a solution of the nonhomogeneous system.

yes

1. (2.5 points) Use the results in parts (a) and (b) to find a general solution of the nonhomogeneous system.

(1,1,1)+ s(-11/5,2/5,1)

1. (2.5 points) Check your result in part (c) by solving the nonhomogeneous system directly

By rref

x =+,when s=1,x=(1,1,1)

1. Let 𝑇 :𝑅5 →𝑅3 be the linear transformation defined by the formula

𝑇(*x1 , x2 , x3 , x4 , x5* ) = (*x1 + x2 , x2 + x3 + x4 , x4 + x5* )

a. (5 points) Find the rank of the standard matrix for 𝑇. 3

b. (5 points) Find the nullity of the standard matrix for 𝑇. 5

10.a. (2.5 points) If 𝐴 is a 3 × 5 matrix, then the rank of 𝐴 is at most 3 . Why?

b. (2.5 points)If 𝐴 is a 3 × 5 matrix, then the nullity of 𝐴 is at most 5 . Why?

c. (2.5 points)If 𝐴 is a 3 × 5 matrix, then the rank of 𝐴𝑇 is at most 3 . Why?

d. (2.5 points)If 𝐴 is a 3 × 5 matrix, then the nullity of 𝐴𝑇 is at most 3 .Why?

評分標準：

**每題10分，每小題配分已標注，答錯即0分。**

**每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.**

繳交期限：11/8 （週二）00:00 遲交分數\*0.8

Reference:

