Assignment #5

題目範圍: section 5.1~5.2

1. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix.

*A*=

1. Find the eigenvalues and a basis for each eigenspace of the linear operator defined by the stated formula. [Suggestion: Work with the standard matrix for the operator.]

𝑇 *(x, y, z) = (2x − y − z, x − z, −x + y + 2z)*

1. In each part of Exercises 4, find the eigenvalues and the corresponding eigenspaces of the stated matrix operator on 𝑅2 . Use geometric reasoning to find the answers. No computations are needed.

a. Reflection about the line *y = x*. (2 points)

b. Orthogonal projection onto the *x*-axis. (2 points)

c. Rotation about the origin through a positive angle of 90∘ . (2 points)

d. Contraction with factor *k* (0 *≤ k <* 1)*.* (2 points)

e. Shear in the *x*-direction by a factor *k* (*k* ≠ 0). (2 points)

1. Suppose that the characteristic polynomial of some matrix *A* is found to be *p*(𝜆) = (𝜆 − 1)(𝜆 − 3)2 (𝜆 − 4)3 . In each part, answer the question and explain your reasoning.

a. What is the size of *A* ? (3 points)

b. Is *A* invertible ? (3 points)

c. How many eigenspaces does *A* have ?(3 points)

1. Let

*A=*

a. Find the eigenvalues of *A*. (3 points)

b. For each eigenvalue 𝜆, find the rank of the matrix 𝜆*I-A* (3 points)

c. Is 𝐴 diagonalizable? Justify your conclusion.(4 points)

1. Find the geometric and algebraic multiplicity of each eigenvalue of the matrix 𝐴, and determine whether *A* is diagonalizable. If 𝐴 is diagonalizable, then find a matrix *P* that diagonalizes *A*, and find *P* −1 *A* *P*.

*A=*

1. If *A* , *B* and *C* are n × n matrices such that *A* is similar to *B* and *B* is similar to *C*, do you think that *A* must be similar to *C*? Justify your answer.
2. Let

*A=* and *P=*

Confirm that *P* diagonalizes *A*, and then compute each of the following powers of *A*.

a. 𝐴1000 b. 𝐴−1000 c. 𝐴2301 d. 𝐴−2301

9. Find a 3 × 3 matrix 𝐴 that has eigenvalues 1, −1, and 0,

and for which

are their corresponding eigenvectors.

10. .a. Is it possible for an n × n matrix to be similar to itself? Justify your answer. (3 points)

b. What can you say about an n × n matrix that is similar to **0**n×n ? Justify your answer. (3 points)

c. Is it possible for a nonsingular matrix to be similar to a singular matrix? Justify your answer.(4 points)

評分標準：

**每題10分，每小題配分已標注，答錯即0分。**

**每題都需寫心得，不單獨算分，但缺一題心得-2 point,最多-20.**

**！！**如果不會請去請教同學，並在作業裡說明你請教了誰。如未說明且被發現答案相似度過高（包括過程，心得，結果），則按抄襲處理！

**繳交期限**：11/16 （週三）00:00 遲交分數\*0.8