Formula Sheet:

Definition 1.1: mean (central tendency)

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Definition 1.2: Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$$

Definition 1.3: Standard deviation

$$s = \sqrt{s^2}$$

Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's Law:

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

Definition 2.6:

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = \sum_{i=1}^{\infty} P(A_i)$$

Probability formulas:

$$P(\overline{A}) = 1 - P(A)$$

$$P(S) = P(A) + P(\overline{A})$$

$$1 = P(A) + P(\overline{A}) \text{ and } P(\overline{A}) = 1 - P(A)$$

$$P(\emptyset) = 0 \text{ and } P(S) = 1$$

$$P(A) \leq 1$$

$$1 = P(S) = P(A) + P(\overline{A}) \geq P(A)$$

Theorem 2.1: MN rule

With m elements $a_1, a_2, \dots, a_m$ $and\ n\ elements$ $b_1, b_2, \dots, b_n$. It is possible to form $mn = m * n$ pairs containing one element from each group.

$$mn = m * n$$

Theorem 2.2: Permutation

$$P_r^n = \frac{n!}{(n-r)!}$$

Theorem 2.3: Multinomial Distribution

$$N = \frac{n!}{n_1!\ n_2!\ \dots\ n_k!}$$

Theorem 2.4 Combinations

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

Definition 2.9: Conditional probability of an event A, given that an event B has occurred

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Definition 2.10: Two events A and B are said to be independent if any one of the following holds

$$P(A \mid B) = P(A)$$

$$P(B \mid A) = P(B)$$

$$P(A \cap B) = P(A)P(B)$$

Otherwise, the events are said to be dependent.

Theorem of total probability:

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_n)P(B_n)$$

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

Bayes theorem: For two events A and B in a sample space s

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

If 0 < P(B) < 1, we may write by the theorem of total probability

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P\left(A\middle|\overline{B}\right)P(\overline{B})}$$

Note: $P(A \ \cap B) = P(A|B)P(B)$

Definition 3.4: expected value of Discrete random Y

$$E(Y) \ = \ \sum_{y} yp(y)$$

Definition 3.5: variance

$$V(Y) = E[(Y - \mu)^2]$$

Probability Mass Function (pmf) for Y, p(.)

$$p(y) = P(Y = y)$$

If y is element of Y, p(y) > 0 otherwise p(y) = 0

Definition 3.7 Binomial distribution

$$p(y) \ = \ \binom{n}{y} p^y q^{n-y}$$

$$y = 0,1,2, \dots, n \ and \ 0 \leq p \leq 1$$

3.7: Y is a binomial random variable based on n trials and success probability p

$$\mu = E(Y) = np \ and \ \sigma^2 = V(Y) = npq$$

Definition 3.8: Geometric Distribution

$$P(y) = q^{y-1} * P$$

$$y = 1, 2, 3, \dots, 0 \leq p \leq 1$$

3.8: If Y is a random variable with a geometric distribution

$$\mu = E(Y) = \frac{1}{p} \; and \; \sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Extra Formulas: for geometric distribution

Success occurs on or before the nth trial.

$$P(X \leq n) = 1 - (1-p)^n$$

Success occurs before the nth trial.

$$P(X < n) = 1 - (1-p)^{n-1}$$

Success occurs on or after the nth trial.

$$P(X \geq n) = (1-p)^{n-1}$$

Success occurs after the nth trial.

$$P(X > n) = (1-p)^n$$