Physics 51 Section
Homework 13
Monday, November 23, 2020

**T1.25:** Suppose that a thin film of acetone (index of refraction $n = 1.25$) of thickness $d$ is coating a thick plate of glass (index of refraction = 1.50). Take the magnitude of the amplitude for reflection of a photon from the top or the bottom surface of the acetone at normal incidence to be $r$ and assume that there is an additional phase change $\pi$ in the reflection from the top *and* the bottom surface of the acetone, since at each of these surfaces light is passing from a medium with a lower index of refraction to one with a higher index of refraction. Calculate the probability that a photon of wavelength $\lambda$ is reflected. Assume that amplitudes that involve multiple reflections at the bottom surface of the film can be neglected in your calculation. Express your answer in terms $\lambda$ and $r$ as well as the thickness $d$ and the index of refraction $n$ of the acetone. What is the minimum thickness of the coating necessary to produce zero reflection? *Note*: For the air-acetone and acetone-glass surfaces $r \cong 0.1$.

**T1.26:** Assume that the first beam splitter at A in the Mach-Zehnder interferometer (Fig. 1.23) is a "third-silvered mirror," that is, a mirror that reflects one-third the light and transmits two-thirds. The two mirrors at B and C reflect 100% of the light, and the second beam splitter at D is a traditional half-silvered mirror that reflects one-half the light and transmits one-half. The probability of detecting a photon in either photomultiplier $PM_1$ or $PM_2$ varies with the position of the movable mirror, say mirror B. Determine the maximum probability and the minimum probability of obtaining a count in, say, $PM_1$. What is the visibility

$$V = \frac{P_{max} - P_{min}}{P_{max} + P_{min}}$$

of the interference fringes, where $P_{max}$. and $P_{min}$ are the maximum and minimum probabilities, respectively, that a photon is counted by the detector, as the position of the movable mirror varies? *Note*: in the experiment of Aspect et al. described in Section 1.5 the visibility of the fringes is $0.987 \pm 0.005$.

**T1.27:** Figure 1.43 shows a Michelson interferometer with a movable mirror $M_1$, a fixed mirror $M_2$, and a beam splitter $M_S$, which is a half-silvered mirror that transmits one-half the light and reflects one-half the light incident upon it independent of the direction of the light. The source emits monochromatic light of wavelength $\lambda$. There are two paths that light can follow from the source to the detector, as indicated in the figure. Note that path 1 includes travel from the beam splitter $M_S$ to the movable mirror $M_1$ and back to the beam splitter, while path 2 includes travel from the beam splitter $M_S$ to the fixed mirror $M_2$ and back to the beam splitter. Assume the beam splitter introduces a phase change of $\pi$ for light that follows path 1 from the source to the detector relative to light that follows path 2 from the source to the detector. Also assume the mirrors $M_1$ and $M_2$ reflect 100% of the light incident upon them and the photodetector PM (a photomultiplier) is 100% efficient as well.

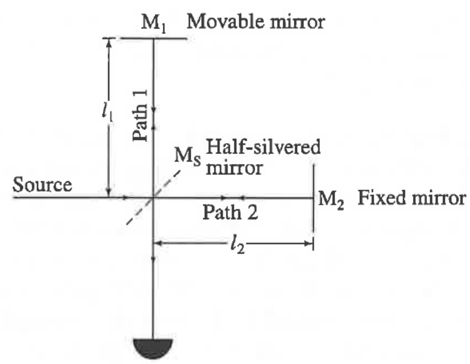


Figure 1.43 The Michelson interferometer.

(a) Use the principles of quantum mechanics to determine the probability that a photon entering the interferometer is detected by the photodetector. Express your answer in terms of the lengths $l_1$, $l_2$ , and $\lambda$.

(b) Find an expression for $l_1$ in terms of $l_2$ and $\lambda$ such that there is 100% probability that the photon is detected by the photodetector.

(c) Suppose that the movable mirror is shifted upward by a distance $\lambda/6$ from the position(s) that you determined in part (b). Find the probability that the photon is detected at the photodetector in this case.

**T1.34:** Starting from first principles, show that the probability that a photon of wavelength $\lambda$ hits a photomultiplier centered on a point P in the detection plane that makes an angle $\theta$ with the horizontal for a grating composed of three very narrow slits each separated by a distance $d$ is given by

$$\text{Prob} = r^2(1 + 4\cos\phi + 4\cos^2\phi)$$

where $r^2$ is the probability that the photon would strike the photomultiplier with a single slit open and $\phi = kd\sin\theta = 2\pi d\sin\theta/\lambda$.

**T1.37:** Determine the probability that a photon is detected at the first minimum of a six-slit grating if the bottom two slits are closed. Assume the magnitude of the probability amplitude due to each slit is $r$. *Suggestion*: Start by showing how the complex probability amplitudes from each slit add up to zero at the first minimum.

**T1.43:** Use the principle of least time to derive Snell's law, namely, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ for light being refracted as it travels from a medium with index of refraction $n_1$ into a medium with index of refraction $n_2$. *Suggestion*: Follow a procedure similar to the one given in Example 1.11 (pp. 42 to 43). Locate the source S in medium 1 and the point P in medium 2.