PHYS 1341: HW 3
Quiz: Thu 3 Feb 2022

1. [After Reif Problem 2.1] A particle of mass m is free to move in one dimension. Denote its position coordinate by x and its momentum by p. Suppose that this particle is confined within a box so as to be located between $x = 0$ and $x = L$, and suppose that its energy is known to lie between E and $E + dE$.

   (a) Draw the classical phase space of this particle, indicating the regions which are accessible to the particle.

   (b) How does the density of states $\Omega(E)$ scale with L?

   (c) How does the density of states $\Omega(E)$ scale with E?

2. Consider N classical, non-interacting particles in an isolated, *one-dimensional* box. How does the density of states $\Omega(E)$ scale with the total energy E for $N \gg 1$?

3. [After Kennett Problem 2.11 and Reif Problem 2.4] Consider an isolated system consisting of a large number N of non-interacting localized particles of spin 1/2. Each particle has a magnetic moment μ which can point either parallel or antiparallel to an applied field H. Each parallel moment contributes $-\mu H$ to the total energy, while each antiparallel moment contributes μH to the total energy.

   (a) Denote by n the number of particles with moments parallel to the applied field. Give an expression for n in terms of N, μ, H, and the total energy E.

   (b) Find the degeneracy $\Omega(E)$. You may leave your expression in terms of N and n because it is understood that n is related to E by part (a).

   (c) Stirling's formula is $\log(x!) \approx x \log x - x$ for $x \gg 1$. Assuming $N \gg n \gg 1$, apply Stirling's formula to your expression in (b) to obtain an approximate expression for $\log \Omega(E)$. Once again, you may leave your expression in terms of N and n.

4. [After Kennett Problem 2.11 and Reif Problem 3.2] Consider your expression from Problem 3(c).

   (a) Using the definition of the entropy $S(E) = k_\mathrm{B} \log \Omega(E)$, and the statistical definition of temperature $1/T = \partial S/\partial E$, find an expression relating the temperature T and the total energy E for this system. It is probably most convenient to write it as an expression for E, which will be a function of $\beta = 1/k_\mathrm{B}T$, N, μ, and H. Hint: since you have $\log \Omega(E)$ in terms of n from Problem 3(c), and n in terms of E from Problem 3(a), it is convenient to do the derivative in two parts using the chain rule, $\partial S/\partial E = (\partial S/\partial n)(\partial n/\partial E)$.

   (b) Sketch a plot of E versus β.

   (c) Under what circumstances is T negative for this system? Is it physically possible for temperature to be negative?