PHY-243 Homework 3 - Brandon Evans

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0.1 PHYS-243 Spring 2019 - Brandon Evans - Homework 3

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```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import math
```

0.1.1 Problem 1 - Probability of l>t

Assumptions/Expectations:

- Needle rotates angles 0 to π radians
- At some length (l) and angle θ, we no longer are concerned with distance from $\frac{t}{2}$, thus probability should increase and approach near 1 with length (l)
- Think about how integration of $l \leq t$ and $l > t$ will have different limits, per Abtin's guidance

Note: Handworked math attached as image in GitHub folder.
Short Needle: Recall in case of $l \leq t$, that probability is within linear proportion, $P = (\frac{l}{t})(\frac{2}{\pi})$
Long Needle: I think it will be easier to set $x = \frac{t}{l}$ Find area a from 0 to curve crossing t: $\int_{\theta=0}^{\arcsin(x)} l\sin(\theta)d\theta \; a = l(1-\sqrt{1-x^2})$
Crossing happens twice, so $2a$ Find area between crossings: $t(\pi - 2(\arcsin(x))$
Compile the area components and divide by total space: $P(l,t) = \frac{t(\pi-2(\arcsin(x))+2l(1-\sqrt{1-x^2})}{t\pi}$

```
In [66]: def prob_long_needle(l,t):
             x = t/l
             return t*(np.pi-2*(np.arcsin(x))+2*l*(1-np.sqrt(1-x**2)))/(t*np.pi)

         def prob_short_needle(l,t):
             return (2*l)/(np.pi*t)

         def prob_needle(l,t):
             if l<=t:
                 return prob_short_needle(l,t)
             else:
                 return prob_long_needle(l,t)
```

In [52]:
```
#Just curious...
print(prob_short_needle(1,1))
print(prob_long_needle(1,1))

print(prob_short_needle(2,1))
print(prob_long_needle(2,1))

print(prob_short_needle(0.5,1))
print(prob_long_needle(0.5,1))
```

```
0.6366197723675814
0.636619772368
1.2732395447351628
0.837248420558
0.3183098861837907
nan
```

```
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:3: RuntimeWarning: invalid value e
  This is separate from the ipykernel package so we can avoid doing imports until
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:3: RuntimeWarning: invalid value e
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```

0.1.2 Problem 2 - Find probability of π

In [129]:
```
#Monte Carlo Simulation for l<t
#Let's iterate across the probability space, with x and theta

class BuffonNeedle:
    def __init__(self, l=1, t=1, name=""):
        self.name = name
        self.l = l
        self.t = t

    def run(self,steps=1000,printOnly=True):
        num_crosses=0
        theta_span= np.pi/2
        x_span= self.t/2
        for i in range(0,steps):
            for j in range(0,steps):
                dist = i*(x_span/steps)
                angle = j*(theta_span/steps)
                if (self.l/2)*np.sin(angle) > dist:
                    num_crosses+=1
        prob = num_crosses/steps**2
        if self.l <= self.t:
            est_pi = (2*steps**2) / num_crosses
```

```
                else:
                    x = self.l / self.t
                    est_pi = 2*(x - np.sqrt(x**2-1) + np.arccos(1/x)) / prob
                if printOnly:
                    print("Crosses:{}".format(num_crosses))
                    print("Probability:{}".format(prob))
                    print("Est PI:{}".format(est_pi))
                else:
                    return prob

        #for x in range(0,10000):
```

Find π for $l < t$ using Monte Carlo

```
In [130]: short_needle = BuffonNeedle(1,1)
          short_needle.run()
```

```
Crosses:636619
Probability:0.636619
Est PI:3.1415964650756574
```

For special case $l > t$ We'll calculate the probability using our function derived from problem #1:

```
In [131]: print(prob_needle(2,1))
```

```
0.837248420558
```

0.1.3 Problem 3 - Find π with $l > t$ with Monte Carlo

```
In [132]: #I took this very similar to previous problem, except perhaps probability only in pr
          long_needle = BuffonNeedle(3,1)
          long_needle.run()
```

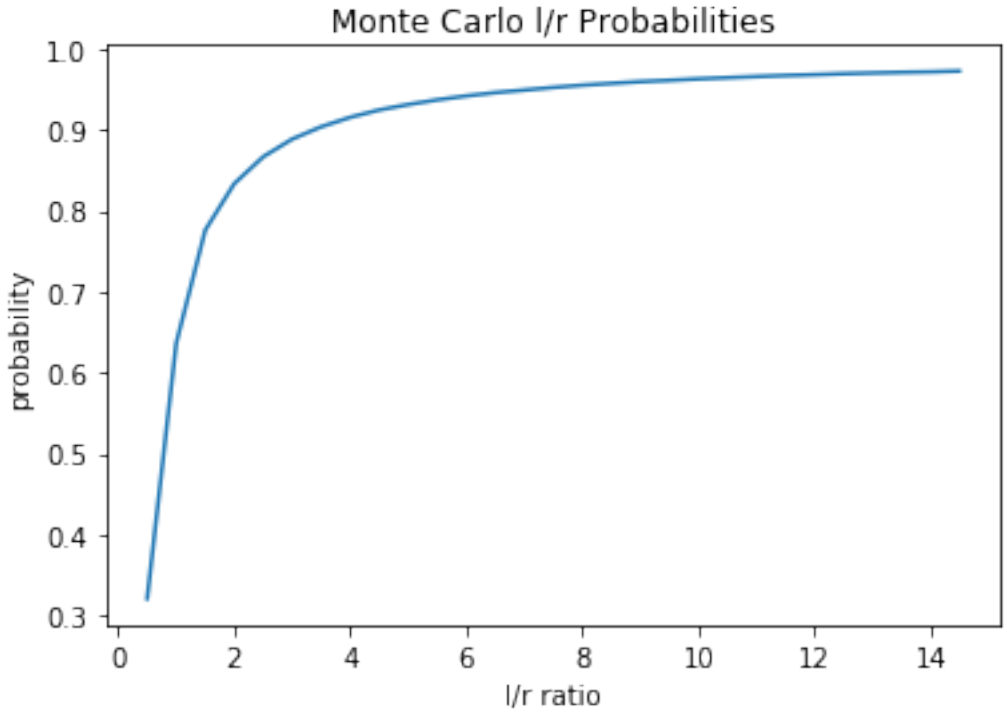
```
Crosses:892489
Probability:0.892489
Est PI:3.1429682440782676
```

0.1.4 Problem 4 - Plot Monte Carlo Probabilities as l/t Changes

```
In [133]: #Let's hold t constant to run the ratio...
          lr_span = np.arange(0.5, 15, 0.5)
          x_data = np.zeros(len(lr_span))
          y_data = np.zeros(len(lr_span))

          sim_needle = BuffonNeedle()
```

```
for i, x in enumerate(lr_span):
    x_data[i]=x
    sim_needle.l = x
    y_data[i]=sim_needle.run(100,False)
plt.plot(x_data,y_data)
plt.title("Monte Carlo l/r Probabilities")
plt.xlabel("l/r ratio")
plt.ylabel("probability")
plt.show()
```

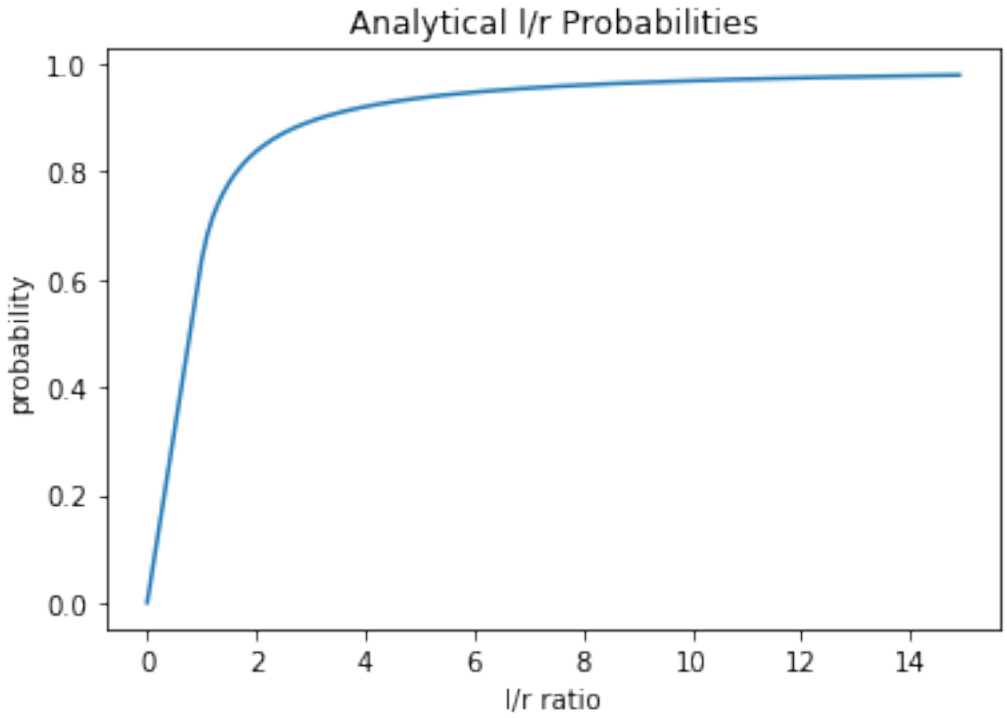


0.1.5 Problem 5 - Plot Analytical Probabilities as l/t Changes

In [134]:
```
#Let's hold t constant to run the ratio...
lr_span = np.arange(0, 15, 0.1)
x_data = np.zeros(len(lr_span))
y_data = np.zeros(len(lr_span))

for i, x in enumerate(lr_span):
    x_data[i]=x
    y_data[i]=prob_needle(x,1)
plt.plot(x_data,y_data)
plt.title("Analytical l/r Probabilities")
plt.xlabel("l/r ratio")
```

```
plt.ylabel("probability")
plt.show()
```



0.1.6 For Fun - Draw Buffon's Experiment (requires drawSvg library):

```
In [163]: #Graphical simulation of needles dropped on a wooden floor
          #I put together fairly quick, didn't confirm exact placement of needles on boards.

          import drawSvg as draw
          import random as random
          DEG2RAD = np.pi/180
          colors = {'wood':'#b77005','needle_miss':'#a09c97','needle_cross':'#d62319'}

          #Inputs
          num_planks = 10
          num_needles = 300
          needle_plank_ratio = 1.0
          plank_width = 10
          plank_length = 350
          board_padding = 10
          plank_padding = 2

          #Calcs
          board_width = plank_length + 2*board_padding
```

```
board_height = num_planks * (plank_width+plank_padding) + 2*board_padding

print("Board width:{} height:{}".format(board_width,board_height))
#Needle length as a ratio of the floor plank width
l = plank_width * needle_plank_ratio

y_padding = 2

#Was origin of center, changing it, my brain can't compute center origin stuff
#d = draw.Drawing(board_width, board_height, origin='center')
d = draw.Drawing(board_width, board_height)

#Draw the planks of the floor
#Technically, the planks are of width, plank_width+padding
for i in range(0,num_planks):
    d.append(draw.Rectangle(board_padding,board_padding+i*(plank_width+plank_padding)
    #d.append(draw.Rectangle(plank_x_offset,i*(plank_width+padding),plank_length+pla

#Drop needles
crossings = 0
for j in range(0,num_needles):
    needle_x = board_padding + ((board_width-2*board_padding) * random.random())
    needle_y = board_padding + ((board_height-2*board_padding) * random.random())
    rand_theta = random.random() * 360 * DEG2RAD
    rand_x = np.cos(rand_theta) * l
    rand_y = np.sin(rand_theta) * l
    #Choose needle color based on whether it crosses the plank
    did_cross = np.abs(rand_y) > ((plank_width+plank_padding)/2)
    needle_color = colors['needle_cross'] if did_cross  else colors['needle_miss']
    if did_cross:
        crossings+=1
    p = draw.Path(stroke_width=1, stroke=needle_color,fill='black', fill_opacity=0.5)
    p.M(needle_x,needle_y)
    p.l(rand_x,rand_y)
    d.append(p)

print("Crossings:{} Total:{}".format(crossings,num_needles))
print("Est PI:{}".format(round(2*num_needles/crossings,5)))

#Render SVG
d.setPixelScale(2)  # Set number of pixels per geometry unit
d  # Display as SVG
```

```
Board width:370 height:140
Crossings:185 Total:300
Est PI:3.24324
```

Out[163]: