Vibration Hw 1

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ME301 - Spring 2021

February 23, 2021

Question 1: Converting to its Equivalent Circuit/Drawing

Problem Statement: Find the equivalent circuit in the electrical domain or mechanical domain given the following diagram. Derive the impedance and admittance matrices.

(a) Mechanical Domain to Electrical Domain I
Here is the mechanical diagram and its electrical equivalent circuit on Figure 1 and 2 respectively.

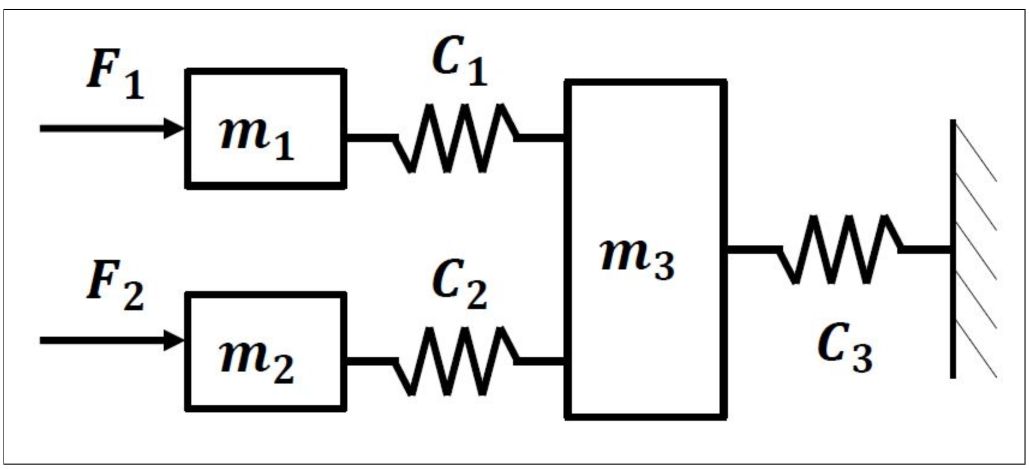

Figure 1: Mechanical Diagram of 1a

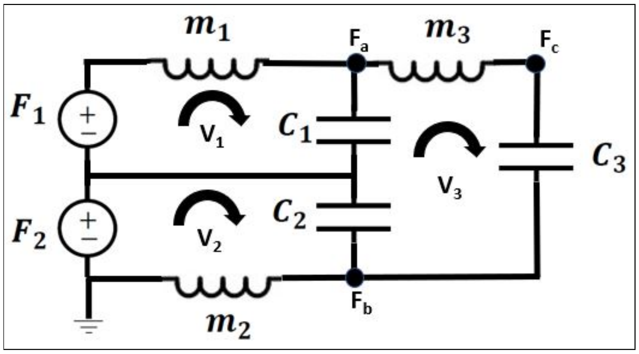

Figure 2: Electrical Equivalent Circuit of 1a

In the LaPlace Domain, unique equations of the circuit can be written using mesh analysis in Equation(1). Look at Figure 2 for the labeled velocities currents: v_1, v_2, v_3.

$$\begin{aligned}
F_1(s) &= m_1 s V_1(s) + \frac{1}{C_1 s}(V_1(s) - V_3(s)) \\
F_2(s) &= \frac{1}{C_2 s}(V_2(s) - V_3(s)) + msV_2(s) \\
0 &= -\frac{1}{C_2 s}(V_2(s) - V_3(s)) - \frac{1}{C_1 s}(V_1(s) - V_3(s)) + msV_3(s) + \frac{1}{C_3 s}V_3(s)
\end{aligned} \tag{1}$$

Equation (1) can be rewritten in matrix form $F = ZV$ where Z is the impedance matrix in Equation (2).

$$\begin{bmatrix} F_1 \\ F_2 \\ 0 \end{bmatrix} = \begin{bmatrix} m_1 s + \frac{1}{C_1 s} & 0 & -\frac{1}{Cs} \\ 0 & m_2 s + \frac{1}{C_2 s} & -\frac{1}{C_2 s} \\ -\frac{1}{C_1 s} & -\frac{1}{C_2 s} & m_3 s + (\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3})\frac{1}{s} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{2}$$

In the LaPlace Domain, unique equations of the circuit can be written using nodal analysis in Equation (3). Look at Figure 2 for the labeled force nodes: F_a, F_b, F_c.

$$\begin{aligned}
\frac{F_1 - F_a}{m_1 s} &= (F_a - F_2)C_1 s + (F_a - F_c)\frac{1}{m_3 s} \\
(F_2 - F_b)C_2 s + (F_c - F_b)C_3 s &= \frac{F_b - 0}{m_2 s} \\
(F_a - F_c)\frac{1}{m_3 s} &= (F_c - F_b)C_3 s
\end{aligned} \tag{3}$$

Equation (3) can be rewritten in matrix form $V = YF$, where Y is the admittance matrix in Equation (4).

$$\begin{bmatrix} \frac{F_1}{m_1 s} + C_1 s F_2 \\ C_2 s F_2 \\ 0 \end{bmatrix} = \begin{bmatrix} C_1 s + (\frac{1}{m_1} + \frac{1}{m_3})\frac{1}{s} & 0 & -\frac{1}{m_3 s} \\ 0 & C_2 s + (\frac{1}{m_2} + \frac{1}{m_3})\frac{1}{s} & -C_3 s \\ -\frac{1}{m_3 s} & -C_3 s & \frac{1}{m_3 s} + C_2 s + C_3 s \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \tag{4}$$

(b) Electrical Domain to Mechanical Domain
Here is the electrical circuit and its mechanical equivalent diagram on Figure 3 and 4 respectively.

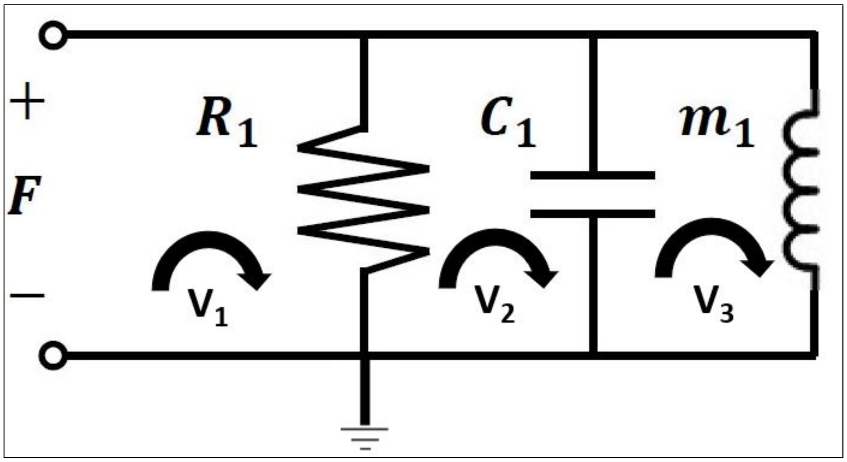

Figure 3: Electrical Equivalent Circuit of 1b

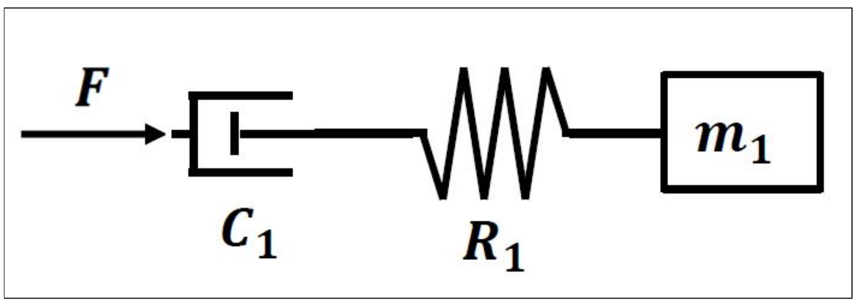

Figure 4: Mechanical Diagram of 1b

In the LaPlace Domain, unique equations of the circuit can be written using mesh analysis in Equation (5). Look at Figure 3 for the labeled velocities currents: v_1, v_2, v_3.

$$\begin{aligned} F(s) &= R(V_1(s) - V_2(s)) \\ 0 &= -R(V_1(s) - V_2(s) + \frac{1}{C_1 s}(V_2(s) - V_3(s)) \\ 0 &= -\frac{1}{C_1 s}(V_2(s) - V_3(s)) + m_1 s V_3(s) \end{aligned} \tag{5}$$

Equation (5) can be rewritten in matrix form $F = ZV$ where Z is the impedance matrix in Equation (6).

$$\begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1 + \frac{1}{C_1 s} & -\frac{1}{C_1 s} \\ 0 & -\frac{1}{C_1 s} & \frac{1}{C_1 s} + ms \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{6}$$

In the LaPlace Domain, unique equations of the circuit can be written using nodal analysis in Equation (7).

$$V(s) = \frac{F(s)}{R_1 + \frac{1}{C_1 s} + m_1 s} \tag{7}$$

Equation (7) can be rewritten in matrix form $V = YF$, where Y is the admittance matrix in Equation (8).

$$\begin{bmatrix} V \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1 + \frac{1}{C_1 s} + m_1 s} \end{bmatrix} \begin{bmatrix} F \end{bmatrix} \tag{8}$$

(c) Mechanical Domain to Electrical Domain II
Here is the mechanical diagram and its electrical equivalent circuit on Figure 5 and 6 respectively.

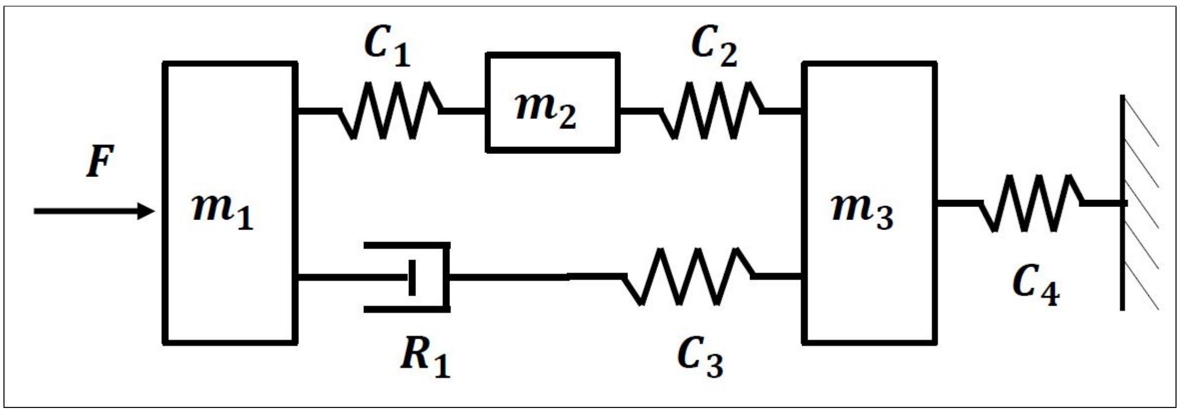

Figure 5: Mechanical Diagram of 1c

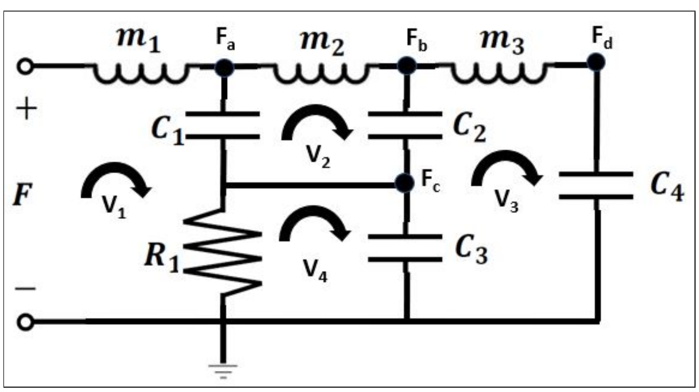

Figure 6: Electrical Equivalent Circuit of 1c

In the LaPlace Domain, unique equations of the circuit can be written using mesh analysis in Equation (9). Look at Figure 5 for the labeled velocities currents: v_1, v_2, v_3, v_4.

$$\begin{aligned}
F(s) &= m_1 s V_1(s) + \frac{1}{C_1 s}(V_1(s) - V_2(s)) + R_1(V_1(s) - V_3(s)) \\
0 &= -\frac{1}{C_1 s}(V_1(s) - V_2(s)) + m_2 s V_2(s) + \frac{1}{C_2 s}(V_2(s) - V_3(s)) \\
0 &= -\frac{1}{C_2 s}(V_2(s) - V_3(s)) + (m_3 s + \frac{1}{C_4 s})V_3(s) + \frac{1}{C_3 s}(V_3(s) - V_4(s)) \\
0 &= -R_1(V_1(s) - V_3(s)) - \frac{1}{C_3 s}(V_3(s) - V_4(s))
\end{aligned} \tag{9}$$

Equation (9) can be rewritten in matrix form $F = ZV$ where Z is the impedance matrix in Equation (10).

$$\begin{bmatrix} F \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m_1 s + \frac{1}{C_1 s} + R_1 & -\frac{1}{C_1 s} & 0 & -R_1 \\ -\frac{1}{C_1 s} & (\frac{1}{C_1} + \frac{1}{C_2})\frac{1}{s} + m_2 s & -\frac{1}{C_2 s} & 0 \\ 0 & -\frac{1}{C_2 s} & (\frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4})\frac{1}{s} + m_3 s & -\frac{1}{C_3 s} \\ -R_1 & 0 & -\frac{1}{C_3 s} & \frac{1}{C_3 s} + R_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \tag{10}$$

In the LaPlace Domain, unique equations of the circuit can be written using nodal analysis in Equation (11). Look at Figure 2 for the labeled force nodes: F_a, F_b, F_c.

$$\begin{aligned}
\frac{F - F_a}{m_1 s} &= (F_a - F_c)C_1 s + \frac{F_a - F_b}{m_2 s} \\
\frac{F_a - F_b}{m_2 s} &= (F_b - F_c)C_2 s + (F_b - F_d)\frac{1}{m_3 s} \\
(F_a - F_c)C_1 s + (F_b - F_c)C_2 s &= (F_c - 0)(\frac{1}{R_1} + C_3 s) \\
(F_b - F_d)\frac{1}{m_3 s} &= (F_d - 0)C_4 s
\end{aligned} \tag{11}$$

Equation (11) can be rewritten in matrix form $V = YF$, where Y is the admittance matrix in Equation (12).

$$\begin{bmatrix} \frac{F}{m_1 s} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (\frac{1}{m_1} + \frac{1}{m_2})\frac{1}{s} + C_1 s + & -\frac{1}{m_2 s} & -C_1 s & 0 \\ -\frac{1}{m_2} & (\frac{1}{m_2} + \frac{1}{m_3 s})\frac{1}{s} + C_2 s & -C_2 s & -\frac{1}{m_3 s} \\ -C_1 s & -C_2 s & (C_1 + C_2 + C_3)s + \frac{1}{R_1} & 0 \\ 0 & -\frac{1}{m_3 s} & 0 & \frac{1}{m_3 s} + C_4 s \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \\ F_d \end{bmatrix} \tag{12}$$

Discussion: In solving the systems to find the velocities of the masses or the forces acting on the masses, finding and solving an equivalent electrical circuit of the system provides a clearer and more visual approach to the problem. This would save time and confusion in setting up Newton's Second Law equations. With an equivalent electrical circuit, one can use mesh analysis to calculated the masses' velocity or use loop analysis to find the forces acting on a mass or at a junction.

Question 2: Transformers + LaGrange Method

Problem Statement: Construct the mechanical system as an equivalent electrical circuit with a transformers and then as one single circuit. Use loop analysis to determine the impedance matrix **Z**. Use LaGrange method to verify result.
Here is the mechanical diagram, its electrical equivalent circuit with transformers, and its electrical equivalent circuit as a single circuit on Figure 7, 8, and 9 respectively.

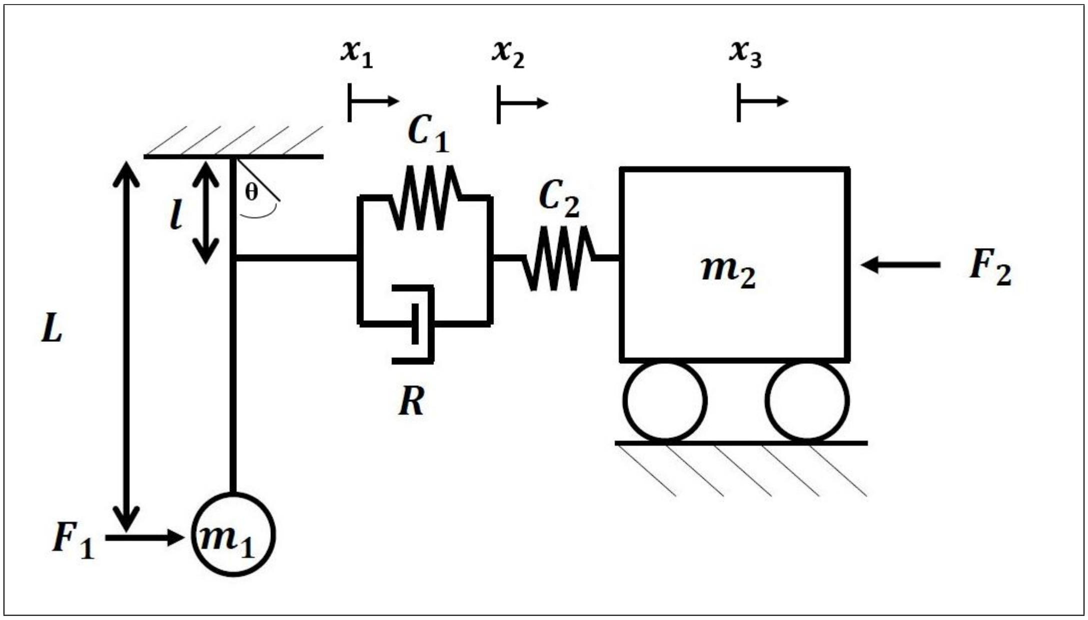

Figure 7: Mechanical Diagram of 2

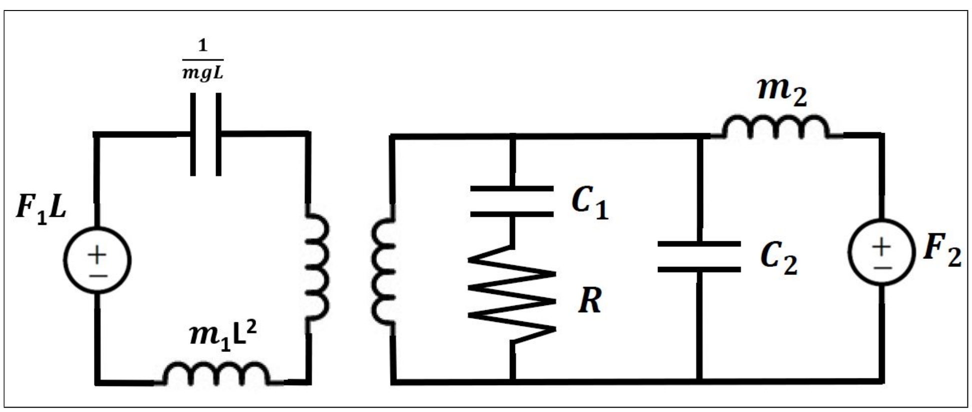

Figure 8: Electrical Equivalent Circuit of 2 with Transformers

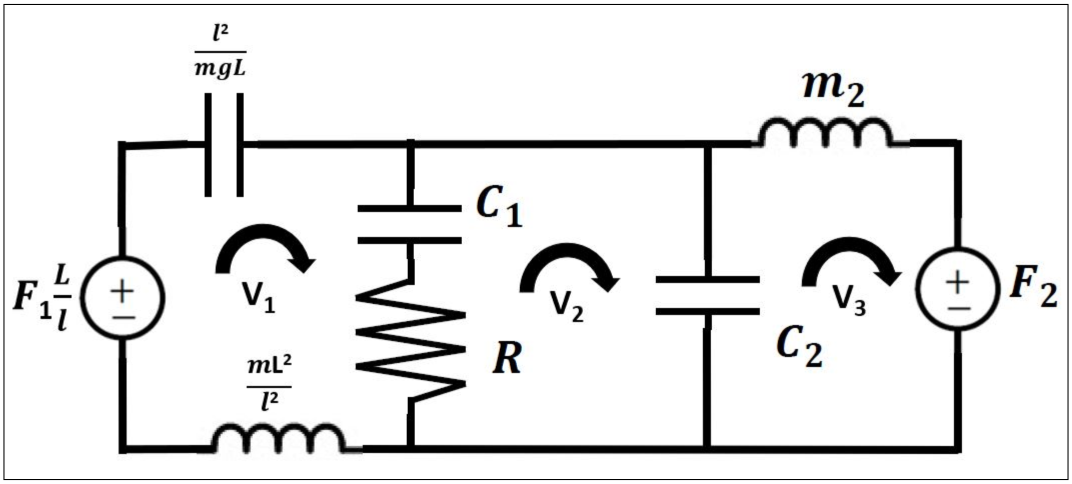

Figure 9: Electrical Equivalent Circuit of 2 as one circuit

In the LaPlace Domain, unique equations of the circuit can be written using mesh analysis in Equation (13). Look at Figure 9 for the labeled velocities currents: v_1, v_2, v_3.

$$\begin{aligned} F_1\frac{L}{l} &= (\frac{mgL}{l^2 s} + m_1\frac{L^2}{l^2}s)V_1\frac{l}{L} + (\frac{1}{C_1 s} + R)(V_1\frac{l}{L} - V_2) \\ 0 &= -(\frac{1}{C_1 s} + R)(V_1\frac{l}{L} - V_2) + \frac{1}{C_2 s}(V_2 - V_3) \\ -F_2 &= -\frac{1}{C_2 s}(V_2 - V_3) + m_2 s V_3 \end{aligned} \tag{13}$$

Equation (13) can be rewritten in matrix form $F = ZV$ where Z is the impedance matrix in Equation (14).

$$\begin{bmatrix} F_1\frac{L}{l} \\ 0 \\ -F_2 \end{bmatrix} = \begin{bmatrix} m_1\frac{L^2}{l^2}s + R + (\frac{mgL}{l^2}s + \frac{1}{C_1})\frac{1}{s} & -(R_1 + \frac{1}{C_1 s}) & 0 \\ -(R_1 + \frac{1}{C_1 s}) & R_1 + (\frac{1}{C_1} + \frac{1}{C_2})\frac{1}{s} & -\frac{1}{C_2 s} \\ 0 & -\frac{1}{C_2 s} & m_2 s + \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{14}$$

The LaGrange Method is shown as the following,

$$F_i = \frac{d}{dt}(\frac{\partial KE}{\partial \dot{x}_i}) + \frac{\partial PE}{\partial x_i} + \frac{\partial DP}{\partial \dot{x}_i} \tag{15}$$

where F_i is the input force of the system, KE is the kinetic energy of the system, PE is the potential energy of the system, DP is the dissipated power of the system. x_i is the coordinates of the system. The coordinates in the system are θ, x_2, and x_3. Look at Figure 7 for the coordinates. Note that $\theta \approx \frac{x_1}{l}$ for small angle.

The kinetic energy, potential energy, dissipate power term are shown in the Equation

(16).

$$\begin{aligned} KE &= \frac{1}{2}m_1 L^2\dot{\theta}^2 + \frac{1}{2}m_2\dot{x_3}^2 \\ PE &= mgL(1-cos(\theta)) + \frac{1}{2}\frac{1}{C_1}(x_2 - lsin(\theta))^2 + \frac{1}{2}\frac{1}{C_2}(x_3 - x_2)^2 \\ DP &= \frac{1}{2}R(\dot{x_2} - lcos(\theta)\dot{\theta})^2 \end{aligned} \tag{16}$$

Next is to substitute these terms into the LaGrange Method. For $x_i = \theta$.

$$\begin{aligned} F_1 L &= \frac{d}{dt}(\frac{\partial KE}{\partial \dot{\theta}}) + \frac{\partial PE}{\partial \theta} + \frac{\partial DP}{\partial \dot{\theta}} \\ &= m_1 L^2\ddot{\theta} + mgLsin(\theta) + \frac{1}{C_1}(x_2 - lsin(\theta))(-lcos(\theta)) + R(\dot{x_2} - lcos(\theta)\dot{\theta})(-lcos(\theta)) \\ &\approx m_1 L^2\ddot{\theta} + mgL\theta + \frac{1}{C_1}(x_2 - l\theta)(-l) + R(\dot{x_2} - l\dot{\theta})(-l) \end{aligned} \tag{17}$$

Small angle approximation was used after taking the derivatives. Now Equation (17) can be rewritten in the LaPlace Domain in terms of velocity $V_2(s)$ and angular velocity $\omega(s)$ in Equation (18).

$$F_1 L = m_1 L^2 s\omega + \frac{mgL}{s}\omega - \frac{l}{C_1 s}(V_2 - l\omega) - Rl(V_2 - l\omega) \tag{18}$$

Note that the ω term Equation (18) can be written into $V_1(s)$ shown in Equation (19).

$$\begin{aligned} F_1 L &= m_1 L^2 s\frac{V_1}{l} + \frac{mgL}{s}\frac{V_1}{l} - \frac{l}{C_1 s}(V_2 - l\frac{V_1}{l}) - Rl(V_2 - l\frac{V_1}{l}) \\ F_1\frac{L}{l} &= m_1 s\frac{L^2}{l^2}V_1 + \frac{mgL}{l^2 s}V_1 - \frac{1}{C_1 s}(V_2 - V_1) - R(V_2 - V_1) \end{aligned} \tag{19}$$

Next is to repeat the steps for $x_i = x_2$.

$$\begin{aligned} 0 &= \frac{d}{dt}(\frac{\partial KE}{\partial \dot{x_2}}) + \frac{\partial PE}{\partial x_2} + \frac{\partial DP}{\partial \dot{x_2}} \\ &= \frac{1}{C_1}(x_2 - lsin(\theta)) + \frac{1}{C_2}(x_3 - x_2)(-1) + R(\dot{x_2} - lcos(\theta)\dot{\theta}) \\ &\approx \frac{1}{C_1}(x_2 - l\theta) + \frac{1}{C_2}(x_3 - x_2)(-1) + R(\dot{x_2} - l\dot{\theta}) \\ 0 &= \frac{1}{C_1 s}(V_2 - V_1) + \frac{1}{C_2 s}(V_3 - V_2)(-1) + Rs(V_2 - V_1) \end{aligned} \tag{20}$$

Next is to repeat the steps for $x_i = x_3$.

$$\begin{aligned} -F_2 &= \frac{d}{dt}(\frac{\partial KE}{\partial \dot{x_3}}) + \frac{\partial PE}{\partial x_3} + \frac{\partial DP}{\partial \dot{x_3}} \\ &= m_2 \ddot{x_3} + \frac{1}{C_2}(x_3 - x_2) \\ -F_2 &= m_2 s V_3 + \frac{1}{C_2 s}(V_3 - V_2) \end{aligned} \tag{21}$$

Equation (19), (20), (21) can be recombined in matrix form $F = ZV$ where Z is the impedance matrix in Equation (22).

$$\begin{bmatrix} F_1 \frac{L}{l} \\ 0 \\ -F_2 \end{bmatrix} = \begin{bmatrix} m_1 \frac{L^2}{l^2} s + R + (\frac{mgL}{l^2} s + \frac{1}{C_1}) \frac{1}{s} & -(R_1 + \frac{1}{C_1 s}) & 0 \\ -(R_1 + \frac{1}{C_1 s}) & R_1 + (\frac{1}{C_1} + \frac{1}{C_2}) \frac{1}{s} & -\frac{1}{C_2 s} \\ 0 & -\frac{1}{C_2 s} & m_2 s + \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{22}$$

Discussion: The loop analysis on a single equivalent circuit and LaGrange Method both lead to the same answer to this problem. However, the LaGrange Method was more tedious to preform with all the derivative involved for each x_i than to preform the loop analysis on a single equivalent circuit. For having multiple coordinate systems and multiple domains, doing the loop analysis on a single equivalent circuit is the recommended method to preform for swiftness and less confusion.

Question 3: Analysis of Mechanical System

Here is the mechanical diagram and its equivalent circuit on Figure 10.

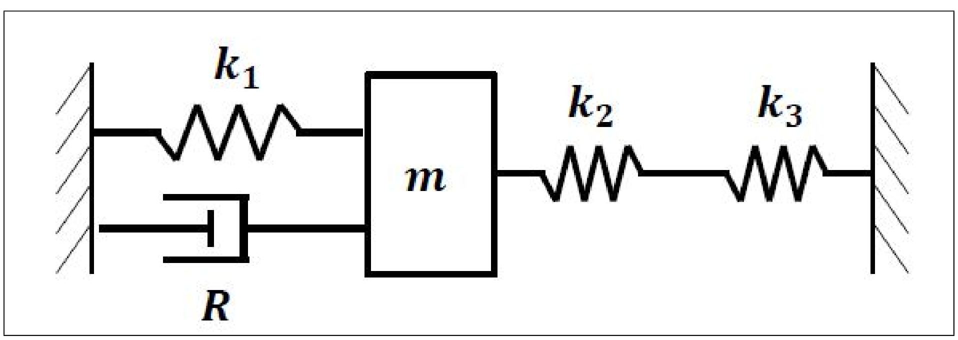

Figure 10: Mechanical Diagram of 3

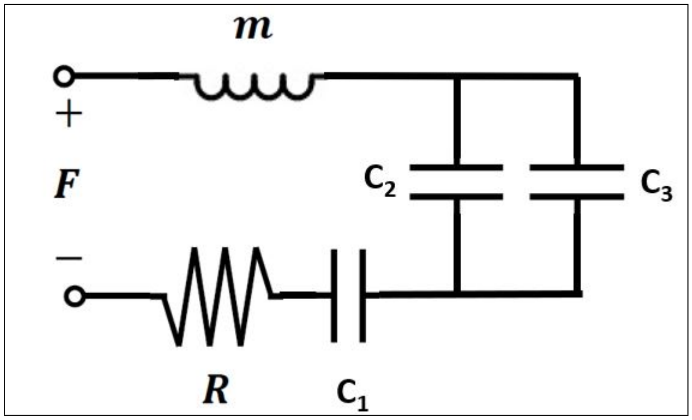

Figure 11: Electrical Equivalent Circuit of 3

(a) The Natural Frequency of the System
Problem Statement: Find the natural frequency of the system.
First step is to find the impedance transfer function of the circuit. The equivalence capacitance for two capacitance in parallel is given in (23) .

$$C_{//} = C_2 + C_3 \tag{23}$$

where $C_i = \frac{1}{k_i}$ for $i = 1, 2, 3$. Then all the components are in series, so the impedance transfer function Y can be found in (24).

$$\begin{aligned} Z &= \frac{1}{C_{//}s} + \frac{1}{C_1 s} + R + ms \\ &= \frac{1}{(C_2 + C_3)s} + \frac{1}{C_1 s} + R + ms \end{aligned} \tag{24}$$

where Z is the impedance transfer function. To find the natural frequency ω_n, set the imaginary term of Equation (24) to 0 and solve the quadratic equation shown in (25). Note that $s = j\omega$.

$$\begin{aligned} 0 &= Im\{Z\} \\ &= Im\{\frac{1}{(C_2 + C_3)j\omega} + \frac{1}{C_1 j\omega} + R + mj\omega\} \\ &= -\frac{1}{\omega C_1} - \frac{1}{\omega(C_2 + C_3)} + m\omega \\ &= -\frac{1}{C_1} - \frac{1}{(C_3 + C_3)} + m\omega^2 \\ &= -k_1 - \frac{k_2 k_3}{(k_3 + k_3)} + m\omega^2 \\ \omega &= \sqrt{\frac{1}{m}(k_1 + \frac{k_2 k_3}{k_2 + k_3})} \end{aligned} \tag{25}$$

With $m = 0.65kg$, $R = 4.35\frac{kg}{s}$, $K_1 = 1.2\frac{N}{m}$, $K_2 = 4\frac{N}{m}$, $K_3 = 0.33\frac{N}{m}$, the natural frequency ω_n is $1.52\frac{rad}{s}$

(b) The Damping Ratio of the System
Problem Statement: Find the damping ratio of the system.
The damping ratio ζ of the system can be calculated in Equation (26).

$$\zeta = \frac{R}{2m\omega_n} \tag{26}$$

The damping ratio ζ is 2.20.

(c) Overdamped, Underdamped, or Critically Damped
Problem Statement: Is the system underdamped, underdamped, or critically damped?
The value of ζ deteremine if the system is overdamped, underdamped, or critically damped. Since ζ is greater than 1, then the system is overdamped.

Discussion: Since the system is overdamped, the system won't reach 0 displacement the fastest. To improve the system settling time, the damping coefficient ζ must decreases to 1. Thus decreasing the damping coefficient R or increasing the natural frequency ω_n or the mass m will decrease ζ. A way to increase the ω_n of the system without changing any parameter value is to put the springs with spring constant k_2 and k_1 in parallel with wall to increase the natural frequency.

Question 4: Skyscraper Damping

(a) Equivalent Electrical Circuit between Mechanical and Rotational Domain
Problem Statement: Draw out the equivalent electrical circuit of the mechanical and rotational system. Here is the mechanical/rotational diagram and its electrical equivalent circuit on Figure 12 and 13 respectively.

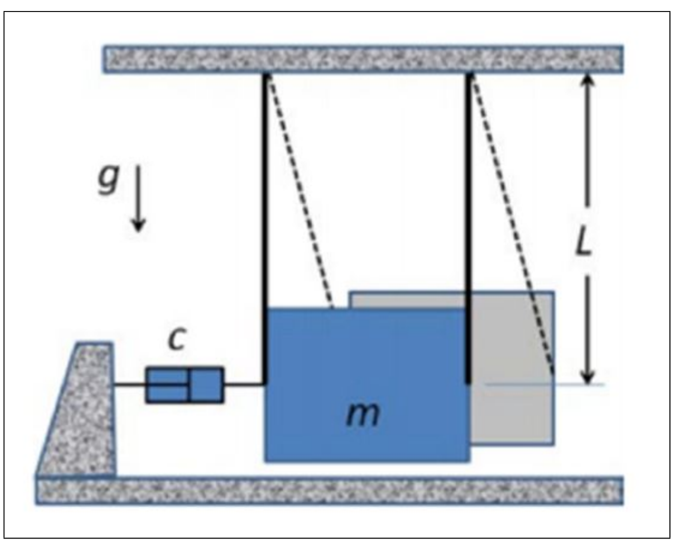

Figure 12: Mechanical Diagram of 4

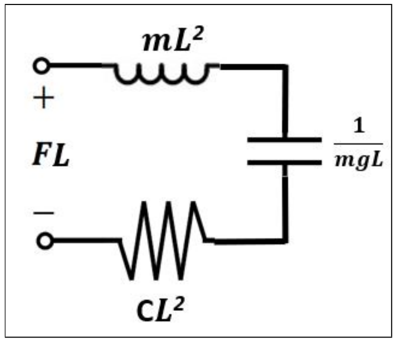

Figure 13: Electrical Equivalent Circuit of 4

(b) Admittance Bode Plot
Problem Statement: Plot the admittance Bode plot.
Given that $L = 12.6m$, $m = 6.6 \times 10^5 kg$, and $C = 8.0 \times 10^5 \frac{kg}{s}$, here is the plot for the admittance Bode plot in Figure 14 . Here is the derivation of the admittance transfer function $Y(s)$ in Equation (27).

$$\begin{aligned} Z &= mL^2s^2 + \frac{mgL}{s} + CL^2 \\ Y &= \frac{1}{Z} \\ Y &= \frac{1}{mL^2s + \frac{mgL}{s} + CL^2} \end{aligned} \tag{27}$$

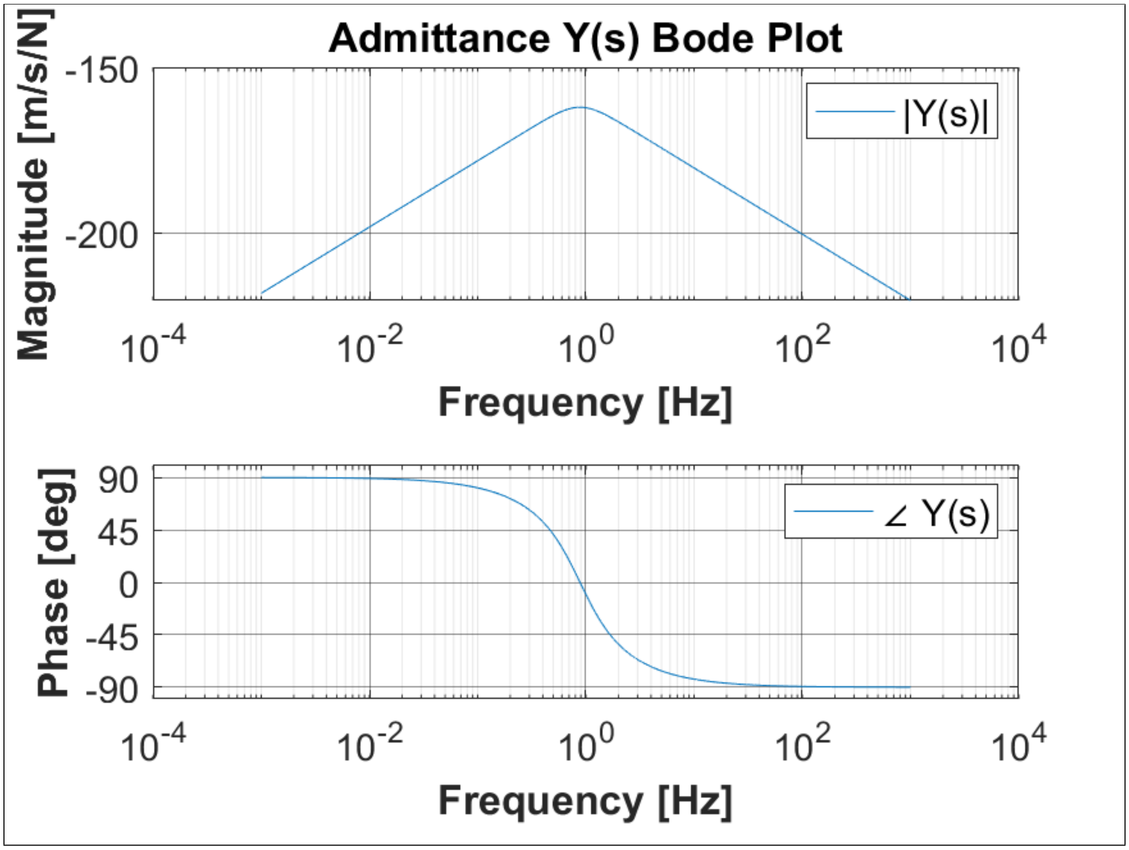

Figure 14: Bode Plot of Y(s). $\omega_n = 5.542\frac{rad}{s}$

(c) Modifying Design Parameters for a Lower Natural Frequency
Problem Statement: Modify any design parameters of the system beside the length of the pendulum to decrease the damped frequency.
In Equation (28), it shows the equation for the damped frequency of the system.

$$\begin{aligned}\omega_d &= \sqrt{\omega_n^2 - \beta^2} \\ &= \sqrt{\frac{g}{L} - \frac{C}{2m}}\end{aligned} \tag{28}$$

Note that the natural frequency of the system is the natural frequency of the pendulum. With $L = 12.6m$, $m = 6.6 \times 10^5 kg$, and $C = 8.0 \times 10^5 \frac{kg}{s}$, $\omega_d = 0.641\frac{rad}{s}$. Then to decrease the damped frequency to $0.620\frac{rad}{s}$, one way to increases the damping coefficient C. Equation (29) shows the equation for C.

$$C = 2m\sqrt{\frac{g}{L} - \omega_d^2} \tag{29}$$

With $\omega_d = 0.620 rad/s$ and the given parameters, $C = 8.329 \times 10^5 \frac{kg}{s}$

Discussion: In order to lower the damped frequency to the desired frequency, increasing the damping coefficient C was a solution. One solution was to decrease the gravitational acceleration imposed on the system by either placing the pendulum above or below ground. Another solution was to decrease the mass. However, both solutions are impractical compared to changing C. In order to have a damped frequency of $0.620\frac{rad}{s}$, $C = 8.329 \times 10^5 \frac{kg}{s}$

Appendix

Below is the MATLAB code used for making the plots.

```
clc;
clear all;
close all;

%% Question 3: Analysis of Mechanical System

%Parameter
R = 4.35; % in [kg/s]
C1 = 1/1.2; % in [m/N]
C2 = 1/4; % in [m/N]
C3 = 1/0.33; % in [m/N]
m = 0.65; % in [kg]

%Frequency Data Points
omega = 0:0.01:1000; % Hz
ss = 1j*omega;

%Solving for Admittance T.F Y(s)
Ceq = C2+C3;
Z = 1./(Ceq*ss) + 1./(C1*ss) + R + m*ss;
Y = 1./Z;

% Y(s) Plot
figure(1)
semilogx(omega,abs(Y));
xlabel('Angular Frequency (rad/s)');
ylabel('Admittance Magnitude(m/(s*N))');

% Finding the natural frequency and damping ratio
[~,wn] = max(real(Y));
[~,w1] = max(imag(Y));
[~,w2] = min(imag(Y));

omega_n = omega(wn);
omega_1 = omega(w1);
omega_2 = omega(w2);

Q = omega_n/(omega_2-omega_1);
zeta = 1/2/Q;

fprintf('wn = %f rad/s\n',omega_n);
fprintf('zeta = %f rad/s\n',zeta);

```

```
%% Question 4: Skyscraper Damping

%Parameter
L = 12.6; % Length of the pendulum in [m]
m = 6.6e5; % Mass in [kg]
C = 8.0e5; % Damping coefficient in [kg/s]
g = 9.81; % gravity in [N/kg]
w_des = 0.620;

%Frequency Data Points
omega = (0:0.001:1000);
ss = 1j*omega;

%Solving for Admittance T.F Y(s)
Z = m*L^2*ss+m*g*L./ss+C*L^2;
Y = 1./Z;

mag = 20*log10(abs(Y));
phase = angle(Y)*180/pi;

figure();
ax1 = subplot(2,1,1);
semilogx(omega,mag);
grid on;
xlabel('Frequency [Hz]', 'Fontsize',16, 'Fontweight', 'bold');
ylabel('Magnitude [m/s/N]', 'Fontsize', 16, 'Fontweight', 'bold');
title('Admittance Y(s) Bode Plot', 'Fontsize', 16, 'Fontweight', 'bold');
set(gca, 'Fontsize', 14, 'GridAlpha', 0.5, 'MinorGridAlpha', 0.1, '
    MinorGridLineStyle', '-');
legend('|Y(s)|', 'Fontsize', 14)

ax2 = subplot(2,1,2);
semilogx(omega,phase);
grid on;
xlabel('Frequency [Hz]', 'Fontsize', 16, 'Fontweight', 'bold');
ylabel('Phase [deg]', 'Fontsize', 16, 'Fontweight', 'bold');
set(gca, 'Fontsize', 14, 'GridAlpha', 0.5, 'MinorGridAlpha', 0.1, '
    MinorGridLineStyle', '-', 'YTick', (-2:2)*45);
legend('\angle Y(s)', 'Fontsize', 14)
saveas(gcf,'Q4Plot.png')

[~,omega_n] = max(mag);
wn = omega(omega_n); %Natural Frequency in [rad/s]

% Finding the natural frequency and damping ratio
[~,wn] = max(real(Y));
[~,w1] = max(imag(Y));
[~,w2] = min(imag(Y));

omega_n = omega(wn);
omega_1 = omega(w1);
omega_2 = omega(w2);

Q = omega_n/(omega_2-omega_1);
```

```
zeta = 1/2/Q;

fprintf('wd = %f rad/s\n',omega_n*sqrt(1-zeta^2));
```