

# Vibration Hw 4

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## Question 1: Mass-Spring Damper System with a Harmonic Excitation

Here is the drawing of the mass-spring system with damper and its circuit analogy system on Figure 1 and 2 respectively.

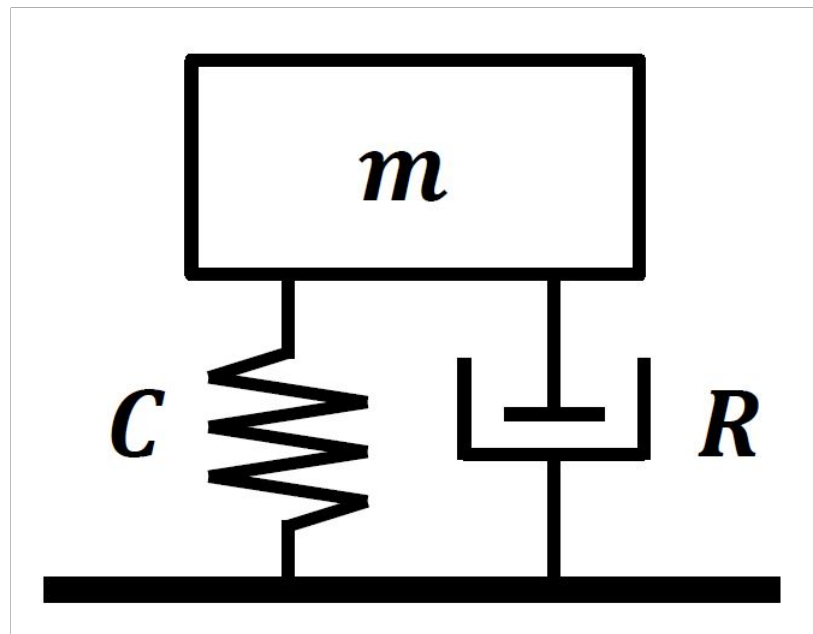


Figure 1: Mass-Spring System Drawing

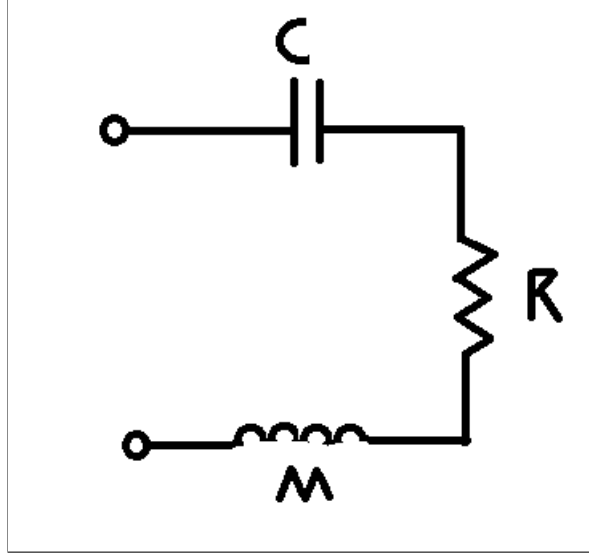


Figure 2: Mass-Spring System Equivalent Circuit Drawing

(a) Mechanical Admittance of the System and its Bode Plot

Problem Statement: Find the mechanical admittance of the system and plot the Bode of the mechanical admittance.

The equation for each individual component impedance in series is listed below.

$$\begin{aligned} Z_m &= ms \\ Z_C &= \frac{1}{C_s} \\ Z_R &= R \end{aligned} \tag{1}$$

Since the velocity or its electrical analogy known as the current is the same for each component, each component is in series, so the equivalent impedance is the sum of the individual impedance in the following equation.

$$\begin{aligned} Z_{eq} &= Z_m + Z_c + Z_R \\ Z_{eq} &= ms + \frac{1}{C_s} + R \end{aligned} \tag{2}$$

To find the mechanical admittance, it is the reciprocal of the equivalent impedance.

$$\begin{aligned} Y &= \frac{1}{Z_{eq}} \\ Y &= \frac{1}{ms + \frac{1}{C_s} + R} \end{aligned} \tag{3}$$

With values  $m = 0.5kg$ ,  $C = 1.2665 * 10^6 m/N$ , and  $R = 10$ , the following magnitude and phase plots are shown.

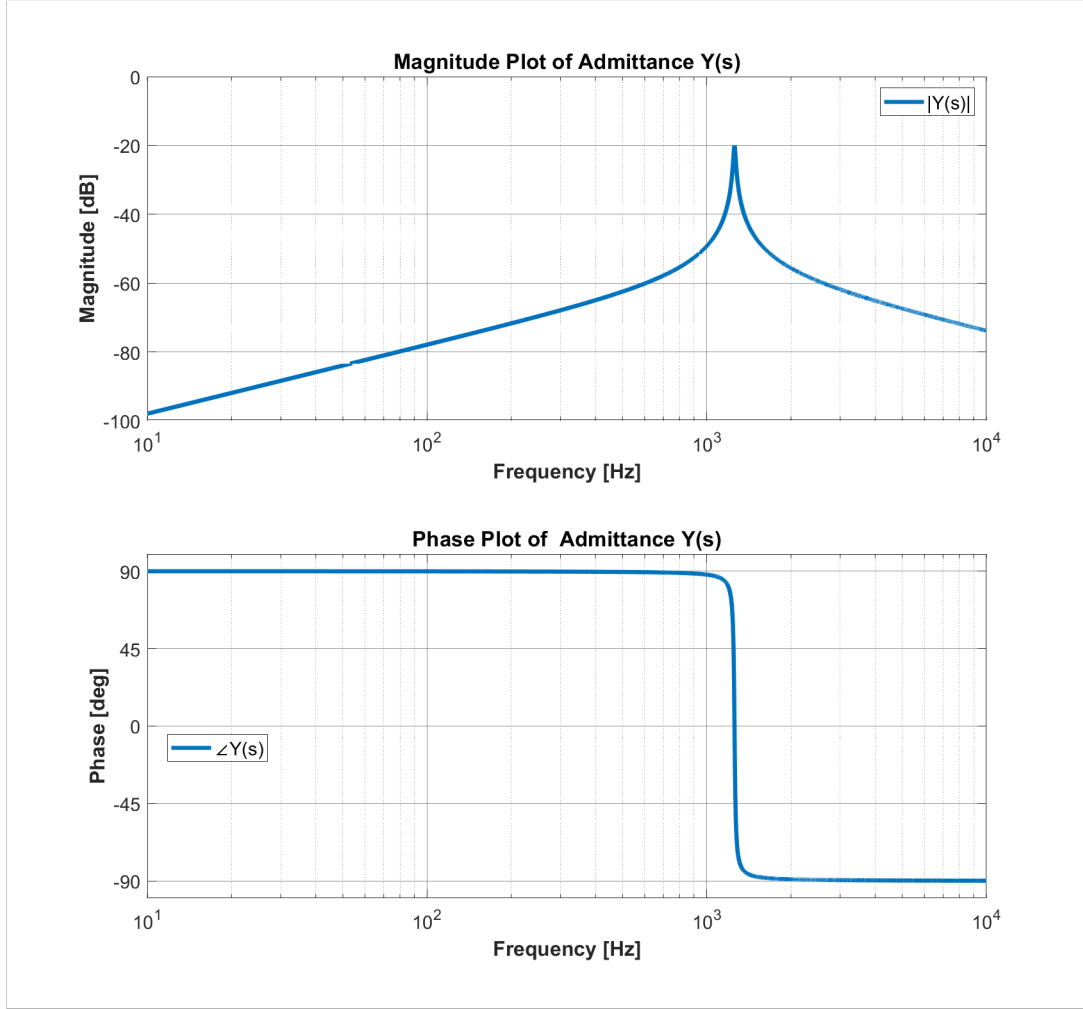


Figure 3: Y(s) Magnitude & Phase plot

(b) Varying Force

Problem Statement: Plot  $v(t)$  and  $x(t)$  of the mass starting from rest with a varying Force of  $F(t) = 2\sin(\omega_f t)$ .

The LaPlace of  $F(t) = 2\sin(\omega_f t)$  is  $F(s) = \frac{2\omega_f}{s^2 + \omega_f^2}$ .  $V(s) = Y(s)F(s)$ .  $X(s) = \frac{V(s)}{s}$ . Note that the initial conditions is 0, so there is no extra terms for  $V(s)$  and  $X(s)$ .

The following displacement and velocity plots of the system starting from rest are shown.

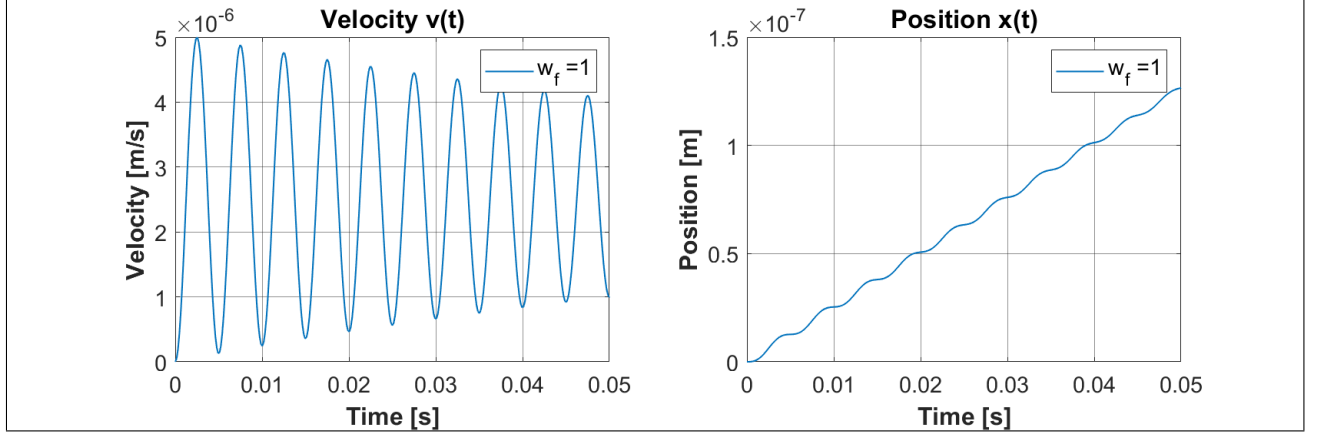


Figure 4: Mass-Spring Velocity & Position Plot for  $\omega_f = 1$

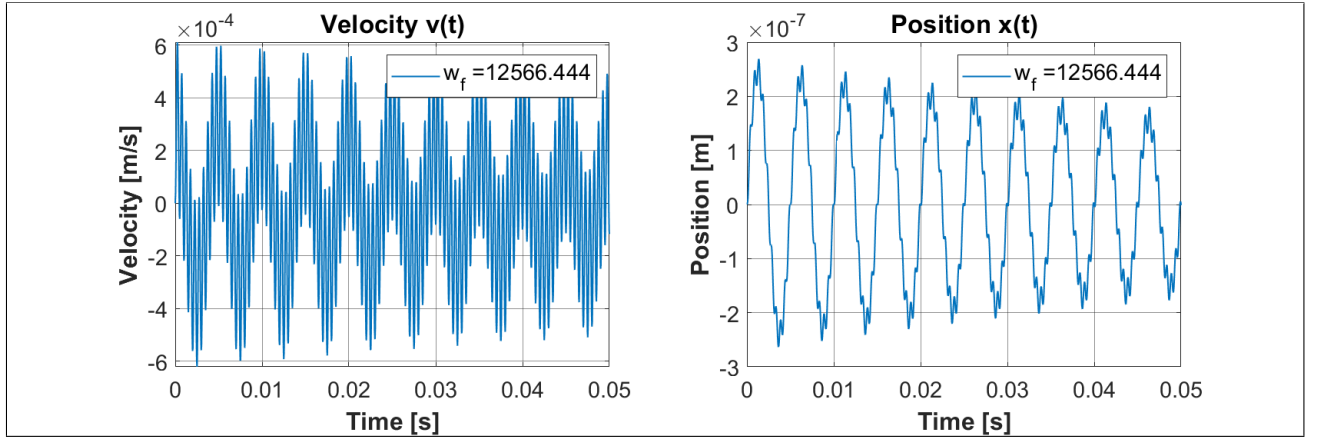


Figure 5: Mass-Spring Velocity & Position Plot for  $\omega_f = 10\omega_n$

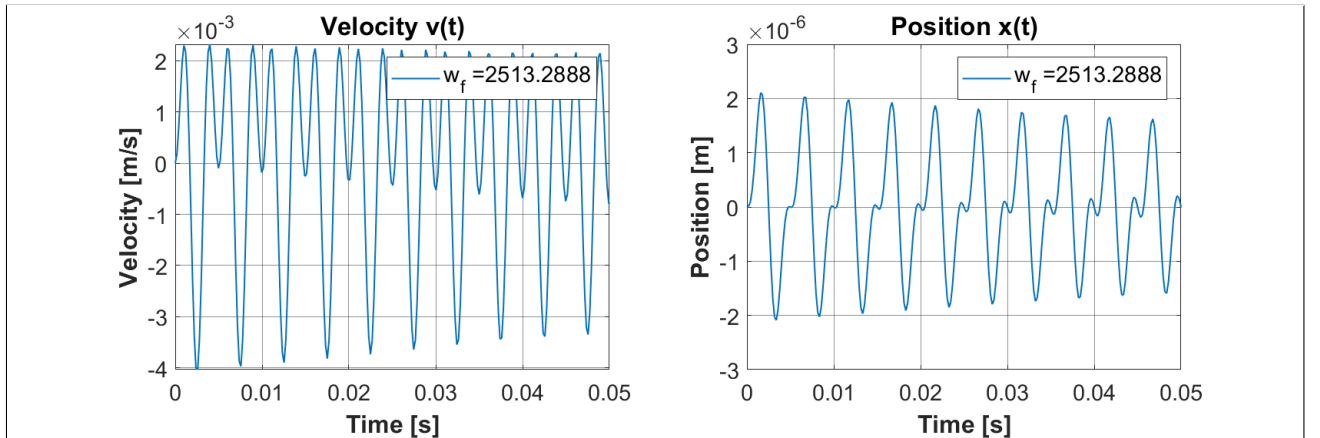


Figure 6: Mass-Spring Velocity & Position Plot for  $\omega_f = 2\omega_n$

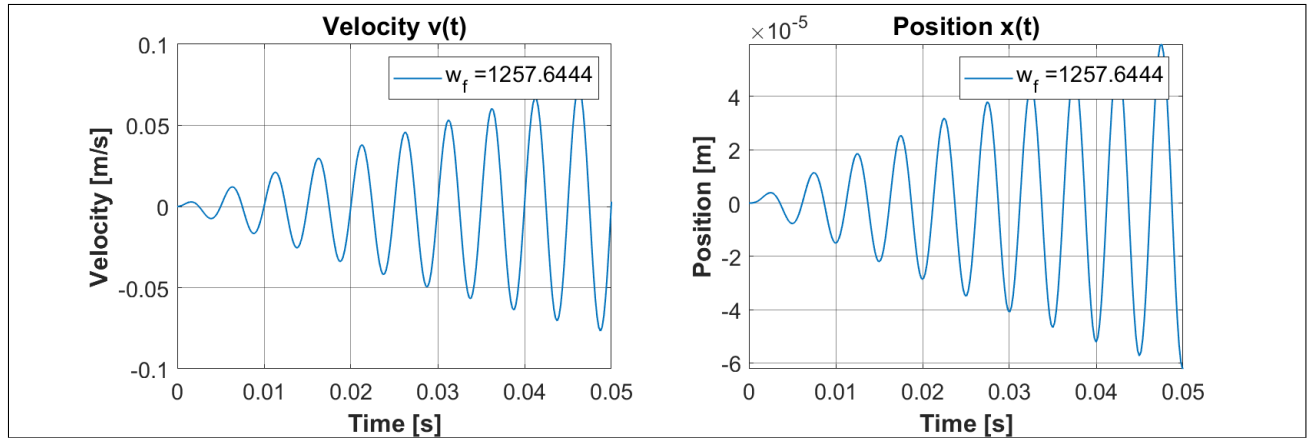


Figure 7: Mass-Spring Velocity & Position Plot for  $\omega_f = \omega_n + 1$

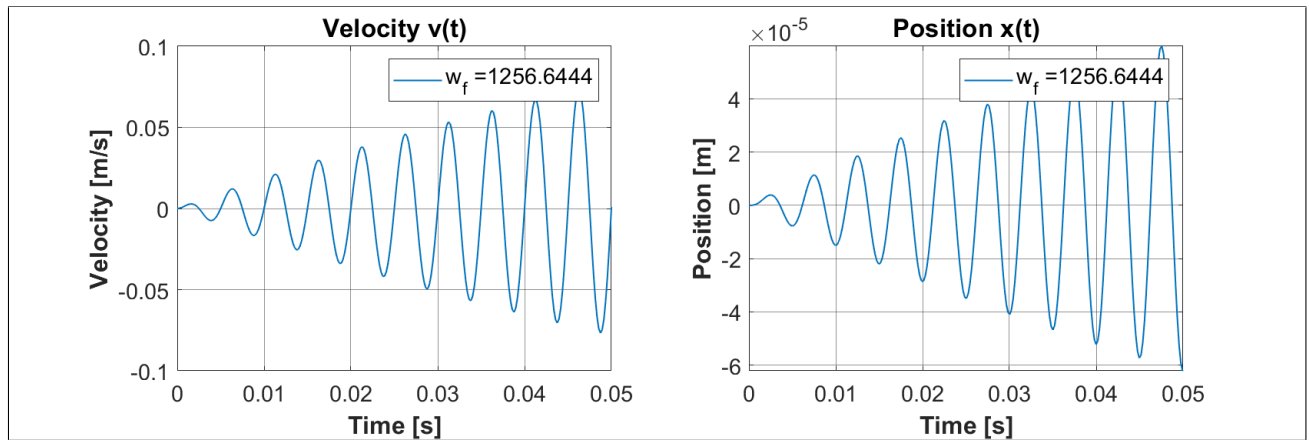


Figure 8: Mass-Spring Velocity & Position Plot for  $\omega_f = \omega_n$

(c) Changing R

Problem Statement: Plot  $Y(\omega)$ ,  $v(t)$ , and  $x(t)$  with varying R. Explain the effect of R on the plots.

From varying  $R$ , the following plots are shown. Note that on the legend of the following plot,  $R_i/R$  is the ratio of the varying resistance to the initial resistance ( $R = 10$ ) stated in the problem.

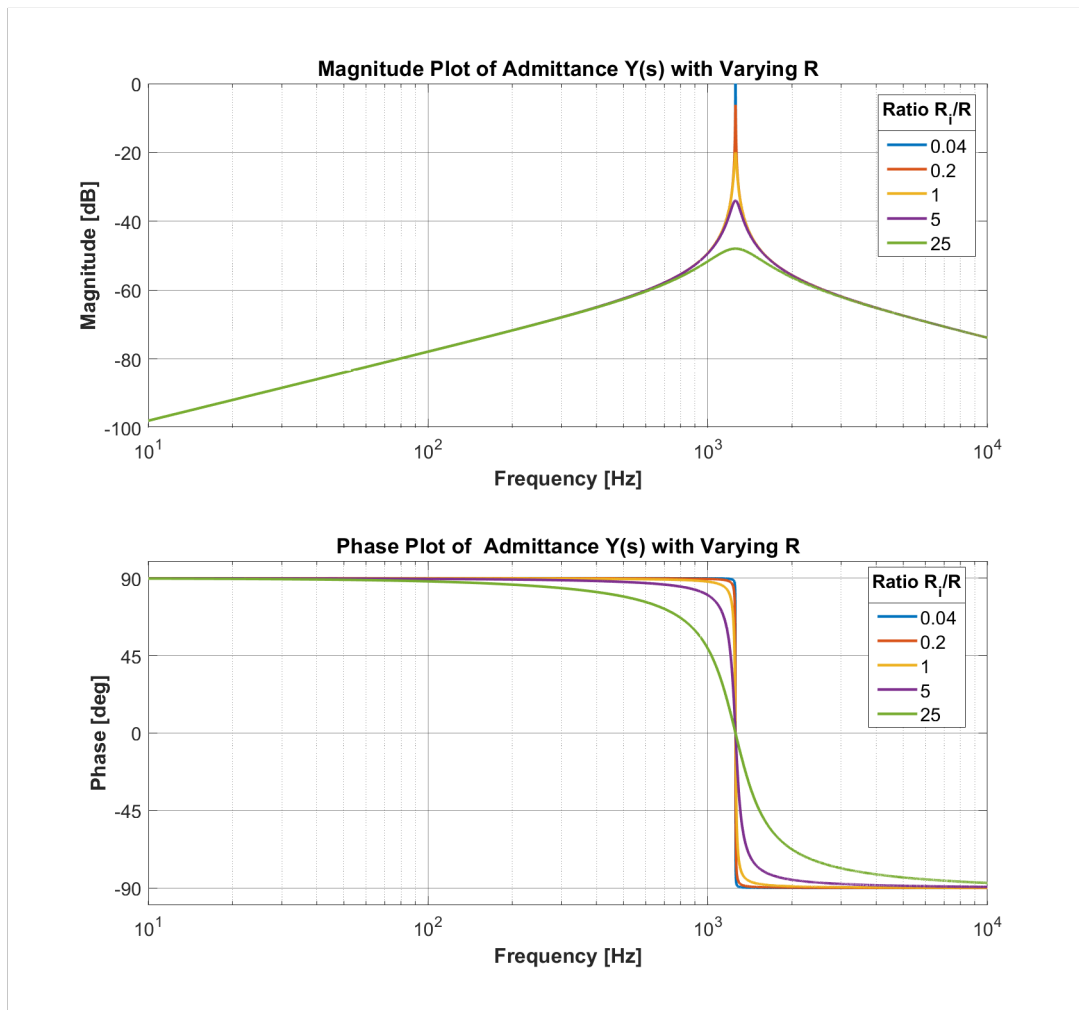


Figure 9:  $Y(s)$  with Varying  $R$

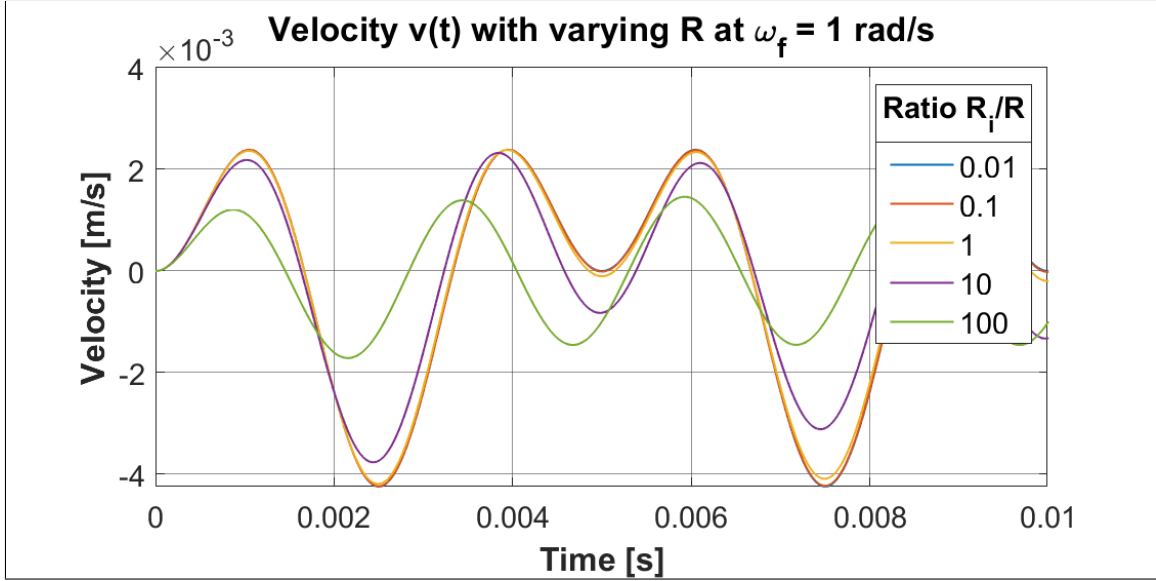


Figure 10: Velocity Plot with Varying  $R$  at  $\omega_f = 2\omega_n$

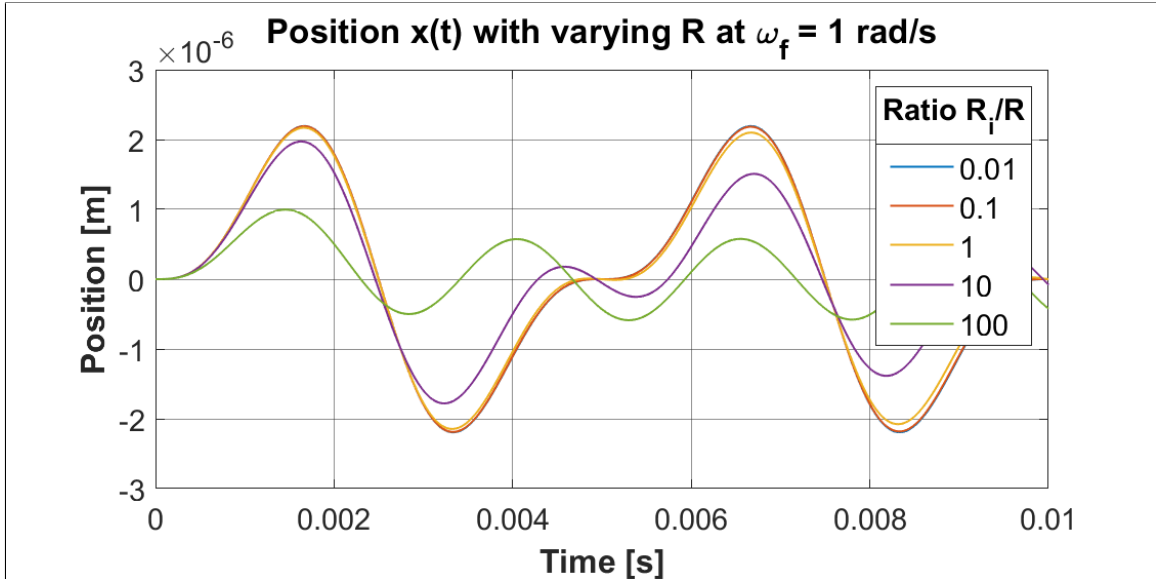


Figure 11: Displacement Plot with Varying  $R$   $\omega_f = 2\omega_n$

From figure 9, the magnitude of  $Y(s)$  deviates from among each other as  $R$  varies except when the frequency is close to the natural frequency. At the natural frequency, the magnitude decreases as  $R$  increases. Near the natural frequency, the phase's curve become more smooth from going  $90^\circ$  to  $-90^\circ$ . As  $R$  increases, the amplitude of the velocity and displacement plot decreases which make senses.

(d) Changing C

Problem Statement: Plot  $Y(\omega)$ ,  $v(t)$ , and  $x(t)$  with varying  $R$ . Explain the effect of  $R$

on the plots. Note that on the legend of the following plot,  $C_i/C$  is the ratio of the varying resistance to the initial resistance ( $C = 1.2664 \times 10^{-6}$ ) stated in the problem.

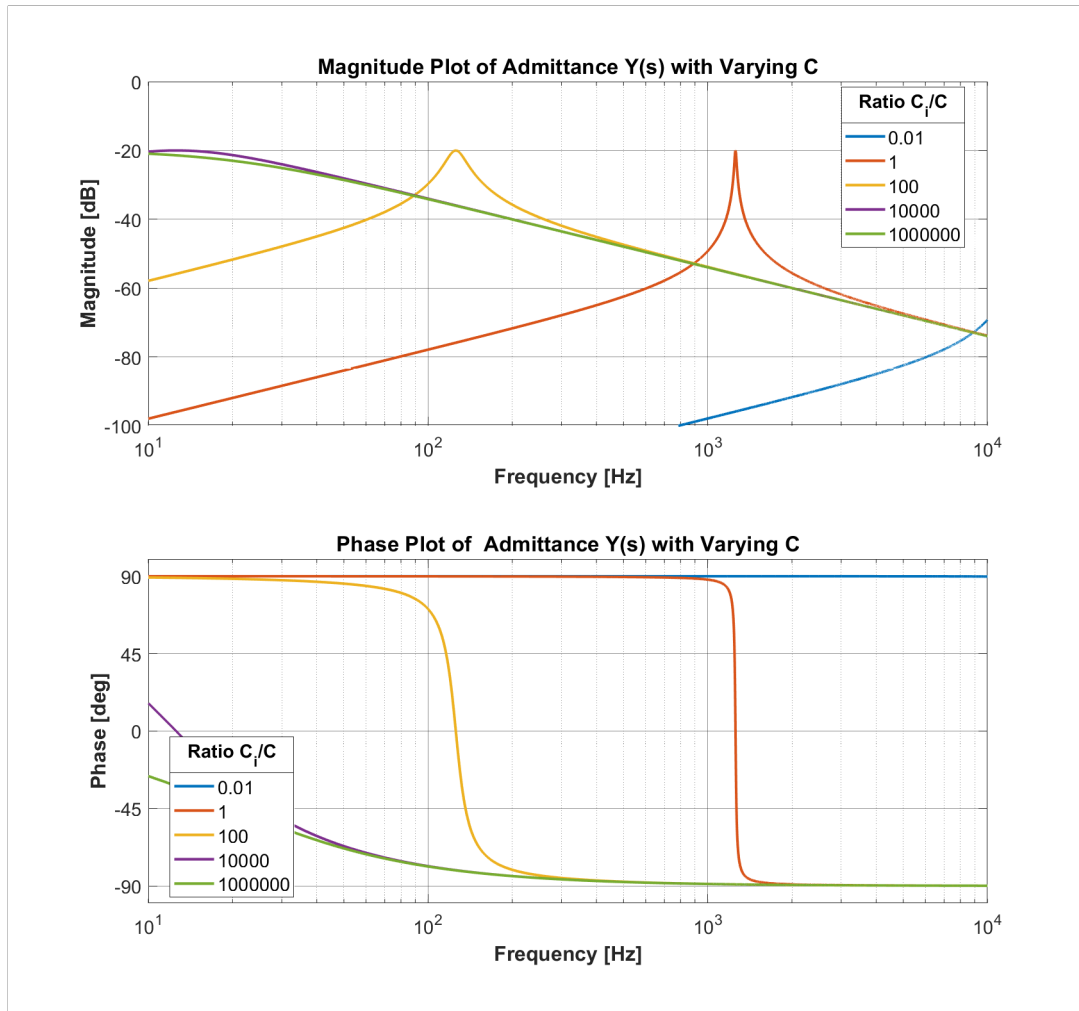


Figure 12: Y(s) with Varying C



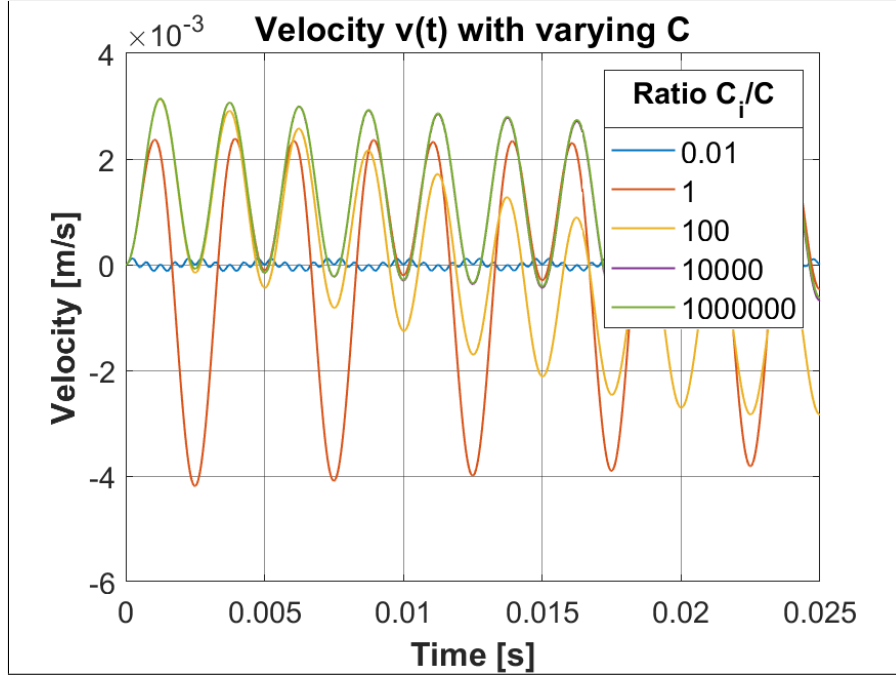


Figure 13: Velocity Plot with Varying  $C$  at  $\omega_f = 2\omega_n$

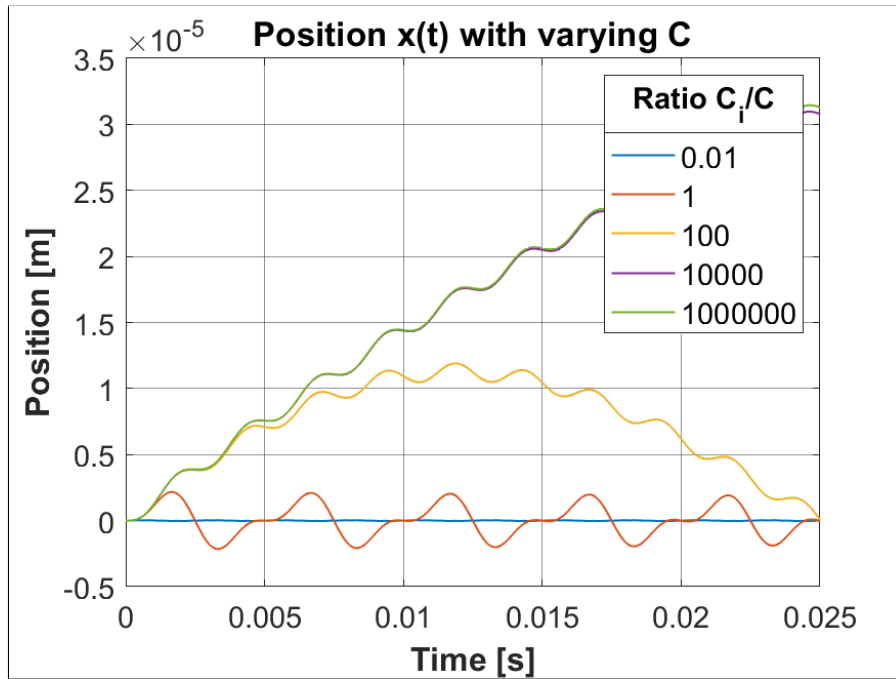


Figure 14: Displacement Plot with Varying  $C$   $\omega_f = 2\omega_n$

From Figure 12, the magnitude and the phase of  $Y(s)$  translate to the left as  $C$  increases. The magnitude and phase plots of  $Y(s)$  near the natural frequency are more

smooth, or there is a less of a peak as  $C$  increases. On Figure 13, the amplitude increases as  $C$  increases, and its frequencies are out of phase among each other.. On Figure 14, the group frequency oscillation increases as  $C$  increases.

Discussion:

The Bode plot shows the behavior of the admittance transfer function when an input force of a given frequency is applied to the system. When the driven frequency is close to the natural frequency, the amplitude of the output has been amplified. When the driven frequency is severely greater than or less than to the natural frequency, the amplitude of the output has been attenuated greatly. In addition, the effects of  $C$  and  $R$  have substantial effects on the behavior of the Bode plot in defining the shape, resonant frequency location, and magnitude. The position and velocity plots represent this behavior in the time domain. It is important to analyze what factors affect the system behavior and what appropriate frequency should the force input be in the system.

## Question 2: A Mass-Spring-Damper System on an Incline

Problem statement : Find out what effect does the angle  $\theta$  have on the magnitude of oscillation. Plot the oscillation.

Here is the drawing of a mass-spring-damper system on an incline with an angle  $\theta$ .

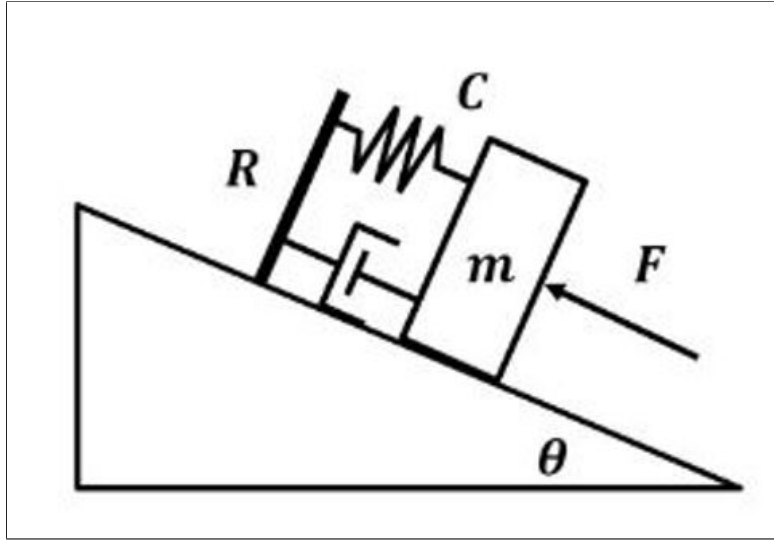


Figure 15: Drawing of the Mass-Spring-Damper System on an Incline with an Angle  $\theta$

Note that Figure 15 can be rotated by an angle  $\theta$  to get the mass-spring-damper system in the horizontal direction. Then the gravitational force is multiplied by  $\sin(\theta)$ , pointing in the opposite direction of  $F$ .

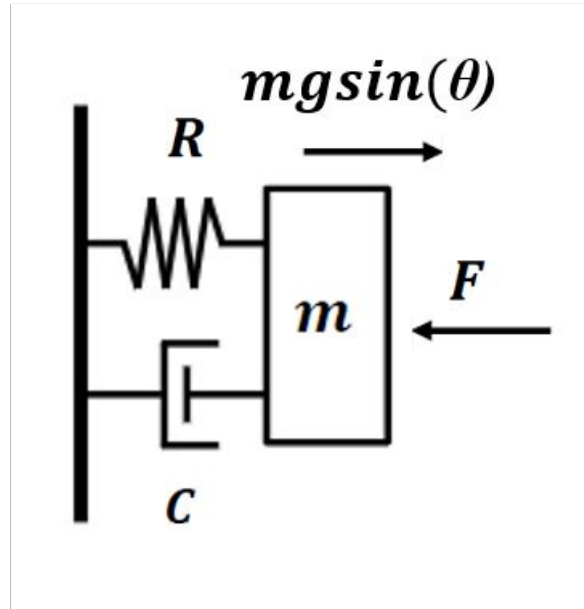


Figure 16: Drawing of an Equivalent & Rotated Figure 16

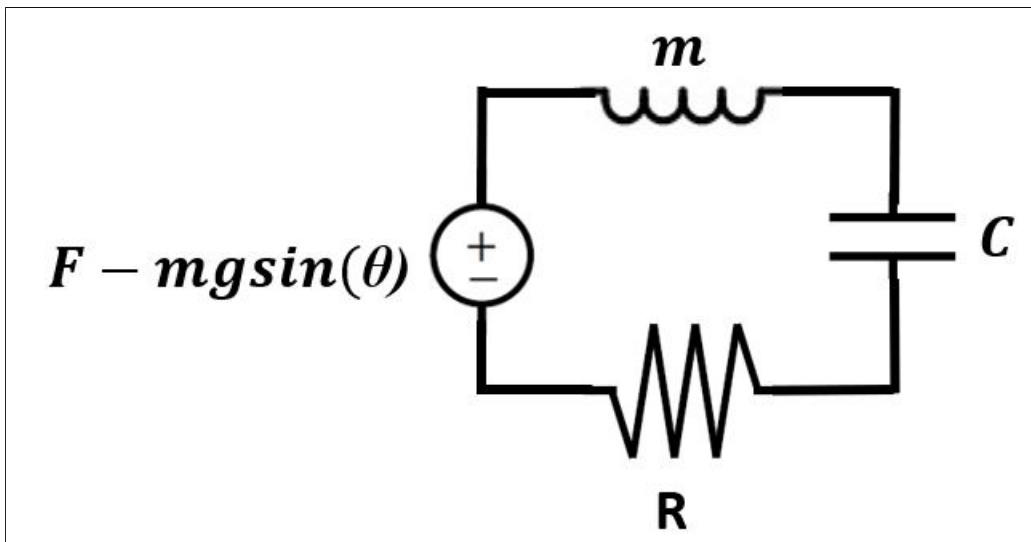


Figure 17: Equivalent Circuit Drawing of Figure 15

Next is to find the admittance transfer function  $Y(s)$  of the system in the following equation.

$$\begin{aligned}
 Z &= ms^2 + \frac{1}{Cs} + R \\
 Y &= \frac{1}{Z} \\
 Y &= \frac{1}{ms + R + \frac{1}{Cs}}
 \end{aligned}
 \tag{4}$$

The force input in the system is assumed to be  $F = 10\cos(10t)$ . The acting force of the system is shown in the following equation.

$$\begin{aligned}
 F_a &= F - mg\sin(\theta) \\
 &= 10\cos(10t) - mg\sin(\theta) \\
 \mathcal{L}\{F_a\} &= \mathcal{L}\{10\cos(10t) - mg\sin(\theta)\} \\
 F_a(s) &= \frac{s}{s^2 + 100} - \frac{mg\sin(\theta)}{s}
 \end{aligned} \tag{5}$$

The position of the system can be rewritten as  $X(s) = \frac{Y(s)F(s)}{s}$ . Note that the velocity is pointing up in the direction of the incline. Here is the plot with varying  $\theta$ .

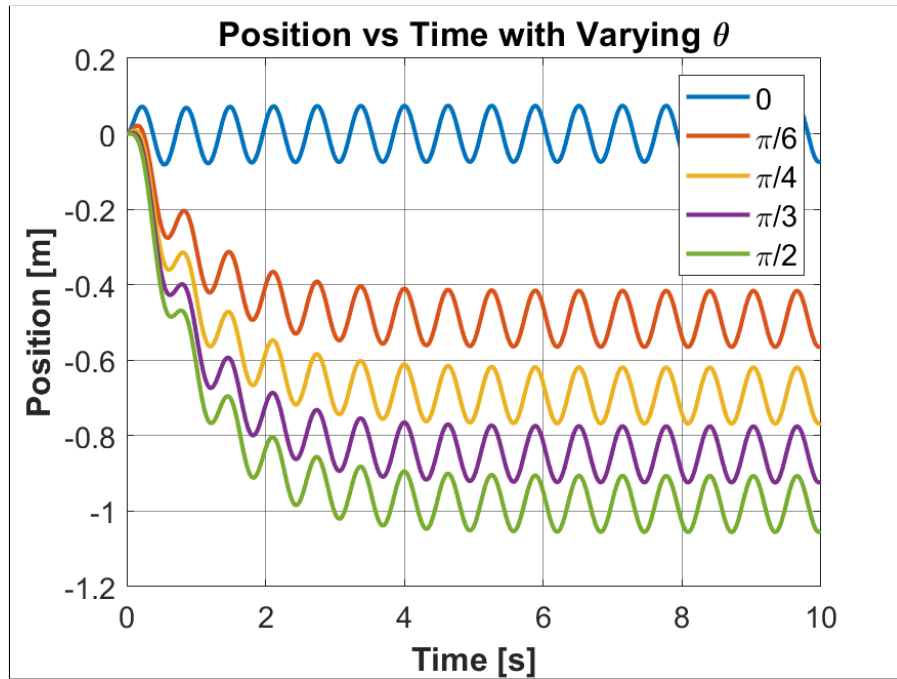


Figure 18: Plots of the displacement vs time over varying  $\theta$

Discussion: Since  $Y(s)$  does not depend on  $\theta$ , then the magnitude of the oscillation is not affected. The inclusion of gravity will only shift the equilibrium point of the mass. On Figure 18, the system will converge in oscillating around a new equilibrium point. The new equilibrium point is dependent on the angle  $\theta$ . The oscillation frequency on the plot remains the same regardless of  $\theta$ .

### Question 3: Shaft & Disk System with a Harmonic Excitation

Problem statement : Compute and plot the response of a shaft and disk system to an applied moment of  $M = 5 \sin \omega_f t$  where  $\omega_f = 215 \frac{rad}{s}$ . Assume that the 0.2-m radius disk is initially

at rest. The 1-m long steel shaft has a diameter of 5 cm, a shear modulus,  $G$ , of  $8.3 \times 10^{10}$  / $m^2$  and a damping ratio of 0.01.

Here is the drawings of a shaft-disk system in the problem.

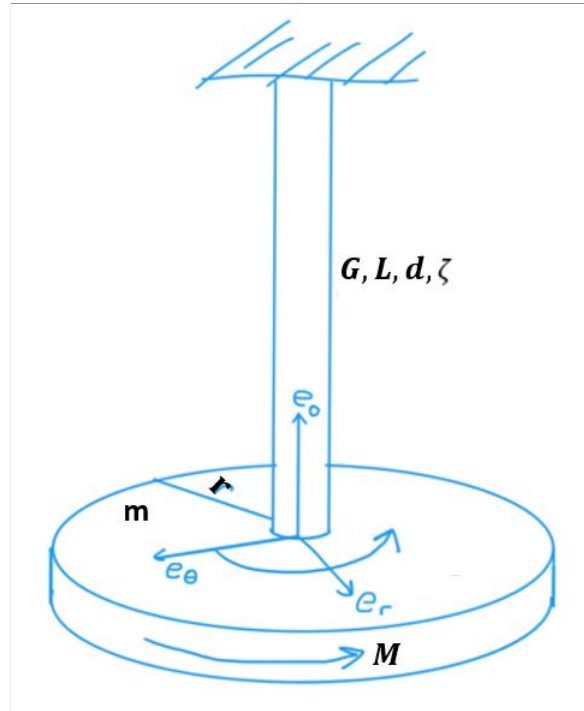


Figure 19: Drawing of the Shaft-Disk System

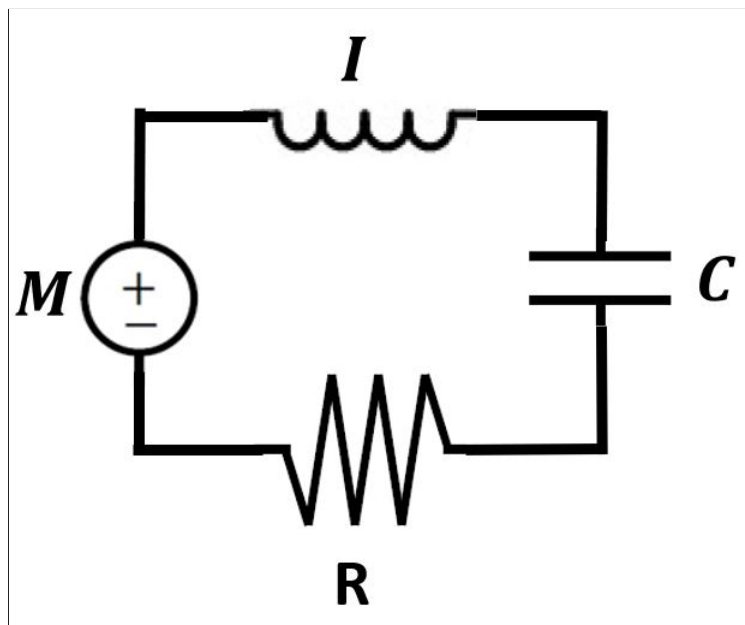


Figure 20: Equivalent Circuit of the Shaft-Disk System

Next is to find the admittance transfer function  $Y(s)$  and  $\Theta(s)$  of the system in the following equation. Note that  $I$  is the moment inertia of the disk.  $I = \frac{1}{2}mr^2$ .

$$\begin{aligned}
Z &= \frac{1}{2}mr^2s + R + \frac{1}{Cs} \\
Y &= \frac{1}{Z} \\
Y &= \frac{1}{\frac{1}{2}mr^2s + R + \frac{1}{Cs}} \\
\Omega(s) &= M(s)Y(s) \\
\mathcal{L}\{M\} &= \frac{5\omega_f}{s^2 + \omega_f^2} \\
\Omega(s) &= \frac{\frac{5\omega_f}{s^2 + \omega_f^2}}{\frac{1}{2}mr^2s + R + \frac{1}{Cs}} \\
\Theta(s) &= \frac{\Omega(s)}{s} \\
\Theta(s) &= \frac{\frac{5\omega_f}{s^2 + \omega_f^2}}{\frac{1}{2}mr^2s + R + \frac{1}{Cs}} \times \frac{1}{s}
\end{aligned} \tag{6}$$

Next is to find the  $C$  which is equal to  $\frac{1}{k}$  where  $k$  is the stiffness. The equation for  $k$  and its sub-variable are in the following equations.

$$\begin{aligned}
k &= \frac{GJ_p}{l} \\
J_p &= \frac{1}{32}\pi d_{shaft}^4 \\
k &= \frac{G\frac{1}{32}\pi d_{shaft}^4}{l} \\
C &= \frac{1}{k} \\
C &= \frac{l}{G\frac{1}{32}\pi d_{shaft}^4}
\end{aligned} \tag{7}$$

$J_p$  is the second of moment of inertia of the shaft.

Next is to find the  $R$ . The equation for  $R$  and its sub-variable are in the following equations.

$$\begin{aligned}
\omega_n &= \sqrt{\frac{1}{\frac{1}{2}mr^2C}} \\
R &= 2I\omega_n\zeta
\end{aligned} \tag{8}$$

With the mass of the shaft to be  $50kg$ ,  $C = 1.9626 \times 10^{-5} \frac{kg*m^2}{s^2}$ ,  $R = 4.5135 \frac{kg*m^2}{s}$ ,  $\omega_n = 225.6 \frac{rad}{s}$ . The plots of the angular velocity and angular displacement are shown in the following figures.

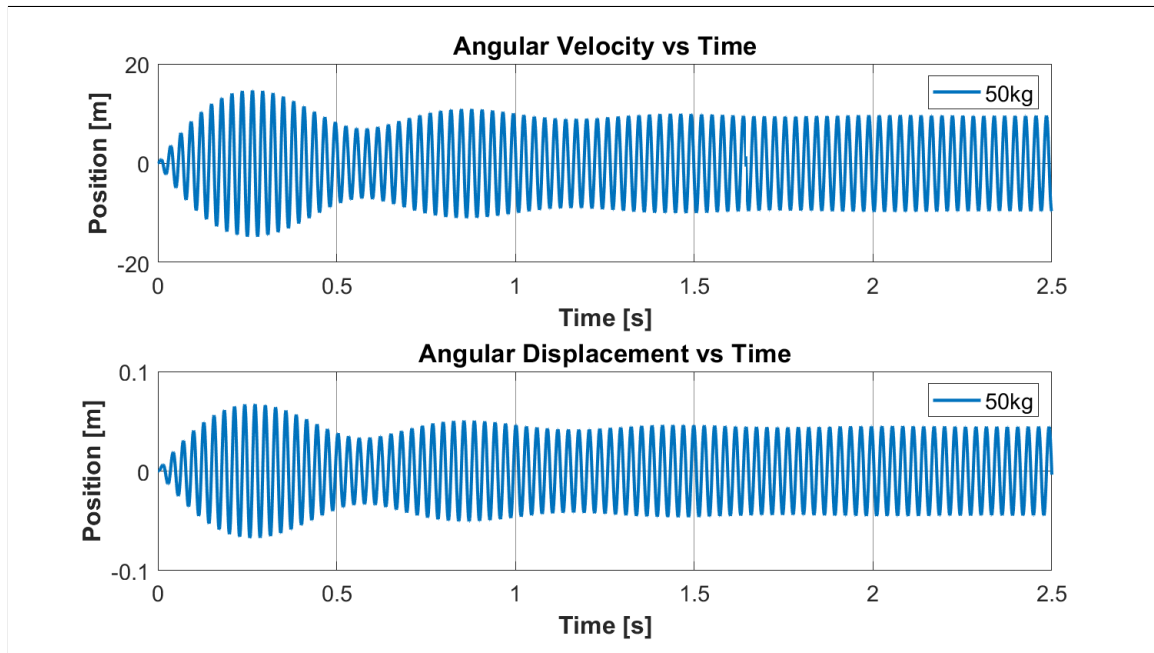


Figure 21: Equivalent Circuit of the Shaft-Disk System

Discussion: Based on the calculation, the natural frequency  $\omega_n$  of the system ( $225.6 \frac{rad}{s}$ ) is very close to the forced frequency  $\omega_f$  of the system ( $215 \frac{rad}{s}$ ). This can be seen in the velocity and displacement plot of Figure 21 by the initial presence of the beat frequency. An envelope is formed initially from with a frequency of about  $10.6 \frac{rad}{s}$  or  $1.69 Hz$ . The period of the envelope would be  $0.6s$  which is shown in the Figure 21. However, the envelope soon stop due to non-zero damping ratio, damping out the natural frequency  $\omega_n$ , so the forced frequency  $\omega_f$  is only present in the plot after some time.

## Appendix

Below is the MATLAB code used for making the plots.

```

1 %Team B2 Hw4
2 %3/2/21
3
4 clc
5 clear all
6 close all
7
8 %%
9 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
10 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%-Question 1%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12 %Parameters
13 m = 0.5; % Mass in [kg]
14 C = 1.2665e-6; %Spring Compliance in [m/N]
15 R = 10;% Damping Coefficient in [Ns/m]
16 freqL = 10;

```

```

17 freqH = 10000;
18
19 %Y(jw) - Mechanical Admittance in Laplace Domain
20 w = (freqL:freqH);
21 ss = (1j)*w;
22 Y = 1./(m.*ss+1./(C.*ss)+R);
23
24 %Y(s) plot
25 mag = 20*log10(abs(Y));
26 phase = angle(Y)*180/pi;
27
28 figure(1)
29 set(gcf,'position',[0,0,1080,1080])
30
31 subplot(2,1,1);
32 mm = semilogx(w,mag);
33 set(mm, 'LineWidth', 3)
34 xlim([freqL,freqH])
35 ylim([-1e2,0])
36 grid on
37 xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
38 ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
39 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
40 legend('|Y(s)|', 'FontSize',14);
41 title('Magnitude Plot of Admittance Y(s)', 'FontSize', 16, 'FontWeight','bold')
42
43 subplot(2,1,2);
44 pp = semilogx(w,phase);
45 set(pp, 'LineWidth', 3)
46 xlim([freqL,freqH])
47 ylim([-1e2,1e2])
48 grid on
49 xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
50 ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
51 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
52 legend('\angle Y(s)', 'FontSize',14,'Location','best')
53 title('Phase Plot of Admittance Y(s)', 'FontSize', 16, 'FontWeight', 'bold')
54 set(gca,'YTick',(-2:2)*45)
55 saveas(gcf,'Y.png')
56 %%
57 %Define s
58 s = tf('s');
59
60 Y = 1/(m*s+1/(C*s)+R);
61 wn = sqrt(1/(C*m));
62 coeff = [25, 100, 20, 20, 20];
63 wf = [1, 10*wn, 2*wn, wn+1,wn];
64 xl = [0.05,0.05,0.05,0.05,0.05];
65
66 for ii = 1:length(wf)
67
68     %Solving v(t) & x(t)

```



```

69     N = round(coeff(ii)*wn); % Numbers of pts
70     dt = 1/(coeff(ii)*wn); %differential time step
71     t = 5*(0:N-1)*dt;
72
73     F = 2*sin(wf(ii)*t);
74
75     v = lsim(Y,F,t);
76     x = lsim(Y/s,F,t);
77
78     %Ploting v(t)
79     figure(ii)
80     set(gcf,'position',[0,0,1080,360])
81     subplot(1,2,1);
82     plot(t,v, 'LineWidth', 1);
83     grid on
84     xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
85     ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
86     set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
87     xlim([0,xl(ii)])
88     legend('w_f =' + string(wf(ii)),'FontSize',14);
89     title('Velocity v(t)', 'FontSize', 16, 'FontWeight','bold')
90
91     %Ploting x(t)
92     subplot(1,2,2);
93     plot(t,x, 'LineWidth', 1);
94     grid on
95     xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
96     ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
97     set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
98     xlim([0,xl(ii)])
99     legend('w_f =' + string(wf(ii)),'FontSize',14);
100    title('Position x(t)', 'FontSize', 16, 'FontWeight','bold')
101    saveas(gcf, string(wf(ii))+'.png');
102
103 end
104
105
106 %% Changing R & C
107 %Define s
108 s = tf('s');
109
110 %Parameters
111 m = 0.5; % Mass in [kg]
112 C = 1.2665e-6; %Spring Compliance in [m/N]
113 R = 10;% Damping Coefficient in [Ns/m]
114
115 %Y(jw) - Mechanical Admittance in Laplace Domain
116 w = (freqL:freqH);
117 ss = (1j)*w;
118
119 % set-up
120 len = 5;
121 Q = (1:len);
122 mag=zeros(length(Q),length(w));

```

```

123 phase=zeros(length(Q),length(w));
124 txt = strings(length(Q),1);
125 RR = zeros(length(Q),1);
126 CC = zeros(length(Q),1);
127 base = 10;
128
129 %DSP set-up
130 wn = sqrt(1/(C*m));
131 N = round(2.5*wn)*20; % Numbers of pts
132 dt = 1/(2.5*wn*20); %differential time step
133 t = (0:N-1)*dt;
134 wf = 2*wn; %rad/s
135 F = 2*sin(wf*t);
136
137 % x(t) and v(t) arrays
138 v = zeros(len, N);
139 x = zeros(len, N);
140
141 V=zeros(length(Q),1);
142 X=zeros(length(Q),1);
143
144 %Solving
145 for ii = 1:length(Q)
146     %Y(s) plot
147     RR(ii) = R*base^(Q(ii)-3);
148     Y = 1./(m.*ss+1./(C.*ss)+RR(ii));
149     YY = 1/(m*s+1/(C*s)+RR(ii));
150     mag(ii,:) = 20*log10(abs(Y));
151     phase(ii,:) = angle(Y)*180/pi;
152     txt(ii) = strcat(string(base^(Q(ii)-3)));
153
154     %Solving v(t) & x(t)
155     v(ii,:) = lsim(YY,F,t);
156     x(ii,:) = lsim(YY/s,F,t);
157 end
158
159 %---R Mag & Angle Plot---%
160 figure(4)
161 set(gcf,'position',[0,0,1080,1080])
162
163 %----R Mag Plot----%
164 subplot(2,1,1);
165 mm = semilogx(w,mag);
166 set(mm, 'LineWidth', 2)
167 xlim([freqL,freqH])
168 ylim([-1e2,0])
169 grid on
170 xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
171 ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
172 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
173 leg = legend(txt,'FontSize',14);
174 htitle = get(leg,'Title');
175 set(htitle, 'String','Ratio R_i/R')

```

```

176     title('Magnitude Plot of Admittance Y(s) with Varying R', 'FontSize',
177           16, 'FontWeight','bold')
178
179     %----R Angle Plot----%
180     subplot(2,1,2);
181     pp = semilogx(w,phase);
182     set(pp, 'LineWidth', 2)
183     xlim([freqL,freqH])
184     ylim([-1e2,1e2])
185     grid on
186     xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
187     ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
188     set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
189     leg = legend(txt, 'FontSize',14,'Location','best');
190     htitle = get(leg,'Title');
191     set(htitle, 'String','Ratio R_i/R')
192     title('Phase Plot of Admittance Y(s) with Varying R', 'FontSize', 16,
193           'FontWeight', 'bold')
194     set(gca,'YTick',(-2:2)*45)
195     saveas(gcf,'R.png')
196 %%
197 % %----R Velocity Plot----%
198 figure(5)
199 set(gcf,'position',[0,0,720,360])
200 plot(t,v, 'LineWidth', 1);
201 grid on
202 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
203 ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
204 xlim([0,0.01])
205 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
206 leg = legend(txt,'FontSize',14);
207 htitle = get(leg,'Title');
208 set(htitle, 'String','Ratio R_i/R')
209 title('Velocity v(t) with varying R at \omega_f = 1 rad/s', 'FontSize',
210       16, 'FontWeight','bold')
211 saveas(gcf,'Rv.png');
212 %%
213 %----R Position Plot----%
214 figure(6)
215 set(gcf,'position',[0,0,720,360])
216 plot(t,x, 'LineWidth', 1);
217 grid on
218 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
219 ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
220 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
221 leg = legend(txt,'FontSize',14);
222 htitle = get(leg,'Title');
223 set(htitle, 'String','Ratio R_i/R')
224 title('Position x(t) with varying R at \omega_f = 1 rad/s', 'FontSize',
225       16, 'FontWeight','bold')
226 saveas(gcf,'Rx.png');

```

```

226 %%
227 %-----Varying C-----%
228
229 %DSP set-up
230 wn = sqrt(1/(C*m));
231 N = round(2.5*wn)*25^2; % Numbers of pts
232 dt = 1/(2.5*wn*25); %differential time step
233 t = (0:N-1)*dt;
234 base = 100;
235 wf = 2*wn;
236 F = 2*sin(wf*t);
237
238 v = zeros(len, N);
239 x = zeros(len, N);
240
241 %Solving
242 for ii = 1:length(Q)
243     CC(ii) = C*base^(ii-2);
244     Y = 1./(m.*ss+1./(CC(ii).*ss)+R);
245     YY = 1/(m*s+1/(CC(ii)*s)+R);
246     mag(ii,:) = 20*log10(abs(Y));
247     phase(ii,:) = angle(Y)*180/pi;
248     txt(ii) = strcat(string(base^(Q(ii)-2)));
249
250     %Solving v(t) & x(t)
251     v(ii,:) = lsim(YY,F,t);
252     x(ii,:) = lsim(YY/s,F,t);
253
254 end
255
256 %---C Mag & Angle Plot---%
257 figure(7)
258 set(gcf,'position',[0,0,1080,1080])
259
260 %----C Mag Plot----%
261 subplot(2,1,1);
262 mm = semilogx(w,mag);
263 set(mm, 'LineWidth', 2)
264 xlim([freqL,freqH])
265 ylim([-1e2,0])
266 grid on
267 xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
268 ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
269 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
270 leg = legend(txt,'FontSize',14, 'Location', 'best');
271 htitle = get(leg,'Title');
272 set(htitle, 'String','Ratio C_i/C')
273 title('Magnitude Plot of Admittance Y(s) with Varying C', 'FontSize',
16, 'FontWeight','bold')
274
275 %----C Angle Plot----%
276 subplot(2,1,2);
277 pp = semilogx(w,phase);
278 set(pp, 'LineWidth', 2)

```

```

279     xlim([freqL,freqH])
280     ylim([-1e2,1e2])
281     grid on
282     xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
283     ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
284     set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
285     leg = legend(txt, 'FontSize',14,'Location','best');
286     htitle = get(leg,'Title');
287     set(htitle, 'String','Ratio Ci/C')
288     title('Phase Plot of Admittance Y(s) with Varying C', 'FontSize', 16,
           'FontWeight', 'bold')
289     set(gca,'YTick',(-2:2)*45)
290     saveas(gcf,'C.png')
291
292 %----C Velocity Plot----%
293 figure(8)
294 plot(t,v, 'LineWidth', 1);
295 grid on
296 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
297 ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
298 xlim([0,0.025])
299 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
300 leg = legend(txt,'FontSize',14);
301 htitle = get(leg,'Title');
302 set(htitle, 'String','Ratio Ci/C')
303 title('Velocity v(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
304 saveas(gcf,'Cv.png');
305
306 %----C Position Plot----%
307 figure(9)
308 plot(t,x, 'LineWidth', 1);
309 grid on
310 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
311 ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
312 xlim([0,0.025])
313 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
314 leg = legend(txt,'FontSize',14);
315 htitle = get(leg,'Title');
316 set(htitle, 'String','Ratio Ci/C')
317 title('Position x(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
318 saveas(gcf,'Cx.png');
319
320 %%
321 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
322 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%-Question 2%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
323 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
324 m = 1; %kg
325 g = 9.81; %N/kg
326 C = 0.1; %m/N
327 R = 10; %kg/s
328 theta = [0,pi/6,pi/4,2*pi/3,pi/2];
329 wn = sqrt(1/m*C);
330
331 s = tf('s');

```

```

332 Y = 1/(m*s+1/(C*s)+R);
333 wf = 10;
334 dt = 1/(250*wn);
335 t = (0:dt:10);
336
337 figure(1)
338 for ii = 1:length(theta)
339     F = 10*s/(s^2+wf^2) - m*g*sin(theta(ii))/s;
340     X = F*Y/s;
341     x = impulse(X,t);
342     plot(t,x, 'LineWidth', 2)
343     hold on
344 end
345 grid on
346 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
347 ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
348 xlim([0,10])
349 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
350 legend('0', '\pi/6', '\pi/4', '\pi/3', '\pi/2', 'FontSize',14);
351 title('Position vs Time with Varying \theta', 'FontSize', 16, 'FontWeight',
    , 'bold')
352 saveas(gcf, 'Q2x.png');
353
354 %%
355 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
356 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%-Question 3%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
357 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
358
359 s = tf('s');
360
361 m = 50; %kg
362 L = 1; %m
363 d = 0.05; %m
364 r = 0.2; %m
365 G = 8.3*10^10; %Pa
366 zeta = 0.01;
367 w_f = 215; %rad/s
368
369 J = pi/32*d^4;
370 k = G*J/L;
371 C = 1/k;
372
373 I = 1/2*m*r^2;
374 wn = sqrt(1/(I*C));
375 R = 2*I*wn*zeta;
376
377 Y = 1/(I*s+R+1/(C*s));
378 M = w_f^2/(s^2+w_f^2);
379
380 Omega = Y*M;
381 Theta = Omega/s;
382
383 t = (0:0.001:2.5);
384 omega = impulse(Omega,t);

```

```

385 theta = impulse(Theta,t);
386
387 figure()
388 set(gcf,'position',[0,0,960,540])
389 subplot(2,1,1);
390 plot(t,omega, 'LineWidth', 2)
391 grid on
392 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
393 ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
394 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
395 legend('50kg','FontSize',14);
396 title('Angular Velocity vs Time', 'FontSize', 16, 'FontWeight','bold')
397
398 subplot(2,1,2);
399 plot(t,theta, 'LineWidth', 2)
400 grid on
401 xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
402 ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
403 set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
404 legend('50kg','FontSize',14);
405 title('Angular Displacement vs Time', 'FontSize', 16, 'FontWeight','bold')
406 saveas(gcf,'Q3.png');

```