Vibration Hw 1

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The LaGrange Method

$$F_i = \frac{d}{dt}(\frac{\partial KE}{\partial \dot{x}_i}) + \frac{\partial PE}{\partial x_i} + \frac{\partial DP}{\partial \dot{x}_i} \tag{1}$$

where F_i is the input force of the system, KE is the kinetic energy of the system, PE is the potential energy of the system, and DP is the dissipated power of the system.

Question 1: EOM & Frequency of Vibration

Problem Statement: Derive the equation of motion of the following systems and their frequency of vibration.

Note: For all parts in this problem, there is no input force, so $F_i = 0$

(a) Simple Pendulum
The equation for kinetic energy and potential energy is listed below.

$$\begin{aligned} KE &= \frac{1}{2}I\dot{\theta}^2 \\ PE &= mgl(1-\cos\theta) \\ DP &= 0 \end{aligned} \tag{2}$$

where I is the moment of inertia of the pendulum ($I = ml^2$), m is mass of the pendulum bob, l is the length of the pendulum, g is gravity, and θ is the angle with respect to the . For this problem, the mass of the pendulum rod is ignored, and the pendulum bob is considered as a point particle. Here is the FBD in Figure 1,

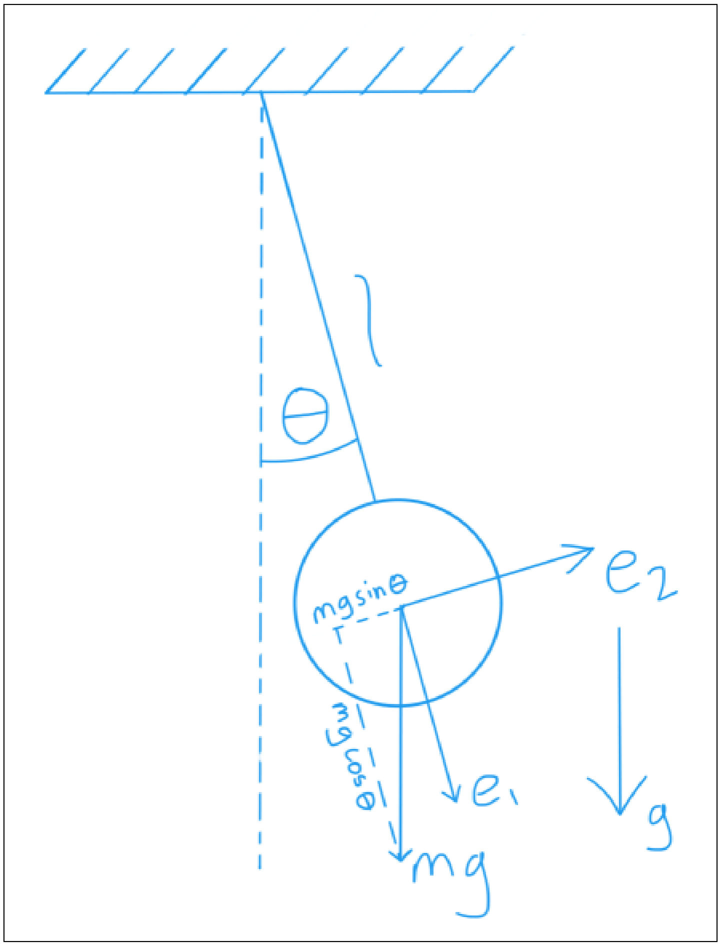

Figure 1: FBD of the simple Pendulum

x_i in the LaGrange Method is θ. With the equation for kinetic energy and potential known from (2), the LaGrange Method can be rewritten in the following equation.

$$0 = \frac{d}{dt}(\frac{\partial}{\partial \dot{\theta}}\left[\frac{1}{2}I\dot{\theta}^2\right]) + \frac{\partial}{\partial \theta}\left[mgl(1-\cos(\theta)\right] \tag{3}$$

By taking the derivatives, the equation can then be simplified in the following equation.

$$0 = I\ddot{\theta} + mgl\sin(\theta) \tag{4}$$

For simplicity, θ will be approximately small, so $\sin\theta \approx \theta$. Then bringing equation (3) over to equation (2), the new equation will be

$$I\ddot{\theta} + mgl\theta = 0 \tag{5}$$

Now, the new equation is a second order linear ODE (ordinary differential equation). Using the guess solution and its derivatives,

$$\theta(t) = Ce^{\lambda t}\dot{\theta}(t) = \lambda Ce^{\lambda t}\ddot{\theta}(t) \quad = \lambda^2 Ce^{\lambda t} \tag{6}$$

where λ and C are constants. The new equation with the guess solution will now be

$$I\lambda^2 Ce^{\lambda t} + mglCe^{\lambda t} = 0 \tag{7}$$

The following steps below show how to find λ from equation (7):

$$\begin{aligned} I\lambda^2 Ce^{\lambda t} + mglCe^{\lambda t} &= 0 \\ I\lambda^2 + mgl &= 0 \\ \lambda^2 &= \frac{-mgl}{I} \\ \lambda^2 &= \frac{-g}{l} \\ \lambda = \pm j\sqrt{\frac{g}{l}}, \quad \omega_n &= \sqrt{\frac{g}{l}} \end{aligned} \tag{8}$$

The frequency of vibration, ω_n, is $\sqrt{\frac{g}{l}}$. To increase the frequency of vibration, l must decrease, or g increases . With this relation between ω_n and l, one can measure time with a simple pendulum, such as a grandfather clock.

(b) Shaft and disk The equation for kinetic energy and potential energy is listed below.

$$\begin{aligned} KE &= \frac{1}{2}I\dot{\theta}^2 \\ PE &= \frac{1}{2}k\theta^2 \\ DP &= 0 \end{aligned} \tag{9}$$

where I is the moment of inertia of the disk ($I = \frac{1}{2}mr^2$), m is the mass of the disk, r is the radius of the disk, and θ is the displaced angle of the shaft. For this problem, the mass of the shaft is ignored. Here is the FBD in Figure2,

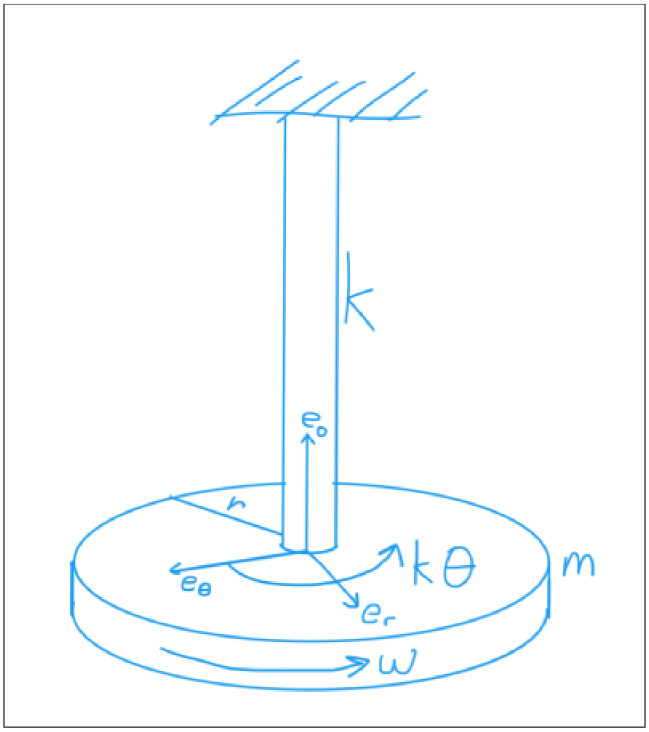

Figure 2: FBD of the Shaft & Disk

x_i in the LaGrange Method is θ. With the equation for kinetic energy and potential known from (9), the LaGrange Method can be rewritten in the following equation.

$$0 = \frac{d}{dt}(\frac{\partial}{\partial \dot{\theta}}\left[\frac{1}{2}I\dot{\theta}^2\right]) + \frac{\partial}{\partial \theta}\left[\frac{1}{2}k\theta^2\right] \tag{10}$$

By taking the derivatives, the equation can then be simplified in the following equation.

$$I\ddot{\theta} + k\theta = 0 \tag{11}$$

Now, the new equation is a second order linear ODE. Using the guess solution and its derivatives from equation (6), the new equation will now be

$$I\lambda^2 Ce^{\lambda t} + kCe^{\lambda t} = 0 \tag{12}$$

The following steps below show how to find λ from equation (12):

$$\begin{aligned} I\lambda^2 Ce^{\lambda t} + kCe^{\lambda t} &= 0 \\ I\lambda^2 + k &= 0 \\ \lambda^2 &= \frac{-k}{I} \\ \lambda^2 &= \frac{-2k}{mr^2} \\ \lambda = \pm j\sqrt{\frac{2k}{mr^2}}, \quad \omega_n &= \sqrt{\frac{2k}{mr^2}} \end{aligned} \tag{13}$$

The frequency of vibration, ω_n, is $\sqrt{\frac{2k}{mr^2}}$. To increase ω_n, k must increase, or m or r must decrease. This seems reasonable because having a bigger m or r means a larger I or a larger inertia, lowering ω_n

(c) Inverted Pendulum The equation for kinetic energy and potential energy is listed below.

$$\begin{aligned} KE &= \frac{1}{2}I\dot{\theta}^2 \\ PE &= mgl(1-\cos\theta) + \frac{1}{2}kx_{spring}^2 \\ DP &= \frac{1}{2}R\dot{x}_{damper}^2 \end{aligned} \tag{14}$$

where T is the net torque, I is the moment of inertia of the disk ($I = ml^2$), m is the mass of the disk, l is the length of the inverted pendulum, θ is the angle of the inverted pendulum with respect to the vertical, x_{spring} is the displaced spring displacement, and x_{damper} is the the displaced damper displacement. For this problem, the mass of the pendulum rod is ignored, and the pendulum bob is considered as a point particle. Here is the FBD in Figure 3,

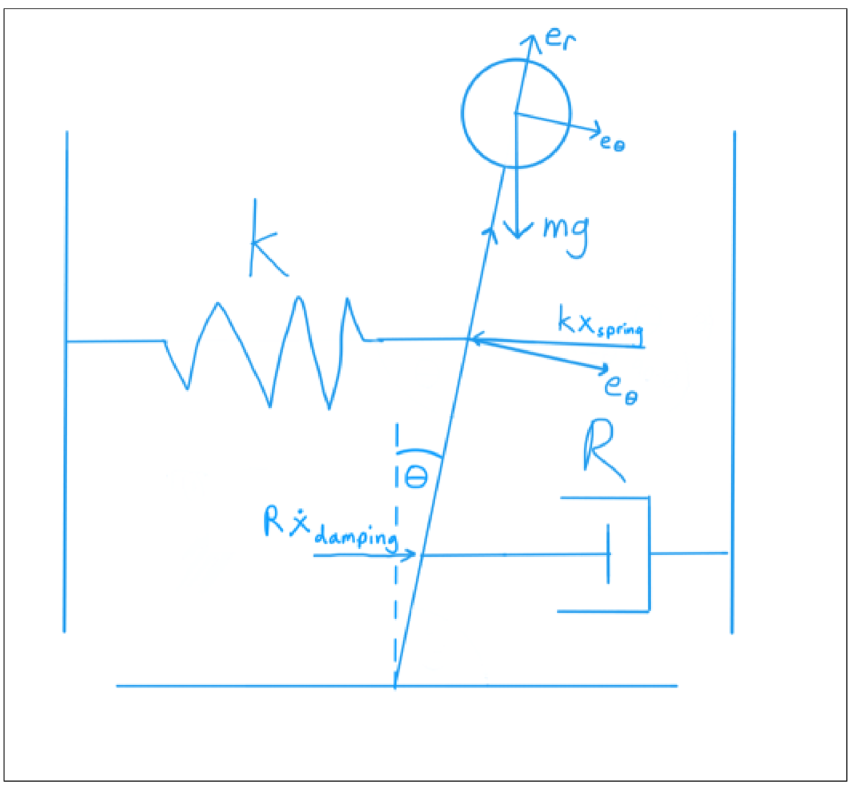

Figure 3: FBD of the Shaft & Disk

The displaced spring distance is listed below:

$$x_{spring} = \frac{2l}{3}\sin(\theta) \tag{15}$$

The velocity that the damper feels is listed below:

$$\dot{x}_{damper} = \frac{d}{dt}[\frac{l}{3}\sin(\theta)] \tag{16}$$
$$= \frac{l}{3}\cos(\theta)\dot{\theta} \tag{17}$$

x_i in the LaGrange Method is θ. With the equation for kinetic energy, potential energy, and damping power known from (2), the LaGrange Method can be rewritten in the following equation.

$$\begin{aligned} 0 &= \frac{d}{dt}(\frac{\partial}{\partial\dot{\theta}}\left[\frac{1}{2}I\dot{\theta}^2\right]) \\ &+ \frac{\partial}{\partial\theta}\left[mgl(\cos(\theta)) + \frac{1}{2}k(\frac{2l}{3}\sin(\theta))^2\right] \\ &+ \frac{\partial}{\partial\dot{\theta}}\left[\frac{1}{2}R(\frac{l}{3}\cos(\theta)\dot{\theta})^2\right] \end{aligned} \tag{18}$$

By taking the derivatives, the equation can then be simplified in the following equation.

$$0 = I\ddot{\theta} - mgl\sin(\theta) + \frac{1}{2}k(\frac{2l}{3})^2(2sin(\theta)cos(\theta)) + \frac{1}{2}R(\frac{l}{3})^2cos^2(\theta)(2\dot{\theta}) \tag{19}$$

For simplicity, θ will be approximately small, so $\sin\theta \approx \theta$ and $\cos(\theta) \approx 1$, the new equation will be the following:

$$I\ddot{\theta} + R(\frac{l}{3})^2\dot{\theta} + (k(\frac{2l}{3})^2 - mgl)\theta = 0 \tag{20}$$

Now, the new equation is a second order linear ODE. Using the guess solution and its derivatives from equation (6), the new equation will now be

$$I\lambda^2Ce^{\lambda t} + [R(\frac{l}{3})^2]\lambda Ce^{\lambda t} + [k(\frac{2l}{3})^2 - mgl]Ce^{\lambda t} = 0 \tag{21}$$

The following step below shows how to find λ from equation(21):

$$\begin{aligned} I\lambda^2Ce^{\lambda t} + [R(\frac{l}{3})^2)]\lambda Ce^{\lambda t} + [k(\frac{2l}{3})^2 - mgl]Ce^{\lambda t} &= 0 \\ I\lambda^2 + (R(\frac{l}{3})^2))\lambda + (k(\frac{2l}{3})^2 - mgl) &= 0 \\ \lambda = \frac{-R(\frac{l}{3})^2}{2I} \pm \sqrt{(\frac{R(\frac{l}{3})^2}{2I})^2 - 4I\frac{(k(\frac{2l}{3})^2 - mgl)}{4I^2}} \\ \lambda = \frac{-R(\frac{l}{3})^2}{2ml^2} \pm \sqrt{(\frac{R(\frac{l}{3})^2}{2ml^2})^2 - \frac{(k(\frac{2l}{3})^2) - mgl}{ml^2}} \\ \lambda = \frac{-R}{18m} \pm j\sqrt{\frac{4k}{9m} - \frac{g}{l} - (\frac{R}{18m})^2}, \quad \omega_n = \sqrt{\frac{4k}{9m} - \frac{g}{l}} \end{aligned} \tag{22}$$

The frequency of vibration, ω_n, is $\sqrt{\frac{4k}{9m} - \frac{g}{l}}$. To increase ω_n, l or k, must increase, or m , R, g must decrease. Since torque due to gravity points in the same direction as with angular displacement, that is why there is a negative sign in the square root.

Discussion: All the parts in this problem were solved with the LaGrange Method and have a restoring force and inertia, which the frequency of vibration depends on. In the equations for frequency of vibration, the numerator is the restoring forces, and the denominator is the inertia. A larger restoring force means a larger frequency of vibration, and a larger inertia means a smaller frequency of vibration. Thus, the LaGrange Method was consistent with Newton equations.

Question 2: Circuit Analogy of Mass-Spring System

Here is the drawing of the mass-spring system with damper and its circuit analogy system on Figure 4 and 5 respectively.

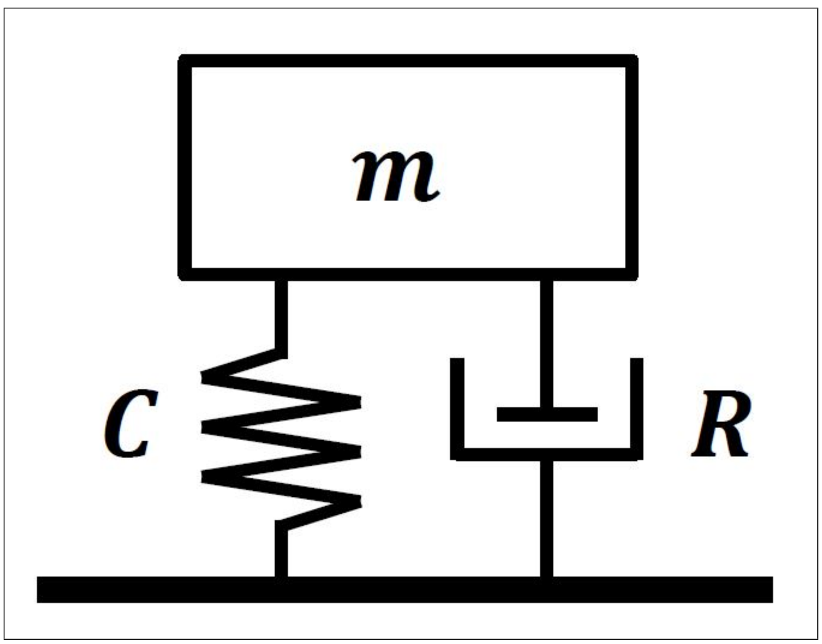

Figure 4: Mass-Spring System Drawing

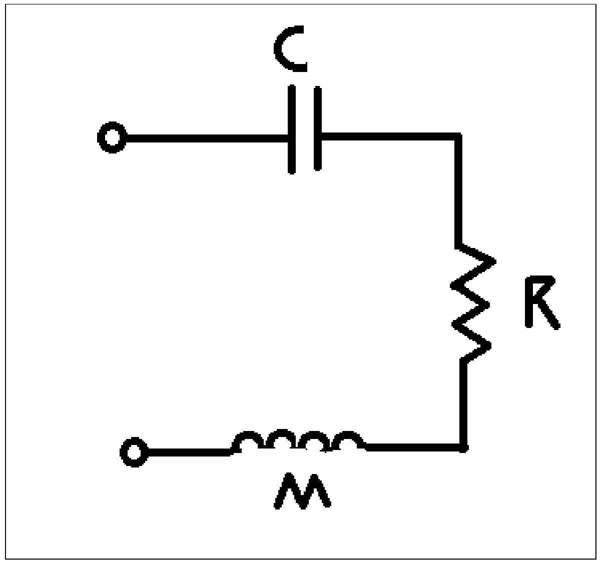

Figure 5: Mass-Spring System Equivalent Circuit Drawing

(a) Mechanical Admittance of the System
Problem Statement: Find the mechanical admittance of the system. The mechanical

The equation for each individual component impedance in series is listed below.

$$\begin{aligned} Z_m &= ms \\ Z_C &= \frac{1}{Cs} \\ Z_R &= R \end{aligned} \tag{23}$$

Since the velocity or its electrical analogy known as the current is the same for each component, each component is in series, so the equivalent impedance is the sum of the individual impedance in the following equation.

$$\begin{aligned} Z_{eq} &= Z_m + Z_c + Z_R \\ Z_{eq} &= ms + \frac{1}{Cs} + R \end{aligned} \tag{24}$$

To find the mechanical admittance, it is the reciprocal of the equivalent impedance.

$$\begin{aligned} Y &= \frac{1}{Z_{eq}} \\ Y &= \frac{1}{ms + \frac{1}{Cs} + R} \end{aligned} \tag{25}$$

(b) Magnitude & Phase Plot of Admittance With values $m = 0.5kg$, $C = 1.2665 * 10^6 m/N$, and $R = 10$, the following magnitude and phase plots are shown.

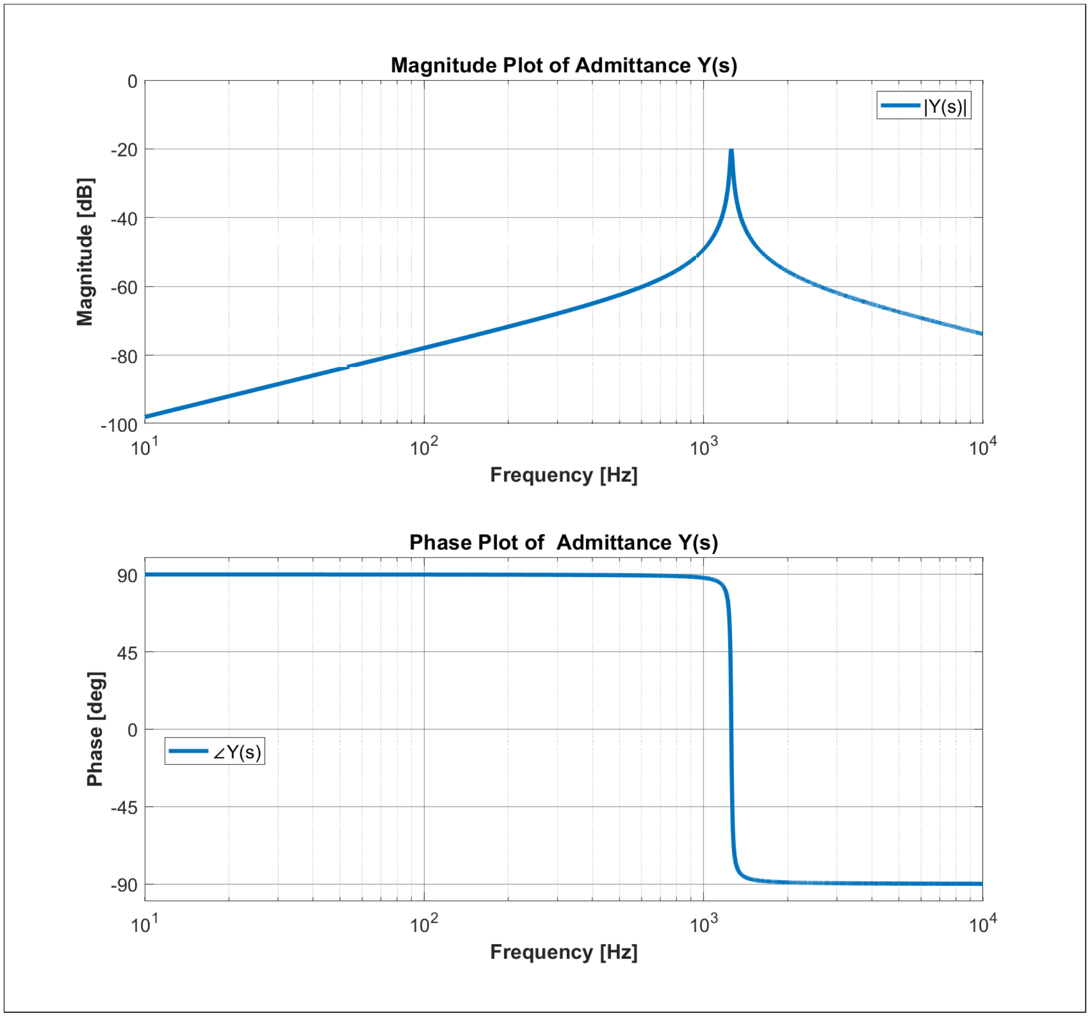

Figure 6: Y(s) Magnitude & Phase plot

At 10 Hz, the magnitude of Y(s) starts at -100 dB and then increase to its maximum till the frequency passes the natural frequency in which it will start increases.. This means that at low and high frequency, the ratio between the inputted force and the mass velocity is small. At the natural frequency, the ratio between the inputted force and the mass velocity is at its maximum. At frequency between 10 Hz and natural frequency, the phase of Y(s) is about 90 degrees. At frequency between natural frequency and 10,000 Hz, the phase of Y(s) is about -90 degrees. At low frequency till the natural frequency, the mass velocity is ahead of the force inputted by 90 degrees. At the natural frequency to high frequency, the mass velocity lags behind of the force inputted by 90 degrees.

(c) Initial Conditions With initial conditions of $x_0 = 0.3m$ and $v_0 = 1\frac{m}{s}$, the following plot of the model is shown.

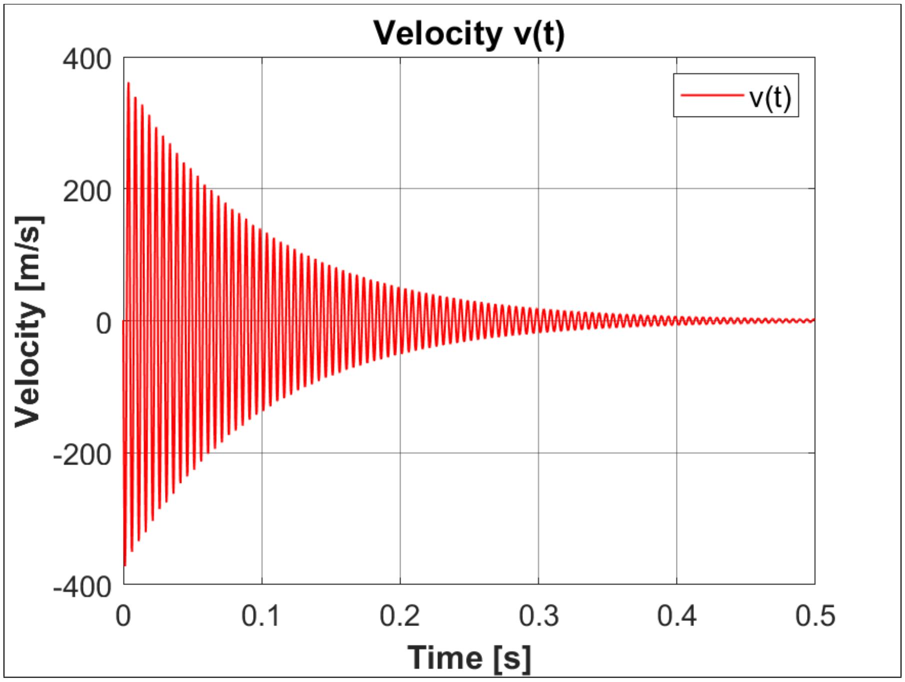

Figure 7: Mass-Spring Velocity Plot

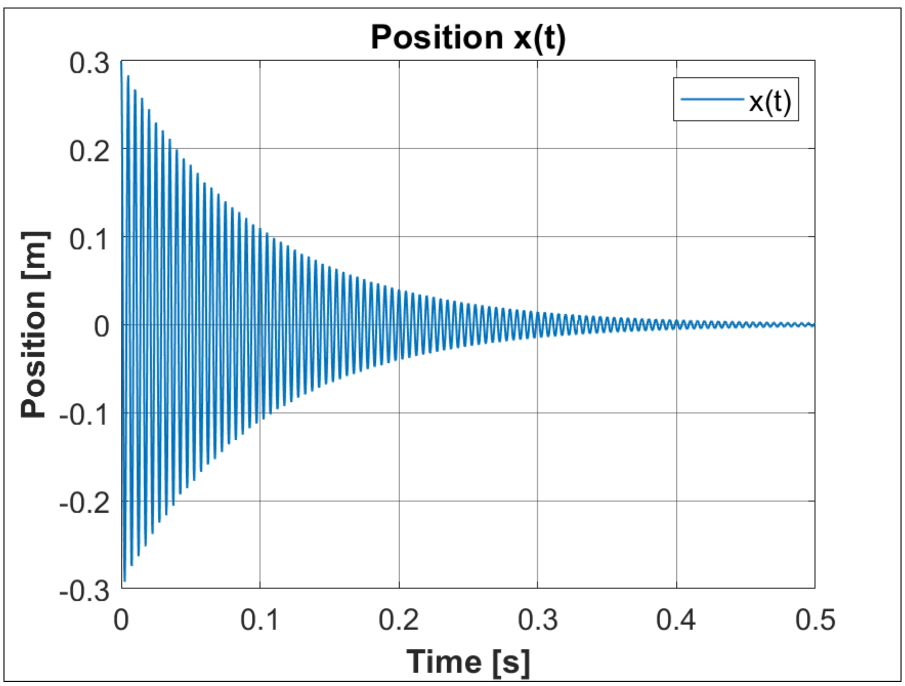

Figure 8: Mass-Spring Displacement Plot

(d) Changing R From varying R, the following plots are shown.

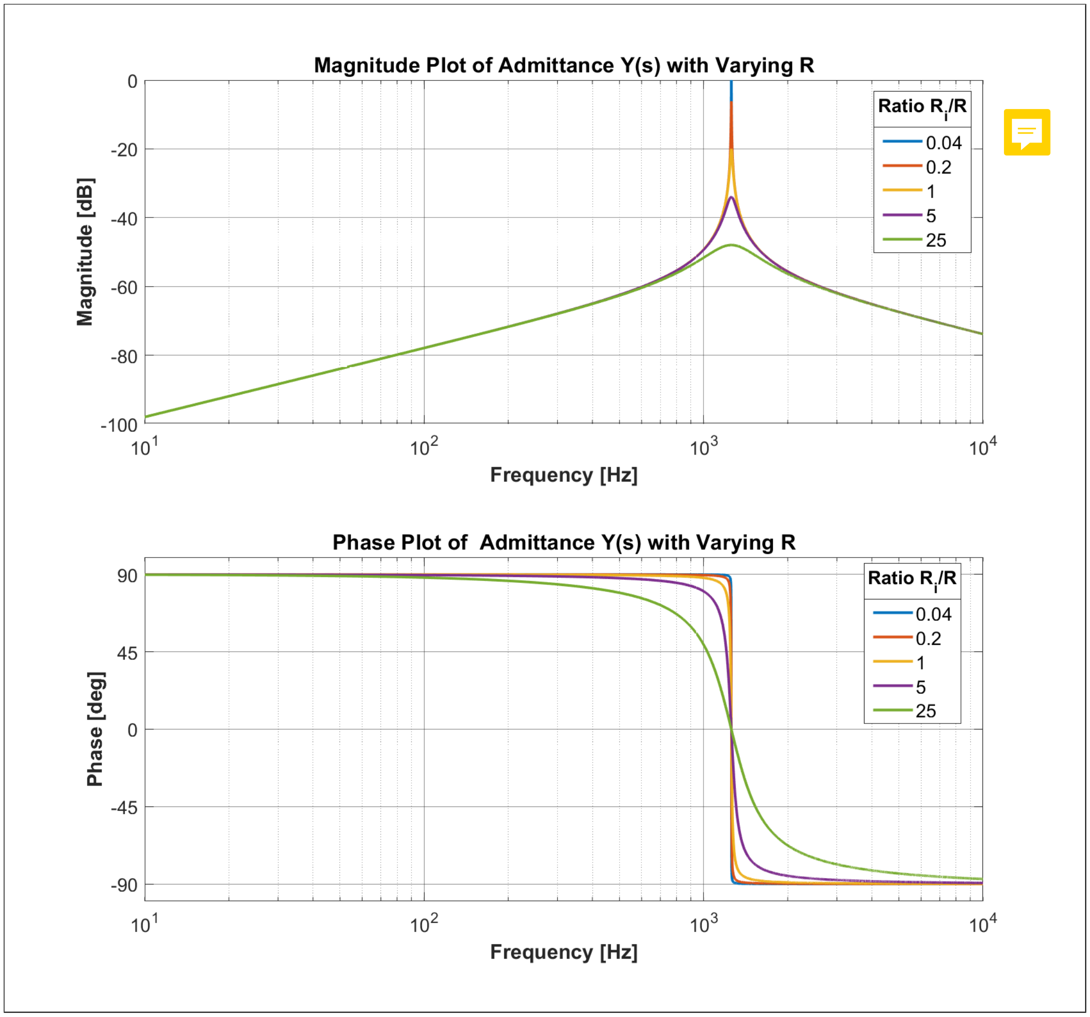

Figure 9: Y(s) with Varying R

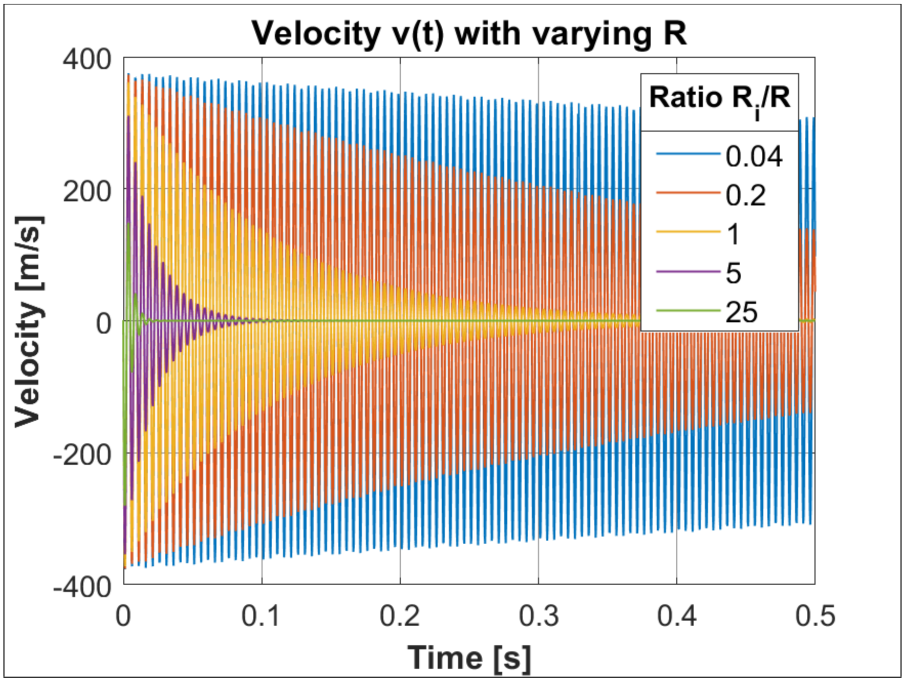

Figure 10: Velocity Plot with Varying R

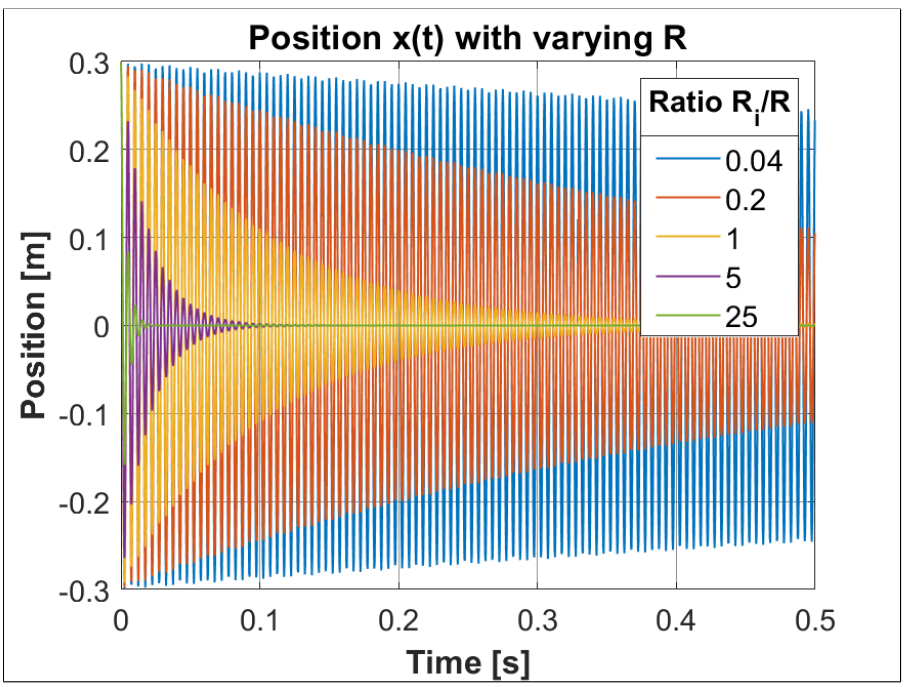

Figure 11: Displacement Plot with Varying R

From figure 9, the magnitude of $Y(s)$ deviates from among each other as R varies except when the frequency is close to the natural frequency. At the natural, the magnitude deceases as R increases. Near the natural frequency, the phase's curve become more smooth from going 90° to -90°. From figure 10 and figure 11, the velocity and displacement plot reach to $0\frac{m}{s}$ and $0m$ respectively quickly as R increases. Also, the maximum velocity of among the plots do not differ as R varies.

(e) Changing C From varying C, the following plots are shown.

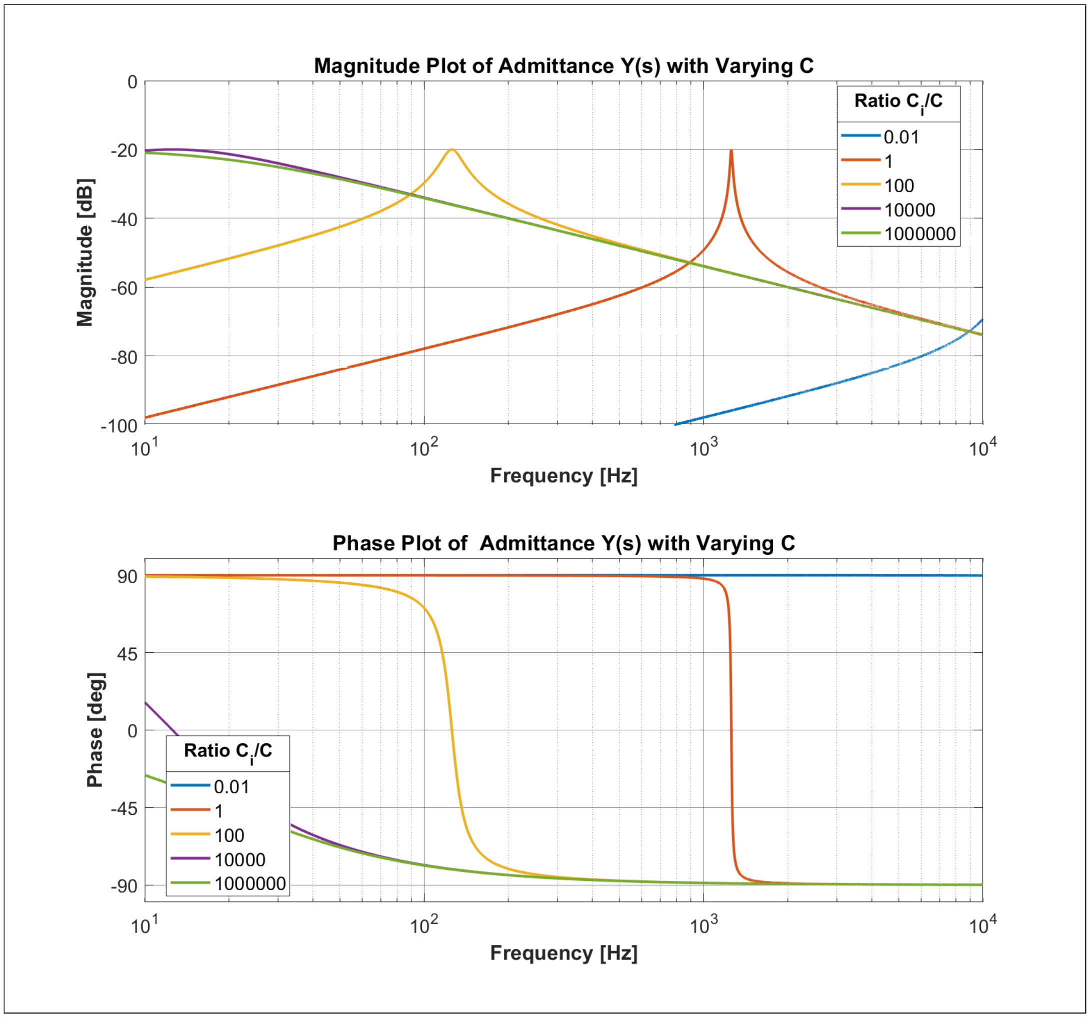

Figure 12: Y(s) with Varying C

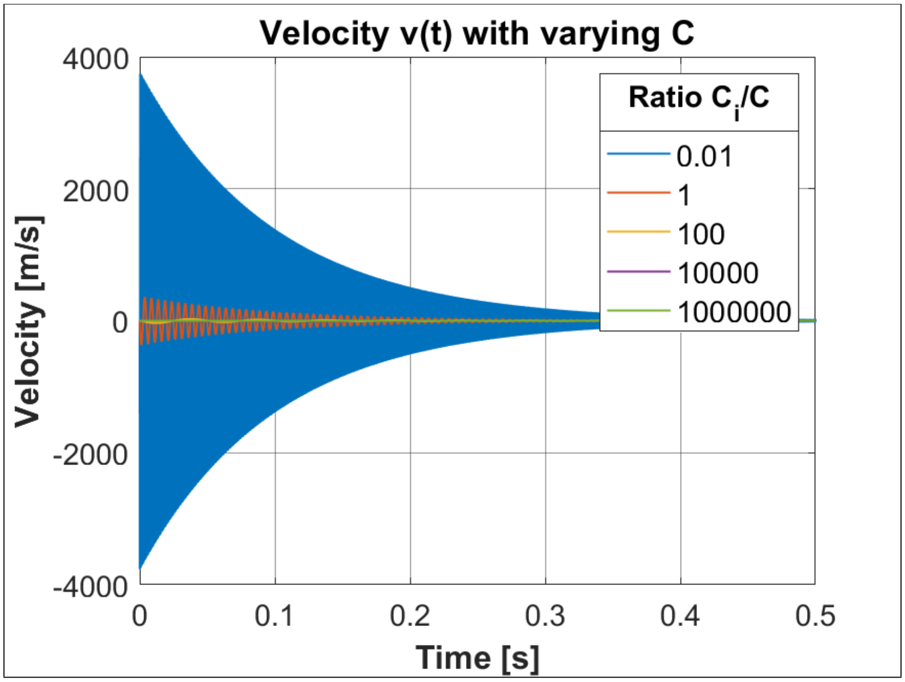

Figure 13: Velocity Plot with Varying C

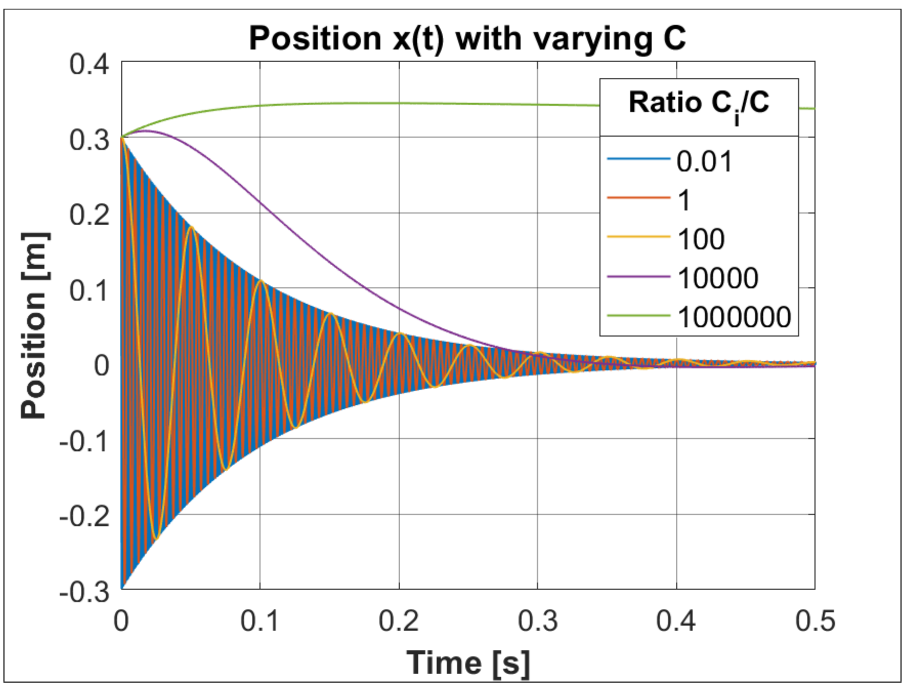

Figure 14: Displacement Plot with Varying C

From Figure 12, the magnitude and the phase of $Y(s)$ translate to the left as C increases. The magnitude and phase plots of $Y(s)$ near the natural frequency are more smooth as C increases. From Figure 13, the the amplitude of velocity plot decreases as C increases, but the time settling for reaching $0\frac{m}{s}$ decreases as C increases. From Figure 14. the time settling for reaching $0m$ increases as C increases. From Figure 14 and 13, the frequency of oscillation of the displacement and velocity plot decreases as C increase.

Discussion: To have the velocity and displacement to reach to $0\frac{m}{s}$ quickly (or slowly), R must increase (or decrease) which means more (or less) damping. To increase (or decrease) the maximum velocity of the mass-spring system, increase (or decrease) C which means more (or less) spring potential energy being converted into kinetic energy. Since the resonant frequency depends on C, this explains why the magnitude plot translates as C varies. For low frequency application, increase C to have the natural frequency be large to lower the chance of reaching the natural frequency by accidentally. Increase R to have the magnitude at the natural frequency be small to minimize damages if the frequency reaches the natural frequency.

Appendix

Below is the MATLAB code used for making the plots.

```
%Team B2 Hw2 Q2
%2/13/21

clc
clear all
close all

%%
%Parameters
m = 0.5; % Mass in [kg]
C = 1.2665e-6; %Spring Compliance in [m/N]
R = 10;% Damping Coefficent in [Ns/m]
freqL  = 10;
freqH  = 10000;

%Y(jw) - Mechanical Admittance in Laplace Domain
w = (freqL:freqH);
ss = (1j)*w;
Y = 1./(m.*ss+1./(C.*ss)+R);

%Y(s) plot
mag = 20*log10(abs(Y));
phase = angle(Y)*180/pi;

figure()
set(gcf,'position',[0,0,1080,1080])

subplot(2,1,1);
mm = semilogx(w,mag);
set(mm, 'LineWidth', 3)
xlim([freqL,freqH])
```

```
ylim([-1e2,0])
grid on
xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('|Y(s)|','FontSize',14);
title('Magnitude Plot of Admittance Y(s)', 'FontSize', 16, 'FontWeight','bold')

subplot(2,1,2);
pp = semilogx(w,phase);
set(pp, 'LineWidth', 3)
xlim([freqL,freqH])
ylim([-1e2,1e2])
grid on
xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('\angleY(s)', 'FontSize',14,'Location','best')
title('Phase Plot of  Admittance Y(s)', 'FontSize', 16, 'FontWeight', 'bold')
set(gca,'YTick',(-2:2)*45)
saveas(gcf,'Y.png')
%%
%Define s
s = tf('s');

% IC's
x0 = 0.3 ; %Inital Displacement of the Mass [m]
v0 = 1; %Inital Velocity of the Mass [m/s]

%V(s) IC's in Laplace Domain
FX0 = -1/C*x0/s;
FV0 = m*v0;

%X(s) IC's in Laplace Domain
X0 = x0/s;

%TF: V(s) = F(s)*Y(s)
F = FX0+FV0;
Y = 1/(m*s+1/(C*s)+R);
V = F*Y;

%TF: X(s)-X(0) = integrate(V(s)) = V(s)/s
X = V/s+X0;

%Solving v(t) & x(t)
wn = sqrt(1/(C*m));
N = round(2.5*wn)/2; % Numbers of pts
dt = 1/(2.5*wn); %differential time step
t = (0:N-1)*dt;
v = impulse(V,t);
x = impulse(X,t);

```

```
%Ploting v(t)
figure()
plot(t,v, 'LineWidth', 1, 'color', 'r');
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('v(t)','FontSize',14);
title('Velocity v(t)', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'v.png');

%Ploting x(t)
figure()
plot(t,x, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('x(t)','FontSize',14);
title('Position x(t)', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'x.png');

%% Changing R & C
%Define s
s = tf('s');

%Parameters
m = 0.5; % Mass in [kg]
C = 1.2665e-6; %Spring Compliance in [m/N]
R = 10;% Damping Coefficent in [Ns/m]

% IC's
x0 = 0.3 ; %Inital Displacement of the Mass [m]
v0 = 1; %Inital Velocity of the Mass [m/s]

%Y(jw) - Mechanical Admittance in Laplace Domain
w = (freqL:freqH);
ss = (1j)*w;

% set-up
len = 5;
Q = (1:len);
mag=zeros(length(Q),length(w));
phase=zeros(length(Q),length(w));
txt = strings(length(Q),1);
RR = zeros(length(Q),1);
CC = zeros(length(Q),1);
base = 5;

%V(s) IC's in Laplace Domain
FX0 = -1/C*x0/s;
FV0 = m*v0;
F = FX0+FV0;

```

```
%X(s) IC's in Laplace Domain
X0 = x0/s;

% x(t) and v(t) arrays
v = zeros(len, N);
x = zeros(len, N);
%------------------------------------------------------------%

%DSP set-up
wn = sqrt(1/(C*m));
N = round(2.5*wn)/2; % Numbers of pts
dt = 1/(2.5*wn); %differential time step
t = (0:N-1)*dt;

V=zeros(length(Q),1);
X=zeros(length(Q),1);

%Solving
for ii = 1:length(Q)
    %Y(s) plot
    RR(ii) = R*base^(Q(ii)-3);
    Y = 1./(m.*ss+1./(C.*ss)+RR(ii));
    mag(ii,:) = 20*log10(abs(Y));
    phase(ii,:) = angle(Y)*180/pi;
    txt(ii) = strcat(string(base^(Q(ii)-3)));

    %TF: V(s) = F(s)*Y(s)
    %TF: X(s)-X(0) = integrate(V(s)) = V(s)/s
    V = F/(m*s+1/(C*s)+RR(ii));
    X = V/s+X0;

    %Solving v(t) & x(t)
    v(ii,:) = impulse(V,t);
    x(ii,:) = impulse(X,t);
end

%---R Mag & Angle Plot---%
figure()
set(gcf,'position',[0,0,1080,1080])

    %----R Mag Plot----%
    subplot(2,1,1);
    mm = semilogx(w,mag);
    set(mm, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,0])
    grid on
    xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
    ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt,'FontSize',14);
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio R_i/R')
```

```
    title('Magnitude Plot of Admittance Y(s) with Varying R', 'FontSize',
   16, 'FontWeight','bold')

    %----R Angle Plot----%
    subplot(2,1,2);
    pp = semilogx(w,phase);
    set(pp, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,1e2])
    grid on
    xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt, 'FontSize',14,'Location','best')
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio R_i/R')
    title('Phase Plot of  Admittance Y(s) with Varying R', 'FontSize', 16,
    'FontWeight', 'bold')
    set(gca,'YTick',(-2:2)*45)
    saveas(gcf,'R.png')

%----R Velocity Plot----%
figure()
plot(t,v, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio R_i/R')
title('Velocity v(t) with varying R', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'v2.png');

%----R Position Plot----%
figure()
plot(t,x, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio R_i/R')
title('Position x(t) with varying R', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'x2.png');





%-------------Varying C--------------------------------%

%DSP set-up
```

```
wn = sqrt(1/(C*m));
N = round(2.5*wn)/2*25; % Numbers of pts
dt = 1/(2.5*wn*25); %differential time step
t = (0:N-1)*dt;
base = 100;

v = zeros(len, N);
x = zeros(len, N);

%Solving
for ii = 1:length(Q)
    CC(ii) = C*base^(ii-2);
    Y = 1./(m.*ss+1./(CC(ii).*ss)+R);
    mag(ii,:) = 20*log10(abs(Y));
    phase(ii,:) = angle(Y)*180/pi;
    txt(ii) = strcat(string(base^(Q(ii)-2)));

    %V(s) IC's in Laplace Domain
    FX0 = -1/CC(ii)*x0/s;
    F = FX0+FV0;

    %TF: V(s) = F(s)*Y(s)
    V = F/(m*s+1/(CC(ii)*s)+R);
    X = V/s+X0;

    %Solving v(t) & x(t)
    v(ii,:) = impulse(V,t);
    x(ii,:) = impulse(X,t);

end

%---C Mag & Angle Plot---%
figure()
set(gcf,'position',[0,0,1080,1080])

    %----C Mag Plot----%
    subplot(2,1,1);
    mm = semilogx(w,mag);
    set(mm, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,0])
    grid on
    xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
    ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt,'FontSize',14, 'Location', 'best');
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio C_i/C')
    title('Magnitude Plot of Admittance Y(s) with Varying C', 'FontSize', 16, 'FontWeight','bold')

    %----C Angle Plot----%
    subplot(2,1,2);
    pp = semilogx(w,phase);
```

```
    set(pp, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,1e2])
    grid on
    xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt, 'FontSize',14,'Location','best')
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio C_i/C')
    title('Phase Plot of  Admittance Y(s) with Varying C', 'FontSize', 16,
     'FontWeight', 'bold')
    set(gca,'YTick',(-2:2)*45)
    saveas(gcf,'C.png')

%----C Velocity Plot----%
figure()
plot(t,v, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,0.5])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio C_i/C')
title('Velocity v(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'v3.png');

%----C Position Plot----%
figure()
plot(t,x, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,0.5])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio C_i/C')
title('Position x(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'x3.png');
```