# Vibration Hw 5

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## Question 1: Improving Washing Machine

Problem Statement: Draw a sketch of and its equivalent circuit of the washing machine system. Plot displacement as a function of time and design a spring-damper system underneath the washing machine such that the amplitude of the vibration is less than 2 cm. Defend your design.

The problem will be analyzed around the equilibrium point of the washing machine, so gravity can be ignored for the problem. The initial displacement and initial velocity of the system with respect to the equilibrium point will be $0m$ and $0\frac{m}{s}$.

(a) Mechanical Drawing
Here is the drawing of the washing machine system on Figure 1.

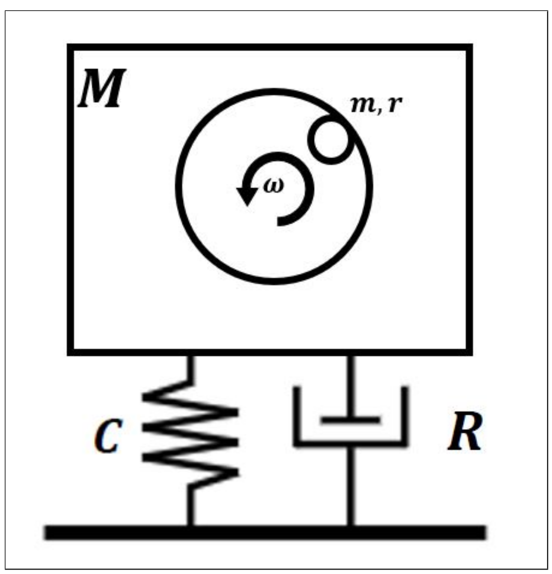


Figure 1: Washing Machine Drawing

(b) Equivalent Circuit
Here is the equivalent circuit of the washing machine system on Figure 2.

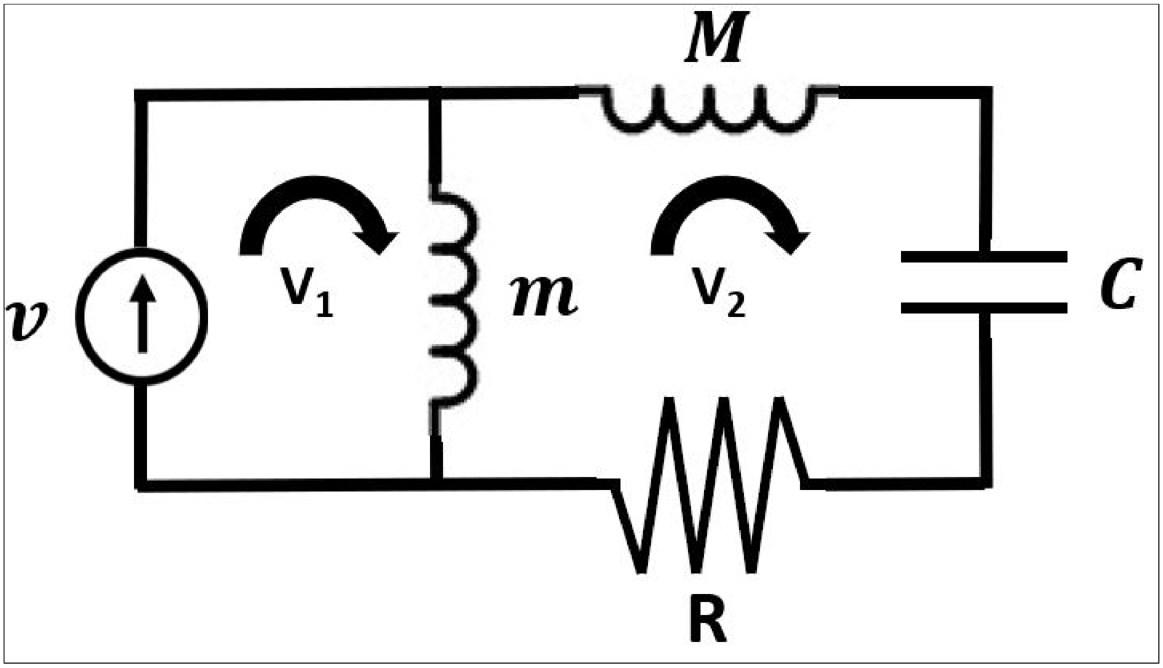


Figure 2: Washing Machine Equivalent Circuit with Velocity Source

For finding $v$ of the velocity source in the time domain, the y-displacement and its derivatives of the clothes with respect to the washing machine is in the following equation.

$$\begin{aligned} y &= rsin(\omega t) \\ \dot{y} &= v = r\omega cos(\omega t) \\ \ddot{y} &= -r\omega^2 sin(\omega t) \end{aligned} \tag{1}$$

where $r$ is distance between the clothes and the center of the washing machine, $\omega$ is the angular velocity produced from the washing machine.

The LaPlace form of Equation (1) is in the following equation.

$$V(s) = \frac{r\omega s}{s^2 + \omega^2} \tag{2}$$

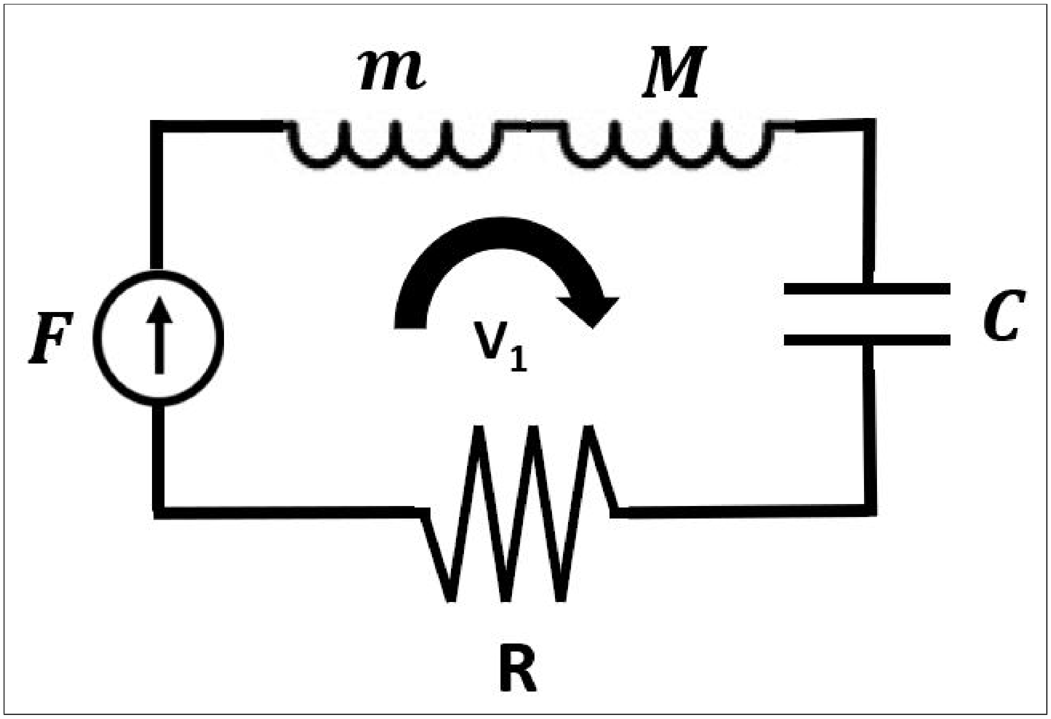


Figure 3: Washing Machine Equivalent Circuit with Force Source

For finding $F$ of the force source in the LaPlace domain, it is in the following equation.

$$F = msV \tag{3}$$

where $m$ is the mass of the clothes, and $V$ is the velocity of the current source in the Laplace Domain from Figure 1. Equation (3) can also be calculated by multiplying $\ddot{y}$ from Equation (1) with $m$.
The LaPlace form of Equation (1) is in the following equation.

$$F(s) = \frac{mr\omega^2 s}{s^2 + \omega^2} \tag{4}$$

(c) Displacement Plot
In the LaPlace Domain, unique equations of the circuit in Figure 2 can be written using mesh analysis in Equation(5). Look at Figure 2 for the labeled velocities currents: $v_1, v_2$.

$$\begin{aligned} V &= V_1 \\ 0 &= (Ms + R + \frac{1}{Cs})V_2 + ms(V_2 - V_1) \end{aligned} \tag{5}$$

Equation (5) can be rewritten in matrix form $F = ZV$ where Z is the impedance matrix in Equation (6).

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -ms & (M+m)s + R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6}$$

In the LaPlace Domain, unique equations of the circuit can be written using mesh analysis in Equation (7) at Figure 1. Since there is one velocity loop, only one unique equation can be made in matrix form $F = ZV$ where Z is the impedance matrix in Equation.

$$F = ((M+m)s + R + \frac{1}{Cs})V_1 \tag{7}$$

Finally, the displacement plot can be calculated from the following equation.

$$X(s) = \frac{V(s)}{s} \tag{8}$$

Note that the initial condition for displacement is $0m$.

Here is the following plot of the displacement curve of the washing machine system.

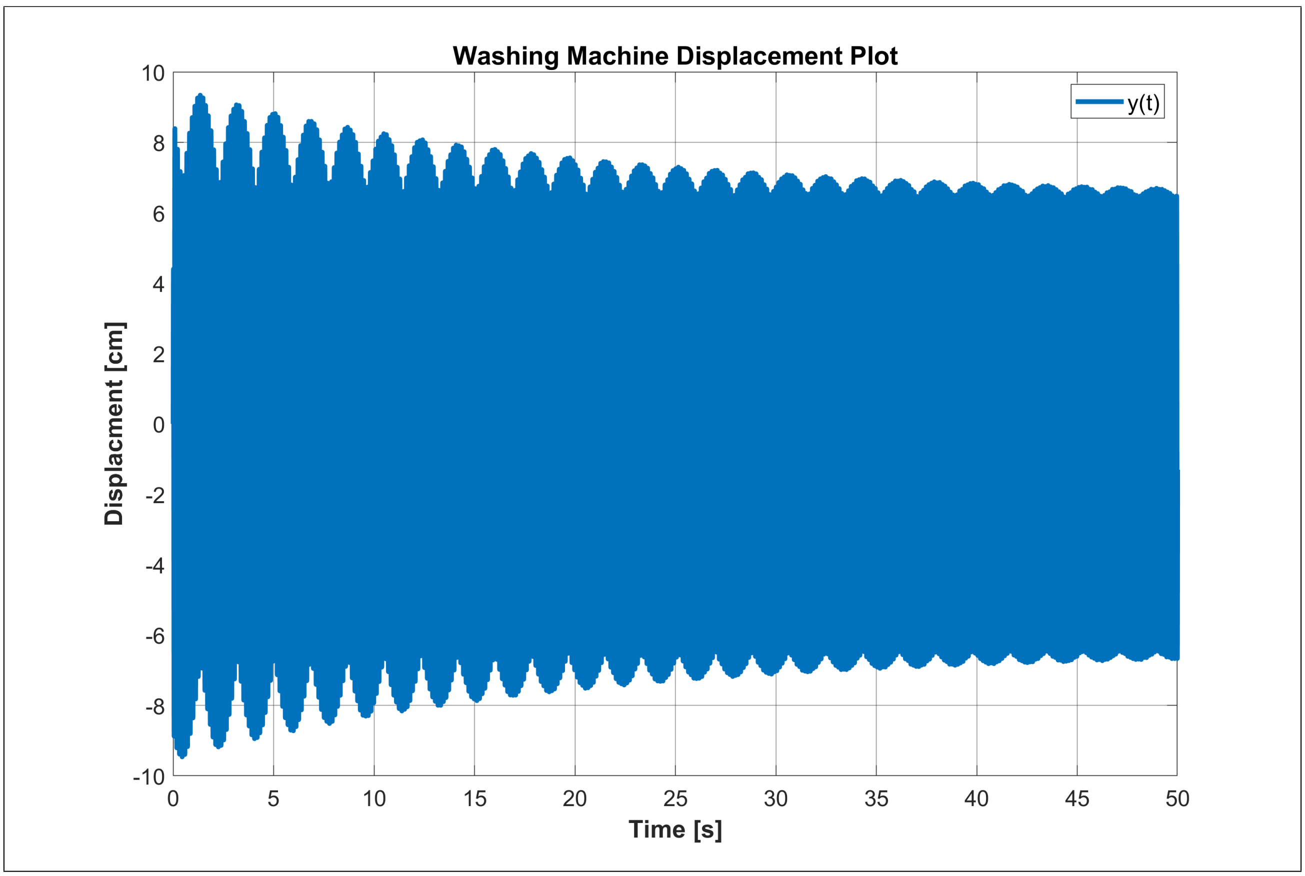


Figure 4: Displacement Plot of the Washing Machine System

(d) Spring-Damper System Addition

A spring board is placed under washing machine. The spring board consists of a spring $k_g$ and damper $R_g$ that are connected in parallel to the board $M_g$. Here is the following drawing and the equivalent circuit of the washing machine system with the added spring board system.

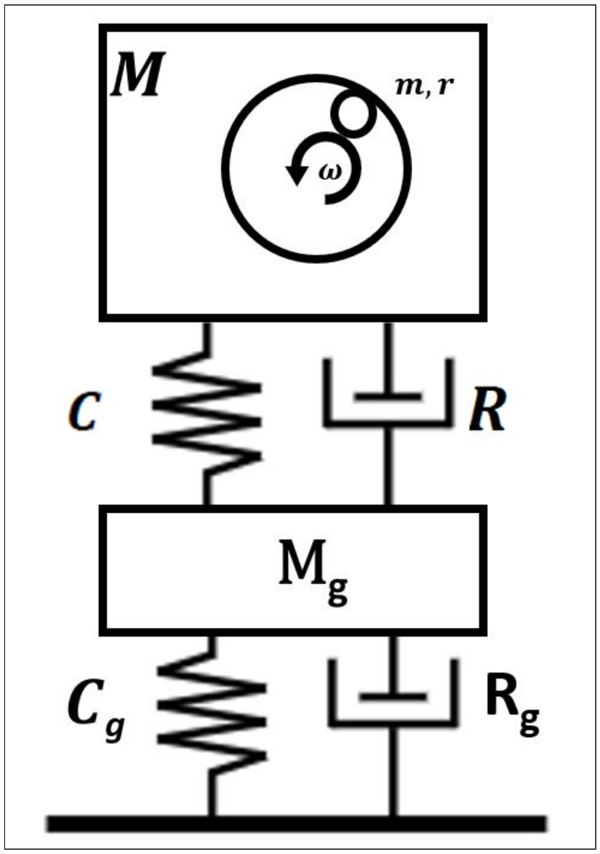


Figure 5: Washing Machine Drawing with the Added Spring Board System

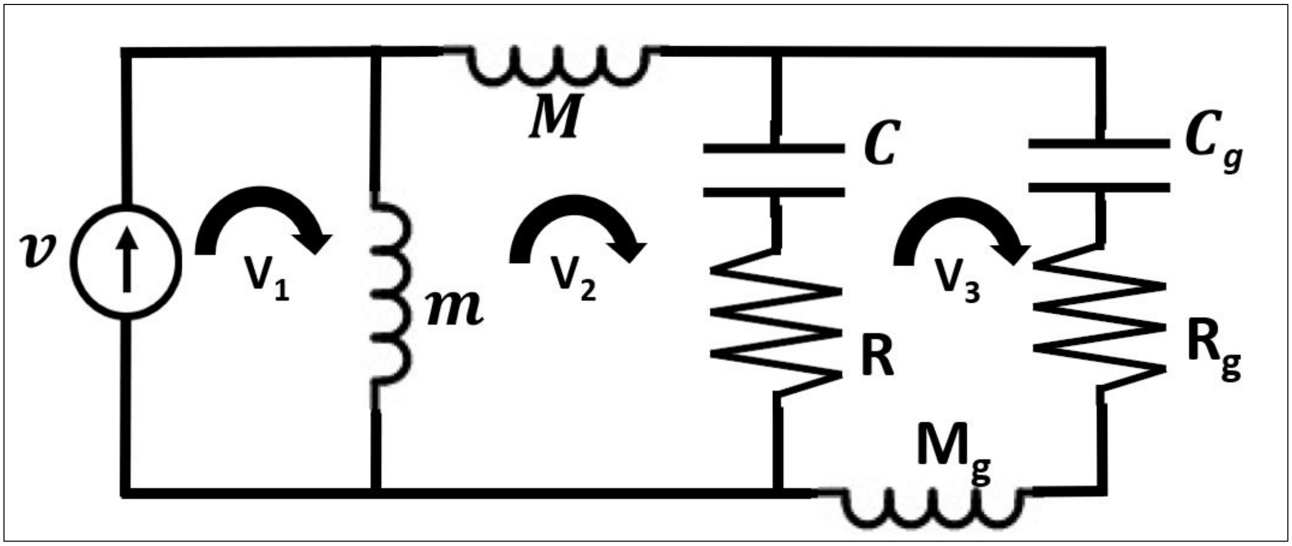


Figure 6: Washing Machine Equivalent Circuit with the Added Spring-Board System

Figure 6 can be rewritten in matrix form F = ZV where Z is the new impedance matrix of the circuit.

$$\begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -ms & (M+m)s + R + \frac{1}{Cs} & -(R + \frac{1}{Cs}) \\ 0 & -(R + \frac{1}{Cs}) & (R + \frac{1}{Cs}) + (R_g + \frac{1}{C_g s} + M_g) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{9}$$

With $R_g = 10\frac{kg}{s}$, $k_g = 100\frac{N}{m}$, and $m_g = 19.5kg$, here is the plot of the new displacement curve. The maximum displacement amplitude at steady-state was calculated to be 1.99cm which is below the 2cm goal.

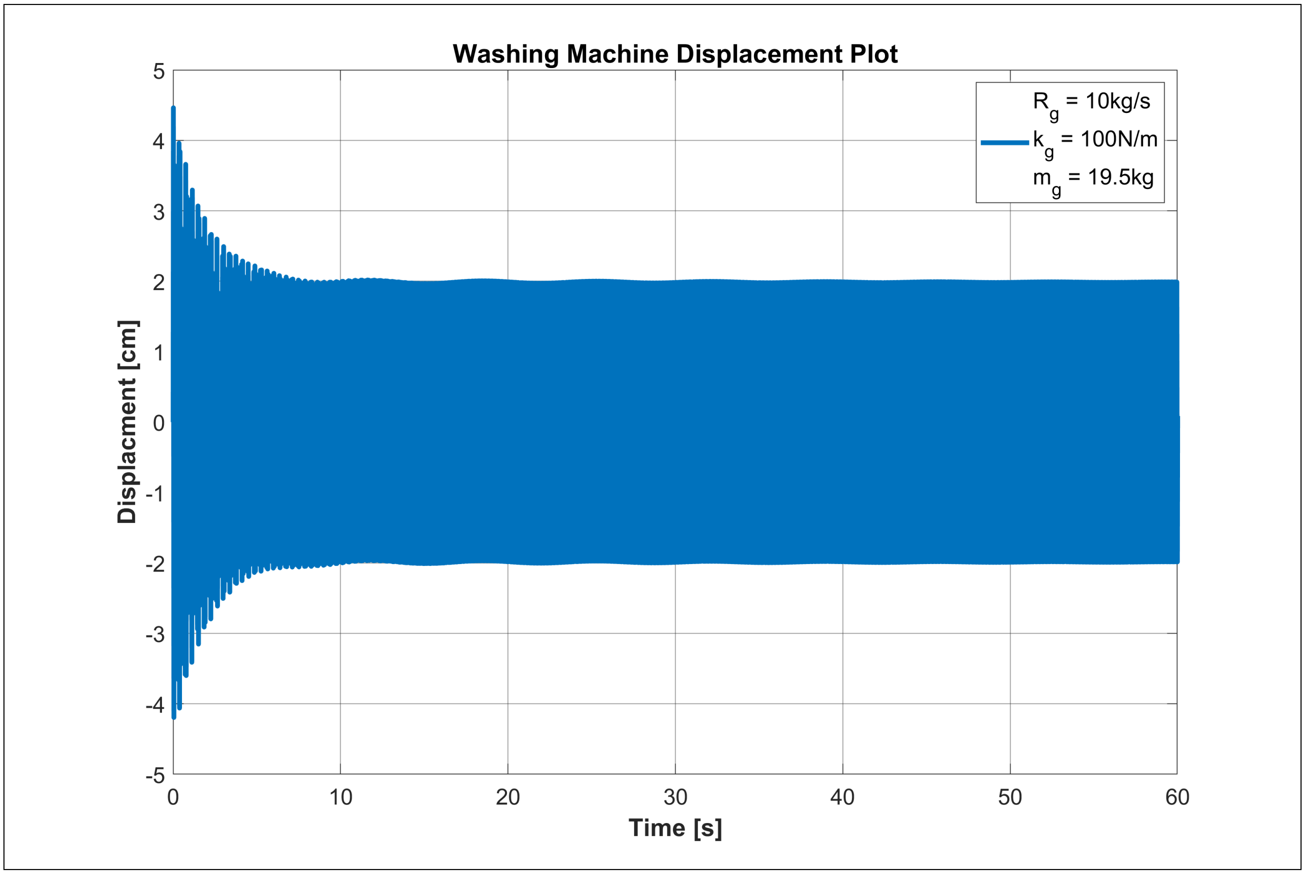


Figure 7: Displacement Plot of the Washing Machine System with Added Spring Board System

Discussion:
Without the spring board system, the maximum displacement amplitude at steady-state was calculated to be 6.67cm which is above the 2cm goal. Thus, the spring board system was added to the washing machine to decrease it maximum displacement amplitude to under 2cm. The major factors that decreased the amplitude were the spring constant $k_g$ and the spring board mass $M_g$. Having a low $k_g$ means that a high $C_g$ which decrease the natural frequency $\omega_n$ of the system. Having a high $M_g$ decrease the natural frequency of the system. When $\omega_n << \omega_f$ the forced frequency, the magnitude of $\frac{X(s)}{F(s)}$ will go to zero. In addition, $M_g$ was minimized as much as possible, so less weight would be required to install the spring board underneath washing machine while fulfilling the 2cm design constraint.

# Appendix

Below is the MATLAB code used for making the plots.