Vibration Hw 4

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Question 1: Mass-Spring Damper System with a Harmonic Excitation

Here is the drawing of the mass-spring system with damper and its circuit analogy system on Figure 1 and 2 respectively.

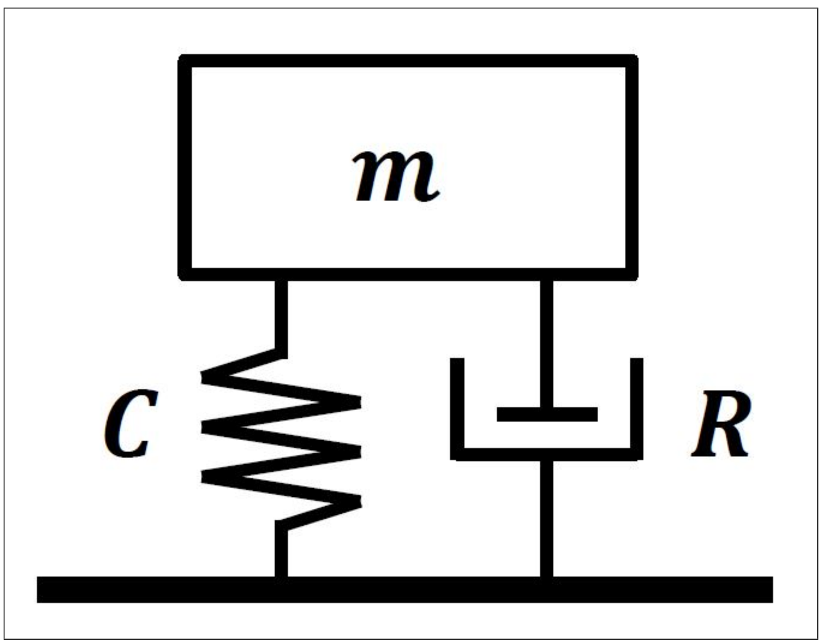

Figure 1: Mass-Spring System Drawing

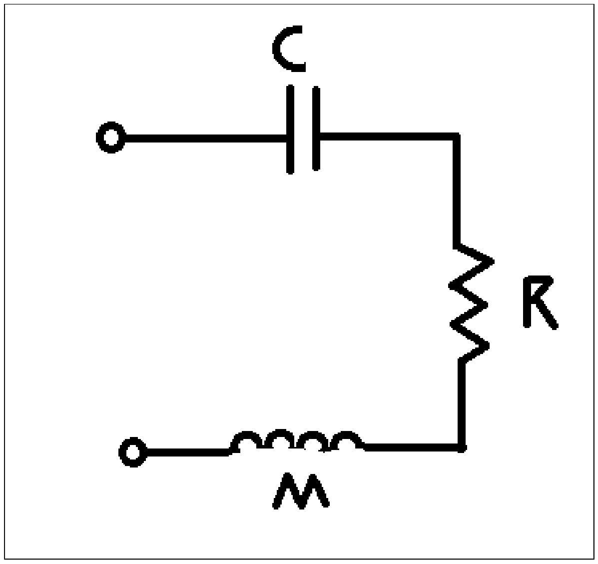

Figure 2: Mass-Spring System Equivalent Circuit Drawing

(a) Mechanical Admittance of the System and its Bode Plot
Problem Statement: Find the mechanical admittance of the system and plot the Bode of the mechanical admittance.

The equation for each individual component impedance in series is listed below.

$$\begin{aligned} Z_m &= ms \\ Z_C &= \frac{1}{Cs} \\ Z_R &= R \end{aligned} \tag{1}$$

Since the velocity or its electrical analogy known as the current is the same for each component, each component is in series, so the equivalent impedance is the sum of the individual impedance in the following equation.

$$\begin{aligned} Z_{eq} &= Z_m + Z_c + Z_R \\ Z_{eq} &= ms + \frac{1}{Cs} + R \end{aligned} \tag{2}$$

To find the mechanical admittance, it is the reciprocal of the equivalent impedance.

$$\begin{aligned} Y &= \frac{1}{Z_{eq}} \\ Y &= \frac{1}{ms + \frac{1}{Cs} + R} \end{aligned} \tag{3}$$

With values $m = 0.5kg$, $C = 1.2665 * 10^6 m/N$, and $R = 10$, the following magnitude and phase plots are shown.

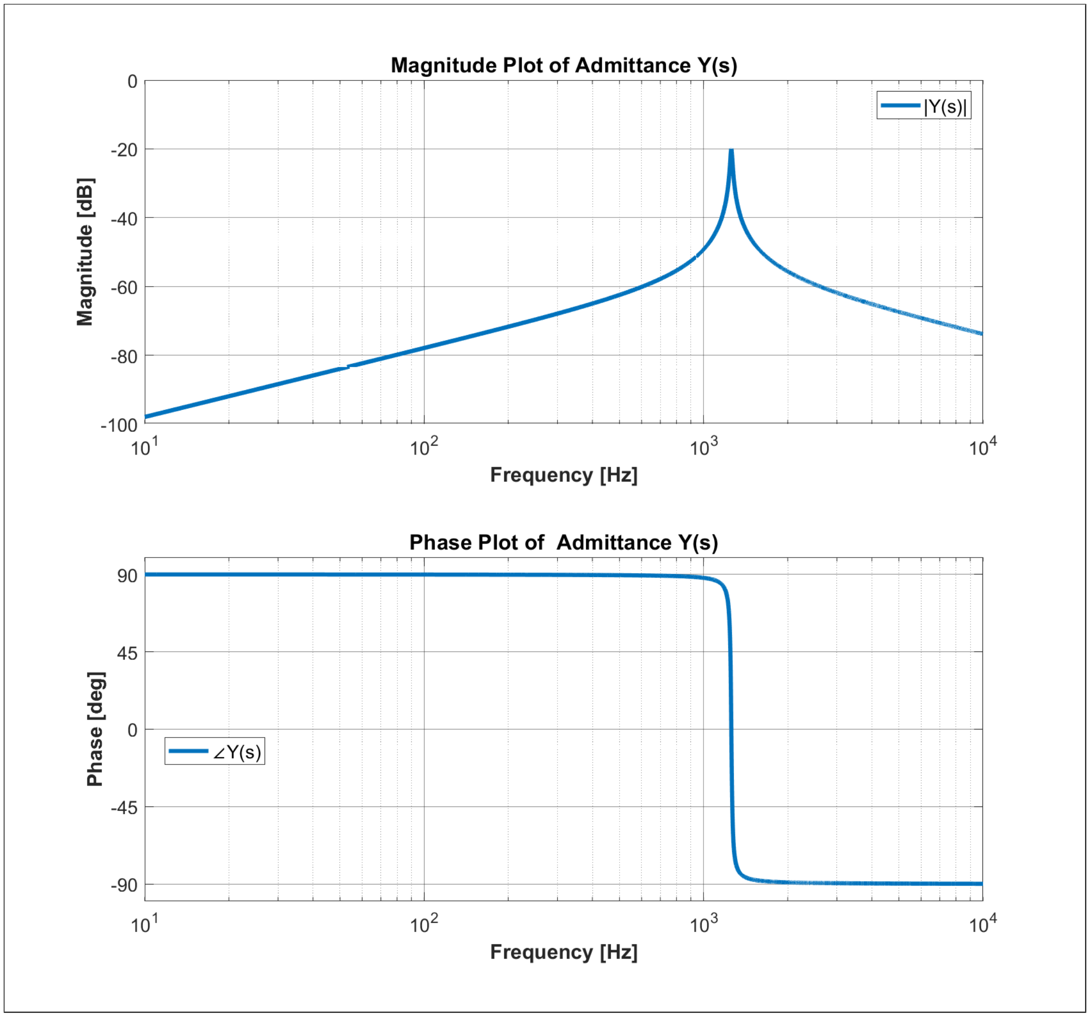

Figure 3: Y(s) Magnitude & Phase plot

(b) Varying Force

Problem Statement: Plot v(t) and x(t) of the mass starting from rest with a varying Force of $F(t) = 2sin(\omega_f t)$.

The LaPlace of $F(t) = 2sin(\omega_f t)$ is $F(s) = \frac{2w_f}{w_f^2+s^2}$. $V(s) = Y(s)F(s)$. $X(s) = \frac{V(s)}{s}$. Note that the initial conditions is 0, so there is no extra terms for $V(s)$ and $X(s)$.

The following displacement and velocity plots of the system starting from rest are shown.

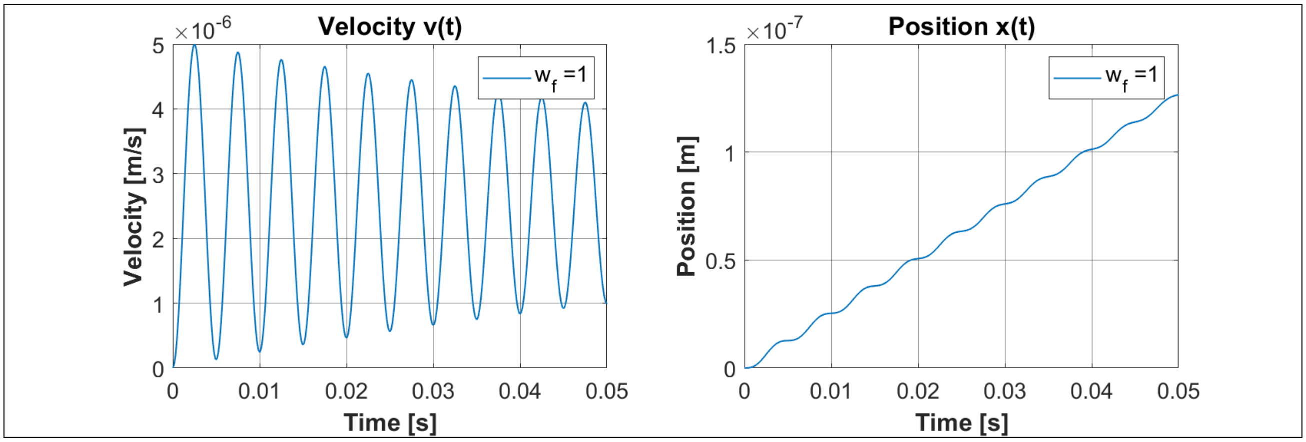

Figure 4: Mass-Spring Velocity & Position Plot for $\omega_f = 1$

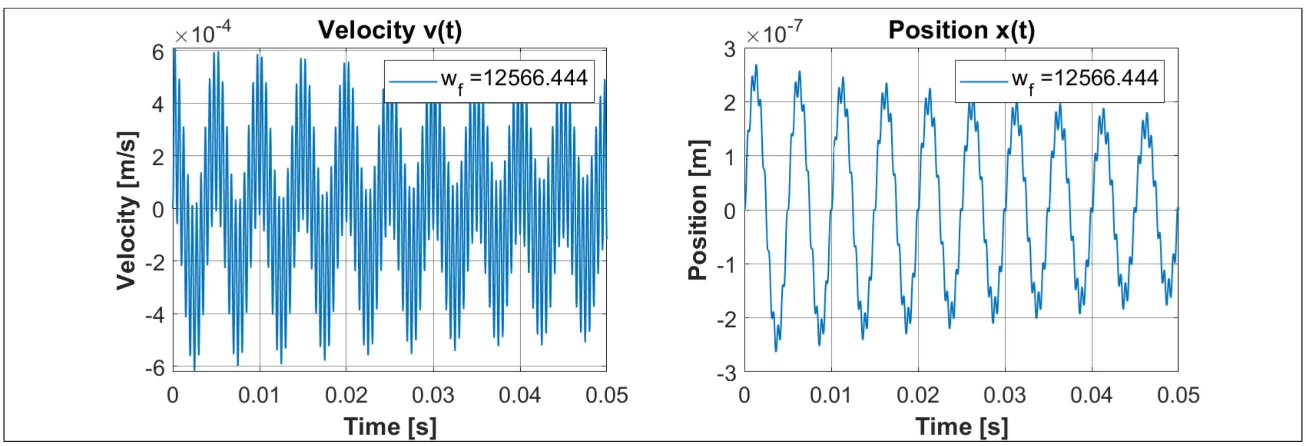

Figure 5: Mass-Spring Velocity & Position Plot for $\omega_f = 10\omega_n$

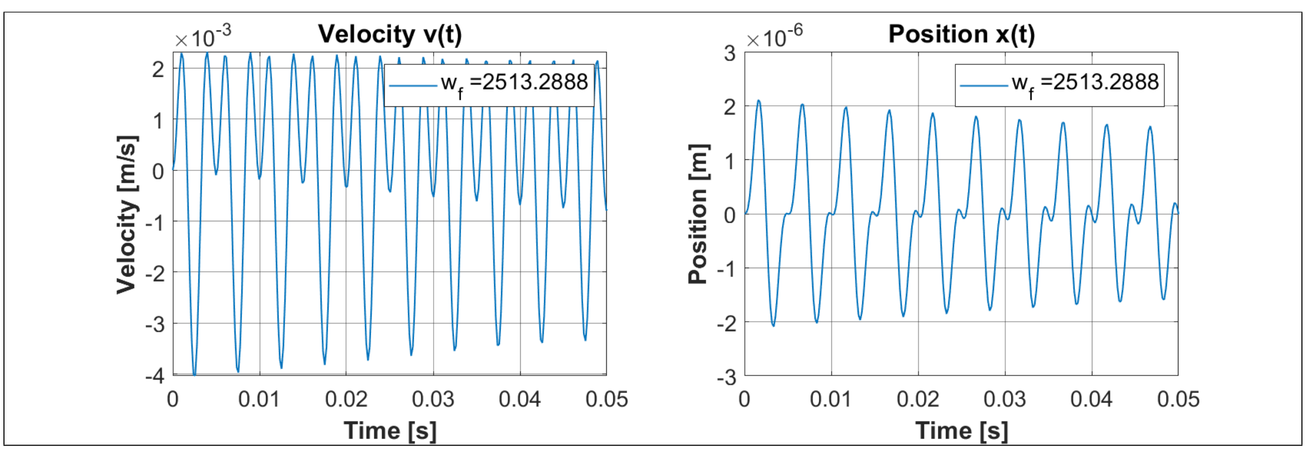

Figure 6: Mass-Spring Velocity & Position Plot for $\omega_f = 2\omega_n$

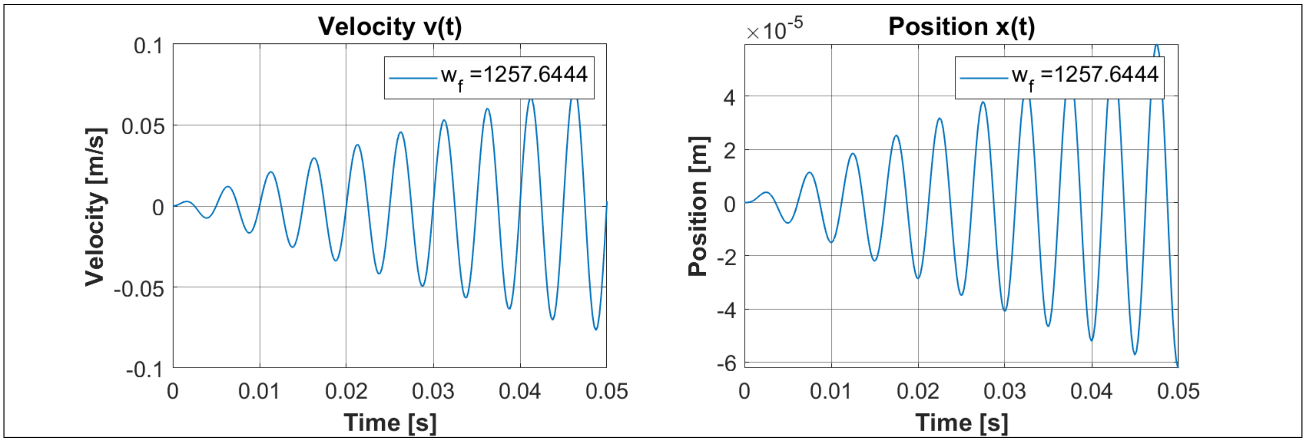

Figure 7: Mass-Spring Velocity & Position Plot for $\omega_f = \omega_n + 1$

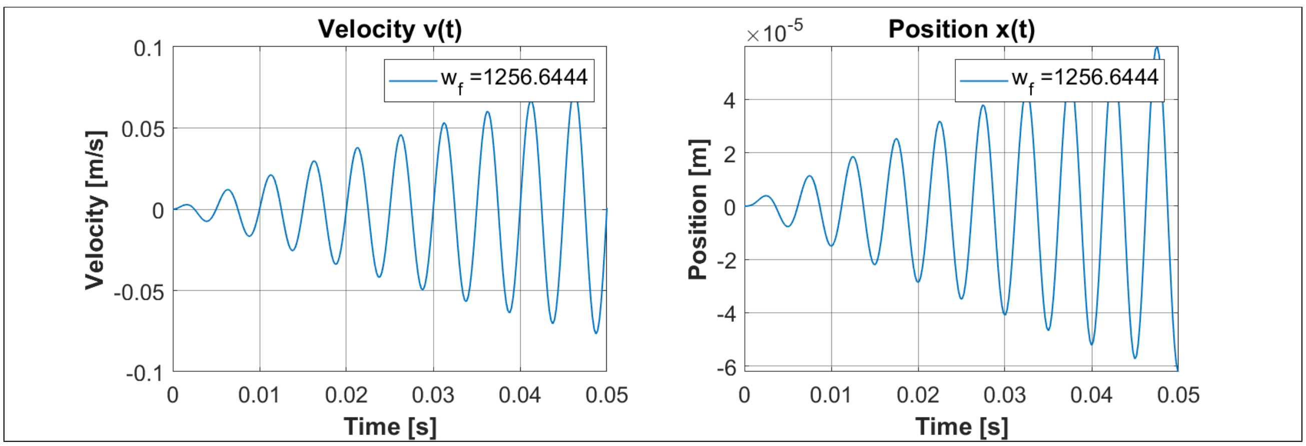

Figure 8: Mass-Spring Velocity & Position Plot for $\omega_f = \omega_n$

(c) Changing R
Problem Statement: Plot $Y(\omega), v(t)$, and $x(t)$ with varying R. Explain the effect of R on the plots.
From varying R, the following plots are shown. Note that on the legend of the following plot, R_i/R is the ratio of the varying resistance to the initial resistance ($R = 10$) stated in the problem.

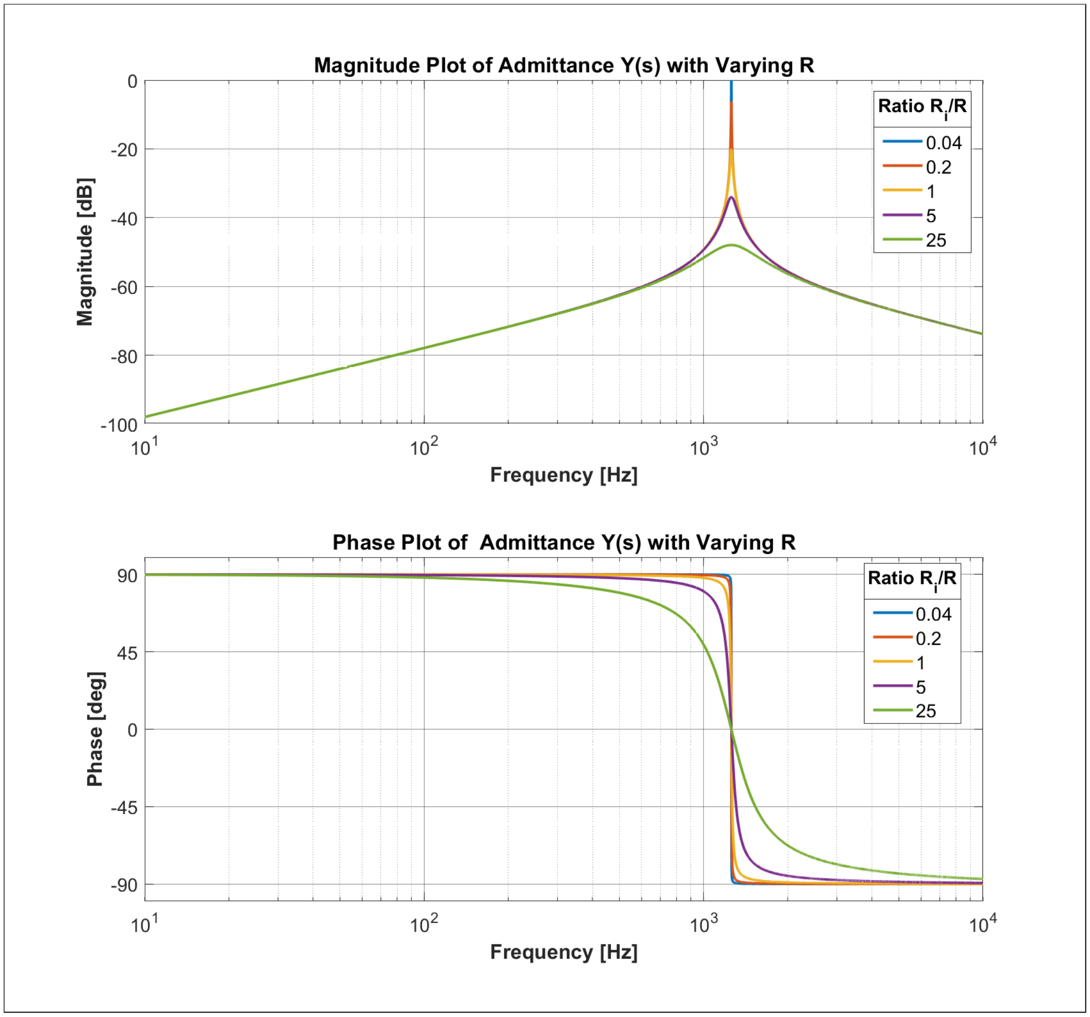

Figure 9: Y(s) with Varying R

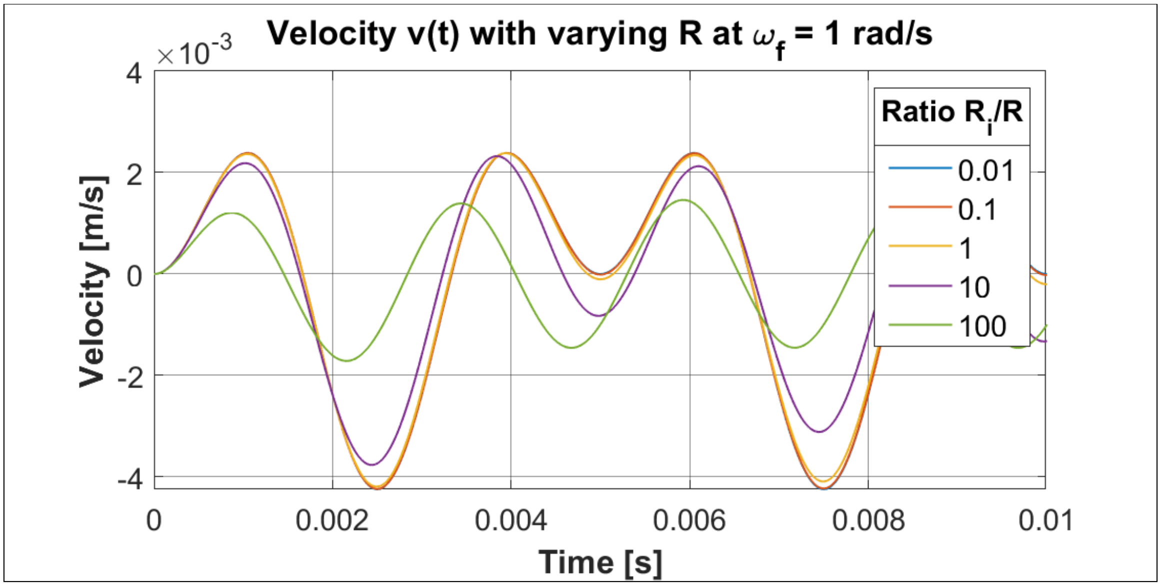

Figure 10: Velocity Plot with Varying R at $\omega_f = 2\omega_n$

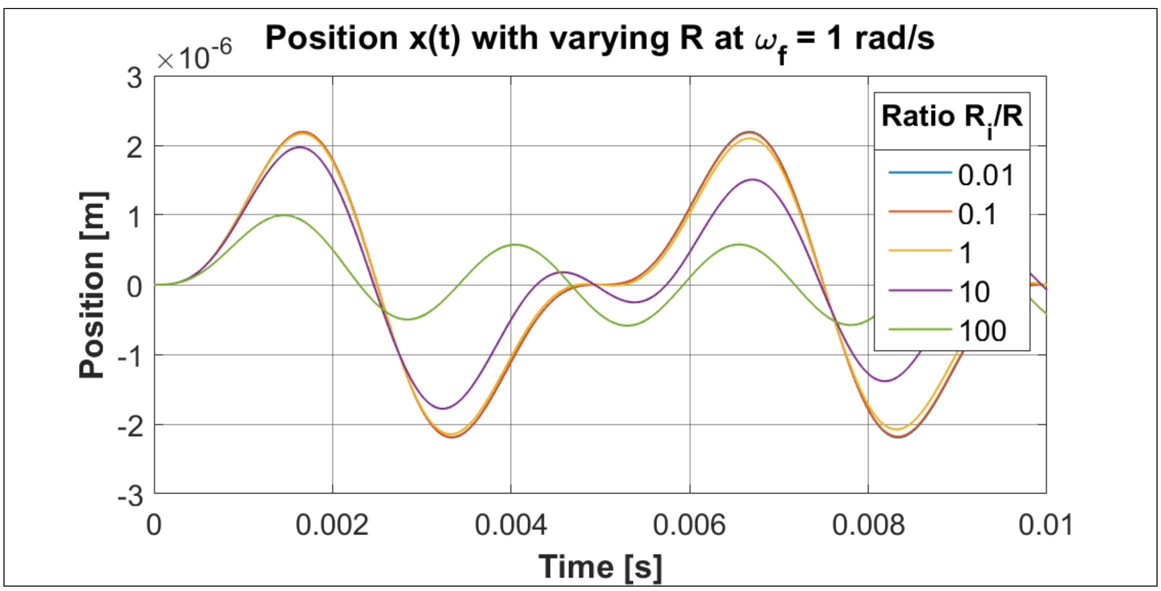

Figure 11: Displacement Plot with Varying R $\omega_f = 2\omega_n$

From figure 9, the magnitude of $Y(s)$ deviates from among each other as R varies except when the frequency is close to the natural frequency. At the natural frequency, the magnitude deceases as R increases. Near the natural frequency, the phase's curve become more smooth from going 90° to -90°. As R increases, the amplitude of the velocity and displacement plot decreases which make senses.

(d) Changing C

Problem Statement: Plot $Y(\omega), v(t), and x(t)$ with varying R. Explain the effect of R on the plots. Note that on the legend of the following plot, C_i/C is the ratio of the varying resistance to the initial resistance ($C = 1.2664 \times 10^{-6}$) stated in the problem.

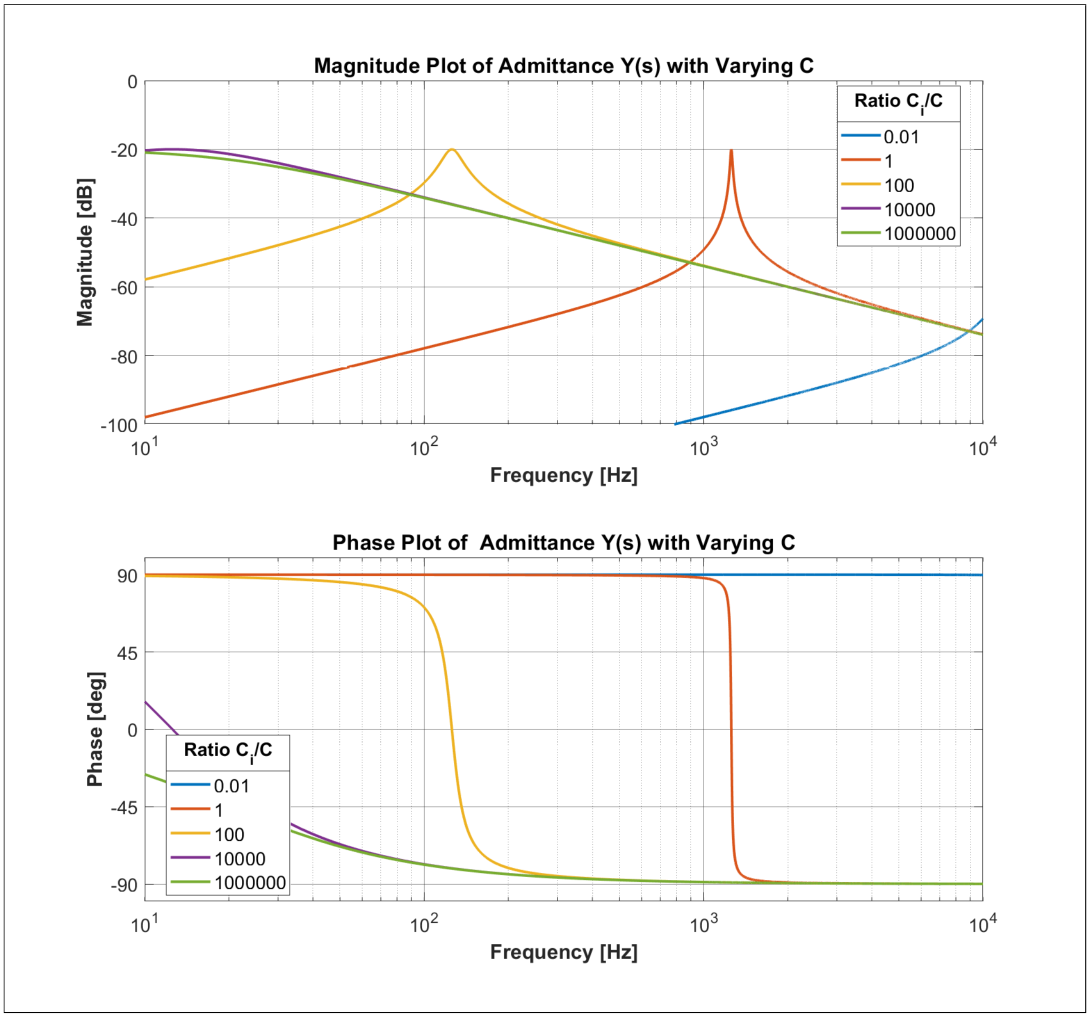

Figure 12: Y(s) with Varying C

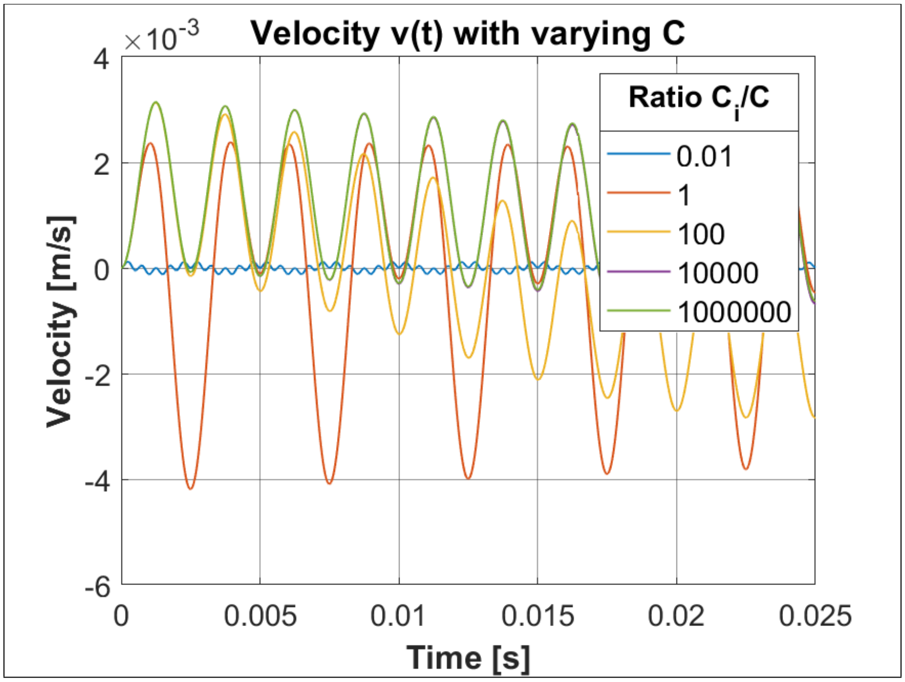

Figure 13: Velocity Plot with Varying C at $\omega_f = 2\omega_n$

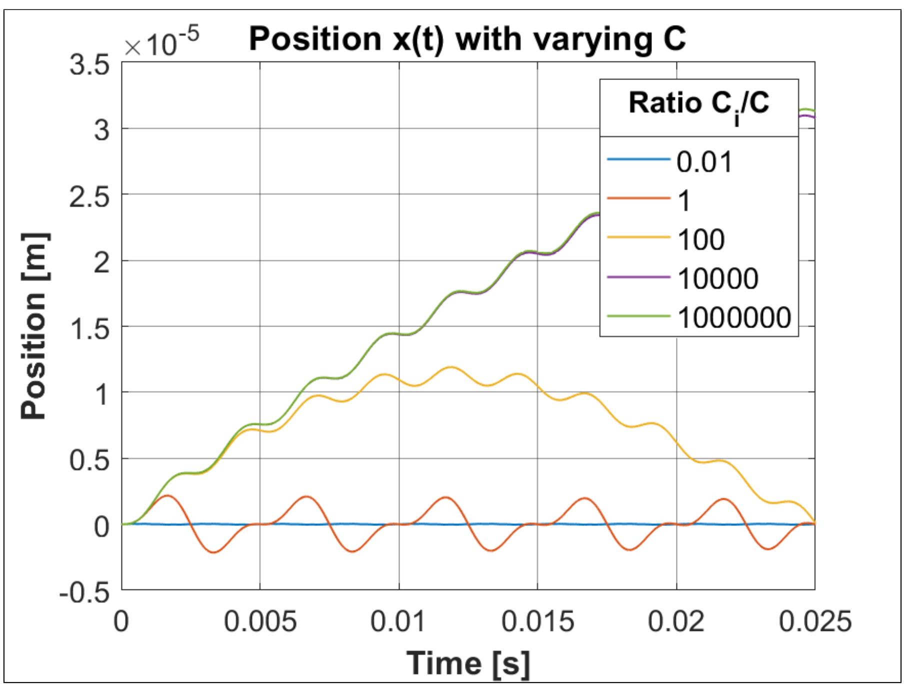

Figure 14: Displacement Plot with Varying C $\omega_f = 2\omega_n$

From Figure 12, the magnitude and the phase of $Y(s)$ translate to the left as C increases. The magnitude and phase plots of $Y(s)$ near the natural frequency are more smooth, or there is a less of a peak as C increases. On Figure 13, the amplitude increases as C increases, and its frequencies are out of phase among each other.. On Figure 14, the group frequency oscillation increases as C increases.

Discussion:
The Bode plot shows the behavior of the admittance transfer function when an input force of a given frequency is applied to the system. When the driven frequency is close to the natural frequency,the amplitude of the output has been amplified. When the driven frequency is severely greater than or less than to the natural frequency,the amplitude of the output has been attenuated greatly. In addition, the effects of C and R have substantial effects on the behavior of the Bode plot in defining the shape, resonant frequency location, and magnitude. The position and velocity plots represent this behavior in the time domain. It is important to analyze what factors affect the system behavior and what appropriate frequency should the force input be in the system.

Question 2: A Mass-Spring-Damper System on an Incline

Problem statement : Find out what effect does the angle θ have on the magnitude of oscillation. Plot the oscillation.

Here is the drawing of a mass-spring-damper system on an incline with an angle θ.

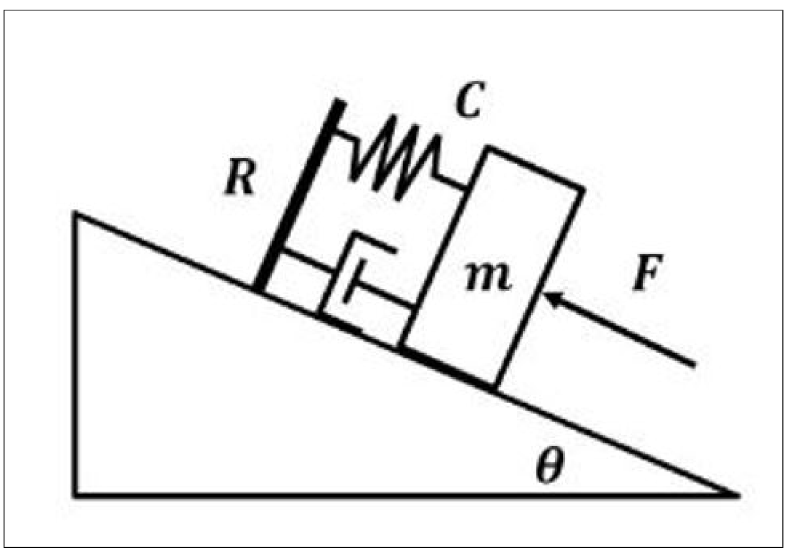

Figure 15: Drawing of the Mass-Spring-Damper System on an Incline with an Angle θ

Note that Figure 15 can be rotated by an angle θ to get the mass-spring-damper system in the horizontal direction. Then the gravitational force is multiplied by $sin(\theta)$, pointing in the opposite direction of F.

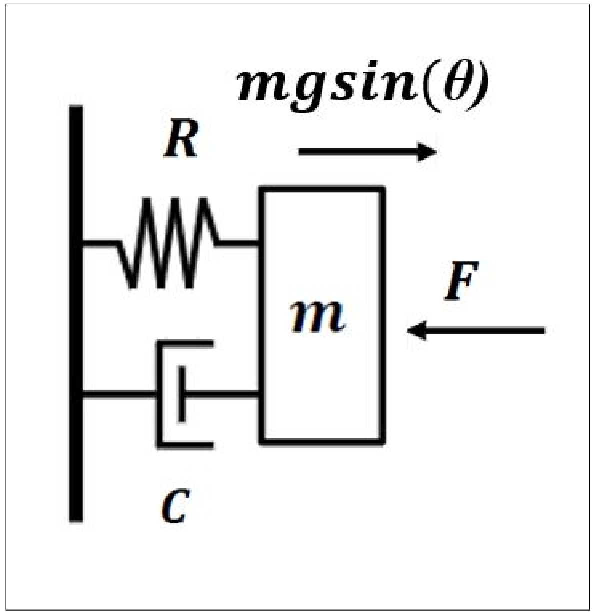

Figure 16: Drawing of an Equivalent & Rotated Figure 16

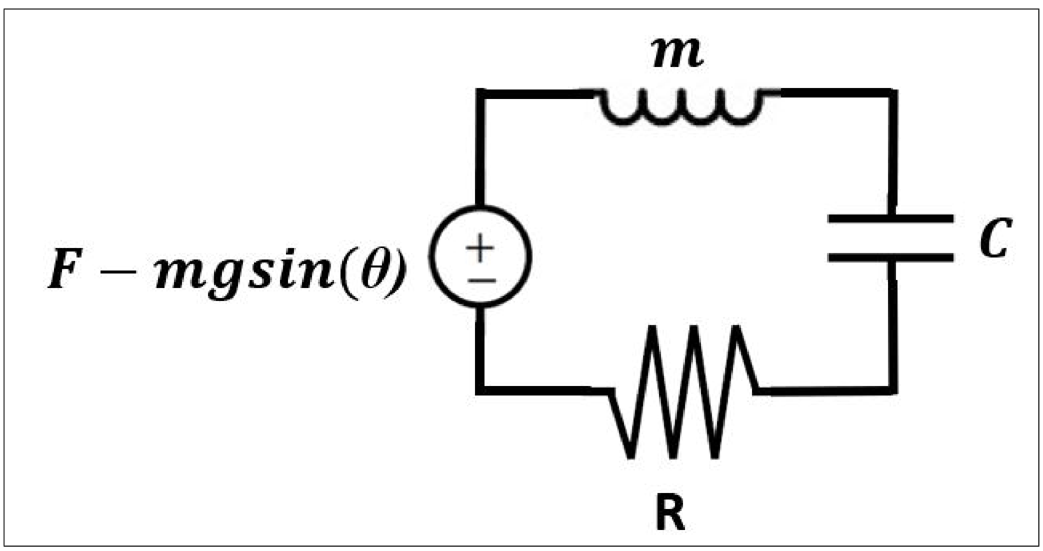

Figure 17: Equivalent Circuit Drawing of Figure 15

Next is to find the admittance transfer function $Y(s)$ of the system in the following equation.

$$
\begin{aligned}
Z &= ms^2 + \frac{1}{Cs} + R \\
Y &= \frac{1}{Z} \\
Y &= \frac{1}{ms + R + \frac{1}{Cs}}
\end{aligned}
\tag{4}
$$

The force input in the system is assumed to be $F = 10cos(10t)$. The acting force of the system is shown in the following equation.

$$\begin{aligned} F_a &= F - mgsin(\theta) \\ &= 10cos(10t) - mgsin(\theta) \\ \mathscr{L}\{F_a\} &= \mathscr{L}\{10cos(10t) - mgsin(\theta)\} \\ F_a(s) &= \frac{s}{s^2+100} - \frac{mgsin(\theta)}{s} \end{aligned} \tag{5}$$

The position of the system can be rewritten as $X(s) = \frac{Y(s)F(s)}{s}$. Note that the velocity is pointing up in the direction of the incline. Here is the plot with varying θ.

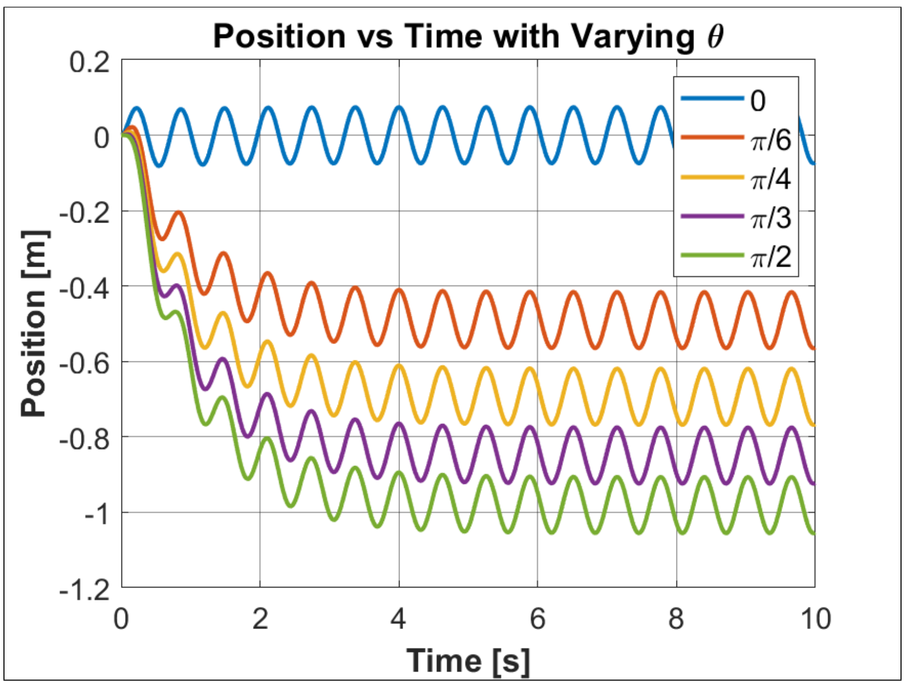

Figure 18: Plots of the displacement vs time over varying θ

Discussion: Since $Y(s)$ does not depends on θ, then the magnitude of the oscillation is not affected. The inclusion of gravity will only shift the equilibrium point of the mass. On Figure 18, the system will converge in oscillating around around a new equilibrium point. The new equilibrium point is depended on the angle θ. The oscillation frequency on the plot remains the same regardless of θ.

Question 3: Shaft & Disk System with a Harmonic Excitation

Problem statement : Compute and plot the response of a shaft and disk system to an applied moment of $M = 5\sin\omega_f t$ where $\omega_f = 215\frac{rad}{s}$. Assume that the 0.2-m radius disk is initially at rest. The 1-m long steel shaft has a diameter of 5 cm, a shear modulus, G, of 8.3×10^{10} $/m^2$ and a damping ratio of 0.01.

Here is the drawings of a shaft-disk system in the problem.

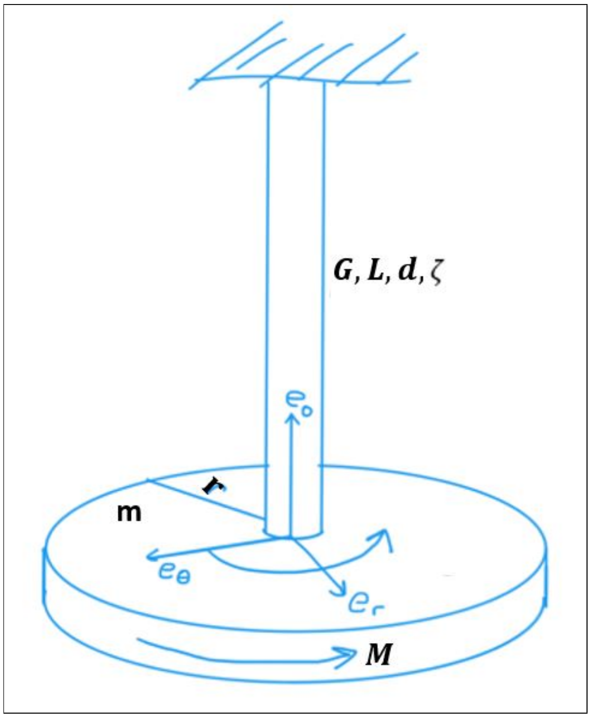

Figure 19: Drawing of the Shaft-Disk System

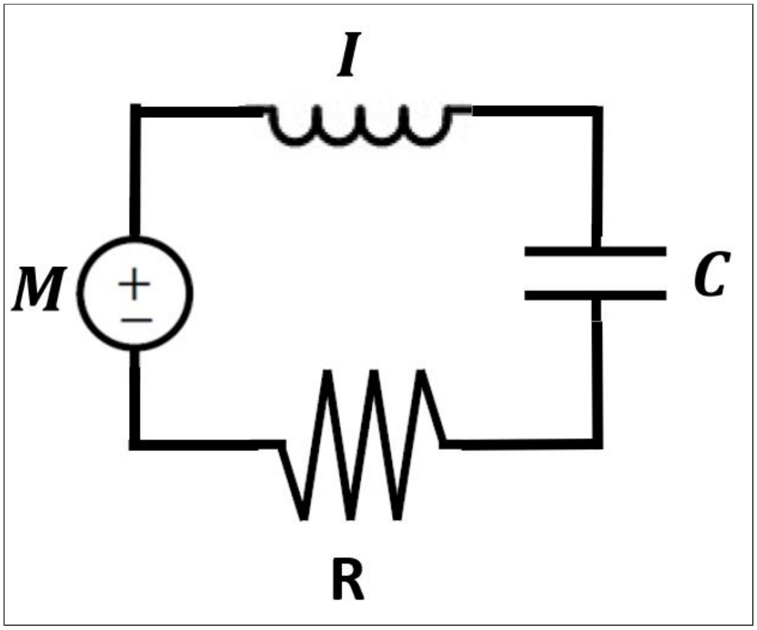

Figure 20: Equivalent Circuit of the Shaft-Disk System

Next is to find the admittance transfer function $Y(s)$ and $\Theta(s)$ of the system in the following equation. Note that I is the moment inertia of the disk. $I = \frac{1}{2}mr^2$.

$$\begin{aligned}
Z &= \frac{1}{2}mr^2 s + R + \frac{1}{Cs} \\
Y &= \frac{1}{Z} \\
Y &= \frac{1}{\frac{1}{2}mr^2 s + R + \frac{1}{Cs}} \\
\Omega(s) &= M(s)Y(s) \\
\mathscr{L}\{M\} &= \frac{5\omega_f}{s^2 + \omega_f^2} \\
\Omega(s) &= \frac{\frac{5\omega_f}{s^2+\omega_f^2}}{\frac{1}{2}mr^2 s + R + \frac{1}{Cs}} \\
\Theta(s) &= \frac{\Omega(s)}{s} \\
\Theta(s) &= \frac{\frac{5\omega_f}{s^2+\omega_f^2}}{\frac{1}{2}mr^2 s + R + \frac{1}{Cs}} \times \frac{1}{s}
\end{aligned} \tag{6}$$

Next is to find the C which is equal to $\frac{1}{k}$ where k is the stiffness. The equation for k and its sub-variable are in the following equations.

$$\begin{aligned}
k &= \frac{GJ_p}{l} \\
J_p &= \frac{1}{32}\pi {d_{shaft}}^4 \\
k &= \frac{G\frac{1}{32}\pi {d_{shaft}}^4}{l} \\
C &= \frac{1}{k} \\
C &= \frac{l}{G\frac{1}{32}\pi {d_{shaft}}^4}
\end{aligned} \tag{7}$$

J_p is the second of moment of inertia of the shaft.

Next is to find the R. The equation for R and its sub-variable are in the following equations.

$$\begin{aligned}
\omega_n &= \sqrt{\frac{1}{\frac{1}{2}mr^2 C}} \\
R &= 2I\omega_n\zeta
\end{aligned} \tag{8}$$

With the mass of the shaft to be $50kg$, $C = 1.9626 \times 10^{-5}\frac{kg*m^2}{s^2}$, $R = 4.5135\frac{kg*m^2}{s}$, $\omega_n = 225.6\frac{rad}{s}$. The plots of the angular velocity and angular displacement are shown n the following figures.

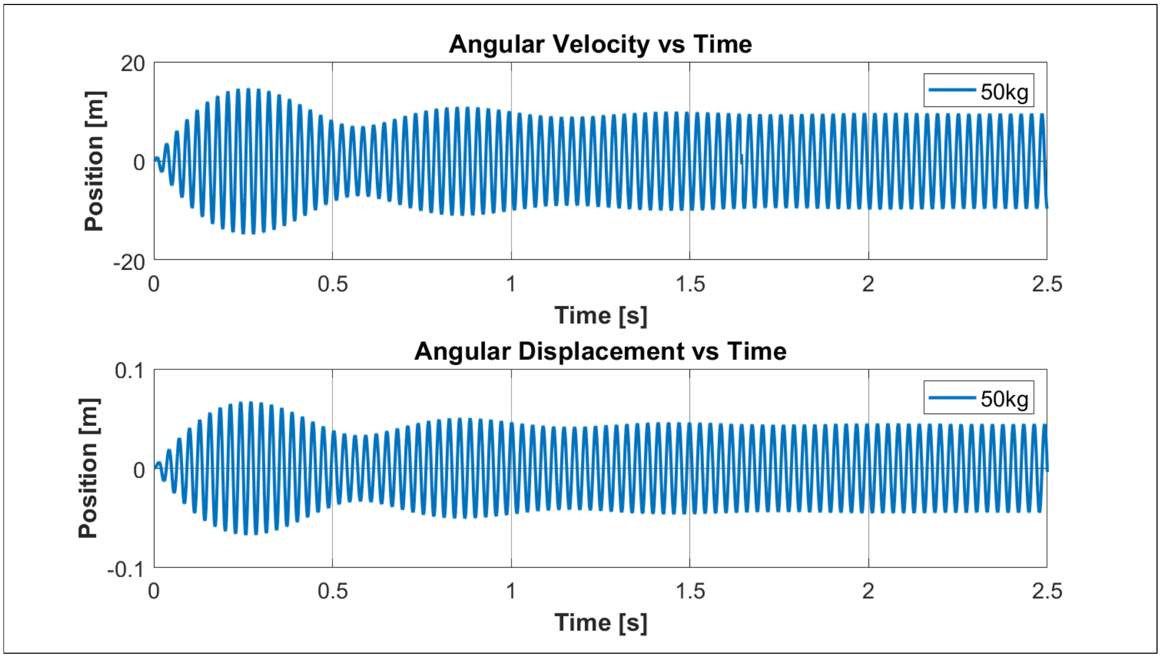

Figure 21: Equivalent Circuit of the Shaft-Disk System

Discussion: Based on the calculation, the natural frequency ω_n of the system ($225.6\frac{rad}{s}$) is very close to the forced frequency ω_f of the system ($215\frac{rad}{s}$). This can be seen in the velocity and displacement plot of Figure 21 by the initial presence of the beat frequency. An envelope is formed initially form with a frequency of about $10.6\frac{rad}{s}$ or $1.69Hz$. The period of the envelope would be $0.6s$ which is shown in the Figure 21. However, the envelope soon stop due to non-zero damping ratio, damping out the natural frequency ω_n, so the forced frequency ω_f is only present in the plot after some time.

Appendix

Below is the MATLAB code used for making the plots.

```
%Team B2 Hw4
%3/2/21

clc
clear all
close all

%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%-Question 1%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters
m = 0.5; % Mass in [kg]
C = 1.2665e-6; %Spring Compliance in [m/N]
R = 10;% Damping Coefficent in [Ns/m]
freqL  = 10;
```

```
freqH  = 10000;

%Y(jw) - Mechanical Admittance in Laplace Domain
w = (freqL:freqH);
ss = (1j)*w;
Y = 1./(m.*ss+1./(C.*ss)+R);

%Y(s) plot
mag = 20*log10(abs(Y));
phase = angle(Y)*180/pi;

figure(1)
set(gcf,'position',[0,0,1080,1080])

subplot(2,1,1);
mm = semilogx(w,mag);
set(mm, 'LineWidth', 3)
xlim([freqL,freqH])
ylim([-1e2,0])
grid on
xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('|Y(s)|','FontSize',14);
title('Magnitude Plot of Admittance Y(s)', 'FontSize', 16, 'FontWeight','bold')

subplot(2,1,2);
pp = semilogx(w,phase);
set(pp, 'LineWidth', 3)
xlim([freqL,freqH])
ylim([-1e2,1e2])
grid on
xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('\angleY(s)', 'FontSize',14,'Location','best')
title('Phase Plot of  Admittance Y(s)', 'FontSize', 16, 'FontWeight', 'bold')
set(gca,'YTick',(-2:2)*45)
saveas(gcf,'Y.png')
%%
%Define s
s = tf('s');

Y = 1/(m*s+1/(C*s)+R);
wn = sqrt(1/(C*m));
coeff = [25, 100, 20, 20, 20];
wf = [1, 10*wn, 2*wn, wn+1,wn];
xl = [0.05,0.05,0.05,0.05,0.05];

for ii = 1:length(wf)

    %Solving v(t) & x(t)
```

```
    N = round(coeff(ii)*wn); % Numbers of pts
    dt = 1/(coeff(ii)*wn); %differential time step
    t = 5*(0:N-1)*dt;

    F = 2*sin(wf(ii)*t);

    v = lsim(Y,F,t);
    x = lsim(Y/s,F,t);

    %Ploting v(t)
    figure(ii)
    set(gcf,'position',[0,0,1080,360])
    subplot(1,2,1);
    plot(t,v, 'LineWidth', 1);
    grid on
    xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
    ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    xlim([0,xl(ii)])
    legend('w_f =' + string(wf(ii)),'FontSize',14);
    title('Velocity v(t)', 'FontSize', 16, 'FontWeight','bold')

    %Ploting x(t)
    subplot(1,2,2);
    plot(t,x, 'LineWidth', 1);
    grid on
    xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
    ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    xlim([0,xl(ii)])
    legend('w_f =' + string(wf(ii)),'FontSize',14);
    title('Position x(t)', 'FontSize', 16, 'FontWeight','bold')
    saveas(gcf, string(wf(ii))+'.png');

end


%% Changing R & C
%Define s
s = tf('s');

%Parameters
m = 0.5; % Mass in [kg]
C = 1.2665e-6; %Spring Compliance in [m/N]
R = 10;% Damping Coefficent in [Ns/m]

%Y(jw) - Mechanical Admittance in Laplace Domain
w = (freqL:freqH);
ss = (1j)*w;

% set-up
len = 5;
Q = (1:len);
mag=zeros(length(Q),length(w));
```

```
phase=zeros(length(Q),length(w));
txt = strings(length(Q),1);
RR = zeros(length(Q),1);
CC = zeros(length(Q),1);
base = 10;

%DSP set-up
wn = sqrt(1/(C*m));
N = round(2.5*wn)*20; % Numbers of pts
dt = 1/(2.5*wn*20); %differential time step
t = (0:N-1)*dt;
wf = 2*wn; %rad/s
F = 2*sin(wf*t);

% x(t) and v(t) arrays
v = zeros(len, N);
x = zeros(len, N);

V=zeros(length(Q),1);
X=zeros(length(Q),1);

%Solving
for ii = 1:length(Q)
    %Y(s) plot
    RR(ii) = R*base^(Q(ii)-3);
    Y = 1./(m.*ss+1./(C.*ss)+RR(ii));
    YY = 1/(m*s+1/(C*s)+RR(ii));
    mag(ii,:) = 20*log10(abs(Y));
    phase(ii,:) = angle(Y)*180/pi;
    txt(ii) = strcat(string(base^(Q(ii)-3)));

    %Solving v(t) & x(t)
    v(ii,:) = lsim(YY,F,t);
    x(ii,:) = lsim(YY/s,F,t);
end

%---R Mag & Angle Plot---%
figure(4)
set(gcf,'position',[0,0,1080,1080])

    %----R Mag Plot----%
    subplot(2,1,1);
    mm = semilogx(w,mag);
    set(mm, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,0])
    grid on
    xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
    ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt,'FontSize',14);
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio R_i/R')
```

```
    title('Magnitude Plot of Admittance Y(s) with Varying R', 'FontSize',
   16, 'FontWeight','bold')

    %----R Angle Plot----%
    subplot(2,1,2);
    pp = semilogx(w,phase);
    set(pp, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,1e2])
    grid on
    xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt, 'FontSize',14,'Location','best');
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio R_i/R')
    title('Phase Plot of  Admittance Y(s) with Varying R', 'FontSize', 16,
    'FontWeight', 'bold')
    set(gca,'YTick',(-2:2)*45)
    saveas(gcf,'R.png')
%%
% %----R Velocity Plot----%
figure(5)
set(gcf,'position',[0,0,720,360])
plot(t,v, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,0.01])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio R_i/R')
title('Velocity v(t) with varying R at \omega_f = 1 rad/s', 'FontSize',
    16, 'FontWeight','bold')
saveas(gcf,'Rv.png');
%%
%----R Position Plot----%
figure(6)
set(gcf,'position',[0,0,720,360])
plot(t,x, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
xlim([0,0.01])
htitle = get(leg,'Title');
set(htitle, 'String','Ratio R_i/R')
title('Position x(t) with varying R at \omega_f = 1 rad/s', 'FontSize',
    16, 'FontWeight','bold')
saveas(gcf,'Rx.png');
```

```
%%
%-------------Varying C---------------------------------%

%DSP set-up
wn = sqrt(1/(C*m));
N = round(2.5*wn)*25^2; % Numbers of pts
dt = 1/(2.5*wn*25); %differential time step
t = (0:N-1)*dt;
base = 100;
wf = 2*wn;
F = 2*sin(wf*t);

v = zeros(len, N);
x = zeros(len, N);

%Solving
for ii = 1:length(Q)
    CC(ii) = C*base^(ii-2);
    Y = 1./(m.*ss+1./(CC(ii).*ss)+R);
    YY = 1/(m*s+1/(CC(ii)*s)+R);
    mag(ii,:) = 20*log10(abs(Y));
    phase(ii,:) = angle(Y)*180/pi;
    txt(ii) = strcat(string(base^(Q(ii)-2)));

    %Solving v(t) & x(t)
    v(ii,:) = lsim(YY,F,t);
    x(ii,:) = lsim(YY/s,F,t);

end

%---C Mag & Angle Plot---%
figure(7)
set(gcf,'position',[0,0,1080,1080])

    %----C Mag Plot----%
    subplot(2,1,1);
    mm = semilogx(w,mag);
    set(mm, 'LineWidth', 2)
    xlim([freqL,freqH])
    ylim([-1e2,0])
    grid on
    xlabel('Frequency [Hz]', 'FontSize',16,'FontWeight','bold')
    ylabel('Magnitude [dB]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt,'FontSize',14, 'Location', 'best');
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio C_i/C')
    title('Magnitude Plot of Admittance Y(s) with Varying C', 'FontSize',
   16, 'FontWeight','bold')

    %----C Angle Plot----%
    subplot(2,1,2);
    pp = semilogx(w,phase);
    set(pp, 'LineWidth', 2)
```

```matlab
    xlim([freqL,freqH])
    ylim([-1e2,1e2])
    grid on
    xlabel('Frequency [Hz]', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Phase [deg]', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
    leg = legend(txt, 'FontSize',14,'Location','best');
    htitle = get(leg,'Title');
    set(htitle, 'String','Ratio C_i/C')
    title('Phase Plot of  Admittance Y(s) with Varying C', 'FontSize', 16,
    'FontWeight', 'bold')
    set(gca,'YTick',(-2:2)*45)
    saveas(gcf,'C.png')

%----C Velocity Plot----%
figure(8)
plot(t,v, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Velocity [m/s]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,0.025])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio C_i/C')
title('Velocity v(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'Cv.png');

%----C Position Plot----%
figure(9)
plot(t,x, 'LineWidth', 1);
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,0.025])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
leg = legend(txt,'FontSize',14);
htitle = get(leg,'Title');
set(htitle, 'String','Ratio C_i/C')
title('Position x(t) with varying C', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'Cx.png');

%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%-Question 2%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 1; %kg
g = 9.81; %N/kg
C = 0.1; %m/N
R = 10; %kg/s
theta = [0,pi/6,pi/4,2*pi/3,pi/2];
wn = sqrt(1/m*C);

s = tf('s');
```

```
Y = 1/(m*s+1/(C*s)+R);
wf = 10;
dt = 1/(250*wn);
t = (0:dt:10);

figure(1)
for ii = 1:length(theta)
    F = 10*s/(s^2+wf^2) - m*g*sin(theta(ii))/s;
    X = F*Y/s;
    x = impulse(X,t);
    plot(t,x, 'LineWidth', 2)
    hold on
end
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
xlim([0,10])
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('0', '\pi/6','\pi/4','\pi/3','\pi/2','FontSize',14);
title('Position vs Time with Varying \theta', 'FontSize', 16, 'FontWeight'
    ,'bold')
saveas(gcf,'Q2x.png');

%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%-Question 3%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

s = tf('s');

m = 50; %kg
L = 1; %m
d = 0.05; %m
r = 0.2; %m
G = 8.3*10^10; %Pa
zeta = 0.01;
w_f = 215; %rad/s

J = pi/32*d^4;
k = G*J/L;
C = 1/k;

I = 1/2*m*r^2;
wn = sqrt(1/(I*C));
R = 2*I*wn*zeta;

Y = 1/(I*s+R+1/(C*s));
M = w_f^2/(s^2+w_f^2);

Omega = Y*M;
Theta = Omega/s;

t = (0:0.001:2.5);
omega = impulse(Omega,t);
```

```
theta = impulse(Theta,t);

figure()
set(gcf,'position',[0,0,960,540])
subplot(2,1,1);
plot(t,omega, 'LineWidth', 2)
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('50kg','FontSize',14);
title('Angular Velocity vs Time', 'FontSize', 16, 'FontWeight','bold')

subplot(2,1,2);
plot(t,theta, 'LineWidth', 2)
grid on
xlabel('Time [s]', 'FontSize',16,'FontWeight','bold')
ylabel('Position [m]', 'FontSize', 16, 'FontWeight', 'bold')
set(gca, 'FontSize',14,'GridAlpha',0.5,'MinorGridAlpha', 0.5);
legend('50kg','FontSize',14);
title('Angular Displacement vs Time', 'FontSize', 16, 'FontWeight','bold')
saveas(gcf,'Q3.png');
```