

Project: Lipkin Model

1. Prove the commutation relations for the quasi-spin operators.
2. Plot the eigenvalues of the LM Hamiltonian matrix in the quasi-spin basis for $\varepsilon=1$, $\Omega=12$, $N=12$ as a function of $0 < V(\Omega - 1)/\varepsilon < 2$. Label eigenvalues using the conserved quantum numbers.
3. What happens with the two lowest eigenvalues around $V(\Omega - 1)/\varepsilon = 1$? Is it reminiscent of something you've seen before?
4. Consider the transition operator

$$\hat{Q} \equiv \hat{K}_x = \frac{1}{2} (\hat{K}_+ + \hat{K}_-)$$

What is expectation value of this operator in the ground state?

5. Calculate the transition matrix element of this operator between the ground state and the first excited state a function of $0 < V(\Omega - 1)/\varepsilon < 2$.
6. Find the eigenvalues of the LM Hamiltonian matrix in the CI basis (occupation number representation basis) for $\varepsilon=1$, $V=0.1$, $\Omega=2$, $N=2$ and for $\Omega=2$, $N=4$. Benchmark this result using the quasi-spin formalism. Identify quasi-spin and signature quantum numbers. Discuss the result.