Multivector Calculus

Brandon Henke

Table of contents

1 Stoke's Theorem

Let $\mathcal{V}$ be a closed interval.

$$\int_{\partial\mathcal{V}} f = \int_{\mathcal{V}} df\,. \tag{1.1}$$

In one dimension, let $\mathcal{V} = [a, b]$. Then $\partial\mathcal{V} = \{a, b\}$, and

$$\begin{aligned} \int_{\partial\mathcal{V}} f &= f(b) - f(a), \\ \therefore \int_{\mathcal{V}} f(x)\,dx &= f(b) - f(a). \end{aligned}$$

Let $a = x$ and $b = x + \Delta x$. Then

$$\lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{\mathcal{V}} f(x)\,dx = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}. \tag{1.2}$$

Thus, define

$$\frac{df}{dx} \equiv \lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{\mathcal{V}} f(x)\,dx\,. \tag{1.3}$$

A bit more generally,

$$\nabla f \equiv \lim_{|\mathcal{V}| \to 0} \frac{1}{|\mathcal{V}|} \int_{\partial\mathcal{V}} f. \tag{1.4}$$