Spherical Harmonic Oscillator

$$\hat{h} = \hat{t} + \frac{m\omega_0^2 r^2}{2} \Rightarrow \quad \varepsilon_N = \left(N + \frac{3}{2}\right)\hbar\omega_0$$

For each shell, the allowed orbital angular momenta are:

$$\ell = N, N-2, \ldots, 1, \text{ or } 0, \quad j = \ell \pm \frac{1}{2}$$

Since each nucleon has an intrinsic spin s=1/2, the maximum number of nucleons in a HO shell is:

$$D_N = \sum_{\ell} 2(2\ell+1) = (N+1)(N+2) \underset{N>>1}{\approx} \left(N + \frac{3}{2}\right)^2$$

The total number of states is:

$$\sum_{N'=0}^{N} (N'+1)(N'+2) = \frac{1}{3}(N+1)(N+2)(N+3)$$

$$\underset{N>>1}{\approx} \frac{1}{3}(N+2)^3$$

N	l	DEGEN.	TOTAL
5	1,3,5	42	112
4	0,2,4	30	70
3	1,3	20	40
2	0,2	12	20
1	1	6	8
0	0	2	2

Dimension of orbits:

$$\left\langle r^2 \right\rangle_{N\ell} = \frac{\hbar}{m\omega_0}\left(N + \frac{3}{2}\right)$$

$$\Rightarrow \quad \hbar\omega_0 \approx \frac{41}{A^{1/3}}(\text{MeV})$$

Project 2: Shell structure of the Modified Spherical Harmonic Oscillator (MHO)

See Sec. 6 of textbook by Nilsson and Ragnarsson on shapes and shells Nucl. Phys. A131, 1 (1969) ; Phys. Scr. 39, 196 (1989)

$$\hbar\omega_0 = \frac{41}{A^{1/3}}\left(1 \pm \frac{N-Z}{3A}\right)\ \mathrm{MeV}$$

the plus sign holds for neutrons and the minus sign for protons

The spin-orbit term is proportional to the Thomas potential

$$V_{\ell s} = \frac{g}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\vec{\ell}\cdot\vec{s}$$

Hence, for the harmonic oscillator, $V_{\ell s} = -2\kappa\hbar\omega_0\vec{\ell}\cdot\vec{s}$

To account for the flat-bottom effect, one modifies the harmonic oscillator potential by adding the so-called ℓ^2 term:

$$V_{\ell\ell} = -\kappa\mu\hbar\omega_0\left(\vec{\ell}^2 - \langle\vec{\ell}^2\rangle_N\right)$$

$$\langle\vec{\ell}^2\rangle_N = N(N+3)/2$$

the average value of ℓ^2 within an N-shell. It is introduced to restore the average position of the shell

MHO single-particle energies:

$$\epsilon(N\ell j) = \hbar\omega_0\left[N + \frac{3}{2} - \kappa\begin{Bmatrix}\ell \\ -(\ell+1)\end{Bmatrix} - \mu'\left(\ell(\ell+1) - \frac{N(N+3)}{2}\right)\right]\begin{cases} j = \ell + \frac{1}{2} \\ j = \ell - \frac{1}{2}\end{cases}$$

1. Consider the “empirical” single-particle energy levels listed in Table II of Schwierz et al. Extract the values of proton and neutron magic gaps at particle numbers 8, 20, 28, 50, 82, and 126. For instance, the size of the neutron (126) gap in ^{208}Pb is -3.94-(-7.37) MeV=3.43 MeV.

^{208}**Pb**				
neutron hole				^{207}Pb
$1h_{9/2}$	-11.40	4.036	0.98	3.400,5.410,5.620 [28]
$2f_{7/2}$	-9.81	2.439	0.95	2,340,4.570 [28]
$1i_{13/2}$	-9.24	1.870	0.91	1.630,5.990 [28]
$3p_{3/2}$	-8.26	0.89	0.88	0.890 [21]
$2f_{5/2}$	-7.94	0.57	0.60	0.570 [21]
$3p_{1/2}$	-7.37	0	0.90	0 [21]
neutron particle				^{209}Pb [22]
$2g_{9/2}$	-3.94	0	0.83	0
$1i_{11/2}$	-3.16	0.779	0.86	0.779
$1j_{15/2}$	-2.51	1.424	0.58	1.424
$3d_{5/2}$	-2.37	1.565	0.98	1.565
$4s_{1/2}$	-1.90	2.033	0.98	2.033
$2g_{7/2}$	-1.44	2.492	1.05	2.492
$3d_{3/2}$	-1.40	2.537	1.09	2.537

2. By means of the chi^2 minimization, adjust the proton and neutron parameters κ and $\mu'=\kappa\mu$ of the MHO potential to the magic gaps extracted in point 1. Reasonable starting values are $\kappa_p = \kappa_n = 0.06$, $\mu'_p=0.04$, and $\mu'_n=0.02$ (see Table 1 of Nucl. Phys. A131, 1 (1969))
3. How uncertain are MHO parameters? Are they correlated?
4. Plot the MHO spectrum. Label the spherical shells.
5. According to the MHO, what should be the next magic nucleus beyond ^{208}Pb? What are associated magic gaps (with uncertainties)?