**PHY493/803 Spring 2022, Intro to Elementary Particle Physics**

**Midterm Exam 2**

***DO NOT TURN THIS PAGE UNTIL THE EXAM STARTS***

*While waiting, carefully fill in the information requested below.*

Your Name : ______________________________

Your Student ID Number : ______________________

*This exam is worth 100 points. There are* **five** *problems and each has 25 points. Partial points are indicated in the first line.* **Choose any 4 of the 5 problems to complete. PHY803 students must do Problem #5.**

*Take a moment and look over the exam before you begin. To receive the full credit for each answer, you must work neatly, show your work and simplify your answer to the extent possible.*

The last page of this exam contains a list of QED Feynman rules, useful for problem 4.

1. (5pt x 5) Answer True or False for each question:

*a)* Fermi's Golden Rule applies to calculations of the decay rate of particles and also of interaction cross sections.

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*b)* QED is a full, quantum mechanical model for spin-1/2 particle interactions.

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c) The cyclotron uses a changing magnetic field to accelerate particles.

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d) Cherenkov detectors make use of the radiation produced when particles exceed the speed of light in the material they are traversing.

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e) At an energy of 10 GeV, charged pions lose energy primarily via ionization when passing through materials.

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2. (10 + 5 + 5 + 5 pts) Provide brief (1-3 sentence) answers.

a) What is the dominant mechanisms by which high-energy electrons lose energy when travelling through material?

b) What is the photo-electric effect?

c) When particles are stored in a synchrotron accelerator, their energy … (mark all correct answers)

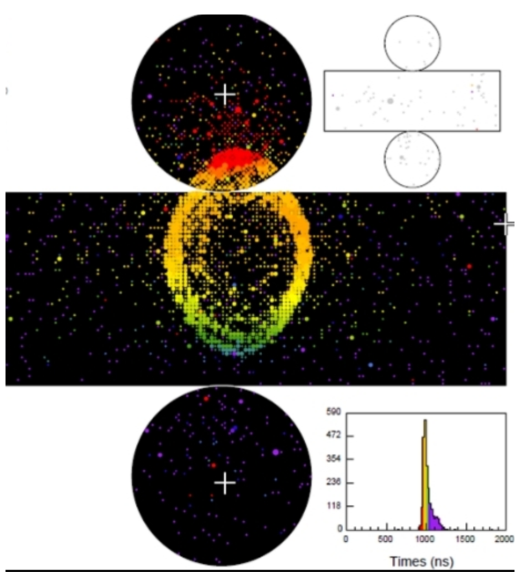
(1) falls due to synchrotron radiation.

(2) is always growing.

(3) grows if RF power is added.

(4) falls due to Lorentz contraction.

d) What is shown in the following picture? What is the detector and what is the interaction?

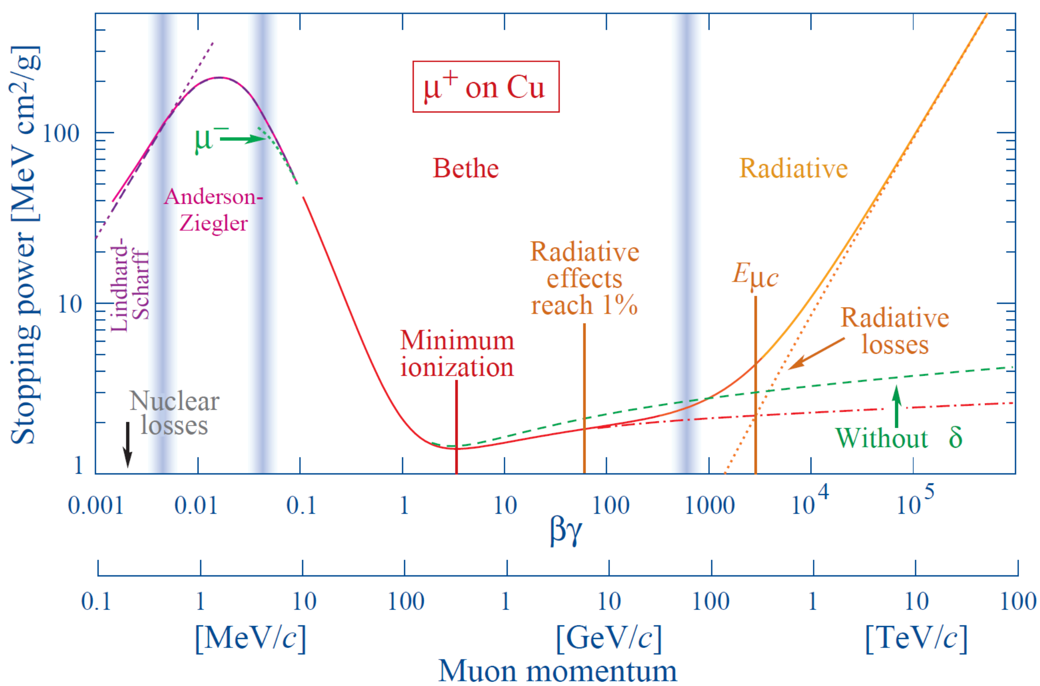


3. (10 + 10 + 5 pts) Kaon decay

The mass of the charged Kaon is about 500 MeV and its lifetime is about 12 ns. The leptonic branching ratio of the Kaon (to muon+neutrino) is about 63%, and the mass of the muon is about 100 MeV.

a) How many muons have been produced in 0.1 microseconds starting from 1E6 Kaons?

b) The Kaons are at rest and surrounded by a Copper absorber (density about 9 g/cm$^3$). How much energy (to within a factor 2) does a muon from a Kaon decay lose when traveling 10 cm in the absorber?




c) Describe the process and draw the Feynman diagram for the dominant process by which these muons lose energy in Copper.

4. (5 + 5 + 5 + 10 pts) $e^+ + \mu^- \rightarrow e^+ + \mu^-$

a) Draw the lowest-order Feynman diagrams for the process $e^+ + \mu^- \rightarrow e^+ + \mu^-$. How many diagrams are there?

b) Using the Feynman rules for QED, set up the formula for the matrix element for the t-channel diagram for $e^+ + \mu^- \rightarrow e^+ + \mu^-$. Just write down the diagram, do not perform the integration at this point. Label the incoming and outgoing muons 1 and 3, respectively. Label the incoming and outgoing positrons 2 and 4, respectively. Feynman rules are given on the last page of the exam handout.

c) Now perform the integration and calculate the matrix element. Don't plug in any traces yet.

d) The matrix element squared, averaged over initial-state spins and summing final state spins is given by $\langle |M|^2 \rangle = \frac{g_e^4}{4t^2} \mathrm{Tr}[\gamma^\mu (\not{p}_1 - m_e)\gamma^\nu (\not{p}_3 - m_e)] \mathrm{Tr}[\gamma_\mu (\not{p}_2 + m_\mu)\gamma_\nu (\not{p}_4 + m_\mu)]$

Use the trace relationships to write $\langle |M|^2 \rangle$ in terms of the momenta of the four particles. You can neglect the electron mass but not the muon mass.

5. (25 pt) $e^- + \gamma$ **Required for 803 students, optional for 493 students.** Draw the lowest-order diagrams for the QED process $e^- + \gamma \rightarrow e^+ + e^- + e^-$. Hint: this is similar to Compton scattering, but with a 3-particle final state. There are 16 lowest-order diagrams, you will get full credit if you can find 8 unique diagrams.

**The Feynman rules for QED are given below. Feel free to tear these pages off.**

1: Draw the Feynman diagrams, including the appropriate arrows for particles and antiparticles.

2: Label incoming and outgoing 4-momenta for each vertex, including the internal momenta of propagators. Conventionally, external 4-momenta are labeled $p_i$ and internal 4-momenta are labeled $q_i$.

3: Each external line gets a factor for the wave function, sandwiching the vertex in a current.

Fermions:

- Incoming particle: $u(p)$ Incoming antiparticle: $\bar{v}(p)$
- Outgoing particle: $\bar{u}(p)$ Outgoing antiparticle: $v(p)$

Photons:

- Incoming photon: $\varepsilon^{\mu}(p)$ Outgoing photon: $\varepsilon^{\mu}(p)^*$

4: Each vertex is assigned a factor of $(ig_e\gamma^{\mu})$, specifying the coupling strength of the interaction at that vertex.

5: Each internal propagator line gets a factor of:

$\frac{-ig_{\mu\nu}}{q^2}$ for photons, $\frac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$ for fermions.

6: Each vertex gets a delta function to enforce conservation of energy and momentum ($k_i$ are the 4-momenta into/out of the vertex). The sign of each 4-momentum must be properly assigned, as necessary:

$$(2\pi)^4\delta^4(k_1 + k_2 + k_3)$$

7:  Each internal propagator line gets a phase-space integration factor:

$$\frac{d^4 q_j}{(2\pi)^4}$$

8:  The final matrix element is obtained by integrating over the propagator 4-momenta.  Cancel out any remaining delta function factors (and factors of $(2\pi)^4$) and multiply by another factor ($i$).  What remains is the matrix element.

9: Antisymmetrization: Include a minus sign between diagrams that differ only in the interchange of two incoming (or outgoing) electrons (or positrons), or of an incoming electron with an outgoing positron (or vice versa).