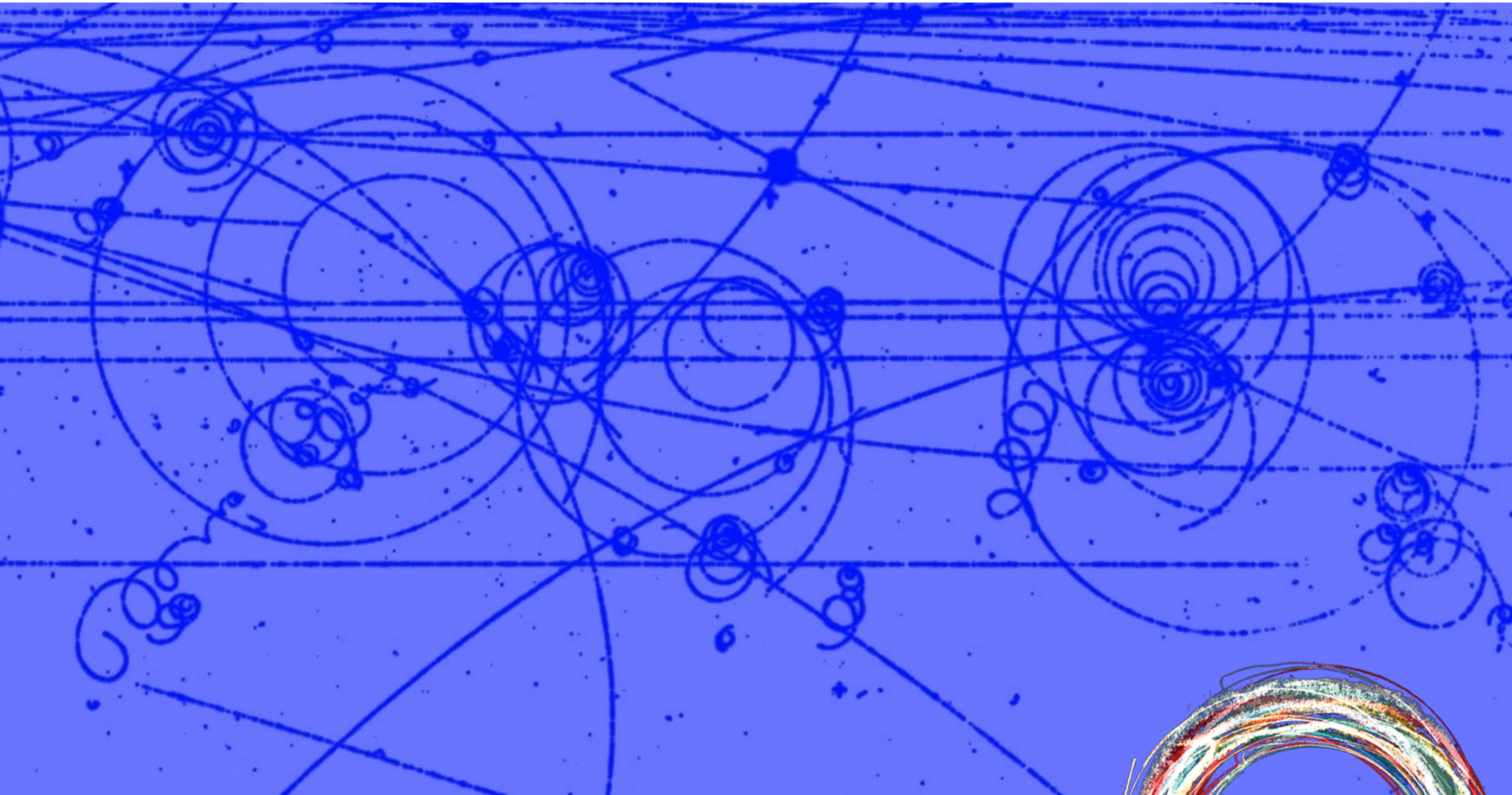


QED OF QUARKS

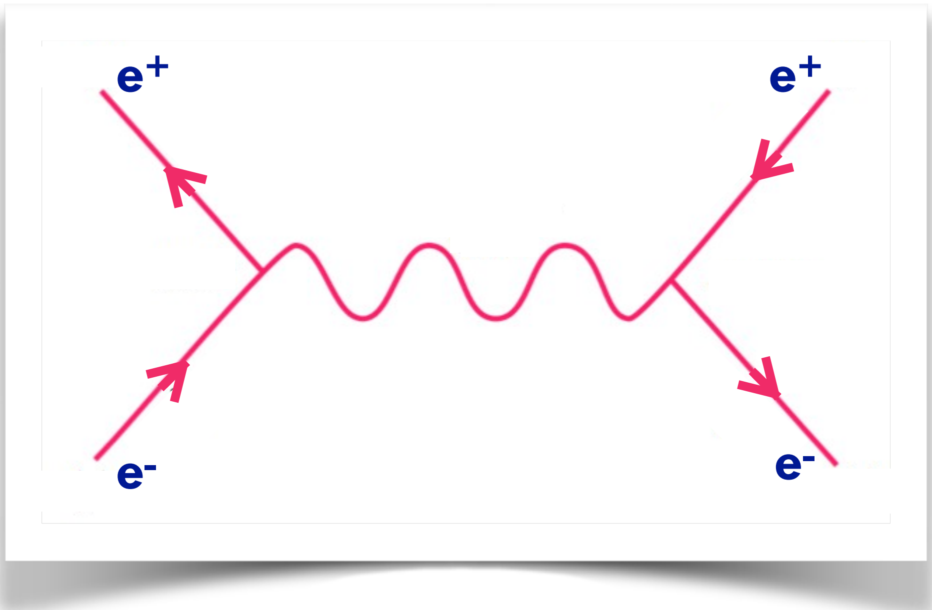
PHY 493/803

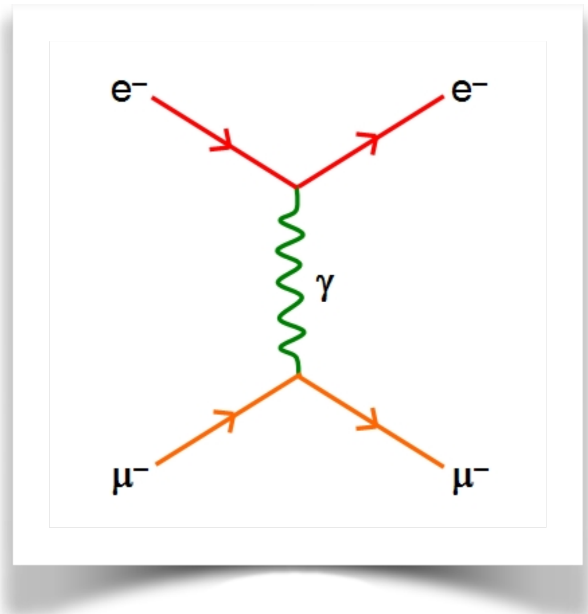
Book chapters covered this week

- Chapter 8, Electrodynamics and Chromodynamics of quarks
-

Quarks in QED

We saw previously how quantum electrodynamics describes the relationship between charged particles and photons.



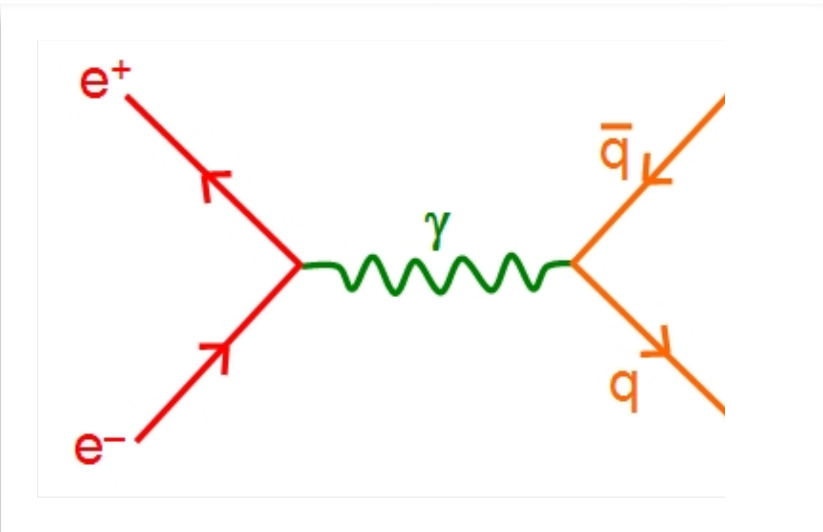


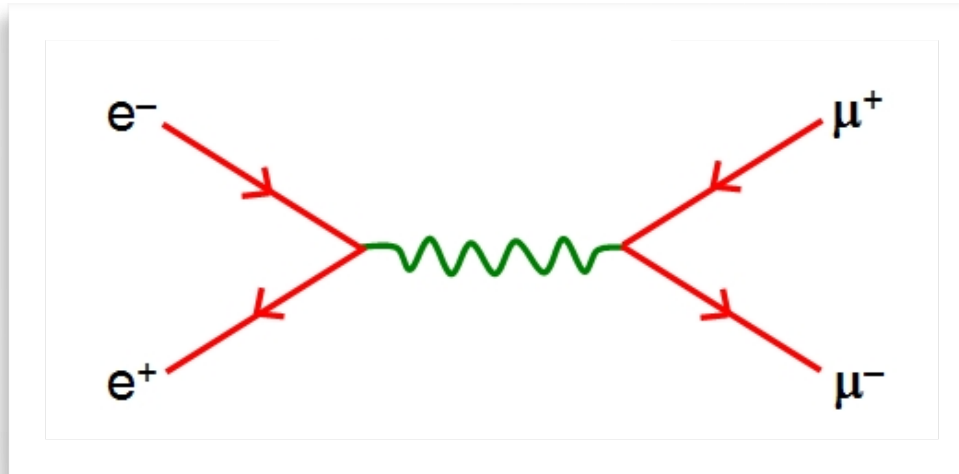
The QED couplings (vertex factors) only depend on the charge involved, not the flavor (or type) of the particle itself.

Cross sections and decay rates obviously depend on particle mass, but that's the only difference between electron and muon currents.

Quarks in QED

Thus we should expect that QED interactions with quarks should have an identical behavior as with electrons or muons!

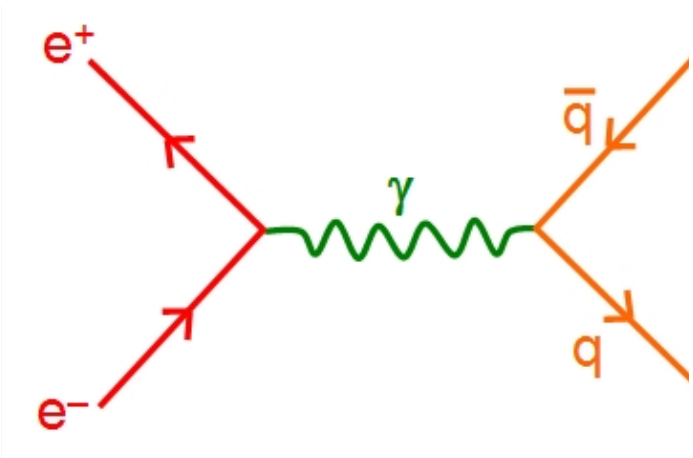


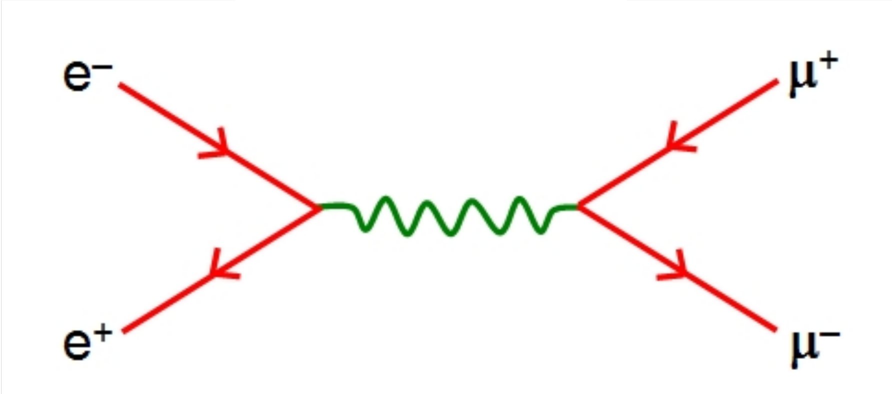


For diagrams that are identical aside from the particle type (and charge!), we should expect that the matrix element will be highly similar.

- This was almost trivial for electrons vs muons
- For quark vs charged lepton, it's also simple but there are some extra considerations!

QED s-Channel Matrix Element

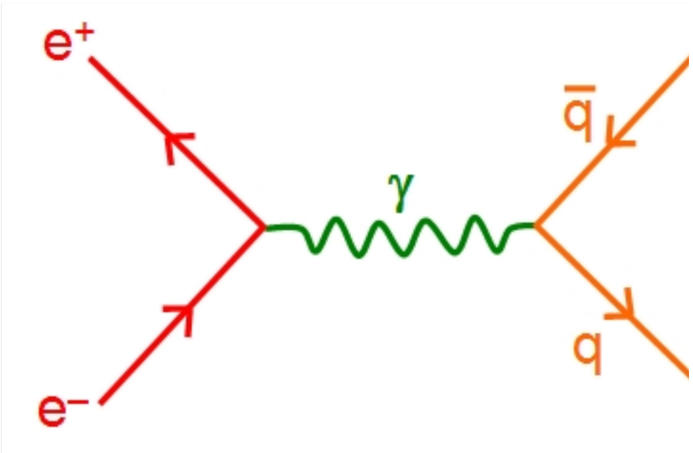


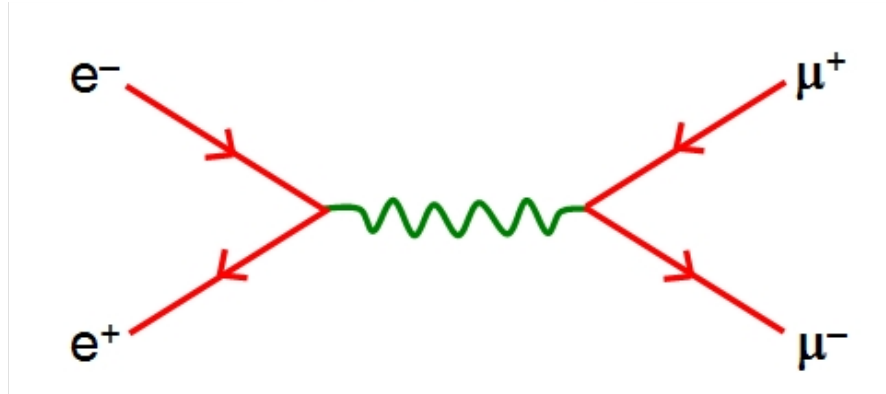


$$\mathcal{M} = (2\pi)^8 \int (J_1) \left(\frac{-ig_{\mu\nu}}{q^2} \right) (J_2)\, \delta^4(p_1 + p_2 - q)\, \delta^4(q - p_3 - p_4)\, \frac{d^4q}{(2\pi)^4}$$

$$\mathcal{M} = \frac{-g^2}{(p_1 + p_2)^2} [\bar{v}(p_1)\gamma^\mu u(p_2)][\bar{u}(p_4)\gamma_\mu v(p_3)]$$

QED s-Channel Matrix Element

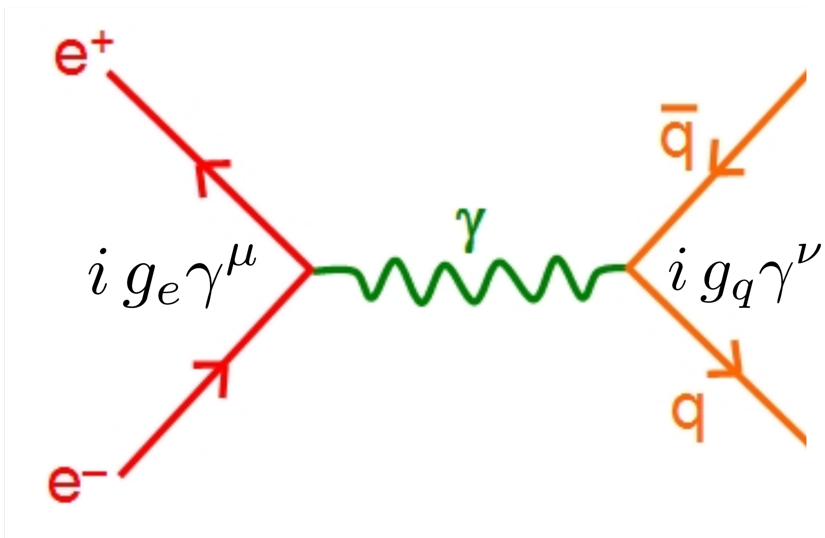


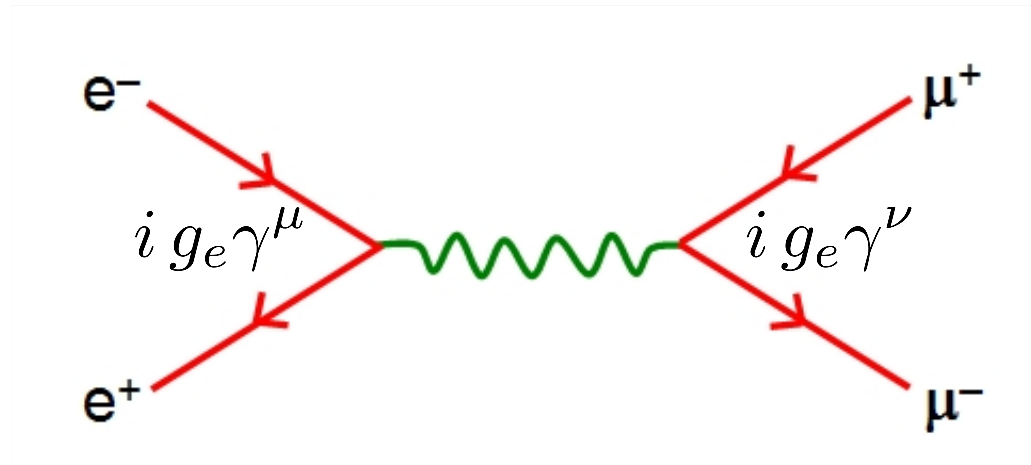


$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{g_e^4}{48\pi E^2} = \frac{\alpha^2\pi}{3E^2} = \frac{4\pi\alpha^2}{3s^2}$$

$$\sigma(e^+e^- \to q\bar{q}) = ??$$

Comparing Vertex Factors





Because the photon couples to charge, the vertex factors only differ by the magnitude of the charge involved!

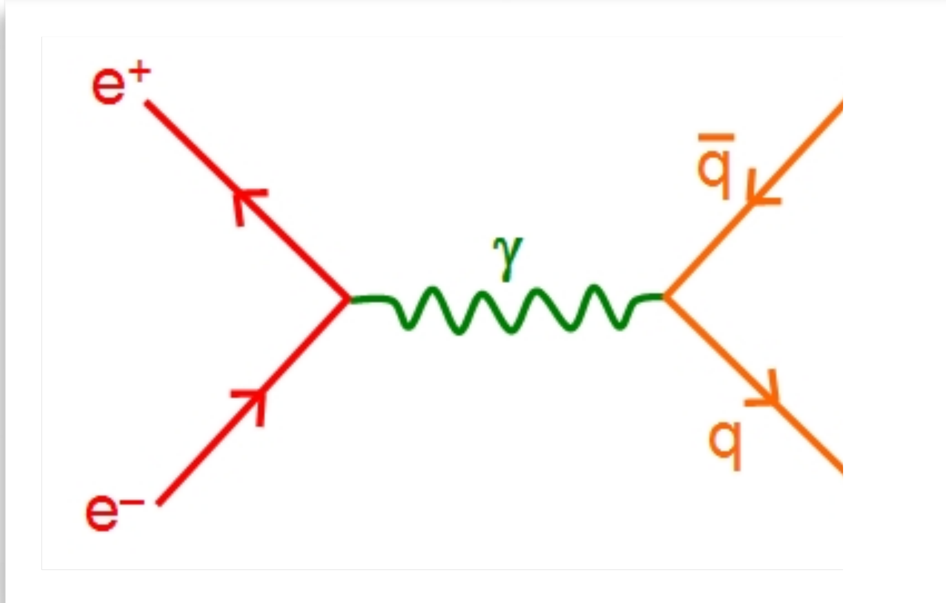
$$g_e = e\sqrt{4\pi/\hbar c} = \sqrt{4\pi\alpha}$$
$$g_q = Q_q\, e\sqrt{4\pi/\hbar c} = Q_q\sqrt{4\pi\alpha}$$

What is the final state?

We also have to consider what our final state is!

Which quarks? u, d, c, s, t, b?

It matters! The charge of up-type quarks is +2/3e and down-type is -1/3e



Which quarks are involved?

All of them? u, d, c, s, t, b? How do we decide?

Conservation of energy tells us that for a given quark flavor:

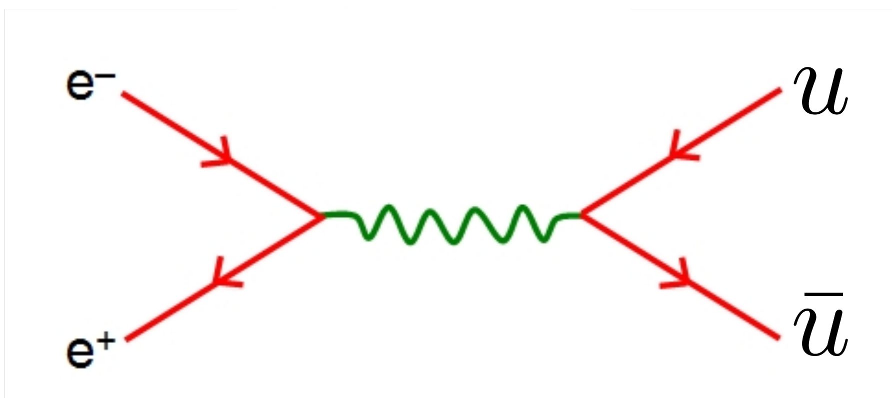
$$\sqrt{s} \geq 2m_q$$

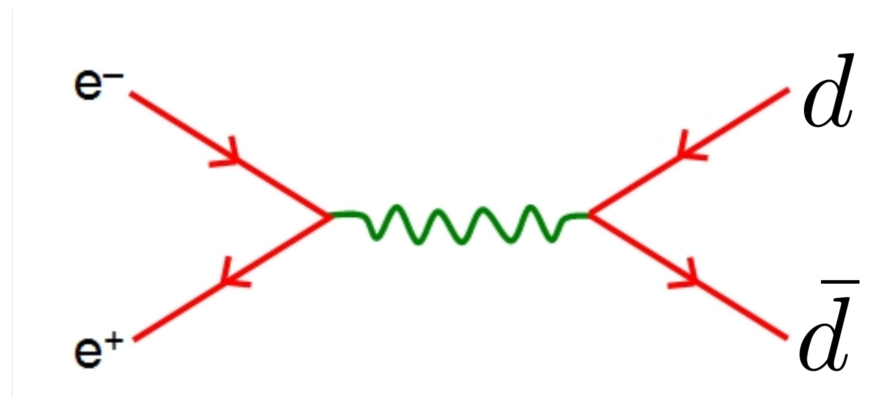
How do we sum over quark flavors?

For multiple Feynman diagrams that contribute to the same final state, we would normally add them up with the appropriate symmetrization factors.

For example, s-channel and t-channel electron/positron scattering

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_1 \pm \mathcal{M}_2 \pm \cdots$$





For final states that aren't identical particles, we do not sum matrix elements.

Instead, we have to sum observables!

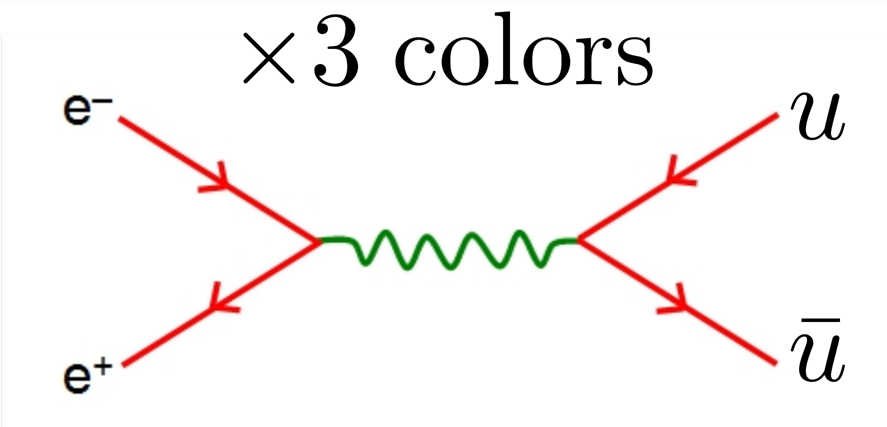
Eg, cross sections

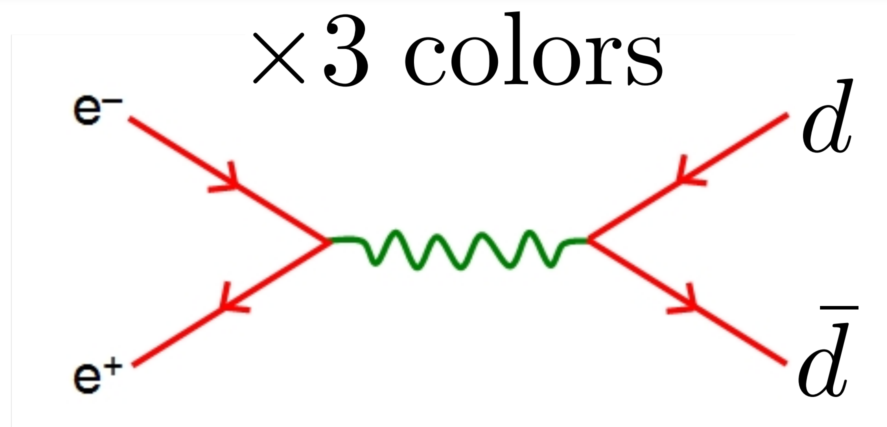
$$\sigma_{\text{tot}} = \sigma_1 + \sigma_2 + \cdots$$

You still need to do individual matrix elements correctly, though!

QED cross-section to quarks

For multiple Feynman diagrams that contribute to the same final state, we would normally add up the matrix elements up with the appropriate symmetrization factors. For example, s-channel and t-channel electron/positron scattering.



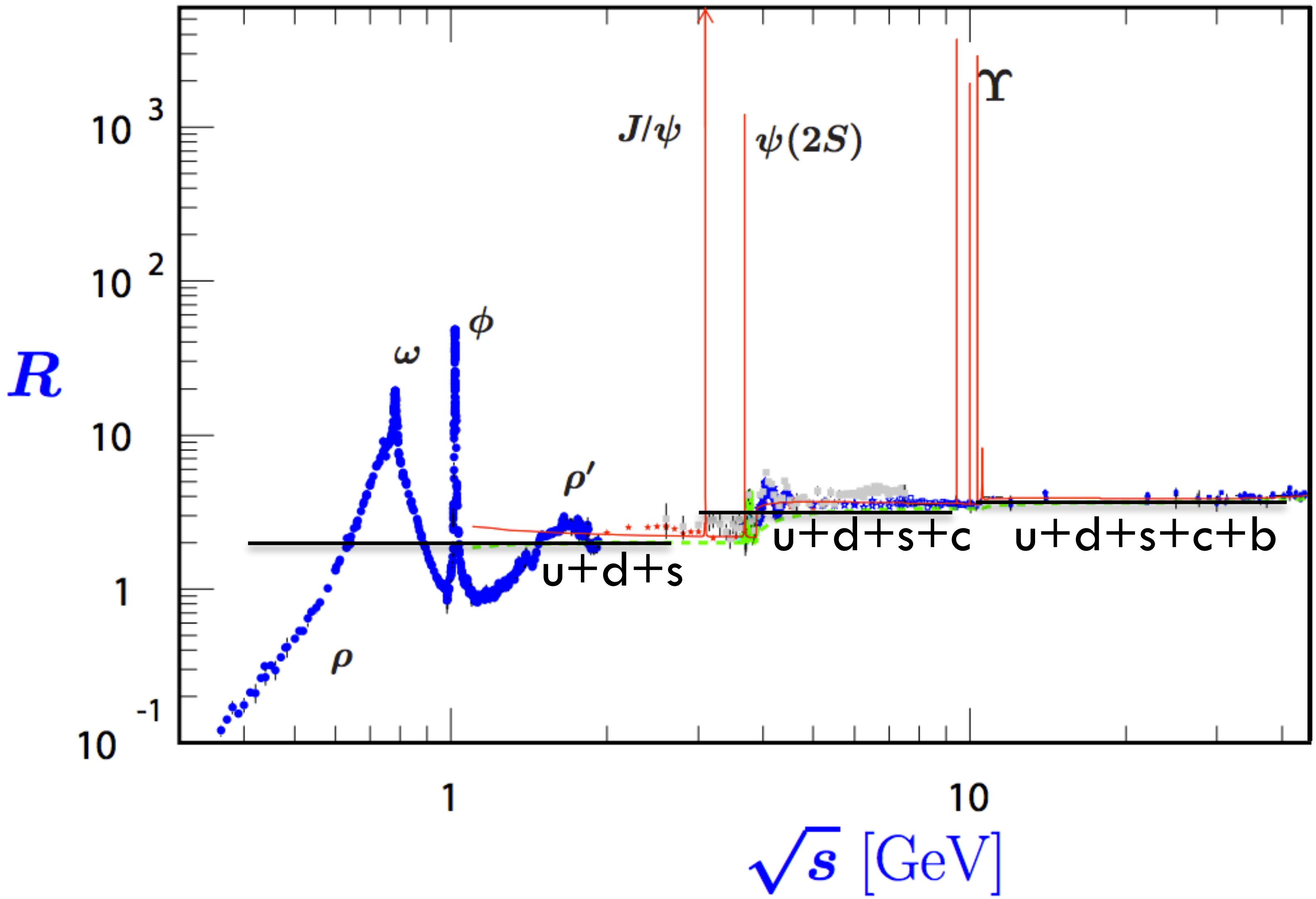


For quark production, the total cross section is the sum over cross-sections for different color states (3), flavor states (depends on energy). Interference (summing matrix elements) is independent of color and flavor for QED interactions of quarks.

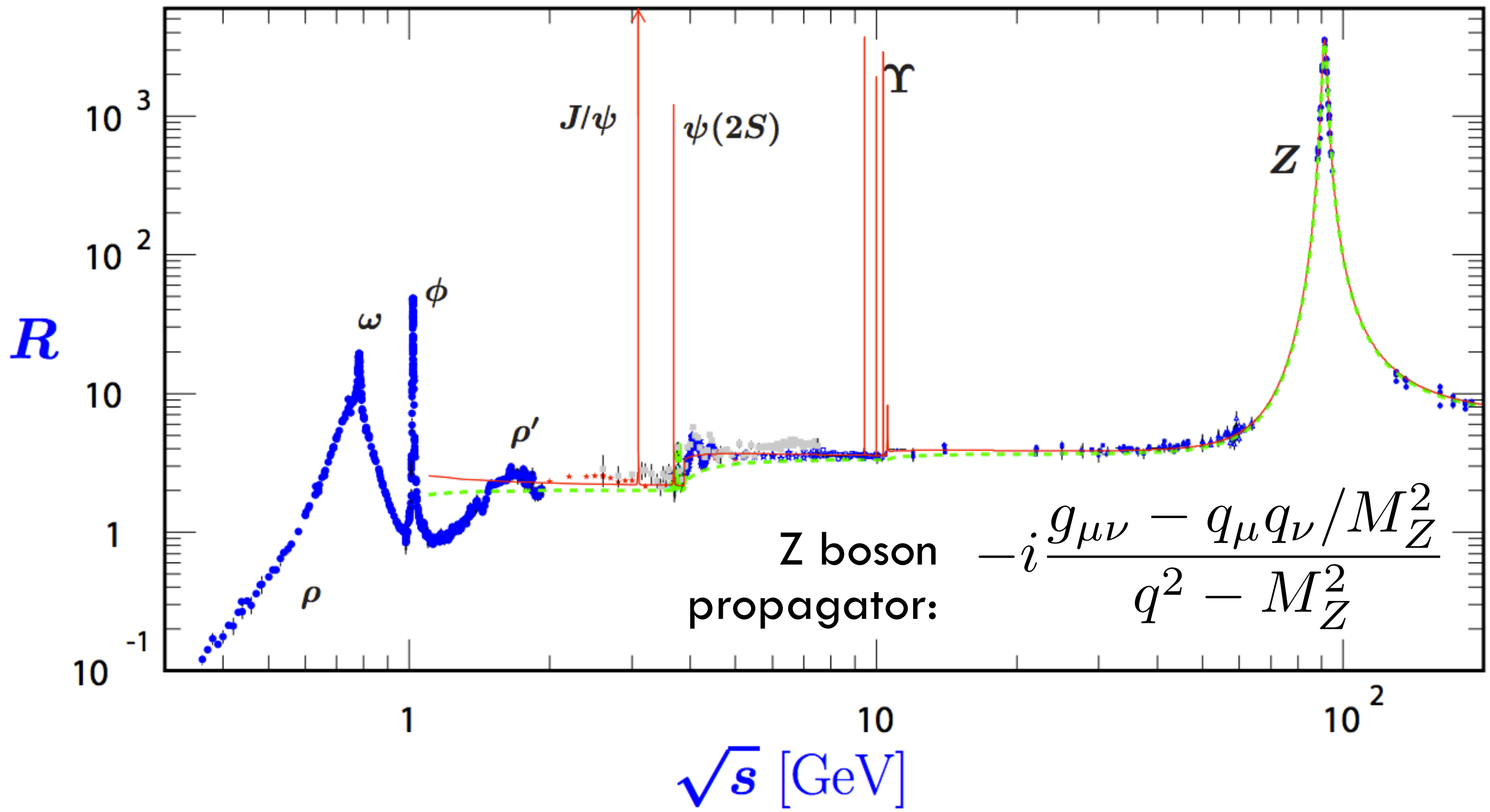
$$\sigma_{\text{tot}} = \sum_{\text{c=color}} \sum_{\text{f=flavor}} \sigma_{\text{cf}}(Q_q^2)$$

Ratio to $\mu^+\mu^-$

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



High-energy behavior of R: Z boson



Z boson propagator: $-i\dfrac{g_{\mu\nu} - q_\mu q_\nu / M_Z^2}{q^2 - M_Z^2}$

- For E≫m the denominator (for the Z contribution) looks like $E^2-(M_Z)^2$ which blows up at $E=M_Z$.
- We will see that this "pole" does not cause problems since M_Z has a width associated with it.
- The Z boson coupling to quarks is larger than the one to muons, hence the peak in the ratio.
- The photon propagator goes like $\sim q^{-2}$ so it's contribution is small at very high energies.
- At very very high energies (that is, for $q^2 \gg (M_Z)^2$) the contributions from the photon and the Z become similar.