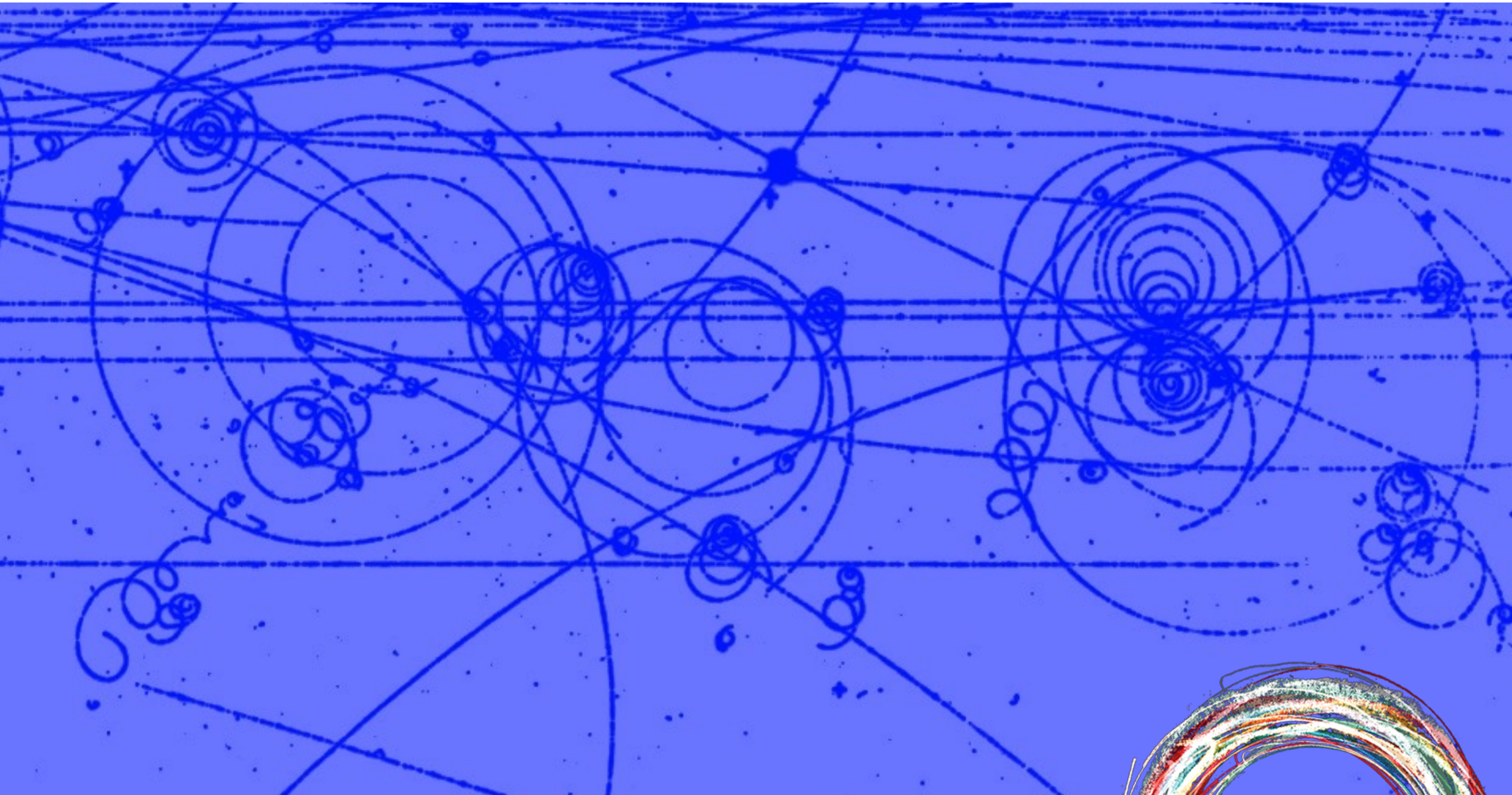


# 3-FLAVOR NEUTRINO MIXING

PHY 493/803

# Three-flavor Neutrino mixing

Supposing that neutrinos have mass, we must now allow a mixing between the weak eigenstates and the mass eigenstates, just as we have done for the quarks.

The neutrino analogue of the CKM matrix is the Maki-Nakagawa-Sakata (MNS) matrix. *Also known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix*.

Like the CKM matrix, the MNS matrix can be parametrized in terms of 3 mixing angles and 1 CP-violating complex phase.

We label the neutrino mass eigenstates (in order of ascending mass) as $\nu_1$, $\nu_2$ and $\nu_3$.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

# Neutrino Mixing Matrix

Parameterization of the PMNS Matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Parameterized in terms of mixing angles $\theta_{23}$, $\theta_{13}$, $\theta_{12}$,
two diagonal phases $\alpha_1$ and $\alpha_2$ and one off-diagonal, CP violating phase $\delta$
$c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$

# Neutrino Mixing Matrix

Parameterization of the PMNS Matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Mixing between $\nu_1$ and $\nu_2$.**

Solar neutrinos, measure disappearance of electron neutrinos.
long baseline, small energies.

# Neutrino Mixing Matrix

Parameterization of the PMNS Matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Mixing between $\nu_2$ and $\nu_3$.**

Atmospheric neutrino oscillations, accelerator-based experiments observe the disappearance of muon neutrinos - measure survival probability

# Interactions and Measurements

Parameterization of the PMNS Matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Mixing between $\nu_1$ and $\nu_3$.** $\theta_{13}$

Can be observed in the fraction of electron neutrinos (or anti-neutrinos) that remain in a reactor-based neutrino experiment. Hence the term "survival probability" in reference to neutrino "disappearance" experiments.

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \, \sin^2 \left(1.27 \, \Delta m^2_{13} \, \frac{L}{E}\right)$$

# Interactions and Measurements

## Parameterization of the PMNS Matrix

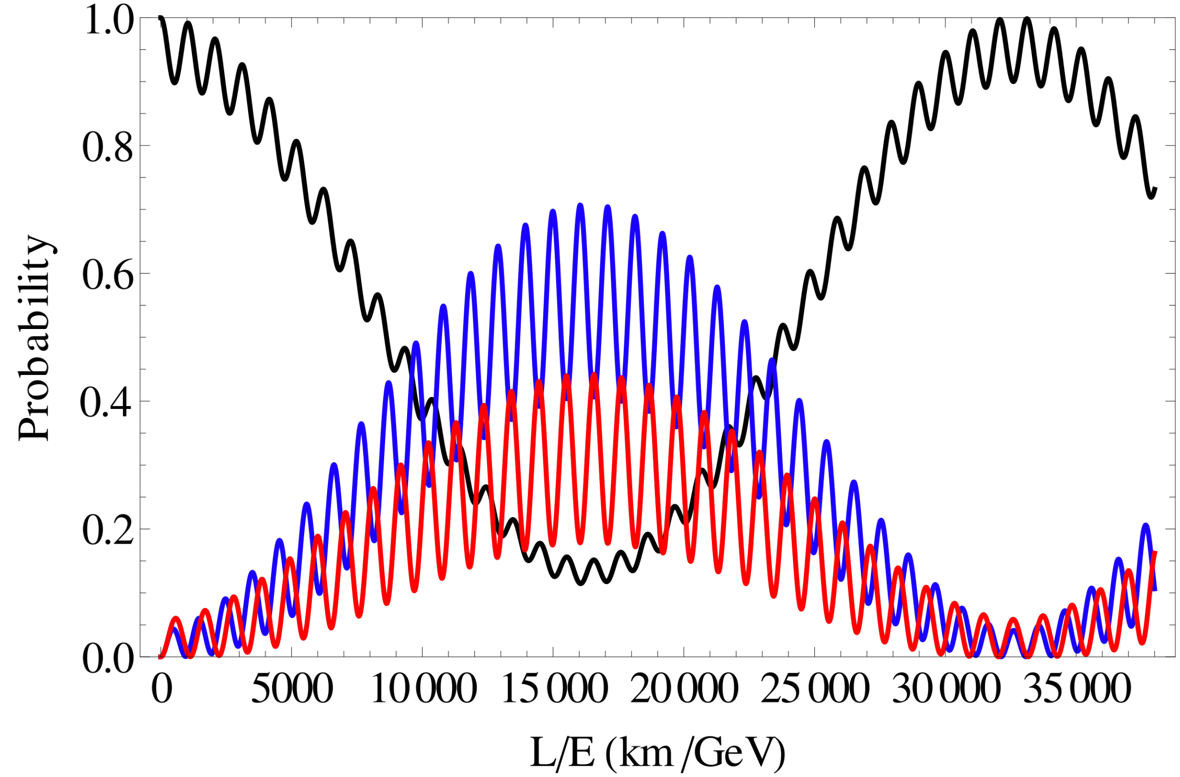
$$
\begin{aligned}
U &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

**Phases $\delta$ and $\alpha_1$, $\alpha_2$**

The phase $\delta$ is a CP-violating phase that has not yet been measured but will be in the next generation of neutrino experiments.
The phases $\alpha_1$, $\alpha_2$ (on the diagonal) are not relevant for neutrino oscillations, but could be determined in neutrinoless double-beta decay experiments (searches for Majorana neutrinos).

# 3-generation mixing

Oscillation probabilities for an initial electron neutrino



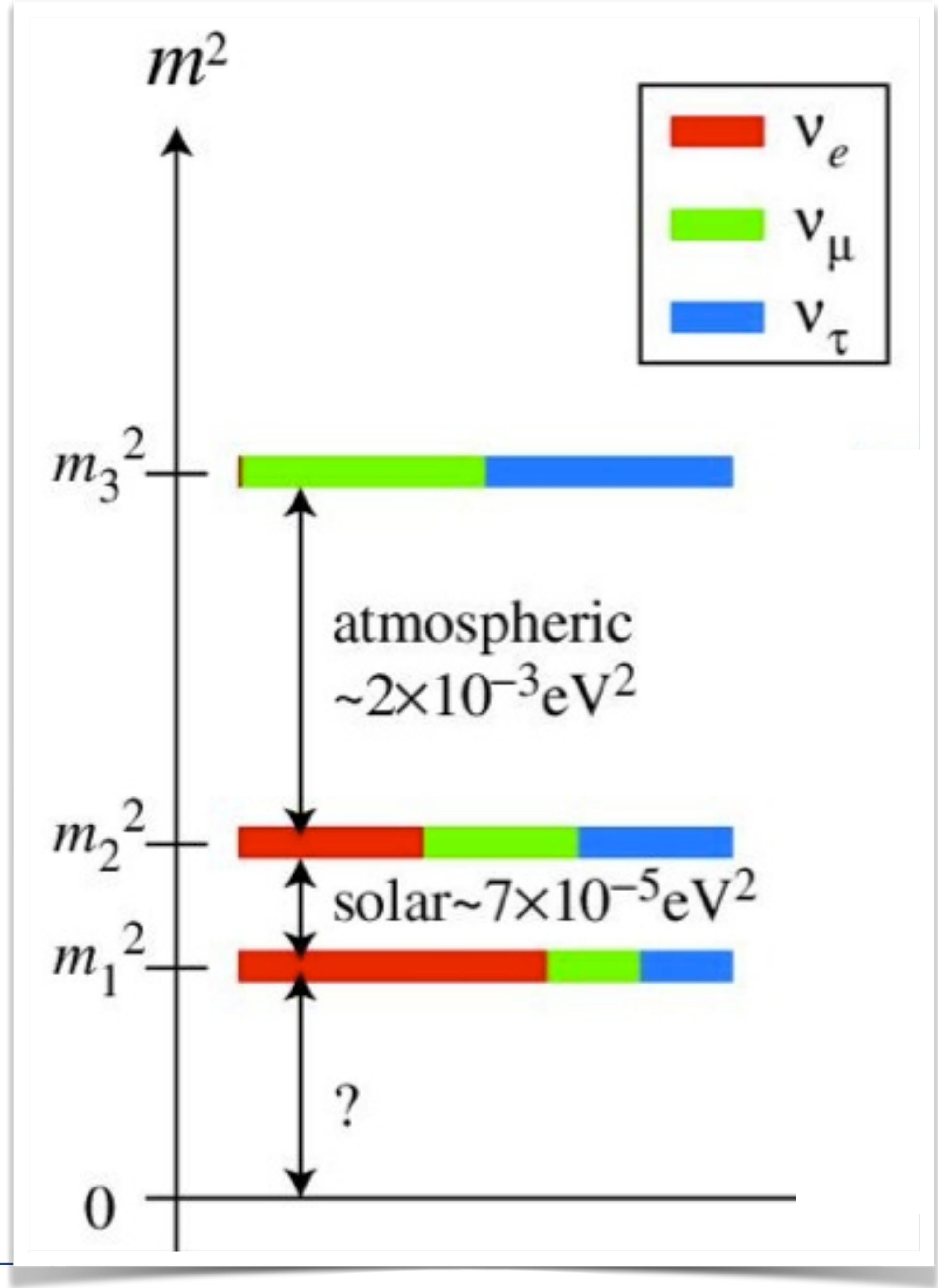

- Solar neutrino oscillations average over many wavelengths → observe average effect
- Reactor neutrino experiments have L/E of (few km)/(0.01 GeV)
  - Oscillations ($\nu_e$ deficit) are just starting to be measurable
  - Sensitive to $\theta_{13}$

# The Neutrino Mass Hierarchy

Oscillation measurements are sensitive to the difference in squared mass, not the mass itself.

This means that we don't know the absolute magnitude of the smallest mass.

And we also do not know the relative sign of each mass splitting unambiguously.



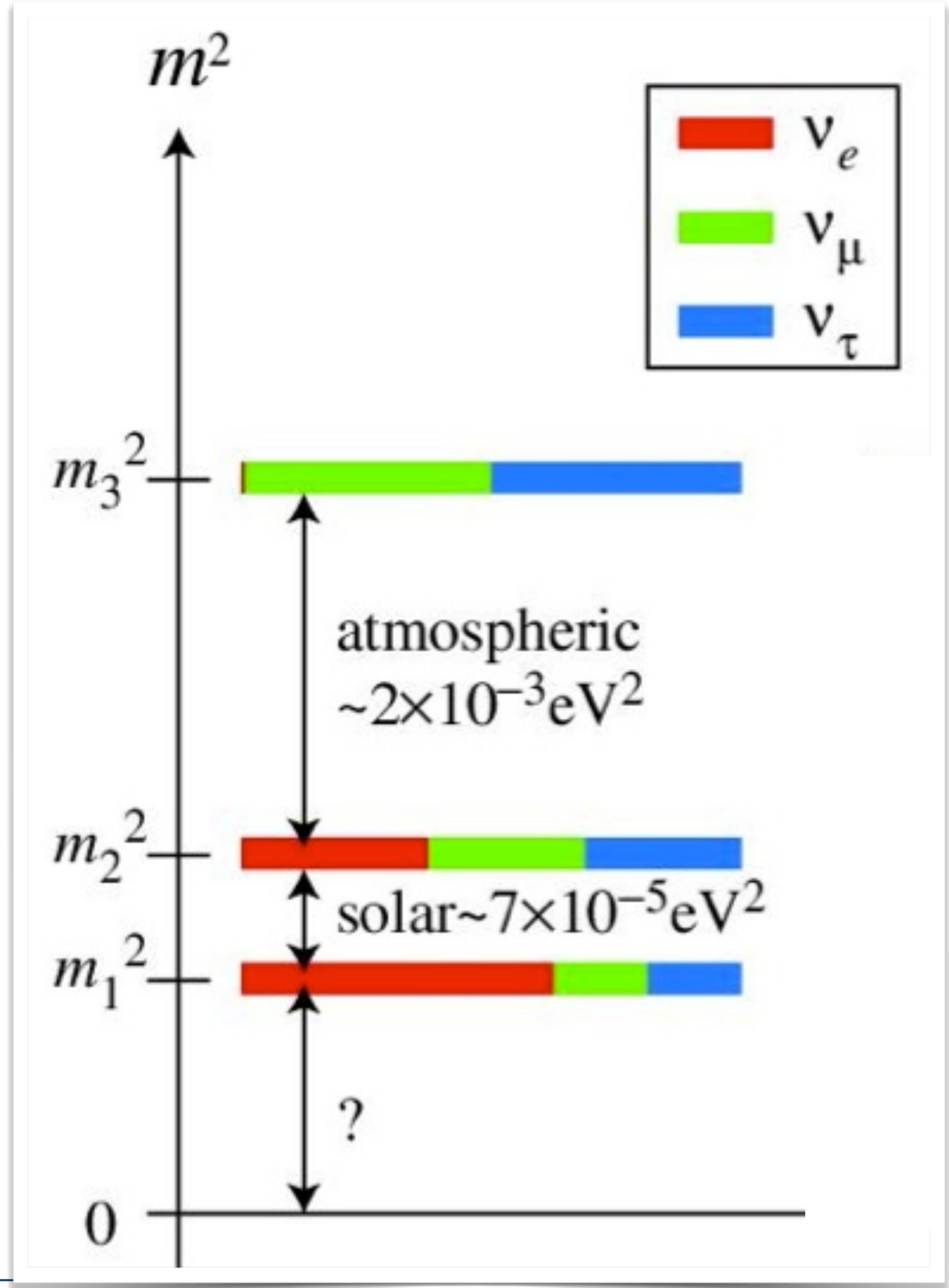
# The Neutrino Mass Hierarchy

Comparison to quark masses:

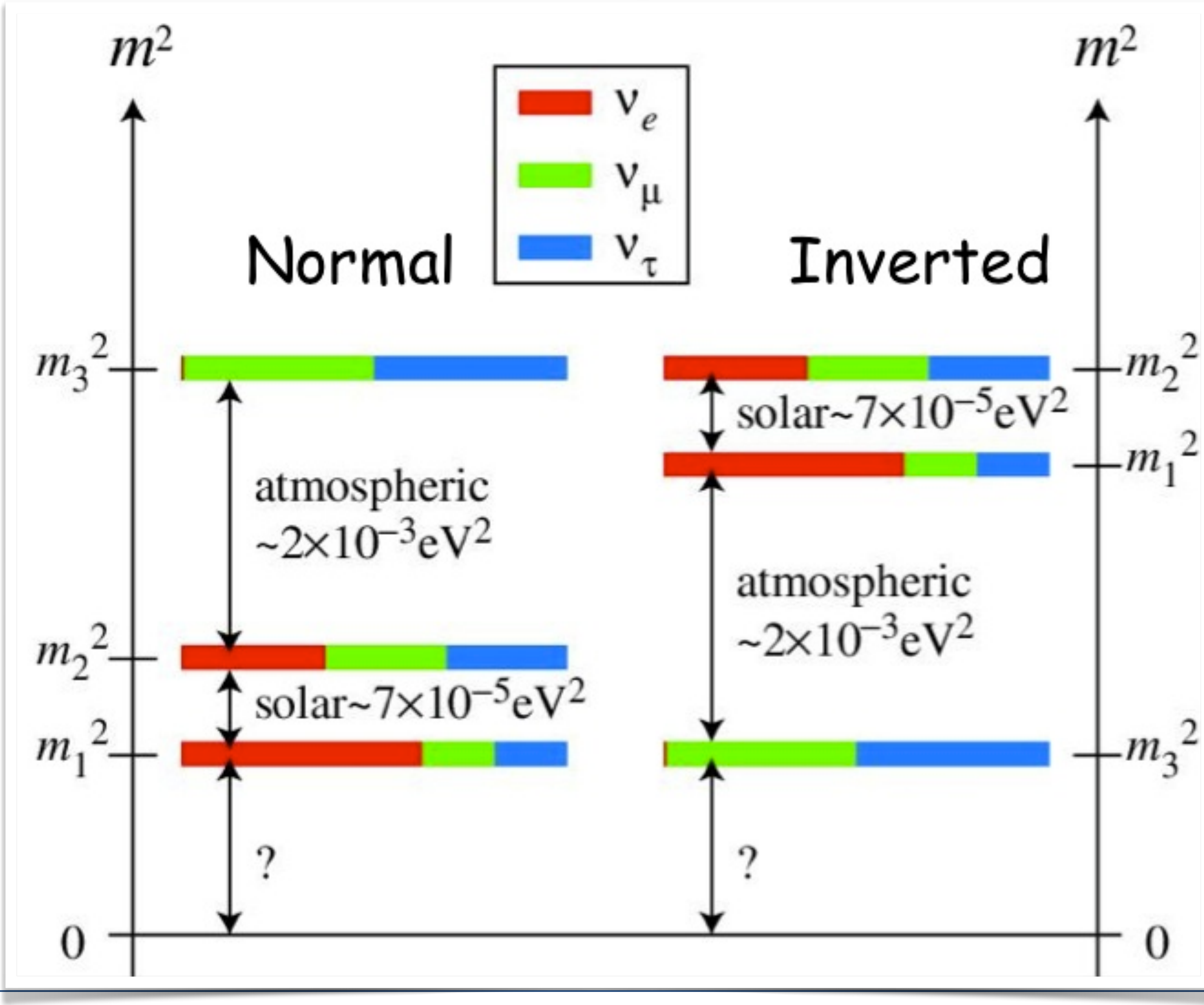
Bottom quark mass: 4500 MeV

Strange quark mass: 100 MeV

Down quark mass: ~5 MeV



# The Neutrino Mass Hierarchy



# Current Status

The mixing matrix appears to feature very large mixing angles; the following gives a crude view of the sizes of the matrix elements:

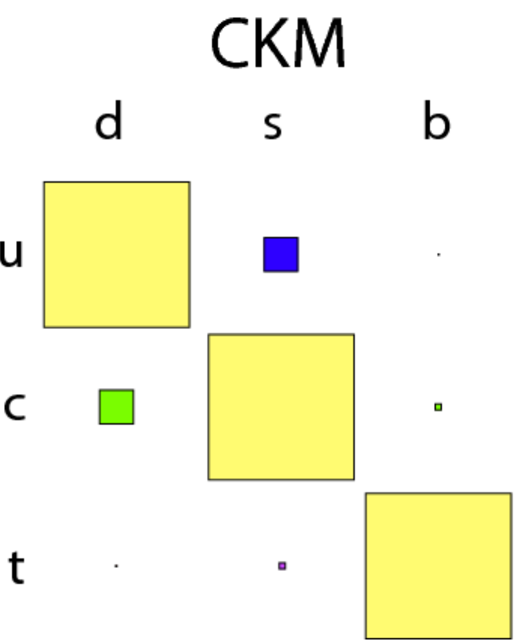
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

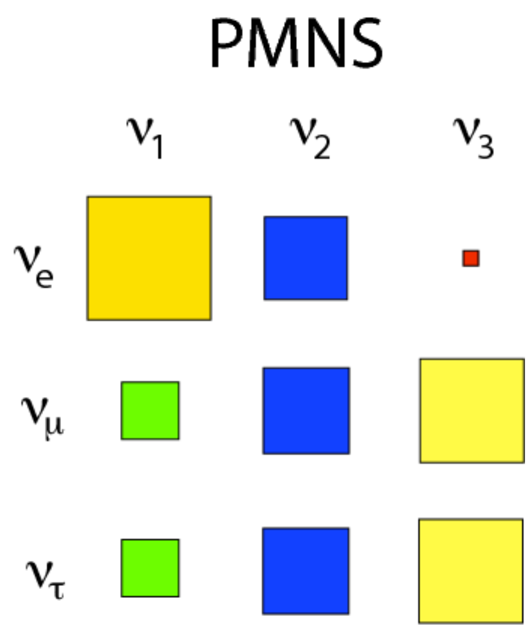
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 0.35 & 0.95 & < 0.2e^{i\delta} \\ -0.35 & 0.25 & 0.70 \\ 0.65 & -0.25 & 0.70 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

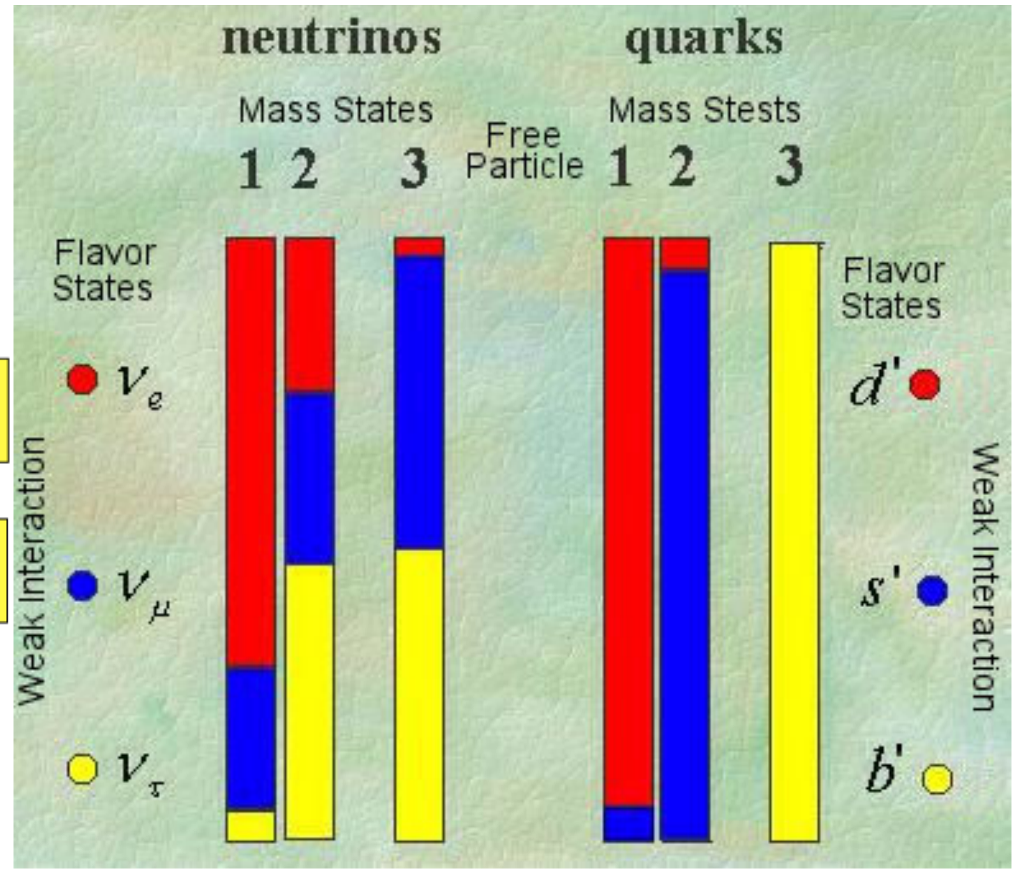
# Comparing Quarks and Neutrinos

Considering that we have a similar type of mixing between weak eigenstates and mass eigenstates in both the neutrino and quark sectors, should we expect that the mixing is identical?

Turns out that it's very different!



# Current Status

| Parameter | best-fit ($\pm 1\sigma$) | $3\sigma$ |
|---|---|---|
| $\Delta m^2_{21}$ [$10^{-5}$ eV $^2$] | $7.54^{+0.26}_{-0.22}$ | $6.99 - 8.18$ |
| $\lvert\Delta m^2\rvert$ [$10^{-3}$ eV $^2$] | $2.43 \pm 0.06$ ($2.38 \pm 0.06$) | $2.23 - 2.61$ ($2.19 - 2.56$) |
| $\sin^2\theta_{12}$ | $0.308 \pm 0.017$ | $0.259 - 0.359$ |
| $\sin^2\theta_{23}$, $\Delta m^2 > 0$ | $0.437^{+0.033}_{-0.023}$ | $0.374 - 0.628$ |
| $\sin^2\theta_{23}$, $\Delta m^2 < 0$ | $0.455^{+0.039}_{-0.031}$, | $0.380 - 0.641$ |
| $\sin^2\theta_{13}$, $\Delta m^2 > 0$ | $0.0234^{+0.0020}_{-0.0019}$ | $0.0176 - 0.0295$ |
| $\sin^2\theta_{13}$, $\Delta m^2 < 0$ | $0.0240^{+0.0019}_{-0.0022}$ | $0.0178 - 0.0298$ |
| $\delta/\pi$ ($2\sigma$ range quoted) | $1.39^{+0.38}_{-0.27}$ ($1.31^{+0.29}_{-0.33}$) | $(0.00 - 0.16) \oplus (0.86 - 2.00)$<br>$((0.00 - 0.02) \oplus (0.70 - 2.00))$ |