PHY493/803, Intro to Elementary Particle Physics

Homework 7

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

(4 x 20pts = 80 pts total)

In the CHOOZ experiment (see http://arxiv.org/abs/hep-ex/0301017 for more information) a neutrino detector was positioned a distance L ≈ 1 km from a nuclear reactor emitting antineutrinos of mean energy E ≈ 3 MeV. The number of antineutrino interactions observed was consistent with the number predicted assuming no antineutrino oscillations, giving the ratio of observed over predicted events as $R = 1.01 \pm 0.04$ (the uncertainty is reported at the 68% Confidence Level). This ratio is interpreted as the survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$, which cannot be larger than one, and thus leads to a lower bound on the survival probability of 0.92 (the upper bound is 1). Consider 2-flavor neutrino oscillations.

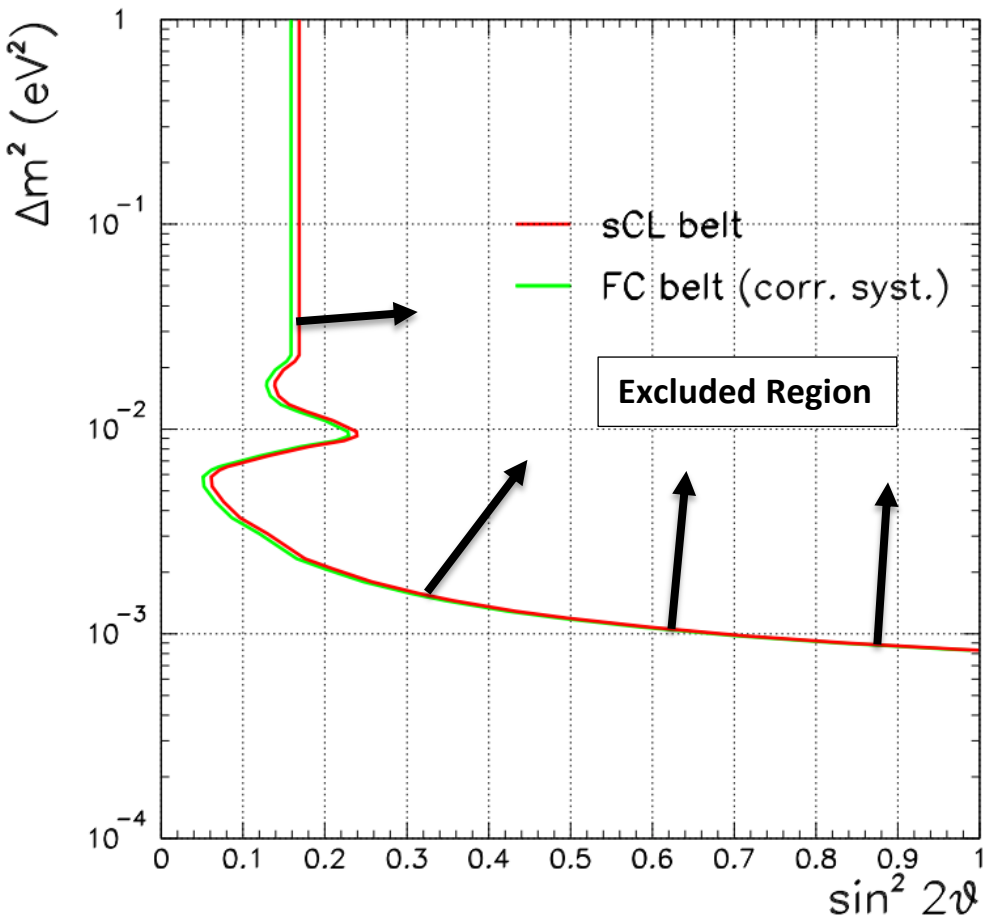
1. In the limit $|\Delta m^2| \gg (E/L)$, draw a sketch of $\sin^2(1.27\ \Delta m^2 L/E)$ as a function of *L* in the volume of the detector for a fixed energy *E*. , i.e. draw the $\sin^2$ function from L to L+ΔL, where ΔL is the length of the detector.

2. In the same limit $|\Delta m^2| \gg (E/L)$, show that a measurement of the survival probability $P_{surv} = P(\bar{\nu}_e \to \bar{\nu}_e)$ determines the neutrino mixing angle to be $\sin^2 2\theta = 2(1 - P_{surv})$.
   Also compute the value of $\sin^2 2\theta$ for the parameters given. Use the lower limit of 0.92 for the survival probability (i.e. $P_{surv} = 0.92$).

3. In the other limit, when $|\Delta m^2| \ll (E/L)$, you can use the small angle approximation ($\sin \Delta = \Delta$). Show that a given measurement of the survival probability $P_{surv} = P(\bar{\nu}_e \to \bar{\nu}_e)$ determines the neutrino mixing

to be $\sin^2 2\theta = C\left(\frac{1}{\Delta m^2}\right)^2$, with a constant of proportionality $C = (1 - P_{surv})(E/1.27L)^2$.

4. The Null result from the CHOOZ experiment, $R = 1.01 \pm 0.04$, can be used to exclude a region of the $(\sin^2 2\theta,\ \Delta m^2)$ parameter space. This is conventionally presented as the region which can be excluded at the 90% Confidence Level, which for the CHOOZ experiment encompasses all values of $(\sin^2 2\theta_{13},\ \Delta m^2_{13})$ that would give a survival probability $P(\bar{\nu}_e \to \bar{\nu}_e) < 0.92$. In the figure given below, published by the CHOOZ collaboration, the curves correspond to the contour $P(\bar{\nu}_e \to \bar{\nu}_e) = 0.92$ and the excluded region lies above and to the right of the curves. The two similar curves correspond to slightly different statistical approaches to the analysis of the data.
   The measured values for these oscillation parameters are $\sin^2 2\theta = 0.1$ and Δm^2 =2.4E-3 eV (as measured by Double-CHOOZ, Daya-Bay and RENO).
   a) What is the oscillation probability for these parameters?
   b) Is the point corresponding to these two values in the allowed or the excluded region in the plot?


Δm^2 (eV^2)
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$\sin^2 2\vartheta$
sCL belt
FC belt (corr. syst.)
Excluded Region