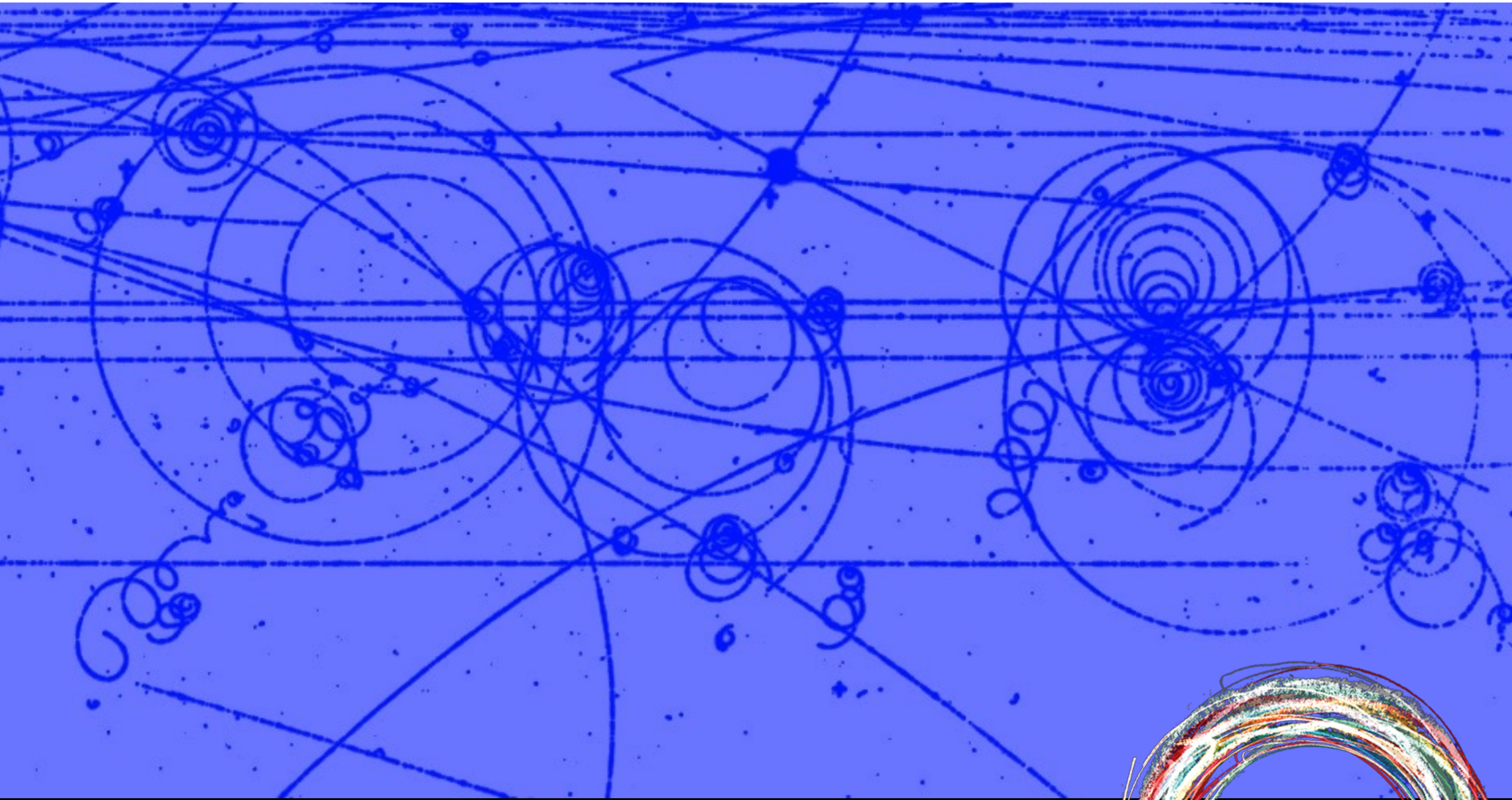

WEAK ISOSPIN

PHY 493/803

Weak Doublets

Since the W couples left-handed leptons and their neutrinos together, it seems natural to define the weak doublet:
Note that ν_e and e are Dirac 4-vectors

$$\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

In terms of the weak doublets,
the charged weak currents:

$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L \qquad j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L$$

can be written as:

$$j_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L$$

Where:

$$\tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Weak Isospin

These "new" weak isospin matrices:

$$\tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

can be constructed from the Pauli spin matrices:

$$\tau^\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y) \longleftrightarrow \tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$$

This is looking a lot like isospin (i.e., an internal SU(2) symmetry), so it's dubbed **weak isospin**. Suppose we define a third matrix to complete the symmetry:

$$\tau^3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Weak isospin is $T^i = I_W^i = \frac{1}{2}\tau^i$ with Eigenvalues $T^3(\nu_L) = \frac{1}{2}$ and $T^3(e_L) = -\frac{1}{2}$

Weak isospin

- The upper entry in the doublet is the neutrino with third component of the weak isospin
$$T^3 = I_W^3(\nu_{e,L}) = \frac{1}{2}$$
  - The anti-neutrino has $T^3(\bar{\nu}_{e,R}) = -\frac{1}{2}$
- The lower entry in the doublet is the electron with third component of the weak isospin $T^3(e_L^-) = -\frac{1}{2}$
  - The positron has $T^3(e_R^+) = \frac{1}{2}$

Spin-½ Representations

FLASHBACK!

Use vectors as spin-½ representations

$$\chi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Then spin observable operators are matrices, the Pauli spin matrices:

Spin vectors transform via the 2-D representation of the SU(2) group

The Lie algebra is spanned by 3, 2×2 Hermitian, unitary, complex matrices.

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\mathbf{S}} = \frac{\hbar}{2}(\sigma_x \ \sigma_y \ \sigma_z) \longrightarrow \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\tau^3 \rightarrow$ neutral current

From τ^3, we can construct a current (with a factor of ½ for consistency with the charged currents):

$$\begin{aligned} j^3_\mu &= \bar{\chi}_L \gamma_\mu \tfrac{1}{2}\tau^3 \chi_L \\ &= \tfrac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \tfrac{1}{2}\bar{e}_L \gamma_\mu e_L \end{aligned}$$

Aha! Here is a neutral current!

<u>2 Problems:</u> This neutral current is pure V-A because it only involves left-handed particles. The Z current,

$$\gamma^\mu (c_V - c_A \gamma^5)$$

conversely, has a more complicated structure and, consequently, it also couples to right-handed particles.

Quick Recap in 3 Steps

(1)

$$u = \frac{1}{2}(1+\gamma^5)u + \frac{1}{2}(1-\gamma^5)u \qquad u = u_R + u_L$$

$$j^{\mu}_{weak} \sim \bar{u}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)u = \bar{u}_L\gamma^{\mu}u_L$$

(2)

$$\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \tau^3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{T} = \vec{I}_W = \frac{1}{2}\vec{\tau} \qquad T^3(\nu_{e,L}) = \frac{1}{2} \qquad T^3(e_L) = -\frac{1}{2}$$

(3)

$$j^{\pm}_{\mu} = \bar{\chi}_L\gamma_{\mu}\tau^{\pm}\chi_L$$

$$\begin{aligned} j^3_{\mu} &= \bar{\chi}_L\gamma_{\mu}\tfrac{1}{2}\tau^3\chi_L \\ &= \tfrac{1}{2}\bar{\nu}_L\gamma_{\mu}\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_{\mu}e_L \end{aligned}$$

Generalizing to Other Weak Doublets

Although we have set up this formalism in terms of the electron and its neutrino, we can just as easily use weak doublets for other leptons

$$\chi_L \longrightarrow \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

or for quarks, provided we account for the CKM rotations which distinguish the weak eigenstates from the mass eigenstates:

$$\chi_L \longrightarrow \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$d' = d\,V_{ud} + s\,V_{us} + b\,V_{ub}$$