HOMEWORK 2

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1 MYSTERY PARTICLE DECAY

Consider a mystery particle described by the 4-vector $p_\mu = (200, 30, 100, 150)$GeV in the lab frame.

A

What is the mass of this particle? Which elementary particle is it?

$$m_0 = \sqrt{p_\mu p^\mu} = \sqrt{E^2 - \mathbf{p}\cdot\mathbf{p}}, \tag{A.1}$$
$$= 81.24\text{GeV}. \tag{A.2}$$

The elementary particle with the closest mass is the W boson (let $m_w = 81.24$GeV).

B

What are β and γ for this particle?

$$\gamma = \frac{E}{m_0}, \tag{B.1}$$
$$= 2.46. \tag{B.2}$$
$$\beta = \frac{\mathbf{p}}{\gamma m_0}, \tag{B.3}$$
$$= 0.91. \tag{B.4}$$

C

Now boost the particle into its own rest frame. The particle decays into an electron and a neutrino. Assume the neutrino travels in the $\hat{\mathbf{z}}$-direction in this mystery particle rest frame. Write down the energy/momentum 4-vectors for the particle and for the electron and the neutrino (in GeV).

In the particle's rest frame, $\mathbf{p} = \mathbf{0}$, and $E = m_w$. Thus,

$$\begin{pmatrix} m_w \\ \mathbf{0} \end{pmatrix} = p_\nu^\mu + p_e^\mu. \tag{C.1}$$

Since the neutrino travels in the $\hat{\mathbf{z}}$-direction, and $\mathbf{p}_\nu + \mathbf{p}_e = \mathbf{0}$, $\mathbf{p}_\nu = -\mathbf{p}_e = p_\nu \hat{\mathbf{z}}$. Additionally, the mass of the neutrino is approximately zero, so its energy is effectively equal to the magnitude of its momentum. Thus, the equality becomes

$$m_w = p_\nu + \sqrt{m_e^2 + p_\nu^2}, \tag{C.2}$$
$$p_\nu = 40.62\text{GeV}. \tag{C.3}$$

Hence, the 4-vectors for each particle, in the rest frame of the original particle, are

$$p_w = \begin{pmatrix} 81.24 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{GeV}, \tag{C.4}$$
$$p_\nu = \begin{pmatrix} 40.62 \\ 0 \\ 0 \\ 40.62 \end{pmatrix} \text{GeV}, \tag{C.5}$$
$$p_e = \begin{pmatrix} 40.62 \\ 0 \\ 0 \\ -40.62 \end{pmatrix} \text{GeV}. \tag{C.6}$$

D

Now consider the decay of the mystery particle into an electron and a neutrino in the lab frame, i.e. start from the particle 4-vector from part (a). Assume the decay products could go in any direction. What is the maximum and the minimum magnitude of the 3-momentum that the electron can have?

Assuming mass of neutrino is 0GeV, then the bounds for the magnitude for the electron's 3-momentum is

$$0 \leq \sqrt{\mathbf{p}_e \cdot \mathbf{p}_e} < \sqrt{\mathbf{p}_w \cdot \mathbf{p}_w} = 182.757\text{GeV}, \tag{D.1}$$

since the neutrino must move with moment > 0.

2 PARTICLE COLLISION

A

Consider the collision of two particles, A and B, which interact and create n final state particles $C_1, C_2, \ldots C_n$. For this reaction to occur, there must be a minimum total energy available, which depends on the final state particles. This minimum (or threshold) energy corresponds to a final state of zero kinetic energy in the center-of-momentum frame. Assuming particle A has total energy E (4-vector $p_A^\mu = (E, \mathbf{p}_A)$) and particle B is at rest (4-vector $p_B^\mu = (m_B, \mathbf{0})$), find an expression for the threshold energy.

Due to conservation of momentum, the COM frame for the final state is the same as the COM frame for the initial state. In the COM frame for the initial state, the 4-momentum of particles A and B are

$$p_A^\mu = \begin{pmatrix} E_A \\ \frac{\mathbf{p}_A}{2} \end{pmatrix}, \tag{A.1}$$

and

$$p_B^\mu = \begin{pmatrix} E_B \\ -\frac{\mathbf{p}_A}{2} \end{pmatrix}. \tag{A.2}$$

B

Use your answer from part (a) to find the threshold energies for the following reactions. In each case, the proton is the at-rest target particle:

I

II

III

3 THE DECAY OF A MESON

The $\eta(549)$ meson has spin-0 and is observed to decay to three-pion final states by the electromagnetic processes $\eta \to \pi^0 + \pi^0 + \pi^0$ and $\eta \to \pi^+ + \pi^- + \pi^0$. Use this information to deduce the parity of the $\eta(549)$, and hence explain why the decays $\eta \to \pi^0 + \pi^0$ and $\eta \to \pi^+ + \pi^-$ have never been observed.

Each pion has a parity of -1. Parity is multiplicative, so three-pion final states have a parity of -1, as well. Since these two three-pion final state decays have been observed, and parity is conserved during electromagnetic processes, the $\eta(549)$ meson must have a parity of -1. The last two decays mentioned cannot occur through an electromagnetic process because their final states have a parity of 1, so parity wouldn't be conserved.

4 SQUARE SYMMETRY GROUP

Following the discussion of the triangle symmetry group in Griffiths (Ch 4.1, pp 117-118), work out the symmetry group of the square.

A

How many elements does it have?

There are eight elements of the group. They are as follows:
e: Perform no transformations.
r: Rotate by $\pi/2$ radians.
r^2: Rotate by π radians.
r^3: Rotate by $3\pi/2$ radians.
d_1: flip along diagonal with positive slope.
d_2: Flip along diagonal with negative slope.
f_1: Flip along the $\hat{\mathbf{x}}$-axis.
f_2: Flip along the $\hat{\mathbf{y}}$-axis.

B

Determine if the square symmetry group is Abelian or non-Abelian.

The group is not abelian since rotating by $\pi/2$ and then flipping along either diagonal does not result in flipping first, then rotating by $\pi/2$. That is

$$rd_1 = f_2 \neq f_1 = d_1 r. \tag{B.1}$$