PHY493/803 Intro to Elementary Particle Physics
Homework 2

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

1. (5+5+10+10 pts)
   Consider a mystery particle described by the 4-vector
   $p_\mu = (200,30,100,150)$ GeV in the lab frame.
   (a) What is the mass of this particle? Which elementary particle is it?

   (b) What are β and γ for this particle?

   (c) Now boost the particle into its own rest frame. The particle decays into an electron and a neutrino. Assume the neutrino travels in the z-direction in this mystery particle rest frame. Write down the energy/momentum 4-vectors for the particle and for the electron and the neutrino (in GeV).

   (d) Now consider the decay of the mystery particle into an electron and a neutrino in the lab frame, i.e. start from the particle 4-vector from part (a). Assume the decay products could go in any direction. What is the maximum and the minimum magnitude of the 3-momentum that the electron can have?

2. (11+9 pts)

a) Consider the collision of two particles, A and B, which interact and create *n* final state particles C_1, C_2….C_n. For this reaction to occur, there must be a minimum total energy available, which depends on the final state particles. This minimum (or threshold) energy corresponds to a final state of zero kinetic energy in the center-of-momentum frame. Assuming particle A has total energy E (4-vector $p_A{}^{\mu} = (E_A, \overrightarrow{p_A})$) and particle B is at rest (4-vector $p_B{}^{\mu} = \left(m_B, \vec{0}\right)$), find an expression for the threshold energy.

b) Use your answer from part (a) to find the threshold energies for the following reactions. In each case, the proton is the at-rest target particle:

i. $\pi^- + p \rightarrow K^0 + \Sigma^0$

ii. $p + p \rightarrow p + p + \pi^0$

iii. $\pi^- + p \rightarrow p + \bar{p} + n$

3. (20 pts)

   The $\eta(549)$ meson has spin-0 and is observed to decay to three-pion final states by the electromagnetic processes $\eta \to \pi^0 + \pi^0 + \pi^0$ and $\eta \to \pi^+ + \pi^- + \pi^0$. Use this information to deduce the parity of the $\eta(549)$, and hence explain why the decays $\eta \to \pi^0 + \pi^0$ and $\eta \to \pi^+ + \pi^-$ have never been observed.

4. (10 pts + 10 pts) {**Required for PHY803 students only. +20 pts extra credit for PHY493 students.}**
   Following the discussion of the triangle symmetry group in Griffiths (Ch 4.1, pp 117-118), work out the symmetry group of the square.
   a) How many elements does it have? [Hint: Draw a diagram of the axes of symmetry. Each symmetry group element for the square must symmetrically preserve the orientation of the original square.]

   b) Determine if the square symmetry group is Abelian or non-Abelian.