HOMEWORK 5

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A

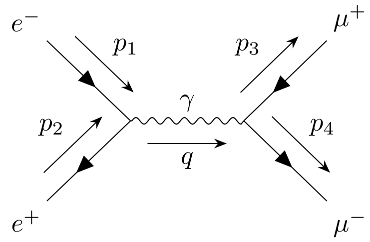

Figure A.1: This is the s-channel diagram for the scattering problem.

B

$$M = (2\pi)^4 \int \left((\bar{v}(p_2)\gamma^\mu u(p_1)) \left(\frac{i g_e g_{\mu\nu}}{q^2} \right) (\bar{u}(p_4)\gamma^\nu v(p_3)) \delta^4(p_1 + p_2 - q)\delta^4(q - p_3 - p_4) \mathrm{d}^4 q \right), \tag{B.1}$$

$$= \frac{i(2\pi)^4 g_e^2}{(p_1 + p_2)^2} (\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3))\delta^4(p_1 + p_2 - p_3 - p_4), \tag{B.2}$$

$$\rightarrow M = -\frac{g_e^2}{s} (\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{B.3}$$

C

From B.3,

$$|M|^2 = \frac{g_e^4}{s^2} (\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3))(\bar{v}(p_2)\gamma^\mu u(p_1))^* (\bar{u}(p_4)\gamma_\mu v(p_3))^*, \tag{C.1}$$

$$\left\langle |M|^2 \right\rangle = \frac{g_e^4}{4s^2} \operatorname{Tr}\left\{ \gamma^\mu (\not{p}_1 + m_e) \gamma^\nu (\not{p}_2 - m_e) \right\} \operatorname{Tr}\left\{ \gamma_\mu (\not{p}_3 - m_\mu) \gamma_\nu (\not{p}_4 + m_\mu) \right\}, \tag{C.2}$$

$$= \frac{4g_e^4}{s^2} \left(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \right) - g^{\mu\nu}(p_1 \cdot p_2) \right) \left(\left(p_{3,\mu} p_{4,\nu} + p_{4,\mu} p_{3,\nu} \right) - g_{\mu\nu}(p_3 \cdot p_4) \right), \tag{C.3}$$

$$= \frac{4g_e^4}{s^2} \Big(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \right) \left(p_{3,\mu} p_{4,\nu} + p_{4,\mu} p_{3,\nu} \right) + 4(p_1 \cdot p_2)(p_3 \cdot p_4)$$
$$- g^{\mu\nu}(p_1 \cdot p_2) \left(p_{3,\mu} p_{4,\nu} + p_{4,\mu} p_{3,\nu} \right) - \left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \right) g_{\mu\nu}(p_3 \cdot p_4) \Big), \tag{C.4}$$

$$= \frac{4g_e^4}{s^2} \left(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \right) \left(p_{3,\mu} p_{4,\nu} + p_{4,\mu} p_{3,\nu} \right) \right), \tag{C.5}$$

$$= \frac{4g_e^4}{s^2} \left(2(p_1 \cdot p_3)(p_2 \cdot p_4) + 2(p_1 \cdot p_4)(p_2 \cdot p_3) \right), \tag{C.6}$$

$$= \frac{8g_e^4}{s^2} \left((p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \right) \tag{C.7}$$

D

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{(8\pi)^2}\frac{\left\langle |M|^2 \right\rangle}{s}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}, &\text{(D.1)}\\
&= \frac{8g_e^4}{(8\pi)^2 s^2}\frac{((p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3))}{s}, &\text{(D.2)}\\
&= \frac{g_e^4}{8\pi^2 s^3}\left((p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\right), &\text{(D.3)}\\
&= \frac{g_e^4}{8\pi^2 s^3}\Big((E_e^2-|\mathbf{p}_1||\mathbf{p}_3|\cos\theta)(E_e^2-|\mathbf{p}_2||\mathbf{p}_4|\cos\theta)+(E_e^2+|\mathbf{p}_1||\mathbf{p}_4|\cos\theta)(E_e^2+|\mathbf{p}_2||\mathbf{p}_3|\cos\theta)\Big), &\text{(D.4)}\\
&= \frac{g_e^4}{8\pi^2 s^3}\Big((E_e^2-E_e^2\cos\theta)(E_e^2-E_e^2\cos\theta)+(E_e^2+E_e^2\cos\theta)(E_e^2+E_e^2\cos\theta)\Big), &\text{(D.5)}\\
&= \frac{g_e^4}{8\pi^2 s^3}E_e^4\Big((1-\cos\theta)^2+(1+\cos\theta)^2\Big), &\text{(D.6)}\\
&= \boxed{\frac{g_e^4}{512\pi^2 E_e^2}(3+\cos 2\theta)\,.} &\text{(D.7)}
\end{aligned}
$$

E

Both go as $1/E_e^2$, however, the interaction we've calculated has a different angular dependence and a magnitude one forth that of the solution given for problem 7.38 in Griffiths.

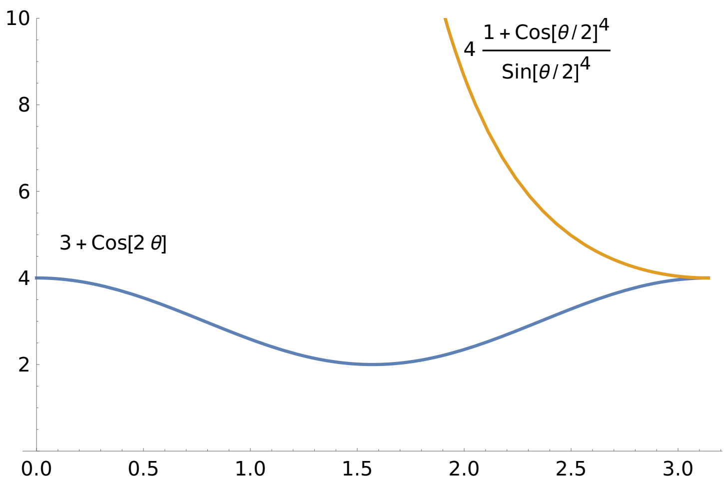

Figure E.1: This figure shows the angular dependence as found in this problem compared to the angular dependence of problem 7.38 in Griffiths. Notice that the dependence found in this problem has an amplitude of one forth that of the dependence in problem 7.38 of Griffiths. That is, the two dependencies are equal at $\theta = \pi$, when multiplying one the book's dependence by 4.

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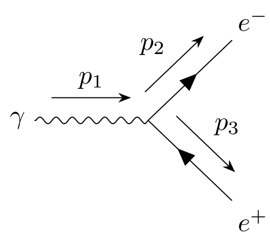

Figure .1: This is the s-channel diagram for the decay problem.

A

For this instance,

$$M = i(\bar{u}(p_2) i g_e v(p_3)), \tag{A.1}$$
$$= -g_e \bar{u}(p_2) v(p_3). \tag{A.2}$$

Thus,

$$\Gamma = \frac{S|\mathbf{p}_2|}{8\pi m_\gamma^2} \left\langle |M|^2 \right\rangle, \tag{A.3}$$
$$= \frac{|\mathbf{p}_2|}{8\pi m_\gamma^2} g_e^2 \operatorname{Tr}\big\{(\not{p}_3 - m_e)(\not{p}_2 + m_e)\big\}, \tag{A.4}$$
$$= \frac{g_e^2 |\mathbf{p}_2|}{8\pi m_\gamma^2} (p_2 \cdot p_3 - 4m_e^2), \tag{A.5}$$
$$= \frac{g_e^2 |\mathbf{p}_2|}{8\pi m_\gamma^2} (E_e^2 + \mathbf{p}_2^2 - 4m_e^2), \tag{A.6}$$
$$= \frac{g_e^2 |\mathbf{p}_2|}{8\pi m_\gamma^2} (2\mathbf{p}_2^2 - 3m_e^2), \tag{A.7}$$
$$= \frac{g_e^2 \sqrt{m_1^2 - 4m_2^2}}{8\pi m_\gamma^3} \left(\frac{1}{4} m_\gamma^2 - m_e^2 - \frac{3}{2} m_e^2\right). \tag{A.8}$$

Either I did something wrong (likely) or equation A.8 simplifies in some way I'm not seeing and is equal to the requested expression.

B

The lifetime is $\tau = 1/\Gamma$, so if $m_\gamma = 300\text{MeV}$, then $\tau = 6.01 \times 10^{-22}\text{s}$