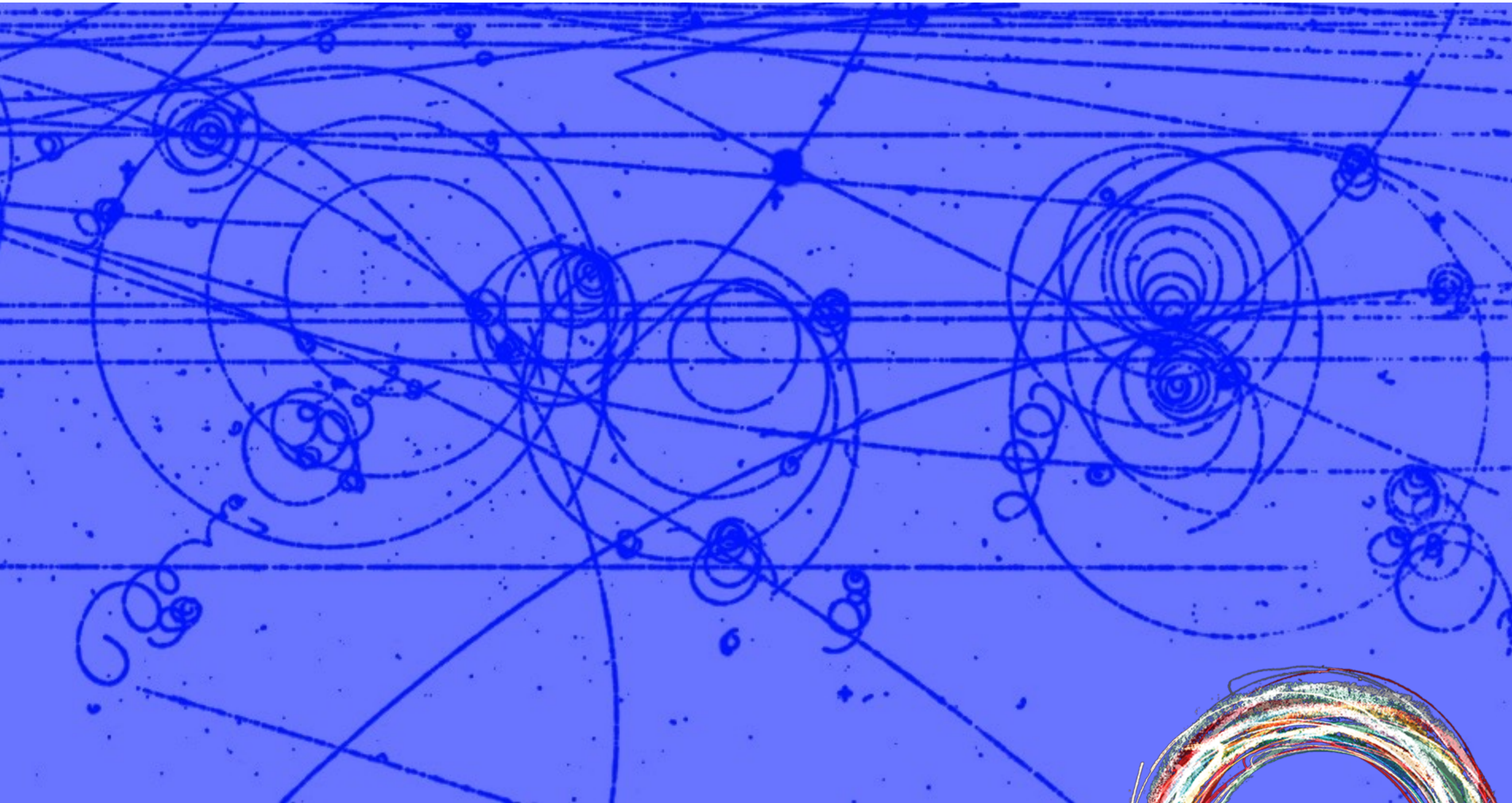


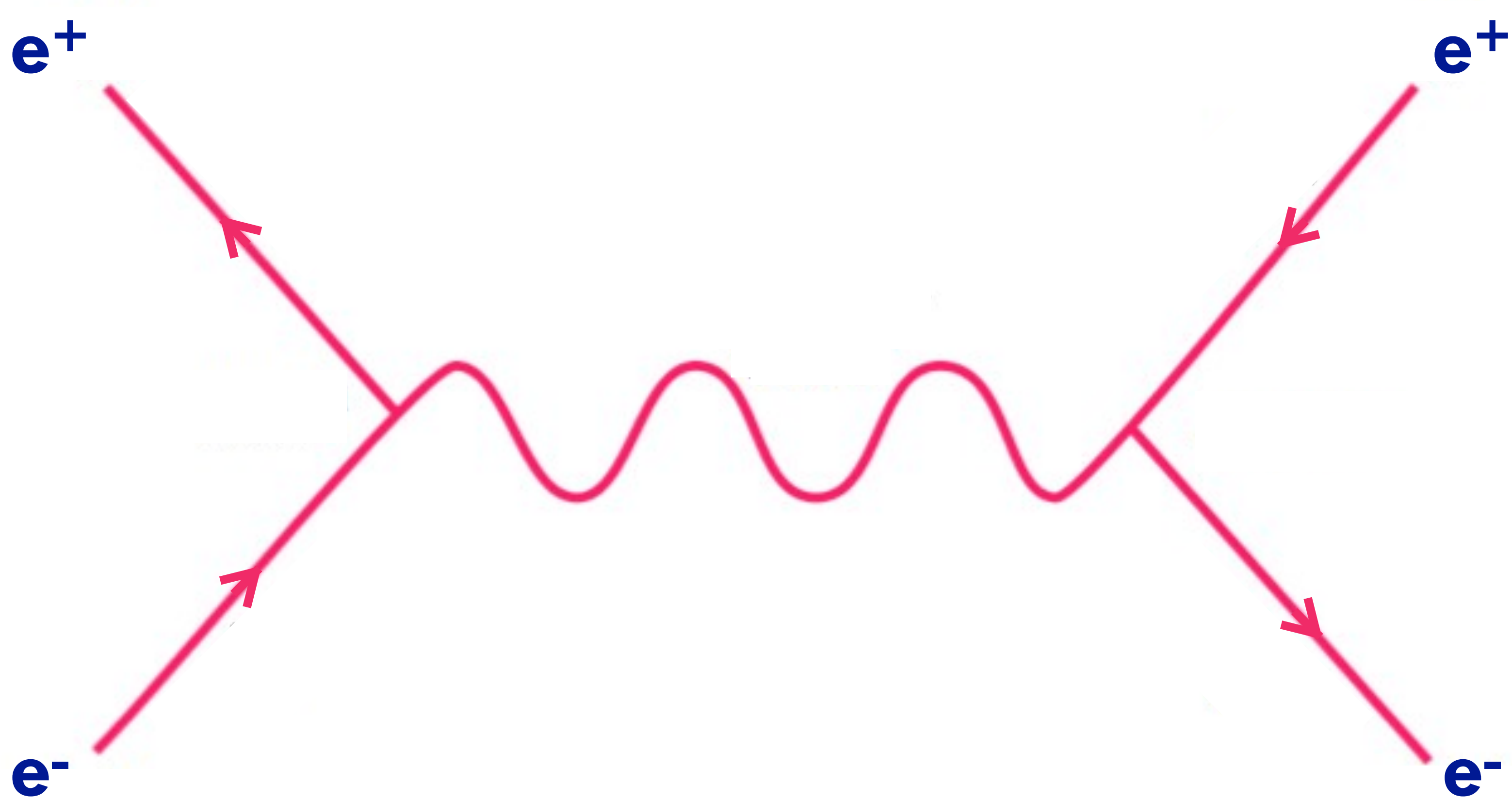
QED FEYNMAN RULES

PHY 493/803

Feynman Rules for QED

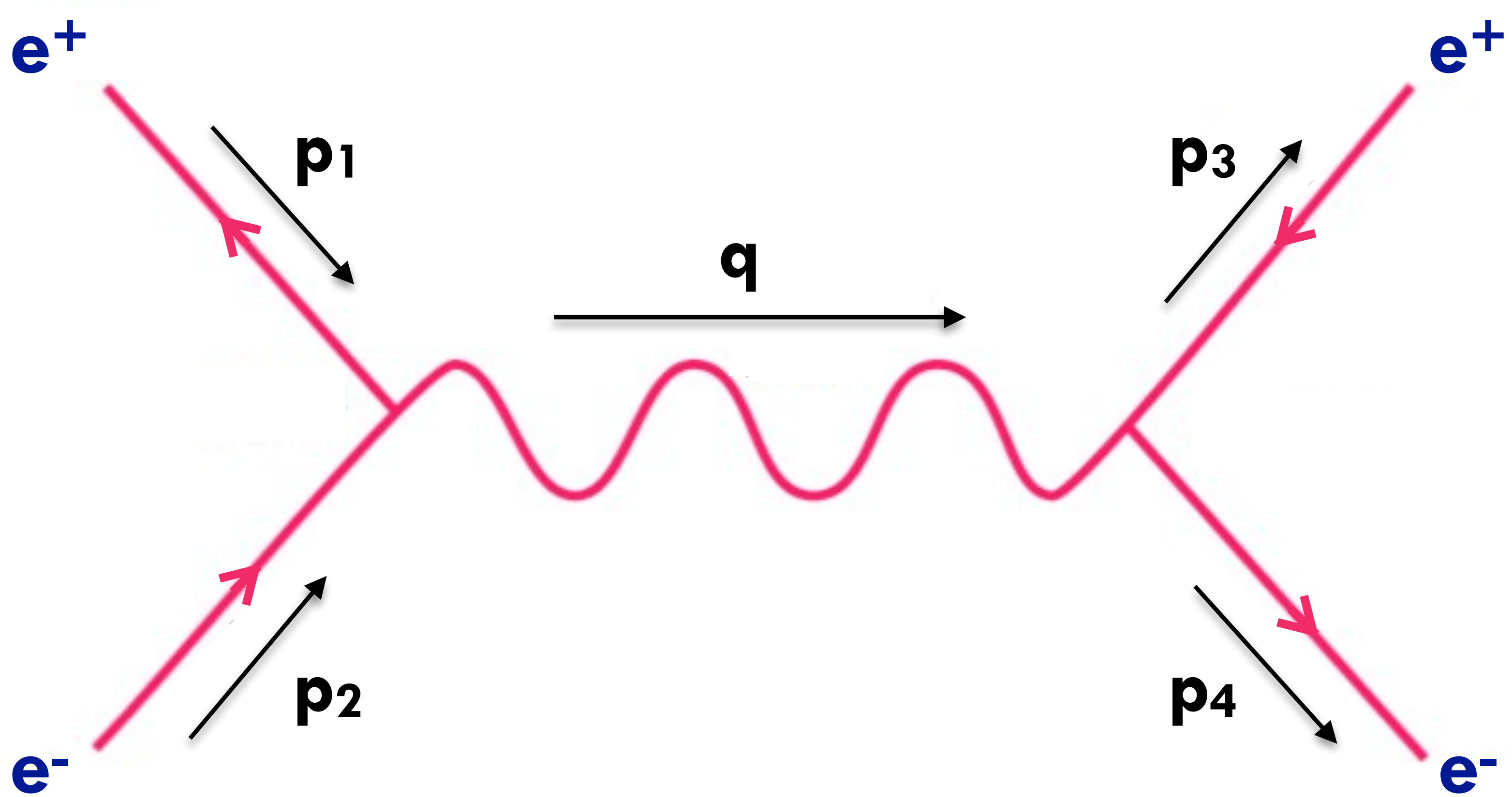
Recall: The Feynman rules provide the recipe for constructing an amplitude $\mathscr{M}$ from a Feynman diagram.

Step 1: For a particular process of interest, draw a Feynman diagram with the minimum number of vertices. There may be more than one.



Feynman Rules for QED

Step 2:
For each Feynman diagram, label the four-momentum of each line, enforcing four-momentum conservation at every vertex.
Note that arrows are only present on fermion lines and they represent particle flow, not momentum.



Feynman Rules for QED

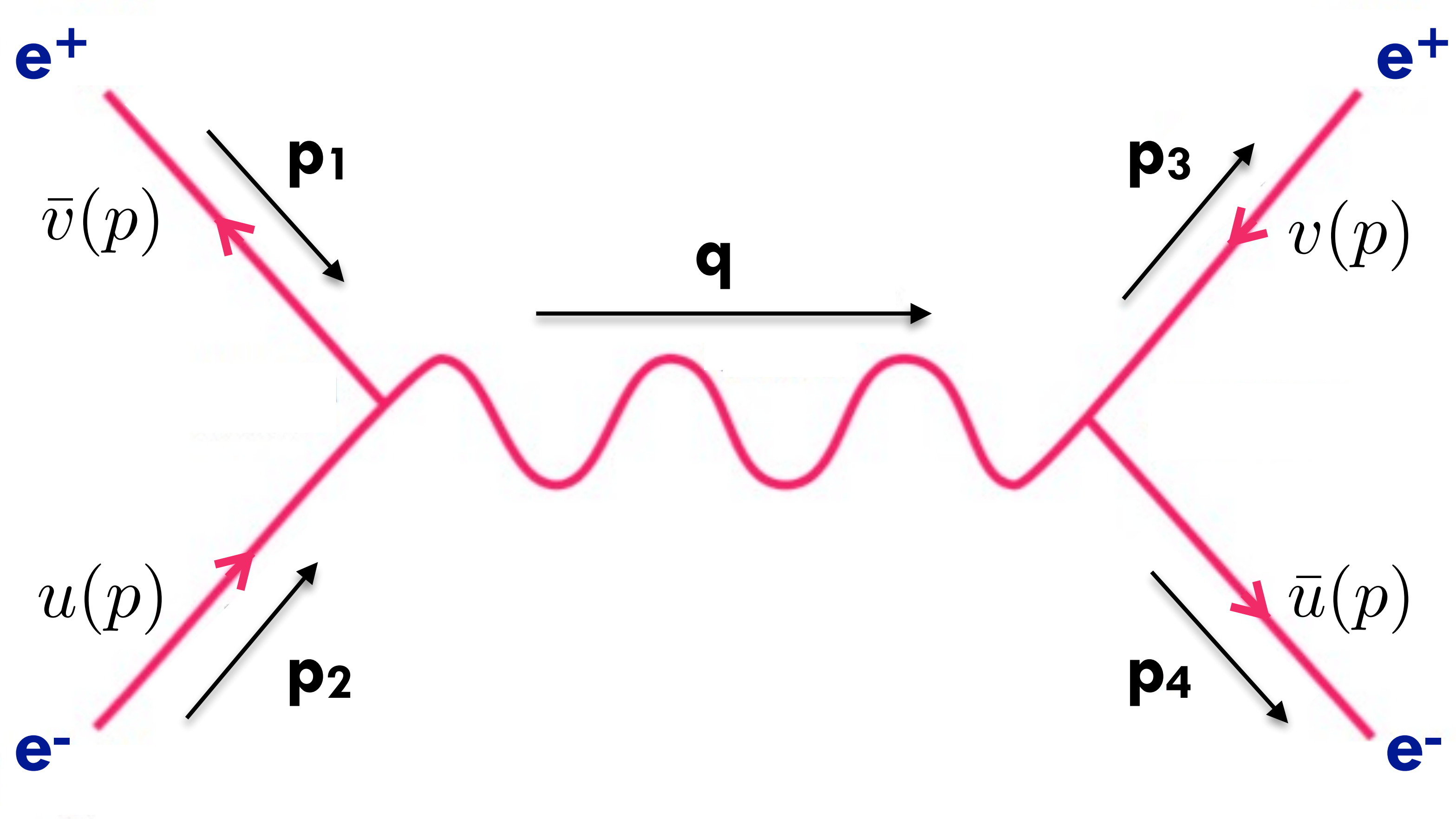
Step 3: For each external line, include a factor for the particle wave function:

spin 1/2

incoming particle	$u(p)$
outgoing particle	$\bar{u}(p)$
incoming antiparticle	$\bar{v}(p)$
outgoing antiparticle	$v(p)$

spin 1

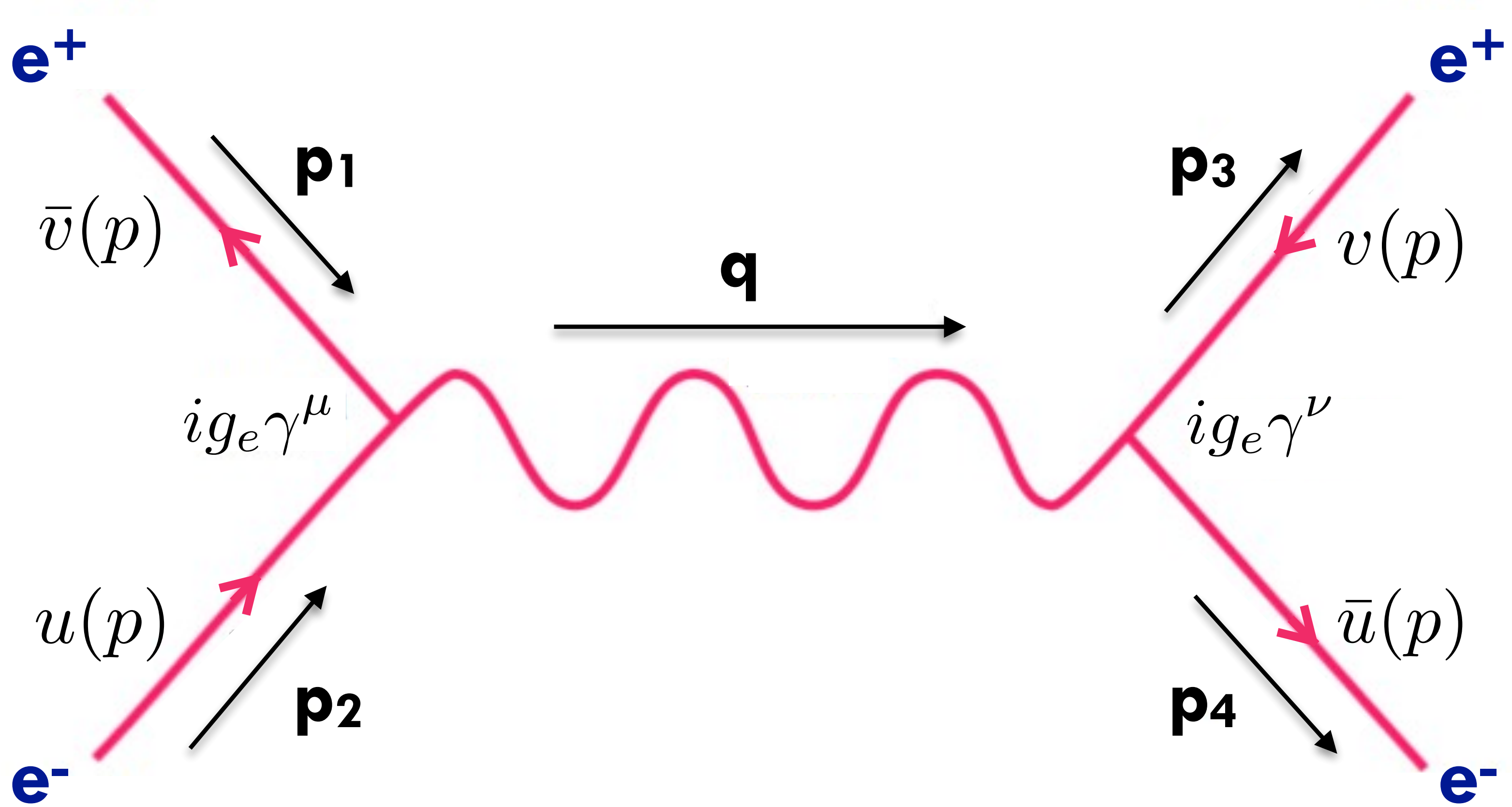
incoming photon	$\varepsilon^{\mu}(p)$
outgoing photon	$\varepsilon^{\mu}(p)^{*}$



Feynman Rules for QED

Step 4:
Every QED vertex contributes a factor of $ig_e\gamma^\mu$

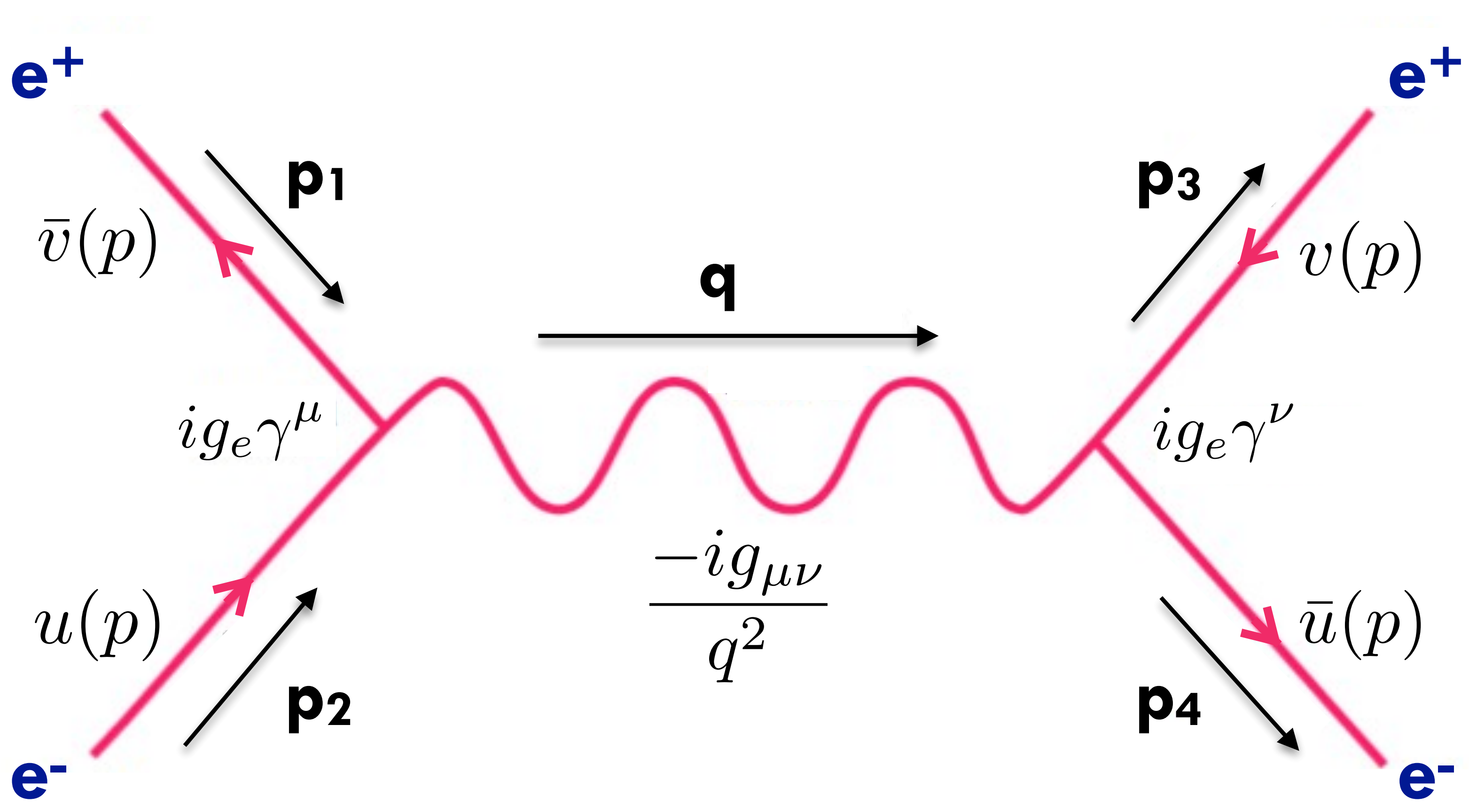
where g_e is a dimensionless coupling constant and is related to the fine-structure constant by $\alpha = \frac{g_e^2}{4\pi}$



Feynman Rules for QED

Step 5:
Each internal line contributes a factor as follows:

Photons: $$\frac{-ig_{\mu\nu}}{q^2}$$ **Fermions:** $$\frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$$



Feynman Rules for QED

Step 6:
Each vertex gets a delta function over the 4-momenta into/out of the vertex. Take care to get the 4-momentum signs right!!

$$(2\pi)^4\,\delta^4(k_1 + k_2 + k_3)$$

Step 7:
Each internal momentum gets a phase space integral factor.

$$\frac{d^4q}{(2\pi)^4}$$

Step 8:
After integrating, the result will include a delta function reflecting total energy/momentum conservation. Cancel this factor (and the $(2\pi)^4$) and multiply by *i*.
The result is the matrix element.

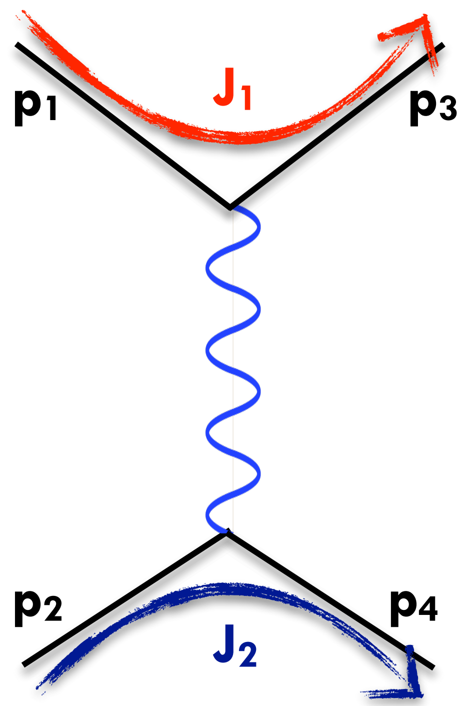
Step 9:
Antisymmetrization: Include a minus sign between diagrams that differ only in the interchange of two incoming (or outgoing) electrons (or positrons), or of an incoming electron with an outgoing positron (or vice versa).

Anti-symmetrization

The anti-symmetrization issue is hiding a more important aspect of QFT.

What we're really doing on some level is tracing the "current" in question.

This can be electric charge, probability, weak hypercharge, etc.

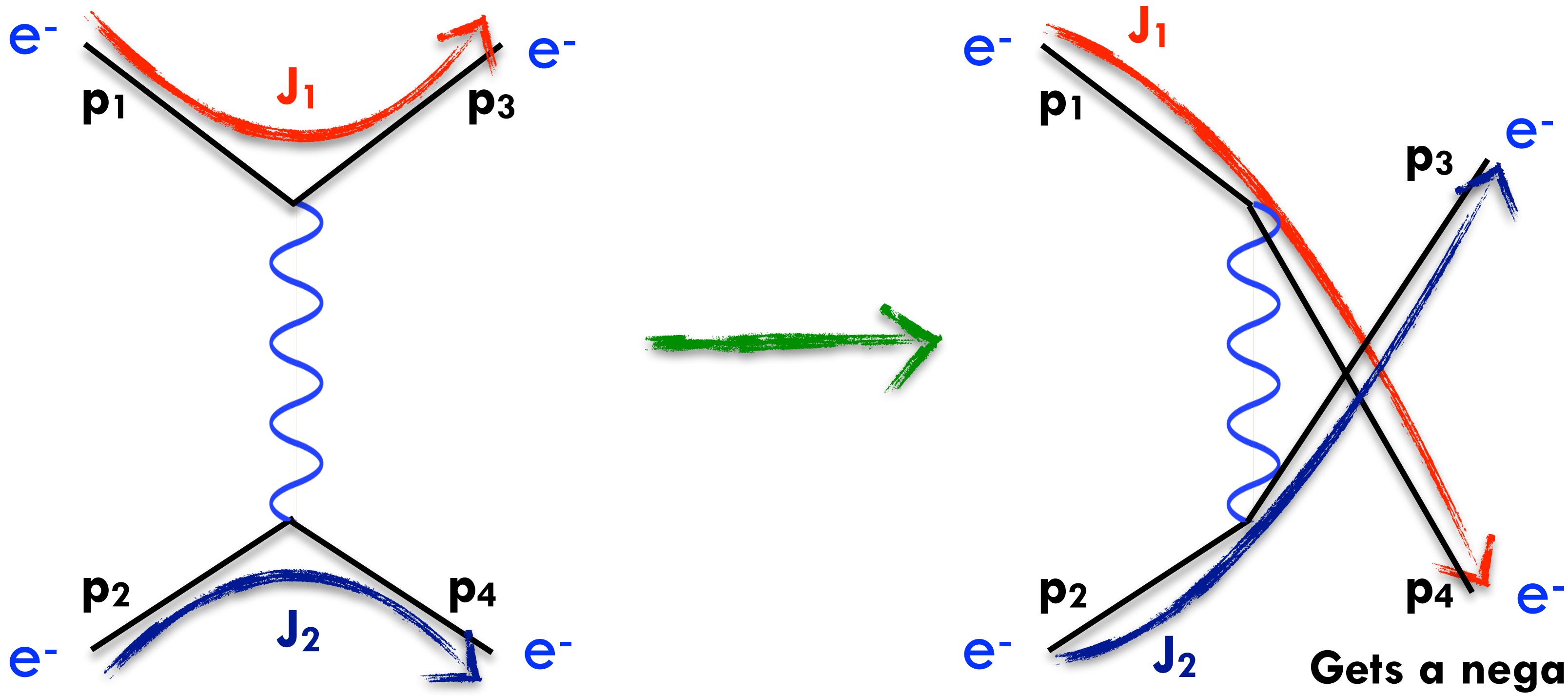


Anti-symmetrization - u-channel

What matters is the "current"

Black = electron **Magenta = positron**

<u>This case:</u> electron-electron scattering (Møller scattering)



Gets a negative sign in the matrix element sum!

The exchange of the final state electrons interchanges momentum definitions.

J_1 goes from (p_3-p_1) to (p_4-p_1)

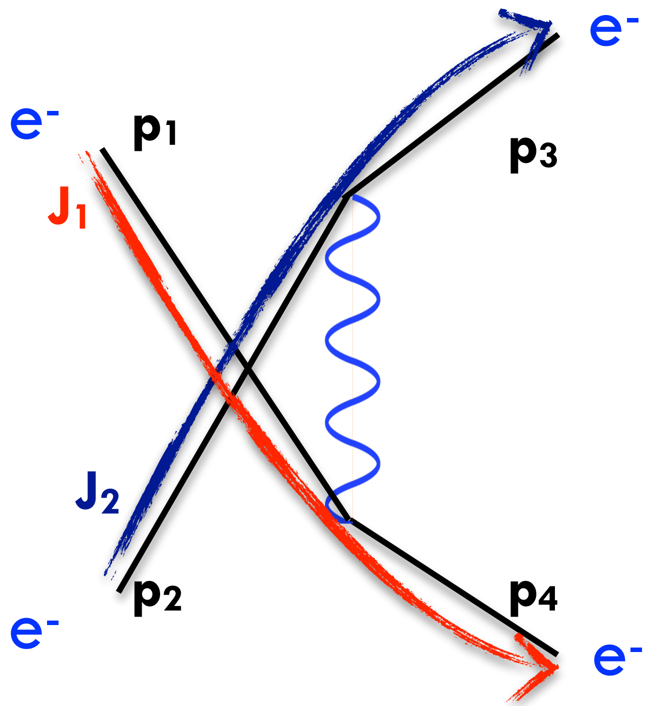
J_2 goes from (p_4-p_2) to (p_3-p_2)

Anti-symmetrization - not for the initial state!

What matters is the "current"

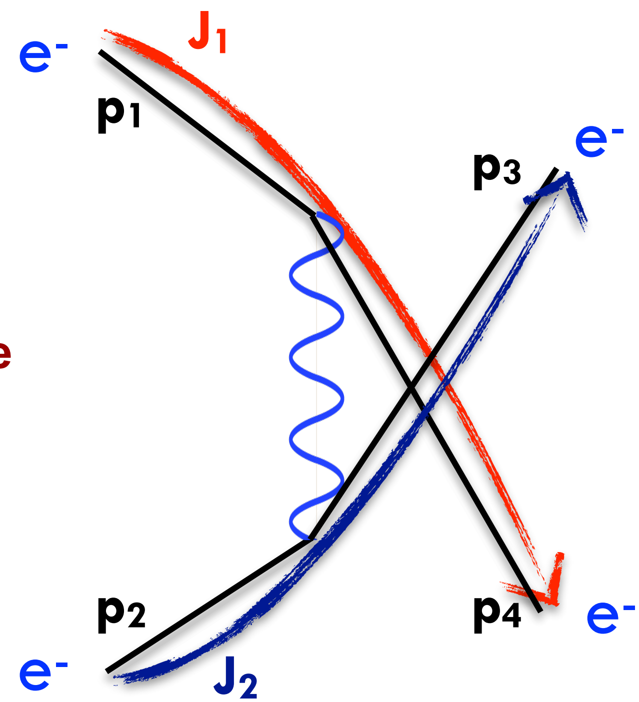
Black = electron **Magenta = positron**

This case: electron-electron scattering (Møller scattering)



NO!

We don't do both diagrams because they arrive at the same current definitions! The convention is the one on the right.



The exchange of the final state electrons interchanges momentum definitions.

J_1 goes from (p_3-p_1) to (p_4-p_1)

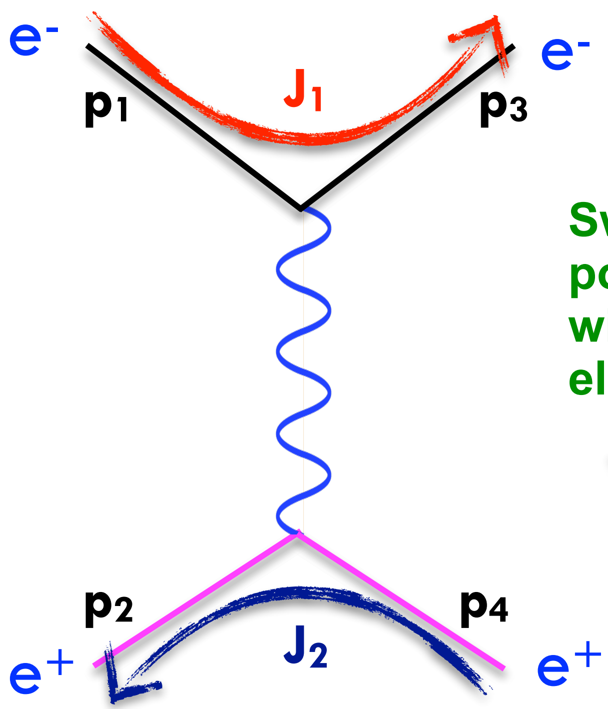
J_2 goes from (p_4-p_2) to (p_3-p_2)

Anti-symmetrization

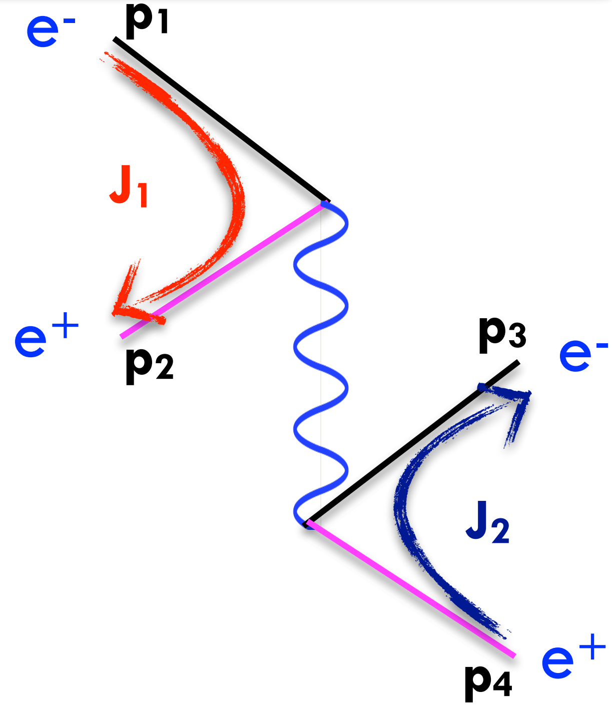
What matters is the "current"

Black = electron **Magenta = positron**

<u>This case:</u> electron-positron scattering (Møller scattering)



Swap incoming positron from J2 with outgoing electron from J1



Gets a negative sign in the matrix element sum!

The exchange of the initial state electron with final state positron interchanges:

J_1 goes from (p_3-p_1) to (p_2-p_1)

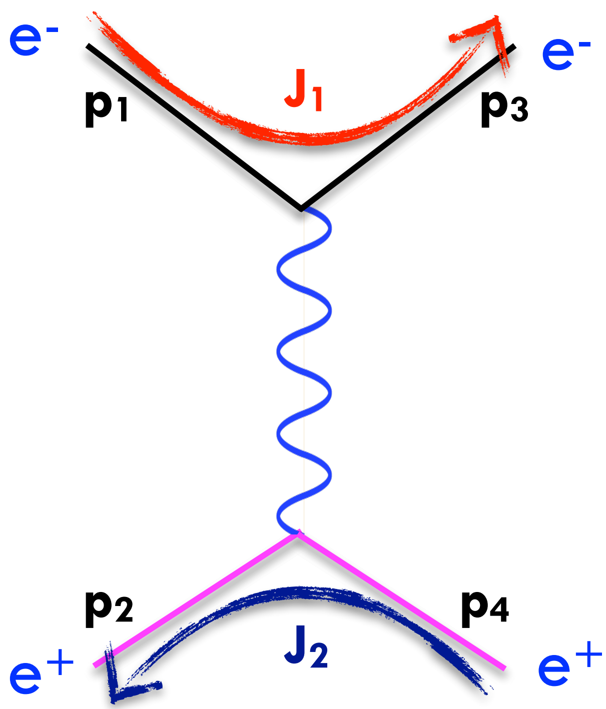
J_2 goes from (p_4-p_2) to (p_4-p_3)

Anti-symmetrization

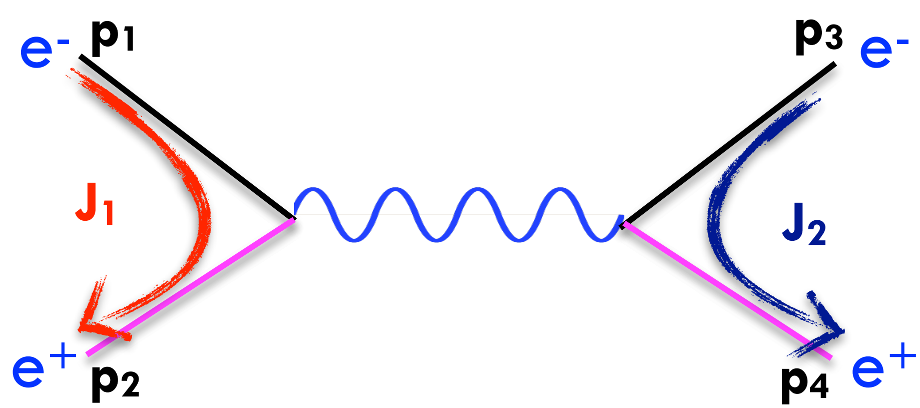
What matters is the "current"

Black = electron **Magenta = positron**

<u>This case:</u> electron-positron scattering (Møller scattering)



Rearranging gets you here!



Gets a negative sign in the matrix element sum!

The exchange of the initial state electron with final state positron interchanges:

J_1 goes from (p_3-p_1) to (p_2-p_1)

J_2 goes from (p_4-p_2) to (p_4-p_3)

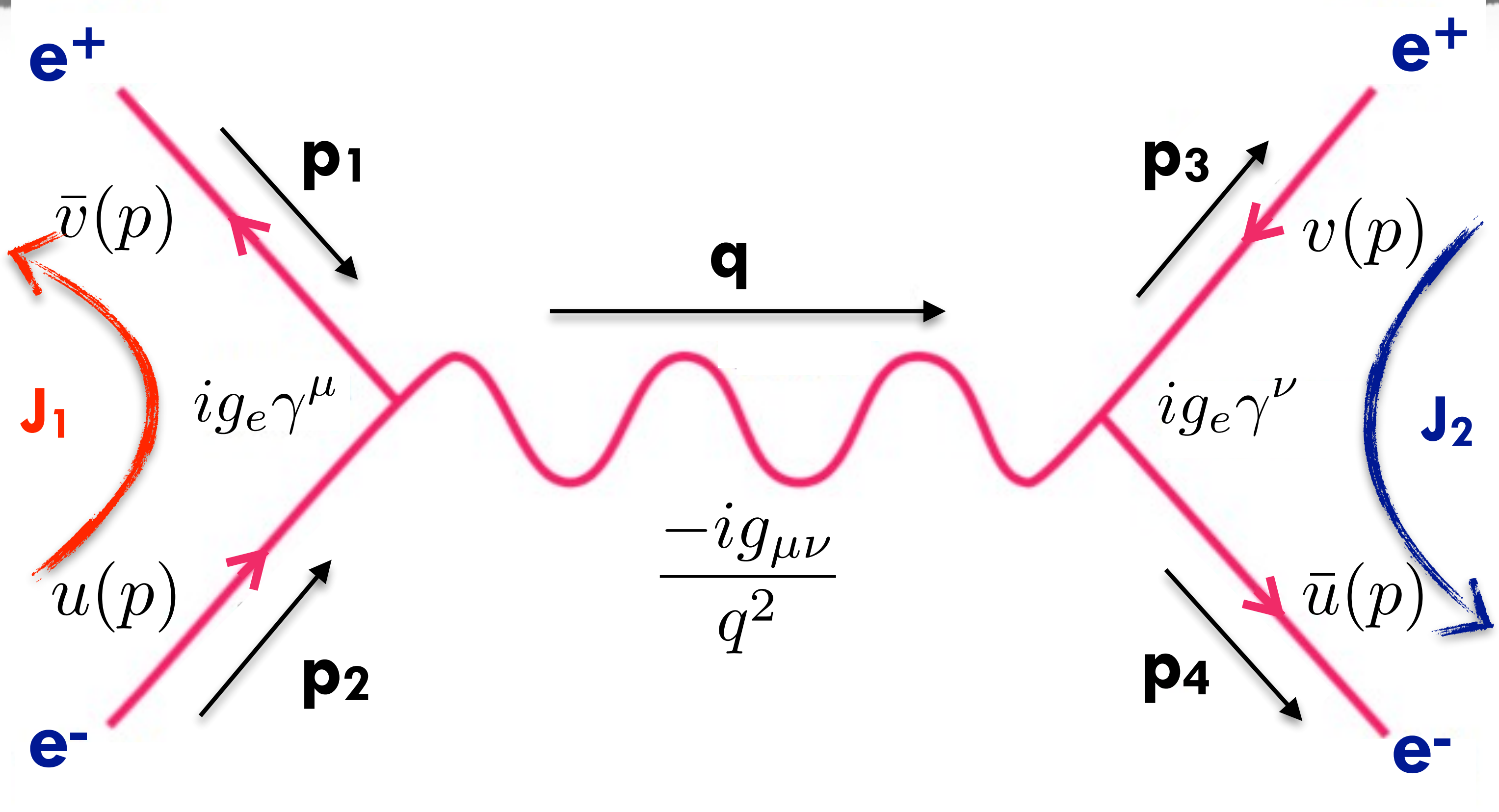
Current "Sandwiches"

When building QED matrix elements, it's easiest to think of following currents and building current "sandwiches".

Follow the particle current from end to start, sandwich the vertex factor in the middle! Adjoint spinors on the left, spinors on the right.

J₁: $\bar{v}(p_1)(ig_e\gamma^\mu)u(p_2)$

J₂: $\bar{u}(p_4)(ig_e\gamma^\nu)v(p_3)$

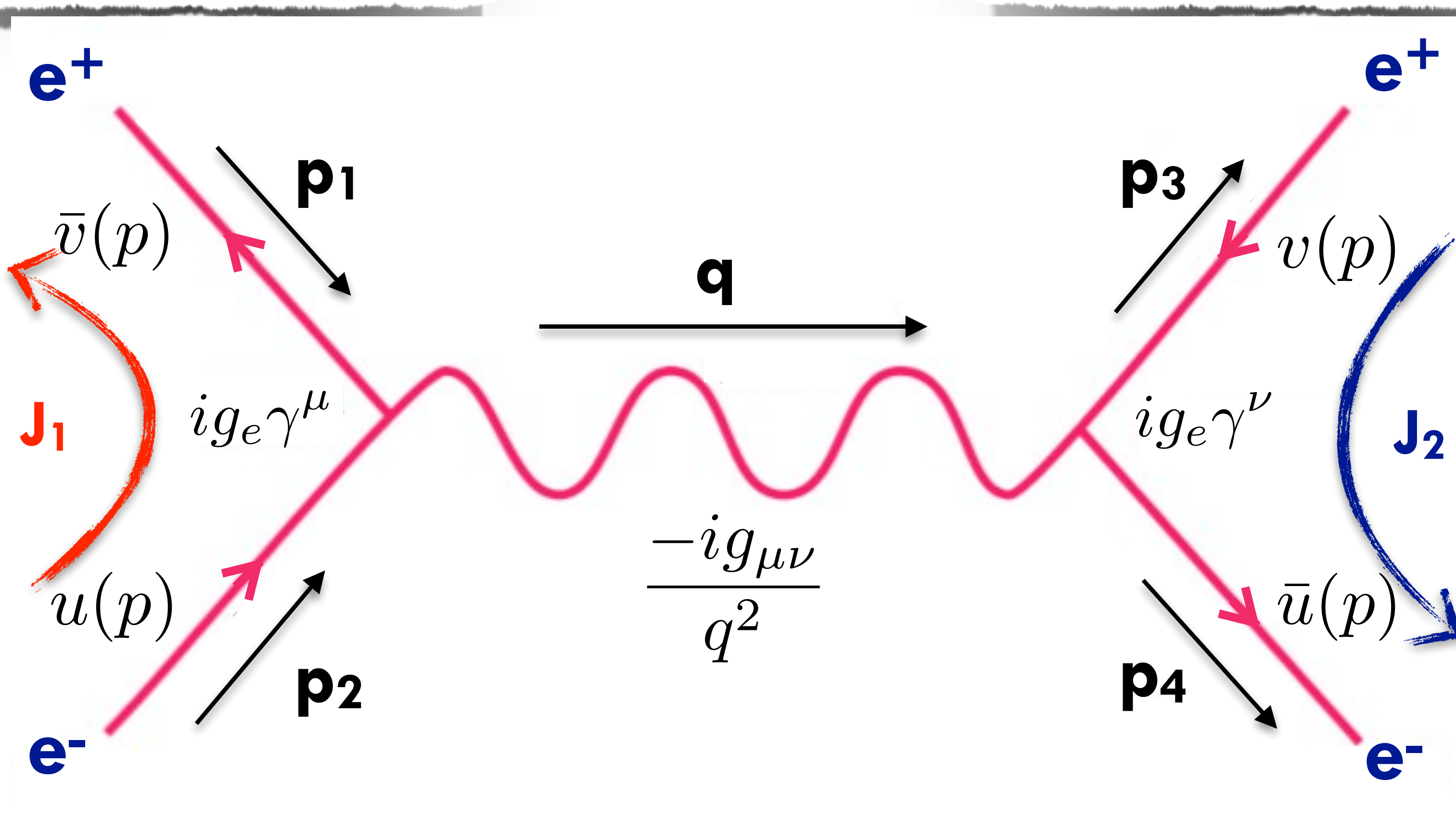


The Matrix Element

$$\mathcal{M} = (2\pi)^8 \int (J_1) \left(\frac{-ig_{\mu\nu}}{q^2} \right) (J_2)\, \delta^4(p_1 + p_2 - q)\, \delta^4(q - p_3 - p_4)\, \frac{d^4q}{(2\pi)^4}$$

J₁: $\bar{v}(p_1)(ig_e\gamma^\mu)u(p_2)$

J₂: $\bar{u}(p_4)(ig_e\gamma^\nu)v(p_3)$



The Matrix Element

$$\mathcal{M} = (2\pi)^8 \int (J_1) \left(\frac{-i g_{\mu\nu}}{q^2} \right) (J_2)\, \delta^4(p_1 + p_2 - q)\, \delta^4(q - p_3 - p_4)\, \frac{d^4 q}{(2\pi)^4}$$

1) Substitute the forms of the currents:

$$= i(2\pi)^4\, g_e^2 \int [\bar{v}(p_1)\gamma^\mu u(p_2)] \left(\frac{g_{\mu\nu}}{q^2} \right) [\bar{u}(p_4)\gamma^\nu v(p_3)] \quad \delta^4(p_1 + p_2 - q)\, \delta^4(q - p_3 - p_4)\, d^4 q$$

2) Do the integration over the propagator momentum: $q \rightarrow p_1 + p_2$

$$= \frac{i(2\pi)^4\, g_e^2}{(p_1 + p_2)^2} [\bar{v}(p_1)\gamma^\mu u(p_2)]\ [\bar{u}(p_4)\gamma_\mu v(p_3)]\ \delta^4(p_1 + p_2 - p_3 - p_4)$$

3) Cancel the remaining delta function (and its $(2\pi)^4$ factor!), multiply by i

$$= -\frac{g_e^2}{(p_1 + p_2)^2} [\bar{v}(p_1)\gamma^\mu u(p_2)]\ [\bar{u}(p_4)\gamma_\mu v(p_3)]$$

Anti-Symmetrization and its effect on the cross section

- Cross section is the square of the matrix element, which normally is
- $|M|^2 = |M_1|^2 + |M_2|^2 + 2|M_1M_2|$
- Anti-symmetrization results in destructive interference
- $|M|^2 = |M_1|^2 + |M_2|^2 - 2|M_1M_2|$
- Total and differential cross-sections are reduced if interference effects are large