

Homework 7

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1 Sketch in the Large Δm Limit

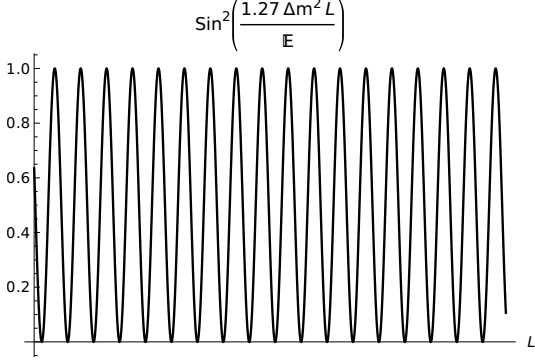


Figure 1.1: This shows a qualitative sketch of $\sin^2\left(1.27\Delta m^2 L/E\right)$, from L to $L + \Delta L$, where ΔL is the size of the detector. In the limit where $|\Delta m|^2 \gg E/L$, the spatial frequency is very high, as depicted by the large number of cycles shown in the graph.

2 Large Δm Limit

Since the probability of an oscillation occurring is

$$P_{osc}(t) = \sin^2(2\theta) \sin^2\left(1.27\Delta m^2 \frac{L}{E}\right), \quad (2.1)$$

the survival probability is just $1 - P_{osc}(t)$. Additionally, for $|\Delta m|^2 \gg E/L$, one can say that the limit of \sin^2 goes to the average value of $1/2$. Thus

$$2(1 - P_{surv}) = \sin^2(2\theta). \quad (2.2)$$

With $P_{surv} = 0.92$, $\sin^2(2\theta) = 0.16$.

3 Small Δm Limit

In the limit $|\Delta m|^2 \ll E/L$,

$$\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right) \rightarrow \left(1.27\Delta m^2 \frac{L}{E}\right)^2. \quad (3.1)$$

Hence

$$1 - P_{surv} = \sin^2(2\theta) \left(1.27\Delta m^2 \frac{L}{E}\right)^2, \quad (3.2)$$

$$\sin^2(2\theta) = C \left(\frac{1}{\Delta m^2}\right)^2, \quad (3.3)$$

where $C = (1 - P_{surv})(E/L)^2$.

4 The Null result from the CHOOZ experiment

A Oscillation Probability

$$P_{osc} = \sin^2(2\theta) \sin^2\left(1.27\Delta m^2 \frac{L}{E}\right), \quad (4.1)$$

$$= 0.085. \quad (4.2)$$

B Allowed or Excluded?

The point is just barely inside the allowed region, being slightly below the green curve at $\sin^2(2\theta) = 0.1$.