PHY493/803 Spring 2022, Intro to Elementary Particle Physics

Homework 1

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

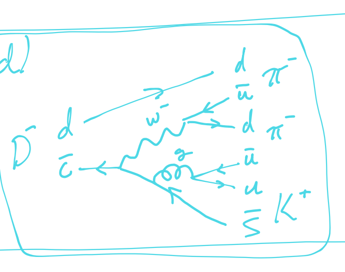
1. (3pt x 8)
   Decays of fundamental particles (particles with no internal structure) that are on-shell (mass corresponds to their rest mass) must strictly follow conservation rules. One of the following decays is allowed and the rest violate at least one conservation rule. For each process, list all violated conservation rules or identify it as allowed.
   Consider: energy conservation, charge conservation, lepton number (each of the three lepton numbers), baryon number, isospin, charmness, strangeness, bottomness), spin, weak isospin T_3.
   (a) $\gamma \rightarrow e^- + \mu^+$ a) lepton # not conserved (L_e + L_μ); energy not conserved
   (b) $W^+ \rightarrow t + \bar{b}$ b) energy not conserved
   (c) $Z^0 \rightarrow \mu^+ + \mu^+$ c) charge not conserved, neither is L_μ.
   (d) $t \rightarrow W^+ + b$ d) allowed
   (e) $W^- \rightarrow e^- + \nu_e$ e) lepton # not conserved (L_e)
   (f) $\gamma \rightarrow \tau^- + \tau^+$ f) energy not conserved
   (g) $b \rightarrow c + e^-$ g) beauty, charmness, L_e, and charge not conserved.
   (h) $\bar{b} \rightarrow Z + \bar{s}$
   h) beauty, strangeness, and energy not conserved.

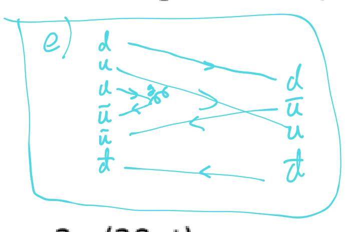
2. (3pt x 6)
   Decays of hadrons (particles made up of quarks) can proceed via many different processes. Evaluate the following reactions and determine which are physically possible. Draw Feynman diagrams at the quark level for the reactions that are allowed. For those that are forbidden, what conservation laws are violated? If the reactions are allowed, which interactions (strong, weak, and electromagnetic forces) should be involved?
   If necessary, visit the following web page at
   http://pdg.lbl.gov/2021/tables/contents_tables.html
   (a) $\Omega^- \rightarrow \Xi^- + \pi^-$
   (b) $\Sigma^+ \rightarrow \pi^+ + \pi^0$
   (c) $\pi^0 \rightarrow \mu^+ + e^- + \bar{\nu}_e$
   (d) $D^- \rightarrow K^+ + \pi^- + \pi^-$
   (e) $p + \bar{p} \rightarrow \pi^+ + \pi^-$
   (f) $p \rightarrow e^+ + \gamma$

a) Weak isospin isn't conserved, so it can't be a weak interaction, but flavor is changing, indicating it can't be any other type of interaction. Not allowed!

b) Baryon # not conserved. Not allowed!

c) lepton # not conserved.
$L_e = 0 \rightarrow L_e = 0$
$L_\mu = 0 \quad\; L_\mu = -1$
Not allowed!

d)


e)


f) Baryon # & lepton # not conserved. Not allowed!

3. (20pt)
   What would be the approximate counting rate observed in the Rutherford scattering of 10MeV α-particles off gold foil at an angle of $\theta=\pi/4$ in the laboratory? Assume an incident flux of 10^5 α-particles per second on the foil, a foil of 0.1 cm thickness, and a detector of transverse area 1 cm x 1 cm placed 100 cm from the interaction point, the atomic (mass) number of gold of 79 (197), and the density of gold of 19.7 g/cm^3.

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZ}{4}\frac{\alpha\hbar c}{E_{kin}}\right)^2 \csc^4\left(\frac{\pi}{8}\right) = 1504.3\ \text{fm}^2$$

$$N_{target} = t \cdot \rho_{Au} = 0.1\text{cm} \cdot 19.7\,\text{g/cm}^3 = 1.97\,\tfrac{\text{g}}{\text{cm}^2} \cdot (196.9665\,\text{g/mol})^{-1} \cdot N_A = 6.023 \times 10^{21}\,\frac{1}{\text{cm}^2}$$

$$d\Omega = \frac{1\,\text{cm}^2}{(100\,\text{cm})^2} = \frac{1}{10000}$$

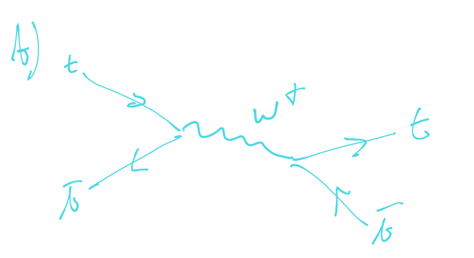
$$dW = N_{target}\frac{d\sigma}{d\Omega}\,d\Omega\,I = \left(6.023 \cdot 10^{21}\,\tfrac{1}{\text{cm}^2}\right)(10^{-4})(1504.3\,\text{fm}^2)\left(10^5\,\tfrac{1}{\text{s}}\right)$$
$$\boxed{= 9.06 \cdot 10^{25}\ \text{Hz}}$$

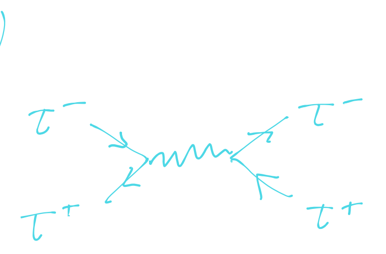
4. (5 pt + 15 pt) **{Required for PHY803 students only. +10 pts extra credit for PHY493 students.}**
   A) The $\Lambda^0(1116)$ is a hadronic resonance with mass 1116 MeV. Explain why the decay $\Lambda^0 \rightarrow p + \pi^-$ is allowed, but $\Lambda^0 \rightarrow \pi^+ \pi^-$ is not.

   Baryon # is conserved in the first interaction, but not the second.

   B) Some of the decays listed in problem 1 actually do occur at the LHC at CERN where they are part of a more extensive Feynman diagram in a collisions (b, f), or at b-factories as part of a particle decay Feynman diagram (h). Give an example of a complete Feynman diagram for each of these three processes and explain what condition needs to be fulfilled for each process to occur.

   b, f, and h all violate energy conservation, so they are allowed if they're off their mass-shell.

b)


f)


h)