HOMEWORK 3

Brandon Henke[1]

[1] *Michigan State University - Department of Physics & Astronomy*

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1 PION PAIR PRODUCTION

Consider the production of a pair of pions in e^+e^- collisions. This is an electromagnetic process that proceeds via a virtual photon.

A

What are the possible final states and what is the isospin of these pions?

All pions have total isospin of 1, so the options, when conserving charge gives two neutral pions ($|1,0\rangle |1,0\rangle$) or a positive and a negative pion ($|1,1\rangle |1,-1\rangle$ or $|1,-1\rangle |1,1\rangle$)

B

The virtual photon can have isospin 0 or 1 because EM interactions don't preserve isospin. From the Clebsch-Gordan tables, find the decay amplitudes for both photon isospin states, $|0,0\rangle$ and $|1,0\rangle$. Write down the particles corresponding to the isospin states for each coefficient.

$$|1,0\rangle \to \frac{1}{\sqrt{2}}\left(|1,1\rangle |1,-1\rangle - |1,-1\rangle |1,1\rangle\right). \tag{B.1}$$

$$|0,0\rangle \to \frac{1}{\sqrt{3}}\left(|1,1\rangle |1,-1\rangle - |1,0\rangle |1,0\rangle + |1,-1\rangle |1,1\rangle\right). \tag{B.2}$$

C

Only one of the two decays ($|0,0\rangle$ and $|1,0\rangle$) is actually allowed. Determine which one and why.

The charge parity of the two neutral pions is 1, but it's -1 for the photon. Thus this charge conjugation is not conserved in that decay, so it ($|0,0\rangle$) is not allowed.

2 MUON COLLIDER

It would be great if we could store and accelerate muons to produce powerful neutrino beams or for a muon collider. The circumference of the Tevatron ring at Fermilab is about 3.9mi. For a particle traveling at the speed of light, it takes $2.2 \cdot 10^{-5}$s to go around once. Suppose you start out with a million muons. Calculate how many muons are left after $2.2 \cdot 10^{-5}$s for different muon beam energies.

A

What is the muon lifetime? Round to two significant digits.

According to the PDG, the mean lifetime of the μ is $\tau = 2.2 \cdot 10^{-6}$s.

B

If the million muons start out at rest, how many will still be around $2.2 \cdot 10^{-5}$s later?

The number after some time is given by

$$\begin{aligned} N(t) &= N_0 e^{-t/\tau}, && \text{(B.1)} \\ &= 10^6 e^{-10}, && \text{(B.2)} \\ &= 45.399. && \text{(B.3)} \end{aligned}$$

C

If one million muons start out with an energy of 1GeV, i.e. traveling in a beam inside an accelerator, how many will still be around $2.2 \cdot 10^{-5}$s later? Use a muon mass of 100MeV.

For a muon with 1GeV, the value of γ is $\gamma = 1\text{GeV}/0.1\text{GeV} = 10$. Thus, the number left is

$$\begin{aligned} N(t) &= N_0 e^{-\frac{t}{\gamma\tau}}, && \text{(C.1)} \\ &= 10^6 e^{-1}, && \text{(C.2)} \\ &= 367879. && \text{(C.3)} \end{aligned}$$

D

If one million muons start out with an energy of 10GeV, i.e. traveling in a beam inside a high-powered accelerator, how many would still be around $2.2 \cdot 10^{-5}$s later?

Similar to before,

$$N(t = 2.2 \cdot 10^5\text{s}) = 904837. \tag{D.1}$$

3 Z BOSON DECAYS TO NEUTRINOS

The LEP e^+e^- collider determined the number of neutrino generations interacting with a Z boson indirectly. The total number of Z bosons produced in the process $e^+e^- \to Z$ can be determined by measuring the width of the Z boson peak through a scan of the beam energy. The peak is wide enough for the measurement not to be limited by experimental resolution. The width of the resonance peak together with knowing the number of e^+e^- collisions determines the total number of Z bosons produced. We start from 5000 Z bosons produced in one such measurement and count the observed Z boson decays. The visible decays are those that can be observed by the detector. The observed number of events produced for each final state ($e^+e^-, \mu^+\mu^-, \tau^+\tau^-$,quarks) is given below:

Particle	Observed Event Count
Electron	168
Muon	172
Tau	164
Quarks	3480

Table .1

These event counts follow Poisson statistics. The uncertainty σ on the number of events N is given by $\sigma_N = \sqrt{N}$. The rate of those Z boson decays that are not observed by the detector (invisible decays) can then be obtained by summing up all of the visible decays of the Z boson (to leptons and quarks) and subtracting the number of visible decays from the total number of Z bosons produced. The invisible decays include Z boson decays to neutrinos as well as Z boson decays to particles beyond the Standard Model (BSM particles).

A

What is the total number of observed Z boson decays? What is the uncertainty σ_N (visible) on the number of observed Z boson decays?

The number of observed Z boson decays is the sum of the observed event counts in table .1: 3984. The uncertainty on the number of observed Z boson decays is $\sqrt{3984} = 63.12$.

B

The total number of Z bosons produced is 5000, with a negligible uncertainty. What is the number of invisible Z boson decays N(invisible)?

It's just the 5000 less the number found in part a: 1016.

C

The uncertainty on the number of invisible decays can be obtained from error propagation, adding the uncertainty you determined in a) in quadrature with the uncertainty on the number of produced events. However, since the latter is negligible, this simplifies and the uncertainty on the number of invisible decays is equal to the uncertainty on the number of visible decays, σ_N(invisible) $= \sigma_N$ (visible). Calculate the relative uncertainty on the invisible decays, σ_N(invisible)$/N$(invisible). Give your result in percent.

The relative uncertainty is 6.2%.

D

The predicted number of Z boson decays to neutrinos for one generation is 334 events. How many generations of neutrinos does the number of invisible decays correspond to? Is this consistent with the Standard Model expectation or is there evidence for BSM physics?

This corresponds to all three generations of neutrinos ($3 \cdot 334 \approx 1016$). This is consistent with the standard model.

4 SCATTERING

Consider the elastic scattering reaction $A + B \to A + B$ in the lab frame (B initially at rest) and assume that the initial energy E_1 of the incoming A particle satisfies $E_1 \ll m_B$ so that the recoil of the target can be neglected.

A

Use the Golden Rule for scattering to show that the differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{|M|^2}{(8\pi m_B)^2} \tag{A.1}$$

Since $E_1 \ll E_2 = m_B$, $E_1 + E_2 = m_B$. Since the knockback on particle B is negligible, $|\mathbf{p}_f|/|\mathbf{p}_i| = 1$. Since there are no identical products, $S = 1/0! = 1$.

B

Write down the lowest order diagram(s) for this scattering process in ABC theory.

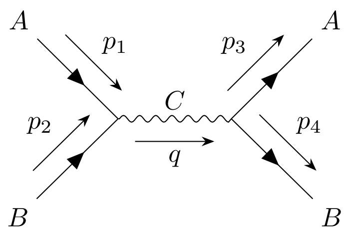

Figure B.1: This is the s-channel diagram for the scattering problem.

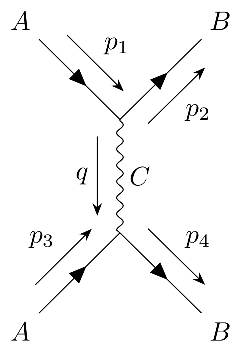

Figure B.2: This is the t-channel diagram for the scattering problem.

C

Calculate the scattering amplitude using the Feynman rules for ABC theory (express your result using the Mandelstam variables s, t, and/or u as relevant).

From the Feynman rules,

$$M = g^2 \left(\frac{1}{s} + \frac{1}{t}\right), \tag{C.1}$$

where $s = (p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$.

D

Combine the results from (a) and (c) to obtain the differential cross section (in the limit $E_1 \ll m_B$ and assuming that m_A and m_C are tiny compared to m_B).

$$s = (p_1 + p_2)^2, \tag{D.1}$$
$$= p_1^2 + p_2^2 + 2p_{1\mu}p_2^\mu, \tag{D.2}$$
$$\approx m_B^2. \tag{D.3}$$
$$t = (p_1 - p_3)^2, \tag{D.4}$$
$$= p_1^2 + p_3^2 - 2p_{1\mu}p_3^\mu, \tag{D.5}$$
$$\approx m_B^2. \tag{D.6}$$
$$\therefore \frac{d\sigma}{d\Omega} = \frac{g^4}{16\pi^2 m_B^6}. \tag{D.7}$$

E

Show that the total cross-section is

$$\sigma = \frac{g^4}{4\pi m_B^6}, \tag{E.1}$$

under the conditions stated in part (d).

Integrating D.7 over the spherical shell gives

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega\,, \tag{E.2}$$
$$= 4\pi \frac{d\sigma}{d\Omega}, \tag{E.3}$$
$$= \frac{g^4}{4\pi m_B^6}. \tag{E.4}$$

5 SCATTERING AGAIN

A

Repeat problem 4(d) assuming the reaction $A + B \to A + B$ occurs in the center-of-momentum frame, and take $m_A = m_B = m$ and $m_C = 0$. You should report your answer in terms of the incident particle energy ($E_A = E_B = E$) and the scattering angle for particle A (θ).

Making the changes from question 4 gives

$$s = 4m^2, \tag{A.1}$$
$$t = -4p^2 \sin^2\left(\frac{\theta}{2}\right), \tag{A.2}$$
$$M = g^2 \left(\frac{(m + p\sin(\theta/2))(m - p\sin(\theta/2))}{4m^2p^2\sin^2(\theta/2)} \right), \tag{A.3}$$

and

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{(64\pi E)^2} \left(\frac{(m + p\sin(\theta/2))(m - p\sin(\theta/2))}{m^2p^2\sin^2(\theta/2)} \right)^2. \tag{A.4}$$

B

Without actually performing the integral, explain what the total cross section would be for this process. Does it converge or is it divergent? Compare to the solution in problem (4e) and explain any differences.

Since the integrand is proportional to $1/\sin^3(\theta)$, this cannot converge on the interval $[0, \pi]$. This implies an infinitely large cross section. This is different from problem 4 because we cannot remove the dependence on θ. In problem 4, the dot products all vanished due to the negligible momentum of particle B. However, that cannot be done here.