Homework 6

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A

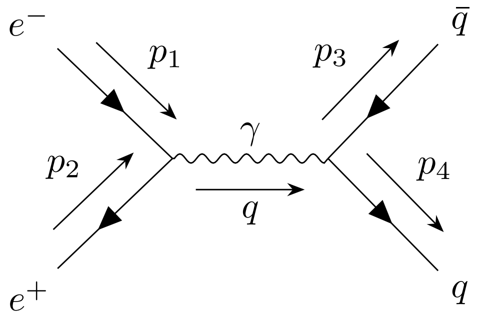

Figure 1.1: This is the s-channel diagram for the scattering problem.

B

$$M = (2\pi)^4 \int \left((\bar{v}(p_2)\gamma^\mu u(p_1)) \left(\frac{ig_e g_{\mu\nu}}{q^2}\right) (\bar{u}(p_4)\gamma^\nu v(p_3))\delta^4(p_1+p_2-q)\delta^4(q-p_3-p_4)\mathrm{d}^4 q \right), \tag{1.1}$$

$$= \frac{i(2\pi)^4 g_e g_q}{(p_1+p_2)^2}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3))\delta^4(p_1+p_2-p_3-p_4), \tag{1.2}$$

$$\to M_u = -\frac{2g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.3}$$

$$M_d = \frac{g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.4}$$

$$M_c = -\frac{2g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.5}$$

$$M_s = \frac{g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.6}$$

$$M_t = -\frac{2g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.7}$$

$$M_b = \frac{g_e^2}{3s}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3)). \tag{1.8}$$

Only some of the equations 1.3-1.8 contribute, and it depends on the energy.

$$
\begin{aligned}
|M|^2 &= \frac{g_e^2 g_q^2}{s^2}(\bar{v}(p_2)\gamma^\mu u(p_1))(\bar{u}(p_4)\gamma_\mu v(p_3))(\bar{v}(p_2)\gamma^\mu u(p_1))^*(\bar{u}(p_4)\gamma_\mu v(p_3))^*, && (1.9)\\
\left\langle |M|^2 \right\rangle &= \frac{g_e^2 g_q^2}{4s^2}\mathrm{Tr}\{\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu(\not{p}_2 - m_e)\}\,\mathrm{Tr}\{\gamma_\mu(\not{p}_3 - m_\mu)\gamma_\nu(\not{p}_4 + m_\mu)\}, && (1.10)\\
&= \frac{4g_e^2 g_q^2}{s^2}\left(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right) - g^{\mu\nu}(p_1\cdot p_2)\right)\left(\left(p_{3,\mu}p_{4,\nu} + p_{4,\mu}p_{3,\nu}\right) - g_{\mu\nu}(p_3\cdot p_4)\right), && (1.11)\\
&= \frac{4g_e^2 g_q^2}{s^2}\Big(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right)\left(p_{3,\mu}p_{4,\nu} + p_{4,\mu}p_{3,\nu}\right) + 4(p_1\cdot p_2)(p_3\cdot p_4) \\
&\qquad - g^{\mu\nu}(p_1\cdot p_2)\left(p_{3,\mu}p_{4,\nu} + p_{4,\mu}p_{3,\nu}\right) - \left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right)g_{\mu\nu}(p_3\cdot p_4)\Big), && (1.12)\\
&= \frac{4g_e^2 g_q^2}{s^2}\left(\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right)\left(p_{3,\mu}p_{4,\nu} + p_{4,\mu}p_{3,\nu}\right)\right), && (1.13)\\
&= \frac{4g_e^2 g_q^2}{s^2}\left(2(p_1\cdot p_3)(p_2\cdot p_4) + 2(p_1\cdot p_4)(p_2\cdot p_3)\right), && (1.14)\\
&= \frac{8g_e^2 g_q^2}{s^2}\left((p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right) && (1.15)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{(8\pi)^2}\frac{\left\langle |M|^2 \right\rangle}{s}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}, && (1.16)\\
&= \frac{8g_e^2 g_q^2}{(8\pi)^2 s^2}\frac{((p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3))}{s}, && (1.17)\\
&= \frac{g_e^2 g_q^2}{8\pi^2 s^3}\left((p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right), && (1.18)\\
&= \frac{g_e^2 g_q^2}{8\pi^2 s^3}\left((E_e^2 - |\mathbf{p}_1||\mathbf{p}_3|\cos\theta)(E_e^2 - |\mathbf{p}_2||\mathbf{p}_4|\cos\theta) + (E_e^2 + |\mathbf{p}_1||\mathbf{p}_4|\cos\theta)(E_e^2 + |\mathbf{p}_2||\mathbf{p}_3|\cos\theta)\right), && (1.19)\\
&= \frac{g_e^2 g_q^2}{8\pi^2 s^3}\left((E_e^2 - E_e^2\cos\theta)(E_e^2 - E_e^2\cos\theta) + (E_e^2 + E_e^2\cos\theta)(E_e^2 + E_e^2\cos\theta)\right), && (1.20)\\
&= \frac{g_e^2 g_q^2}{8\pi^2 s^3}E_e^4\left((1-\cos\theta)^2 + (1+\cos\theta)^2\right), && (1.21)\\
&= \frac{g_e^2 g_q^2}{512\pi^2 E_e^2}\left(3 + \cos 2\theta\right). && (1.22)
\end{aligned}
$$

Additionally, one needs to sum equation 1.22 over each of the colors, of which there are three, so just a factor of three is picked up. Hence, the differential cross section, including colors, is

$$
\frac{\mathrm{d}\sigma_{col}}{\mathrm{d}\Omega} = \frac{3g_e^2 g_q^2}{512\pi^2 E_e^2}\left(3 + \cos 2\theta\right). \tag{1.23}
$$

C

Equation 1.23 needs to be summed for all possible quark flavors. In the problem statement, it says the center-of-momentum energy is 30GeV. This means that only quarks with a mass $m_q \le 15\text{GeV}$. Hence, the allowed quarks are u, d, c, s, and b. The u and c contribute $g_q = 2g_e/3$, and the d, s, and b contribute $g_q = -g_e/3$. Thus, for this problem, the total differential cross section is

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{tot}}{\mathrm{d}\Omega} &= \frac{3(8+3)g_e^4}{9\times 512\pi^2 E_e^2}\left(3 + \cos 2\theta\right), && (1.24)\\
&= \frac{11g_e^4}{3\times 512\pi^2 E_e^2}\left(3 + \cos 2\theta\right), && (1.25)\\
&= \frac{11g_e^4}{1536\pi^2 E_e^2}\left(3 + \cos 2\theta\right). && (1.26)
\end{aligned}
$$

Compared to the cross section of the $e^+e^- \to \mu^+\mu^-$ interaction, it's just a factor of 11/3 larger.

2

A

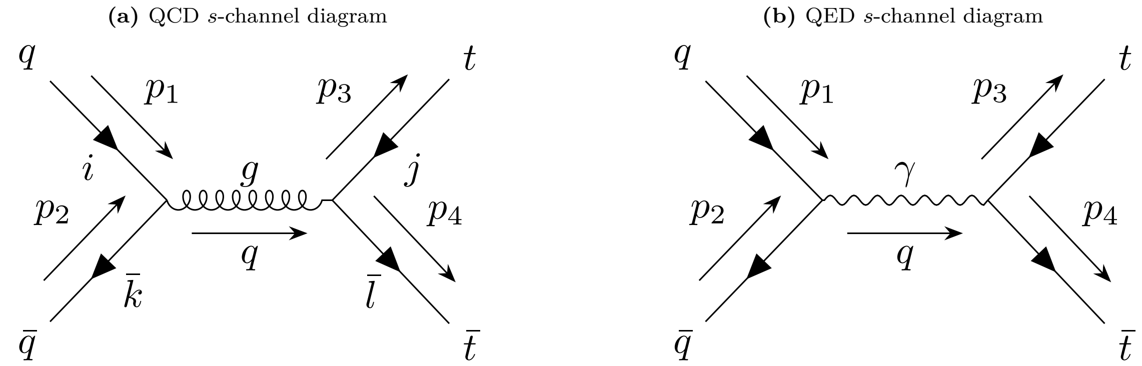

Figure 2.1: These are the s-channel diagrams for problem 2.

B

Coincidently, this is the same as slide 12 of the 11-02b lecture slides, since the initial state particles can have any of the three colors ($3^2 = 9$ color combinations), and the final state is the same way, but with color conserved:

$$\left\langle |C|^2 \right\rangle = \frac{1}{9} \sum_{i,j,k,l=1}^{3} \left| C(ij \to kl) \right|^2, \tag{2.1}$$

$$= \frac{1}{9} \left[3 \left(\frac{1}{3} \right)^2 + 6 \left(-\frac{1}{6} \right)^2 + 6 \left(\frac{1}{2} \right)^2 \right], \tag{2.2}$$

$$= \frac{2}{9}. \tag{2.3}$$

C

$$E_{CM} = 2m_t, \tag{2.4}$$

$$E = \frac{1}{2} E_{CM}, \tag{2.5}$$

$$= m_t = 173\text{GeV}. \tag{2.6}$$

D

$$\frac{\sigma(q\bar{q} \to t\bar{t})}{\sigma(e^+e^- \to t\bar{t})} \approx \frac{\alpha_s^2 \left\langle |C|^2 \right\rangle}{\alpha_{EM}^2}, \tag{2.7}$$

$$\approx 41.7. \tag{2.8}$$