PHY493/803 Intro to Elementary Particle Physics

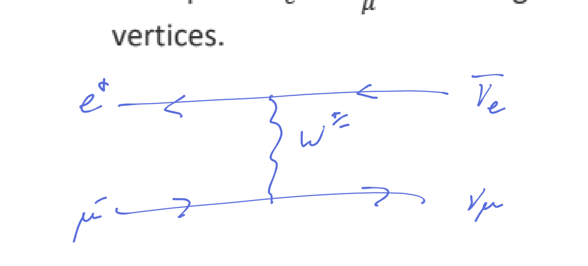
Midterm Exam 1

This exam is worth 100 points. There are ***five*** *problems and each has 25 points. Partial points are indicated in the first line.* **Choose any 4 of the 5 problems to complete. PHY803 students must do Problem #5.**

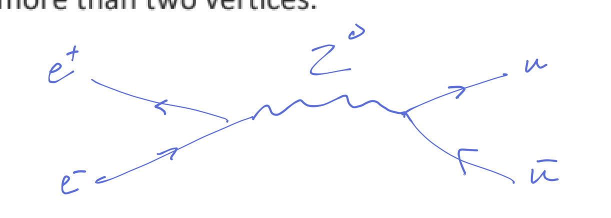
Take a moment and look over the exam before you begin. To receive the full credit for each answer, you must work neatly, show your work and simplify your answer to the extent possible.

1. (10+10+5pt) Feynman diagrams

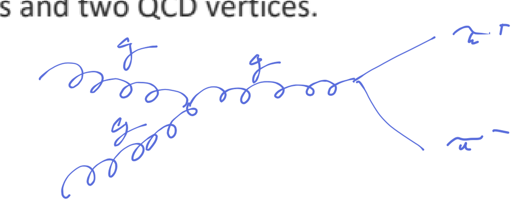
a) Draw a Feynman diagram contributing to the process $e^+ + \mu^- \to \bar{\nu}_e + \nu_\mu$. Your diagram should contain no more than two vertices.



b) Draw a Feynman diagram for the production of one neutral pion at an electron-positron collider, $e^+ + e^- \to \pi^0$. Your diagram should contain no more than two vertices.



c) Draw at least one Feynman diagram for the production of two charged pions at a hadron collider through the collision of two gluons, $g + g \to \pi^+ + \pi^-$. Hint: there is one diagram with two should be two QED vertices and two QCD vertices.



2. (5pt x 5) Mark each statement as true or false.

a) The total energy of a relativistic object is $E = \sqrt{p^2 - m^2}$

False

b) Parity violation is a small effect in weak interactions.

True? It happens in weak interactions but what do you mean "small effect"?

c) A circle of unit length is the complex plane is an Abelian U(1) group.

True

d) Since the quark content of the neutral pion is $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$, neutral pions can only be produced if four quarks come together.

False

e) The symmetry associated with translation along the x-axis gives rise to the conservation of momentum p_x.

True

3. (15+10 pt) quantum numbers and C, P, T

a) Compute isospin I_3 in terms of upness U and downness D. First, write down the upness and downness of the up and down quarks and antiquarks. Then set up the formula to compute I_3 from U and D. Then show that this works by comparing the result of your formula to the PDG isospin values for the proton, the neutron, the π^+ and the π^0.

$u: U = 1$ $\quad I_3 = \frac{1}{2}(U + D)$

$d: D = -1$

These are the same as the PDG: proton: $I_3 = \frac{1}{2}(U+D) = \frac{1}{2}(2-1) = \frac{1}{2}$ neutron: $I_3 = \frac{1}{2}(1-2) = -\frac{1}{2}$ $\quad \pi^+: I_3 = \frac{1}{2}(1+1) = 1$ $\quad \pi^0: I_3 = \frac{1}{2}(0) = 0$

b) A proton-proton collision strong interaction can produce a final state of three pions ($\pi^+ + \pi^+ + \pi^0$), but not two ($\pi^+ + \pi^+$). Why?

$p: P = +1$ $\quad \pi^\pm \,\&\, \pi^0: P = -1$

$p + p \to \pi^+ + \pi^+ + \pi^0$ $\quad \bar{p} + \bar{p} \to \pi^- + \pi^- + \pi^0$

$P = 1^2 = 1$ $\quad P = (-1)^3 = -1$

Parity isn't conserved, but this is an allowed strong interaction, so charge conjugation must also be violated.

$\hat{C} \otimes \hat{C} |p\rangle|p\rangle = |\bar{p}\rangle|\bar{p}\rangle$

$\hat{C} \otimes \hat{C} \otimes \hat{C} |\pi^+\rangle|\pi^+\rangle|\pi^0\rangle = -|\pi^-\rangle|\pi^-\rangle|\pi^0\rangle$

In the two pion case, parity is conserved but C-symmetry is still violated, which is only allowed in weak interations. Hence $p + p \to \pi^+ + \pi^+$ isn't allowed.

4. (10+10+5 pt) Isospin

Compare the two proton-pion collision processes:

(i) $\pi^- + p \to K^0 + \Sigma^0$ and (ii) $\pi^- + p \to K^+ + \Sigma^-$. Use isospin to find the ratio of the two cross-sections. You can assume that the initial state has isospin $|1/2, -1/2>$.

a) Write down the quark content and the isospin of the four final state particles.

$K^0: d\bar{s}$ $\quad I = \frac{1}{2}$ $\quad I_3 = -\frac{1}{2}$

$\Sigma^0: uds$ $\quad I = 1$ $\quad I_3 = 0$

$K^+: u\bar{s}$ $\quad I = \frac{1}{2}$ $\quad I_3 = \frac{1}{2}$

$\Sigma^-: dds$ $\quad I = 1$ $\quad I_3 = -1$

b) Look up the coefficients in the Clebsch-Gordan tables in the PDG for each of the two processes.

$A_1 |\frac{1}{2}, -\frac{1}{2}\rangle |1, 0\rangle + A_2 |\frac{1}{2}, \frac{1}{2}\rangle |1, -1\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$

$A_1 = \sqrt{\frac{1}{3}}$ $\quad A_2 = -\sqrt{\frac{2}{3}}$

c) What is the ratio of the cross-sections?

$\frac{1}{3} \Big/ \frac{2}{3} = \frac{1}{2} \to 2:1$

$K^+\Sigma^- : K^0\Sigma^0$

5. (5+5+10+5 pt) **Required for 803 students, optional for 493 students.**

a) The process listed in question 1.b), $e^+ + e^- \to \pi^0$, is not allowed. Why not?

Energy isn't conserved.

b) A top-quark pair is produced in a proton-proton collision at the LHC at CERN. Assume a top quark mass of m_t = 172.5 GeV. What is the minimum COM energy to produce the top-quark pair?

$\boxed{\text{min}: 2m_t = 345 \text{ GeV}}$

c) The top quark then decays to a W boson (m_W = 80 GeV) and a bottom quark (m_b = 5 GeV). In the top-quark rest frame, what is the energy of the bottom quark E_b? Hint: You can assume that energy E_b is much larger than the bottom-quark mass.

$\begin{pmatrix} m_t \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} E_b \\ \vec{p}_b \end{pmatrix} + \begin{pmatrix} E_W \\ -\vec{p}_b \end{pmatrix}$ $\quad E_b \approx \sqrt{\vec{p}_b^{\,2}}$

$m_t = \sqrt{\vec{p}_b^{\,2}} + E_W$

$= \sqrt{\vec{p}_b^{\,2}} + \sqrt{\vec{p}_b^{\,2} + m_W^2}$

$= E_b + \sqrt{E_b^2 + m_W^2}$

$E_b = \frac{m_t^2 - m_W^2}{2m_t} \approx 62.77 \text{ GeV}$

d) What is the energy of the W boson from the top-quark decay in the top-quark rest frame?

$E_W = m_t - E_b = 109.73 \text{ GeV}$