PHY493/803, Intro to Elementary Particle Physics

Homework 5

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

1. (10pts + 15pts + 15pts + 15pts + 15pts = 70 pts total)
   Calculate the cross section for the reaction $e^+e^- \to \mu^+\mu^-$. Assume that the incoming particles are not polarized and that the spin projections of the outgoing muons are not measured. Further assume that the energy of the incoming electrons in the center-of-momentum frame is much larger than the electron or muon masses.
   Hint: For this problem, it will be highly instructive to work through Griffiths' examples 7.1, 7.3, 7.5 and 7.7 first. Griffiths' problems 7.26 and 7.38 will also be of use to peruse first. Note the "crossing" symmetry with the $e^+\mu^- \to e^+\mu^-$ scattering process.

   a) Draw the relevant Feynman diagram(s) for the lowest order process. Label the particles as follows: 1 is the incoming electron, 2 the incoming positron, 3 is the outgoing μ^+, 4 is the outgoing μ^-.

   b) Use the Feynman rules to determine the matrix element.

   c) Use Casimir's trick to calculate the spin-averaged square of the matrix element $\langle|\mathcal{M}^2|\rangle$. You can use the approximation $E_e \gg m_\mu$.
   Hint: You will need the following trace relationships:
   $Tr[\gamma^\mu(\not{p}_1+m_e)\gamma^\nu(\not{p}_2-m_e)] = 4\{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_2)\}$
   $Tr[\gamma_\mu(\not{p}_3+m_\mu)\gamma_\nu(\not{p}_4-m_\mu)] = 4\{p_{3,\mu}p_{4,\nu} + p_{4,\mu}p_{3,\nu} - g_{\mu\nu}(p_3 \cdot p_4)\}$
   And remember to average over initial state spins and sum over final state spins. And $g^{\mu\nu}g_{\mu\nu} = 4$.

   d) Calculate the cross section, starting from the average matrix element $\langle|M|^2\rangle = \frac{8g_e^4}{s^2}((p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)$ and remember that the cross section is given by the golden rule, $\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{\langle |M|^2 \rangle}{(E_1+E_2)^2} \frac{|p_f|}{|p_i|}$. Give your result in the center-of-momentum frame. You can use the approximation $E_e \gg m_\mu$.

e) How does the differential cross section for $e^+e^- \to \mu^+\mu^-$ compare with the cross section for $e^+\mu^- \to e^+\mu^-$ scattering at the same electron energy (see Griffiths problem 7.38)?

2. (25+15 pts) {**Required for PHY803 students only. +40 pts extra credit for PHY493 students.}**

   a) Griffiths' problem 7.48(a), show that the result is $\Gamma = \frac{g^2 (m_\gamma^2 - 4m_e^2)^{3/2}}{8\pi m_\gamma^2}$:

      Imagine that the photon, instead of being a massless vector (spin 1) particle, were a massive scalar (spin 0). Specifically, suppose the QED vertex factor were $ig_e 1$
      (where 1 is the 4 by 4 unitary matrix), and the 'photon' propagator were
      $\frac{-i}{q^2 - (m_\gamma c)^2}$.
      There is no photon polarization vector now, and hence no factor for external photon lines. Apart from that, the Feynman rules for QED are unchanged. Assuming it is heavy enough, this 'photon' can decay. Calculate the decay rate for $\gamma \to e^+ + e^-$.
      Notes:
      a. The description of the problem begins on the previous page (pg 272 in my book).
      b. It is helpful to calculate the width in the COM frame.
      c. The trace to compute in this case is

$$Tr(\gamma_\mu p_2^\mu + m_e)(\gamma_\mu p_2^\mu - m_e) = p_2 \cdot p_3 - 4m_e^2$$

d. Use the golden rule for 2-particle decays (book Eq. 6.35) to determine the width. In the CM frame, your result should depend only on $\mathrm{g_e}$, m_γ and m_e.

b) Griffiths' problem 7.48(b): If $m_\gamma = 300$ MeV, find the lifetime of the 'photon', in seconds.
Notes:
   a. You can use $\alpha = \frac{1}{137} = \frac{g^2}{4\pi}$.
   b. To get the correct units, you will have $\hbar = 6.58 \times 10^{-22}$ MeV·s to get the time in seconds.