PHY493/803, Intro to Elementary Particle Physics

Example Midterm Exam 2

This example exam is worth 100 points. There are **five** *problems and each has 25 points. Partial points are indicated in the first line.* **Choose any 4 of the 5 problems to complete. PHY803 students must do Problem #5.**

Take a moment and look over the exam before you begin. To receive the full credit for each answer, you must work neatly, show your work and simplify your answer to the extent possible.

The last 2 pages of this example exam contain a list of Feynman rules, useful for problem 4. This will also be provided at the midterm exam.

1. (5pts x 5) Answer True or False for each question:

a) The cross section and the decay rate both depend on the matrix element in the same way.
_____________TRUE____

b) QED is a full, quantum mechanical model for spin-1/2 particle interactions.
____________TRUE.

c) Dirac spinors in QED have two components. ____________FALSE______

d) Synchrotrons use magnetic fields to maintain a linear path for particles moving at high velocity.

____________FALSE (the particles maintain a circular path)

e) At very relativistic energies, electrons primarily lose energy via Bremsstrahlung when passing through materials.
________TRUE____

2. (10 + 5 + 5 + 5 pts)

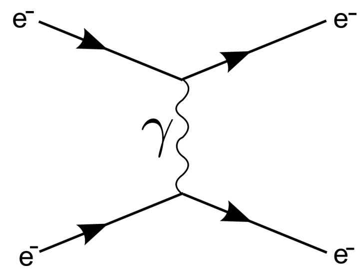
   a) What is the length L of the drift chamber in a linear accelerator that operates with a frequency F of 30 MHz and accelerates electrons to a momentum of 0.5 GeV? Assume that the electrons will travel the full length of the chamber before the voltage swings from positive to negative (or vice versa).

      Answer: This is similar to the homework problem (which was also worked out in class), except here it is specified that the particles are electrons. Since p>>m, these electrons are relativistic, thus the length is simply l = c/2f. Thus, the drift chamber has to be 5 m long.

   b) What is the dominant process by which photons lose energy at low energy (photon energies around 1 keV)? Provide a brief (1-3 sentences) answer.
      Answer: Photo-electric effect, which is a photon scattering of an atom and knocking out an electron.

   c) What is the dominant process by which photons lose energy at high energy (photon energies of more than 1 GeV)? Provide a brief (1-3 sentences) answer.
      Answer: pair-production, which is the creation of an electron-positron pair when the high-energy photon passes near a nucleus.

d) What class of Feynman diagram is illustrated below?



a) s-channel

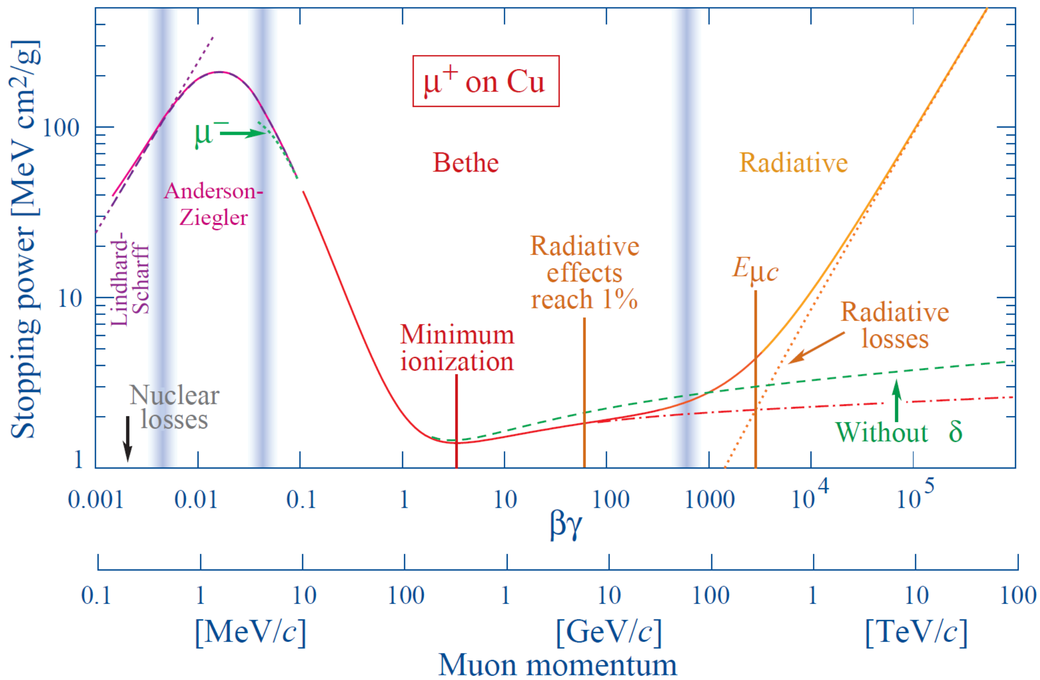
b) u-channel

c) t-channel

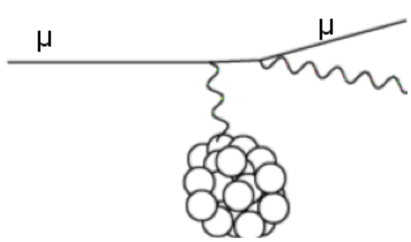
d) m-channel

3. (8 + 8 + 9 pts)

A muon with an energy of 1 TeV enters a Copper absorber (density about 9 g/cm^3).



a) Describe and/or draw the Feynman diagram for the dominant mode by which this muon loses energy in the copper.
Answer: At 1 TeV, the muon energy loss is in the radiative regime, meaning the muon emits photons as it passes through the copper.



b) What is the energy lost by the muon after passing through 1 m of copper (to within a factor 2)?
Answer: From the graph, the stopping power is about 10 MeV cm^2/g. The density is 9g/cm^3. Multiply the two together to get the energy loss

per distance, thus the muon loses 90 MeV/cm. Thus, for 1 m = 100 cm, the muon loses about 9,000 MeV = 9 GeV.

c) After the copper absorber, the muon passes through a gas ionization chamber where it loses 2 MeV energy. The energy required to create an electron-ion pair is 20 eV. The chamber has a 50% charge collection efficiency. Calculate the charge collected by the chamber in Coulomb.

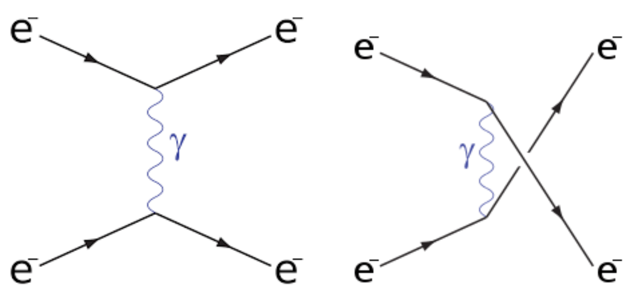
Answer: 1e = 1.6E-19 Coulomb. There are 2E6 eV/20 eV electron-ion pairs created, or 1E5. This corresponds to 1.6E-14 Coulomb. Collecting half of this charge means the total charge collected is 0.8E-14 Coulomb.

4. (10 + 5 + 5 + 5 pts)

a) Draw the two lowest-order Feynman diagrams for the process

$$e^- + e^- \rightarrow e^- + e^-$$

Answer: the two Feynman diagrams are given below.



b) Using the Feynman rules, label the particles and set up the formula for the matrix element of your t-channel diagram in part (4a). Do not perform the integral at this point. Feynman rules are given on the last 2 pages of the exam.

Answer: The matrix element for the t-channel is given by

$$i \int [\bar{u}_3 i g_e \gamma^\mu u_1] \frac{-i g_{\mu\nu}}{q^2} [\bar{u}_4 i g_e \gamma^\nu u_2] (2\,\pi)^4 \delta^4(p_1 - p_3 - q)(2\,\pi)^4 \delta^4(p_2 + q - p_4) \frac{d^4 q}{(2\pi)^4}$$

where 1 and 2 are the two incoming particles, 3 and 4 are the two outgoing ones.
Not required, but can do as well: the matrix element for the u-channel can be similarly obtained by swapping p_3 and p_4:

$$i \int [\bar{u}_4 i g_e\, \gamma^\mu u_1] \frac{-i g_{\mu\nu}}{q^2} [\bar{u}_3 i g_e\, \gamma^\nu u_2] (2\,\pi)^4 \delta^4(p_1 - p_4 - q)(2\,\pi)^4 \delta^4(p_2 + q - p_3) \frac{d^4 q}{(2\pi)^4}$$

c) Now perform the integral, get rid of the delta functions, and calculate the final matrix element for the t-channel process in part b.
Answer: The matrix element for the t-channel is $-\frac{g_e^2}{t}[\bar{u}_3 \gamma^\mu u_1][\bar{u}_4 \gamma_\mu u_2]$.

d) Start from the matrix element for an electron-electron scattering t-channel process, $M = -\frac{g_e^2}{t}[\bar{u}_3 \gamma^\mu u_1][\bar{u}_4 \gamma_\mu u_2]$. Use Casimir's trick to write the spin-averaged matrix element $\langle |M|^2 \rangle$ in terms of traces. You don't need to solve the resulting traces.

Answer: We average over the initial state spins, which gives factor ¼ because it is two fermions, and we sum over the final state spins. We get

$$\langle |M|^2 \rangle = \frac{g_e^4}{4t^2} \sum_{initial\ spins} \sum_{\substack{\text{final} \\ spins}} [\bar{u}_3\, \gamma^\mu u_1][\bar{u}_4 \gamma_\mu u_2]([\bar{u}_3 \gamma^\nu u_1][\bar{u}_4 \gamma_\nu u_2])^*$$

Grouping terms so that the upper line terms are together (1 and 2) and the lower lines are together (2 and 4), we can then replace this by the traces. This gives

$$\langle |M|^2 \rangle = \frac{g_e^4}{4t^2} \mathrm{Tr}[\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu(\not{p}_3 + m_e)]\mathrm{Tr}[\gamma^\mu(\not{p}_2 + m_e)\gamma_\nu(\not{p}_4 + m_e)]$$

5. (10 + 5 + 10 pts) **Required for 803 students, optional for 493 students.**

   a) Starting from the average matrix element squared for a t-channel scattering process for a particle of mass m,
   $\langle |M|^2 \rangle = \frac{g_e^4}{4t^2} \mathrm{Tr}[\gamma^\mu(\not{p}_1 + m)\gamma^\nu(\not{p}_3 + m)]\mathrm{Tr}[\gamma_\mu(\not{p}_2 + m)\gamma_\nu(\not{p}_4 + m)]$,
   where 1 and 2 are the initial state particles and 3 and 4 are the final state particles and $t = (p_1 - p_3)^2$, solve the traces and simplify the result. You can assume $E \gg m$, i.e. that the mass is negligible. Express your result in terms of the 4-momenta of the four particles and simplify as much as you can.

   Answer: The two traces result in $4\{p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_3)\}$ and $4\{p_{2,\mu}p_{4,\nu} + p_{4,\mu}p_{2,\nu} - g_{\mu\nu}(p_2 \cdot p_4)\}$.
   With these, the average matrix element is now

$$\langle |M|^2 \rangle = \frac{g_e^4}{4t^2} 4 \cdot 4\{p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_3)\}\{p_{2,\mu}p_{4,\nu} + p_{4,\mu}p_{2,\nu} - g_{\mu\nu}(p_2 \cdot p_4)\}$$

$$= \frac{4g_e^4}{t^2}[2(p_1 \cdot p_2)(p_3 \cdot p_4) + 2(p_1 \cdot p_4)(p_2 \cdot p_3) - 2(p_1 \cdot p_3)(p_2 \cdot p_4) - 2(p_2 \cdot p_4)(p_1 \cdot p_3) + 4(p_1 \cdot p_3)(p_2 \cdot p_4)]$$

$$= \frac{8g_e^4}{t^2}[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

   b) Write down the Mandelstam variables s, t, u in terms of the momenta p_1, p_2, p_3, p_4, for each pair of momenta (i.e. for s, write it down both in terms of p_1 and p_2 and in terms of p_3 and p_4.
   Answer:

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2
\end{aligned}
$$

c) Now substitute these s, t, u into the expression for $\langle |M|^2 \rangle$. You can assume $E \gg m$, i.e. that the mass is negligible.

Answer:

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = 2p_1 \cdot p_2 = 2p_3 \cdot p_4 \\
t &= (p_1 - p_3)^2 = -2p_1 \cdot p_3 = -2p_2 \cdot p_4 \\
u &= (p_1 - p_4)^2 = -2p_1 \cdot p_4 = -2p_2 \cdot p_3
\end{aligned}
$$

With these expressions, the average matrix element becomes

$$
\langle |M|^2 \rangle = \frac{8g_e^4}{t^2}\left(-\frac{1}{4}s^2 - \frac{1}{4}u^2\right) = 2g_e^4 \frac{s^2 + u^2}{t^2}
$$

The Feynman rules for QED are given below. Feel free to tear these pages off.

1: Draw the Feynman diagrams, including the appropriate arrows for particles and antiparticles.

2: Label incoming and outgoing 4-momenta for each vertex, including the internal momenta of propagators. Conventionally, external 4-momenta are labeled p_i and internal 4-momenta are labeled q_i.

3: Each external line gets a factor for the wave function, sandwiching the vertex in a current.

Fermions:

- Incoming particle: $u(p)$ Incoming antiparticle: $\bar{v}(p)$
- Outgoing particle: $\bar{\mathrm{u}}(p)$ Outgoing antiparticle: $\mathrm{v}(p)$

Photons:

- Incoming photon: $\varepsilon^{\mu}(p)$ Outgoing photon: $\varepsilon^{\mu}(p)^{*}$

4: Each vertex is assigned a factor of $(ig_e\gamma^{\mu})$, specifying the coupling strength of the interaction at that vertex.

5: Each internal propagator line gets a factor of:
$\frac{-ig_{\mu\nu}}{q^2}$ for photons, $\frac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$ for fermions.

6: Each vertex gets a delta function to enforce conservation of energy and momentum (k_i are the 4-momenta into/out of the vertex). The sign of each 4-momentum must be properly assigned, as necessary:

$$(2\pi)^4\delta^4(k_1+k_2+k_3)$$

7: Each internal propagator line gets a phase-space integration factor:

$$\frac{d^4 q_j}{(2\pi)^4}$$

8: The final matrix element is obtained by integrating over the propagator 4-momenta. Cancel out any remaining delta function factors (and factors of $(2\pi)^4$) and multiply by another factor (i). What remains is the matrix element.

9: Antisymmetrization: Include a minus sign between diagrams that differ only in the interchange of two incoming (or outgoing) electrons (or positrons), or of an incoming electron with an outgoing positron (or vice versa).