PHY493/803, Intro to Elementary Particle Physics
Homework 6

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

1. (10pts + 15pts + 15pts = 40 pts total)
   Calculate the QED scattering cross-section for the reaction $e^+e^- \to q\bar{q}$, where q is an individual quark species of charge e_q. Assume that the incoming particles are not polarized and that the spin projections of the outgoing quarks are not measured. Further assume that the energy of the incoming electrons in the center-of-mass frame is much larger than the electron or quark masses.
   Hint 1: You should start with your solution to problem 1 from HW #5. If you're clever, you will not need to repeat all of the calculations. You should be able to add in the relevant factors describing the difference between quarks and muons.
   Hint 2: If the previous hint isn't 100% clear, set up the matrix element for $e^+e^- \to q\bar{q}$ first and look at the differences relative to what you had in HW #5 and in-class exercise 10-2 for $e^+e^- \to \mu^+\mu^-$.

   a) Draw the relevant Feynman diagram(s) for the lowest order process.

   b) Calculate the scattering cross-section under the conditions listed above. You can use the approximation $E_e \gg m_q$. Give your result in the center-of-mass frame. Don't forget that quarks come in three distinct colors and that they have fractional charge (i.e. the QED coupling is different from that for charged leptons).

   c) The result rom (b) is for one quark flavor, now compute the total cross-section. Assume the center-of-momentum energy is 30 GeV; what is the total cross-section, i.e. which quark flavors are produced and what is the sum over the corresponding cross-sections? How does the total cross section for scattering into quarks (all flavors) compare with the cross-section for $e^+e^- \to \mu^+\mu^-$ at this energy? Note: you don't need to evaluate the cross section numerically. The center-of-momentum

energy is important for deciding which quark types should contribute to the total cross section.

2. (5 + 5 + 5 + 5 pts = 20 pts total) {**Required for PHY803 students only. +20 pts extra credit for PHY493 students.**}
   The Tevatron collider at Fermilab was a proton-antiproton collider. One of the main physics achievements was the study of the top quark, which was produced mainly in pairs through the strong interaction, $q\bar{q} \to t\bar{t}$ (ttbar production).

   a) Draw the possible Feynman diagrams for ttbar production from a $q\bar{q}$ initial state.

   b) Determine the average color factor squared $\langle |C|^2 \rangle$ for this interaction, averaging over initial state color combinations and summing over all final state color combinations.

   c) The Tevatron $p\bar{p}$ collider energy is $\sqrt{s} = 1.96$ TeV, but ttbar production happens mainly at threshold, i.e. twice the top-quark mass. What is the CM energy of the collision, and what is therefore *E*, the energy of one of the incoming quarks, at threshold?

   d) Compare QCD and QED ttbar production. What is the ratio of $\sigma(q\bar{q} \to t\bar{t})$ over $\sigma(e^+e^- \to t\bar{t})$ above the ttbar production threshold? Assume that the strong coupling constant at such high energies is $\alpha_s = 0.1$. Don't worry about the kinematics of the four-vectors, just compare the

QCD coupling and color factors to the QED coupling in the expression for the cross-section.