PHY493/803, Intro to Elementary Particle Physics

Homework 4

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

1. (20 + 20 pts) Ionization chamber

   a) A 50-μCi source emits 5-MeV α particles and is suspended between the plates of a large gas ionization chamber. If the emitted α particles lose all their energy in the chamber, calculate the current measured at the output. Assume that the average energy to create an electron-ion pair in the gas is 30 eV and that the charge collection efficiency is 89%.

      1. How many alphas per second? $5 \cdot 10^{-5} \cdot 3.7 \cdot 10^{10} s^{-1} = 18.5 \cdot 10^{5} s^{-1}$
      2. How many electrons-ion pairs are produced per alpha? $5 \cdot 10^{6} / 30 = 1.67 \cdot 10^{5}$
      3. What is the total charge of the electrons produced? $1.67 \cdot 10^{5} \cdot q_e \approx 2.67 \cdot 10^{-14} C$
      4. Taking into account the collection efficiency, what is the current?

      $0.89 \cdot 18.5 \cdot 10^{5} s^{-1} \cdot 1.67 \cdot 10^{5} q_e \approx 4.40 \cdot 10^{-8} A$
      $= 44.0\ nA$

   b) When a 5-MeV α particle is stopped in the gas ionization chamber, the voltage at the detector output undergoes a step change of 2 mV. Assume that the average energy to create an electron-ion pair in the gas is 30 eV and that the charge collection efficiency is 89% (like in part A). Calculate the capacitance of the chamber, assuming that the time constant of the chamber and electronics is long compared to the duration of charge deposition by the α particles.

$$C = \frac{dQ}{dV} = \frac{1.67 \cdot 10^{5} q_e\, 0.89}{2 \cdot 10^{-3} V}$$
$$= 1.19 \cdot 10^{-11} f$$
$$= 1.19 \cdot 10^{-2} nf$$

2. (20pts + 20pts) Linear accelerator

   a) A linear accelerator, consisting of a series of drift tubes, operates with a frequency F of 30 MHz and accelerates particles to a velocity v_i. What is the length of the following drift tube? Assume that the particle will travel the full length of the drift tube as the voltage swings from positive to negative (i.e. ½ period).

$$l = \frac{1}{60\times10^6}s \cdot V_i$$
$$= 16.7\,ns \cdot V_i$$

   b) If the particles above are ^{16}O ions and are accelerated to a momentum of 80 MeV, what is the length of the chamber?

$p = 80MeV$

$p = \gamma m v$
$= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mv$

$l = 16.7\,ns \cdot V_i$

$m = 14.9\,GeV$

$$\boxed{l = 16.7_{ns} \cdot 5.37 \cdot 3 \cdot 10^5 \tfrac{m}{s} = 2.6\,cm}$$

$p^2(1-\frac{v^2}{c^2}) = m^2v^2$
$1-\frac{v^2}{c^2} = \frac{m^2}{p^2}v^2$
$1 = \frac{m^2}{p^2}v^2 + \frac{v^2}{c^2} \rightarrow v^2 = (\frac{m^2}{p^2}+\frac{1}{c^2})^{-1}$
$v = (\frac{m^2}{p^2}+\frac{1}{c^2})^{-1/2} = 0.00537c$

3. (4*10 pts) CERN LHC storage ring

The CERN LHC has a circumference of 26660 m and runs at a RF frequency of $\omega = 400$ MHz. Round each answer to two significant digits.

angular frequencies are rad/s not Hz

   a) Assuming protons circulate at the speed of light, calculate the ring revolution frequency Ω. $\Omega = \frac{2\pi c}{26660m} = 71 \frac{krad}{s}$

   b) Calculate the harmonic number *N* of the LHC.

$\omega = N\Omega$
$N = \frac{\omega}{\Omega} = 5700$

   c) Calculate the distance between buckets in length (cm) and in time (ns).

$l = \lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega} = 470cm$

or $16ns$

d) The distribution of protons in each bunch has a Gaussian profile with an RMS of about 7.5 cm in the longitudinal direction (along the beam). The limits on the horizontal beam profile (in the plane of the ring) can be determined from this because a proton that is further away from the center of the ring travels a larger distance. For a proton that is circulating around the ring with a radius that is 1 cm larger than the nominal center of the bunch, how much further will this proton have traveled than the nominal 26660 m after one rotation around the ring? Is it still within the RMS of the center of the center of the bunch in the longitudinal direction?

$r \to r + 0.01\,m$

$C \to C + 0.1\,m$

No the particle isn't within the RMS. It is a distance (-10) cm from the center.

4. (4pts + 4pts + 4pts + 4pts + 4pts) Synchrotron radiation **{Required for PHY803 students only. +20 pts extra credit for PHY493 students.}**
The total power radiated by a point charge (q) as it accelerates at non-relativistic speeds is given by the Larmor formula:

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

a) The non-relativistic Larmor formula can be modified for a particle in a synchrotron traveling in a circular orbit with a constant velocity magnitude and constant radius r. Evaluate the relativistic acceleration

$$\vec{a} = \frac{1}{m}\frac{d\vec{p}}{d\tau} = \frac{1}{m}\gamma\frac{d(\gamma m\vec{v})}{dt} = \gamma^2 \vec{a}$$

centripital acceleration $= \frac{v^2}{r}$

where τ is the proper time. You can take advantage of the constant velocity magnitude to simplify the time derivative. Show the radiated power becomes:

$$P = \frac{q^2 \gamma^4 v^4}{6\pi\varepsilon_0 c^3 r^2}$$ ✓

$$P = \frac{q^2 \gamma^4 v^4}{6\pi\varepsilon_0 c^3 r^2}$$

b) The Large Electron Positron (LEP) collider was located in the LHC tunnel electron-positron collisions at CERN at a center-of-momentum energy of 200 GeV. It had a radius of 4300 m. Calculate the energy lost by one electron in each turn.

$\gamma = \frac{E}{m} = \frac{100 \text{ GeV}}{5.11 \times 10^{-3} \text{ GeV}}$

$P = \frac{q^2 c \gamma^4}{6\pi\varepsilon_0 r^2} = 0.36 \text{ nJ/turn}$

c) The electron and positron beams were comprised of 4 bunches each and each bunch contained 10^{11} particles. Show that the total energy lost by the two beams over one rotation is 262 J/turn.

$8 \cdot 10^{11} \cdot P = 292 \text{ J/turn}$

another typo?

d) Now calculate the total energy lost by the LEP beams during one hour of continuous operation.

$292 \text{ J/turn} \cdot N \frac{\text{turns}}{\text{h}} = 292 \frac{\text{J}}{\text{turn}} \cdot 4.00 \times 10^{7} \frac{\text{turns}}{\text{h}} = 1.17 \times 10^{10} \text{ J/h}$

e) For comparison, calculate the total energy lost by the LHC proton beams (at 6.5 TeV energy per beam) during one hour of continuous operation. The LHC has the same radius of 4300 m and the same number of particles per bunch (10^{11}). For simplicity, assume there are also 4 bunches in each beam.

In one hour, the energy lost by the LHC is 1.84×10^{8} J/h.