PHY493/803 Intro to Elementary Particle Physics

Example midterm Exam 1

This exam is worth 100 points. There are **five** *problems and each has 25 points. Partial points are indicated in the first line.* **Choose any 4 of the 5 problems to complete. PHY803 students must do Problem #5.**

Take a moment and look over the exam before you begin. To receive the full credit for each answer, you must work neatly, show your work and simplify your answer to the extent possible.

Upload your answers to gradescope when finished.

1. (5pt x 5) Mark each statement as true or false.

a) Time-like 4-vectors describe the displacement between events that can be causally connected by particles traveling slower than the speed of light

   _True. Time-like 4-vectors can be causally connected.

b) The parity symmetry is conserved in electromagnetic interactions but not weak and strong interactions.

   False. The parity symmetry s conserved in EM and strong interactions, but not in the weak interaction.

c) The elements of a group are not required commute.

   _True. If they commute, then it is an Abelian group.

d) Hadrons are made of either only quarks or anti-quarks, but not both.

   False. Mesons are bound quark-antiquark states.

e) The symmetry associated with translation in time gives rise to the conservation of energy.

   True

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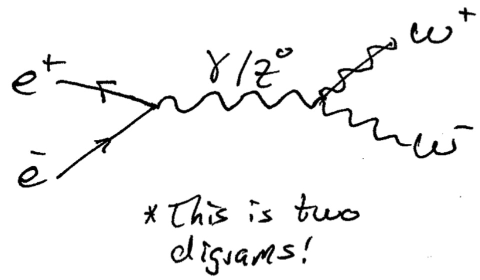
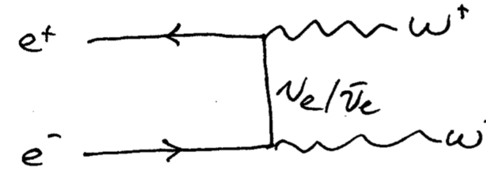
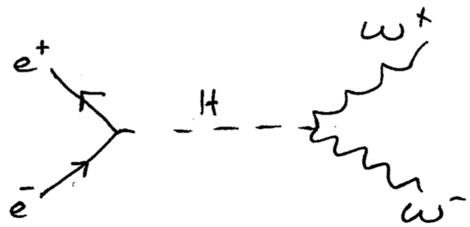
2. (10pt + 15pt)

   a) Draw Feynman diagrams contributing to the process $\mathrm{e}^+ + \mathrm{e}^- \rightarrow W^+ + W^-$. Each diagram should contain no more than two vertices.





Higgs diagram:

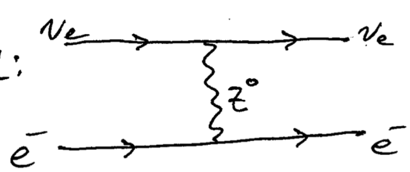


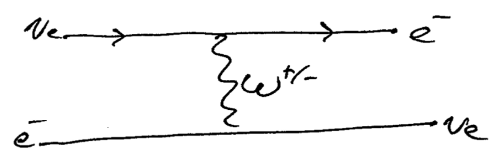
b) Electron neutrinos can interact with electrons in a different manner than muon neutrinos or tau neutrinos interact with electrons. Demonstrate this by drawing the lowest order Feynman diagrams for the following two reactions:

1: $\nu_e + \mathrm{e}^- \rightarrow \nu_e + \mathrm{e}^-$ (two diagrams)

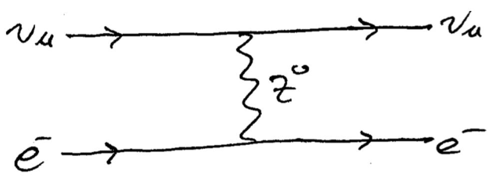
2: $\nu_\mu + \mathrm{e}^- \rightarrow \nu_\mu + \mathrm{e}^-$ (one diagram)

1:





2:



3. (10pt + 10 + 5 pt)

a) The $\eta(549)$ meson has spin-0 and is observed to decay to three-pion final states by the electromagnetic processes $\eta \rightarrow \pi^0 + \pi^0 + \pi^0$ and

$\eta \to \pi^+ + \pi^- + \pi^0$. Use this information to deduce the parity of the $\eta(549)$, and hence explain why the decays $\eta \to \pi^0 + \pi^0$ and $\eta \to \pi^+ + \pi^-$ have never been observed.

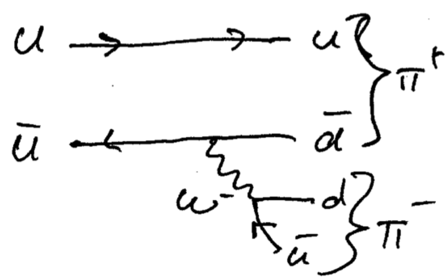
$\boxed{\pi^\pm}$ $\qquad I^G(J^P) = 1^-(0^-)$

$\boxed{\pi^0}$ $\qquad I^G(J^{PC}) = 1^-(0^{-+})$

The EM decay conserves parity, and the parity of a pion is -1. Thus the parity of the 3-pion system is -1, thus the parity of the $\eta(549)$ is -1.

The parity of a 2-pion final state is +1 and thus this decay cannot happen via the EM or strong interaction.

Not part of the requested answer but for your information: It can happen via the weak interaction, see the Feynman diagram below:



However, the branching ratio for this process is so small (weak interaction) that it has never been observed.

b) Some particles, but not all, are eigenstates of the charge conjugation operator ($\hat{C}$). Give an example of a boson and an example of a meson that is an eigenstate of $\hat{C}$. What do you get when you apply $\hat{C}$ to each of these particles? Use the notation $|p >$, where *p* specifies the particle.

For a particle that it is its own antiparticle, $\hat{C}\,|p \geq \pm 1\,|p >$.

Neutral mesons are $q\bar{q}$ and are eigenstates of $\hat{\mathrm{C}}$. (only one example required)
The neutral pion has $\hat{\mathrm{C}}$ eigenvalue +1: $\hat{\mathrm{C}}|\pi^0 >= 1|\pi^0 >$
The J/Psi has $\hat{\mathrm{C}}$ eigenvalue -1.
The photon and the Z boson are eigenstates of $\hat{\mathrm{C}}$. (only one example required)
The photon has $\hat{\mathrm{C}}$ eigenvalue -1: $\hat{\mathrm{C}}\ |\gamma \geq -1\ |\gamma >$.

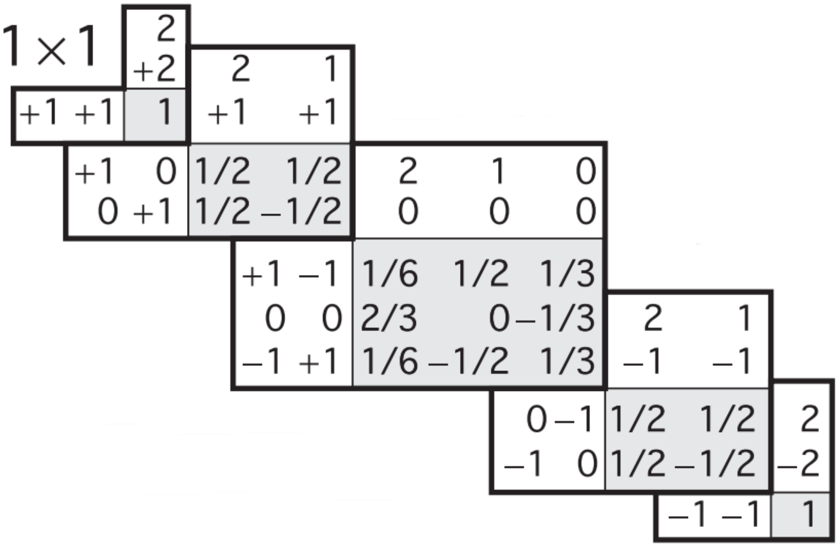
c) Give an example of a particle that is not an eigenstate of $\hat{\mathrm{C}}$. What do you get when you apply $\hat{\mathrm{C}}$ to this particle?

   For a particle that it is not its own antiparticle, for example any fermion, $\hat{\mathrm{C}}\ |p \geq\ |\ \bar{\mathrm{p}} >$, where $\bar{\mathrm{p}}$ is the antiparticle.

4. (25pt)
   The Σ^{*0} can decay into ($\pi^- + \Sigma^+$), ($\pi^0 + \Sigma^0$) or ($\pi^+ + \Sigma^-$). Suppose that your experiment has observed 10,000 such decays. Use isospin conservation to predict what fractions of each type of decay you would expect to find. The π and Σ systems both form isospin-1 triplets.

   The particle content of these hadrons is as follows:

   $\Sigma^{*0} = (uds)$ $\quad$ $\Sigma^+ = (uus)$ $\quad$ $\Sigma^- = (dds)$
   $\Sigma^0 = (uds)$ $\quad$ $\pi^\pm = (u\bar{d}\ , \bar{u}d)$ $\quad$ $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$



$\Sigma^{*0} = (uds)$, isospin |1,0> $\Sigma^{+} = (uus)$, isospin |1,1>
$\Sigma^{-} = (dds)$, isospin |1,-1> $\Sigma^{0} = (uds)$, isospin |1,0>
$\pi^{+} = (u\bar{d})$, isospin |1,1> $\pi^{-} = (\bar{u}d)$, isospin |1,-1>
$\pi^{0} = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, isospin |1,0>

From the CG table, we see that for the column |1,0> we have three contributions,

|1,0> = $\sqrt{1/2}$|1,1>|1,-1> + 0 |1,0>|1,0> - $\sqrt{1/2}$ |1,-1> |1,1>

Thus each of the two decays $(\pi^{-} + \Sigma^{+})$ and $(\pi^{+} + \Sigma^{-})$ are equally likely, and we expect 5000 events of each decay. $(\pi^{0} + \Sigma^{0})$ will not occur.

5. (25 pt) **Required for 803 students, optional for 493 students.**
At an electron-positron collider, an electron beam (with energy E_{beam}) and a positron beam (with energy E_{beam}) collide head-on. Explore the process where the collision results in a Higgs boson (m_H = 125 GeV) together with a Z boson (m_Z = 90 GeV), i.e. the final state is ZH.
   a) (5 pt) What is the minimum beam energy (E_{beam}) required to produce ZH?

The minimum energy corresponds to both particles being produced at rest, so 125 + 90 GeV = 215 GeV. Thus the beam energy should be half of that, Ebeam = 107.5 GeV

b) (10 pt) Now assume that each beam has an energy of E_{beam} = 200 GeV. Determine the energy of the Z boson in the ZH final state assuming the Z boson is emitted perpendicular to the beam.
Answer:

b) $E_{beam} = 200\,GeV$ what is E_Z?

$$p_i^\mu = p_H^\mu + p_Z^\mu \Rightarrow p_H^\mu = p_i^\mu - p_Z^\mu$$

square both sides:

$$p_H^2 = (p_i^\mu - p_Z^\mu)(p_{i,\mu} - p_{Z,\mu})$$

$$\Rightarrow \underbrace{p_H^2}_{m_H^2} = \underbrace{p_i^2}_{(2E_{beam})^2} + \underbrace{p_Z^2}_{m_Z^2} - 2 \underbrace{p_i^\mu p_{Z,\mu}}_{2E_{beam}E_Z - \underbrace{\vec{p}_i \cdot \vec{p}_Z}_{=0}}$$

$$\Rightarrow m_H^2 = (2E_{beam})^2 + m_Z^2 - 4E_{beam}E_Z$$

$$\Rightarrow E_Z = \frac{4E_{beam}^2 + m_Z^2 - m_H^2}{4E_{beam}} = 190.6\,GeV$$

c) (5 pt) Still assuming E_{beam} = 200 GeV and that the Z boson is emitted perpendicular to the beam. In the lab frame, what is the transverse momentum of the Z boson?

Answer:

The Z boson momentum is calculated from the energy in part b, $E_Z = 190$ GeV, thus $\vec{p}^2 = 190^2 - 90^2$, thus $|\vec{p}| = 167$ GeV.

d) (5 pt) The Higgs boson decays to two b-quarks (m_b = 5 GeV). What is the magnitude of the b-quark momentum in the Higgs boson rest frame?

Possible answer: The b-quark mass is much smaller than the Higgs boson mass and can thus be ignored. Then the b-quark momentum is simply 62.5 GeV.

Possible answer:

$E_b = \frac{1}{2} m_H$ in Higgs rest frame

$\Rightarrow E^2 = p^2 + m^2 \Rightarrow p^2 = E^2 - m^2$

$\Rightarrow p^2 = \left(\left(\frac{125}{2}\right)^2 - 5^2\right)$ GeV $= 62.3$ GeV

Both answers are valid.