f04

Wednesday, December 16, 2020 9:36 AM

$$r^2 = x^2+y^2+z^2$$

$$I_{ij} = \int d^3r\, \rho(\vec{r}) (\delta_{ij} r^2 - r_i r_j) \qquad \rho(\vec{r}) = H(-x) H(l-x)\, \delta(y) \delta(z) \frac{l}{M} + H(\tfrac{l}{2}-y) H(\tfrac{l}{2}-y) \delta(x-l)\, \delta(z) \frac{l}{M}$$

$$I_{xx} = \frac{l}{M}\int_0^l dx\, (0+0) + \frac{l}{M}\int_{-\frac{l}{2}}^{\frac{l}{2}} dy\; y^2$$

$$= \frac{2}{M} \frac{l^3/8}{3} l$$

$$= \frac{l^4}{12M}$$

$$I_{yy} = \frac{l}{M}\int_0^l dx \left(x^2\right) + \frac{l}{M}\int_{-\frac{l}{2}}^{l/2} dy \left(l^2\right)$$

$$= \frac{1}{M}\frac{l^4}{3} + \frac{l^3}{M} l$$

$$= \frac{4l^4}{3M}$$

$$I_{zz} = \frac{l}{M}\int_0^l dx\; x^2 + \frac{l}{M}\int_{-l/2}^{l/2} dy \left(l^2 + y^2\right)$$

$$= \frac{1}{M}\frac{l^4}{3} + \frac{l}{M}\left(l^2 l + \frac{2 l^3/8}{3}\right)$$

$$= \frac{l}{M}\frac{l^3}{3} + \frac{l}{M}\left(\frac{13 l^3}{12}\right)$$

$$= \frac{1}{M} l^4 \frac{17}{12}$$

$$I_{xy} = \frac{l}{M}\int_0^l dx\, (-x\cdot 0) + \frac{l}{M}\int_{-\frac{l}{2}}^{l/2} dy\, (-ly)$$

$$= 0$$

$$I_{xz} = \frac{l}{M}\int_0^l dx\, (-x\cdot 0) + \frac{l}{M}\int_{-\frac{l}{2}}^{l/2} dy\, (-l\cdot 0)$$

$$= 0$$

$$I_{yz} = \frac{l}{M}\int_0^l dx\; 0 + \frac{l}{M}\int_{-\frac{l}{2}}^{l/2} dy\, (y\cdot 0)$$

$$= 0$$

$$\tilde{I} = \frac{l^4}{12M}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 17 \end{pmatrix}$$

Center of mass, for the pendulum is at $\vec{p} = \frac{3l}{4}\begin{pmatrix}\cos\vartheta \\ \sin\vartheta\end{pmatrix}$.

$$m = 2M$$

$$\mathcal{L} = \frac{1}{2}\dot{\vec{\vartheta}}^T \tilde{I} \dot{\vec{\vartheta}} + Mg\frac{3}{2} l\cos\vartheta$$

$$\dot{\vec{\vartheta}} = \begin{pmatrix} 0 \\ 0 \\ \dot{\vartheta}\end{pmatrix}$$

$$= \frac{17}{24}\frac{l^4}{M}\dot{\vartheta}^2 + mg\frac{3}{4} l\cos\vartheta$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\vartheta}} - \frac{\partial\mathcal{L}}{\partial\vartheta} = 0 = \frac{17}{12}\frac{l^4}{M}\ddot{\vartheta} + \frac{3}{2} Mgl\sin\vartheta$$

$$\approx \frac{17}{12}\frac{l^4}{M}\ddot{\vartheta} + \frac{3}{2} Mgl\,\vartheta$$

$$\omega^2 = \frac{18}{17}\frac{M^2 g}{l^3}$$

I don't think this is right. It shouldn't depend on mass. Not sure what's wrong...