f01

Wednesday, December 16, 2020 8:02 AM

$$V = mgh = mgl\cos\vartheta \qquad r = \begin{pmatrix} x + l\sin\vartheta \\ l\cos\vartheta \end{pmatrix}$$

$$T = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}m\left((\dot{x} + l\cos\vartheta\,\dot{\vartheta})^2 + l^2\sin^2\vartheta\,\dot{\vartheta}^2\right)$$

$$\mathcal{L} = \tfrac{1}{2}(M+m)\dot{x}^2 + m\dot{x}l\cos\vartheta\,\dot{\vartheta} + \tfrac{1}{2}ml^2\dot{\vartheta}^2 - mgl\cos\vartheta$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} - \frac{\partial\mathcal{L}}{\partial x} = 0 = (M+m)\ddot{x} - ml\sin\vartheta\,\dot{\vartheta}^2 + ml\cos\vartheta\,\ddot{\vartheta} \overset{\text{small angle}}{\approx} (M+m)\ddot{x} - ml\vartheta\dot{\vartheta}^2 + ml\ddot{\vartheta}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\vartheta}} - \frac{\partial\mathcal{L}}{\partial\vartheta} = 0 = \frac{d}{dt}\left(m\dot{x}l\cos\vartheta + ml^2\dot{\vartheta}\right) + m\dot{x}l\sin\vartheta\,\dot{\vartheta} - mgl\sin\vartheta$$

$$= m\ddot{x}l\cos\vartheta - \cancel{m\dot{x}l\sin\vartheta\,\dot{\vartheta}} + ml^2\ddot{\vartheta} + \cancel{m\dot{x}l\sin\vartheta\,\dot{\vartheta}} - mgl\sin\vartheta$$

$$= m\ddot{x}l\cos\vartheta + ml^2\ddot{\vartheta} - mgl\sin\vartheta$$

$$\approx m\ddot{x}l + ml^2\ddot{\vartheta} - mgl\vartheta \quad \text{(small angle)}$$

$$(M+m)\ddot{x} - ml\vartheta\dot{\vartheta}^2 + ml\ddot{\vartheta} = 0 \rightarrow \ddot{x} = \frac{1}{(M+m)}\left(ml\vartheta\dot{\vartheta}^2 - ml\ddot{\vartheta}\right)$$

$$0 = \frac{ml}{(M+m)}\left(ml\vartheta\dot{\vartheta}^2 - ml\ddot{\vartheta}\right) + ml^2\ddot{\vartheta} - mgl\vartheta$$

$$\underbrace{\left(ml^2 - \frac{(ml)^2}{M+m}\right)\ddot{\vartheta} + ml\left(\frac{ml}{M+m}\dot{\vartheta}^2 - g\right)\vartheta = 0}$$

This is more complicated than I think it's supposed to be... Something went wrong, but I don't have time to fix it.