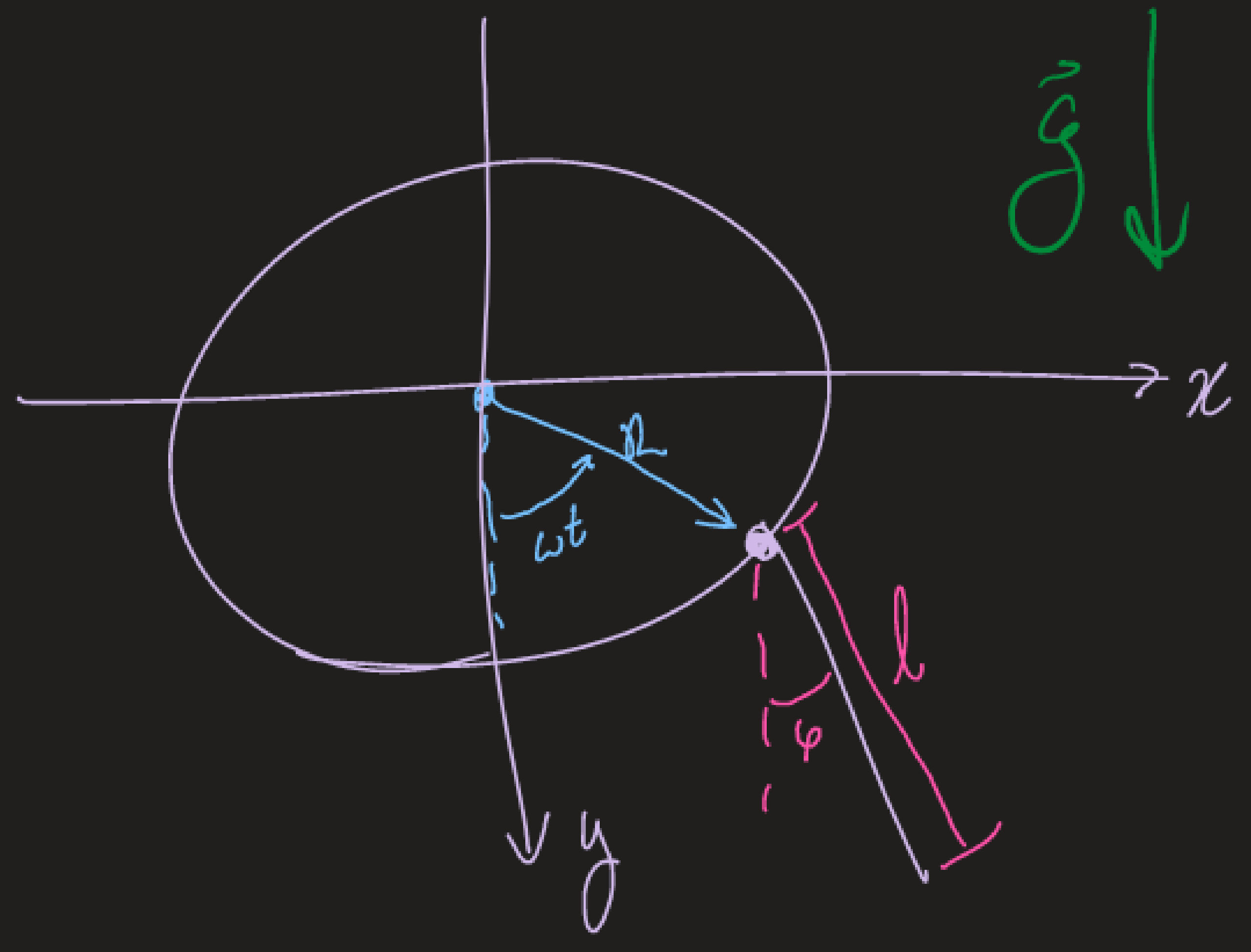
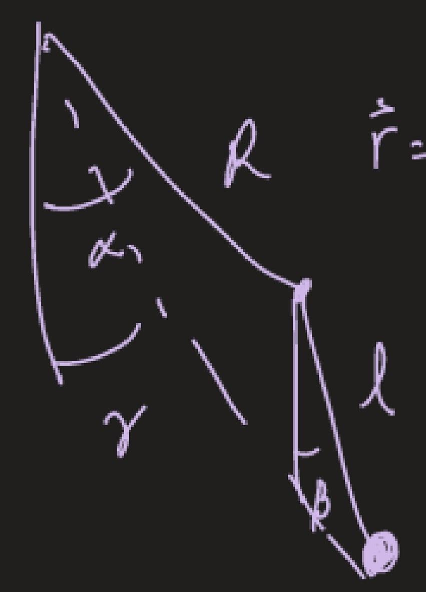
# p02

Friday, November 13, 2020 4:25 PM




$$V = -mgh$$
$$= -mg\left(R\cos\omega t + \tfrac{1}{2} l \cos\varphi\right)$$

$$T = \frac{1}{2} m \dot{\vec{r}}^2$$
$$= \frac{1}{2} m\left(\omega^2 R^2 + \frac{l^2}{4}\dot{\varphi}^2 + \omega R l \dot{\varphi}\cos(\varphi - \omega t)\right)$$




$$\vec{r} = \begin{pmatrix} R\sin\alpha + \frac{l}{2}\sin\beta \\ R\cos\alpha + \frac{l}{2}\cos\beta \end{pmatrix} \rightarrow \dot{\vec{r}} = \begin{pmatrix} \omega R\cos\omega t + \frac{l}{2}\cos\varphi\,\dot{\varphi} \\ -\omega R\sin\omega t - \frac{l}{2}\sin\varphi\,\dot{\varphi} \end{pmatrix}$$

$$\dot{\vec{r}}^2 = \left(\omega R\cos\omega t + \frac{l}{2}\cos\varphi\,\dot{\varphi}\right)^2 + \left(\omega R\sin\omega t + \frac{l}{2}\sin\varphi\,\dot{\varphi}\right)^2$$
$$= \omega^2 R^2\cos^2\omega t + \frac{l^2}{4}\cos^2\varphi\,\dot{\varphi}^2 + \omega R l\cos\omega t\cos\varphi\,\dot{\varphi}$$
$$+ \omega^2 R^2\sin^2\omega t + \frac{l^2}{4}\sin^2\varphi\,\dot{\varphi}^2 + \omega R l\sin\omega t\sin\varphi\,\dot{\varphi}$$
$$= \omega^2 R^2 + \frac{l^2}{4}\dot{\varphi}^2 + \omega R l\dot{\varphi}\cos(\varphi - \omega t)$$

$$\mathcal{L} = \frac{l^2}{8} m\dot{\varphi}^2 + \underbrace{\cancel{\frac{m\omega^2 R^2}{2}}}_{\text{"irrelevant"}} + \frac{1}{2} m\omega R l\dot{\varphi}\cos(\varphi - \omega t)$$
$$+ \underbrace{\cancel{mgR\cos\omega t}}_{\text{"irrelevant"}} + \frac{1}{2} mgl\cos\varphi$$

$$= \frac{1}{8} ml^2\dot{\varphi}^2 + \frac{1}{2} m\omega R l\dot{\varphi}\cos(\varphi - \omega t) + \frac{1}{2} mgl\cos\varphi$$

Supposed to be a 6? How?

From here on, I use eq. 2 from the exam.

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} - \frac{\partial\mathcal{L}}{\partial\varphi} = 0 = \frac{d}{dt}\left[\frac{1}{3} ml^2\dot{\varphi} + \frac{1}{2} mRl\omega\cos(\varphi - \omega t)\right] + \frac{1}{2} mRl\omega\dot{\varphi}\sin(\varphi - \omega t) + \frac{1}{2} mgl\sin\varphi$$
$$= \frac{1}{3} ml^2\ddot{\varphi} - \frac{1}{2} mRl\omega\sin(\varphi - \omega t)(\dot{\varphi} - \omega) + \frac{1}{2} mRl\omega\dot{\varphi}\sin(\varphi - \omega t) + \frac{1}{2} mgl\sin\varphi$$

For $R\omega^2 \ll g$: I