f03

Wednesday, December 16, 2020 8:47 AM

$$V = \frac{1}{2} k \eta_1^2 + \frac{1}{2} k \eta_2^2 + \frac{1}{2}(2k)(\eta_2 - \eta_1)^2$$

$$\eta_1^2 + \eta_2^2 - 2\eta_1\eta_2 = \sum_{ij} \delta_{ij}\eta_i\eta_j - (1-\delta_{ij})\eta_i\eta_j$$

$$= 2\delta_{ij}\eta_i\eta_j - \eta_i\eta_j$$

$$= \eta_i(2\delta_{ij} - 1)\eta_j$$

$$= \frac{1}{2}\vec{\eta}^T \tilde{V} \vec{\eta}$$

$$\tilde{V} = \begin{pmatrix} 5 & -2 \\ -2 & 5 \end{pmatrix} k$$

$$T = \frac{1}{2} m(\dot{\eta}_1^2 + \dot{\eta}_2^2) = \frac{1}{2}\dot{\vec{\eta}}^T \tilde{T} \dot{\vec{\eta}}$$

$$\tilde{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} m$$

$$\mathcal{L} = \frac{1}{2}\dot{\vec{\eta}}^T \tilde{T} \dot{\vec{\eta}} - \frac{1}{2}\vec{\eta}^T \tilde{V} \vec{\eta}$$

$\downarrow$

Normal modes: $\det(\tilde{V} - \omega^2 \tilde{T}) = 0$

$$\begin{vmatrix} 5k - \omega^2 m & -2k \\ -2k & 5k - \omega^2 m \end{vmatrix} = 0$$

$$(5k - \omega^2 m)^2 - 4k^2 = 0$$

$$\omega^2 = \left(\mp\sqrt{4k^2} + 5k\right)\frac{1}{m}$$

Normal frequencies $\rightarrow$ $\omega_\pm = \left(\sqrt{5k \pm 2k}\right)\frac{1}{\sqrt{m}}$

$$\begin{pmatrix} 5k - (5k \pm 2k) & -2k \\ -2k & 5k - (5k \pm 2k) \end{pmatrix} \vec{\rho}_\pm = 0$$

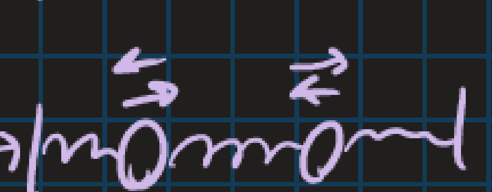
$$[5k - (5 \pm 2)k]\rho_1 - 2k\rho_2 = 0$$

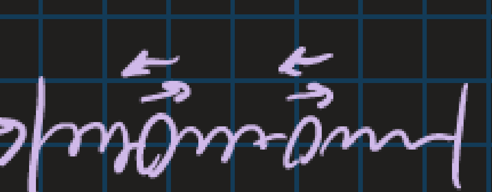
$$-2k\rho_1 + [5k - (5 \pm 2)k]\rho_2 = 0$$

$$\rho_2 = \frac{5 - (5 \pm 2)}{2}\rho_1$$

$\downarrow$

Normal vectors $\rightarrow$ $\vec{\rho}_\pm = \begin{pmatrix} 1 \\ \mp 1 \end{pmatrix}$

$\vec{\rho}_+ = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow$  (motion in opposite directions)

$\vec{\rho}_- = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow$  (motion in same direction)