p03

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$$\vec{N} = \frac{d}{dt}\vec{L} + \vec{\omega} \times \vec{L}, \quad \text{where } \vec{L} = I\vec{\omega} = \begin{pmatrix} A\omega_{x'} \\ B\omega_{y'} \\ C\omega_{z'} \end{pmatrix}$$

$$\frac{d}{dt}\vec{L} = \begin{pmatrix} \dot{A}\omega_{x'} + A\dot{\omega}_{x'} \\ \dot{B}\omega_{y'} + B\dot{\omega}_{y'} \\ \dot{C}\omega_{z'} + C\dot{\omega}_{z'} \end{pmatrix}$$

$$\vec{N}_z = \dot{C}\omega_{z'} + C\dot{\omega}_{z'} + \left(\omega_{x'}B\omega_{y'} - A\omega_{x'}\omega_{y'}\right)$$

$$= \dot{C}\omega_{z'} + C\dot{\omega}_{z'} + \cancel{(B-A)\omega_{x'}\omega_{y'}} \to 0$$

$$= \dot{C}\omega_{z'} + C\dot{\omega}_{z'}$$

$$C'(t) = -\frac{2}{5}Ma^2\varepsilon\cos\Omega t\,\Omega$$

$$N_{z'} = -\frac{2}{5}Ma^2\varepsilon\Omega\cos(\Omega t)\omega_{z'} + \frac{2}{5}Ma^2(1+\varepsilon\sin\Omega t)\dot{\omega}_{z'}$$

Star in space, so $\vec{N} = \vec{0}$:

$$\dot{\omega}_{z'} = \frac{\varepsilon\Omega\cos(\Omega t)\omega_{z'}}{1+\varepsilon\sin(\Omega t)}$$

For $\varepsilon \ll 1$: $\dot{\omega}_{z'} \approx 0$. ✓

In the case where derivatives of A, B, and C can be ignored ($\Omega \ll \omega_{z'}$),

$$A\dot{\omega}_{x'} + (A-C)\omega_{y'}\omega_{z'} = 0,$$

$$A\dot{\omega}_{y'} + (C-A)\omega_{z'}\omega_{x'} = 0,$$

$$C\dot{\omega}_{z'} = 0.$$

Then the value Ω_p can be defined as follows:

$$\Omega_p = \frac{C-A}{A}\omega_{z'}.$$

From the Euler equations above,

$$\dot{\omega}_{x'} = -\Omega_p\omega_{y'}, \quad \dot{\omega}_{y'} = \Omega_p\omega_{x'}.$$

This gives

$$\ddot{\omega}_{x'} + \Omega_p\omega_{x'} = 0,$$

$$\omega_{x'} = \Gamma\cos\Omega_p t,$$

$$\omega_{y'} = \Gamma\sin\Omega_p t,$$

where Γ is just some constant.

So the angular velocity is precessing around the z' axis with a frequency of Ω_p.

$$\Omega_p = \frac{\frac{2}{5}Ma^2(1+\varepsilon\sin\Omega t) - \frac{1}{5}M(a^2+b^2)(1-\frac{\varepsilon}{2}\sin\Omega t)}{\frac{1}{5}M(a^2+b^2)(1-\frac{\varepsilon}{2}\sin\Omega t)}\omega_{z'}$$

$$= \left[\frac{2a^2(1+\varepsilon\sin\Omega t)}{(a^2+b^2)(1-\frac{\varepsilon}{2}\sin\Omega t)} - 1\right]\omega_{z'}(t)$$

$$\approx \left[\frac{a^2-b^2}{a^2+b^2} + \frac{3a^2\sin\Omega t\,\varepsilon}{a^2+b^2} + O(\varepsilon^2)\right]\omega_{z'}(t)$$

If $a=b=R$ the star is spherical (?),
the $O(1)$ term vanishes,
and the $O(\varepsilon)$ term simplifies to,

$$\frac{3}{2}\sin\Omega t\,\varepsilon.$$

The first term dictates precession in non ellipsoidal bodies, and the second term is a correction for more spherical bodies.