p01

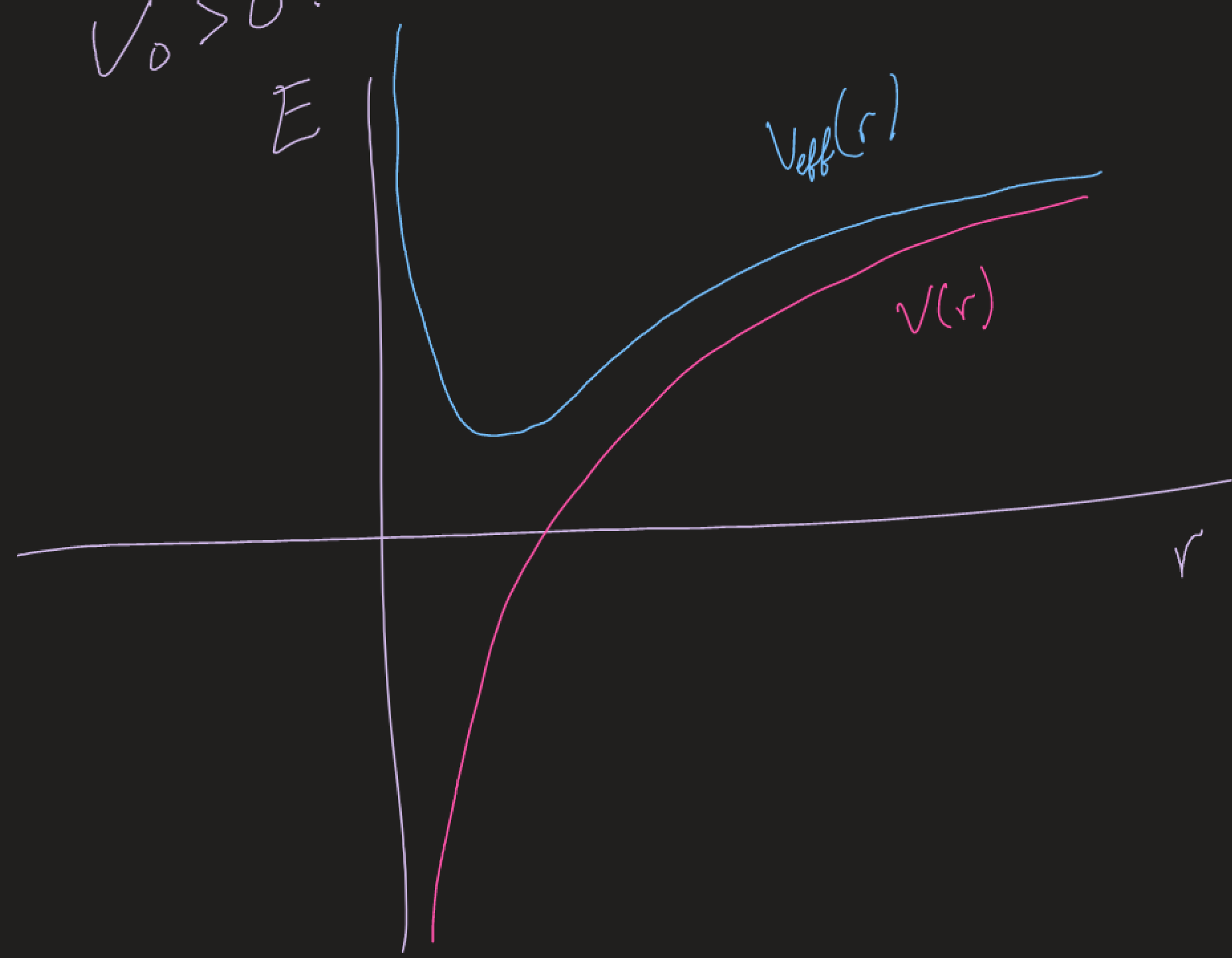
Friday, November 13, 2020 4:49 PM

$$V(r) = V_0 \ln(r/r_0)$$

$$\vec{F}(r) = -\frac{\partial}{\partial r} V(r)\,\hat{r} = -V_0 \frac{r_0}{r}\frac{1}{r_0}\,\hat{r}$$

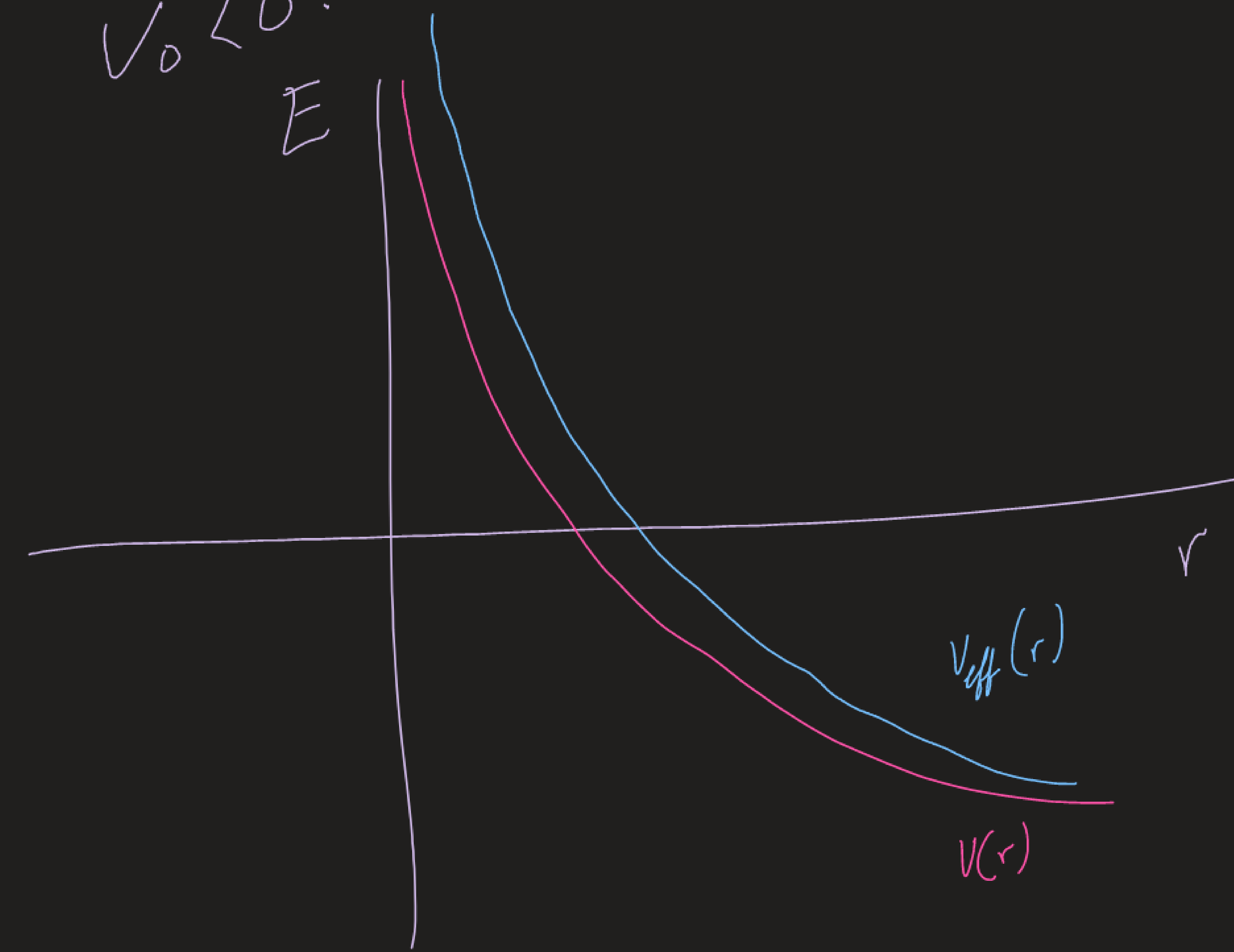
$$= -\frac{V_0}{r}\,\hat{r}$$

$V_0 > 0$:



allowed trajectories:
- circular
- bound orbit

$V_0 < 0$:



allowed trajectories:
- scattering

$$V_{eff}(r) = \frac{l^2}{2mr^2} + V_0 \ln(r/r_0)$$

Bounded orbits are possible if $V_0 > 0$.

This is true for all orbit energies, since $\lim_{x\to\infty} \ln(x) = \infty$.

$$\mathcal{L} = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\varphi}^2) - V(r)$$

$\hookrightarrow \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{r}} - \frac{\partial\mathcal{L}}{\partial r} = 0 = m\ddot{r} - mr\dot{\varphi}^2 + V'(r)$

$\hookrightarrow \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} - \frac{\partial\mathcal{L}}{\partial\varphi} = 0 = \frac{d}{dt}(mr^2\dot{\varphi})$

$\therefore$ $mr^2\dot{\varphi} = l$ is a constant.

$$\dot{\varphi} = \frac{l}{mr^2} \rightarrow m\ddot{r} = \frac{l^2}{mr^3} - V'(r)$$

In a circular orbit, $\dot{r} = 0$ and $r = R$.

$$m\ddot{R} = 0 = \frac{l^2}{mR^3} - \frac{V_0}{R}$$

$\hookrightarrow$ $mR^2V_0 = l^2$

$$R = \frac{l}{\sqrt{mV_0}}$$

If $r(t) = R + \varepsilon(t)$:

$$m\ddot{\varepsilon} = \frac{l^2}{m(R+\varepsilon)^3} - \frac{V_0}{R+\varepsilon}$$

$$\approx \frac{l^2}{mR^3}\left(1 - 3\frac{\varepsilon}{R}\right) - \frac{V_0}{R}\left(1 - \frac{\varepsilon}{R}\right)$$

$$= \underbrace{\cancel{\frac{l^2}{mR^3} - \frac{V_0}{R}}}_{0\ \text{(from before)}} - \underbrace{\left(\frac{3l^2}{mR^4} - \frac{V_0}{R^2}\right)}_{m\omega^2}\varepsilon$$

So,

$$\ddot{\varepsilon} + \omega^2\varepsilon = 0,$$

where

$$\omega^2 = \frac{1}{m}\left(\frac{3l^2}{mR^3} - \frac{V_0}{R^2}\right) \geq 0.$$