# f02

Wednesday, December 16, 2020 8:29 AM

$$\vec{\nabla}\times\vec{F} = -\frac{nk}{r^{n+1}}\vec{\nabla}\times\hat{r}$$
$$= \vec{0}$$

Since the curl is zero, $\vec{F}$ is conservative.

$$V(r) = \int_r^\infty \frac{nk}{\rho^{n+1}}\,d\rho$$
$$= \left[-\frac{n^2k}{\rho^n}\right|_r^\infty$$
$$= -\frac{n^2k}{r^n}$$

$$V_{eff}(r) = \frac{l^2}{2mr^2} - \frac{n^2k}{r^n}$$

If $\lim_{r\to 0} V_{eff}(r) < +\infty$, the mass can reach the center:

$$\lim_{r\to 0} V_{eff}(r) = \lim_{r\to 0}\left(\frac{l^2}{2mr^2} - \frac{n^2k}{r^n}\right)$$
$$= \lim_{r\to 0}\left(\frac{l^2r^n - 2mr^2n^2k}{2mr^{n+2}}\right)$$
$$= \lim_{r\to 0}\left(\frac{n(n-1)lr^{n-2} - 4mn^2k}{(n+2)(n+1)2mr^n}\right)$$
$$= \lim_{r\to 0}\left(\frac{n(n-1)l}{(n+2)(n+1)2mr^2} - \frac{4mn^2k}{(n+2)(n+1)2mr^n}\right)$$

I don't know...

For circular orbits:

$$V_{eff}'(R) = 0 = -\frac{l^2}{mR^3} - \frac{n^2k}{R^{n+1}}$$
$$\frac{l^2}{m}R^{n-2} = -n^2k$$
$$R = \left(-\frac{n^2km}{l^2}\right)^{\frac{1}{n-2}}$$ if $n$ is odd, these are stable.