PHY 831: Statistical Mechanics
Homework 7

Due Wed Nov. 24 , 2021

1. Find the 2nd virial coefficients b_2 for non-relativistic *non-interacting* quantum Fermi and Boson gases, working in the grand canonical ensemble. Now imagine a classical gas with some inter-particle potential $u(r)$ where r is the separation between two particles. Assuming $u(r)$ is a gaussian form, how would its range and strength have to be to chosen reproduce the results you got for the non-interacting Boson/Fermion gases?

2. Consider a gas in d dimensions where the particles interact with the pairwise potential

$$\begin{aligned} u(r) &= +\infty \qquad \text{for } 0 < r < a \\ &= -u_0 \qquad \text{for } a < r < b \\ &= 0 \qquad \text{for } b < r < \infty \end{aligned} \tag{1}$$

   (a) Calculate b_2 and comment on the high- and low-temperature behavior.

   (b) Calculate the first correction to the isothermal compressibility

$$\kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N} \tag{2}$$

   (c) In the high-T limit, cast the equation of state into van der Waals form and identify the van der Waals parameters

   (d) For $b = a$ (i.e., hard sphere potential) in $d = 1$, calculate the third virial coefficient b_3.

3. Given a system with the two-particle (or density-density) correlation function

$$g(R) = 1 + Ae^{-R/\ell}, \tag{3}$$

   where A, and ℓ are constants, find the number fluctuations of the system,

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{\langle N^2 - \langle N \rangle^2 \rangle}{\langle N \rangle}.$$

   What is $\langle \Delta N^2 \rangle / N$ for a Boltzmann gas? What is $\langle \Delta N^2 \rangle / N$ for a non-interacting Fermi gas? [Calculate these directly from the grand canonical distribution function] Qualitatively, what does this tell us about spatial correlations in a Fermi gas?