domenica 5 dicembre 2021 12:59

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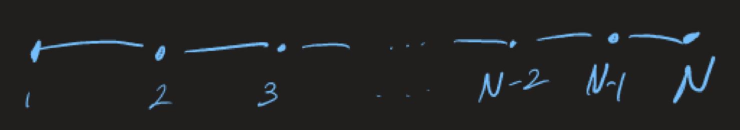

a) $\int_0^{\varepsilon_F} a(\varepsilon)\,d\varepsilon = N$

$a(\varepsilon) = \frac{2L}{2\pi\hbar}\oint d\Omega_1 \, p_0^{-1} m$

$= \frac{L}{2\pi\hbar} 2 \frac{m}{\sqrt{2m\varepsilon}}$

$\int_0^{\varepsilon_F} \frac{Lm}{\pi\hbar}\frac{1}{\sqrt{2m}}\varepsilon^{-1/2} d\varepsilon = \frac{2Lm}{\pi\hbar\sqrt{2m}}\varepsilon_F = N$

$\boxed{\varepsilon_F = \frac{\pi\hbar N}{\sqrt{2mL^2}}}$

b) At $T=0$, phonons are nonexistent, so all of the energy comes from the electrons. (I think).

c) $\int_0^{\nu_D} g(\omega)\,d\omega = N$ (There are N normal modes)

$\int_0^{\nu_D} \frac{L}{2\pi c_s} d\omega = N$

$\frac{L\nu_D}{2\pi c_s} = N$

$\boxed{\nu_D = \frac{2\pi c_s N}{L}}$

$\Theta_D = \frac{\hbar \nu_D c_s}{\omega}$

d) For $0 < T \ll \Theta_D$, all of the energy is from the phonons. (I think).

2

$N \approx \underbrace{\frac{e^{\beta\mu}}{1-e^{\beta\mu}}}_{>0 \text{ for B.E.C. to form}} + \int d\varepsilon\, g(\varepsilon)\frac{1}{e^{\beta(\varepsilon-\mu)}-1}$

IDK why it isn't 3, and that it depends on ζ (and D??)

$N_0 = \frac{z}{1-z}$

$\mu \to 0^-$: $\frac{1}{N_0} \to \frac{1}{z} - 1 \approx -\beta\mu \leftarrow \mu < 0 \to -\beta\mu > 0$

$\frac{N_0}{N} = -\frac{1}{N\beta\mu}$ $\quad \frac{N_c}{L^D} \approx \zeta\left(\frac{D}{2}\right)\left(\frac{m}{2\pi\hbar^2\beta}\right)^{D/2}$

3

a) s.p. DOS : $a(\varepsilon) = 2\left(\frac{L}{2\pi\hbar}\right)^2 \oint d\Omega_2 \, p_0^0 m$

$= 2\left(\frac{L}{2\pi\hbar}\right)^2 2\pi m$

$= \frac{Am}{\pi\hbar^2}$

$\int_0^{\varepsilon_F} a(\varepsilon)\,d\varepsilon = N \to \frac{Am}{\pi\hbar^2}\varepsilon_F = N$

$\varepsilon_F = \frac{\pi\hbar^2 N}{mA}$

$\rho = \frac{Nm}{A} \to \boxed{\varepsilon_F = \frac{\pi\hbar^2 \rho}{m^2}}$

b) $\int_0^{p_F} \frac{A}{\pi\hbar^2} p\,dp = \frac{A p_F^2}{2\pi\hbar^2} = N \to p_F = \sqrt{\frac{2\pi\hbar^2 N}{A}}$

$E_0 = \frac{2\pi 2A}{(2\pi\hbar)^2}\int_0^{p_F} \frac{p^3}{2m} dp$

$= \frac{A}{\pi\hbar^2}\frac{1}{2m}\frac{p_F^4}{4}$

$= \frac{A}{8\pi\hbar^2 m}\left(\frac{2\pi\hbar^2 N}{A}\right)^2$

$= \frac{\pi\hbar^2 N^2}{2Am}$

$= \frac{\pi\hbar^2 N}{2m^2}\rho = \boxed{\frac{N\varepsilon_F}{2}}$

c) $P_0 = -\frac{\partial E_0}{\partial A} = -\frac{N}{2}\frac{\partial \varepsilon_F}{\partial A}$

$= -\frac{N}{2}\frac{\pi\hbar^2}{m^2}\frac{\partial \rho}{\partial A}$

$\boxed{P_0 = \frac{N}{2}\frac{\pi\hbar^2}{m^2}\frac{\rho}{A}}$

d) $U = \frac{N\varepsilon_F}{2}\left(1 + \frac{5\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2 + O\left(\left(\frac{kT}{\varepsilon_F}\right)^3\right)\right)$

$\boxed{= \frac{N\varepsilon_F}{2} + \frac{5\pi^2 N k^2 T^2}{24\,\varepsilon_F}}$

4

$u(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$

a) $b_2 = -\frac{2\pi}{\lambda_a^3}\int_0^\infty \left(1 - e^{-\beta u(r)}\right) r^2 dr$

$= -\frac{2\pi}{\lambda_a^3}\int_0^a r^2 dr$

$\boxed{= -\frac{2\pi}{\lambda_a^3}\frac{a^3}{3}}$

b) $b_2 = \frac{\lambda_a^3}{V}\left(Z_2 - \frac{1}{2}Z_1^2\right)$

$= \frac{\lambda_a^3}{V}\left(Z_2 - \frac{1}{2}\frac{V^2}{\lambda_a^6}\right)$

$= \sqrt{2}\, Z_{2,rel} - \frac{1}{2}\frac{V}{\lambda_a^3}$

$Z_{2,rel} = \sum_n e^{-\beta\varepsilon_n} + \int d\varepsilon\, e^{-\beta\varepsilon} g(\varepsilon)$