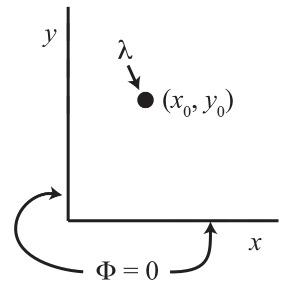
Problems

1. **Line charge and conducting half-planes:**
A straight-line charge with constant linear charge density λ is parallel to the z-axis and intersects the xy-plane in the first quadrant at the point (x_0, y_0). In addition, two conducting surfaces along the xz-plane ($x \geq 0$) and the yz-plane ($y \geq 0$) meet at the z-axis and are held at zero potential. The cross section of this configuration is seen in the figure. Consider the potential, fields, and surface charges in the region bounded by the surfaces (that is, $x \geq 0$, $y \geq 0$).

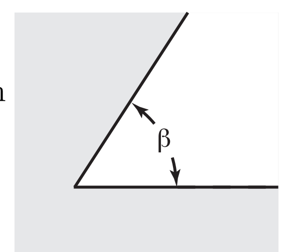


   (a) (15 pts) Show that the potential of a very long isolated line charge at (x_0, y_0) can be written as $\phi(x, y, z) = \frac{1}{4\pi\epsilon_0}\lambda \ln(R^2/\rho^2)$, where $\rho^2 = (x-x_0)^2+(y-y_0)^2$ and R is a constant.

   (b) (20 pts) Derive a simple expression for the potential of the line charge in the presence of the conducting half-planes. Verify explicitly that the potential and the tangential electric field vanish on the boundary surfaces.

   (c) (15 pts) Determine the surface charge density $\sigma(x, z)$ on the xz half-plane.

2. **A corner:**
Consider the potential in the region bounded by two half-planes, which meet at the origin at an angle β as shown in the figure. Near the origin there are no charges, although we presume there are other charges (not shown) away from the origin. For simplicity, we shall also assume that nothing depends on the variable z. Then the potential near the origin can be expressed in cylindrical coordinates, and it satisfies the Laplace equation, $\Delta\phi = 0$, with a solution of the form



$$\begin{aligned}\phi(\rho, \varphi) \quad = \quad & a_0 + b_0\varphi + (c_0 + d_0\varphi)\ln\rho + \sum_{n=1}^{\infty}\Big[\rho^{\nu_n}(a_n\cos\nu_n\varphi + b_n\sin\nu_n\varphi) \\ & +\rho^{-\nu_n}(c_n\cos\nu_n\varphi + d_n\sin\nu_n\varphi)\Big] \ .\end{aligned}$$

In this case, however, the parameters ν_n are not integers, since φ doesn't run periodically from 0 to 2π in the charge-free region. We have also included terms linear in φ, which may appear if the solution is not assumed periodic in φ.

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(a) (20 pts) Assume that the half-planes are grounded, so that $\phi(\rho, 0) = \phi(\rho, \beta) = 0$. Use this to obtain the possible values of ν_n and to determine which of the coefficients are nonzero.

(b) (15 pts) Since we are interested in the field near the corner, we include the point $\rho = 0$ in the region and assume that the potential is finite as $\rho \to 0$ (no charge singularities). Write the general solution for the potential near the corner. What is the leading behavior of the potential as $\rho \to 0$? (We can assume that the coefficient of the leading term is nonzero due to the presence of the other charges away from the origin.)

(c) (15 pts) Keeping only the leading term of the solution near $\rho = 0$, determine the $\vec{E}$ field and surface charge density. Discuss the behavior of the field and surface charge near the corner as $\rho \to 0$ for the values of $\beta \approx 0$, $\beta = \pi$ and $\beta \approx 2\pi$.