Problems

1. **Plane waves:**

   (a) (10 pts) Consider a monochromatic plane wave electric field given by

   $$E_x = 0, \qquad E_y = E_0 \cos\alpha, \qquad E_z = E_0 \sin\alpha, \qquad \alpha = \omega\left(t - \frac{x}{c}\right) \ . \quad (1)$$

   State the direction of propagation, the character of the polarization, and find the associated magnetic field.

   (b) (10 pts) Consider the superposition of two waves propagating along the z-direction with equal frequencies ω and wave vectors $\mathbf{k} = \hat{\mathbf{z}}(\omega/c)$, but with different amplitudes, polarizations, and phases,

   $$\vec{E}_1 = a_1\,\hat{x}\,\cos(\mathbf{k}\cdot\mathbf{r} - \omega t) \qquad (2)$$
   $$\vec{E}_2 = a_2\,\hat{y}\,\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \beta) \ . \qquad (3)$$

   Give examples for the values of a_1, a_2 and β (with $0 \le \beta \le \pi$) such that the sum of the two plane waves, $\vec{E} = \vec{E}_1 + \vec{E}_2$, has (i) linear, (ii) right-handed circular or (iii) right-handed elliptical polarization. For the case of linear polarization, state the axis of the polarization.

2. **Pulse:** A pulse of electromagnetic field propagates in vacuum along the positive direction of the z-axis. The initial shape of the pulse is a Gaussian,

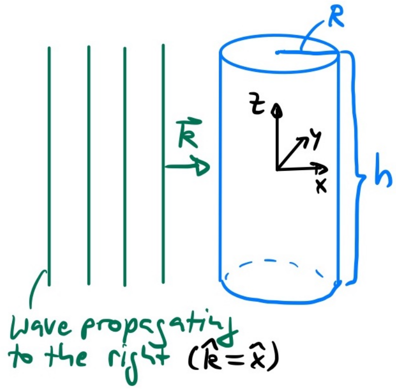
   $$F(z, t = 0) \ = \ \frac{a}{\sqrt{2\pi\sigma^2}} e^{-z^2/(2\sigma^2)} \ .$$

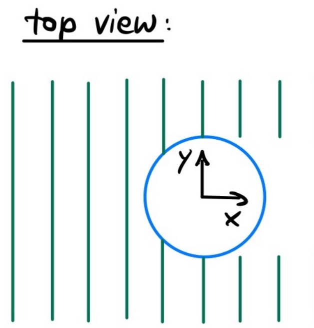
   (a) (10 pts) Find the shape of the pulse $F(z,t)$ for $t > 0$ and determine if there is a spreading of the pulse.

! PLEASE SEE NEXT PAGE FOR PROBLEMS 3 AND 4 !

3. **Plane wave and cylinder:** Consider a plane wave electric field given by

$$E_x = 0, \qquad E_y = E_0 \cos\alpha, \qquad E_z = E_0 \sin\alpha, \qquad \alpha = \omega\left(t - \frac{x}{c}\right) \; .$$





(a) (30 pts) Suppose that a cylinder of radius R and height h is placed perpendicularly to the direction of wave propagation, and the wave is totally absorbed by the cylinder surface. Neglecting diffraction effects, derive the force applied to the cylinder averaged over the period of the wave.

4. **Fields in a hollow cylinder:** Using cylindrical coordinates (ρ, φ, z), consider electric and magnetic fields

$$\vec{E} = \hat{\rho}\frac{c_1}{\rho}, \qquad \vec{B} = \hat{\varphi}\frac{c_2}{\rho},$$

with constant c_1 and c_2 inside the volume bounded by $a \leq \rho \leq b$, i.e. inside an infinitely long cylinder with a hole.

(a) (5 pts) Determine the Poynting vector $\vec{S}$ inside the volume.

(b) (10 pts) Determine the total flux of energy in the fields through a cross-sectional surface with $a \leq \rho \leq b$.

(c) (20 pts) Determine the energy per unit length $d\mathcal{E}/dz$ and the momentum per unit length $d\vec{p}/dz$ in the fields (for $a \leq \rho \leq b$).

(d) (5 pts) Consider a new problem, where sources contained within a restricted volume generate electric and magnetic fields. Outside of this region the Poynting vector is non-vanishing. Does that imply that the sources lose energy due to radiation ?