Problems

1. **Inhomogeneously charged sphere:** Consider a sphere with charge density
$$\rho(\vec{r}) = \rho_0 (r/R)^2 \qquad \text{for } r \equiv |\vec{r}| \leq R, \tag{1}$$
and no charge outside.
   (a) (10 pts) Calculate the total charge in terms of ρ_0 and R.
   (b) (10 pts) Derive the electric field (both magnitude and direction) at an arbitrary point in terms of the total charge.
   (c) (10 pts) Draw $|\vec{E}|$ as a function of r. With which power of r does it fall off outside of the sphere ? Which simpler charge density would generate the same electric field for $r > R$?

2. **Spherical cavity:** Consider a sphere of radius R that has a hollow spherical cavity of radius b inside it. The center of the big sphere is at the origin, the center of the cavity is at $\vec{a}$. The volume of the big sphere (excluding the cavity) is uniformly charged with a charge density ρ_0.
   (a) (15 pts) Derive the electric field (both magnitude and direction) at an arbitrary point inside the cavity. (Find a compact expression in terms of the given parameters.)

3. **Charge density of a special field:** A system of charges creates the electric field
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right) e^{-2r/a}\frac{\vec{r}}{r^3} \tag{2}$$
where a is a positive constant and $r = |\vec{r}|$.
   (a) (20 pts) Find the charge density $\rho(\vec{r})$.
   (b) (5 pts) Calculate the total charge of the system.
   (c) (honors only) Suggest a physical system which could be modeled by this charge distribution.

4. **Field of a thin disc:** An infinitely thin round disk of radius R has its symmetry axis on the z-axis. It is uniformly charged with total charge q.
   (a) (10 pts) Write an expression for the charge density of the disk $\rho(\vec{r})$ using appropriate coordinate variables. Determine the Cartesian surface density $\sigma(x, y)$ from your expression for $\rho(\vec{r})$.
   (b) (15 pts) Calculate by direct integration the electric field $\vec{E}(\vec{r})$ at an arbitrary point on the z-axis (from your expression for $\rho(\vec{r}\,')$).
   (c) (5 pts) Find the limits of the field for $z \gg R$ and for $z \ll R$ and explain the results.