Problems

1. **Cylindrical conductor with current:** A cylindrical conductor of radius R has zero net electric charge. The current density $\vec{j}$ is uniform throughout the metal of the cylinder and parallel to the axis of the cylinder.

   (a) (20 pts) Find the magnitude and direction of the magnetic field at an abritrary point. To be specific, let the current run in the positive z direction ($\vec{j} = j\hat{z}$) and let the center of the cylinder pass through the origin.

   (b) (20 pts) If this system is observed in a reference frame moving at a constant velocity $\vec{\beta} = \beta\hat{z}$, what will be the electric and magnetic fields ?

2. **Rotating cylinder:** A homogeneously charged cylinder with charge density ρ_0 is rotating around its symmetry axis, which coincides with the z axis. The radius of the cylinder is R, its height is h, and its center is at the origin. The angular velocity is $\vec{\omega} = \omega\hat{z}$.

   (a) (20 pts) Calculate the magnetic moment of the rotating cylinder. [Hint: $\vec{r} = s\hat{s} + z\hat{z}$, $\vec{r} \times \hat{\varphi} = s\hat{z} - z\hat{s}$.]

   (b) (20 pts) A magnetic dipole with moment $\vec{m}_D = m_D\hat{z}$ ($m_D > 0$) is placed at $\vec{r}_D = (0, 0, z_D)$ ($z_D > 0$), far away from the rotating cylinder. If the orientation of the magnetic dipole moment $\vec{m}_D$ is reversed, $\vec{m}_D \to -\vec{m}_D$, will the dipole-cylinder system have a lower or higher energy than before ? What is the magnitude of the difference in energy ?

3. **Summary of course topics:** (20 pts) Please compile your personal summary of the course topics covered since your last summary (homework 5). Ideally, your summary should reflect key concepts, the relations between them, and problem solving strategies. If you prefer to keep it short without giving much details (say, one page), that is fine. Please make sure to mention spherical multipoles, general solutions of Laplace's equation and expansions, boundary value problems, and Green's method for electrostatics as well as basic concepts of magnetostatics and magnetic dipoles. (This assignment is intended to support your preparations for the exam, please avoid spending a lot of time on the write-up itself.)