Please read all of the following before starting the exam:

- Please access the exam problems on Gradescope only after we agree to start *in the Zoom meeting*
- You then have 2 hours to solve the problems.
- Please upload your solutions to Gradescope and indicate for each problem where the relevant pages are.
- Please stay connected to Zoom during the entire duration of your exam and switch on your camera.
- To solve the problems, you may use a simple calculator, but no computer algebra systems, external notes, books, etc.
- All problems are in S.I. units unless stated otherwise. Please give your answers in terms of the given variables and units.
- A complete answer usually includes a derivation of the result (unless stated otherwise). Show all work as neatly and logically as possible to maximize your credit. State clearly which equations were used. Circle or otherwise indicate your final answers.
- Please ask if the problem description is unclear.
- *Good luck !*

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\begin{aligned}
\vec{a} \times (\vec{b} \times \vec{c}) &= \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}), \\
\vec{a} \cdot (\vec{b} \times \vec{c}) &= \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b}), \\
(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) &= (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c}), \\
\vec{\nabla} \times (\vec{\nabla}\psi) &= 0, \\
\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) &= 0, \\
\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) &= \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}, \\
\vec{\nabla} \cdot (\psi \vec{a}) &= \vec{a} \cdot \vec{\nabla}\psi + \psi \vec{\nabla} \cdot \vec{a}, \\
\vec{\nabla} \times (\psi \vec{a}) &= \vec{\nabla}\psi \times \vec{a} + \psi \vec{\nabla} \times \vec{a}, \\
\vec{\nabla}(\vec{a} \cdot \vec{b}) &= (\vec{a} \cdot \vec{\nabla})\vec{b} + (\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a}), \\
\vec{\nabla} \cdot (\vec{a} \times \vec{b}) &= \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b}), \\
\vec{\nabla} \times (\vec{a} \times \vec{b}) &= \vec{a}(\vec{\nabla} \times \vec{b}) - \vec{b}(\vec{\nabla} \times \vec{a}) + (\vec{b} \cdot \vec{\nabla})\vec{a} - (\vec{a} \cdot \vec{\nabla})\vec{b}, \\
\vec{\nabla} \cdot \vec{r} &= 3, \\
\vec{\nabla} \times \vec{r} &= 0, \\
\vec{\nabla} \cdot \hat{r} &= 2/r, \\
\vec{\nabla} \times \hat{r} &= 0, \\
\vec{\nabla} r &= \hat{r}, \\
\vec{\nabla} \frac{1}{r} &= -\frac{\hat{r}}{r^2}, \\
\vec{\nabla} \cdot (\hat{r} f(r)) &= \frac{2}{r} f + \frac{df}{dr}, \\
(\vec{a} \cdot \vec{\nabla})\hat{r} &= \frac{1}{r}[\vec{a} - \hat{r}(\vec{a} \cdot \hat{r})] = \frac{\vec{a}_\perp}{r}, \\
\vec{\nabla}^2 \left(\frac{1}{r}\right) &= -4\pi\delta(\vec{r}), \\
\int_V d^3r \; \vec{\nabla} \cdot \vec{A} &= \int_S d\vec{S} \cdot \vec{A}, \\
\int_V d^3r \; \vec{\nabla}\psi &= \int_S \psi \; d\vec{S}, \\
\int_V d^3r \; \vec{\nabla} \times \vec{A} &= \int_S d\vec{S} \times \vec{A}, \\
\int_V d^3r \; (\phi \nabla^2 \psi + \vec{\nabla}\phi \cdot \vec{\nabla}\psi) &= \int_S \phi \; d\vec{S} \cdot \vec{\nabla}\psi, \\
\int_V d^3r \; (\phi \nabla^2 \psi - \psi \nabla^2 \phi) &= \int_S (\phi \vec{\nabla}\psi - \psi \vec{\nabla}\phi) \cdot d\vec{S}, \\
\int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} &= \oint d\vec{\ell} \cdot \vec{A}, \\
\int_S d\vec{S} \times \vec{\nabla}\psi &= \oint_C d\vec{\ell} \; \psi.
\end{aligned}
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\begin{aligned}
\mathscr{L} &= -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu \\
L &= \frac{1}{\gamma}(-mc^2 - qA_\mu u^\mu) \\
\frac{dp^\mu}{d\tau} &= qF^{\mu\nu} u_\nu \\
\partial_\mu F^{\mu\nu} &= \mu_0 J^\nu \\
\partial_\mu \tilde{F}^{\mu\nu} &= 0 \\
\partial_\mu J^\mu &= 0 \\
(g_{\mu\nu}) &= \mathrm{diag}(1, -1, -1, -1), \\
\vec{\beta} &= \vec{v}/c, \\
\gamma &= 1/\sqrt{1-\beta^2}, \\
x'^\mu &= \Lambda^\mu{}_\nu x^\nu \\
(\Lambda^\mu{}_\nu) &= \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
\end{aligned}
$$

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\begin{aligned}
x^\mu &= (ct, x, y, z), \\
\partial^\mu &= ((1/c)\partial/\partial t, -\vec{\nabla}) \\
k^\mu &= (\omega/c, \vec{k}), \\
u^\mu &= (\gamma c, \gamma \vec{v}), \\
p^\mu &= (E/c, \vec{p}), \\
A^\mu &= (\phi/c, \vec{A}), \\
J^\mu &= (c\rho, \vec{j}), \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu, \\
\tilde{F}^{\mu\nu} &= \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}
\end{aligned}
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\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho, \qquad \vec{\nabla}\cdot\vec{B} = 0,
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\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \mu_0 \vec{j}, \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0,
$$

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\begin{aligned}
\frac{d\vec{p}}{dt} &= q(\vec{E} + \vec{v}\times\vec{B}) \\
\vec{E} &= -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t} \\
\vec{B} &= \vec{\nabla}\times\vec{A} \\
p_i &= \int d^3\, r' \rho(\vec{r}')\, r'_i \\
Q_{ij} &= \int d^3 r'\, \rho(\vec{r}')\, (3r'_i r'_j - \delta_{ij} r'^2) \\
\phi &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q_{\mathrm{tot}}}{r} + \frac{p_i \hat{r}_i}{r^2} + \frac{1}{2!} Q_{ij}\frac{\hat{r}_i \hat{r}_j}{r^3} + \dots\right) \\
U &= Q_{\mathrm{tot}}\phi(\vec{r}) - p_i E_i(\vec{r}) - \frac{1}{6} Q_{ij}\partial_i E_j + \dots \\
\vec{E} &= \frac{1}{4\pi\epsilon_0}\left(Q_{\mathrm{tot}}\frac{\hat{r}}{r^2} + \frac{3(\vec{p}\cdot\vec{r})\hat{r} - \vec{p}}{r^3} + \dots\right)
\end{aligned}
$$