Problems

1. **Charged kaon decay:** A charged kaon K^+ is produced in a collision at FRIB. The K^+ is an unstable particle, which decays after some time, see e.g. the particle listings of the particle data group, `https://pdg.lbl.gov`.

   (a) (10 pts) At a speed of $0.99c$, how far will the K^+ travel on average before it decays ?

2. **Alarm clock:** At exactly 6:00 am Eastern Standard Time your alarm clock goes off. At a time $\Delta t = 10^{-5}$ s later the alarm clock of a University of Michigan graduate student goes off, exactly 100 km away.

   (a) (10 pts) Find a Lorentz frame in which the UM student's alarm clock goes off before yours.

   (b) (10 pts) For each of the following cases, draw a Minkowski diagram ($c\,t$ vs. x) , where x is the distance from East Lansing, and denote two events in it.

      (i) [A] your alarm clock goes off, [B] the UM alarm clock goes off at a time, which is later in any frame of reference.

      (ii) [A] your alarm clock goes off, [B] the UM alarm clock goes off, such that it depends on the frame which one is first.

   (c) (10 pts) What is the largest time-difference Δt for which it is possible to find a Lorentz frame that can swap the time-ordering of the alarms?

3. **Colliding rockets:** A person on Earth observes two rocket ships moving directly toward each other and colliding. At time $t = 0$ in the Earth frame, the Earth observer determines that rocket A, travelling to the right at $v_A = 0.8c$, is at point a and rocket B is at point b, travelling to the left at $v_B = 0.6c$. They are separated by a distance $l = 4.2 \cdot 10^8$m.

   (a) (15 pts) How fast is rocket B approaching in A's frame ? How fast is rocket A approaching in B's frame ?

   (b) (15 pts) How much time will elapse in A's frame from the time rocket A passes point a until collision ? How much time will elapse in B's frame from the time rocket B passes point b until collision ?

4. **General boost:** Given a four vector $x^\mu = (5, 4, 3, 2)$m (with $x^0 = ct$) in a frame F, consider how it is observed in a frame F' which moves with velocity $\vec{v}_0 = (0.4, 0.5, 0.6)c$ with respect to F.

   (a) (10 pts) Calculate the four vector x'^μ observed in the frame F'. You may just look up the necessary formula and use it without derivation, but please indicate your reference.

(b) (10 pts) Determine the rapidity of the Lorentz boost from F to F'.

(c) (10 pts) Is the four vector x^μ time-like, space-like or light-like respectively ? What about x'^μ ? Suppose $x^\mu = x^\mu_A - x^\mu_B$ for two events A and B with four-vectors x^μ_A and x^μ_B, respectively (and the value for x^μ as before). Could there be a causal connection between the two events ?