Problems

1. **Greetings to the traveler:** You say goodbye to your friend who leaves Earth with her spaceship traveling at a constant speed. You stay on Earth and, after $10h$, you send her a greeting using radio waves. As soon as she receives your message, she answers back. You receive her answer $1h$ after you sent your message.

   (a) (15 pts) Draw a Minkowski (ct,x) diagram in your frame and denote the following events: (i) when you say goodbye at (0,0), (ii) when you send the greeting at (ct_i, x_i) , (iii) when your friend receives the greeting at (ct_m, x_m), (iv) when you receive her answer at (ct_f, x_f). Please also denote four different worldlines (paths in space-time) for (I) you, (II) your friend, (III) the radio signal you sent, (IV) the radio signal your friend sent.

   (b) (15 pts) At which relativistic speed β is your friend traveling ?

   (c) (5 pts) If your friend's frame coincides with yours at $(0, 0)$, what will her clock show when she receives your message ? Express the answer first in terms of t_m and β, then plug in the number you obtained in the previous step and give the result to 5 significant digits.

2. **Field tensor and four-potential:** The field tensor $F^{\mu\nu}$ and the electromagnetic four-potential A^μ allow to formulate electromagnetism in a Lorentz covariant way.

   (a) (10 pts) Derive F^{01},F^{10},F^{12}, and F^{21} in terms of the electric field (E_x, E_y, E_z) and the magnetic field (B_x, B_y, B_z).

   (b) (5 pts) Consider a four-potential A^μ, which describes some specific electric and magnetic fields. Is it possible that there is a different four-potential A'^μ which produces the same fields ? If so, by which kind of transformation would A^μ and A'^μ need to be related ?

3. **Particle decay:** A positively-charged sigma-hyperon Σ^+ with momentum 900 MeV/c decays in flight into a positive pion π^+ and a neutral particle. The pion is detected with momentum 200 MeV/c at an angle 60° with respect to the hyperon momentum.

   (a) (20 pts) Determine the mass of the undetected particle, and deduce what type of particle it is. [Data: $m_{\Sigma^+} = 1189$ MeV/c^2, $m_{\pi^+} = 140$ MeV/c^2.]

(please see next page for problem 4)

4. **Electron-positron pair creation in photon-photon annihilation:** Consider two photons with different energies that annihilate and produce an electron-positron pair, e.g. the process under consideration is $\gamma(p_1)\gamma(p_2) \to e^-(p_3)e^+(p_4)$, where the p_1 and p_2 are the incoming 4-momenta, p_3 and p_4 are the outgoing 4-momenta. The angle between the incoming 3-momenta is θ_{12}.

   (a) (20 pts) Suppose photon 1 has some given energy E_1. What is the minimal energy of photon 2, E_2, for the creation of an electron-positron pair to become possible ? Evaluate the expression for $\theta_{12} = \pi$, the mass of the electron 511 keV/c^2, and the two cases, $E_1 = 10^{-4}$ eV and $E_1 = 10^6$ eV, respectively.

   (b) (10 pts) Derive that the boost to the center of mass frame of the incoming particles is given by

$$\vec{\beta} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}. \tag{1}$$