Problems

1. **A simple Poisson equation:** Consider the charge distribution $\rho(\vec{r}) = \rho_0 e^{-\mu r}$ with $r = |\vec{r}|$.

   (a) (10 pts) First, derive the action of the Laplace operator on a general function $f(r)$ which depends only on $r = |\vec{r}|$. Show that it can be written as

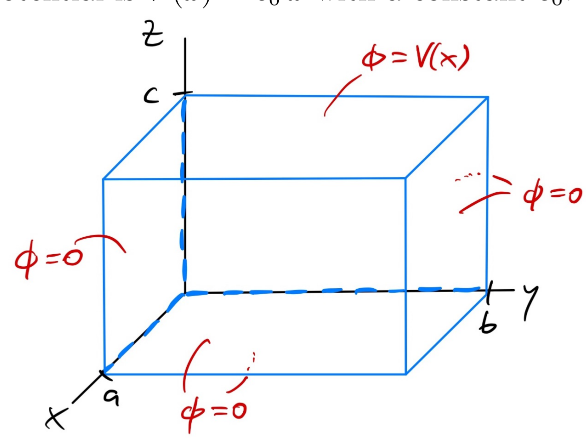
   $$\Delta f(r) = \frac{1}{r}\frac{\partial^2}{\partial r^2}\left(r f(r)\right) . \tag{1}$$

   If you like, you can also remind yourself how to derive the complete Laplace operator in spherical coordinates.

   (b) (20 pts) Next, consider the charge distribution $\rho(\vec{r})$ given above and solve the Poisson equation for the potential in spherical coordinates. Derive a general solution for the potential with generic boundary constants.

   (c) (20 pts) Last, determine the boundary constants by studying the large distance behavior of the potential.

2. **Potential for a box:** Consider a rectangular empty box with lengths (a, b, c) in (x, y, z) direction. All surfaces of the box have zero potential, except for the side at $z = c$, where the potential is $V(x) = c_0\, x$ with a constant c_0.



   (a) (30 pts) Solve the Laplace equation in Cartesian coordinates using a product ansatz and separation of variables. Derive a general solution for the potential with generic boundary constants.

   (b) (20 pts) Determine the boundary constants by requiring the potential to take the prescribed values on the surfaces of the box. [It is fine to use Wolfram Alpha or similar for the integration, but please state what you used.]