Homework 1

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PHY841
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1.1

a

For a kaon, K^+, whose average lifetime is $(1.2379 \pm 0.0021) \times 10^{-8}$s, traveling at $v = 0.99c$, the Lorentz Factor is given as

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 7.088.$$

Thus the distance the kaon travelled in the lab's reference frame is

$$x = \gamma v t \approx 25.977\text{m}$$

1.2

a

In the East Lansing reference frame, K, the spacetime coordinates of the events are

$$\begin{aligned} E_1 : &\qquad (ct = 0\text{km}, x = 0\text{km}), \\ E_2 : &\qquad (ct = 3 \times 10^4\text{km}, x = 100\text{km}). \end{aligned}$$

For the alarm clock in Ann Arbor to go off before the alarm clock in East Lansing, the reference frame, K', the $ct' = x'_0$ coordinate must be negative:

$$x'_0 = \gamma(x_0 - \beta x) < 0 \to v > \frac{c^2 t}{x}.$$

Plugging in the numbers gives

$$v > 9 \times 10^6 \frac{\text{m}}{\text{s}}.$$

b

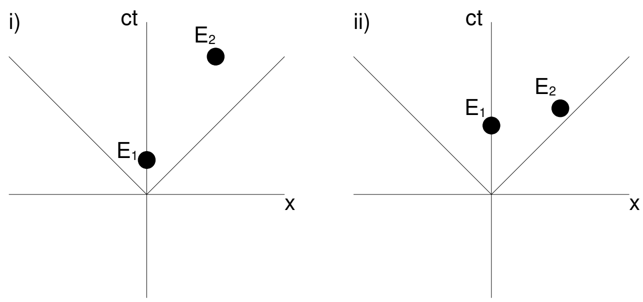

Figure 1.2.1

c

If $c\Delta t = 100$km, then the reference frame would have to be moving faster than light, to switch the order of the events, which is illegal. Thus, the maximum Δt would be $\Delta t = 10^5/c \approx 3.34 \times 10^{-4}$s.

1.3

For the following problem, let the velocity of rocket A, in the observer's reference frame, be $\mathbf{u}_A = 0.8c\hat{\mathbf{x}}$, and let the velocity of rocket B, in the observer's reference frame, be $\mathbf{u}_B = -0.6c\hat{\mathbf{x}}$.

a

From equation 11.17 in Jackson, the differential expressions dx'_0, dx'_1, dx'_2, and dx'_3, are given by

$$\begin{aligned} dx'_0 &= \gamma_v(dx_0 - \beta dx_1), \\ dx'_1 &= \gamma_v(dx_1 - \beta dx_0), \\ dx'_2 &= dx_2, \\ dx'_3 &= dx_3. \end{aligned}$$

If $u_i = c dx_i/dx_0$ and $u'_i = c dx'_i/dx'_0$, then

$$u'_{||} = \frac{u_{||} - v}{1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}}, \tag{1.3.1}$$

$$\mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp}}{1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}}. \tag{1.3.2}$$

In order to find the velocity of rocket B in the reference frame of rocket A, let $\mathbf{v} = \mathbf{u}_A$ and $\mathbf{u} = \mathbf{u}_B$. Since there is no perpendicular component, equation 1.3.2 vanishes. Thus, 1.3.1 becomes

$$u'_B = \frac{u_B - u_A}{1 - \frac{u_A u_B}{c^2}} = \frac{-1.4c}{1 + 0.48} = -0.\overline{945}c.$$

Similarly, the velocity of rocket A, in the reference frame of rocket B is

$$u'_A = 0.\overline{945}c$$

b

The amount of time it takes for the two rockets to collide is given by

$$u_A t = -u_B t + l \rightarrow t = \frac{l}{u_A + u_B} \approx 1.$$

Thus, the amount of time that passes, for rocket A, before collision is

$$t_A = \gamma_A t \approx 1.668\text{s},$$

and the amount of time that passes, for rocket B, before collision is

$$t_B = \gamma_B t \approx 1.251\text{s}.$$

1.4

a

From equation 11.98 in Jackson, the Lorentz boost matrix, for a given boost, $\boldsymbol{\beta}$, is given by

$$A(\boldsymbol{\beta}) = \begin{pmatrix} \gamma & -\gamma\beta_1 & -\gamma\beta_2 & -\gamma\beta_3 \\ -\gamma\beta_1 & 1 + \frac{(\gamma-1)\beta_1^2}{\beta^2} & \frac{(\gamma-1)\beta_1\beta_2}{\beta^2} & \frac{(\gamma-1)\beta_1\beta_3}{\beta^2} \\ -\gamma\beta_2 & \frac{(\gamma-1)\beta_1\beta_2}{\beta^2} & 1 + \frac{(\gamma-1)\beta_2^2}{\beta^2} & \frac{(\gamma-1)\beta_2\beta_3}{\beta^2} \\ -\gamma\beta_3 & \frac{(\gamma-1)\beta_1\beta_3}{\beta^2} & \frac{(\gamma-1)\beta_2\beta_3}{\beta^2} & 1 + \frac{(\gamma-1)\beta_3^2}{\beta^2} \end{pmatrix}.$$

Plugging in the given values to $A(\boldsymbol{\beta})x^\mu$ gives

$$x'^\mu = A(\boldsymbol{\beta})x^\mu = \begin{pmatrix} 1.4596 \\ 2.25367 \\ 0.817088 \\ -0.619495 \end{pmatrix}.$$

b

By definition, the rapidity of the boost is

$$\boldsymbol{\zeta} = \hat{\boldsymbol{\beta}} \tanh^{-1}(\beta) \approx 1.36478\hat{\boldsymbol{\beta}}.$$

c

The quantity $x_\mu x^\mu$ is invariant, so $x'_\mu x'^\mu = x_\mu x^\mu = -4 < 0$. Thus, both four-vectors are spacelike. If this is a four-vector representing separation between two events, these two events could not have a causal connection between them.