Homework 2

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PHY841
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2.1

a

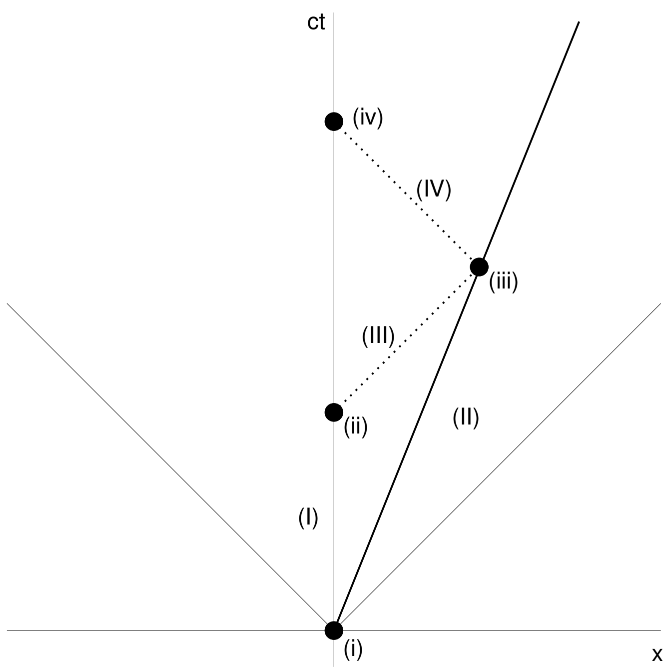

Figure 2.1.1: This is not scaled properly. I'm sorry.

b

The relativistic velocity is easily found to be

$$\beta = \frac{x_m}{ct_m} = \frac{c0.5\text{h}}{c10.5\text{h}} = \frac{1}{21}.$$

c

Her clock, when the signal is received, will show

$$t'_m = \frac{t_m}{\sqrt{1-\beta^2}} = 10.512\text{h}.$$

2.2

a

From equation 11.136, in Jackson,

$$F^{\alpha\beta} = -F^{\beta\alpha},$$

so

$$F^{10} = -F^{01} \quad \text{and} \quad F^{21} = -F^{12}.$$

Using the same equation from Jackson,

$$F^{01} = \partial^0 A^1 - \partial^1 A^0 = -E_x \rightarrow F^{10} = E_x,$$

and

$$F^{12} = \partial^1 A^2 - \partial^2 A^1 = -B_z \rightarrow F^{21} = B_z.$$

b

Two four potentials, A^μ and A'^μ, can both produce the same electric and magnetic fields, if they only differ by a gauge transformation:

$$A'^\mu = A^\mu + \boldsymbol{\nabla}\Lambda$$

2.3

a

Since the pion has a momentum of 200MeV/c, at an angle of 60° from the hyperon momentum, then the pion momentum perpendicular to this is $100\sqrt{3}$MeV/c, and the pion momentum parallel to this is 100MeV/c. Thus, the momentum of the neutral particle is

$$\mathbf{p}_? = \begin{pmatrix} 800 \\ -100\sqrt{3} \end{pmatrix} \text{MeV}/c \rightarrow p_? \approx 806.23\text{MeV}/c.$$

In all of the cases, the total energy can be related to the momentum by using the following equation:

$$E = \sqrt{p^2c^2 + m^2c^4}.$$

Due to the units, the factors of c vanish, though. The final equation for the mass of the neutral particle, $m_?$, is

$$m_? = \sqrt{(E_{\Sigma+} - E_{\pi+})^2 - p_?^2} \approx 951.43\text{MeV}/c^2,$$

where

$$E_{\Sigma+} = \sqrt{p_{\Sigma+}^2 + m_{\Sigma+}^2} \approx 1491.21\text{MeV}$$

and

$$E_{\pi+} = \sqrt{p_{\pi+}^2 + m_{\pi+}^2} \approx 241.13\text{MeV}.$$

The neutral particle is probably a neutron, even though the "accepted" value is 20MeV/c^2 lower than the value found here.

2.4

a

The 4-momentum has the invarient quantity $p_\mu p^\mu$. Thus

$$(p_1 + p_2)_\mu (p_1 + p_2)^\mu = (p_3 + p_4)_\mu (p_3 + p_4)^\mu = (m_- c)^2.$$

The right hand side of this equation simplifies to

$$\begin{aligned}(p_1 + p_2)_\mu (p_1 + p_2)^\mu = (E_1^2 + E_2^2) \left(\frac{1}{c^2} - c^2 \right) \\ + 2E_1 E_2 \left(\frac{1}{c^2} - c^2 \cos\theta_{12} \right).\end{aligned}$$

Thus, solving the conservation equation gives

$$E_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A},$$

where

$$\begin{aligned} A &= \left(\frac{1}{c^2} - c^2 \right), \\ B &= 2E_1 \left(\frac{1}{c^2} - c^2 \cos\theta_{12} \right), \\ \text{and} \quad C &= E_1^2 \left(\frac{1}{c^2} - c^2 \right) - m_-^2 c^2. \end{aligned}$$

I think I did this problem wrong.

b

It's late. I give up.