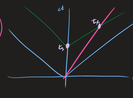
Exam 1

Thursday, February 18, 2021 12:26 PM

Missing equations make this

Alice's Birthday

a)



$x_A = \frac{1}{\sqrt{3}} c\,t_A$

$(c t_A, x_A)$

$t_A = \frac{24\,hr}{\gamma_A}$ $\quad \gamma_A = \frac{1}{\sqrt{1-\frac{1}{3}}} = \sqrt{\frac{3}{2}}$

$= \frac{24\,hr}{\sqrt{\frac{2}{3}}}$

$c = \frac{x_A}{t_A - t_B} \rightarrow t_B = -\frac{x_A}{c} + t_A$

$= -\frac{1}{\sqrt{3}} t_A + t_A$

$= \frac{1}{\sqrt{3}} t_A$

$= \frac{24\,hr}{\sqrt{6}}$

b) $\approx 0.5\,hr$ I don't think that is right... ?

Particle Decay

$E_f = E_1 + E_2$

$E_i = \sqrt{p_1^2 + m^2}$

$\sqrt{p_1^2 + m^2} = E_1 + E_2$

$p_1^2 = (P_1 + P_2)^2$

$= (104\ 78\ 104)\begin{pmatrix} 104 \\ 78 \\ 104 \end{pmatrix}$

$= 27716\ \frac{GeV^2}{c^2}$

$m = \sqrt{(E_1 + E_2)^2 - p_1^2}$

$= \sqrt{(161\,GeV + 154\,GeV)^2 - 27716\,\frac{GeV^2}{c^2}}$ $\quad c^2 = 1$

$m \approx 124.69\ \frac{GeV}{c^2}$

Three-Vectors

Since $\hat{v} = \frac{d\hat{x}}{dt}$, it can not transform like $\hat{x}$.

$\hat{A}$ only depends on $\hat{x}$, so it transforms the same way.

Similarly, $\hat{E}$ and $\hat{B} = \hat{v} \times \hat{A}$ transform like $\hat{x}$ since they only depend on $\hat{x}$ or $\frac{\partial}{\partial \hat{x}}$.

Like $\hat{v}$, $\hat{j}$ depends on a time derivative of $\hat{x}$, so it doesn't transform the same as $\hat{x}$.

← Reasoning

$\hat{A}, \hat{E}, \hat{B}$ (I hope...)

Flipping a Dipole

$W = -\Delta U$ (energy)

$= (\vec{p}_B \vec{E}_B(\vec{r}_B) - (\vec{p}_A \vec{E}_A(\vec{r}_A))$

(not energy)

$= (p_{Bx} - p_{Ax}) E_x(\vec{r})$

$= -2 p_{Bx} \frac{p_x}{4\pi\epsilon_0 d^3}$

$\vec{r} = \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix}$

$E_x(\vec{r}) = \frac{1}{4\pi\epsilon_0 d^3} 3(\hat{p}\cdot\hat{r})\hat{r}^0 - \hat{p}_x$

$= -\frac{p_x}{4\pi\epsilon_0 d^3} = \frac{-p_x}{4\pi\epsilon_0 d^3}$

$\vec{p}_B = \begin{pmatrix} p_{Bx} \\ p_{By} \\ p_{Bz} \end{pmatrix}$ $\quad |\vec{p}_B| = |\vec{p}| = \sqrt{p_{Bx}^2 + p_{By}^2 + p_{Bz}^2}$

$p_{Bx} = -p_{Ax}$

Charged Spherical Cavity

$\oint_{\partial\Omega} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}(\Omega)}{\epsilon_0}$

If the region, Ω, is a sphere centered at the origin, $\vec{E} \cdot d\vec{A} = c\,dA^2$, and $\rho(\vec{r}) = 0$, for $|\vec{r}| < R_c$. Thus

$E \oint dA^2 = 0 \rightarrow \vec{E} = \vec{0}$.

Charged Cylinder

Total charge:

$Q = \int \rho(\vec{r})\, d^3r$

$= \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^R \rho_0 \frac{s^2}{R^2}\, s\, ds\, dz\, d\phi$

$= \frac{\rho_0}{R^2} 2\pi \frac{h}{2} \frac{R^3}{5}$

$Q_{tot} = \frac{2\pi h \rho_0 R^3}{5}$

$\vec{p} = \int \rho(\vec{r})\, \vec{r}\, d^3r$

$= \frac{\rho_0}{R^3} \int_0^R \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} s^3 \begin{pmatrix} s\cos\phi \\ s\sin\phi \\ z \end{pmatrix} ds\, d\phi\, dz$ (all three triple integrals vanish)

$\vec{p} = 0$

$Q_{zz} = Q_{yy} = \int \rho(\vec{r})(3 r_i r_j - r^2 \delta_{ij})\, d^3r$ $\quad i = j = z$

$Q_{zz} = \int \rho(\vec{r})(2z^2 - s^2)\, d^3r$

$= \frac{\rho_0}{R^3} \int_0^R \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} (2z^2 - s^2)\, s\, dz\, d\phi\, ds$

$= 2\pi\frac{\rho_0}{R^3} \left[2\frac{z^3}{3} s - \frac{s^4}{4} z \right]_{-\frac{h}{2}}^{\frac{h}{2}} ds$

$= 2\pi\frac{\rho_0}{R^3} \left[2\frac{h^3}{24} s - \frac{h}{4} s^4 \right]_0^R$

$= 2\pi h R^2 \rho_0 \left[\frac{1}{12} - \frac{1}{20} \right]$

$Q_{xx} = Q_{yy} = -\frac{Q_{zz}}{2}$ $\quad (\sum_i Q_{ii} = 0)$

$= -\pi h R^2 \rho_0 \left[\frac{1}{20} - \frac{1}{12} \right]$

(same reasoning, radially symmetric)

$Q_{xz} = Q_{yz} = \int \rho(\vec{r})(3 r_i r_z)\, d^3r$

$= h\frac{\rho_0}{R^3} \int_0^R \int_0^{2\pi} s^3 \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} s\, ds\, d\phi$ $\quad 2\sin\phi\cos\phi = \sin 2\phi$

$= \frac{h\rho_0}{7} R^4 \int_0^{2\pi} \frac{1}{2} \begin{pmatrix} \sin 2\phi \\ 1 - \cos 2\phi \end{pmatrix} d\phi$ $\quad u = 2\phi,\ \frac{du}{2} = d\phi$

$= \frac{1}{7} h\rho_0 R^4 \begin{pmatrix} 0 \\ 2\pi \end{pmatrix}$

$= 0$

$Q_{xy} = \frac{\rho_0}{R^3} \int_0^R \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} s^3 \sin\phi\cos\phi\, z\, dz\, d\phi\, ds$ (the z integral $= 0$)

$= 0$

$Q_{yx} = 0$ ← (radial symmetry)

$$Q = Q_{zz} \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where

$Q_{zz} = 2\pi h R^2 \rho_0 \left[\frac{1}{12} - \frac{1}{20} \right]$