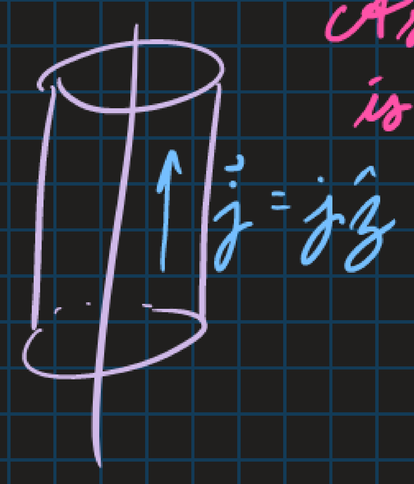
Homework 9

mercredi 28 avril 2021 21:46

1. Assume the cylinder is infinitely long.



The magnetic field is found using Ampère's law:

$$2\pi r B = \pi r^2 j$$

$$\vec{B} = \begin{cases} \frac{r}{2} j \hat{\varphi} & r \leq R \\ \frac{R^2}{2r} j \hat{\varphi} & r \geq R \end{cases}$$

If an observer is moving like $\vec{\beta} = \beta\hat{z}$, then

$$E'_z = B'_z = 0,$$

Since $\vec{E}'_{\parallel} = \vec{E}_{\parallel}$ and $\vec{B}'_{\parallel} = \vec{B}_{\parallel}$.

Additionally,

$$\begin{aligned} \vec{E} &= \gamma(c\vec{\beta} \times \vec{B}), \\ &= -\gamma c \beta \hat{r} \begin{cases} \frac{r}{2} j & r \leq R \\ \frac{R^2}{2r} j & r \geq R \end{cases}, \\ \vec{B} &= \gamma \vec{B}. \end{aligned}$$

2. The current density is $\vec{j} = \omega s \rho_0 \hat{\varphi}$.

Ainsi the magnetic moment is

$$\begin{aligned} \vec{m} &= \frac{1}{2}\int \vec{r} \times \vec{j}\, d^3r, \\ &= \frac{1}{2}\hat{z}\int s^3 \omega \rho_0\, d^3r, \\ &= \underbrace{\frac{\pi\omega\rho_0 h R^4}{4}}_{=m}\hat{z}. \end{aligned}$$

If a magnetic dipole moment is placed far away from the cylindar, $\vec{m}_D = m_D\hat{z}$, (at $(0, 0, z_D > 0)$) then the energy of the system would be $-\vec{m}_D \cdot \vec{B}$ (ignoring angular momentum). Flipping the dipole ($\vec{m}_D \to -\vec{m}_D$) would increase the energy of the system since the moments are now antialigned. The change in potential energy is

$$\begin{aligned} \Delta U &= 2\vec{m}_D \cdot \vec{B}, \\ &= \frac{\mu_0}{\pi}\frac{m m_D}{z_D^3}. \end{aligned}$$