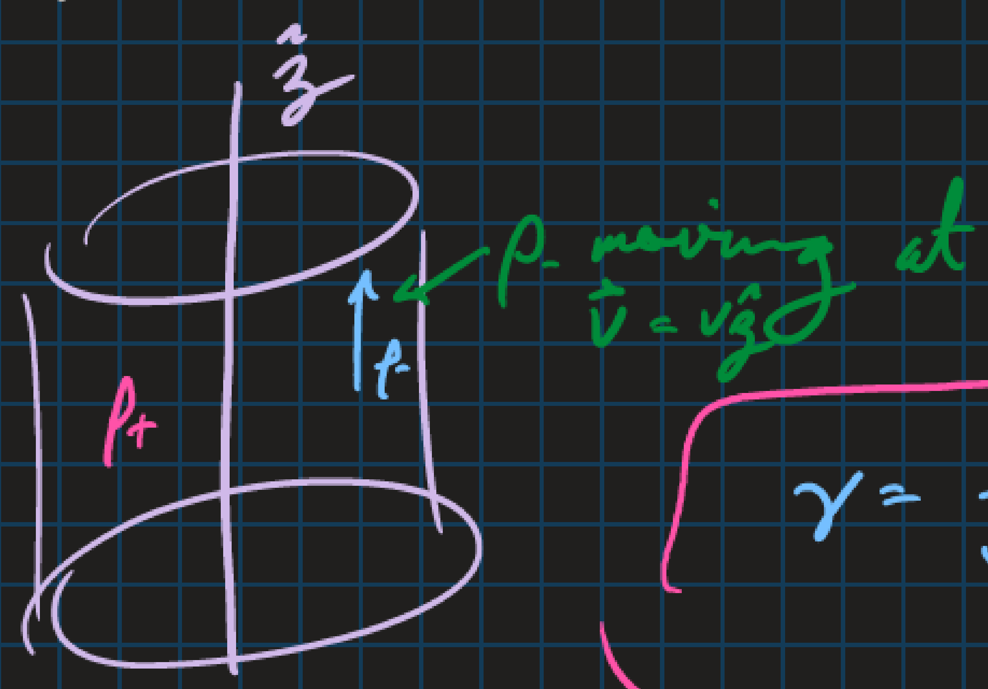
Final p01

jeudi 29 avril 2021 07:46



$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \vec{\beta} = \frac{\vec{v}}{c} = \frac{v}{c}\hat{z}}$$

$$\int d\vec{l} \cdot \vec{B} = \int d\vec{A} \cdot \rho_- \vec{v}$$

$$\int dl\, B = \int dA\, \rho_- v$$

$$\int_0^{2\pi s} B\, dl = 2\pi \int_0^s \rho_- v\, r\, dr$$

$$2\pi s B = \pi \rho_- v s^2$$

$$\vec{B} = \frac{\rho_- v s}{2} \hat{\varphi}$$

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} = \frac{\rho_+ + \rho_-}{2\pi\varepsilon_0} s \hat{s}$$

$$\vec{E}'_{\perp} = \gamma (\vec{E}_{\perp} + c\vec{\beta} \times \vec{B})$$

$$= \gamma (\vec{v} \times \vec{B})$$

$$\boxed{= -\gamma\, v B\, \hat{s}}$$

$$\boxed{\vec{B}'_{\parallel} = \vec{B}_{\parallel} = 0}$$

$$\vec{B}'_{\perp} = \gamma (\vec{B}_{\perp} - \tfrac{1}{c} \vec{\beta} \times \vec{E})$$

$$\boxed{= \gamma \vec{B}}$$

probably ignore all this...

$$\boxed{\begin{aligned} B_s &= 0 \\ E_s &= \frac{\rho_+ + \rho_-}{2\pi\varepsilon_0} s \end{aligned}}$$

If q is moving with the negative charges and is under no radial force

$$q v B_\varphi = q E_s$$

$$\cancel{q} \frac{v^2 \rho_- \cancel{s}}{2} = \cancel{q} \frac{\rho_+ + \rho_-}{2\pi\varepsilon_0} \cancel{s}$$

$$\boxed{2\pi\varepsilon_0 \rho_- \left(\frac{v^2}{2} - \frac{1}{2\pi\varepsilon_0} \right) = \rho_+}$$

Final p02

jeudi 29 avril 2021 08:17

$$\sigma = \sigma_0 \left(1 - \tfrac{3}{2}(1-\cos^2\vartheta)\right)$$
$$= \sigma_0 \left(-\tfrac{1}{2} + \tfrac{3}{2}\cos^2\vartheta\right)$$
$$= \frac{\sigma_0}{2}\left(-1+3\cos^2\vartheta\right) = \sigma_0 P_2(\cos\vartheta)$$

$$\phi = \sum_{l=0}^{\infty} \left(a_l r^l + b_l r^{-l-1}\right) P_l(\cos\vartheta)$$

at $r=0$ ϕ is finite,
as $r\to\infty$ $\phi\to 0$, and
at $r=R$ $\frac{\partial\phi_{in}}{\partial r} - \frac{\partial\phi_{out}}{\partial r} = -\frac{\sigma_0}{\varepsilon_0} P_2(\cos\vartheta)$

$$\phi_{in} = \sum_{l=0}^{\infty} a_l r^l P_l(\cos\vartheta)$$
$$\phi_{out} = \sum_{l=0}^{\infty} b_l r^{-l-1} P_l(\cos\vartheta)$$

$$\frac{\partial\phi_{in}}{\partial r} = \sum_{l=0}^{\infty} l\, a_l r^{l-1} P_l(\cos\vartheta)$$

$$\frac{\partial\phi_{out}}{\partial r} = \sum_{l=0}^{\infty} (-l-1)\, b_l r^{-l-2} P_l(\cos\vartheta)$$

$$\left.\frac{\partial\phi_{out}}{\partial r}\right|_{r=R} - \left.\frac{\partial\phi_{in}}{\partial r}\right|_{r=R} = \sum_{l=0}^{\infty}\left[-(l+1)\, b_l R^{-l-2} - l\, a_l R^{l-1}\right] P_l(\cos\vartheta) = \frac{\sigma_0}{\varepsilon_0} P_2(\cos\vartheta)$$

$$a_l = b_l = 0 \quad \text{for } l \neq 2$$

$$\underbrace{\left[-3 b_2 R^{-4} - 2 a_2 R\right]}_{=\frac{\sigma_0}{\varepsilon_0}} P_2(\cos\vartheta) = \frac{\sigma_0}{\varepsilon_0} P_2(\cos\vartheta)$$

$$b_2 = -3R^4\left(\frac{\sigma_0}{\varepsilon_0} + 2 a_2 R\right)$$

Final p03

jeudi 29 avril 2021 08:33

If $\Delta t < 10^6 \, m/c$, then the two events can switch order in time.

Final p04

jeudi 29 avril 2021 08:36

Left figure: there is no dipole moment.

$$\vec{m} = \lambda_0 \vec{\Omega}(t) R^2$$

$$\vec{m} = \frac{1}{2}\int \vec{r} \times \vec{j}\, d^3r \qquad \vec{j}(\vec{r}) = \Omega(t)\, r \left(\lambda_0 \delta(z)\, \delta(r-R)\right) \hat{\varphi}$$

$$= \frac{1}{2}\int_0^\infty \int_{-\infty}^\infty \int_0^{2\pi} \vec{r}' \times \vec{j}(\vec{r}')\, r\, d\varphi\, dz\, dr$$

$$= \frac{1}{2}\int_0^\infty \int_{-\infty}^\infty \int_0^{2\pi} r^3\, \Omega(t) \left(\lambda_0 \delta(z)\, \delta(r-R)\right) \hat{z}\, d\varphi\, dz\, dr$$

$$= \pi R^3 \Omega(t) \lambda_0 \hat{z}$$

$$\ddot{\vec{m}} = \pi R^3 \Omega''(t) \lambda_0 \hat{z} \qquad \Omega''(t) = -\omega^2 \Omega(t)$$

$$P(t) = \frac{\mu_0}{4\pi c}\left(\frac{2}{3}|\ddot{\vec{m}}|^2\right)$$

$$= \frac{\mu_0}{4\pi c}\left(\frac{2}{3}\left(\pi R^3 \omega^2 \Omega(t) \lambda_0\right)^2\right)$$

$$= \frac{\mu_0 \pi}{2c} R^6 \omega^4 \Omega_0^2 \cos^2(\omega t) \lambda_0^2$$

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} P(t)\, dt = \boxed{\frac{\mu_0 \pi^2}{4c} \omega^4 \lambda_0^2 R^6 \Omega_0^2}$$

$$\frac{1}{\tau}\int \cos^2(\omega t)\, dt$$
$$= \frac{1}{2\tau}\int 1 + \cos 2\omega t\, dt$$
$$= \frac{1}{2\tau}\left(t + \frac{\sin 2\omega t}{2\omega}\right)\Big|_0^{2\pi/\omega}$$
$$= \frac{\omega}{2\pi}\, \frac{1}{2}\, \frac{2\pi}{\omega} = \frac{1}{2}$$

$$\frac{dP}{d\Omega} = \frac{c}{\mu_0}|B|^2 r^2$$

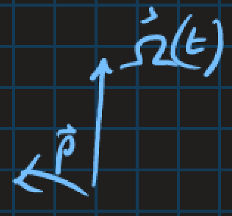
$$= \frac{c}{\mu_0}\frac{\mu_0^2}{16\pi^2 c^2}\frac{1}{r^2 c^2}|\ddot{\vec{m}}|^2 r^2$$

$$= \frac{\mu_0}{16\pi^2 c^3}\left(\pi R^3 \omega^2 \Omega(t) \lambda_0\right)^2$$

Comparing the left figure to the right one, instead of having no electric dipole and having a magnetic moment, there is no magnetic moment and there is an electric dipole. With this change, however, the solution remains the same, since the factor of c involved in this change gets cancelled out.

total power radiated

The pattern of radiation changes, though. In the left case, $\ddot{\vec{m}}$ is in the $\hat{z}$ direction, and in the right case $\ddot{\vec{p}}$ is in the $\hat{\varphi}$ direction. If the radiation is most intense perpendicular to these, then there is more energy radiated in the $\hat{z}$ direction for the right case than for the left case. However, in both cases, the radial direction is perpendicular so no change is seen there. In particular, the radiation in the $\hat{x}$ direction doesn't change.



$\vartheta = \Omega(t)\, t + \delta$

not important

$$\vec{p}(t) = p_0 \begin{pmatrix} \cos(\vartheta) \\ \sin(\vartheta) \\ 0 \end{pmatrix} \qquad \vec{\Omega}(t) = \Omega(t)\hat{z}$$

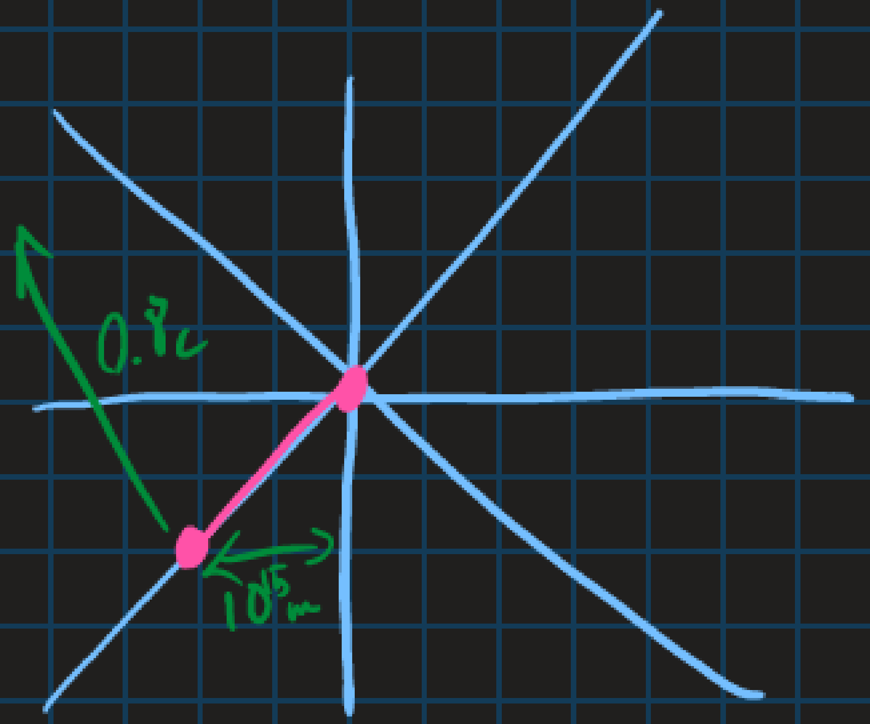
$$\dot{\vec{p}}(t) = p_0 \begin{pmatrix} -\sin(\vartheta) \\ \cos(\vartheta) \\ 0 \end{pmatrix} \dot{\vartheta}$$

$$\ddot{\vec{p}}(t) = p_0\left[\begin{pmatrix} -\cos\vartheta \\ -\sin\vartheta \\ 0 \end{pmatrix} \underbrace{\dot{\vartheta}}_{\Omega(t)} + \begin{pmatrix} -\sin\vartheta \\ \cos\vartheta \\ 0 \end{pmatrix} \underbrace{\ddot{\vartheta}}_{\dot{\Omega}(t)}\right]$$

so this is in the $\hat{\varphi}$ direction.

Final p05

jeudi 29 avril 2021 08:36



$\Delta t_0 = 10^{15}\text{m}/c$

$\Delta t_s = \gamma \Delta t_0$

$\gamma = \frac{1}{\sqrt{1-0.8^2}} = \frac{10}{6}$

$$\boxed{\Delta t_s = \frac{10^{16}\text{m}}{6c}}$$

$\lambda_0 = 666\text{nm}$

$\lambda_s = \lambda_0/\gamma$

$$\boxed{\lambda_s = \frac{6 \cdot 666\text{nm}}{10}}$$

Final p06

jeudi 29 avril 2021 08:36

Yes it is possible. Propogating EM fields carry momentum. The momentum flux through the area, A, has to be non-zero, so the EM fields need to radiate energy out that area.

$$\boxed{\int_A \varepsilon_0 \vec{E} \times \vec{B} \cdot d\vec{A} > 0}$$