Homework 02

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1.10

1.10.1

$$|45\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle + i\,|L\rangle\right), \tag{1.10.1}$$
$$|135\rangle = \frac{1}{\sqrt{2}}\left(-\,|R\rangle + i\,|L\rangle\right). \tag{1.10.2}$$

1.10.2

$$\frac{e^{-i\pi/4}}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \tag{1.10.3}$$

The factor of $e^{-i\pi/4}$ is there to rotate the state into a nicer (but identical) form, since relative phase is all that matters.

1.10.3

For a unitary transformation, the adjoint is equal to the inverse.

$$\begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{1.10.4}$$

Since it has been shown that the adjoint is the transformation's inverse, the transformation is unitary.

1.11

1.11.1

$$P_x = |X\rangle\langle X| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{1.11.1}$$

1.11.2

The eigenvalues of P_x are

$$\lambda = 1, 0. \tag{1.11.2}$$

The eigenstates are

$$|\lambda\rangle = |X\rangle\,, |Y\rangle\,, \tag{1.11.3}$$

respectively.

1.11.3

In the RL basis, P_x is given by

$$P_x = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}(\mathbb{1} + \sigma_1). \tag{1.11.4}$$

From this, it's easy to see that the eigenvalues are, again,

$$\lambda = 1, 0. \tag{1.11.5}$$

1.12

1.12.1

$$\begin{aligned} \mathrm{Tr}\left(U^\dagger A U\right) &= U^*_{ij} A_{jk} U_{ki}, && (1.12.1) \\ &= U^*_{ij} U_{ki} A_{jk}, && (1.12.2) \\ &= \delta_{jk} A_{jk}, && (1.12.3) \\ &= A_{jj} = \mathrm{Tr}(A). && (1.12.4) \end{aligned}$$

1.12.2

$$\begin{aligned}
\text{Tr}(AB) &= A_{ij}B_{ji}, && (1.12.5)\\
&= B_{ji}A_{ij}, && (1.12.6)\\
&= \text{Tr}(BA). && (1.12.7)
\end{aligned}$$

1.13

Let $\phi(z,t)$ be the operator

$$\phi(z,t) = e^{-i\omega t}\begin{pmatrix} e^{ik_x z} & 0 \\ 0 & e^{-ik_y z} \end{pmatrix}, \tag{1.13.1}$$

where z is the distance into the crystal. Then

$$\begin{aligned}
\phi(z,t)\left|\psi\right\rangle &= \frac{1}{\sqrt{2}}e^{-i\omega t}\begin{pmatrix} e^{ik_x z} \\ e^{ik_y z} \end{pmatrix}, && (1.13.2)\\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(k_y-k_x)z} \end{pmatrix} && (1.13.3)
\end{aligned}$$

is the polarization of the photon at a distance z inside the crystal. If the photon comes out as righthand circularly polarized, then

$$e^{i(k_y-k_x)z} = i, \tag{1.13.4}$$

or

$$(k_y-k_x)z = \frac{\pi}{2}. \tag{1.13.5}$$

Thus

$$z = \frac{c}{4|n_y-n_x|\nu} = \tag{1.13.6}$$

1.14

1.15

1.15.1

1.15.2

1.15.3

1.15.4

1.17

1.17.1

1.17.2

1.17.3

1.17.4

1.18

2.1

2.2

2.3