Problem Set 2

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I PAULI MATRICES

A

The proof is tedious but trivial.

B

The eigenvalues for all of the following are ± 1.

$$\left|S_x, \pm\right\rangle = \frac{1}{\sqrt{2}}\big(\left|-\right\rangle \pm \left|+\right\rangle\big). \tag{I.1}$$

$$\left|S_y, \pm\right\rangle = \frac{1}{\sqrt{2}}\big(\left|-\right\rangle \pm i\left|+\right\rangle\big). \tag{I.2}$$

$$\left|S_z, \pm\right\rangle = \left|\pm\right\rangle. \tag{I.3}$$

C

The proof is tedious but trivial.

II SAKURAI 1.4

A

Since all Pauli matrices are traceless, $\mathrm{tr}\{X\} = 2a_0$. Additionally, $\mathrm{tr}\{\sigma_k X\} = 0 + \sum_{i=1}^{3} \sigma_k \sigma_i = 2a_k$.

B

$$a_0 = \frac{1}{2}\left(X_{00} + X_{11}\right), \tag{II.1}$$

$$a_1 = \frac{1}{2}\left(X_{10} + X_{01}\right), \tag{II.2}$$

$$a_2 = -\frac{i}{2}\left(X_{10} - X_{01}\right), \tag{II.3}$$

$$a_3 = \frac{1}{2}\left(X_{00} - X_{11}\right). \tag{II.4}$$

III SAKURAI 1.10

Since $\sigma_i\sigma_j = 1$ for $i = j$ and $\sigma_i\sigma_j = -\sigma_j\sigma_i$ for $i \neq j$:

$$\big\{\sigma_i, \sigma_j\big\} = 2\delta_{ij}. \tag{III.1}$$

$$\big[\sigma_i, \sigma_j\big] = 0. \quad (i = j) \tag{III.2}$$

$$\tag{III.3}$$

Since $-i = \sigma_3\sigma_2\sigma_1$,

$$\begin{aligned} \big[\sigma_i, \sigma_j\big] &= 2\sigma_i\sigma_j, && \text{(III.4)}\\ &= 2i\left[\sigma_i\sigma_j\sigma_k\sigma_3\sigma_2\sigma_1\right]\sigma_k, && \text{(III.5)}\\ &= 2i\epsilon_{ijk}\sigma_k. \quad (i \neq j \neq k) && \text{(III.6)} \end{aligned}$$

Since $S_k = \frac{\hbar}{2}\sigma_k$, $\big[S_i, S_j\big] = \frac{\hbar^2}{4}\big[\sigma_i, \sigma_j\big]$, and the same for the anticommutator:

$$\therefore \big[S_i, S_j\big] = \hbar i\epsilon_{ijk}S_k. \tag{III.7}$$

$$\big\{S_i, S_j\big\} = \frac{\hbar^2}{2}\delta_{ij}. \tag{III.8}$$

IV SAKURAI 1.11

Since $n = n^k\sigma_k$ is a unit vector,

$$n = \sin\theta(\cos\phi\sigma_1 + \sin\phi\sigma_2) + \cos\theta\sigma_3. \tag{IV.1}$$

Additionally

$$\langle n\rangle = n^k\left\langle\psi\middle|\sigma_k\middle|\psi\right\rangle. \tag{IV.2}$$