Homework 04

Brandon Henke
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Scott Pratt

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2.13

2.13.1

$$
\begin{aligned}
a(t) &= U^\dagger(t) a U(t), && (2.13.1)\\
\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{i}{\hbar} U^\dagger(t) \big[H(t), a\big] U(t), && (2.13.2)\\
&= -i\omega U^\dagger(t) a U(t), && (2.13.3)\\
&= -i\omega a(t). && (2.13.4)\\
a(t) &= a_0 e^{-i\omega t}, && (2.13.5)\\
a^\dagger(t) &= a_0^\dagger e^{i\omega t}. && (2.13.6)
\end{aligned}
$$

2.13.2

$$
\begin{aligned}
\Big[a(t), a^\dagger(t')\Big] &= e^{i\omega(t'-t)}(a_0 a_0^\dagger - a_0^\dagger a_0), && (2.13.7)\\
&= e^{i\omega(t'-t)} \Big[a_0, a_0^\dagger\Big], && (2.13.8)\\
&= e^{i\omega(t'-t)}. && (2.13.9)
\end{aligned}
$$

2.14

$$
\begin{aligned}
\langle 0|\hat{x}(t)\hat{x}(t')|0\rangle &= \frac{\hbar}{2m\omega} \langle 0|(a(t) + a^\dagger(t))(a(t) + a^\dagger(t))|0\rangle, && (2.14.1)\\
&= \frac{\hbar}{2m\omega} \langle 0|a(t) a^\dagger(t)|0\rangle, && (2.14.2)\\
&= \frac{\hbar}{2m\omega} e^{i\omega(t'-t)}. && (2.14.3)
\end{aligned}
$$

2.15

$$\langle \mathbf{r}|\frac{1}{|\hat{p}|}|\mathbf{r}'\rangle = \int \frac{\mathrm{d}^3q'\,\mathrm{d}^3q}{(2\pi)^6}\,\langle \mathbf{r}|\mathbf{q}\rangle\,\langle \mathbf{q}|\frac{1}{\hat{p}}|\mathbf{q}'\rangle\,\langle \mathbf{q}'|\mathbf{r}'\rangle\,, \tag{2.15.1}$$

$$= \int \frac{\mathrm{d}^3q\,\mathrm{d}^3q'}{(2\pi)^3} e^{\frac{i}{\hbar}(\mathbf{r}\cdot\mathbf{q}-\mathbf{r}'\cdot\mathbf{q}')}\frac{1}{\hbar|q'|}\delta^3(q-q'), \tag{2.15.2}$$

$$= -\frac{1}{2\pi^2\hbar|r-r'|^2}. \tag{2.15.3}$$

3.1

Let Λ be such that $\boldsymbol{\nabla}\cdot\mathbf{A}=0$.

$$\frac{\partial\rho}{\partial t} = \left(\frac{\partial}{\partial t}\psi^*\psi + \psi^*\frac{\partial}{\partial t}\psi\right), \tag{3.1.1}$$

$$= \frac{i}{\hbar}\left(-H\psi^*\psi + \psi^* H\psi\right), \tag{3.1.2}$$

$$= \frac{i}{2m\hbar}\left(-\hbar^2\nabla^2\psi^*\psi + \hbar^2\psi^*\nabla^2\psi + \frac{i\hbar e}{c}\mathbf{A}\cdot\boldsymbol{\nabla}\psi^*\psi + \frac{i\hbar e}{c}\psi^*\mathbf{A}\cdot\boldsymbol{\nabla}\psi\right), \tag{3.1.3}$$

$$= \frac{i\hbar}{2m}\left(\psi^*\nabla^2\psi - \nabla^2\psi^*\psi\right) + \frac{e}{mc}\mathbf{A}\cdot\left(\boldsymbol{\nabla}\psi^*\psi + \psi^*\boldsymbol{\nabla}\psi\right). \tag{3.1.4}$$

$$\boldsymbol{\nabla}\cdot\mathbf{j} = \boldsymbol{\nabla}\cdot\left(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^*\psi\right) - \frac{e}{mc}\boldsymbol{\nabla}\cdot(\mathbf{A}\rho), \tag{3.1.5}$$

$$= \frac{i\hbar}{2m}\left(\psi^*\nabla^2\psi - \nabla^2\psi^*\psi\right) - \frac{e}{mc}\mathbf{A}\cdot\left(\boldsymbol{\nabla}\psi^*\psi + \psi^*\boldsymbol{\nabla}\psi\right). \tag{3.1.6}$$

Since 3.1.4 and 3.1.6 are negatives of each other, the continuity equation holds.

3.2

3.2.1

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial\mathbf{A}}{\partial t}, \tag{3.2.1}$$

$$= -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{\nabla}\Lambda + \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{\nabla}\Lambda, \tag{3.2.2}$$

$$= 0. \tag{3.2.3}$$

$$B = \boldsymbol{\nabla}\wedge\mathbf{A}, \tag{3.2.4}$$

$$= -\boldsymbol{\nabla}\wedge\boldsymbol{\nabla}\Lambda, \tag{3.2.5}$$

$$= 0. \tag{3.2.6}$$

3.2.2

3.3

3.4