

# Problem Set 1

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## I THE BOHR ATOM

### A

$$m\ddot{r} = f = -\frac{ke^2}{r^2} = -\frac{n\tilde{h}}{r^2}\dot{r}, \quad (\text{I.1})$$

$$\dot{r}_n = \frac{ke^2}{n\tilde{h}}. \quad (\text{I.2})$$

$$r_n = \frac{n^2\tilde{h}^2}{ke^2m}. \quad (\text{I.3})$$

$$E_n = \frac{1}{2}m\dot{r}_n^2 - \frac{ke^2}{r_n}, \quad (\text{I.4})$$

$$= -\frac{mk^2e^4}{2n^2\tilde{h}^2}. \quad (\text{I.5})$$

$$\nu = \frac{E_n - E_{n'}}{h}, \quad (\text{I.6})$$

$$= \frac{(n')^2 - n^2}{(nn')^2} \frac{mk^2e^4}{2h\tilde{h}}. \quad (\text{I.7})$$

### B

For  $n \gg 1$  and  $n' = n - 1$ :

$$\nu = \frac{\dot{r}}{2\pi r} = \frac{k^2e^4m}{2\pi(n\tilde{h})^3} \approx \frac{2k^2e^4m}{2n^3\tilde{h}^2}, \quad (\text{I.8})$$

$$\tilde{h} = \frac{h}{2\pi}. \quad (\text{I.9})$$

## II SIMPLE WAVE FUNCTION PROBLEM

### A

$$1 = A^2 \int_0^\infty e^{-2x/\lambda} dx, \quad (\text{II.1})$$

$$= A^2 \frac{\lambda}{2}, \quad (\text{II.2})$$

$$A = \sqrt{\frac{2}{\lambda}}. \quad (\text{II.3})$$

### B

$$P(\in [0, \lambda]) = \frac{2}{\lambda} \int_0^\lambda e^{-2x/\lambda} dx, \quad (\text{II.4})$$

$$= 1 - \frac{1}{e^2}. \quad (\text{II.5})$$

## III WAVE MECHANICS PROBLEM

### A

$$-\frac{\hbar}{2m}\nabla^2\psi = E\psi, \quad (\text{III.1})$$

$$\therefore \psi_n(x) = A_n \cos(\alpha_n x) + B_n \sin(\alpha_n x). \quad (\text{III.2})$$

$$\left( \alpha_n = \sqrt{\frac{2E_n m}{\hbar}} \right) \quad (\text{III.3})$$

$$\psi(0) = \psi(L) = 0, \quad (\text{III.4})$$

$$\therefore A_n = 0, \quad (\text{III.5})$$

$$E_n = \frac{n^2\pi^2\hbar}{2mL^2}. \quad (\text{III.6})$$

$$1 = \int_0^L B_n^2 \sin^2(\alpha x) dx, \quad (\text{III.7})$$

$$= \frac{B_n^2}{2}(L), \quad (\text{III.8})$$

$$B_n = \sqrt{\frac{2}{L}}. \quad (\text{III.9})$$

$$\therefore \psi_n(x) = \sqrt{\frac{2}{L}} \sin(\alpha x). \quad \left( \alpha_n = \sqrt{\frac{2E_n m}{\hbar}} \right) \quad (\text{III.10})$$

### B

$$1 = \int_0^L N^2 x^2 (L-x)^2 dx, \quad (\text{III.11})$$

$$\rightarrow N = \sqrt{\frac{30}{L^5}}. \quad (\text{III.12})$$

$$P(E = E_1) = \int_0^L \sqrt{\frac{60}{L^6}} x(L-x) \sin\left(\frac{\pi x}{L}\right) dx, \quad (\text{III.13})$$

$$P(E = E_1) = \frac{8\sqrt{15}}{\pi^3}. \quad (\text{III.14})$$

## IV EIGENVALUES AND EIGENVECTORS

$$A = \begin{pmatrix} 0 & a & 0 \\ a & 0 & a \\ 0 & a & 0 \end{pmatrix}. \quad (\text{IV.1})$$

$$\det(\lambda \mathbb{1} - A) = 0 = \lambda(\lambda^2 - 2a^2), \quad (\text{IV.2})$$

$$\lambda \in \{0, \pm\sqrt{2}\}. \quad (\text{IV.3})$$

$$(\lambda \mathbb{1} - A)(v) = \mathbf{0} = \begin{pmatrix} \lambda & -a & 0 \\ -a & \lambda & -a \\ 0 & -a & \lambda \end{pmatrix} \begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} \quad (\text{IV.4})$$

$$v^1 + v^3 = \frac{\lambda}{2}v^2, \quad (\text{IV.5})$$

$$\text{or } v^2 = \frac{\lambda}{2}v^1 = \frac{\lambda}{2}v^3 \quad (\text{IV.6})$$

$$v \in \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1 \\ \pm\sqrt{2} \\ 1 \end{pmatrix} \right\} \quad (\text{IV.7})$$