Physics 851 **Problem Set 4** **Fall 2022**

NOTE: All problem numbers from Sakurai correspond to the 3^{rd} Edition.

1. For position and momentum operators X and P, using the commutation relation, $[X, P] = i\hbar$, we can prove $[P, F(X)] = -i\hbar \frac{dF}{dX}$, for some function $F(X)$ in the following steps:

   (a) Prove/verify $[P, X^n] = -i\hbar X^{n-1} + X[P, X^{n-1}]$.

   (b) Use the result of part (a) and induction to prove $[P, X^n] = -i\hbar n X^{n-1}$.

   (c) Use the result of part (b) to prove $[P, F(X)] = -i\hbar \frac{dF}{dX}$.

   (d) In a similar manner, prove $[X, G(P)] = i\hbar \frac{dG}{dP}$, for some function $G(P)$.

2. Sakurai 1.19

   [Hint: Consider the state $[A_1, A_2]|E\rangle$, where $|E\rangle$ is an eigenstate of H.]

3. Sakurai 1.23

   [Work in position-space, using the eigenfunctions $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$, for $n = 1, 2, \ldots$. Remember, in position-space the momentum operator acts on wave-functions by $P|\psi\rangle \doteq -i\hbar \frac{d\psi}{dx}$. Any method to obtain the integrals is fine, but obtain the results for arbitrary n.]

4. Sakurai 1.32

   [In part (b), the expectation value is taken in the transformed state.]