Problem Set 1

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I THE BOHR ATOM

A

$$m\ddot{r} = f = -\frac{ke^2}{r^2} = -\frac{n\tilde{h}}{r^2}\dot{r}, \tag{I.1}$$

$$\boxed{\dot{r}_n = \frac{ke^2}{n\tilde{h}}.} \tag{I.2}$$

$$\boxed{r_n = \frac{n^2\tilde{h}^2}{ke^2 m}.} \tag{I.3}$$

$$E_n = \frac{1}{2}m\dot{r}_n^2 - \frac{ke^2}{r_n}, \tag{I.4}$$

$$\boxed{= -\frac{mk^2e^4}{2n^2\tilde{h}^2}.} \tag{I.5}$$

$$\nu = \frac{E_n - E_{n'}}{h}, \tag{I.6}$$

$$\boxed{= \frac{(n')^2 - n^2}{(nn')^2}\frac{mk^2e^4}{2h\tilde{h}}.} \tag{I.7}$$

B

For $n \gg 1$ and $n' = n - 1$:

$$\nu = \frac{\dot{r}}{2\pi r} = \frac{k^2e^4m}{2\pi(n\tilde{h})^3} \approx \frac{2k^2e^4m}{2n^3\tilde{h}^2}, \tag{I.8}$$

$$\boxed{\tilde{h} = \frac{h}{2\pi}.} \tag{I.9}$$

II SIMPLE WAVE FUNCTION PROBLEM

A

$$1 = A^2\int_0^\infty e^{-2x/\lambda}\,\mathrm{d}x\,, \tag{II.1}$$

$$= A^2\frac{\lambda}{2}, \tag{II.2}$$

$$\boxed{A = \sqrt{\frac{2}{\lambda}}.} \tag{II.3}$$

B

$$P(\in[0,\lambda]) = \frac{2}{\lambda}\int_0^\lambda e^{-2x/\lambda}\,\mathrm{d}x\,, \tag{II.4}$$

$$\boxed{= 1 - \frac{1}{e^2}.} \tag{II.5}$$

III WAVE MECHANICS PROBLEM

A

$$-\frac{\hbar}{2m}\nabla^2\psi = E\psi, \tag{III.1}$$

$$\therefore \psi_n(x) = A_n\cos(\alpha_n x) + B_n\sin(\alpha_n x). \tag{III.2}$$

$$\left(\alpha_n = \sqrt{\frac{2E_n m}{\hbar}}\right) \tag{III.3}$$

$$\psi(0) = \psi(L) = 0, \tag{III.4}$$

$$\therefore A_n = 0, \tag{III.5}$$

$$E_n = \frac{n^2\pi^2\hbar}{2mL^2}. \tag{III.6}$$

$$1 = \int_0^L B_n^2\sin^2(\alpha x)\,\mathrm{d}x\,, \tag{III.7}$$

$$= \frac{B_n^2}{2}(L)\,, \tag{III.8}$$

$$B_n = \sqrt{\frac{2}{L}}. \tag{III.9}$$

$$\boxed{\therefore \psi_n(x) = \sqrt{\frac{2}{L}}\sin(\alpha x). \quad \left(\alpha_n = \sqrt{\frac{2E_n m}{\hbar}}\right)} \tag{III.10}$$

B

$$1 = \int_0^L N^2x^2(L-x)^2\,\mathrm{d}x\,, \tag{III.11}$$

$$\to N = \sqrt{\frac{30}{L^5}}. \tag{III.12}$$

$$P(E = E_1) = \int_0^L \sqrt{\frac{60}{L^6}}x(L-x)\sin\left(\frac{\pi x}{L}\right)\mathrm{d}x\,, \tag{III.13}$$

$$\boxed{P(E = E_1) = \frac{8\sqrt{15}}{\pi^3}.} \tag{III.14}$$

IV EIGENVALUES AND EIGENVECTORS

$$A = \begin{pmatrix} 0 & a & 0 \\ a & 0 & a \\ 0 & a & 0 \end{pmatrix}. \tag{IV.1}$$

$$det(\lambda\mathbb{1} - A) = 0 = \lambda(\lambda^2 - 2a^2), \tag{IV.2}$$

$$\boxed{\lambda \in \{0, \pm\sqrt{2}\}.} \tag{IV.3}$$

$$(\lambda \mathbb{1} - A)(v) = \mathbf{0} = \begin{pmatrix} \lambda & -a & 0 \\ -a & \lambda & -a \\ 0 & -a & \lambda \end{pmatrix} \begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} \tag{IV.4}$$

$$v^1 + v^3 = \frac{\lambda}{2} v^2, \tag{IV.5}$$

$$\text{or} \quad v^2 = \frac{\lambda}{2} v^1 = \frac{\lambda}{2} v^3 \tag{IV.6}$$

$$\boxed{v \in \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1 \\ \pm\sqrt{2} \\ 1 \end{pmatrix} \right\}} \tag{IV.7}$$