# Problem Set 9

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Last updated: April 20, 2023

## I SAKURAI 6.3

$$q = 2k\sin\left(\frac{\theta}{2}\right), \tag{I.1}$$
$$800\text{MeV} = \frac{\hbar^2 k^2}{2m}, \qquad \text{(from figure)} \tag{I.2}$$
$$k = 6.11\text{fm}^{-1}. \tag{I.3}$$
$$qa = (4.49, 7.73, 10.9, ...), \tag{I.4}$$
$$\theta = (7.7, 14.25, 20.95)^\circ, \tag{I.5}$$
$$a = (5.47, 5.10, 4.91)\text{fm}. \tag{I.6}$$
$$1.4A^{1/3} = 4.79\text{fm}. \tag{I.7}$$

My estimates are a bit large.

## II SCATTERING OFF OF A SPHERICALLY-SYMMETRIC POTENTIAL

### A

$$f = -\frac{m}{2\pi\hbar^2}\int \mathrm{d}^3x\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} V_0 e^{-\frac{r^2}{2a^2}}, \tag{II.1}$$
$$= -\frac{mV_0}{2\pi\hbar^2}\left(\int \mathrm{d}x\, e^{-iq_x x} e^{-\frac{x^2}{2a^2}}\right)^3, \tag{II.2}$$
$$= -\frac{\sqrt{2\pi}a^3 m V_0}{\hbar^2} e^{-3a^2q^2/2}. \tag{II.3}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f|^2, \tag{II.4}$$
$$= \frac{2\pi a^6 m^2 V_0^2}{\hbar^4} e^{-3a^2q^2}. \tag{II.5}$$

### B

$$\sigma = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,\mathrm{d}\Omega\,, \tag{II.6}$$
$$= \int_0^{2\pi}\mathrm{d}\phi \int_{-1}^{1}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,\mathrm{d}\cos\theta\,, \tag{II.7}$$
$$= \frac{4\pi^2 a^6 m^2 V_0^2}{\hbar^4} e^{-3a^2}\int_{-1}^{1} e^{q^2}\,\mathrm{d}\cos\theta\,, \tag{II.8}$$
$$= \frac{4\pi^2 a^6 m^2 V_0^2}{\hbar^4}\int_{-1}^{1} e^{-6a^2k^2(1-\cos\theta)}\,\mathrm{d}\cos\theta\,, \tag{II.9}$$
$$= \frac{2\pi^2 a^4 m^2 V_0^2(1-e^{-12a^2k^2})}{3k^2\hbar^4}. \tag{II.10}$$

## III SCATTERING OFF OF A "SEPARABLE POTENTIAL"

### A

$$T = |v\rangle\langle v|\,(1 + \langle v|G_0|v\rangle + \langle v|G_0|v\rangle^2 + \dots), \tag{III.1}$$
$$= |v\rangle\langle v|\sum_{n=0}^{\infty}\langle v|G_0|v\rangle^n\,, \tag{III.2}$$
$$= \frac{|v\rangle\langle v|}{1 - \langle v|G_0|v\rangle}, \tag{III.3}$$
$$\langle v|G_0|v\rangle = \frac{2m}{\hbar^2}\int \mathrm{d}^3\boldsymbol{q}\,\frac{|v(\boldsymbol{k}')|^2}{k^2 - q^2 + i\epsilon}. \tag{III.4}$$
$$T(\boldsymbol{k}', \boldsymbol{k}) = \frac{v(\boldsymbol{k}')v^*(\boldsymbol{k})}{1 - \langle v|G_0|v\rangle}. \tag{III.5}$$

### B

The Fourier transform of a constant is a dirac delta function:

$$\langle \boldsymbol{r}'|V|\boldsymbol{r}\rangle = g\int \mathrm{d}^3\boldsymbol{k}'\,\mathrm{d}^3\boldsymbol{k}\,\frac{e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(2\pi)^3}, \tag{III.6}$$
$$= (2\pi)^3 g\delta(\boldsymbol{r})\delta(\boldsymbol{r}'). \tag{III.7}$$

### C

$$T(0,0) = a = \frac{g}{1 - \langle v|G_0|v\rangle}, \tag{III.8}$$
$$g = a(1 - \langle v|G_0|v\rangle), \tag{III.9}$$
$$= a\left(1 - \frac{2m}{\hbar^2}g\int_0^{\Lambda}\mathrm{d}^3\boldsymbol{q}\,\frac{1}{-q^2 + i\epsilon}\right), \tag{III.10}$$
$$= a\left(1 - \frac{8\pi m g}{\hbar^2}\Lambda\right), \tag{III.11}$$
$$g(\Lambda) = \frac{a}{1 - \frac{8\pi m a\Lambda}{\hbar^2}}. \tag{III.12}$$

### D

$$\frac{\mathrm{d}}{\mathrm{d}\Lambda}\frac{1}{T(\boldsymbol{k}', \boldsymbol{k})} = -\frac{8\pi m}{\hbar^2} + \frac{8\pi m}{\hbar^2}\frac{\mathrm{d}}{\mathrm{d}\Lambda}\int_0^{\Lambda}\frac{q^2\,\mathrm{d}q}{q^2 - k^2}, \tag{III.13}$$
$$= \frac{8\pi m}{\hbar^2}\frac{k^2}{\Lambda^2 - k^2}. \tag{III.14}$$

For $k \ll \Lambda$, this expression goes to zero.