Problem Set 2

Brandon Henke[1]

[1] *Michigan State University - Department of Physics & Astronomy*

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I SAKURAI 5.1

$$H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}\omega^2 m^2 \hat{x}^2, \tag{I.1}$$

$$H = H_0 + b\hat{x}. \tag{I.2}$$

$$\Delta_0 = \langle 0^{(0)}|b\hat{x}|0^{(0)}\rangle + \sum_{k\neq 0} \frac{2b^2 \left| \langle 1|x|0\rangle \right|^2}{\hbar\omega - (2k+1)\hbar\omega}, \tag{I.3}$$

$$= 0 + \sum_{k\neq 0} \frac{\hbar}{2m\omega} \frac{2b^2 \left| \langle k|1\rangle \right|^2}{\hbar\omega - (2k+1)\hbar\omega}, \tag{I.4}$$

$$= \frac{\hbar}{2m\omega} \frac{2b^2}{-2\hbar\omega}, \tag{I.5}$$

$$= -\frac{b^2}{2m\omega^2} \tag{I.6}$$

$$V(x) \to \frac{1}{2}m\omega^2 \left(x + \frac{b}{m\omega^2}\right)^2 - \frac{b^2}{2m\omega^2}. \tag{I.7}$$

$$x' = x + \frac{b}{m\omega^2}, \tag{I.8}$$

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}\omega^2 m^2 \hat{x}'^2 - \frac{b^2}{2m\omega^2}. \tag{I.9}$$

This is a simple harmonic oscillator with an energy shift of $-\frac{b^2}{2m\omega^2}$, so the perturbation theory result gives the exact energy shift.

II SAKURAI 5.7

A

The energy for the 2D Simple Harmonic Oscillator are

$$E_{n_x,n_y} = \hbar\omega \left(n_x + n_y + 1\right). \tag{II.1}$$

Hence, the three lowest lying states have energies of $E \in \{\hbar\omega, 2\hbar\omega, 2\hbar\omega\}$. The degeneracy for each energy level, $n = n_x + n_y$, is $\binom{n+1}{1} = n + 1$.

B

C

III SECOND ORDER GROUND STATE CORRECTION

IV SAKURAI 5.16