# Problem Set 2

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## I  PERTURBATIVE HYDROGEN FINE STRUCTURE

### A

### B

### C

## II  SAKURAI 5.6

The energy eigenstates of the infinite square well in two dimensions are:

$$\psi(\mathbf{r}) = \frac{2}{L} \sin\left(x \frac{n_x \pi}{L}\right) \sin\left(y \frac{n_y \pi}{L}\right). \tag{II.1}$$

The energies of these states are

$$E_{n_x,n_y} = \frac{\hbar^2}{2m} \left(\frac{(n_x + n_y)\pi}{L}\right)^2 \tag{II.2}$$

For a total $N = n_x + n_y$, there are $N + 1$ states with the same energy. The ground state is $n_x = n_y = 1$, and the first excited state is $n_x = 2$ and $n_y = 1$ or vice versa. Denote the energy states as $|n_x, n_y\rangle$.

$$\langle 1,1|\hat{V}_1|1,1\rangle = \lambda \langle 1,1|\hat{x}\hat{y}|1,1\rangle, \tag{II.3}$$

$$= \lambda \frac{L^2}{4}. \tag{II.4}$$

$$V = \lambda \begin{pmatrix} \langle 1,2|xy|1,2\rangle & \langle 1,2|xy|2,1\rangle \\ \langle 2,1|xy|1,2\rangle & \langle 2,1|xy|2,1\rangle \end{pmatrix}, \tag{II.5}$$

$$= \frac{\lambda L^2}{4\pi^4} \begin{pmatrix} \pi^4 & 1024/81 \\ 1024/81 & \pi^4 \end{pmatrix}, \tag{II.6}$$

$$\therefore \Delta_{1\pm}^{(1)} = \frac{1}{81}(81\pi^4 \pm 1024). \tag{II.7}$$

The eigenvectors (zeroth-order energy eigenkets) are

$$|\Delta_{1\pm}^{(1)}\rangle = \frac{1}{\sqrt{2}} \left(|1,2\rangle \pm |2,1\rangle\right). \tag{II.8}$$

## III  SAKURAI 5.19

$$H = \begin{pmatrix} E_2 + h\delta & 3ea_0\mathcal{E} \\ 3ea_0\mathcal{E} & E_2 \end{pmatrix}. \tag{III.1}$$

The eigenvalues, $E$, are

$$E_\pm = -\frac{h\delta}{2} \pm \frac{1}{2}\sqrt{h^2\delta^2 + 36e^2\mathcal{E}^2 a_0^2} + E_2. \tag{III.2}$$

Taking the limit where $\delta \ll 3ea_0\mathcal{E}$,

$$E_\pm \approx -\frac{h\delta}{2} \pm 3ea_0\mathcal{E} + E_2, \tag{III.3}$$

which is indeed linear in $\mathcal{E}$.

In the limit where $\delta \gg 3ea_0\mathcal{E}$,

$$E_\pm = -\frac{h\delta}{2} \pm \frac{1}{2}\left(h\delta + \frac{9e^2a_0^2\mathcal{E}^2}{2h\delta}\right) + E_2, \tag{III.4}$$

which is quadratic in $\mathcal{E}$.