

NOTE: All problem numbers from Sakurai correspond to the 3<sup>rd</sup> Edition.

1. In class, the third perturbative contribution to the hydrogen fine structure, the Darwin term, was given as

$$H'_D = \frac{\pi \hbar^3 \alpha}{2m^2 c} \delta^{(3)}(\vec{r}).$$

- (a) Show that the first order correction to the energies due to  $H'_D$  only affects  $\ell = 0$  states.
- (b) Evaluate this first order correction, for a state with  $\ell = 0$  and arbitrary  $n$ .
- (c) Combine the Darwin correction with the relativistic kinetic energy correction for  $\ell = 0$  to show that the fine structure formula

$$E_{n\ell j}^{(1)} = E_n^{(0)} \alpha^2 \left[ \frac{1}{n(j + \frac{1}{2})} - \frac{3}{4n^2} \right]$$

remains valid for  $\ell = 0$ .

2. Sakurai 5.6
3. Sakurai 5.19