

Problem Set 9

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I SAKURAI 6.3

$$q = 2k \sin\left(\frac{\theta}{2}\right), \quad (\text{I.1})$$

$$800\text{MeV} = \frac{\hbar^2 k^2}{2m}, \quad (\text{from figure}) \quad (\text{I.2})$$

$$k = 6.11\text{fm}^{-1}. \quad (\text{I.3})$$

$$qa = (4.49, 7.73, 10.9, \dots), \quad (\text{I.4})$$

$$\theta = (7.7, 14.25, 20.95)^\circ, \quad (\text{I.5})$$

$$a = (5.47, 5.10, 4.91)\text{fm}. \quad (\text{I.6})$$

$$1.4A^{1/3} = 4.79\text{fm}. \quad (\text{I.7})$$

My estimates are a bit large.

II SCATTERING OFF OF A SPHERICALLY-SYMMETRIC POTENTIAL

A

$$f = -\frac{m}{2\pi\hbar^2} \int d^3x e^{-i\mathbf{q}\cdot\mathbf{x}} V_0 e^{-\frac{r^2}{2a^2}}, \quad (\text{II.1})$$

$$= -\frac{mV_0}{2\pi\hbar^2} \left(\int dx e^{-iq_x x} e^{-\frac{x^2}{2a^2}} \right)^3, \quad (\text{II.2})$$

$$= -\frac{\sqrt{2\pi}a^3 mV_0}{\hbar^2} e^{-3a^2 q^2/2}. \quad (\text{II.3})$$

$$\frac{d\sigma}{d\Omega} = |f|^2, \quad (\text{II.4})$$

$$= \frac{2\pi a^6 m^2 V_0^2}{\hbar^4} e^{-3a^2 q^2}. \quad (\text{II.5})$$

B

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega, \quad (\text{II.6})$$

$$= \int_0^{2\pi} d\phi \int_{-1}^1 \frac{d\sigma}{d\Omega} d\cos\theta, \quad (\text{II.7})$$

$$= \frac{4\pi^2 a^6 m^2 V_0^2}{\hbar^4} e^{-3a^2} \int_{-1}^1 e^{q^2} d\cos\theta, \quad (\text{II.8})$$

$$= \frac{4\pi^2 a^6 m^2 V_0^2}{\hbar^4} \int_{-1}^1 e^{-6a^2 k^2 (1-\cos\theta)} d\cos\theta, \quad (\text{II.9})$$

$$= \frac{2\pi^2 a^4 m^2 V_0^2 (1 - e^{-12a^2 k^2})}{3k^2 \hbar^4}. \quad (\text{II.10})$$

III SCATTERING OFF OF A "SEPARABLE POTENTIAL"

A

$$T = |v\rangle\langle v| (1 + \langle v|G_0|v\rangle + \langle v|G_0|v\rangle^2 + \dots), \quad (\text{III.1})$$

$$= |v\rangle\langle v| \sum_{n=0}^{\infty} \langle v|G_0|v\rangle^n, \quad (\text{III.2})$$

$$= \frac{|v\rangle\langle v|}{1 - \langle v|G_0|v\rangle}, \quad (\text{III.3})$$

$$\langle v|G_0|v\rangle = \frac{2m}{\hbar^2} \int d^3q \frac{|v(\mathbf{k}')|^2}{k^2 - q^2 + i\epsilon}. \quad (\text{III.4})$$

$$T(\mathbf{k}', \mathbf{k}) = \frac{v(\mathbf{k}')v^*(\mathbf{k})}{1 - \langle v|G_0|v\rangle}. \quad (\text{III.5})$$

B

The Fourier transform of a constant is a dirac delta function:

$$\langle \mathbf{r}'|V|\mathbf{r}\rangle = g \int d^3\mathbf{k}' d^3\mathbf{k} \frac{e^{i\mathbf{k}'\cdot\mathbf{r}'} e^{-i\mathbf{k}\cdot\mathbf{r}}}{(2\pi)^3}, \quad (\text{III.6})$$

$$= (2\pi)^3 g \delta(\mathbf{r}) \delta(\mathbf{r}'). \quad (\text{III.7})$$

C

$$T(0, 0) = a = \frac{g}{1 - \langle v|G_0|v\rangle}, \quad (\text{III.8})$$

$$g = a(1 - \langle v|G_0|v\rangle), \quad (\text{III.9})$$

$$= a \left(1 - \frac{2m}{\hbar^2} g \int_0^\Lambda d^3q \frac{1}{-q^2 + i\epsilon} \right), \quad (\text{III.10})$$

$$= a \left(1 - \frac{8\pi m g}{\hbar^2} \Lambda \right), \quad (\text{III.11})$$

$$g(\Lambda) = \frac{a}{1 - \frac{8\pi m a \Lambda}{\hbar^2}}. \quad (\text{III.12})$$

D

$$\frac{d}{d\Lambda} \frac{1}{T(\mathbf{k}', \mathbf{k})} = -\frac{8\pi m}{\hbar^2} + \frac{8\pi m}{\hbar^2} \frac{d}{d\Lambda} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2}, \quad (\text{III.13})$$

$$= \frac{8\pi m}{\hbar^2} \frac{k^2}{\Lambda^2 - k^2}. \quad (\text{III.14})$$

(II.10) For $k \ll \Lambda$, this expression goes to zero.