Problem Set 5

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I DEUTERIUM HYPERFINE TRANSITION

II TIME-DEPENDENT ELECTRIC FIELD

III SAKURAI 5.29

$$
\begin{aligned}
c_0^{(0)} &= 1. && \text{(III.1)}\\
c_n^{(1)} &= -\frac{i}{\hbar}\int_0^t e^{i(E-E_0)t'/\hbar}\left\langle n\middle|F_0 x\cos\omega t'\middle|0\right\rangle \mathrm{d}t'\,, && \text{(III.2)}\\
&= -\frac{i}{\hbar}F_0\sqrt{\frac{\hbar}{2m\omega_0}}\delta_{n1}\int_0^t e^{-i\omega_0 t'}\cos\omega t'\,\mathrm{d}t'\,. && \text{(III.3)}\\
c_1^{(1)} &= -\frac{i}{\hbar}F_0\sqrt{\frac{\hbar}{2m\omega_0}}\int_0^t e^{-i\omega_0 t'}\cos\omega t'\,\mathrm{d}t'\,, && \text{(III.4)}\\
&= -\frac{i}{2\hbar}F_0\sqrt{\frac{\hbar}{2m\omega_0}}\\
&\qquad\times\left(\frac{e^{i(\omega_0+\omega)t}-1}{\omega_0+\omega}+\frac{e^{i(\omega_0-\omega)t}-1}{\omega_0-\omega}\right)\equiv c_1(t). && \text{(III.5)}
\end{aligned}
$$

$$
\begin{aligned}
\langle\hat{x}\rangle &= \langle\alpha,t|\hat{x}|\alpha,t\rangle_S\,, && \text{(III.6)}\\
&= \left(e^{i\omega_0 t/2}\langle 0| - c_1^* e^{3i\omega_0 t/2}\langle 1|\right)\hat{x} && \text{(III.7)}\\
&\qquad\times\left(e^{-i\omega_0 t/2}|0\rangle - c_1 e^{-3i\omega_0 t/2}|1\rangle\right), && \text{(III.8)}\\
&= \sqrt{\frac{\hbar}{2m\omega_0}}\left(c_1 e^{-i\omega_0 t}+c_1^* e^{i\omega_0 t}\right), && \text{(III.9)}\\
&= -\frac{F_0}{m}\frac{\cos\omega t-\cos\omega_0 t}{\omega_0^2-\omega^2}. && \text{(III.10)}
\end{aligned}
$$

This does not work at resonance, $\omega = \omega_0$, since it tends towards infinity. It must be modified from the start for that case.

IV SAKURAI 5.33

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\begin{aligned}
F(t) &= \frac{F_0\tau/\omega}{\tau^2+t^2}, \qquad t\in(-\infty,\infty). && \text{(IV.1)}\\
\therefore V(t) &= -F(t)x. && \text{(IV.2)}
\end{aligned}
$$

$$
\begin{aligned}
c_1^{(1)}(\infty) &= \frac{i}{\hbar}\frac{F_0\tau}{\omega}\langle 1|\hat{x}|0\rangle\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{\tau^2+t^2}\,\mathrm{d}t\,, && \text{(IV.3)}\\
&= \frac{i}{\hbar}\frac{F_0\tau}{\omega}\sqrt{\frac{\hbar}{2m\omega}}\frac{\pi}{\tau}e^{-\omega\tau}. && \text{(IV.4)}
\end{aligned}
$$

This makes sense in the limit $\tau \gg 1/\omega$. The perturbation is essentially turned on then off very slowly, so the system stays in the ground state.