Physics 852 **Problem Set 6** **Spring 2023**

NOTE: All problem numbers from Sakurai correspond to the 3rd Edition.

1. In class (lecture of Feb. 22), we showed that electromagnetic transitions in hydrogen with $\hbar\omega \sim e^2/a_0$ are long-wavelength processes in the sense that $ka_0 \approx \alpha \ll 1$. This was the justification for the electric dipole approximation (EDA), where we took $e^{i\vec{k}\cdot\vec{r}} \approx 1$ in the transition matrix element.

   (a) Suppose the initial state is the hydrogen ground state. Another way to motivate the EDA is to view $e^{i\vec{k}\cdot r}$ as a momentum translation (aka "boost") operator that injects momentum $\hbar\vec{k}$ to the state it acts on. Using simple dimensional analysis, show that this momentum is much less than the typical momentum of the electron in the hydrogen ground state, thereby providing another "justification" for the EDA.

   (b) We have also neglected the coupling between the electron's spin and the EM field (i.e., the $-\vec{\mu}\cdot\vec{B}$ terms) in our discussions of hydrogen in a monochromatic EM field. Make a simple estimate of how big this term is compared to the leading EDA term.

2. Calculate the decay rate and lifetime (an actual number!) for the spontaneous $2p \to 1s+\gamma$ decay in Hydrogen.
   Hints: i) Work in the electric dipole approximation and in a large box with periodic boundary conditions (take $L^3 \to \infty$ at the end), ii) You may assume that the $m_l = -1, 0, 1$ substates of the initial $2p$ state are equally populated and that we are detecting both polarization states of the photon. This means that we average over the initial populations of the m substates and sum over the outgoing photon polarization states so that you actually calculate

$$w_{2p\to 1s} \;=\; \frac{1}{3}\sum_{m=-1,0,1}\sum_{\lambda=1,2} w_{2p\to 1s}(m,\lambda)\,.$$

3. Sakurai 5.47.