Problem Set 6

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I ELECTRIC DIPOLE APPROXIMATION

A

$$\omega = ck, \tag{I.1}$$

$$\hbar\omega = \frac{e^2}{a_0}, \tag{I.2}$$

$$\hbar k = \frac{e^2}{ca_0}, \tag{I.3}$$

$$\alpha \equiv \frac{\hbar k}{p}, \tag{I.4}$$

$$p = \frac{\hbar}{a_0} \to \frac{\hbar k}{p} = \frac{e^2}{\hbar c} \ll 1. \tag{I.5}$$

B

$$\frac{mc}{e}\frac{\boldsymbol{\mu}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{p}} = \frac{mc}{e}\frac{\boldsymbol{\mu}\cdot\mathbf{B}}{A_0\hbar/a_0}. \tag{I.6}$$

$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}, \tag{I.7}$$

$$= i\mathbf{k}\times\mathbf{A}, \tag{I.8}$$

$$\therefore B = kA_0. \tag{I.9}$$

$$\frac{mc}{e}\frac{\boldsymbol{\mu}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{p}} = \frac{mc}{e}\frac{\frac{e\hbar}{mc}kA_0}{A_0\frac{\hbar}{a_0}}, \tag{I.10}$$

$$= ka_0 = \alpha \ll 1. \tag{I.11}$$

II HYDROGEN DECAY RATE AND LIFE TIME

III SAKURAI 5.47

$$\langle \mathbf{k}_f | e^{i(\omega/c)\widehat{\mathbf{n}}\cdot\mathbf{r}}\hat{\boldsymbol{\epsilon}}\cdot\mathbf{p}|i\rangle = \hat{\boldsymbol{\epsilon}}\cdot\int \mathrm{d}^3r\ \langle\mathbf{k}_f|\mathbf{r}\rangle\, e^{i(\omega/c)\widehat{\mathbf{n}}\cdot\mathbf{r}}\,\langle\mathbf{r}|\mathbf{p}|i\rangle\,, \tag{III.1}$$

$$= -i\hbar\hat{\boldsymbol{\epsilon}}\cdot\int \mathrm{d}^3r\,\frac{e^{-i\mathbf{k}_f\cdot\mathbf{r}}}{L^{3/2}}e^{i(\omega/c)\widehat{\mathbf{n}}\cdot\mathbf{r}}\boldsymbol{\nabla}\psi(\mathbf{r}), \tag{III.2}$$

$$= i\hbar(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{k}_f)\frac{1}{L^{3/2}}\left(\frac{m\omega_0}{\pi\hbar}\right)^{3/4} \times\int\mathrm{d}^3r\,e^{-i\mathbf{q}\cdot\mathbf{r}}e^{m\omega_0 r^2/2\hbar}, \tag{III.3}$$

where $\mathbf{q}\equiv\mathbf{k}_f-(\omega/c)\widehat{\mathbf{n}}$. Therefore

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{4\alpha\hbar^2}{m\omega\omega_0}k_f(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{k}_f)^2\left(\frac{\pi\hbar}{m\omega_0}\right)^{1/2}e^{-\frac{\hbar q^2}{m\omega_0}}, \tag{III.4}$$

$$= \frac{4\alpha\hbar^2k_f^2}{m^2\omega\omega_0}\sin^2\theta\cos^2\phi\left(\frac{\pi\hbar}{m\omega_0}\right)^{1/2} \times\exp\!\left(-\frac{\hbar}{m\omega_0}\left(k_f^2-(\omega/c)^2\right)\right) \times\exp\!\left(2\hbar k_f\frac{\omega}{mc\omega_0}\cos\theta\right). \tag{III.5}$$