Problem Set 1

Brandon Henke[1]

[1] *Michigan State University - Department of Physics & Astronomy*

Last updated: January 22, 2023

I SAKURAI 4.4

A

$$\mathcal{Y}_0^{1/2,1/2} = \begin{pmatrix} Y_0^0 \\ 0 \end{pmatrix}, \tag{I.1}$$

$$= \frac{1}{\sqrt{4\pi}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{I.2}$$

B

$$(\boldsymbol{\sigma} \cdot \mathbf{x})\mathcal{Y}_0^{1/2,1/2} = \frac{r}{\sqrt{4\pi}} \begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix}, \tag{I.3}$$

$$= \frac{r}{\sqrt{3}} \begin{pmatrix} Y_1^0 \\ -\sqrt{2}Y_1^1 \end{pmatrix}, \tag{I.4}$$

$$= -r\mathcal{Y}_1^{1/2,1/2}. \tag{I.5}$$

Here, j and m remained the same, but l increased from 0 to 1.

C

Since $\mathbf{S}\cdot\mathbf{x}$ is a scalar under rotations, it can only connect states with $j' = j$ and $m' = m$ (Wigner-Ekhart Theorem). However, the operator connects states of opposite parity. This is done by changing l by one unit.

II SAKURAI 4.5

The symmetry of V is determined by $\mathbf{S}\cdot\mathbf{p}$, which is a pseudoscalar. Therefore the matrix element $C_{n'l'j'm'}$ is zero unless $j' = j$, $m' = m$, and $l' = l \pm 1$. The radial wave functions go as r^l, so, since $V(\mathbf{x}) \propto \delta^3(\mathbf{x})$, the matrix element will only be nonzero for states with $l = 0$. Consequently, this interaction only connects $S_{1/2}$ and $P_{1/2}$ states.

III SAKURAI 4.7

A

$$\psi_p(x,t) = e^{i(p\cdot x - Et)/\hbar}. \tag{III.1}$$

$$\psi_p^*(x,-t) = e^{-i(p\cdot x + Et)/\hbar}, \tag{III.2}$$

$$= e^{i(-p\cdot x - Et)/\hbar}, \tag{III.3}$$

$$= \psi_{-p}(x,t). \quad \checkmark \tag{III.4}$$

B

Option 1

$$\chi_+(n) = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}. \tag{III.5}$$

$$i\sigma_2\chi_+^*(n) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\phi/2}\cos\frac{\theta}{2} \\ e^{-i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \tag{III.6}$$

$$= \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \quad \checkmark \tag{III.7}$$

Option 2

$$\chi(n) = e^{-i\sigma_3\phi/2}e^{-i\sigma_2\theta/2}. \tag{III.8}$$

$$-i\sigma_2\chi^*(n) = -i\sigma_2 e^{i\sigma_2\theta/2}e^{i\sigma_3\phi/2}, \tag{III.9}$$

$$= e^{-i\sigma_2\pi/2}e^{i\sigma_2\theta/2}e^{i\sigma_3\phi/2}. \quad \checkmark \tag{III.10}$$

IV SAKURAI 4.11

$$\langle\widehat{\mathbf{L}}\rangle = \langle E|\widehat{\mathbf{L}}|E\rangle, \tag{IV.1}$$

$$= \langle\tilde{E}|\hat{\Theta}\widehat{\mathbf{L}}\hat{\Theta}^{-1}|\tilde{E}\rangle, \tag{IV.2}$$

$$= -\langle\tilde{E}|\widehat{\mathbf{L}}|\tilde{E}\rangle, \tag{IV.3}$$

$$= -\langle E|\widehat{\mathbf{L}}|E\rangle. \tag{IV.4}$$

$$\therefore \langle\widehat{\mathbf{L}}\rangle = 0. \tag{IV.5}$$

$$\langle\mathbf{x},\tilde{E}|\mathbf{x},\tilde{E}\rangle = e^{-i\delta}\langle\mathbf{x},E|\mathbf{x},E\rangle, \tag{IV.6}$$

$$= e^{-i\delta}\sum_{lm} F_{lm}(r)Y_l^m(\theta,\phi). \tag{IV.7}$$

$$\langle\mathbf{x},\tilde{E}|\mathbf{x},\tilde{E}\rangle = \langle E|\mathbf{x}\rangle, \tag{IV.8}$$

$$= \sum_{lm} F_{lm}^*(r)(Y_l^m)^*(\theta,\phi), \tag{IV.9}$$

$$= \sum_{lm} F_{l,-m}^*(r)(-1)^{-m}Y_l^{-m}(\theta,\phi), \tag{IV.10}$$

$$\therefore F_{lm}(r) = (-1)^m e^{-i\delta}F_{l,-m}^*(r). \tag{IV.11}$$