Intermediate Fluid Mechanics (ME EN 5700/6700)
MidTerm Exam, Fall 2025

Name: **UiD:**

(*Note: The exam is closed notes, book, and notebook. No calculator is needed. Only the provided equation sheet is needed.*)

1. Continuum Hypothesis

(a) [2 points] Explain why the continuum hypothesis is important in the study of fluid dynamics

(b) [2 points] For a gas, when is the continuum hypothesis valid [Hint: think length scales]?

2. Write the following vector quantities in index notation.

(a) [2 points] $\vec{u} \times \left(\vec{\nabla} \times \vec{u}\right)$

(b) [2 points] $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{u}\right)$

(c) [2 points] $\vec{\nabla} \times \vec{\nabla}\phi$

3. Navier-Stokes equations

(a) [5 points] Cauchy's equation of motion is given by: $\rho \frac{Du_i}{Dt} = \rho g_i + \frac{\partial \tau_{ij}}{\partial x_j}$ and represents a system of equations that govern a material's motion. What information about the material is missing that would be required to solve this set of equations?

(b) [8 points] Simplify the following form of the Navier-Stokes equation by assuming the flow is incompressible (show your work).

$$\rho \frac{Du_i}{Dt} = \rho\, g_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial u_i}{\partial x_j} + \frac{\mu}{3}\frac{\partial u_m}{\partial x_m}\delta_{ij}\right] + F_i \tag{1}$$

(c) [2 points] What is the no-slip condition and how does it relate (or becomes useful) to the equation from part b?

4. Consider the 2D velocity field shown below and given by: $u = x^2$ and $v = -2xy$

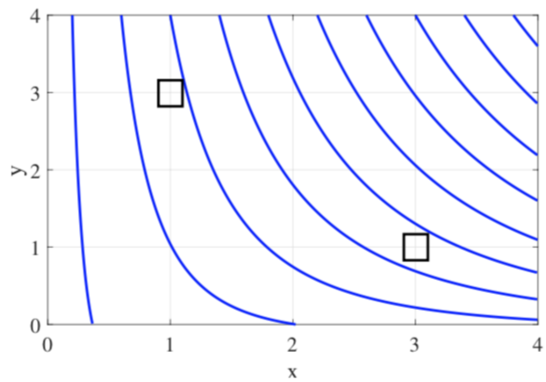

Figure 1: Streamlines of the flow indicated above.

(a) [5 points] Is the flow incompressible (show why or why not)?

(b) [10 points] Calculate the equation for the streamlines in the flow

(c) [5 points] Determine the components in the strain-rate tensor, e_{ij} (show your work).

(d) [5 points] Determine the components in the vorticity vector, ω_i (show your work).

(e) [10 points] Consider two initially square particles with centroid located at positions $(x, y) = (1, 3)$ and $(x, y) = (3, 1)$. How will the fluid particles look after an infinitesmial time Δt later in each case? [Support your reasoning with calculations]

(f) [6 points] Derive expressions for the components of the acceleration (a_x, a_y) of a generic fluid particle.

5. Consider a planar flow in a channel with porous walls. A constant vertical velocity V_w exists at the top and bottom walls as shown. Flow is driven through the channel by a pressure gradient $\partial P/\partial x = -K$.

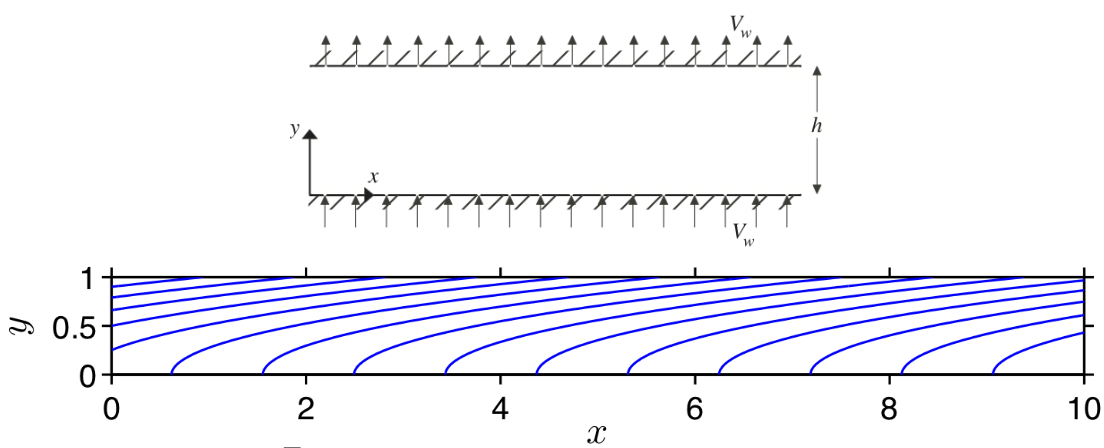

Figure 2: Flow in a channel with porus walls.

(a) [6 points] Use the continuity equation to find the vertical velocity v. [State assumptions]

(b) [10 points] Simplify the Navier-Stokes equation (x-component ONLY) for this flow and provide the appropriate boundary conditions. [State assumptions]

(c) [5 points] For the case where $V_w = 0$, explain the role of viscous diffusion in the fluid dynamics.

(d) [5 points] How would you expect the streamline pattern to change if the viscosity of the fluid was increased? [Suggestion: write the acceleration term in (b) from the Lagrangian viewpoint, then consider the motion of a fluid particle.]

(e) [8 points] Solve for the horizontal velocity component.

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Equations Sheet Mid-Term Exam, Fall 2025

$$\vec{u} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\,\hat{i} + \frac{\partial}{\partial y}\,\hat{j} + \frac{\partial}{\partial z}\,\hat{k}$$

$$e_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)$$

$$\varepsilon_{ijk}\,\varepsilon_{klm} = \delta_{il}\delta_j m - \delta_{im}\delta_{jl}$$

$$\vec{\omega} = \vec{\nabla} \times \vec{u},$$

$$\omega_i = \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j}.$$

$$\Gamma = \int_{\mathcal{S}} \vec{\omega} \cdot d\vec{\mathcal{S}},$$

$$r_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\Big),$$

$$r_{ij} = -\frac{1}{2}\varepsilon_{ijk}\,w_k.$$

$$u \equiv \frac{\partial \psi}{\partial y}, \qquad v \equiv -\frac{\partial \psi}{\partial x}$$

$$\frac{D}{Dt}\int_{V(t)} F(\vec{x},t)\,dV = \int_{V(t)} \frac{\partial F}{\partial t}\,dV + \int_{A(t)} F\,(\vec{u}\cdot d\vec{A})$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho\,u_i)}{\partial x_i} = 0$$

$$\frac{D\,(m\vec{u})}{Dt} = \vec{F}$$

$$\rho\frac{\partial u_i}{\partial t} + \rho\,u_j\frac{\partial u_i}{\partial x_j} = \rho\,g_i + \frac{\partial \tau_{ij}}{\partial x_j}.$$

$$\tau_{ij} = -p\delta_{ij} + \sigma_{ij}$$

$$\tau_{ij} = -p\delta_{ij} + \lambda e_{mm}\delta_{ij} + 2\mu e_{ij}.$$

$$\tau_{ij} = -p\delta_{ij} + \mu(2e_{ij} - \frac{2}{3}e_{mm}\delta_{ij}).$$

$$\rho\frac{Du_i}{Dt} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\mu}{3}\frac{\partial}{\partial x_i}\Big(\frac{\partial u_m}{\partial x_m}\Big)$$

$$\rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

$$\rho\Big[C_p\frac{DT}{Dt}+\frac{T\beta}{\rho}\frac{Dp}{Dt}+p\Big(\frac{\partial u_i}{\partial x_i}\Big)\Big]=-\rho\frac{\partial u_i}{\partial x_i}+\phi-\frac{\partial q_i}{\partial x_i}.$$

$$\rho\,C_p\,u\frac{\partial T}{\partial x}+\rho\,C_p\,v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial x^2}+k\frac{\partial^2 T}{\partial y^2}+\mu\phi$$

$$q_i=-k\,\frac{\partial T}{\partial x_i},$$

$$\phi=2\mu\,e_{ij}\,e_{ij}=2\mu\Big[\Big(\frac{\partial u}{\partial x}\Big)^2+\Big(\frac{\partial v}{\partial y}\Big)^2\Big]+\mu\Big(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\Big)^2.$$