Homework 7 Solutions

1) A 50-mm-diameter ball is pressed into a spherical seat of diameter 75 mm by a force of 500 N. The material is steel ($E = 200\ GPa$, $\nu = 0.3$). Calculate (a) the radius of the contact area, (b) the maximum contact pressure, and (c) the relative displacement of the centers of the ball and seat.

We apply Eqs. (3.63).

(a) $$a = 0.88[\frac{2(500)(0.025\times 0.0375)}{(200\times 10^9)(0.0125)}]^{\frac{1}{3}} = 0.635\ mm$$

(b) $$\sigma_c = 0.62[500(200\times 10^9)^2(\frac{0.0125}{2\times 0.025\times 0.0375})^2]^{\frac{1}{3}} = 596.1\ MPa$$

(c) $$\delta = 1.54[(500)^2(\frac{0.0125}{2(200\times 10^9)^2(0.025\times 0.0375)})]^{\frac{1}{3}} = 5.339(10^{-3})\ mm$$

2) Calculate the maximum contact pressure p_o in Prob 1 for the cases when the 50-mm-diameter ball is pressed against (a) a flat surface and (b) an identical ball. For case (b), determine the yield strength of a material for which the described loading will take the material to the verge of failure, using the maximum distortion energy criterion. Note how the determined yield strength compares to the maximum contact pressure.

(a) Use Eq. (3.62):

$$\sigma_c = 0.62[\frac{500(200\times 10^9)^2}{4(0.025)^2}]^{\frac{1}{3}} = 1233\ MPa$$

(b) Apply Eqs. (3.60) and (3.59) for $r_1 = r_2 = r$ and $E_1 = E_2 = E$ to obtain the formula

$$\sigma_c = 0.617[\frac{PE^2}{r^2}]^{\frac{1}{3}}$$

Thus, $$\sigma_c = 0.617[\frac{500(200\times 10^9)^2}{(0.025)^2}]^{\frac{1}{3}} = 1959\ MPa$$

- $\cancel{2}(\sigma_z - \sigma_y)^2 = \cancel{2}\sigma_Y^2 \Rightarrow \sigma_Y = \sigma_z - \sigma_y$
- $max(\sigma_z - \sigma_y)$ @ $0.5a = 0.619 * p_o$

$\Rightarrow \sigma_Y = 1214$ MPa

* smaller than p_o!

See the HW 07 Hints document for the detailed rationale for the derivation of the expression of von Mises stress.

The final answer is a very rough estimate. Students get full marks for any number.

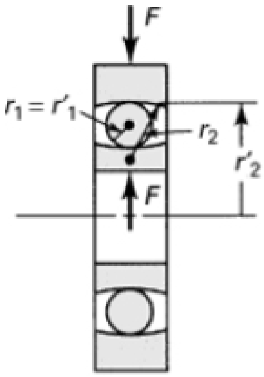
3) A concentrated load of 2.5 kN at the center of a deep steel beam is applied through a 10-mm-diameter steel rod laid across the 100-mm beam width. Compute the maximum contact pressure and the width of the contact between rod and beam surface. Use $E = 200\ GPa$ and $\nu = 0.3$.

Use Eqs. (3.67):

$$\sigma_c = 0.418[\frac{2.5(10^3)(200\times 10^9)}{0.1(0.005)}]^{\frac{1}{2}} = 418\ MPa$$

$$2b = 2\{1.52[\frac{2.5(10^3)(0.005)}{200\times 10^9(0.1)}]^{\frac{1}{2}}\} = 2(0.038) = 0.076\ mm$$

4) Determine the maximum pressure at the contact point between the outer race and a ball in the single-row ball bearing assembly shown. The ball diameter is 40 mm; the radius of the grooves is 22 mm; the diameter of the outer race is 250 mm; and the highest compressive force on the ball is $F = 1.8\ kN$. Take $E = 200\ GPa$ and $\nu = 0.3$.



Now given data is as follows:

$$r_1 = r_1' = 0.02\ m \qquad r_2 = -0.022\ m$$

$$r_2' = -0.125\ m \qquad \nu = 0.3 \qquad E = 200\ GPa$$

Therefore,

$$m = \frac{4}{\frac{1}{0.02} + \frac{1}{0.02} - \frac{1}{0.022} - \frac{1}{0.125}} = 0.08594$$

$$n = \frac{4(200\times10^9)}{3(1-0.09)} = 293.0403(10^9)$$

We have

$$A = \frac{2}{m} = 23.2721$$

$$B = \pm\frac{1}{2}[-\frac{1}{0.022} + \frac{1}{0.125}] = 18.7273$$

$$\alpha = \cos^{-1}\frac{18.7273}{23.2721} = 36.42^o$$

Using Table 3.3

$$c_a = 2.323 \qquad c_b = 0.541$$

Then, the semiaxes are:

$$a = 2.323[\frac{1800(0.08594)}{293.0403\times10^9}]^{\frac{1}{3}} = 0.00188\ m = 1.88\ mm$$

$$b = 0.541[\frac{1800(0.08594)}{293.0403\times10^9}]^{\frac{1}{3}} = 0.00044\ m = 0.44\ mm$$

Maximum contact stress is now obtained as

$$\sigma_c = 1.5\frac{1800}{\pi(1.88\times0.44)10^{-6}} = 1039\ MPa$$