Fluid Mechanics (ME EN 5700/6700)
Exam 1, Fall 2015

(Open Book, Open Notes, Closed neighbor)

0. [1 pt] What is your name?

1. [6 pt] Continuum Hypothesis

(a) How large must the infinitesimal volume of a point be for the continuum hypothesis to apply [explain in words]?

(b) Give an example of a case when the continuum hypothesis would not be valid?

2. [9 pt] Write the following vector quantities in index notation.

(a) $\vec{u} \cdot \left(\vec{\nabla} \cdot \vec{u} \right)$

(b) $\nabla^2 \left[\left(\frac{1}{2} \vec{u} \cdot \vec{u} \right) \cdot \vec{u} \right]$

(c) $\vec{u} \times \left[\vec{\nabla} \times \left(\vec{\nabla} \times \vec{u} \right) \right]$

3. [10 pt] Navier-Stokes equations.

a. List <u>all</u> the assumptions required to derive the following form of the Navier-Stokes equations

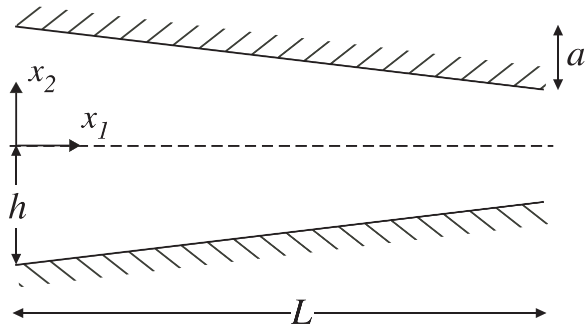
$$\rho \frac{Du_i}{Dt} = \rho g_i - \frac{\partial P}{\partial x_i} + \mu \frac{\partial}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j} + \frac{1}{3}\frac{\partial u_m}{\partial x_m}\right] + F_i$$

- The stress tensor can be split into a isotropic and deviatoric components and the isotropic component is the thermodynamic pressure (ie the fluid is at thermodynamic equilibrium).
- the fluid is a Stokesian fluid with the deviatoric stress a function of the velocity gradient tensor. This also implies that the fluid is an isotropic material and that the stress is isotropic in the absence of motion
- the fluid is a Newtonian fluid. This assumes we have a Stokesian fluid with a bulk coefficient of viscosity equal to 0 (mechanical pressure equals thermodynamic) resulting in a linear relationship between the deviatoric stress and the strain rate tensor.
- The dynamic viscosity is constant in space.

b. Write the equation from part [a.] with the additional assumption that the flow is incompressible.

c. Describe the no-slip condition in words

4. [18 pt] Consider steady, incompressible flow through the plane converging channel shown below. The streamwise velocity on the horizontal centerline (x_1) is given by $u_1 = V_1\left(1 + \frac{x_1}{L}\right)$.



a. Find an expression for the acceleration of a particle moving along the centerline from an Eulerian viewpoint.

b. Find an expression for the acceleration of a Lagrangian particle moving along the centerline [hint: you will have to first find the particle position x_p].

5. [16 pt] Conservation of Mass

(a) Write the conservation of mass equation for a fluid in the Lagrangian view.

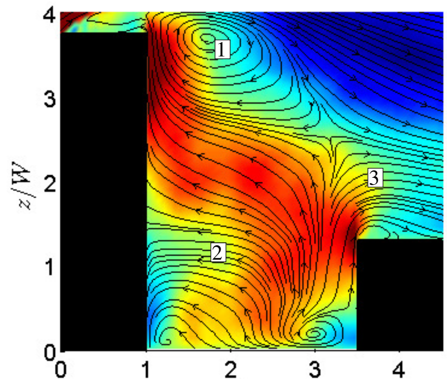
(b) Write the conservation of mass equation for a fluid in the Eulerian view.

(c) Can a *steady* flow with a non-uniform density field of the form $\rho = \rho(x, y)$ be incompressible? Why or Why not? [Show your work.]

(d) Derive an expression for the density as a function of time, $\rho = \rho(t)$, for a a flow with velocity components given below. Assume the density field is uniform.

$$u = \frac{x}{1+t} \quad \text{and} \quad v = \frac{2}{2+t}$$

6. [14 pt] The image below illustrates the streamline pattern in a steady state flow around two buildings.



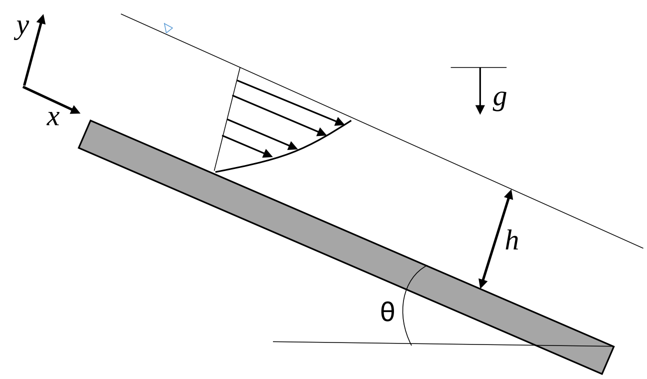
a. How are the streamlines related to the stream function?

b. Describe the pathline of a particle released at point 1 in flow?

c. Will a particle released at point 2 ever leave the space between the two buildings, Why or Why not?

d. Now assume the flow is <u>unsteady</u> instead of steady state. Given this new assumption, will a particle released at point 3 ever enter the space between the two buildings?

7. [26 pt] A liquid flows down a slope in a steady, fully developed laminar film of thickness h.



(a) Solve for the vertical v velocity in the liquid using conservation of mass (state all assumptions).

(b) Simplify the x direction momentum equation (Navier-Stokes) for this problem (state all assumptions).

(c) Obtain an expression for the liquid x direction velocity profile [hint: the upper BC should be no stress, $\frac{\partial u}{\partial y} = 0$ at $y = h$].

(d) Using your velocity profile from the previous part, find an expression for the shear stress distribution σ_{ij} on the surface of the slope ($y = 0$).