# Lab 6: Operational Amplifiers

## 1. Introduction

In this lab, you will explore the basic properties of operational amplifiers, or op-amps. These electronic devices are very useful in analog circuitry. As their name implies, they can be used to create circuits that perform mathematical operations on voltage signals including algebraic, calculus, and logic functions. In this lab, you will experiment with inverting amplifiers, non-inverting amplifiers, and active low-pass filters.

### 1.1 Lab Objectives

- Understand simple operational amplifier circuits and active filters

### 1.2 Project Objectives

- Finalize mechanical construction of robot
- Demonstrate mobility and manipulation
- Demonstrate serial communication method

### 1.3 Lab Hardware

- Wire kit
- Digital multimeter (DMM)
- LM324 op-amp

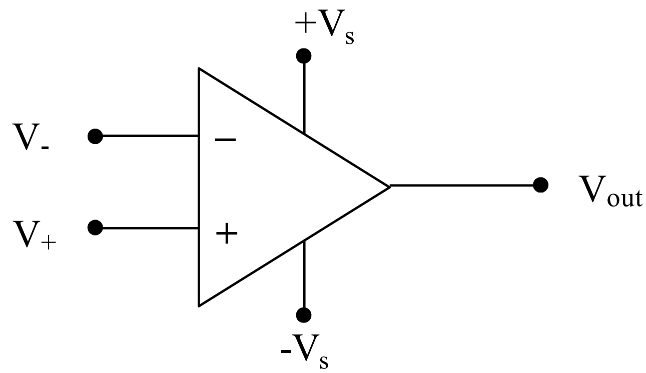
### 1.4 Project hardware

- Students should purchase parts from TA as necessary

# 2. Laboratory Concepts

## 2.1 Op-Amps

When analyzing op-amp circuits, we usually begin with the ideal op-amp model as shown below, which has two inputs $V_-$ (inverting) and $V_+$ (noninverting), a single output $V_{out}$, and two power supply rails, $+V_s$ and $-V_s$.




The ideal op-amp has infinite input impedance, zero output impedance, and an infinite gain G such that $V_{out} = G\Delta V = G(V_+ - V_-)$. Most practical op-amp circuits use the op-amp in a negative feedback configuration, where $V_{out}$ is connected in some manner to $V_-$. When analyzing op-amp circuits, we always begin with the **Golden Rules** of ideal op-amps:

**Rule #1:** The inputs draw no current (due to infinite input impedance)

**Rule #2:** When configured with negative feedback, the op-amp will adjust $V_{out}$ to balance the two inputs ($V_+ = V_-$).

In general, these two rules, along with Kirchoff's Laws can be used to derive the transfer function for any op-amp circuit.

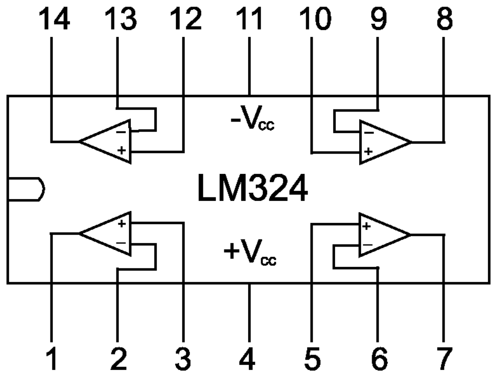
We should also be mindful that in practice, $V_{out}$ is restricted to the range of $+V_s$ to $-V_s$. Some op-amps are designed for single-sided operation ($-V_s$=0), while others are designed for double-sided operation ($|+V_s|=|-V_s|$). When choosing an op-amp, there are many other practical features/specs to be aware of.[1]

## 2.2 Op-Amp Packages/Chips

Op-amps are readily available as inexpensive Integrated Circuits (ICs). The op-amp IC used in the Mechatronics lab is the LM324 quad op-amp. This is a 14-pin *Dual In-line Package* (DIP) chip. When plugged into an electrical breadboard and provided the appropriate $\pm V_{cc}$ (e.g. the ±12V supplied to the chip by an external source), the chip provides four independent op-amps.

When using ICs, it is important to refer to the data sheet provided by the chip manufacturer that describes the device's characteristics and limitations. For example, the maximum and minimum allowable supply voltages for the device. Search the web for a data sheet for the LM324; there are many manufacturers for the chip, so there will be many different data sheets. Each data sheets contain a connection diagram for the LM324, one of which is reproduced in the figure below. The connection diagram, also known as a *pin-out diagram*, indicates how the pins of the DIP correspond to the inputs and outputs of the four op-amps.

[1] See Chapter 12 of Introduction to Mechatronic Design, by Carryer, Ohline and Kenny.

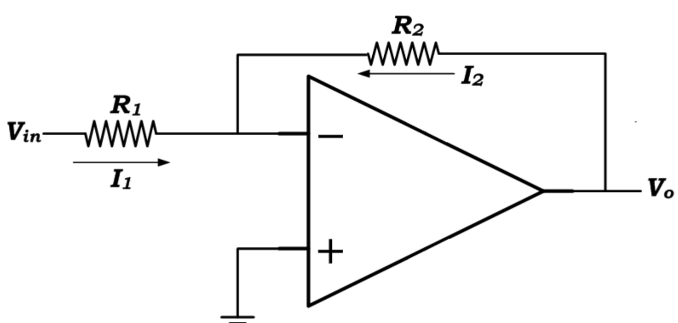



Aside from the applications discussed in this handout, the op-amp has many other uses in analog electronics. Such applications include instrumentation amplifiers, integrators, differentiators, high-pass filters, band-pass filters, Schmidt triggers, and mathematical operations like addition and subtraction. There are many books, datasheets, and online references that you can use to learn about these circuits.

## 2.3 Inverting Amplifier

A basic inverting amplifier circuit is shown in the figure below. To determine the output voltage $V_o$ as a function of $V_{in}$, the first step is to note that the non-inverting input is connected to ground, so $V_+ = 0$. Given Rule #2, $V_- = V_+ = 0$, since the circuit has a negative feedback loop through $R_2$. The voltage at the inverting input is known as virtual ground. The directions of the currents are toward virtual ground. Applying Ohm's law results in the following equations:

$$I_1 = \frac{V_{in}}{R_1} \qquad\qquad I_2 = \frac{V_o}{R_2}$$




The next step is to apply Kirchoff's Current Law (KCL) at the node containing the inverting input, remembering that the current into the inverting terminal is $I_- = 0$ (e.g. Rule #1).

$$I_1 + I_2 = 0$$

Substituting current equations into the equation above yields:

$$\frac{V_{in}}{R_1} + \frac{V_o}{R_2} = 0$$

Finally, the input-output relationship is,

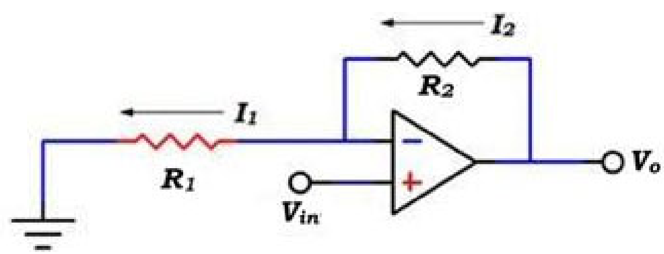
$$\frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \tag{1}$$

The mathematical operation performed by this op-amp circuit is multiplication (i.e. amplification of the input). Notice that the sign of the output is opposite the sign of the input. This sign inversion is why the circuit is called an inverting amplifier.

## 2.4 Non-inverting Amplifier

A basic non-inverting amplifier circuit is shown in the figure below. To determine the output voltage, $V_o$, as a function of $V_{in}$, the first step is to note that the non-inverting input is connected to $V_{in}$. Applying Rule #2, $V_- = V_+ = V_{in}$, since the circuit has a negative feedback loop through $R_2$. Applying Ohm's law results in the following equations:

$$I_1 = \frac{V_{in}}{R_1} \qquad\qquad I_2 = \frac{V_o - V_{in}}{R_2}$$




The next step is to apply KCL at the node containing the inverting input, remembering that the current into the inverting terminal is $I_- = 0$ (e.g. Rule #1), from Equation (3).

$$I_1 - I_2 = 0$$

Substituting current equations into the equation above yields:

$$\frac{V_{in}}{R_1} - \frac{V_o - V_{in}}{R_2} = 0$$

Finally, solving Equation (16) for the non-inverting amplifier yields

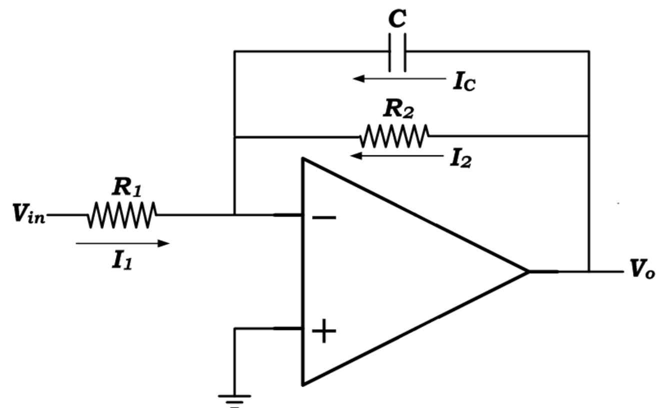
$$\frac{V_o}{V_{in}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{2}$$

The mathematical operation performed by this op-amp circuit is multiplication (i.e. amplification of the input).

## 2.5 First-Order Active Low-Pass Filter

A simple modification can be made to the inverting amplifier to allow it to filter out high frequency noise, as is often found in electronic signals. A prominent source of noise in the lab is AC line noise at 60Hz. High frequency noise is commonly caused by external voltage sources, magnetic sources, or from imperfections in circuit devices or components, and is generally characterized by sporadic, high frequency variations in the voltage signal.

The low pass filter attenuates the amplitude of input frequencies higher than a specified frequency called the cutoff frequency. An active low pass filter can be created by connecting a capacitor, *C*, in parallel with the feedback resistor, $R_2$, of the inverting amplifier as shown in the figure below.




This circuit performs two operations. First, it amplifies the input voltage at low frequencies, and second, it suppresses frequencies higher than its cutoff frequency, where the cutoff frequency, $\omega_c$ in rad/sec, is defined as:

$$\omega_c = \frac{1}{R_2 C} \tag{3}$$

Recall that the frequency in Hz is $f_c = \frac{\omega}{2\pi}$. The low-pass filter's gain and phase as a function of input frequency is described by:

$$\left|\frac{V_O}{V_{in}}\right| = \frac{R_2}{R_1}\frac{1}{\sqrt{1+\left(\omega/\omega_c\right)^2}} \qquad\qquad \emptyset = 180° - \tan^{-1}\left(\frac{\omega}{\omega_c}\right) \tag{4}$$

where ω is the frequency of the input signal $V_{in}$. Note the extra 180º in the phase shift is due to the inverting nature of the amplifier.

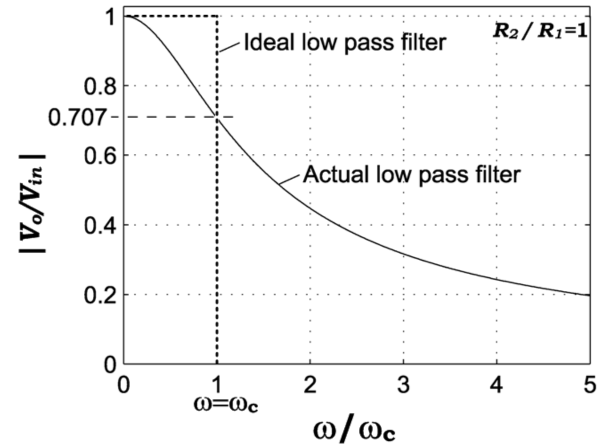
At ω = 0 (DC voltage input), the output voltage is:

$$|V_o| = \frac{R_2}{R_1}|V_{in}| \tag{5}$$

As the frequency of the input signal increases, the amplitude of the output decreases. At the cutoff frequency, $\omega = \omega_c$, the magnitude of the output is defined by the following:

$$|V_o| = \frac{R_2}{R_1\sqrt{2}}|V_{in}| = 0.707\frac{R_2}{R_1}|V_{in}| \tag{6}$$

The following figure shows the frequency response of an active low pass filter, where the resistors have been selected to provide a unity gain. The x-axis is the normalized frequency, and the y-axis is the absolute value of the system gain. Note this is identical to the frequency response of the passive RC low-pass filter we saw in last week's lab, but the figure below uses linear scale instead of logarithmic scale to highlight the difference between an actual low pass filter and an ideal low pass filter as we approach the cutoff frequency.




Ideally, the low pass filter would pass through everything below the cutoff frequency with a gain of 1, and would completely remove everything above the cutoff frequency with a gain of zero. However, the gain of an actual low-pass filter, as in eq. (4), exhibits a gradual decrease as we approach the cutoff frequency. In order to minimize the impact of the low pass filter on the gain of the desired output signal, the cut-off frequency is set as high as possible. The trade-off of using a high cutoff frequency is that more noise is permitted to pass through the filter. A reasonable guideline is to select a cutoff frequency that is at least twice the maximum expected input frequency, and less than one tenth the expected noise frequency.

## 2.6 Second-Order Active Low-Pass Filter

The active filter in the preceding section is a 1st order low-pass filter, such that above the cutoff frequency, the magnitude drops off at a rate that is inversely proportional to frequency ω. In many cases, a 1st order low-pass filter will not attenuate high frequency noise enough. In such cases a higher-order filter must be designed instead. For example, for a 2nd order low-pass filter, the magnitude will drop off a rate that is inversely proportional to $\omega^2$, which means the drop-off is twice as steep on a log-scale.

Many design procedures have been developed to quickly create high-order filters that more aggressively attenuate high frequency noise. The Butterworth filter is one popular choice for designing a higher-order filter. A 2nd-order Butterworth filter is constructed using the Sallen-Key architecture shown in the figure below, where a particular combination of resistor and capacitor values are used. The resulting frequency response of a 2nd-order Butterworth filter not only has a steeper drop-off than a 1st order filter, but also has a sharper cutoff frequency, meaning that the magnitude stays relatively flat below the cutoff frequency, which is usually a desirable trait.

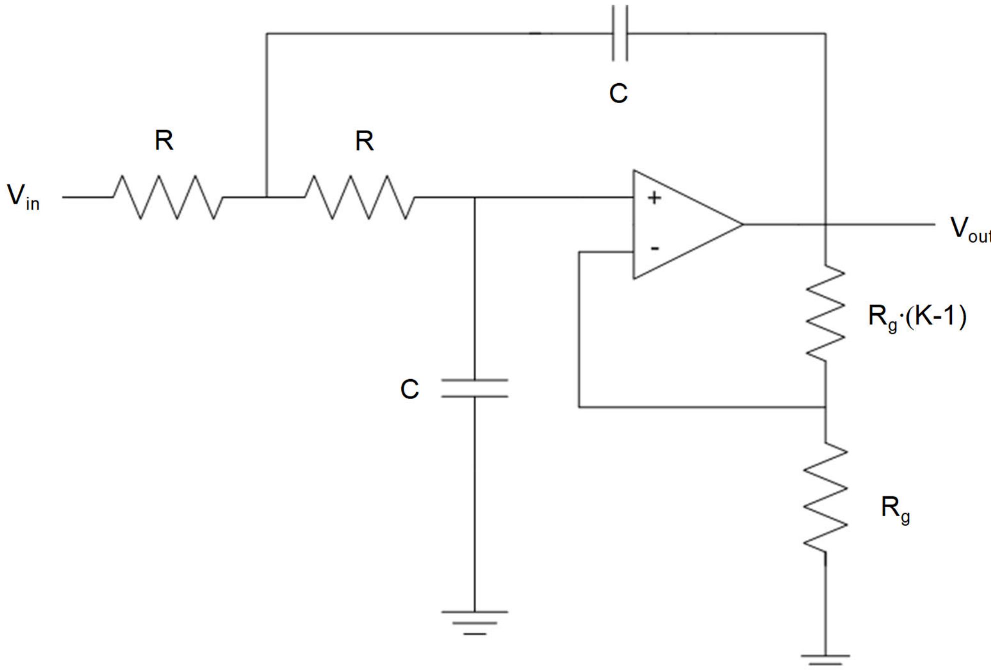



Figure 1. 2nd Order Low Pass Filter with Sallen-Key Architecture.

The resistors and capacitors involved in the filter are defined to have the same values; therefore, they have been labeled identically R or C respectively. To determine the cutoff frequency of the low-pass filter you use the equation below.

$$RC = \frac{1}{2\pi f_c}$$

R is usually chosen in the range 1K to 200K. It is best to avoid large resistor values, because we assume the input impedance of the op-amp is much higher than elements in the circuit. We also want to avoid having the resistors too small, since we want to the limit the current flowing in the circuit to the mA range. It's usually easiest to pick C and then calculate R for a given $f_c$. In addition to acting as a low-pass filter, the circuit shown also has a DC gain factor $K$, which is determined by the ratios of the two resistors on the right. For a 2nd order Butterworth filter, $K$ should be chosen to be 1.586, and the two gain resistors $R_g$ and $R_g * (K - 1)$ should be chosen accordingly.

# 3. Pre-Lab Exercises

Important Notes:

- Resistors are available in the student laboratory cabinet with the following values: 1.0, 2.0, 3.3, 4.7, 6.8, and 8.2, all x$10^x$ Ω, where the power can be 1 through 5. Often, you can use multiple resistors in series and/or parallel to create a desired resistance. When you cannot, a potentiometer (pot), or variable resistor, is a useful tool to produce very precise values of resistance (you will not use potentiometers for this lab).
- When using op-amps, it is wise to choose resistors substantially smaller than the input resistance of the op-amp being used (~10MΩ), otherwise the ideal op-amp assumptions will not hold, but still choose large enough resistors to limit the amount current through connected devices/sensors.
- Capacitors are available in a smaller variety of values than resistors. Some capacitor values available in the lab are: 1μF, 1nF, 10nF, 100nF, 330nF, 470nF. Capacitors can be combined in series or parallel to achieve different capacitances. They are additive in parallel.

1. Design an inverting amplifier which produces a gain of approximately -π (-3.14± 0.1) given the available resistors indicated above. Show your calculations and sketch the inverting amplifier circuit. Record practical resistor (see notes above) values for $R_1$ and $R_2$. Use the 1kΩ-100kΩ range. Hint: you can combine multiple resistors to form $R_1$ and $R_2$ if needed.

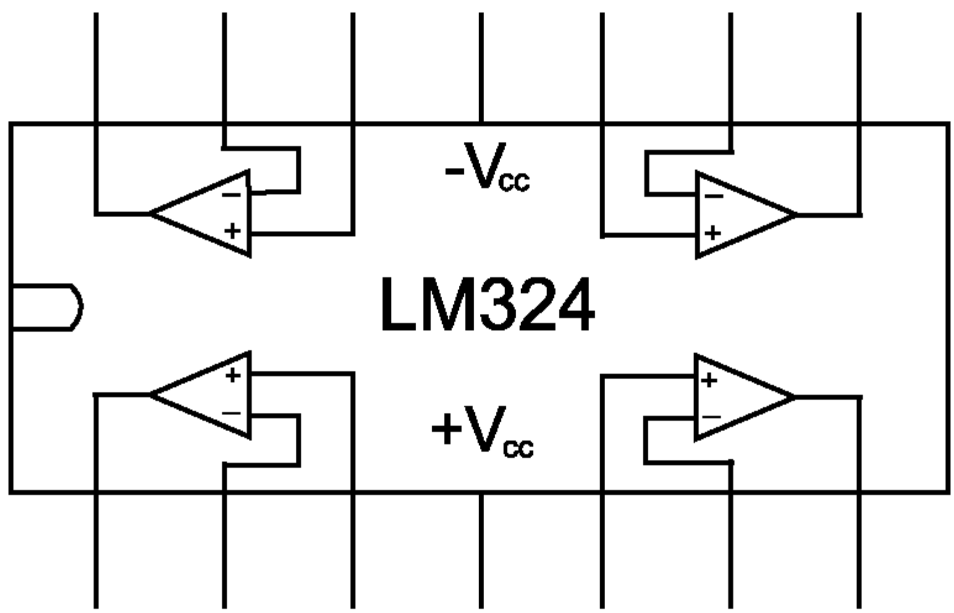
2. If you now constructed a non-inverting amplifier using the same values for $R_1$ and $R_2$ found for the inverting amplifier in Problem 1, what would be the gain of the non-inverting amplifier? Sketch the circuit and show your calculations.

3. Using impedances and Kirchoff's voltage and current Laws, derive the transfer function $V_o/V_{in}$, as a function of frequency for the 1st-order active low pass filter in Section 2.5.

4. The purpose of a low pass filter is to reduce the amplitude of high frequency components of noise present in low frequency signals.
   a. Design a 1st-order low pass filter-amplifier with a gain of approximately -π (like in problem 1) and a cutoff frequency of approximately 10 Hz using the available resistors and capacitors above. Hint: you can combine multiple resistors/capacitors if needed.

   b. Using Equation (4) and MATLAB, plot the gain of your filter for a range of frequencies from 0 to 100 Hz. Use a linear scale for both axes. What would be the gain at f=0, f=10 Hz, and f=100 Hz? (Use data points, not lines, in MATLAB). Include screenshots of your code and properly labeled plots in line with this question with your name indicated. Also submit your .m files to canvas.

5. On the diagram below, sketch the power connections and circuit elements (e.g. resistors, capacitors) for the circuits below. Clearly label the external connections to: $V_{in}$, $V_{out}$, $R_1$, $R_2$, $C$, and Ground. Label +/-$V_{cc}$ with +/- 12 v, respectively. Label which op amp is for which circuit.
   a. Inverting amplifier
   b. Non-inverting amplifier
   c. Active low pass filter



6. Assume you are provided a Sallen-Key filter circuit as shown in Section 2.6, and you are told it is a Butterworth filter. You measure the values of the components in this circuit as the following:

   R = 16.4 kΩ

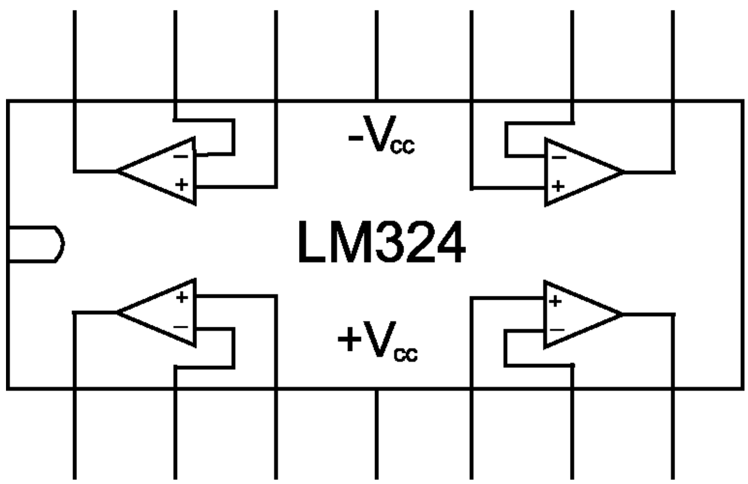
   C = 1 μF

   Rg = 6.8 kΩ

   Rg*(K-1) = 4 kΩ

Based on your knowledge of the circuit architecture, what is the cut-off frequency of this filter? What is the value of the gain K? Do you agree this is a Butterworth filter? Why or why not?

7. Draw the Low-pass Butterworth filter circuit on the picture shown below. Clearly label all resistor and capacitor values as well as the input signal voltage ($V_{in}$), output signal voltage ($V_{out}$), and power supply voltages ($\pm V_{cc}$).



# 4. Laboratory Exercises

Though you have completed your circuit design as part of the pre-lab, you may find that it does not work properly and requires debugging. Troubleshooting circuits is a very useful skill and requires patience and attention to detail. It is very important to build up circuits step by step and verify that each piece is correct as you go along. If the circuit does not work, check voltages at key places where you know what they should be. If they are incorrect, check the connections that should be made to that point. Ask your TA for help if the circuit fails to function after your best efforts to debug the circuit. Make sure to check the voltage of the power supply pins on the op-amp to ensure they are properly connected.

- Make sure the power supply is turned off while you build your circuits.
- Make sure the power connections to the op-amp are made with the correct polarity.
- Power the op-amp with the +/- 12V from the power supply box. Use the power supply COM as common ground for all circuits. **DO NOT** turn on the 12V power supply until a TA checks your circuit.

## 4.1 Non-inverting Amplifier

4.1.1 Turn off the +/- 12V power supply.

4.1.2 Construct a non-inverting amplifier circuit design using an LM324 op-amp. (Hint: pre-lab part 2).

4.1.3 **DO NOT** turn on the +/- 12V power supply until a TA checks your circuit.

4.1.4 The input of the circuit should be a 1V 5Hz sine wave with no bias (no DC offset) from the function generator.

4.1.5 Examine the input and output waveforms using the Lab3_GUI

- Connect the input to CH0 and the output to CH1

4.1.6 Determine and record the theoretical gain for the actual resistances in Section 4.5 Student Work and Tables.

4.1.7 Measure the output amplitudes for the approximate input amplitudes in Section 4.5 Student Work and Tables.

**Have a TA check your progress**

## 4.2 Inverting Amplifier

4.2.1 Turn off the +/- 12V power supply.

4.2.2 Construct an inverting amplifier circuit design using an LM324 op-amp. (Hint: pre-lab part 1).

4.2.3 **DO NOT** turn on 12V power supply until a TA checks your circuit

4.2.4 The input of the circuit should be a 1V 5Hz sine wave with a no bias (no DC offset) from the function generator

4.2.5 Examine the input and output waveforms using the Lab3_GUI

- Connect the input to CH0 and the output to CH1

4.2.6 Determine and record the theoretical gain for the actual resistances in Section 4.5 Student Work and Tables.

4.2.7 Measure the output amplitudes for the approximate input amplitudes in Section 4.5 Student Work and Tables.

**Have a TA check your progress**

## 4.3 First-Order Active Low-Pass Filter

4.3.1 Turn off the +/- 12V power supply.

4.3.2 Construct an active low-pass filter circuit design using an LM324 op-amp. (Hint: prelab part 4. You may be able to use the circuit from the previous part as a basis for the filter)

4.3.3 **DO NOT** turn on the +/- 12V power supply until a TA checks your circuit

4.3.4 The input of the circuit should be a 1V 5Hz sine wave with a no bias (no DC offset) from the function generator

4.3.5 Examine the input and output waveforms using the Lab3_GUI

- Connect the input to CH0 and the output to CH1

4.3.6 Collect amplitude and phase measurements at the frequencies indicated in the table in Section 4.5 Student Work and Tables. You will need this data for the post lab.

**Have a TA check your progress**

## 4.4 Second-Order Active Low-Pass Filter

4.4.1 Turn off the +/- 12V power supply.

4.4.2 Construct a 2nd order active low-pass filter circuit design using an LM324 op-amp. (Hint: prelab part 6. You may be able to use the circuit from the previous part as a basis for the filter)

4.4.3 **DO NOT** turn on the +/- 12V power supply until a TA checks your circuit

4.4.4 The input of the circuit should be a 1V 5Hz sine wave with a no bias (no DC offset) from the function generator

4.4.5 Examine the input and output waveforms using the Lab3_GUI

- Connect the input to CH0 and the output to CH1

4.4.6 Collect amplitude and phase measurements at the frequencies indicated in the table in Section 4.5 Student Work and Tables. You will need this data for the post lab.

**Have a TA check your progress**

## 4.5 Student Work and Tables

**Work for section 4.1**

Determine and record the theoretical gain for the actual resistances. 4.1.6

Theoretical Gain = ______________ $R_1$ = __________Ω $R_2$ = ____________Ω

Measure the output amplitudes for the approximate input amplitudes 4.1.7

| $V_{in}$ | Actual $V_{in}$ | $V_{out}$ | Experimental Gain |
|---|---|---|---|
| **1V** | | | |

**Work for section 4.2**

Determine and record the theoretical gain for the actual resistances. 4.2.6

Theoretical Gain = ______________ $R_1$ = __________Ω $R_2$ = ____________Ω

Measure the output amplitudes for the approximate input amplitudes 4.2.7

| $V_{in}$ | Actual $V_{in}$ | $V_{out}$ | Experimental Gain |
|---|---|---|---|
| **1V** | | | |

**Work for section 4.3**

| $f$ (Hz) | $V_{in}$ (V) * | $V_{out}$ (V) * | $V_{out}/V_{in}$ | $V_{out}/V_{in}$ (dB) † | $\phi$ (°) |
|---|---|---|---|---|---|
| **1** | | | | | |
| **5** | | | | | |
| $f_c$ | | | | | |
| **20** | | | | | |
| **50** | | | | | |
| **100** | | | | | |
| **1000** | | | | | |

## Work for section 4.4

| $f$ (Hz) | $V_{in}$ (V) * | $V_{out}$ (V) * | $V_{out}/V_{in}$ | $V_{out}/V_{in}$ (dB) † | $\phi$ (º) |
|---|---|---|---|---|---|
| **1** | | | | | |
| **5** | | | | | |
| $f_c$ | | | | | |
| **20** | | | | | |
| **50** | | | | | |
| **100** | | | | | |
| **1000** | | | | | |

# Post-Lab Exercises

1. Plot the experimental frequency response (Gain (dB) vs Frequency and Phase (deg) vs. Frequency (Hz)) of the active low pass filter using the data from 4.3.6, as well as the theoretical frequency response using equation (4). Plot experimental (markers) and theoretical (lines) responses. Use log scale on the Frequency. Show where the cutoff frequency is located on the plot. Discuss differences between theoretical and experimental responses in terms of gain, cutoff frequency, and phase shift. Include screenshots of your code and properly labeled plots in line with this question with your name indicated. Also submit your .m files to canvas.

2. Plot the experimental frequency response of your 2$^{nd}$ order filter using the data from 4.4. Discuss the differences between the frequency responses of your 1$^{st}$ and 2$^{nd}$ order filters.

3. A sensor outputs a voltage in the range of ±3mV. Your A/D samples at 1000 Hz, has an input range of ±10V, and has a 12-bit resolution. Design an active 1$^{st}$-order filter to amplify the sensor signal to fit at least 75% of the range of the A/D and filter out anything above the Nyquist frequency. Justify your assumptions and show your work.

# 6. Project Milestone 6

The purpose of Milestone 6 is to present the mechanical elements of your mobility platform, manipulators, and remote solution. You will also demonstrate mobility, mechanisms, and wireless serial communications. For this PM you will present your design and polished prototypes in a detailed PowerPoint (15 mins) during Lab 7. Submit one PowerPoint file per team on canvas before the beginning of Lab 7.

## Presentation (90 Points)

The presentation should have a detailed description of your manufactured prototype. Use pictures or diagrams for clarity. Your presentation should include the following:

- Discuss each team member's contributions for this week.
- An updated CAD model of your robot
    - Show your model from PM 5
    - Show any changes made for PM 6
    - Subassembly models and multiple view when necessary to see all components
    - Including all actuators, wheels, mechanisms, fasteners, hardware, microcontrollers, shields (simple blocks), and battery
    - Provide important dimensions of you robot in each of its configurations
- List and describe each component in the CAD model
- Present updated budget for parts purchased in lab and parts purchased outside of lab
- Describe your wireless communication method
    - Describe your hardware setup
    - Describe your software/packaging process
    - DO NOT put code snapshots in presentation
- Discuss any changes to how the robot will accomplish the competition objectives
- An updated state transition diagram
    - Your diagram should provide a high-level view of each separate action/sub-routine the robot will perform and how the robot transitions between different actions.
    - States are continuous actions that repeat until an event causes a service.
    - Events are what initiates the robot to change states like a flag or a condition. Services are what discrete action is implemented to actually change the state.
- An updated pin table
    - The table should include all known connections to each microcontroller including shields, actuators, and all known electrical hardware.
- Create computer-generated wiring schematics for all components for your robot and wireless transmitter.
    - Include Arduinos, shields, power sources, switches, LEDs, motors, sensors, and other electrical components.
    - For wiring schematics, we recommend using AutoDesk Eagle, which is free. On Canvas, you can find a library of mechatronic components we have created for Eagle.

## Demonstration (100 Points)

During Lab 7 you will demonstrate the robot's mobility, mechanisms, and wireless serial communication between the robot (Mega) and the transmitter (Uno). Demonstrate that your robot can drive and that your robot's mechanisms can move.

Using wireless serial communication, command the robot to:

- Drive forward and reverse
- Turn in both directions (Left and Right)
- Move each manipulator in all DOF

You should be able to send commands from your transmitter (Uno) setup to the robot (Mega) via wireless connections just like we did in lab 1. The commands should initiate the robot to do each of the above actions. The TA may ask to send the inputs to check that the commands are initiating the movement.