Aerospace Propulsion

Lecture 10

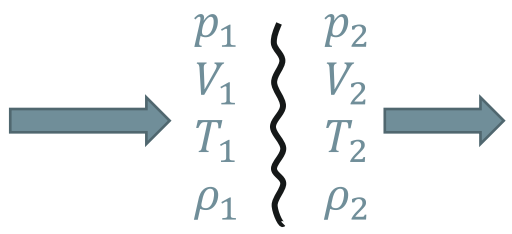
Compressible Flows: Part IV

Compressible Flows: Part IV

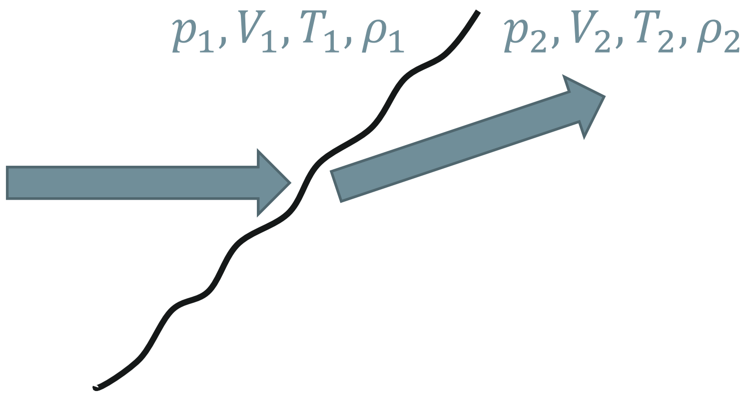
- Isentropic Shocks?
- Expansion Waves
- Advanced Compressible Flows
- Shocks and Expansions + Nozzles

Isentropic Shocks?

- Previously, we discussed normal and oblique shocks, both of which are **non-isentropic**

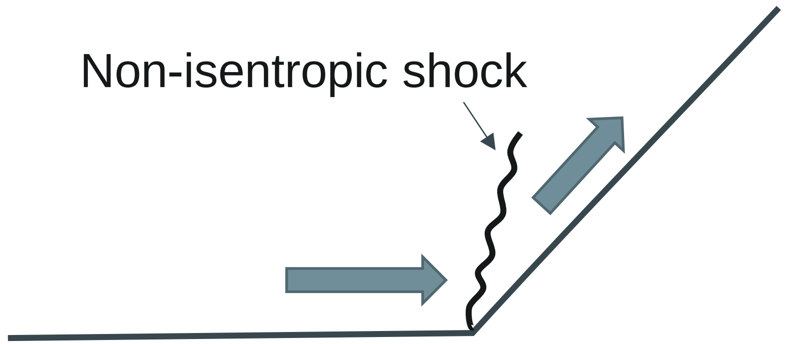


- We can (but will not) show that shocks can be isentropic <u>**if**</u> their angle and therefore effect (e.g., pressure change across shock) is infinitesimal

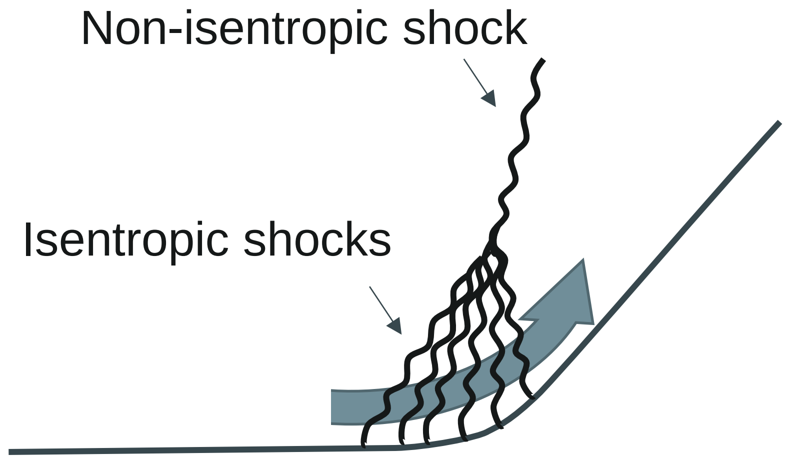


Isentropic Shocks?

- How do we take an infinitesimal effect and make it significant?
  - Many (approaching infinite) shocks
    - Possible with e.g., smoothly curved wall



- Infinitesimal oblique shocks nearly always coalesce into a single oblique shock
  - Rarely consider such shocks because coalescence is so close to wall
    - Assume obliques are always non-isentropic

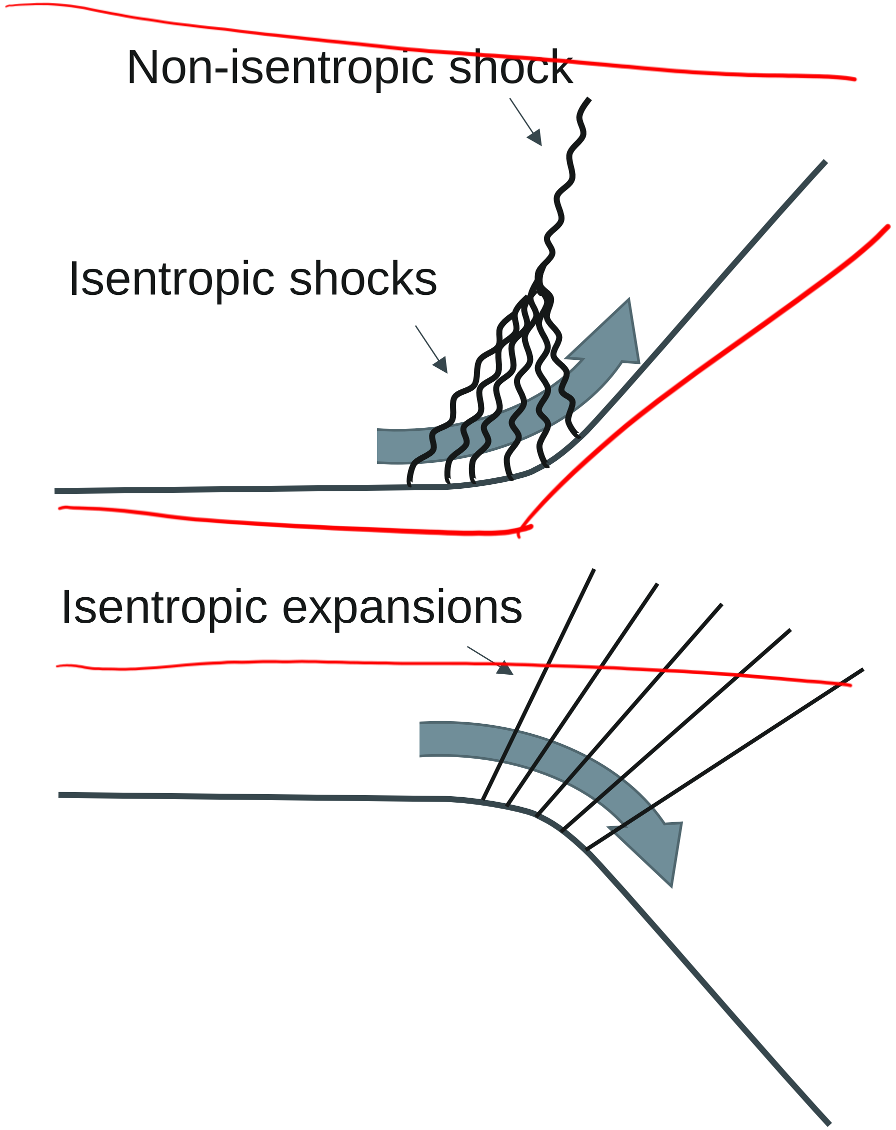


Expansion Waves

- So far, we've only explored compressions
- What about expansions?

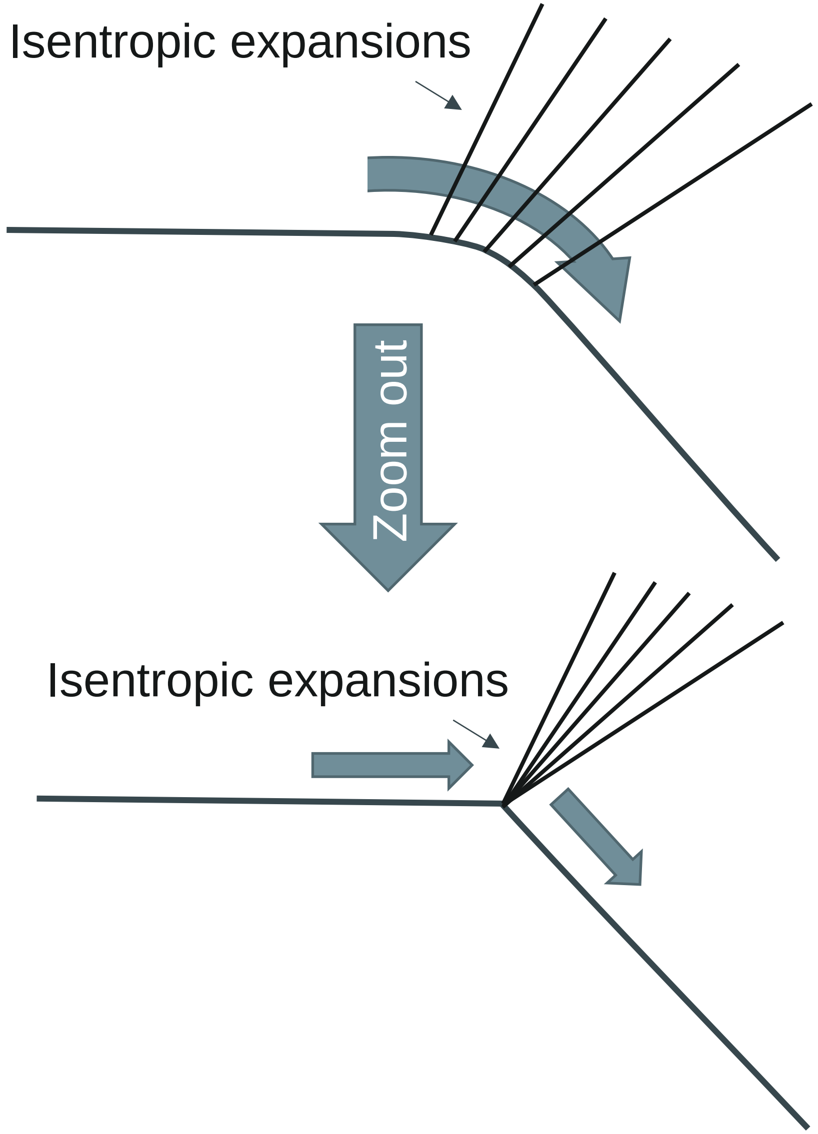
- Just like we had isentropic compression waves (oblique shocks), we can have isentropic expansion waves

- Expansion waves **increase** Mach number and **decrease** pressure



Expansion Waves

- Due to the geometry of expansion waves, they **<u>do not</u>** coalesce
  - **All expansion waves are isentropic**
- Also called Prandtl-Meyer Flow
- We will not calculate state changes across expansion waves in this course
  - See Farokhi 2.14 if interested
  - Generally, isentropic equations hold

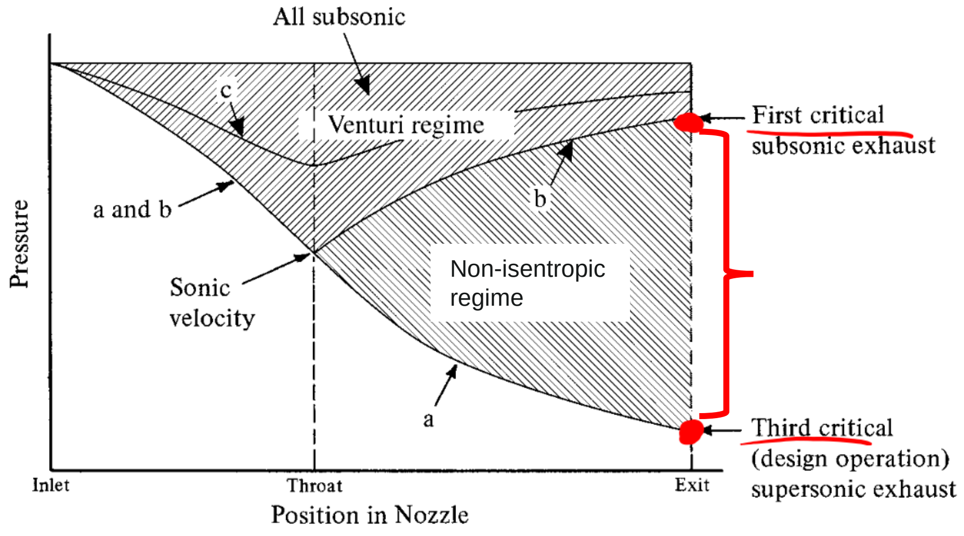


Advanced Compressible Flows

- Only four lectures on compressible flows – focused on essentials
- Some slightly more advanced concepts if interested:
  - Duct flow with Heat Transfer (e.g., combustor) a.k.a *Rayleigh Flow*
    - See Farokhi 2.15
  - Duct flow with Friction (i.e., boundary layers) a.k.a *Fanno Flow*
    - See Farokhi 2.16
- We will stick to our simple (but reasonably accurate) approximations throughout the remainder of this course

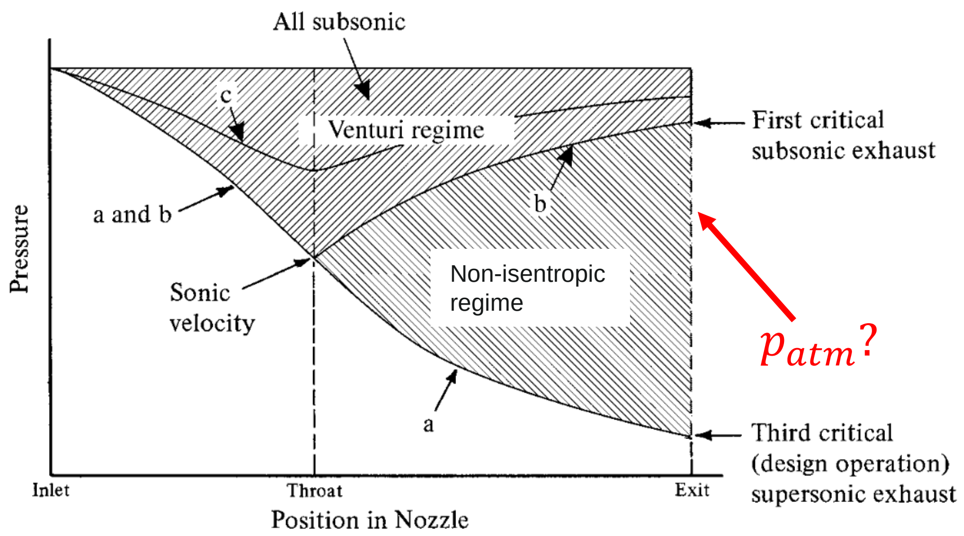
Shocks and Expansions + Nozzles

- In previous discussions about nozzles, we only focused on isentropic behavior
- Quick summary:
  - For a given inlet pressure, if the throat reaches sonic conditions ($M_2 = 1$), there are only two possible outlet Mach numbers for an isentropic nozzle
    - Subsonic outlet – first critical (higher pressure)
    - Supersonic outlet – third critical (lower pressure)
- What happens at outlet pressures between the first and third critical?



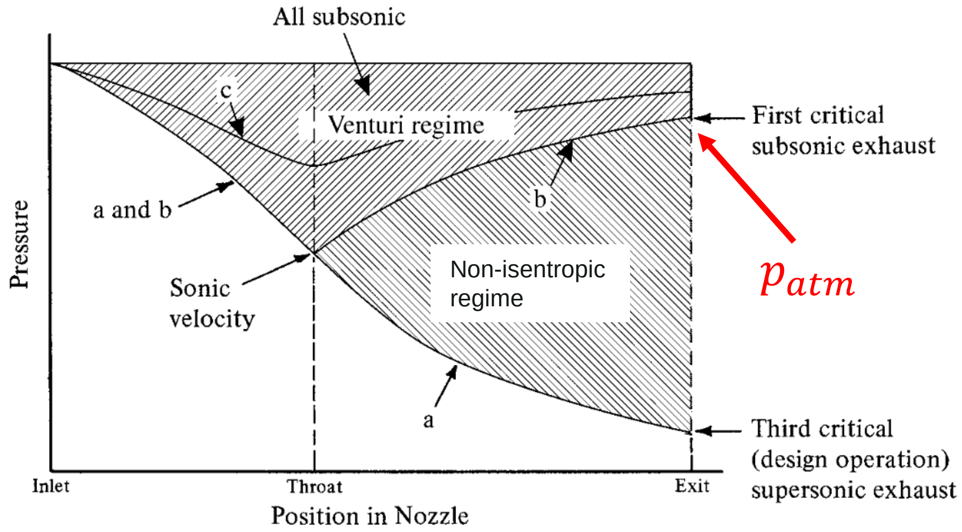
Shocks and Expansions + Nozzles

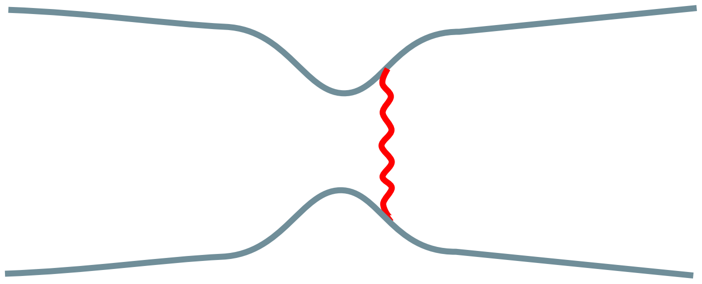
- Remember, we need outlet pressure to match outside pressure
- What if outside pressure is p_{atm} as shown on the right?
  - For the given inlet pressure and area ratio, this is isentropically impossible
  - Luckily, we learned about non-isentropic compressible flows in last lecture



Shocks and Expansions + Nozzles

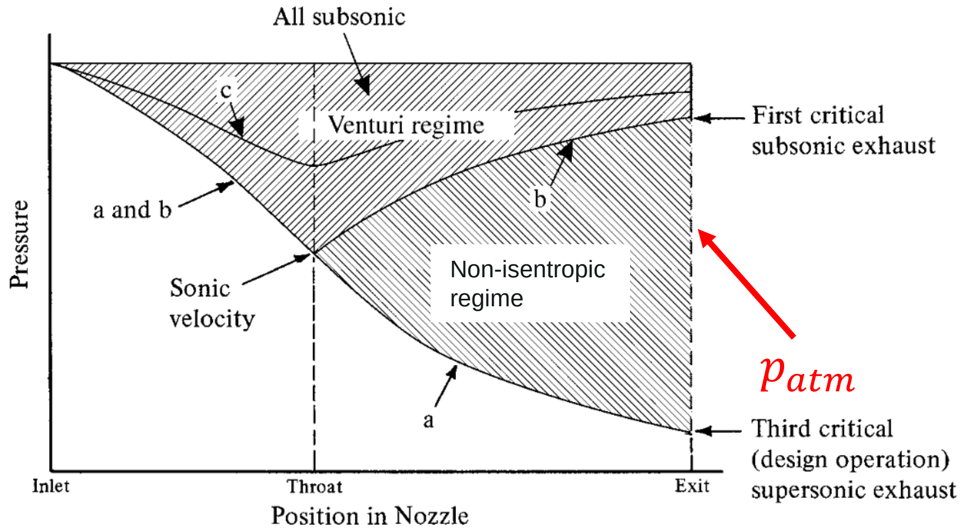
- Imagine an outside pressure slightly below the outlet pressure
  - Need a small non-isentropic adjustment
- A normal shock will form slightly downstream of the throat
  - The Mach number before the shock will be slightly larger than 1
  - The Mach number after the shock will be slightly smaller than 1
  - The pressure after the shock will be slightly higher than before the shock

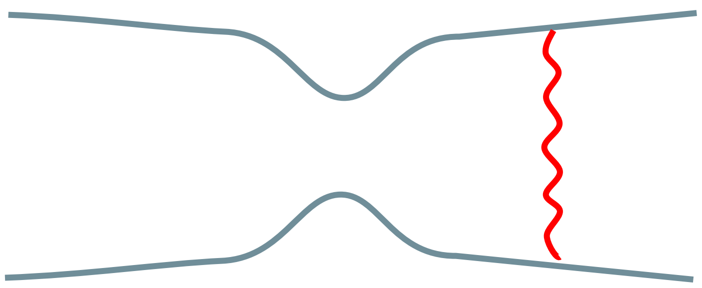




Shocks and Expansions + Nozzles

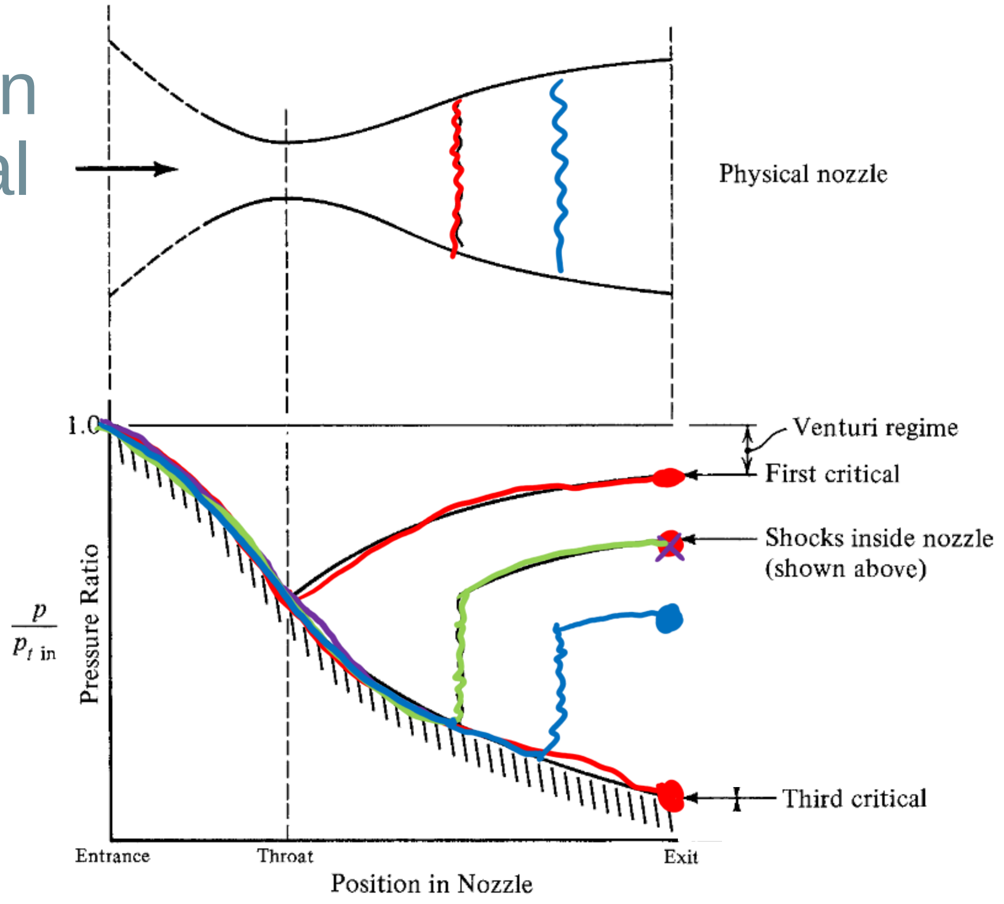
- Imagine an outside pressure far below the outlet pressure
  - Need a larger non-isentropic adjustment
- A normal shock will form far downstream of the throat
  - The Mach number before the shock will be larger than 1
  - The Mach number after the shock will be smaller than 1
  - The pressure after the shock will be higher than before the shock





Shocks and Expansions + Nozzles

- The normal shock occurs at the location that guarantees outlet pressure is equal to outside pressure
- The bigger the difference between first critical and outlet pressure, the further downstream the shock
  - Stronger shock to make up the remaining pressure difference
- After the shockwave, flow is <u>subsonic</u>
  - Subsonic diffuser



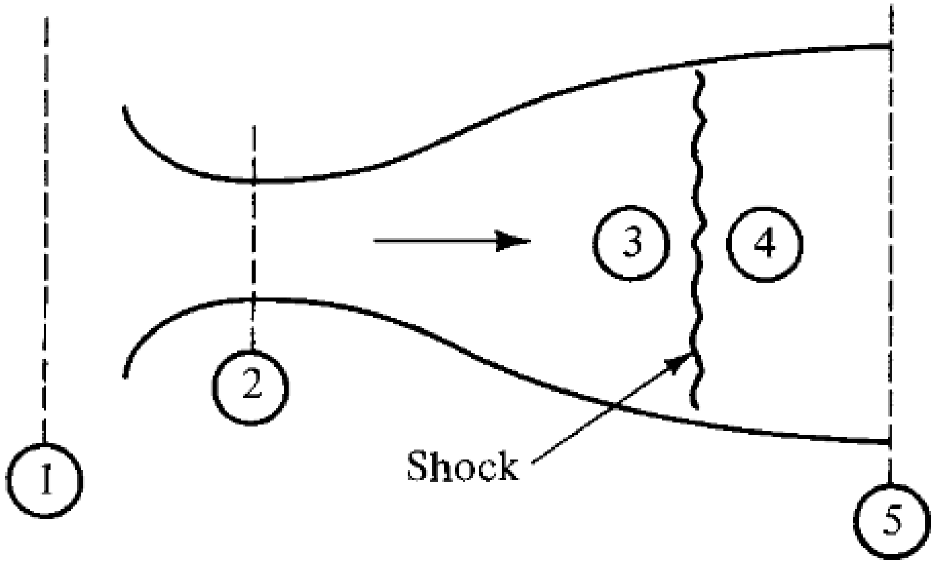
Shocks and Expansions + Nozzles

- Nozzle shock analysis
  - 1 → 2: Subsonic Nozzle
  - 2: Sonic Throat
  - 2 → 3: Supersonic Nozzle
  - 3 → 4: Normal Shock
  - 4 → 5: Subsonic Diffuser
  - 5: Outlet Pressure = Outside Pressure

isentropic

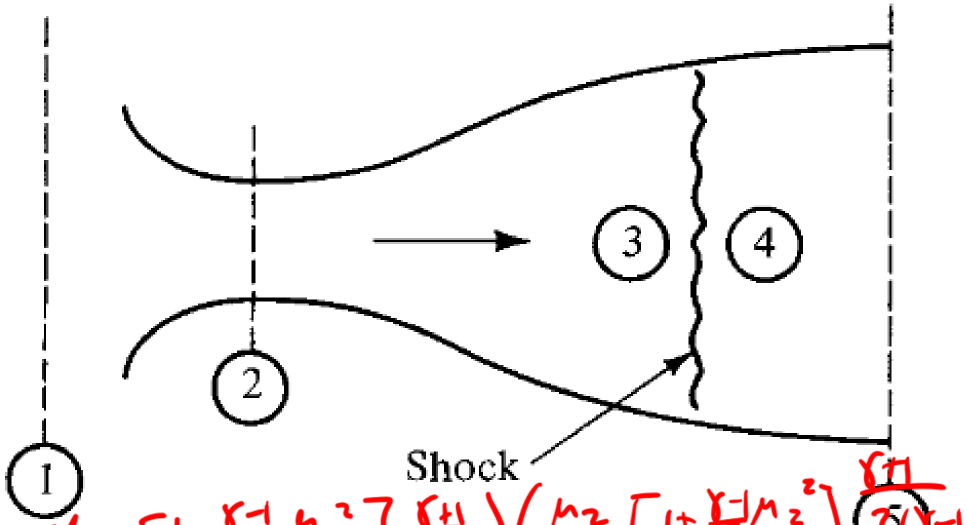
non - isentropic

isentrop



Shocks and Expansions + Nozzles

- Example:
- Find the location of the shock (area ratio at shock)
  - $\frac{A_5}{A_2} = 2.5$
  - $M_5 = 0.38$



$$\frac{A_5}{A_4} = \frac{M_4}{M_5}\left(\frac{1+\frac{\gamma-1}{2}M_5^2}{1+\frac{\gamma-1}{2}M_4^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (1)$$

$$\frac{A_3}{A_2} = \frac{M_2}{M_3}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (2)$$

3 Eqs.

Don't know: $\frac{A_5}{A_4}$, $\frac{A_3}{A_2}$, M_3, M_4

$$\left.\begin{array}{c} \frac{A_5}{A_2} = \frac{A_5}{A_4}\frac{A_4}{A_2} \\ A_3 = A_4 \end{array}\right\} \text{Combine}$$

$$\frac{A_5}{A_2} = \frac{A_5}{A_4}\frac{A_3}{A_2} \quad (3)$$

$$\frac{A_5}{A_2} = \left(\frac{M_4}{M_5}\left[\frac{1+\frac{\gamma-1}{2}M_5^2}{1+\frac{\gamma-1}{2}M_4^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right)\left(\frac{M_2}{M_3}\left[\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right)$$

$$M_4^2 = \frac{M_3^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_3^2 - 1}$$

2 Eq.

2 Un.

Solve for M_3, M_4

Shocks and Expansions + Nozzles

- How to numerically solve the previous example
  - Use whatever solver you're comfortable with

Wolframalpha.com

solve $2.5 = \left(\left(\frac{y}{0.38}\right) * \left(\frac{(1+0.2*(0.38)^2)}{(1+0.2*y^2)}\right)^3\right) * \left(\left(\frac{1}{x}\right) * \left(\frac{(1+0.2*x^2)}{(1+0.2)}\right)^3\right),\ y^2 = \frac{\left(x^2 + \frac{2}{.4}\right)}{\left(\frac{2.8}{0.4} * x^2 - 1\right)}$

NATURAL LANGUAGE | MATH INPUT

$x = -2.19905 - 2.79202\,i$ and $y = 0.366778 - 0.0766685\,i$
$x = -2.19905 + 2.79202\,i$ and $y = 0.366778 + 0.0766685\,i$
$x = -2.12311$ and $y = -0.557837$
$x = -0.893595 - 4.16855\,i$ and $y = -0.326835 + 0.0249879$
$x = -0.893595 + 4.16855\,i$ and $y = -0.326835 - 0.0249879$
$x = -0.579062 - 0.337033\,i$ and $y = -1.09242 + 0.827722\,i$
$x = -0.579062 + 0.337033\,i$ and $y = -1.09242 - 0.827722\,i$
$x = -0.349386 - 0.0962593\,i$ and $y = 1.73061 - 2.63062\,i$
$x = -0.349386 + 0.0962593\,i$ and $y = 1.73061 + 2.63062\,i$
$x = -0.312505 - 0.0289573\,i$ and $y = -0.721048 + 3.76825$
$x = -0.312505 + 0.0289573\,i$ and $y = -0.721048 - 3.76825$
$x = -0.761701\,i$ and $y = -0.93448\,i$
$x = 0.761701\,i$ and $y = 0.93448\,i$
$x = 0.312505 - 0.0289573\,i$ and $y = 0.721048 + 3.76825\,i$
$x = 0.312505 + 0.0289573\,i$ and $y = 0.721048 - 3.76825\,i$
$x = 0.349386 - 0.0962593\,i$ and $y = -1.73061 - 2.63062\,i$
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$x = 2.19905 - 2.79202\,i$ and $y = -0.366778 - 0.0766685\,i$
$x = 2.19905 + 2.79202\,i$ and $y = -0.366778 + 0.0766685\,i$

Find real, non-negative solution

Excel

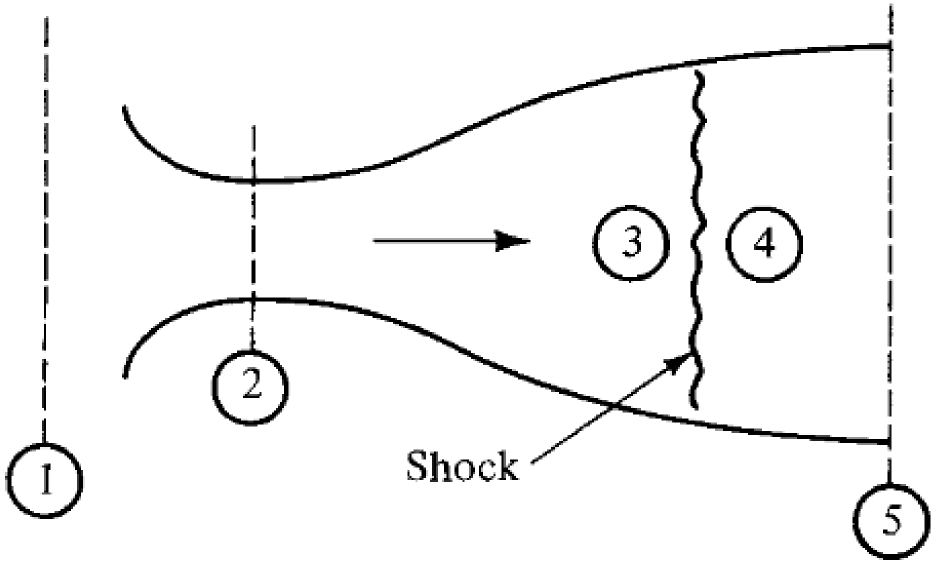
Guess	Normal Shock Relation	A_5/A_2	Other side of equation	
M3	M4	LHS	RHS	DIFF
2	0.57735027	2.5	2.30095202	0.199048
2.1	0.56127694	2.5	2.46023172	0.0397683
2.11	0.55977648	2.5	2.47730087	0.0226991
2.12	0.55829419	2.5	2.49458273	0.0054173
2.13	0.55682978	2.5	2.51207867	-0.0120787
2.14	0.55538294	2.5	2.52979008	-0.0297901
2.15	0.55395342	2.5	2.54771835	-0.0477184

Shocks and Expansions + Nozzles

- Example:
- Find the **location** of the shock (area ratio at shock)
  - $\frac{A_5}{A_2} = 2.5$
  - $M_5 = 0.38$
  - $\boldsymbol{M_3 = 2.12}$
  - $\boldsymbol{M_4 = 0.56}$

$$\frac{A_3}{A_2} = \frac{M_2}{M_3}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

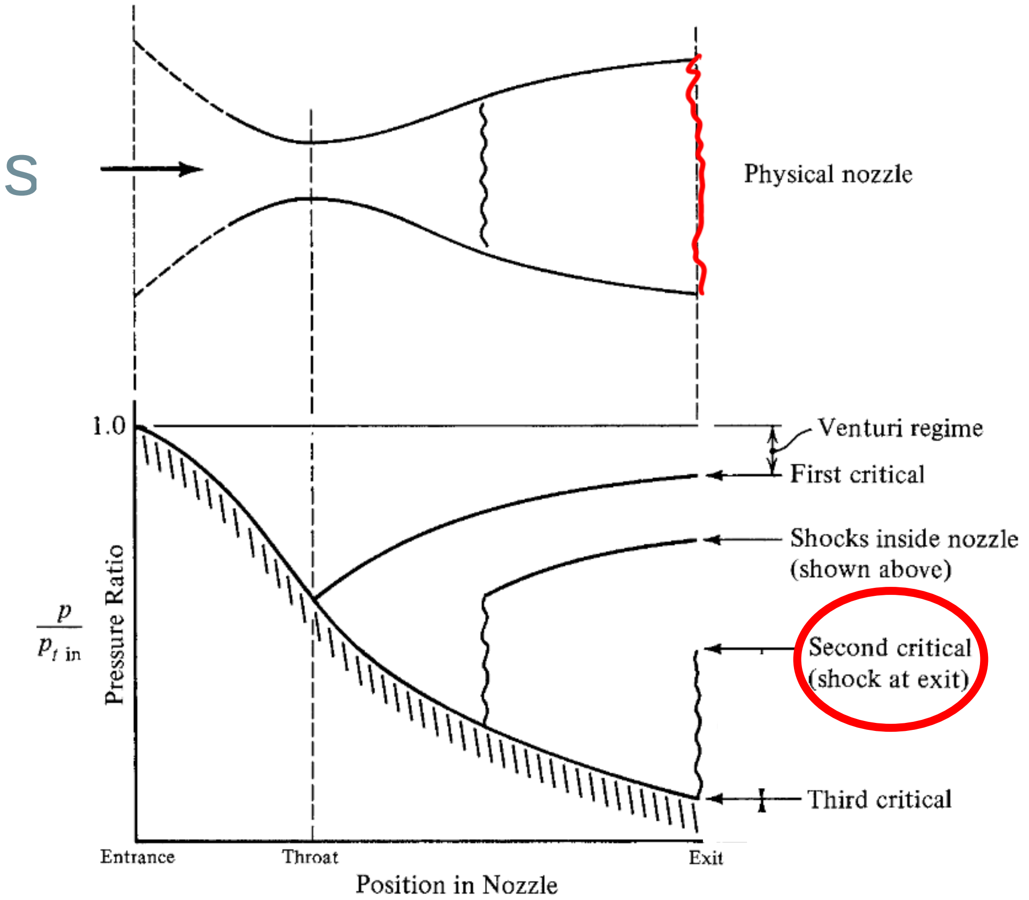
$$\boxed{\frac{A_3}{A_2} = 1.87}$$



Shocks and Expansions + Nozzles

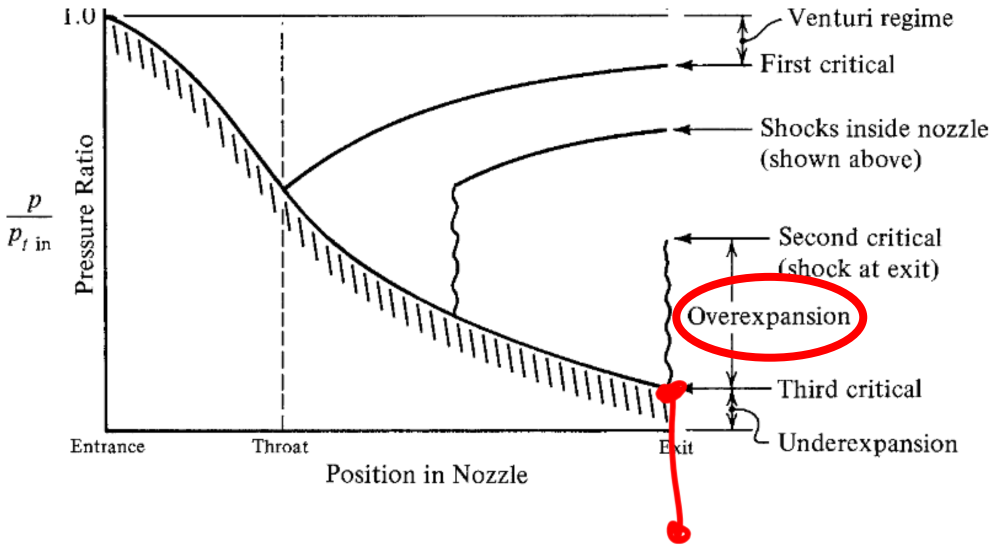
- If we keep lowering the outside pressure, eventually the shock reaches the outlet
  - This is the second critical limit
  - Corresponds to the strongest shock possible within the nozzle

- What if we keep lowering the outside pressure…

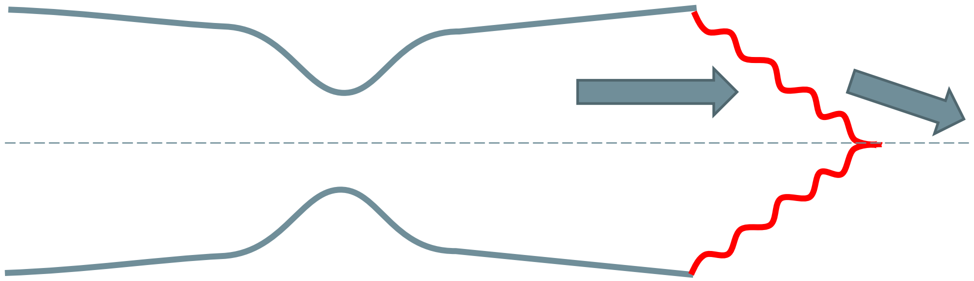


Shocks and Expansions + Nozzles

- Overexpanded nozzle
  - A normal shock within the nozzle can no longer give us the conditions we require
  - Oblique shocks outside the nozzle form

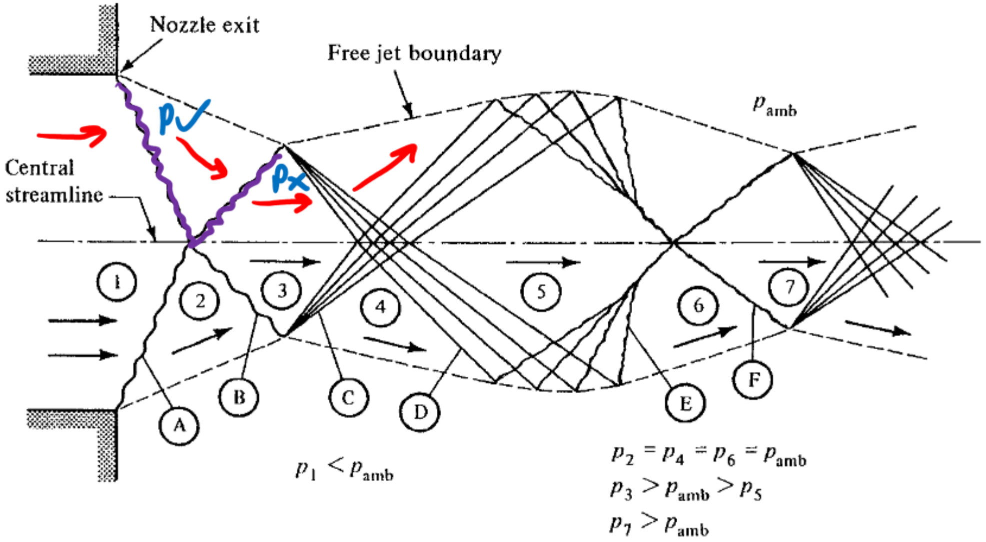


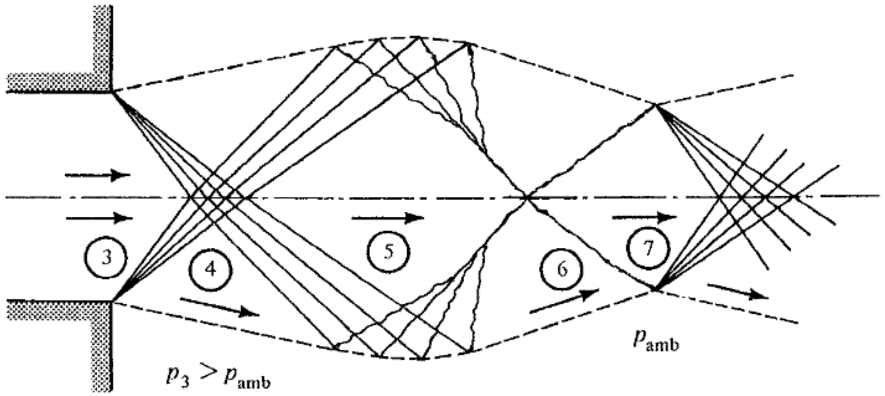
- Outlet pressure equals outside pressure! (This is good)
- Does the flow direction after the shock makes sense…?



Shocks and Expansions + Nozzles

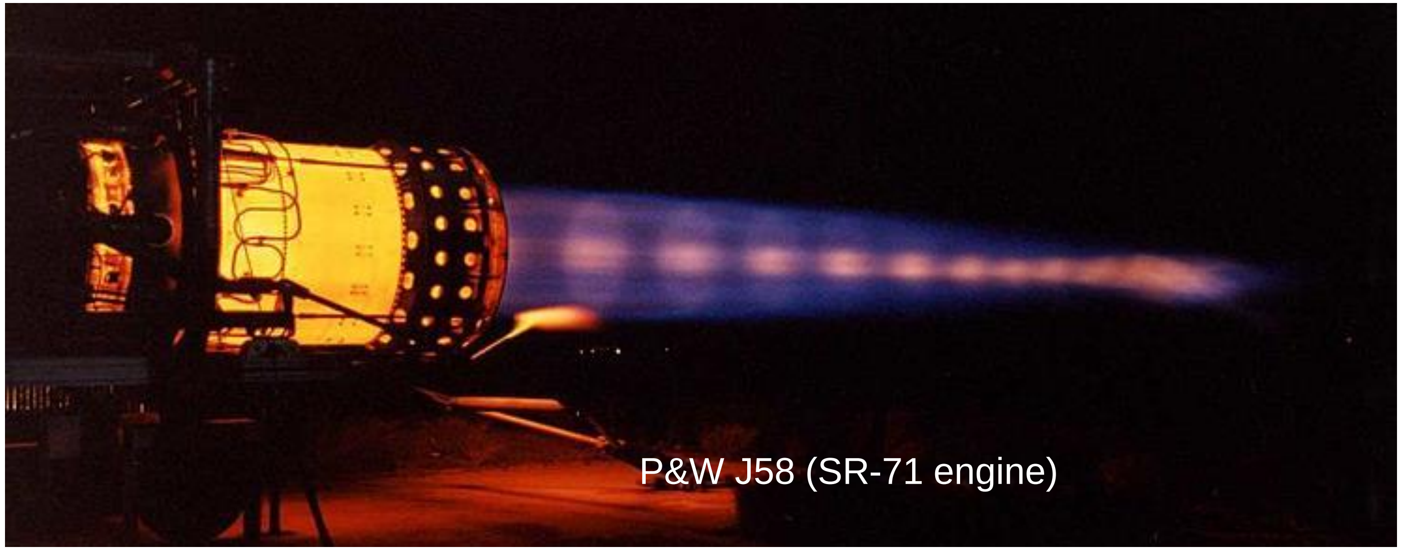
- Overexpanded nozzle
  - An overexpanded nozzle leads to a series of shocks and expansions to alternate matching pressures and flow directions
  - Ends due to non-isentropic effects
- Underexpanded nozzle
  - Outside pressure below third critical (see figure on previous slide)
  - Similar to above, starts with an expansion wave before alternating to shocks





Shocks and Expansions + Nozzles

- Shock/Mach/Thrust Diamonds
  - Common terminology for the visible effect of the pressure waves in over/underexpanded nozzle


P&W J58 (SR-71 engine)

NASA XCOR LOX/Methane

Lunar recovery