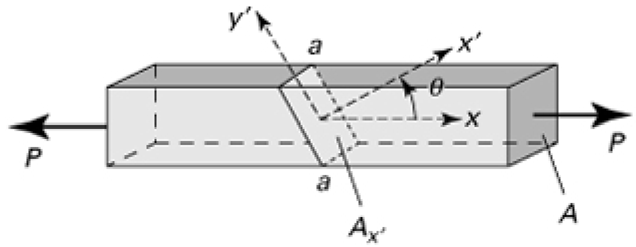
Homework 1
(due Thurs, Jan 18)

1) A square prismatic bar of 1300 mm^2 cross-sectional area is composed of two pieces of wood glued together along the x' plane, which makes an angle θ with the axial direction. Assuming that the normal and shearing stresses acting simultaneously on the joint are 20 and 10 MPa, respectively, determine the applied axial load and the corresponding value of the angle θ.



Refer to Fig. 1.6c. Equations (1.11) by substituting the double angle-trigonometric relations, or Eqs. (1.18) with $\sigma_y = 0$ and $\tau_{xy} = 0$, become

$$\sigma_{x'} = \tfrac{1}{2}\sigma_x + \tfrac{1}{2}\sigma_x \cos 2\theta \qquad \text{and} \qquad |\tau_{x'y'}| = \tfrac{1}{2}\sigma_x \sin 2\theta$$

or

$$20 = \tfrac{P}{2A}(1+\cos 2\theta) \qquad \text{and} \qquad 10 = \tfrac{P}{2A}\sin 2\theta$$

The foregoing lead to

$$2\sin 2\theta - \cos 2\theta = 1 \tag{a}$$

By introducing trigonometric identities, Eq. (a) becomes

$4\sin\theta\cos\theta - 2\cos^2\theta = 0$ or $\tan\theta = 1/2$. Hence

$$\theta = 26.56^\circ \qquad \blacktriangleleft$$

Thus,

$$20 = \tfrac{P}{2(1300)} = (1+0.6)$$

gives

$$P = 32.5\ kN \qquad \blacktriangleleft$$

It can be shown that use of Mohr's circle yields readily the same result.

2) Given zero body forces, use the equilibrium equations to determine whether the following stress distribution can exist for a body in equilibrium

$$\sigma_x = -2c_1xy, \qquad \sigma_y = c_2z^2, \qquad \sigma_z = 0$$
$$\tau_{xy} = c_1(c_2 - y^2) + c_3xz, \qquad \tau_{xz} = -c_3y, \qquad \tau_{yz} = 0$$

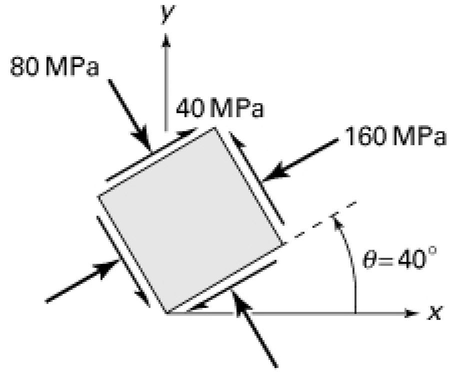
Here the c's are non-zero constants.

Equations (1.14):

$$-2c_1y - 2c_1y + 0 + 0 = 0, \qquad 4c_1y \neq 0$$
$$0 + c_3z + 0 + 0 = 0, \qquad c_3z \neq 0$$
$$0 + 0 + 0 + 0 = 0$$

No. Eqs. (1.14) are not satisfied. ◀

3) At a point in a loaded machine, the normal and shear stresses have the magnitudes and directions acting on the inclined element shown. What are the stresses σ_x, σ_y, and τ_{xy} on an element whose sides are parallel to the xy axes?



Transform from $\theta = 40^o$ to $\theta = 0$. For convenience in computations, Let

$$\sigma_x = -160\ MPa, \qquad \sigma_y = -80\ MPa, \qquad \tau_{xy} = 40\ MPa \text{ and } \theta = -40^o$$

Then

$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(-160-80) + \frac{1}{2}(-160+80)\cos(-80^o) + 40\sin(-80^o)$$

$=$ -166.3 MPa ◀

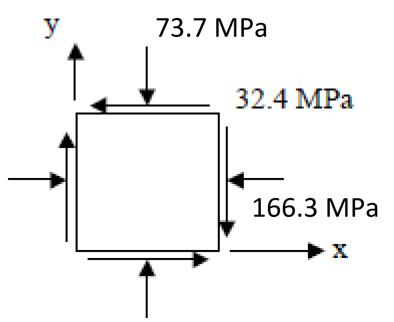
$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}(-160+80)\sin(-80^o) + 40\cos(-80^o)$$

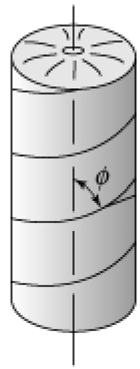
$= -32.4\ MPa$ ◀

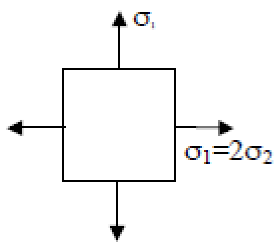
So $\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -160 - 80 + 138.6 =$ -73.7 MPa ◀

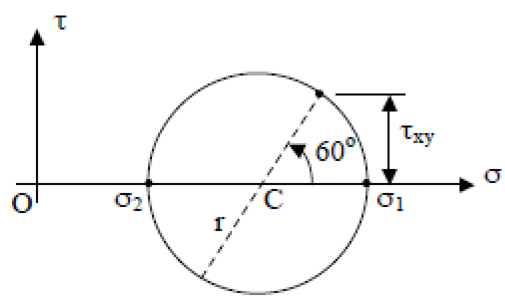
For $\theta = 0^o$:



4) The cylindrical portion of a thin-walled, compressed-air tank is made of 5 mm-thick plate welded along a helix at an angle of $\varphi = 60°$ with the axial direction. The radius of the tank is 250 mm. If the allowable shearing stress parallel to the weld is 30 MPa, calculate the largest internal pressure p that may be applied.







$$r = \frac{2\sigma_2 - \sigma_2}{2} = \frac{\sigma_2}{2}$$

From Mohr's circle,

$$\tau_{x'y'} = \left(\frac{\sigma_2}{2}\right)\sin 60^o = 0.433\sigma_2$$

Therefore

$$30 = 0.433\sigma_2 \qquad \sigma_2 = 69.28\ MPa$$

We have

$$\sigma_2 = \frac{pr}{2t} = 69.28 \qquad p\left(\frac{250}{2}\right) = 69.28$$

Solving

$$p = 2.771\ MPa$$ ◄