Stability: R-H Approach for special cases

Case 1

If a first-column term in any row is zero, but remaining terms in the same row are not zero or there are no remaining terms, then replace zero with a very small number ε and then complete the rest of the table.

Example

$$d(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

1. Create R-H Table

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}\ \varepsilon$	$7/2$	0
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	3	0
s^1	$\frac{42\varepsilon-49-\varepsilon^2}{12\varepsilon-14}$	0	
s^0	3		

$$a_1 = \frac{-\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}}{2}$$

$$a_1 = 0$$

$$a_2 = \frac{-\begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix}}{2} = 7/2$$

Look at first-column for sign changes as $\varepsilon \to 0$

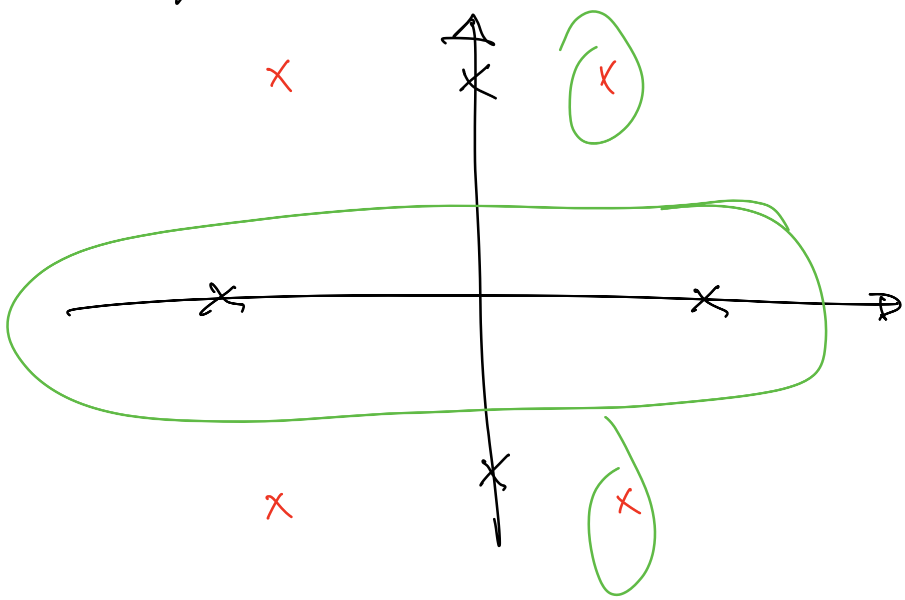
s^5	1	1	
s^4	2	2	
s^3	ε	$+$	1 sign change
s^2	$\dfrac{6\varepsilon-7}{\varepsilon}$	$-$	1 sign change
s^1	$\dfrac{42\varepsilon-49-\varepsilon^2}{12\varepsilon-14}$	$+$	
s^0	3	3	

$\Rightarrow$ 2 signs changes

$\Rightarrow$ $\boxed{\text{2 ORHP poles}}$

<u>Case 2</u> : Row of zeros appears in R-H Table. This occurs when purely even or odd polynomial is a factor of the original polynomial.

Even polynomials only have roots that are symmetrical about the origin.



* if we do not have a row of zeros, we do not have poles on $j\omega$-axis.

<u>example</u>

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

① Create R-H

② we will have row of zeros

③ Deal w/ it

R-H table

s^5	1	6	8
s^4	7	42	56
s^3	$\cancel{0}$ 28	$\cancel{0}$ 84	$\cancel{0}$ 0
s^2	21	56	
s^1	$9\frac{1}{3}$	0	
s^0	56		

Above row of zeros.

we have a row of zeros so there are complex congugate pairs of roots that are mirror images of each other.

To handle this we will create an auxillary polynomial to finish table.

Auxillary Equation: $7s^4 + 42s^2 + 56 = A(s)$

Then, find $\frac{dA(s)}{ds} = 28s^3 + 84s + 0$