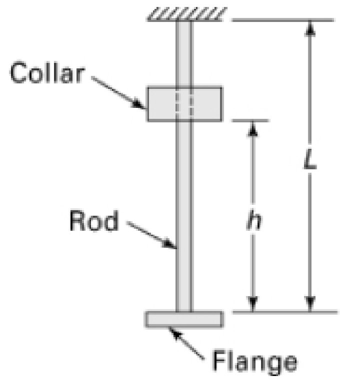
Homework 6 Solutions

1) A sliding collar of $m = 80\ kg$ falls onto a flange at the bottom of a vertical rod. Calculate the height h through which the mass m should drop to produce a maximum stress in the rod of $350\ MPa$. The rod has length $L = 2m$, cross-sectional area $A = 250\ mm^2$, and modulus of elasticity $E = 105\ GPa$.



We have $W = mg = 80 \times 9.81 = 784.8\ N$

From Eq. (4.29);

$$\sigma_{\max} = \left(1 + \sqrt{1 + \tfrac{2h}{\delta_{st}}}\right)\tfrac{W}{A}$$

Solving, with $\delta_{st} = WL/AE$, we obtain

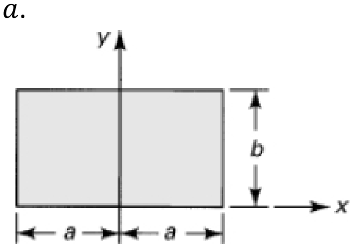
$$h = \tfrac{L\sigma_{\max}}{2EW}(A\sigma_{\max} - 2W) \tag{a}$$

Substituting the given data:

$$h = \frac{(2)(350\times10^6)}{2(105\times10^9)(784.8)}(250 \times 350 - 1569.6) = 0.365\ m$$ ◀

2) If the given stress field acts in the thin plate shown and p is a known constant, determine the c's so that edges $x = \pm a$ are free of shearing stress and no normal stress acts on edge $x = a$.

$$\sigma_x = pyx^3 - 2c_1xy + c_2y$$
$$\sigma_y = pxy^3 - 2px^3y$$
$$\tau_{xy} = -\tfrac{3}{2}px^2y^2 + c_1y^2 + \tfrac{1}{2}px^4 + c_3$$



Edge $x = \pm a$:

$$\tau_{xy} = 0: \qquad -\tfrac{3}{2}pa^2y^2 + c_1y^2 + \tfrac{1}{2}pa^4 + c_3 = 0$$

$$\tau_{xy} = 0: \qquad -\tfrac{3}{2}pa^2y^2 + c_1y^2 + \tfrac{1}{2}pa^4 + c_3 = 0$$

Adding, $(-3pa^2 + 2c_1)y^2 + pa^4 + 2c_3 = 0$

or $\quad c_1 = \tfrac{3}{2}pa^2 \qquad c_3 = -\tfrac{1}{2}pa^4$ ◀

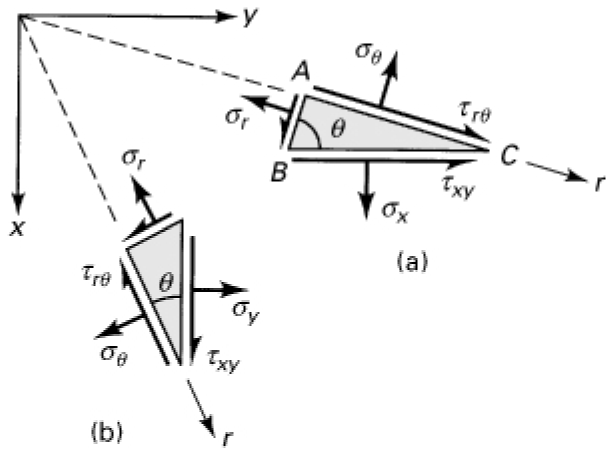
Edge $x = a$:

$$\sigma_x = 0: \qquad pa^3y - 2c_1ay + c_2y = 0$$

or

$$c_2 = 2pa^3$$ ◀

3) Verify that Eqs. (3.37) in the text are determined from the equilibrium of forces acting on the elements shown.



Refer to Fig. P3.26a. Let $A_{BC} = 1$ and hence $A_{AB} = \cos\theta$, $A_{AC} = \sin\theta$.

$\sum F_x = 0$:

$$\sigma_x = \sigma_r \cos\theta \cos\theta + \sigma_\theta \sin\theta \sin\theta - 2\tau_{r\theta} \sin\theta \cos\theta$$

$\sum F_y = 0$:

$$\tau_{xy} = \sigma_r \cos\theta \sin\theta - \sigma_\theta \sin\theta \cos\theta + \tau_{r\theta} \cos\theta \cos\theta - \tau_{r\theta} \sin\theta \sin\theta$$

Similarly, from Fig. P3.26b:

$\sum F_y = 0$:

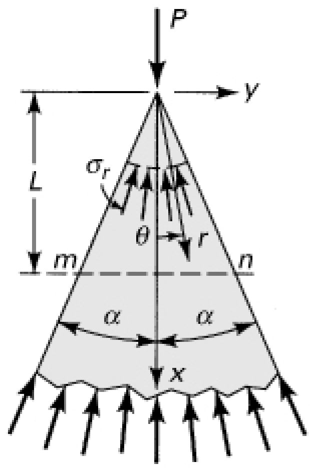
$$\sigma_y = \sigma_r \sin^2\theta + \sigma_\theta \cos^2\theta + 2\tau_{r\theta} \sin\theta \cos\theta$$

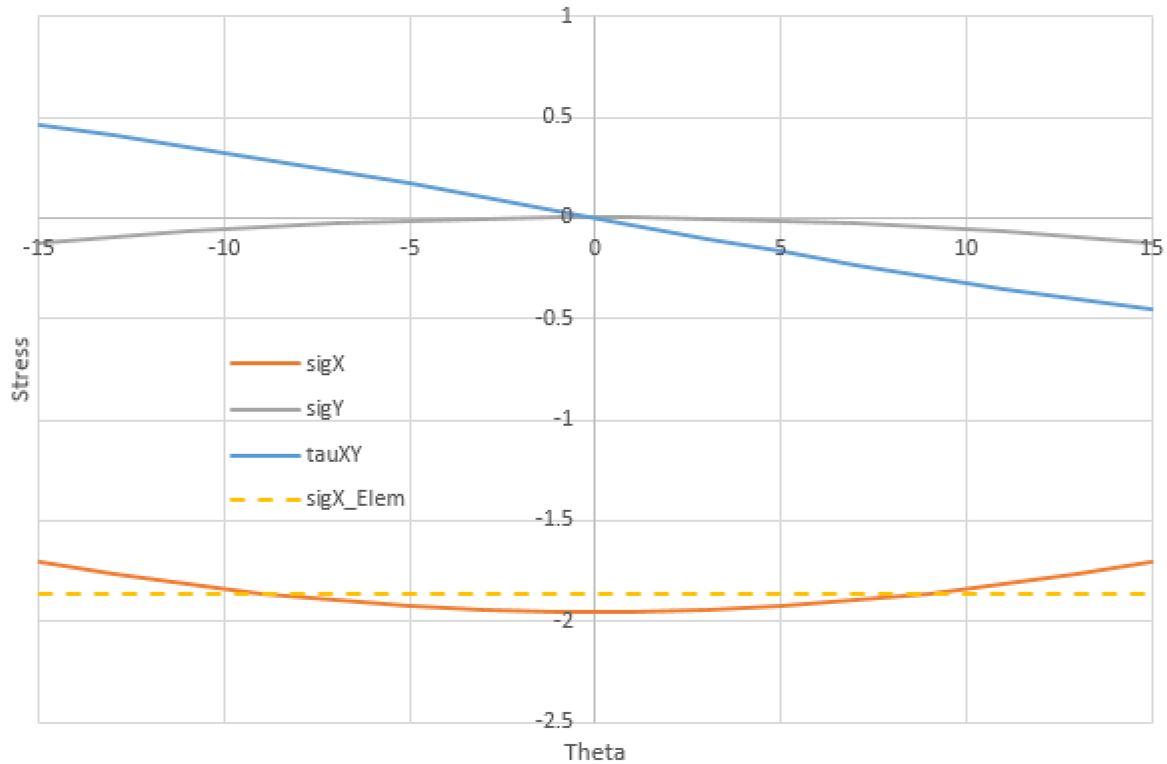
Check: $\sum F_x = 0$:

$$\tau_{xy} = \sigma_r \sin\theta \cos\theta - \sigma_\theta \sin\theta \cos\theta + \tau_{r\theta}(\cos^2\theta - \sin^2\theta)$$

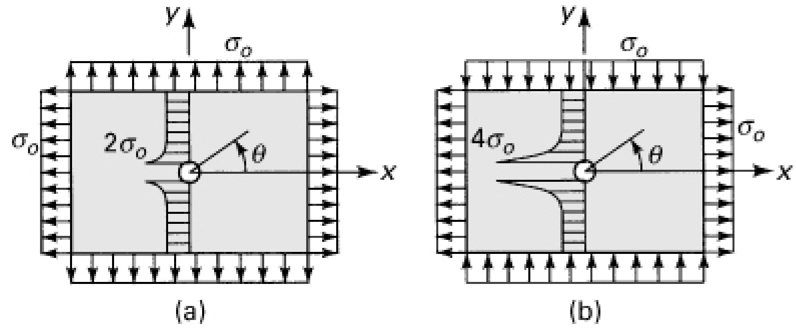
Thus, quoted equations are derived.

4) Consider the pivot of unit thickness subject to force $P = 1\ N$ per unit thickness at its vertex. Plot the values of σ_x, σ_y, and τ_{xy} as a function of θ (in deg) at section m-n a distance $L = 1\ m$ from the apex using Eq'ns. (3.37) and (3.43). Also plot σ_x using the elementary (mechanics of materials) approach for comparison. Take $\alpha = 15°$.

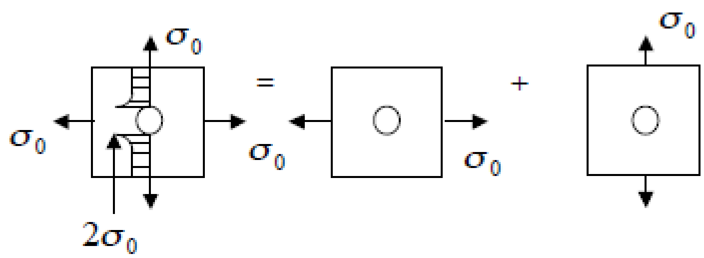




5) Verify the results given below by employing Eq. (3.55b) and the method of superposition.



(a)



$$\sigma_{r1} = \frac{\sigma_0}{2}[(1-\frac{a^2}{r^2}) + (1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2})\cos 2\theta]$$
$$\sigma_{\theta 1} = \frac{\sigma_0}{2}[(1+\frac{a^2}{r^2}) - (1+\frac{3a^4}{r^4})\cos 2\theta]$$
$$\tau_{r\theta 1} = -\frac{\sigma_0}{2}(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2})\sin 2\theta$$

and

$$\sigma_{r2} = \frac{\sigma_0}{2}[(1-\frac{a^2}{r^2}) + (1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2})\cos 2(\theta+90^o)]$$
$$\sigma_{\theta 2} = \frac{\sigma_0}{2}[(1+\frac{a^2}{r^2}) - (1+\frac{3a^4}{r^4})\cos 2(\theta+90^o)]$$
$$\tau_{r\theta 2} = -\frac{\sigma_0}{2}(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2})\cos 2(\theta+90^o)$$

We have, by superposition:

$$\sigma_r = \sigma_{r1} + \sigma_{r2} \qquad \sigma_\theta = \sigma_{\theta 1} + \sigma_{\theta 2} \qquad \tau_{r\theta} = \tau_{r\theta 1} + \tau_{r\theta 2}$$

Hence, at r=a and $\theta = \pi/2$,

$$\sigma_{r1} = 0 \qquad \sigma_{r2} = 0$$
$$\sigma_{\theta 1} = 3\sigma_0 \qquad \sigma_{\theta 2} = -\sigma_0$$
$$\tau_{r\theta 1} = 0 \qquad \tau_{r\theta 2} = 0$$

lead to the solution:

$$\sigma_r = 0 \qquad \sigma_\theta = 2\sigma_0 \qquad \tau_{r\theta} = 0$$ ◀

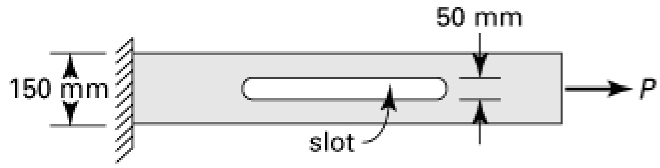
(b) Referring to the results of part (a), we write

$$\sigma_{r1} = 0 \qquad \sigma_{r2} = 0$$
$$\sigma_{\theta 1} = 3\sigma_0 \qquad \sigma_{\theta 2} = \sigma_0$$
$$\tau_{r\theta 1} = 0 \qquad \tau_{r\theta 2} = 0$$

Thus,

$$\sigma_r = 0 \qquad \sigma_\theta = 4\sigma_0 \qquad \tau_{r\theta} = 0$$ ◀

6) A 20 mm-thick steel bar with a slot (25 mm radii at ends) is subjected to an axial load P, as shown. What is the maximum stress for $P = 180\ kN$? Use Appendix D to estimate the value of the K.



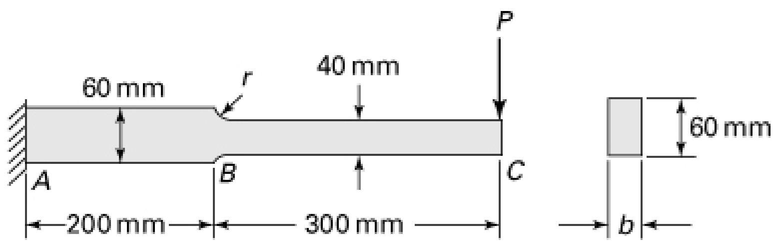
We have

$$\frac{d}{D} = \frac{1}{3}$$

Then, from Fig. D.8: $K \approx 2.3$. Hence

$$\sigma_{max} = K\frac{P}{A} = 2.3\frac{180(10^3)}{(150-50)20} = 207\ MPa$$ ◀

7) The figure depicts a filleted cantilever spring. Find the largest bending stress for two cases: (a) the fillet radius is $r = 5\ mm$; (b) the fillet radius is $r = 10\ mm$. Given: $b = 12\ mm$ and $P = 400\ N$.
NOTE: The parameter r = 5 mm in Part (a) does not any sense in practical engineering (while it is actually possible in theory). You can ignore this part and work on Part (b) only



<u>At a section through B</u>

$$M_B = 400(0.3) = 120\ N\cdot m$$

$$\sigma_{nom} = \frac{M_B c}{I} = \frac{120(0.02)}{\frac{1}{12}(0.012)(0.04)^3} = 37.5\ MPa$$

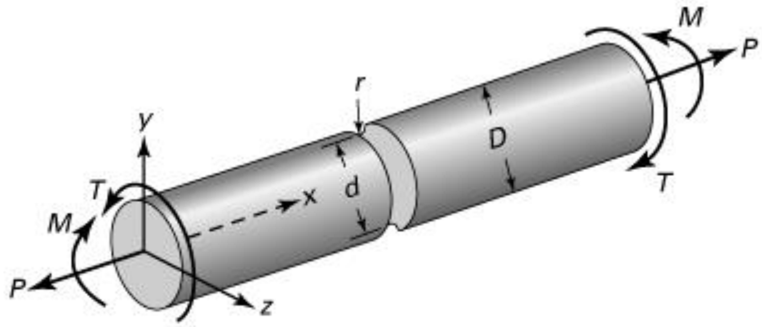
(a) $\frac{r}{d} = \frac{5}{40} = 0.125 \qquad \frac{D}{d} = \frac{60}{40} = 1.5: \qquad K \approx 1.65$ (Fig. D.2)

$$\sigma_{max} = 1.65(37.5) = 61.9\ MPa$$ ◀

(b) $\frac{r}{d} = \frac{10}{40} = 0.25 \qquad \frac{D}{d} = \frac{60}{40} = 1.5: \qquad K \approx 1.41$ (Fig. D.2)

$$\sigma_{max} = 1.41(37.5) = 52.8\ MPa$$ ◀

8) The shaft shown has the following dimensions: $r = 20\ mm$, $d = 400\ mm$, and $D = 440\ mm$. The shaft is subjected simultaneously to a torque $T = 20\ kN \cdot m$, a bending moment $M = 10\ kN \cdot m$, and an axial force $P = 50\ kN$. Calculate at the root of the notch (a) the maximum principal stress, (b) the maximum shear stress, and (c) the octahedral stresses.



(a) We have D/d=1.1 and r/d=0.05. Then, we find from Figs. D.6, D.7, and D.5 that

$$K_t = 1.64 \qquad K_b = 2.2 \qquad K_a = 2.3$$

Then, Eqs. (b) of Example 3.5 yield

$$\sigma_x = 2.3\frac{50(10^3)}{\pi(0.2)^2} + 2.2\frac{4(10\times10^3)}{\pi(0.2)^3} = 4.42\ MPa$$

$$\tau_{xy} = 1.64\frac{2(20\times10^3)}{\pi(0.2)^3} = 2.61\ MPa$$

Equation (a) of Example 3.4 is therefore

$$\sigma_{1,2} = \tfrac{4.42}{2} \pm [(\tfrac{4.42}{2})^2 + (2.61)^2]^{\frac{1}{2}}$$

or $\sigma_1 = 5.63\ MPa \qquad \sigma_2 = -1.21\ MPa$ ◀

(b) $\tau_{max} = \frac{1}{2}(5.63 + 1.21) = 3.42\ MPa$ ◀

(c) $\sigma_{oct} = \frac{1}{3}(5.63 - 1.21) = 1.47\ MPa$ ◀

$\tau_{oct} = \frac{1}{3}[(5.63 + 1.21)^2 + (-1.21)^2 + (-5.63)^2]^{\frac{1}{2}} = 2.98\ MPa$ ◀