# Blackbody Radiation

## Thermal Fluids and Energy Systems Lab

## (ME EN 4650)

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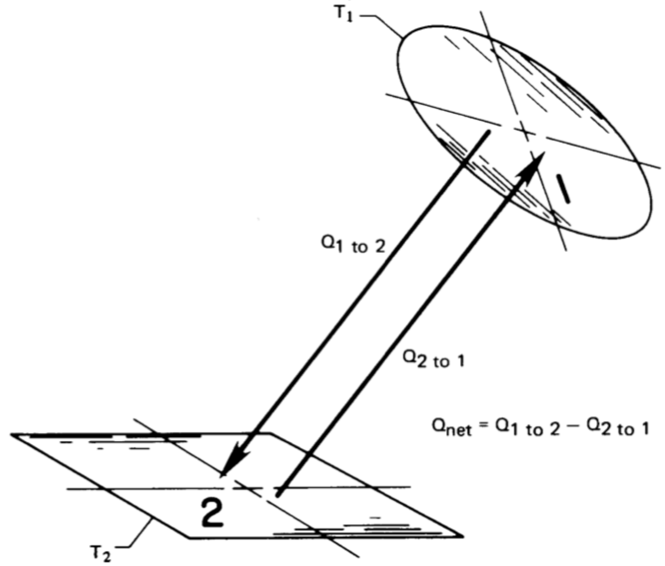
Based on Prof. M's slides

# Net Radiation Exchange




The net radiation exchange from a hotter to cooler surface depends on:

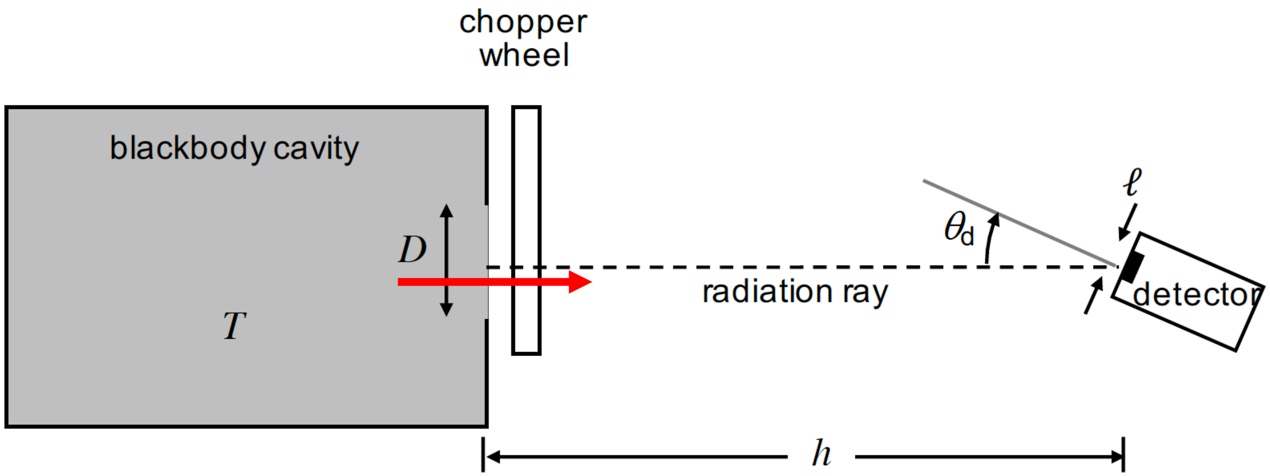
1. The temperatures of surface 1 and 2: $T_1$ and $T_2$
2. The areas of surface 1 and 2: $A_1$ and $A_2$
3. The shape, orientation, and spacing of surfaces 1 and 2
4. The radiative properties of the surface (e.g., $\varepsilon$, $\alpha$, $\rho$)
5. Additional surfaces in the environment that may reflect radiation from surface 1 to surface 2 and vice versa
6. The medium between surfaces 1 and 2 (e.g., if it absorbs or emits radiation)

# Heat Transfer by Thermal Radiation

- What we want to know (simplify the problem):

$$q_{\mathrm{rad}} = f(T, h, D)$$

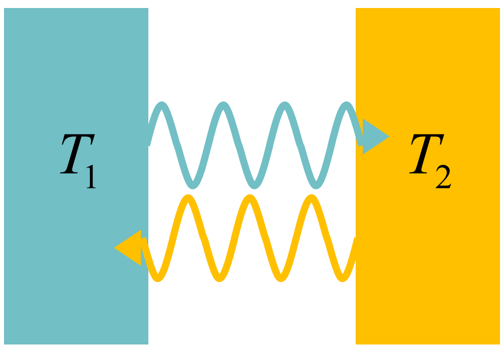
Use our data to validate the theory




- What we can measure:
  - $q_{\mathrm{rad}}$ (pyroelectric radiometer)

# Thermal Radiation & Electromagnetic Waves

**Thermal radiation**: Heat transfer via electromagnetic waves

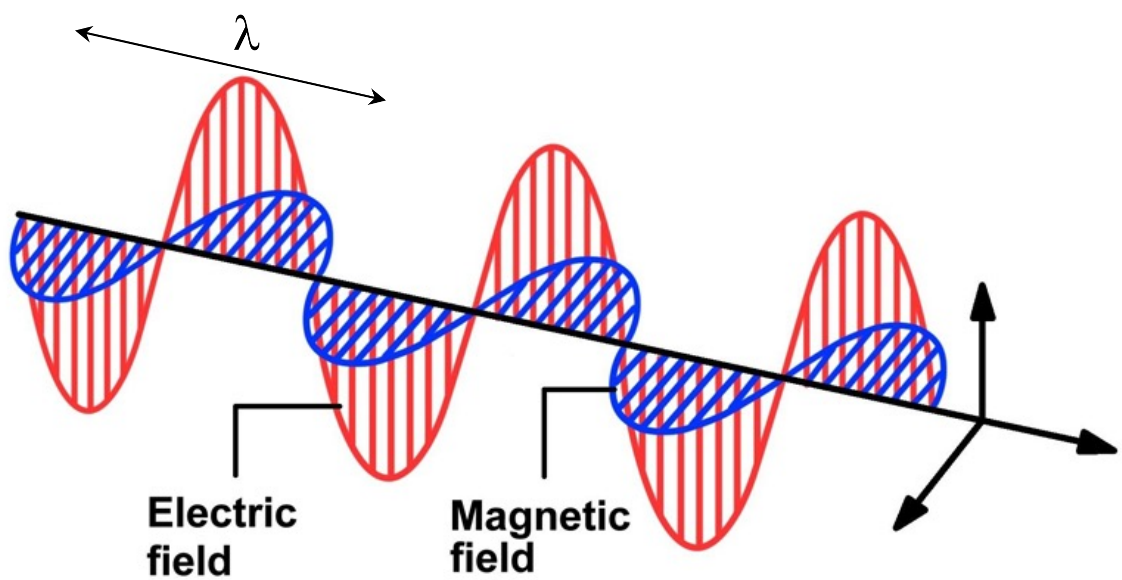



- ***Heat transfer by radiation <u>requires no matter</u>***
- ***Matter at a finite temperature will emit thermal radiation – the mechanism of emission is related to energy released as a result of oscillations or transitions of the electrons – the oscillations are sustained by the internal energy of the matter***

$$\lambda = \frac{c}{\nu} = \frac{c_0 / n}{\nu}$$

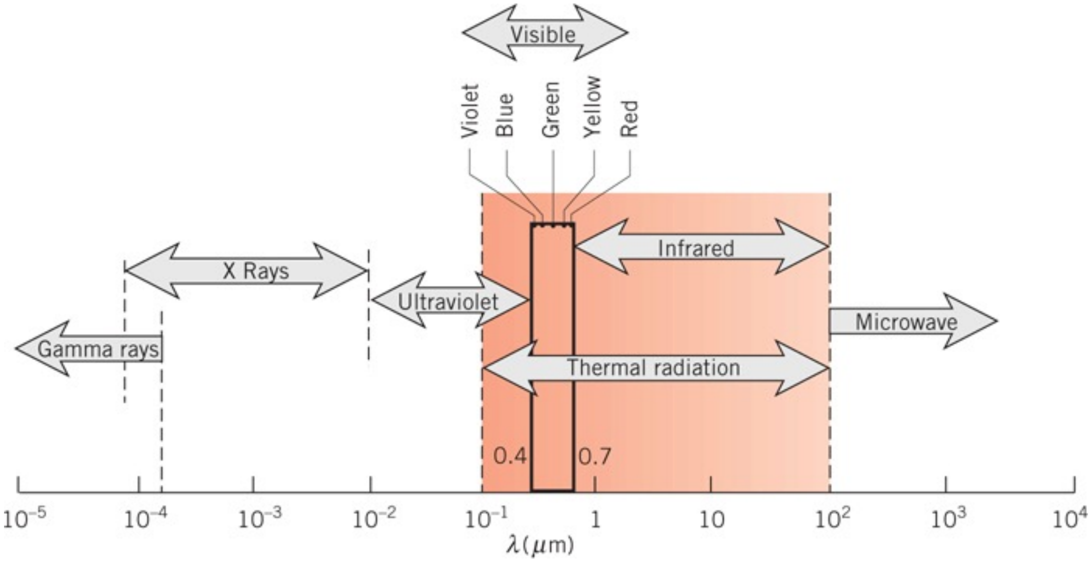
frequency

- $c$ is speed of light in a medium of refractive index $n$
- $c_0$ is speed of light in vacuum ($2.998 \times 10^8$ m/s)

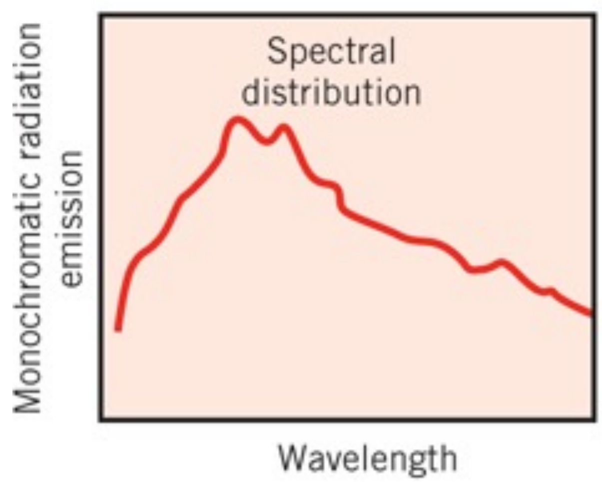


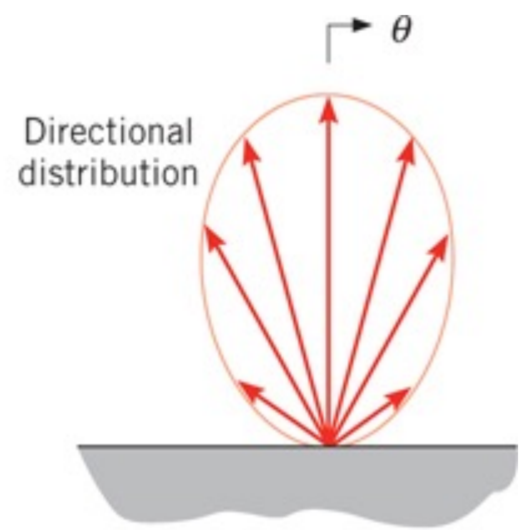
# Electromagnetic Waves

***Thermal radiation:***
***Part of the UV + Visible + IR***




- Complexity of radiation: up to 7 independent variables
  - Space: $x, y, z$
  - Time: $t$
  - Direction: $\theta, \phi$
  - Wavelength: $\lambda$



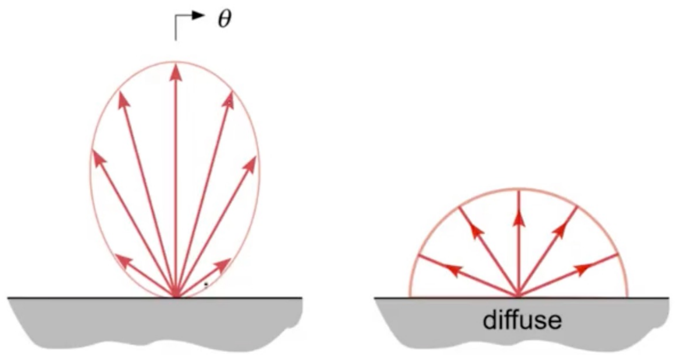
# Blackbody Radiation

**Blackbody** (standard against which radiative properties of real surfaces may be compared):

**(1)** A blackbody absorbs all incident radiation, regardless of wavelength and direction

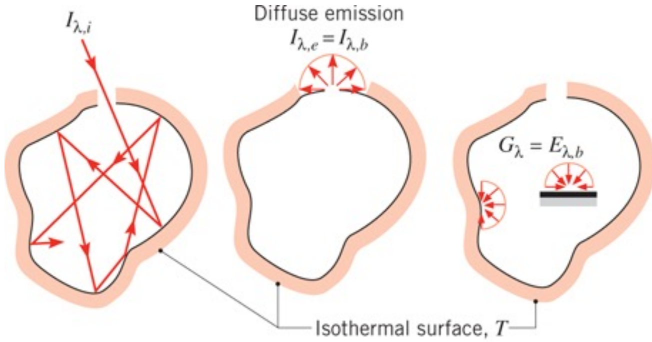
**(2)** For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody

**(3)** Radiation emitted by a blackbody is a function of wavelength and temperature, but it is independent of direction: the blackbody is a diffuse emitter.



# Blackbody Radiation – Planck Distribution

- A blackbody can be approximated by a cavity with inner surface at a uniform temperature




- Blackbody spectral intensity:

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda k_B T)-1]} \quad \left[\frac{W}{\mu m \cdot m^2 \cdot sr}\right]$$

- $h = 6.626 \times 10^{-34}$ J·s is the Planck constant
- $k_B = 1.381 \times 10^{-23}$ J/K is the Boltzmann constant
- $T$ is the absolute temperature of the blackbody [K]

# Planck Distribution

- Blackbody spectral emissive power:

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T)$$

$$\left[\frac{W}{\mu m \cdot m^2}\right]$$

***Diffuse emitter***

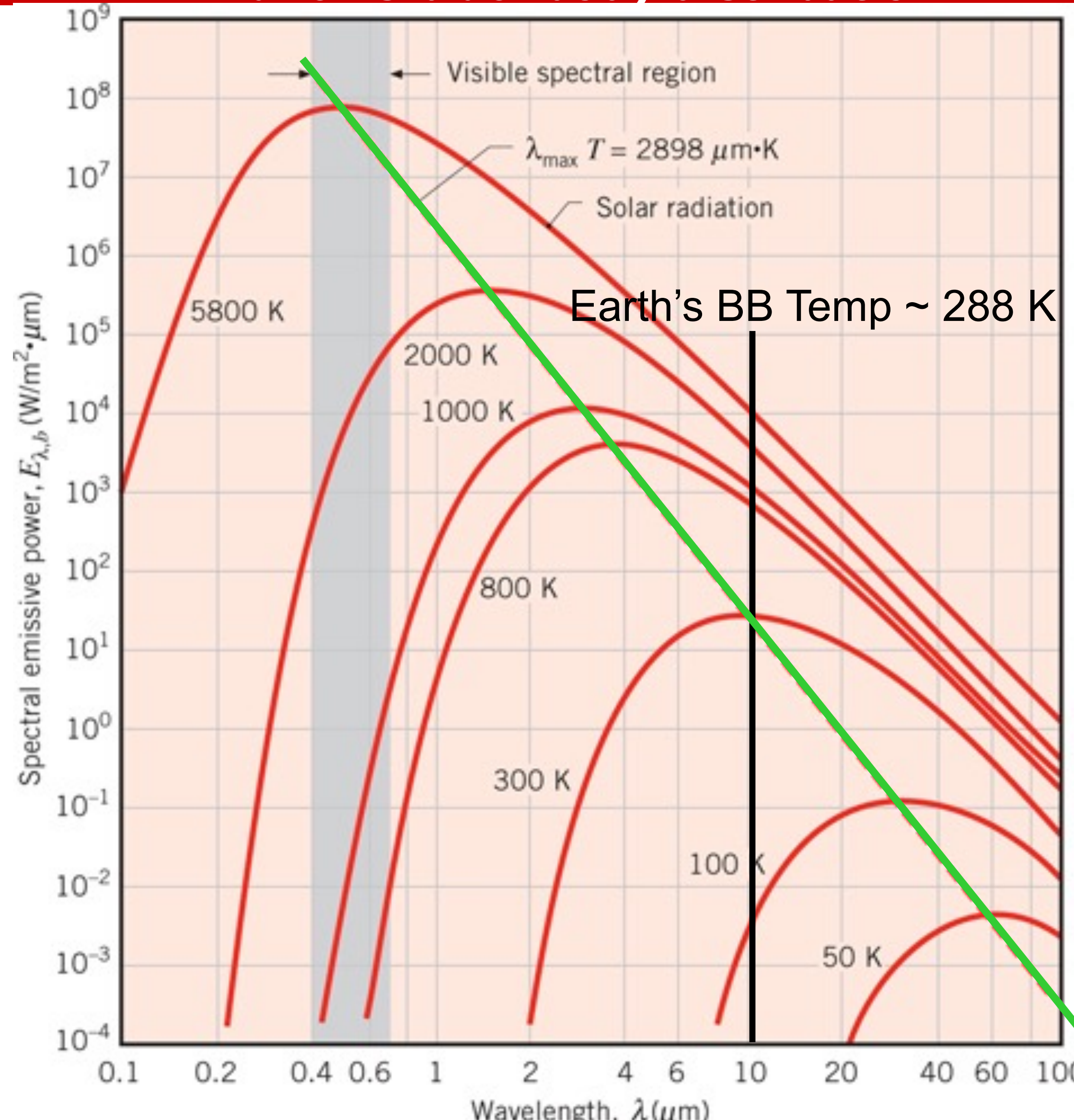
- Wien's displacement law:

$$\lambda_{\max} T = \text{Constant}$$

$$\lambda_{\max} T = 2898 \;\mu m \cdot K$$

***Wavelength leading to the maximum emissive power for a given temperature***

***Planck's blackbody distribution***



# Spherical Coordinates & Solid Angles

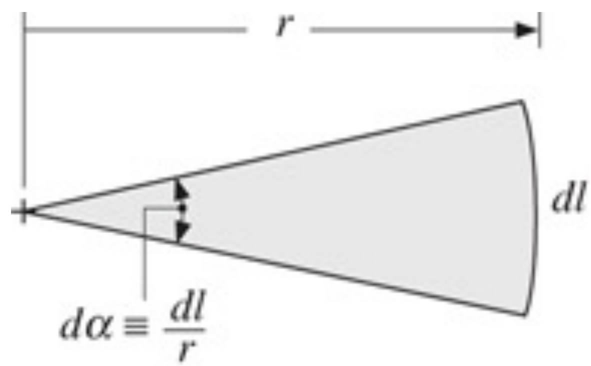
- Directional distribution of thermal radiation is described via solid angles

- Solid angles are 2D angular spaces:

  - 1D angular space: $d\alpha = \dfrac{dl}{r}$

    **radians [rad]**

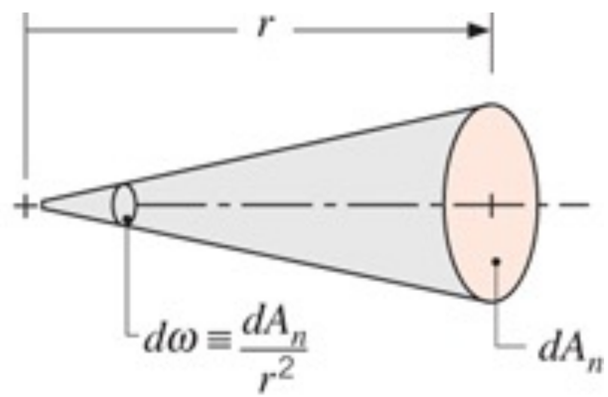
    $dl$: infinitesimal length on a **circle**




  - 2D angular space: $d\omega = \dfrac{dA_n}{r^2}$

    **steradians [sr]**

    $dA_n$: infinitesimal area on a **sphere**



# Spherical Coordinates & Solid Angles

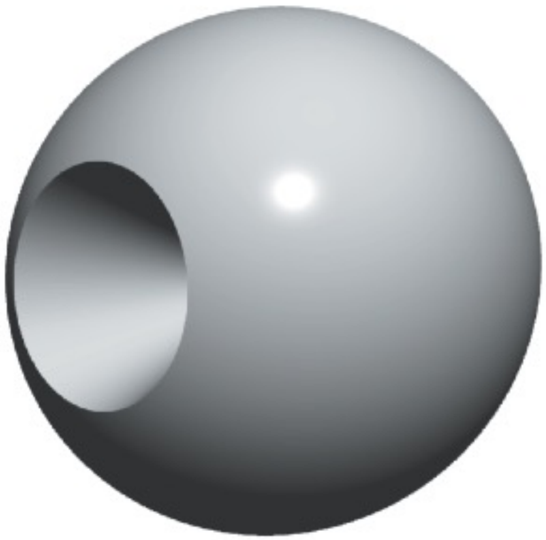


Figure 3.2 — A 1-steradian solid angle removed from a sphere.

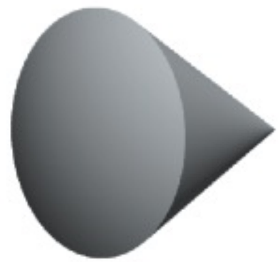


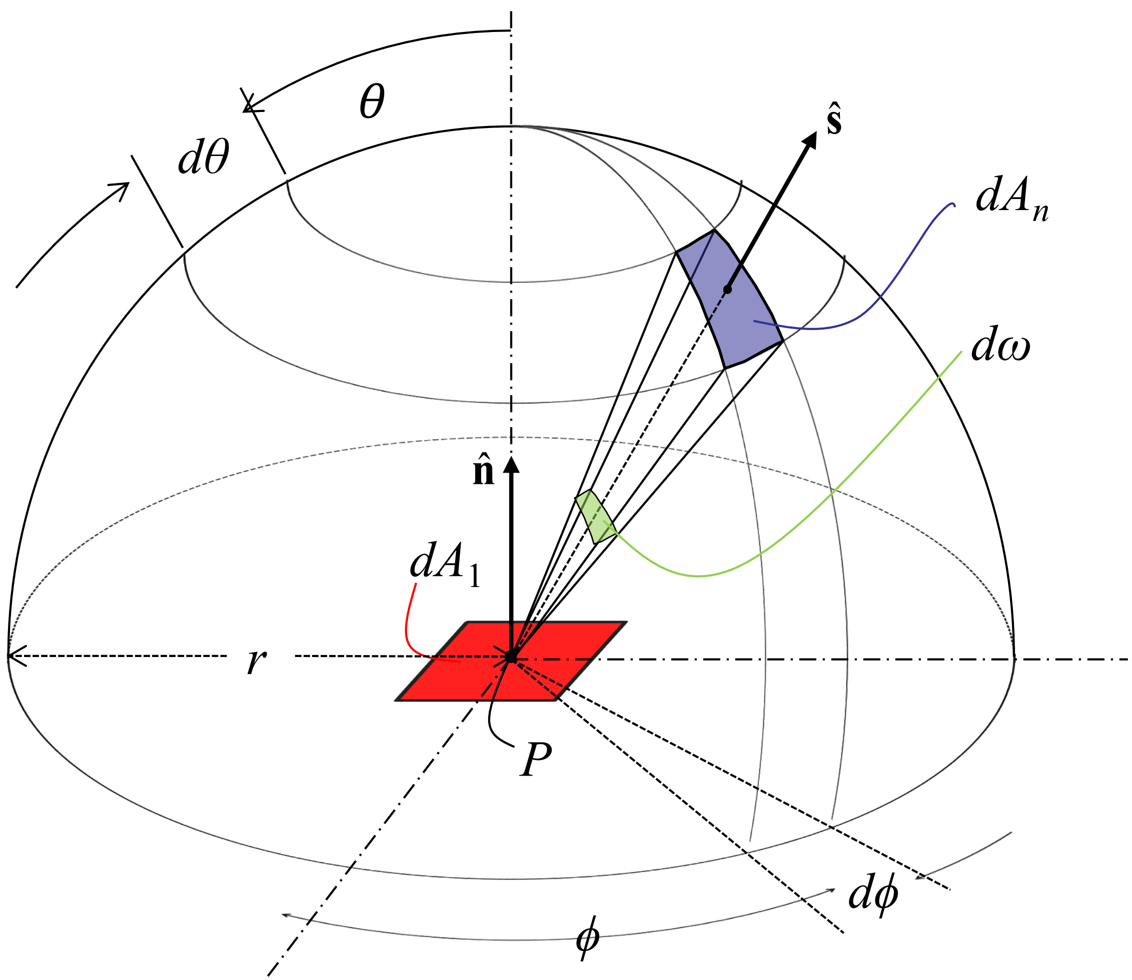
Figure 3.3 — For a solid angle that measures 1 steradian, A = $r^2$.

Adapted by James J. Gross from The Light Measurement Handbook.

$$d\omega = \frac{dA_n}{r^2}$$

# Spherical Coordinates & Solid Angles

- Let's consider an emitting point $P$ on a surface. Point $P$ can radiate into all directions contained within a hemisphere of radius $r$.




$dA_n$: infinitesimal area on the hemisphere of radius $r$

$\theta$: polar angle

$\phi$: azimuthal angle

$d\omega$: infinitesimal solid angle

We say, "The area $dA_n$, through which the radiation passes, **subtends** a differential solid angle $d\omega$ when viewed from a point on $dA_1$"

# Spherical Coordinates & Solid Angles

- Let's take a closer look at a particular direction $\hat{\mathbf{s}}$
  - The infinitesimal area $dA_n$ is given by:

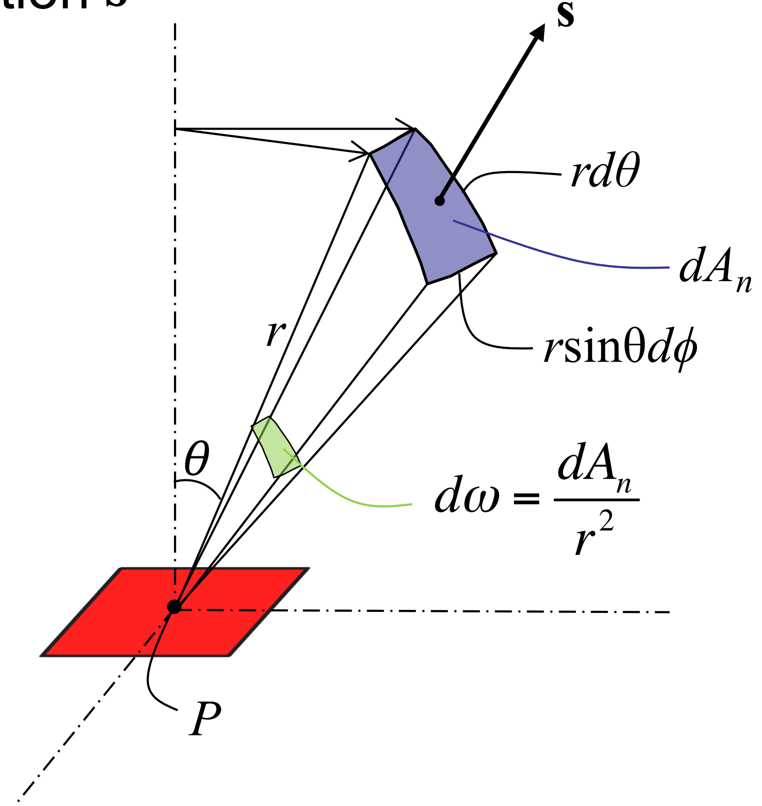
$$dA_n = r^2 \sin\theta d\theta d\phi$$

  - The infinitesimal solid angle is given by:

$$d\omega = \frac{dA_n}{r^2} = \frac{r^2 \sin\theta d\theta d\phi}{r^2}$$

$$\boxed{\therefore d\omega = \sin\theta d\theta d\phi}$$

***Solid angle in spherical coordinates***



# Radiation Intensity, I

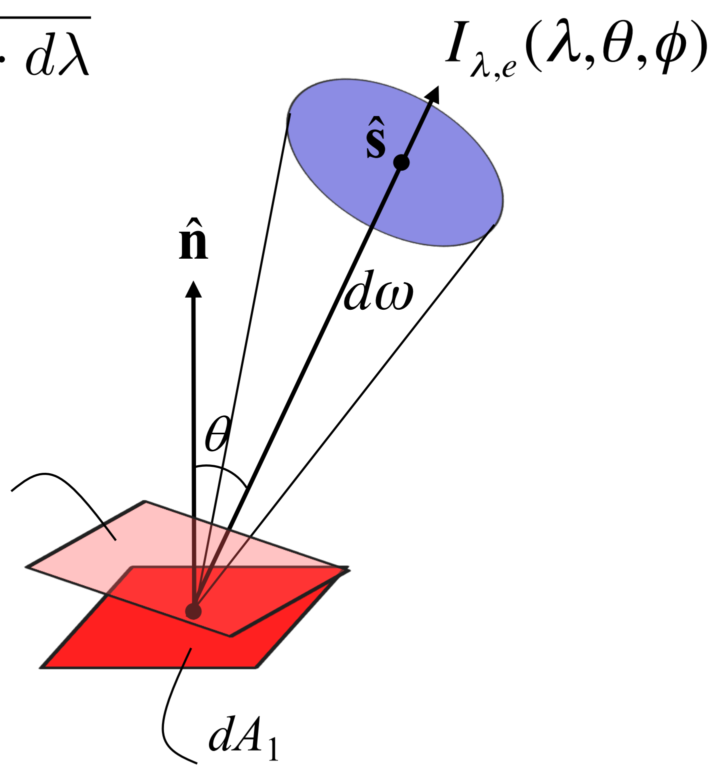
- Rate at which radiant energy is emitted at the wavelength $\lambda$ in the direction $(\theta,\phi)$, per unit wavelength, per unit solid angle and per unit area normal to the direction $(\theta,\phi)$

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{dq}{dA_n \cdot d\omega \cdot d\lambda} = \frac{dq}{dA_1 \cos\theta \cdot d\omega \cdot d\lambda}$$

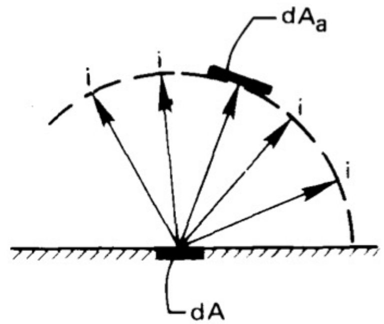
$$\left[\frac{W}{m^2 \cdot sr \cdot \mu m}\right]$$

$$dA_n = dA_1 \cos\theta$$

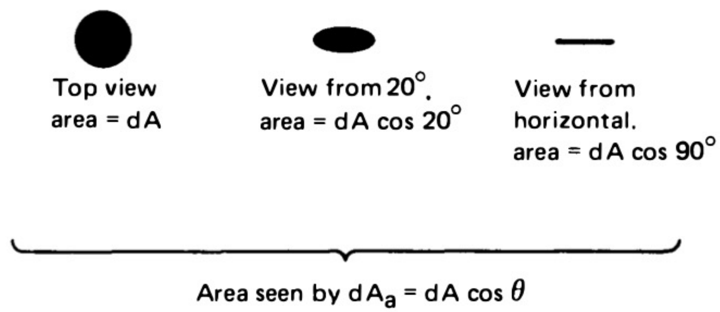
Lambert's Cosine Law



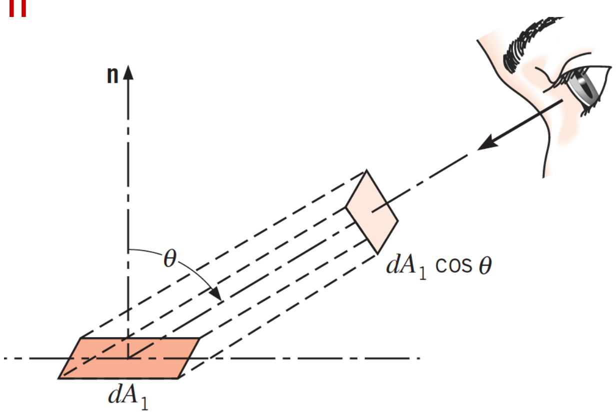
# Lamberts Cosine Law

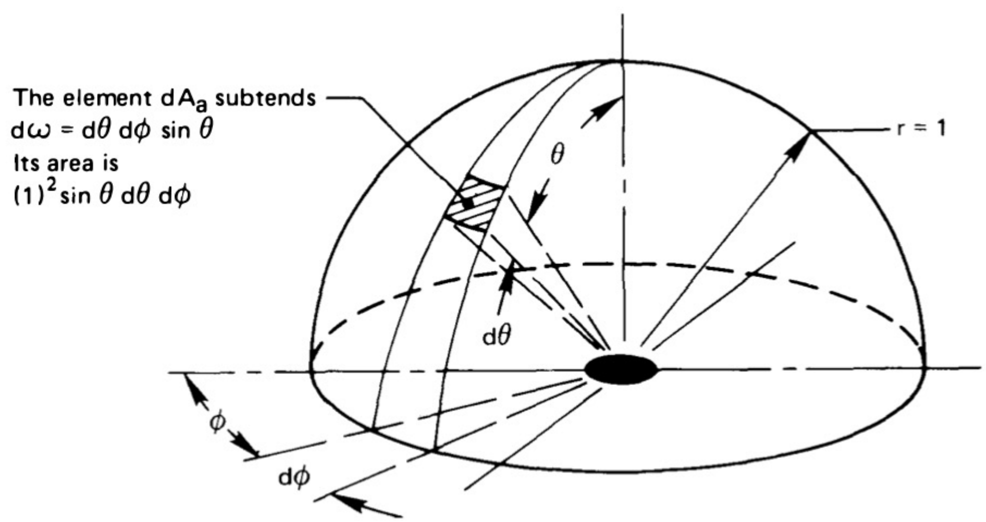



A single area element radiates with equal intensity in all directions




Lambert's Cosine Law



# Radiation Exchange Between 2 Black Bodies

Radiation transfer from source to the detector is given by:

(1) $dq = I_b \, dA_{s_n} \, d\omega \qquad I_b = \int I_{b,\lambda} \, d\lambda$

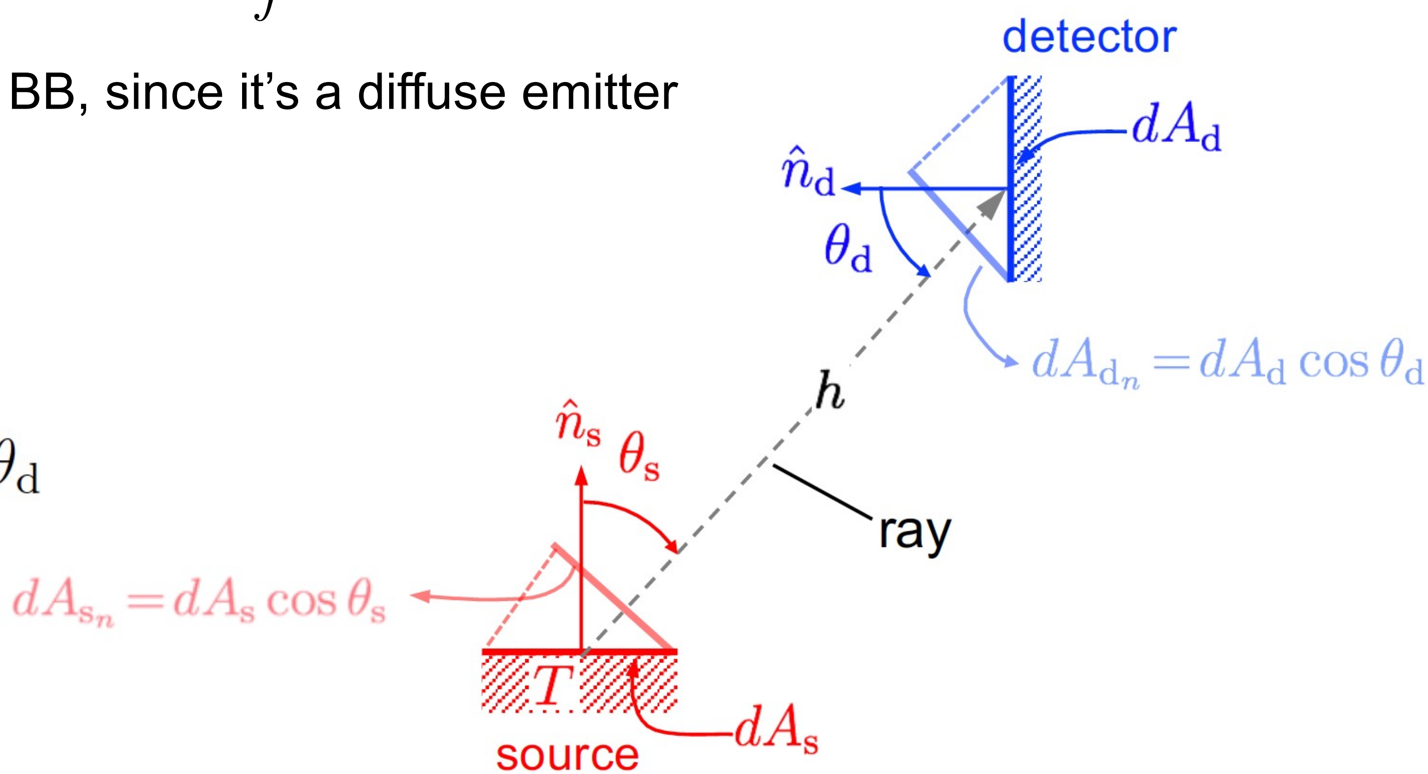
Total Intensity for a BB, since it's a diffuse emitter

(2) $I_b = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$

(3) $d\omega = \frac{dA_{d_n}}{h^2}$

(4) $dA_{d_n} = dA_d \, \cos\theta_d$

(5) $dA_{s_n} = dA_s \, \cos\theta_s$

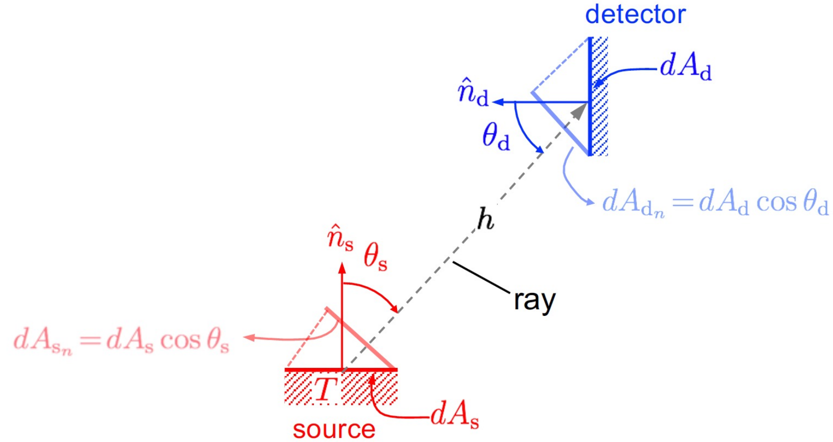



Plug 2-5 into (1)

$$dq = \frac{\sigma T^4}{\pi h^2} \cos\theta_s \, \cos\theta_d \, dA_d \, dA_s$$

# Radiation Exchange Between 2 Black Bodies

$$dq = \frac{\sigma T^4}{\pi h^2} \cos\theta_s \cos\theta_d \, dA_d \, dA_s$$




Integrate over the Areas

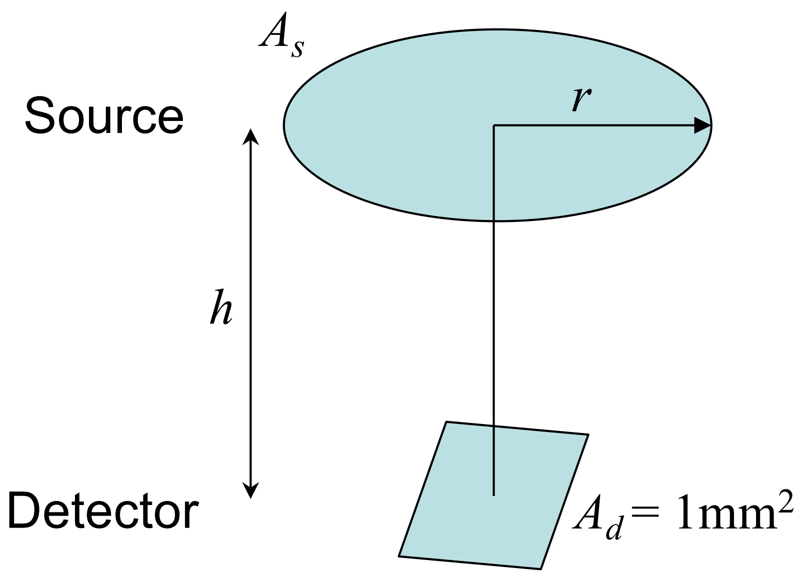
$$q = \sigma T^4 \underbrace{\int_{A_s}\int_{A_d} \frac{\cos\theta_s \cos\theta_d}{\pi h^2} dA_d \, dA_s}_{= A_s F_{s\to d}}$$

Recall, view factor definition

$$\Rightarrow F_{ij} = \frac{q_{i\to j}}{q_i}$$

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

# View Factors




View Factor (from literature): $r = D/2$

$$F_{\mathrm{d}\to\mathrm{s}} = \frac{1}{1 + \left(\frac{h}{r}\right)^2}$$

Using reciprocity:

$$A_{\mathrm{s}}\, F_{\mathrm{s}\to\mathrm{d}} = A_{\mathrm{d}}\, F_{\mathrm{d}\to\mathrm{s}}$$

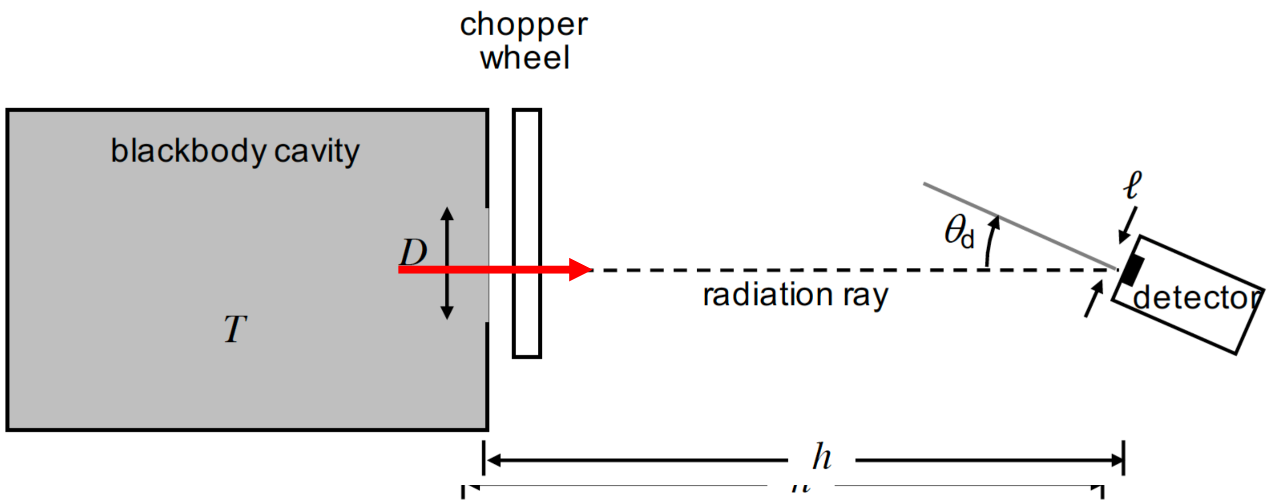
Heat transfer rate:

$$q_{\mathrm{theory}} = \frac{\sigma\, T^4\, A_d\, r^2}{(r^2 + h^2)}$$

Or,

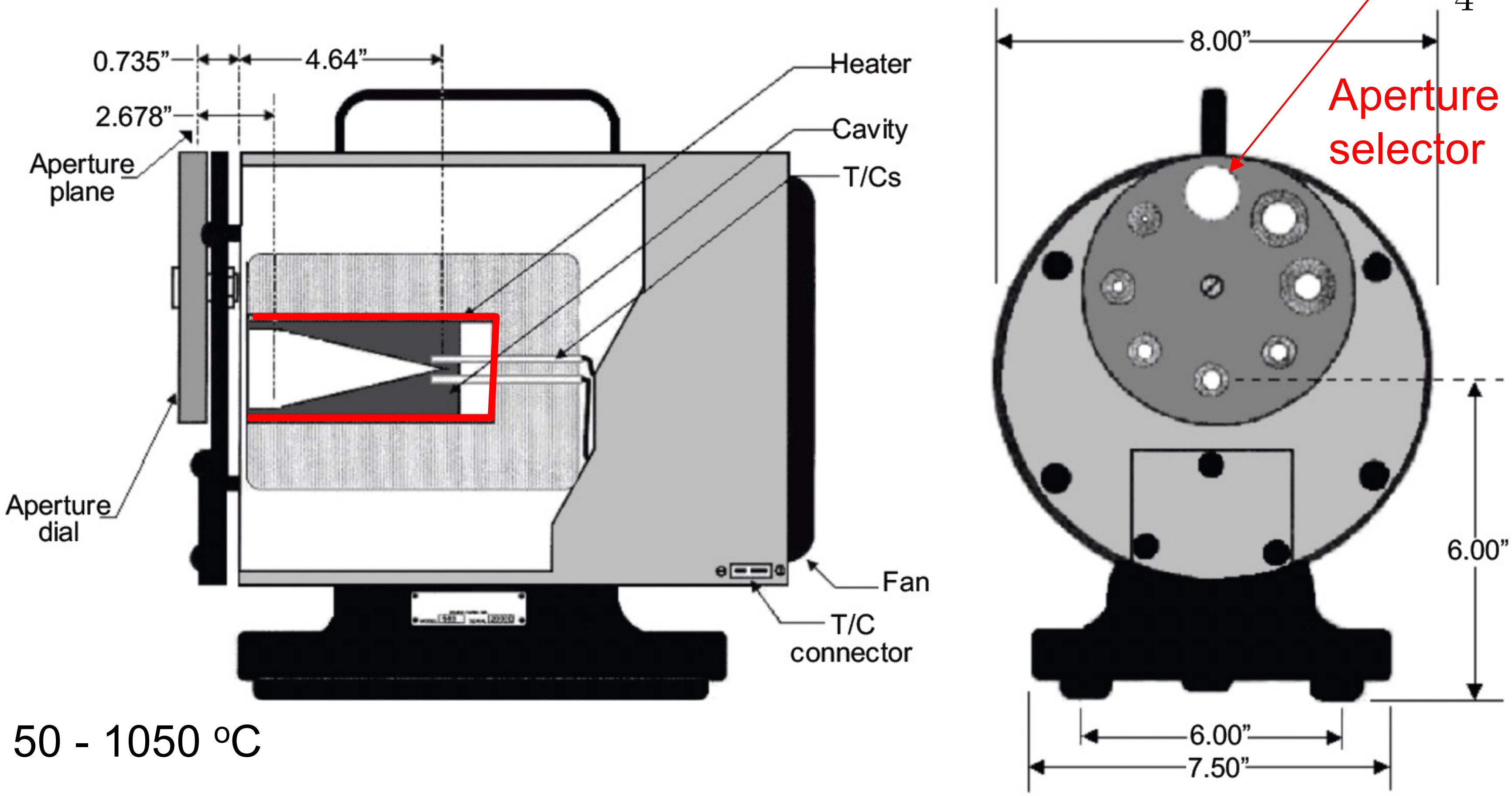
$$q_{\mathrm{theory}} = \frac{\sigma\, T^4\, A_d\, D^2}{(D^2 + 4\, h^2)}$$

# Experimental Setup




Ray - the straight line along which the electromagnetic wave travels between two points

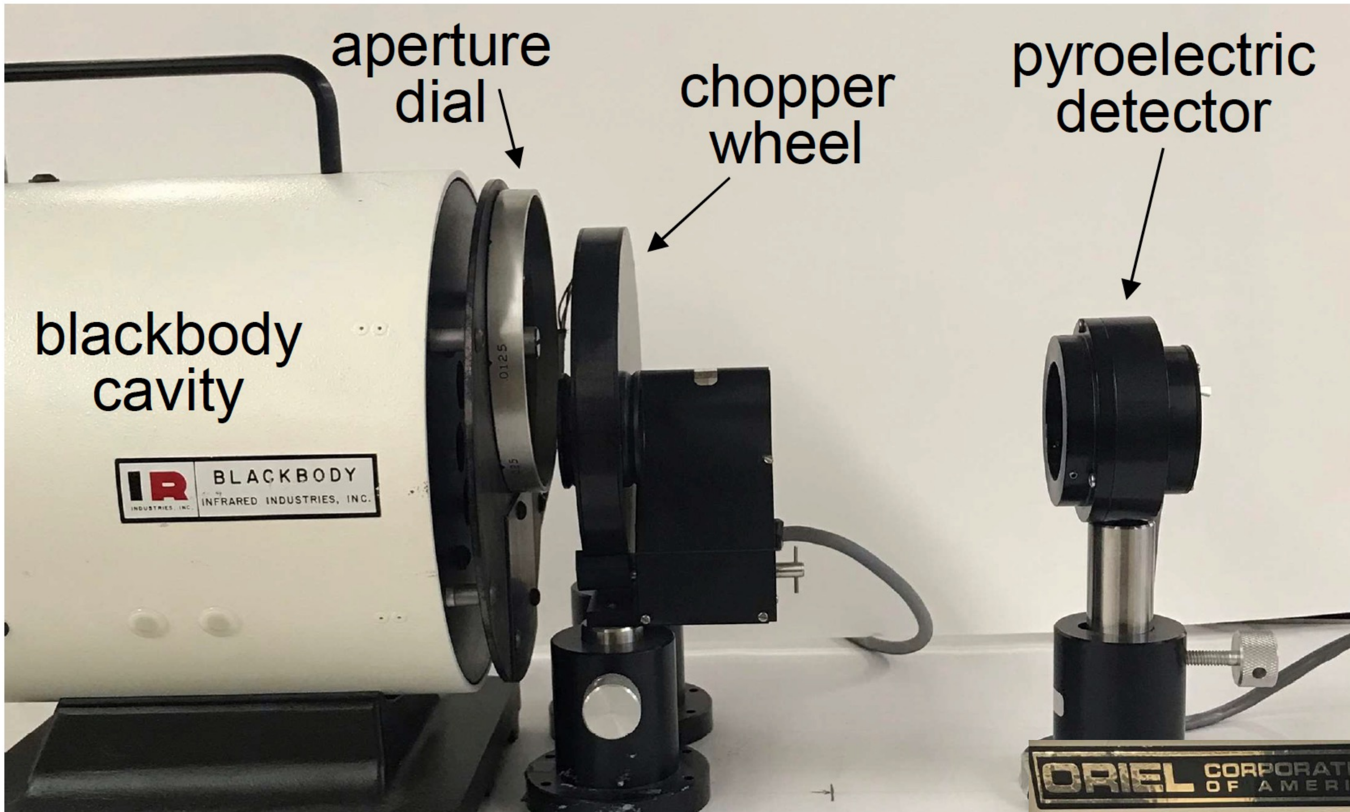
# Blackbody Cavity




$$A_s = \frac{\pi D^2}{4}$$

T = 50 - 1050 $^{o}$C

# Experimental Setup



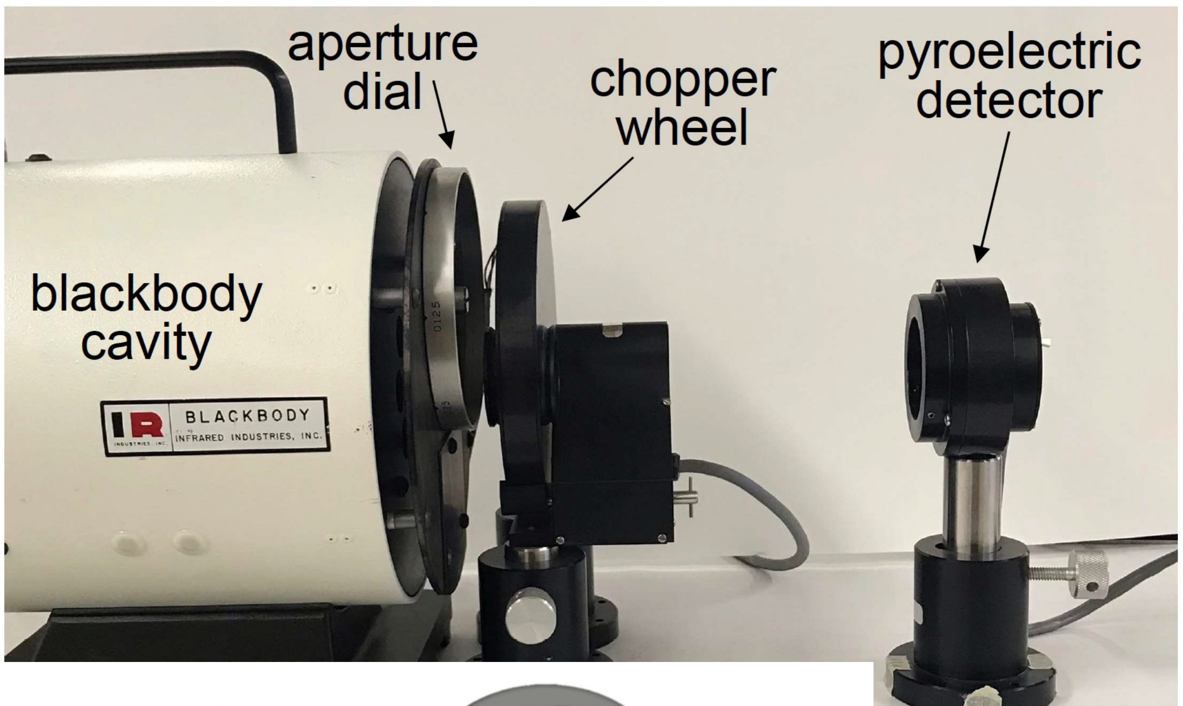

Measures very small heat transfer rates: $10^{-2}$ to $10^{-6}$ W

Instrument box

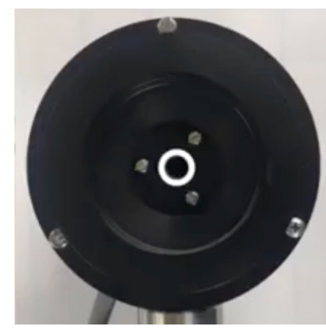
# Experimental Setup – Pyroelectric detector

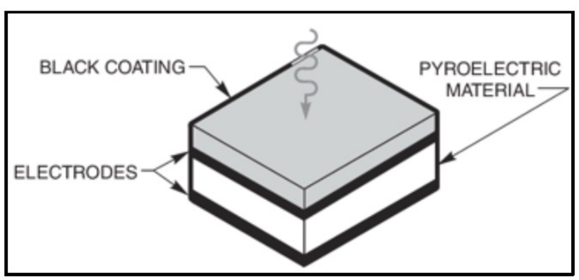
Pyroelectric detector – voltage generated due to a change in temperature

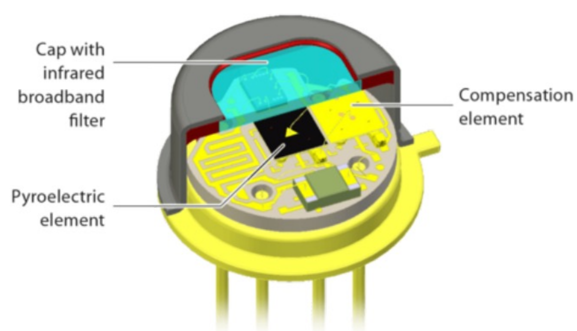
$\lambda$ in the range 2-14 $\mu$m




detector

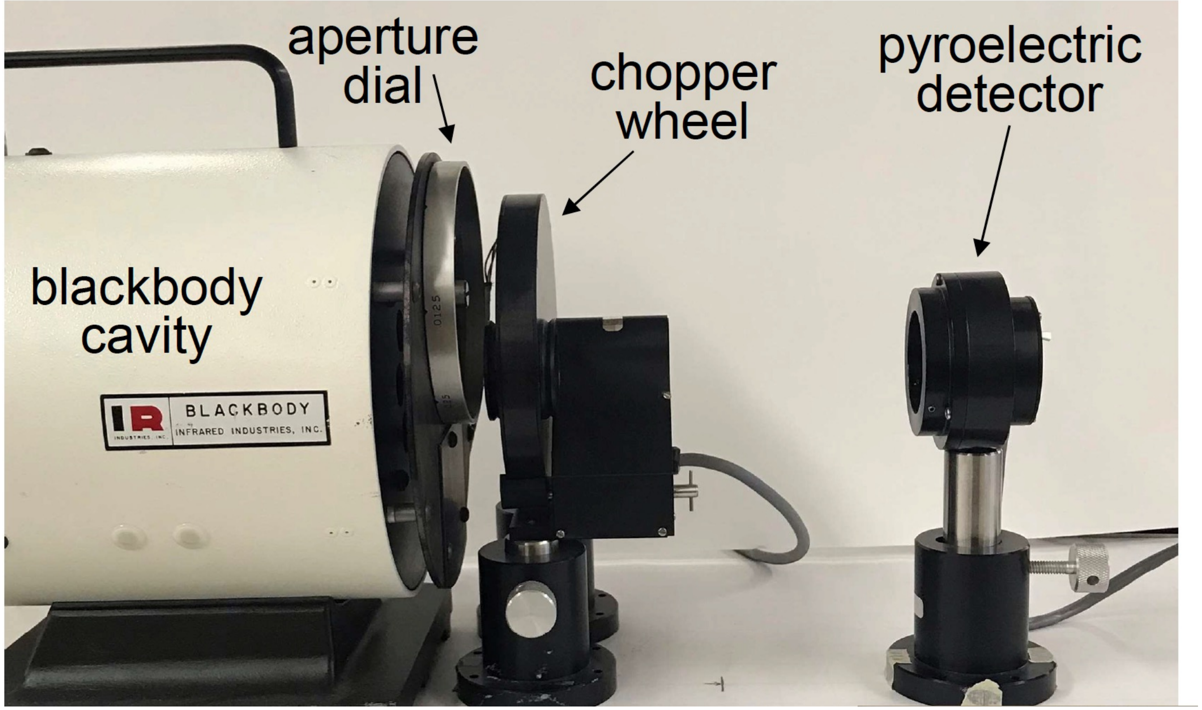


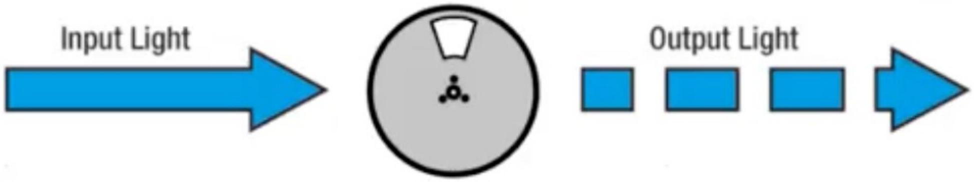


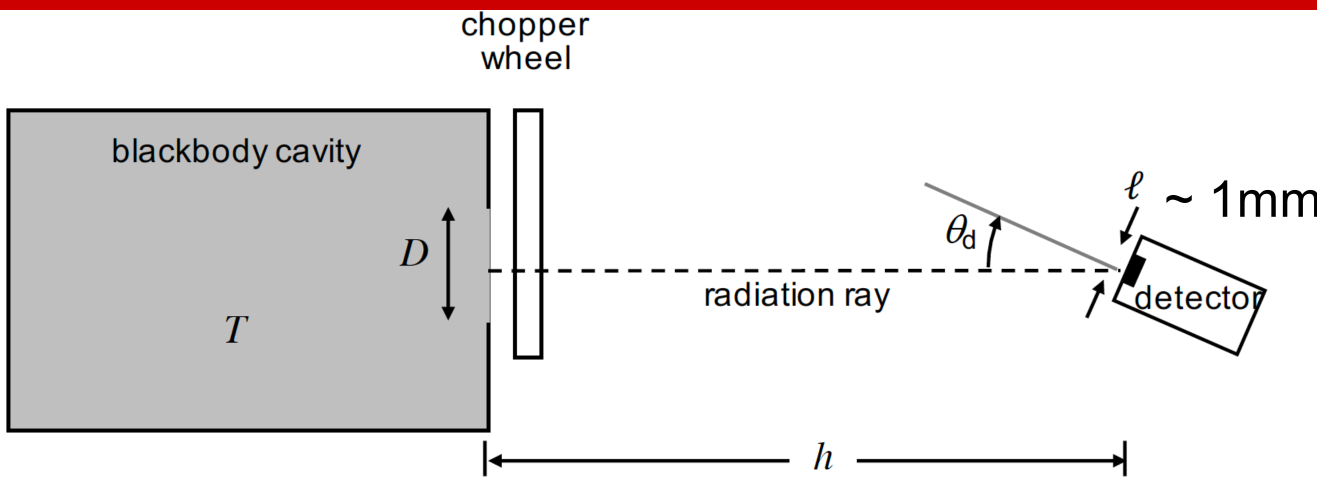
# Experimental Setup – Pyroelectric detector

Pyroelectric detector – voltage generated due to a change in temperature



# Measurements




| Quantity | Symbol | Units | Instrument |
|---|---|---|---|
| Temperature of blackbody | $T$ | °C | thermocouple |
| Separation distance | $h$ | in | linear ruler |
| Angle of detector head | $\theta_{\mathrm{d}}$ | deg | rotation table |
| Aperture diameter of source | $D$ | in | markings on dial |
| Heat transfer rate | $q$ | W | pyroelectric radiometer |

$$q_{\mathrm{rad}} = f(T, h, D)$$

$\theta_{\mathrm{d}} = 0^\circ$ for all experiments

Vary $h$ hold $T$ and $D$ constant
Vary $T$ hold $h$ and $D$ constant
Vary $D$ hold $T$ and $H$ constant

# Data Collection Sheet

**TFES Lab (ME EN 4650)**

**Blackbody Radiation Experiment: Raw Data Sheet**

$T_{atm}$ : ____________ ($^oC$)

$P_{atm}$ : ____________ (mbat/hPa)

**Experiment 1: Variable h**

| T (oC) | D (in) | h (in) | q (μW) |
|---|---|---|---|
| | 0.6 | 8 | |
| | 0.6 | 9 | |
| | 0.6 | 10 | |
| | 0.6 | 11 | |
| | 0.6 | 12 | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

**Experiment 2: Variable T**

| T (oC) | D (in) | h (in) | q (μW) |
|---|---|---|---|
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | 0.6 | 9 | |
| | | | |
| | | | |

**Experiment 3: Variable D**

| T (oC) | D (in) | h (in) | q (μW) |
|---|---|---|---|
| | 0.1 | 9 | |
| | 0.2 | 9 | |
| | 0.4 | 9 | |
| | 0.6 | 9 | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Set T ~ 460 C
vary h

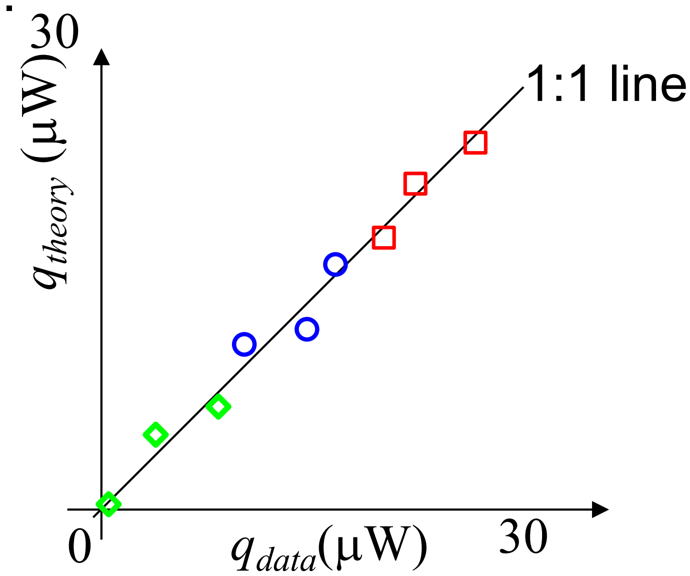
Set T ~ 700 C
Record q every 20C

Set T ~ 700 C
Vary aperture ,D

# Submission Requirements

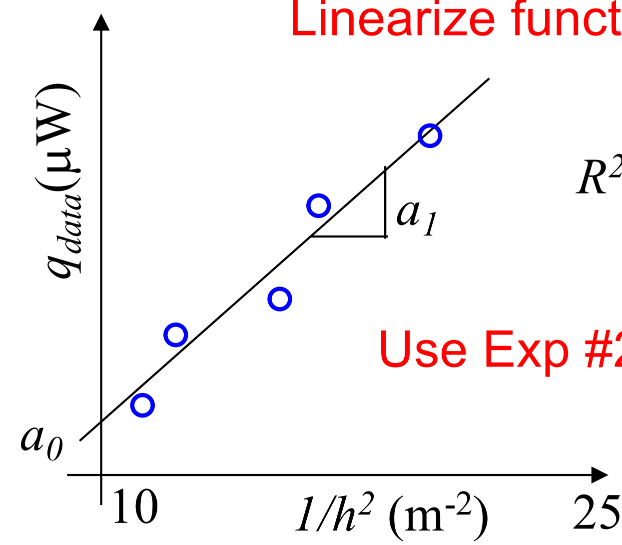
$$q = f(T, h, D)$$

1a.




1b.

Linearize function!



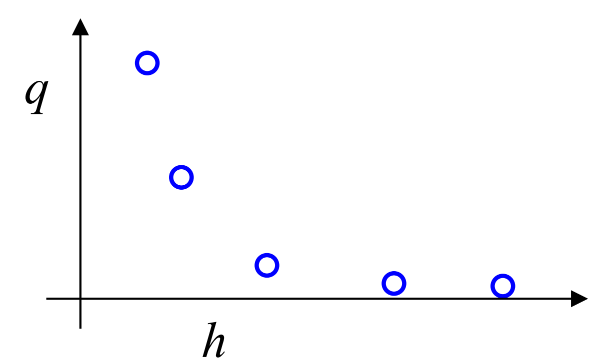

$R^2$ Goodness-of-fit parameter

Use Exp #2 data

$$R^2 = 1 - \frac{S_R}{S_T}$$ Correlation coef/coef of determination

$$S_R = \sum_{i=1}^{N} (y_i - a_0 - a_1 x_i)^2$$

$$S_T = \sum_{i=1}^{N} (y_i - \overline{y})^2$$



# Pseudocode for Linearizing and Computing Linear Regression

```
h2 = 1./(h.^2); %Linearize h
p = polyfit(h2,qdata,1) %Least squares regression
a1 = p(1) %slope
a0 = p(2) %intercept
SR = sum((qdata - a0 - a1*h2).^2) %compute sum of the squares of the residual
ST = sum((qdata - mean(qdata)).^2) %compute
Rsquared = 1 - SR/ST %compute coefficient of determination
```

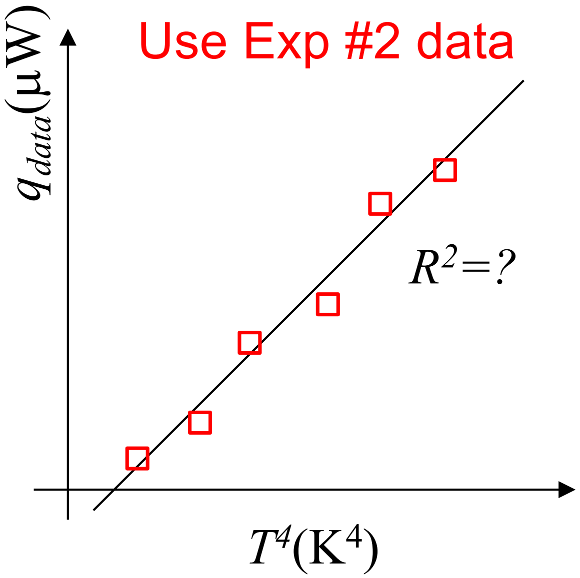
$$R^2 = 1 - \frac{S_R}{S_T}$$

$$S_R = \sum_{i=1}^{N} (y_i - a_0 - a_1 x_i)^2$$

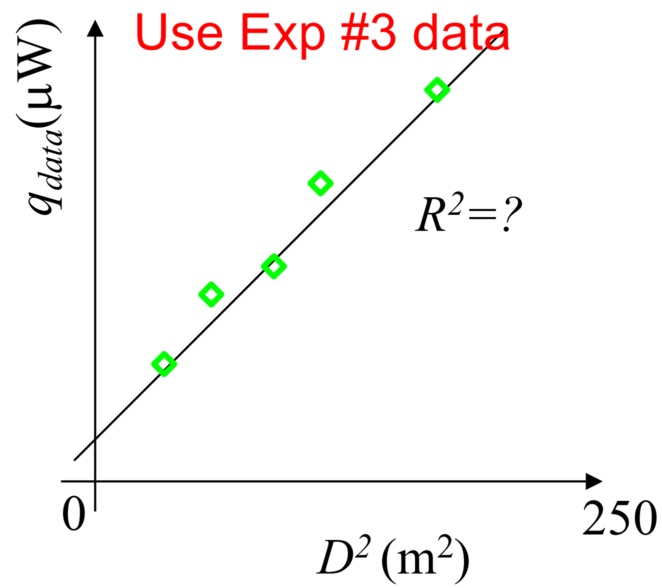
$$S_T = \sum_{i=1}^{N} (y_i - \overline{y})^2$$

# Submission Requirements

1c.




1d.



# Solid Angle of Moon Subtended from Earth