HW 1: Material Derivative and Flow Kinematics

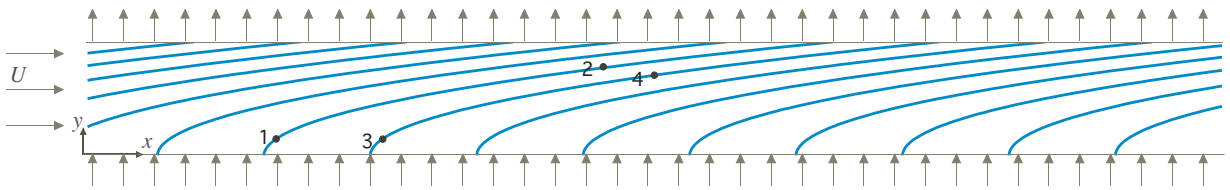
1. Complete the following questions regarding the material derivative.

(a) Mathematically state the definition of the material derivative of the x-component of momentum (Note: this should be in the form of an equation).

(b) Describe in words why the material derivative is important in fluid dynamics and how it is used in practice.

(c) Label each of the terms in your equation from part (a) using a roman numeral. Describe in words the *physical* meaning associated with each of the identified terms.

2. Streamlines in a porous channel are shown below.



(a) Starting with the mathematical definitions of a streamline and the streamfunction, ψ (see your reference book), show that the value of ψ remains constant along a streamline.

(b) On the figure provided, draw the *direction* of the velocity vector at points 1, 2, 3, 4.

(c) What can you say about the relative magnitude of the velocity at point 1 compared to that at points 2, 3, and 4? [Explain your answer]

3. Consider a flow with the following velocity

$$u = x^2 + 2\,x\,y \qquad v = -y^2 - 2\,x\,y\,.$$

The pathline equation for the particle that originates at (x_o, y_o) at time $t = 0$ is given by

$$y = \left(-x^2 + \sqrt{x^4 + 4\,x\,\left(x_o^2\,y_o + y_o^2\,x_o\right)}\right) / \left(2\,x\right)$$

(a) Determine a_x and a_y, which are the x- and y-components of the acceleration of any generic particle in the flow. [Hint: use the definition of the material derivative, i.e., $a_x = Du/Dt$ and $a_y = Dv/Dt$]

(b) Plot the pathline for the particle with initial position $(x_o, y_o) = (0.52, 6)$.

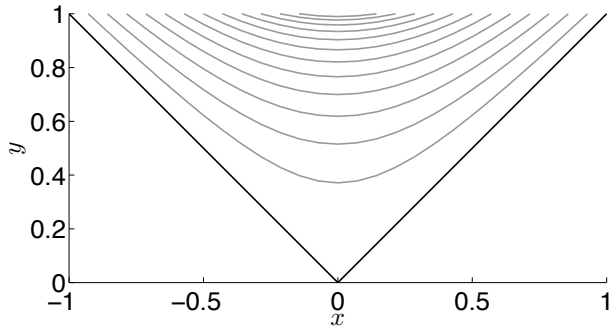
(c) On a separate figure, plot the acceleration *magnitude* versus distance s traveled (along the pathline) for the same particle with initial position $(x_o, y_o) = (0.52, 6)$.

4. A two-dimensional steady flow has velocity components

$$u = y \qquad \text{and} \qquad v = x.$$

(a) Show that the streamlines are rectangular hyperbolas of the form $x^2 - y^2 = \text{const}$.

(b) Plot 10 streamlines in Matlab/Python and convince yourself they represent flow in a 90° corner with the side walls oriented at 45°, as shown below. [Turn in your Matlab code]



5. [**6700 only**] The velocity components in an unsteady plane flow are given by

$$u = \frac{x}{1+t} \qquad \text{and} \qquad v = \frac{2\,y}{2+t}.$$

(a) Calculate the equation for the streamlines.

(b) Calculate the equation for the pathline of the particle that originates at location (x_o, y_o) at time $t = 0$.

(c) In Matlab/Python, plot the streamlines at time $t = 2$ and overlay the pathline of the particle with initial position $(x_o, y_o) = (1, 1)$. [Turn in your Matlab code]