ME EN 5830/6830: Aerospace Propulsion
Problem Set #4: Compressible Flows I and II
Due date: 02/06/2025 by 11:59pm

Submission

Assignments can only be submitted on Gradescope, which can be accessed through Canvas. If you have any questions about submission, please email the class TA, John Gardner at john.w.gardner@utah.edu. Submissions will be automatically locked at the due date given above.

Introduction

This problem set primarily covers the material from Lectures 7 and 8. The goal is for students to work with subsonic and supersonic compressible flows in nozzles and diffusers. Upon completion of this homework, students should be able to:

- Calculate state changes across an isentropic supersonic nozzle.
- Calculate state changes across an isentropic converging-diverging nozzle.
- Select operating conditions for a converging-diverging nozzle to accelerate fluid to supersonic.
- Explain when a flow is choked.

Approximations

While studying combustion, we avoided assuming gases are calorically perfect (constant specific heat) because of the large temperature increases. When working with **non-chemically reacting** compressible flows, we will generally assume that gases are ideal and calorically perfect unless otherwise stated. In this problem set, all gases are assumed to be ideal and calorically perfect. Do not use JANAF tables for property lookups. Instead, for simplicity, we will assume that $T_{ref} = 0$ K and $h(T_{ref})$ = 0 kJ/kg such that $h = c_p T$.

Assignment

Problem #1: Air is flowing through an isentropic <u>supersonic nozzle</u>. Just before the nozzle, the fluid is at $M_1 = 1.3, T_1 = 298$ K, $p_1 = 1$ atm. Just after the nozzle, the fluid is at $M_2 = 3$. Assume air acts as an ideal gas with constant specific heats ($\gamma = 1.4, R = 287\ J/Kg \cdot K$).

a) **Conceptual:** Based on the flow, is the nozzle converging or diverging?

b) Compute the enthalpy h_1 and velocity V_1 before the nozzle.

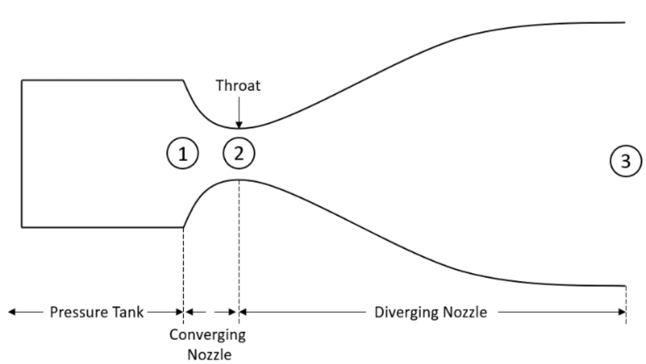
c) Compute the stagnation temperature T_{t1}, stagnation pressure p_{t1}, and stagnation enthalpy h_{t1} before the nozzle.

d) Compute the temperature T_2, pressure p_2, velocity V_2, and enthalpy h_2 after the nozzle.

e) **Conceptual:** You should have previously found that $T_2 < T_1$. Where did this lost thermal (internal) energy go? (What was this lost internal energy converted into by the nozzle?)

f) Prove your answer from part a) mathematically by computing the area ratio between the nozzle outlet and inlet.

Problem #2: Consider a pressure tank connected to a converging-diverging nozzle, pictured below. Assume this tank is so large that the pressure p_1 stays approximately constant and that the velocity of the fluid in the tank V_1 is approximately zero. The nozzle outlet is at sea level where $p_3 = 1$ bar and is designed with an area ratio $A_3/A_2 = 3$. Assume the fluid is air ($\gamma = 1.4, R = 287\ J/Kg \cdot K$).



a) Assuming the system is perfectly isentropic, what should the pressure in the tank p_1 be such that the outlet flow is supersonic (i.e., $M_3 > 1$)?

b) Assuming the system is perfectly isentropic, what should the pressure in the tank p_1 be such that the outlet flow is subsonic (i.e., $M_3 < 1$) **but the throat remains sonic** (i.e., $M_2 = 1$)?

c) If the outlet temperature is given as $T_3 = 300$ K, what is the velocity at the nozzle exit V_3 for the Mach numbers you found in both part a) and b)

Problem #3: Consider the same pressure tank and nozzle system from problem #2. For this problem, the shape of the nozzle and pressure in the tank are constant and allow the flow to reach a sonic velocity in the throat. The nozzle is operated at two different outlet pressures p_3. In the first case, p_3 causes an isentropic supersonic flow to exit the nozzle. In the second case, p_3 causes an isentropic subsonic flow to exit the nozzle.

a) **Conceptual:** How do the magnitudes of p_3 in case 1 and case 2 compare?

b) **Conceptual:** How do the magnitudes of the mass flow rate through the nozzle in case 1 and case 2 compare?