ME EN 5830/6830: Aerospace Propulsion
Problem Set #9: Rocket Propulsion I and II
Due date: 04/03/2025 by 11:59pm

Submission

Assignments can only be submitted on Gradescope, which can be accessed through Canvas. If you have any questions about submission, please email the class TA, John Gardner at john.w.gardner@utah.edu. Submissions will be automatically locked at the due date given above.

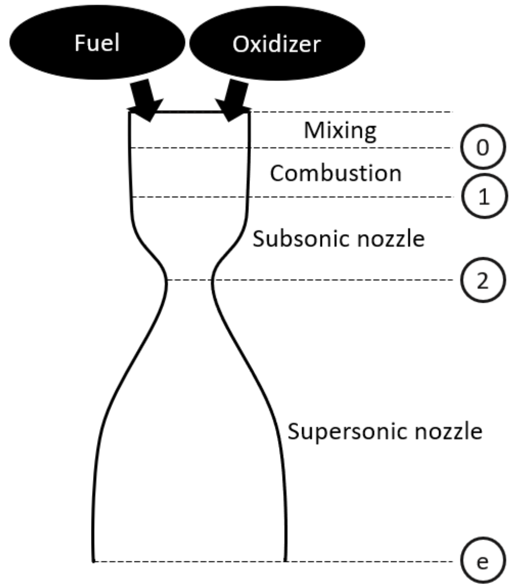
Introduction

This problem set primarily covers the material from Lectures 18 and 19. The goal is for students to perform a basic analysis of rocket thrust and compute various rocket parameters. After completion of the assignment, students should be able to:

- Compute the thrust produced by a rocket engine with and without combustion.
- Calculate the amount of mass needed by a rocket to achieve a total impulse.
- Compute specific impulse, characteristic velocity, and mass ratio for a rocket.

Assignment

Problem #1: Consider a simple rocket engine as shown in the figure on the right. This rocket engine is *loosely* based on the Merlin sea-level rocket engine from SpaceX. Propellant is stored in a pressurized chamber in the rocket behind a closed valve. Once the valve is opened, perfectly mixed *gaseous* propellant leaves this chamber and enters the combustor (state 0) at static pressure $p_0 = 10$ MPa and temperature $T_0 = 300$ K. Consider the flow velocity to be negligible outside of the nozzle (i.e., velocity is zero at state 0 and 1). For this problem, use $\underline{\gamma = 1.2}$ as we can no longer assume the flow is predominantly air. Unless otherwise stated, assume a molar mass of $\bar{m} = 0.03$ kg/mol.



a) Consider a launch of this rocket near sea-level ($p_a = 0.1$ MPa). Assume that the rocket engine is designed such that its operating point is at this elevation (i.e., isentropic flow without shocks, $p_e = p_a$). For now, assume the combustor is broken and no energy is delivered to the flow via combustion. Compute the exit velocity V_e of the rocket engine after the valve is opened (ignoring any transients).

b) For the same assumptions as in part a) and assuming a throat diameter of $D_2 = 0.25$ m, compute the mass flow rate through the rocket engine.

c) Using the results from parts a) and b), compute the thrust generated by this rocket engine. *For reference, the sea-level version of the Merlin engine is listed as having a thrust of 845 kN.*

d) As propellant is expelled from this rocket engine, the pressure in the storage tanks must decrease. However, the mass flow rate of the propellants through the nozzle will remain constant for a long time. Why? Conceptually, at what point will the mass flow rate through the nozzle decrease? *Hint: Think back to compressible flows and Lecture 8.*

e) Assume the mass flow rate remains constant from take-off and until there is zero propellant remaining in the rocket. Also assume that the thrust is constant the entire time. If this rocket engine needs a total impulse of 150,000,000 N-s to complete its mission, how much mass of fuel would need to be carried onboard?

f) An engineer is able to come out and fix the combustor. Using all the same assumptions as in part a), calculate the exit velocity V_e if combustion increases the temperature to $T_1 = 3000\,K$. How many times larger is this exit velocity compared to the non-reacting case in part a)? *For reference, the Merlin sea-level engine is listed as having an effective exhaust velocity of 2700 m/s.*

g) Making the same assumptions as in part b), compute the mass flow rate and thrust generated by this rocket engine.

h) For the assumptions made in this assignment ($p_e = p_a$, isentropic nozzle, negligible flow velocity at state 0 and 1), prove that rocket thrust is <u>not</u> a function of T_1. *Hint: For this derivation, start from the thrust equation and continue expanding out terms. All the temperature terms should eventually cancel out with the correct approximations. Note: We've made several assumptions that make this true, but it's not strictly accurate for real rockets.*

i) Although you just showed that rocket thrust is not a function of combustion temperature, higher temperatures still play a very important role. As in part e), assume a constant rocket thrust and a required total impulse of 150,000,000 N-s. What is the necessary mass of fuel that needs to be carried on this rocket to achieve this goal? What is the ratio of fuel required in the non-burning case to the burning case?

j) Compute the specific impulse of the rocket for both the non-burning and burning case. *For reference, the Merlin sea-level engine is listed as having a specific impulse of 275 s.*

k) Assume our rocket has 9 of the (burning) engines we just analyzed. The initial mass of the entire rocket (including propellant) is $m_0 = 1{,}000{,}000$ kg. Compute the mass ratio of the entire rocket if each engine exhausts the mass of propellant computed in part i).