Homework #3
ME EN 5210/6210 & CH EN 5203/6203 & ECE 5652/6652
Linear Systems & State-Space Control

Use this page as the cover page on your assignment, submitted as a single pdf.

Problem 1

Find the rank, nullity, basis of the range space, and basis of the null space, for each of the following matrices:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2

Consider the linear algebraic equation

$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} = \boldsymbol{y}$$

It has three equations and two unknowns. Does a solution $\boldsymbol{x}$ exist in the equation? Is the solution unique? Does a solution exist if

$$\boldsymbol{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}?$$

Problem 3

Find the general solution of

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

How many parameters do you have?

Problem 4

For the discrete system

$$\boldsymbol{x}[k+1] = A\boldsymbol{x}[k] + Bu[k]$$

$$y[k] = C\boldsymbol{x}[k] + Du[k]$$

where

$$A = \begin{bmatrix} -0.5 & -0.1 \\ 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 5 \end{bmatrix}, C = [1 \quad 10], D = 4$$

(a) Compute the transfer function from in the input $u[k]$ to the output $y[k]$.

(b) Using the answer from part (a), compute the input-output equation.