Lab 01: Stability

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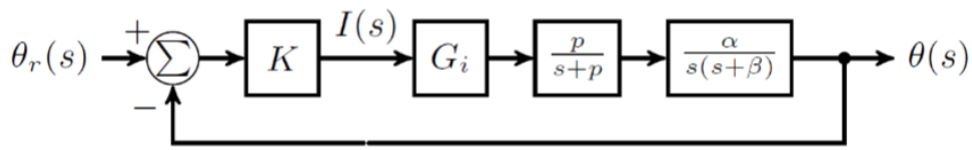

Figure 1: block diagram of the closed-loop feedback controller.

From the closed loop transfer feedback controller block diagram as seen in Figure 1, block K is the controller, block Gi is the amplifier gain, block $\frac{\alpha}{s(s+\beta)}$ is the transfer function of the motor, and block $\frac{p}{s+p}$ is a filter function.

The open loop transfer function of this system is $\frac{\theta(s)}{I(s)} = \frac{Gi \cdot p \cdot \alpha}{s^3+\beta s^2+ps^2+p\beta s}$, where Gi is 10, p is 20, α is 7.5 and β is 1.67. After plugging in these values, we obtain that the open loop transfer function is $\frac{\theta(s)}{I(s)} = \frac{1500}{s^3+21.67s^2+33.4s}$. .Now, considering the closed loop system, the closed loop transfer function is $\frac{\theta(s)}{\theta_r(s)} = \frac{K \cdot Gi \cdot p \cdot \alpha}{s^3+\beta s^2+ps^2+p\beta s + K \cdot Gi \cdot p \cdot \alpha}$. Substituting in the values for their constants, we obtain that the closed loop transfer function is $\frac{\theta(s)}{\theta_r(s)} = \frac{1500K}{s^3+21.67s^2+33.4s + 1500K}$. Therefore, the characteristic equation of the closed loop transfer function is: $s^3 + 21.67s^2 + 33.4s + 1500K = 0$.

S^3	1	33.4
S^2	21.67	1500K
S^1	33.4-K69.22	0
S^0	1500K	0

Figure 2: Routh-Hurwitz stability table for the closed-loop transfer function

For the Routh – Hurwitz table as seen in Figure 2, all the values in the first column must have zero sign change for the transfer function to be considered stable. To ensure that all the terms in the first column are positive, each term in the first column is set to an inequality greater than zero and K is solved. After doing this, it is found that K must be in the range of 0 to 0.483 for the closed loop system to be stable.

Using the characteristic equation of the closed loop transfer function and the max K value at marginal stability, we obtain the characteristic equation $s^3 + 21.67s^2 + 33.4s + 724.5 = 0$. Solving for the roots of the characteristic equation in MATLAB, we find that $s = -21.67$ and $s = 0 \pm 5.78i$. These roots tell us about the oscillatory frequency at marginal stability where the imaginary part of the root is the damped natural frequency. Therefore, this system will have an oscillatory frequency of 5.78 rad/sec or 0.92 Hz at marginal stability.

It is expected that a change in the filter coefficient to greater values will result in the system's stable gain value range increasing as well. Decreasing the filter coefficient likewise decreases the range of stable gain values. The justification of this hypothesis is by analyzing the Routh-Hurwitz table in Fig. 1 with some undefined variable 'P' as the filter coefficient, and examining how it might effect the stability of the system.

Experimental Results

To determine the stability of a system in the real world, it is not practical to rely solely on theoretical analysis of the closed-loop transfer function. Rather, it is necessary to incrementally increase the value of the gain and test the response of the system until it reaches marginal stability. Fig. 3 shows the experimental results of gain values ranging from 0.1 to 0.3, as the system approaches marginal stability.

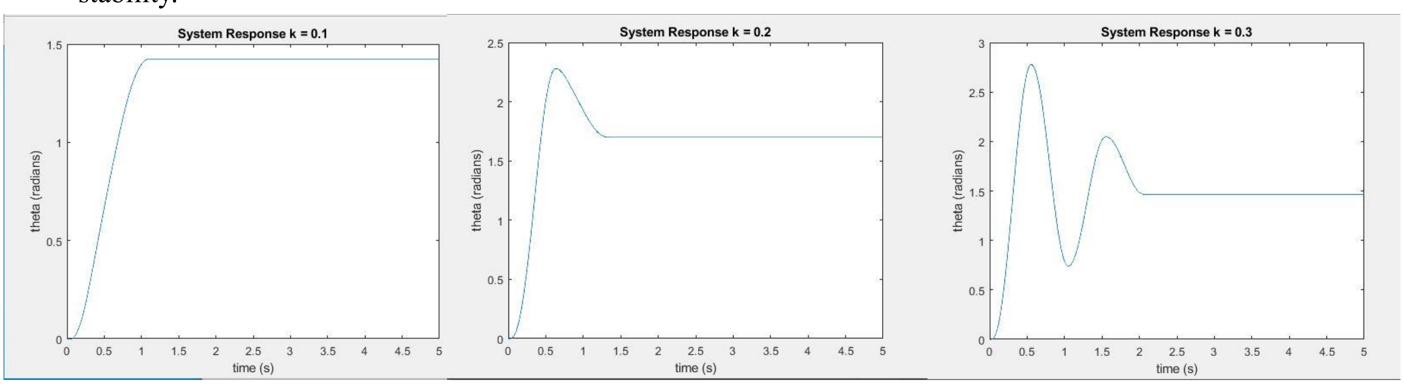

Figure 3. Time domain responses of the system with varying gain values. A lower gain (top left) causes the system to have higher damping. A higher gain (bottom center) causes the system to oscillate more.

Changing the gain values of the system has dramatic effects on the location of the poles. It can be visualized in Fig. 3 that the imaginary poles of the third order transfer function of the system move closer to the imaginary axis as the gain approaches an unstable value. By incrementally increasing the value of K over many experiments, the gain value which causes the system to become marginally stable is about 0.4. The time response of such a case is present in Fig. 4. By measuring the oscillations of Fig. 4, the experimental natural frequency of the system at marginal stability is $\omega_n = \frac{T}{t} = 6.98 \frac{rad}{sec}$.

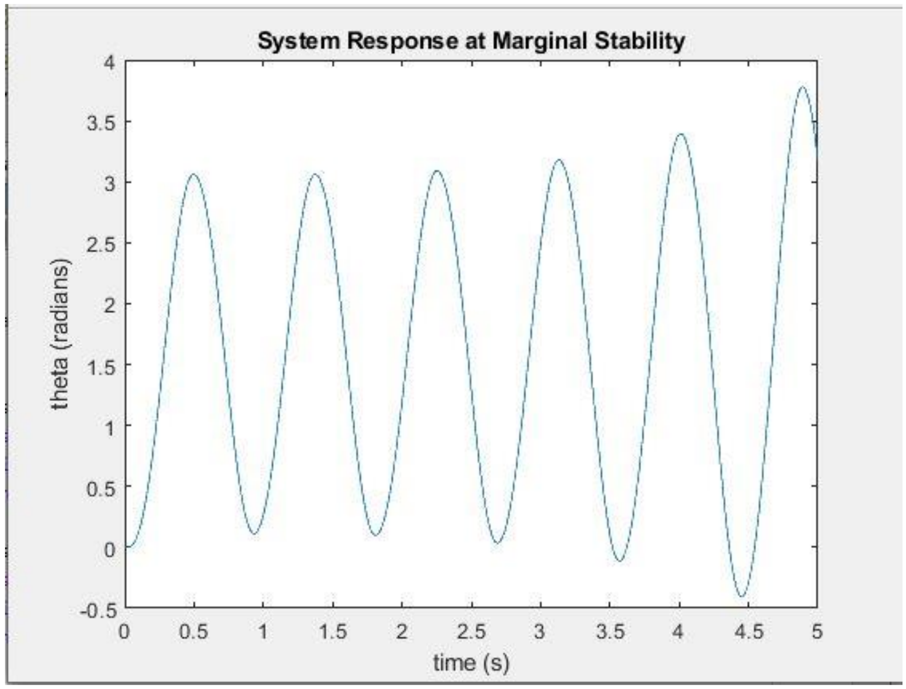

Figure 4: Time domain response of the system at marginal stability. The y-axis represents the actual value of theta (radians) as it attempts to achieve a desired theta value of 90 degrees.

Increasing the filter coefficient (p) during the experiment resulted in the system becoming more stable, while decreasing the filter coefficient caused the system to become unstable. In both images shown in figure 5, the same gain value of K was used but the filter coefficient P was changed. On the right, a filter coefficient of 50 was used while a filter coefficient of 10 was used for the experiment shown on the left. As seen from these results, the filter coefficient changed the range of K values that the system is stable for as expected.

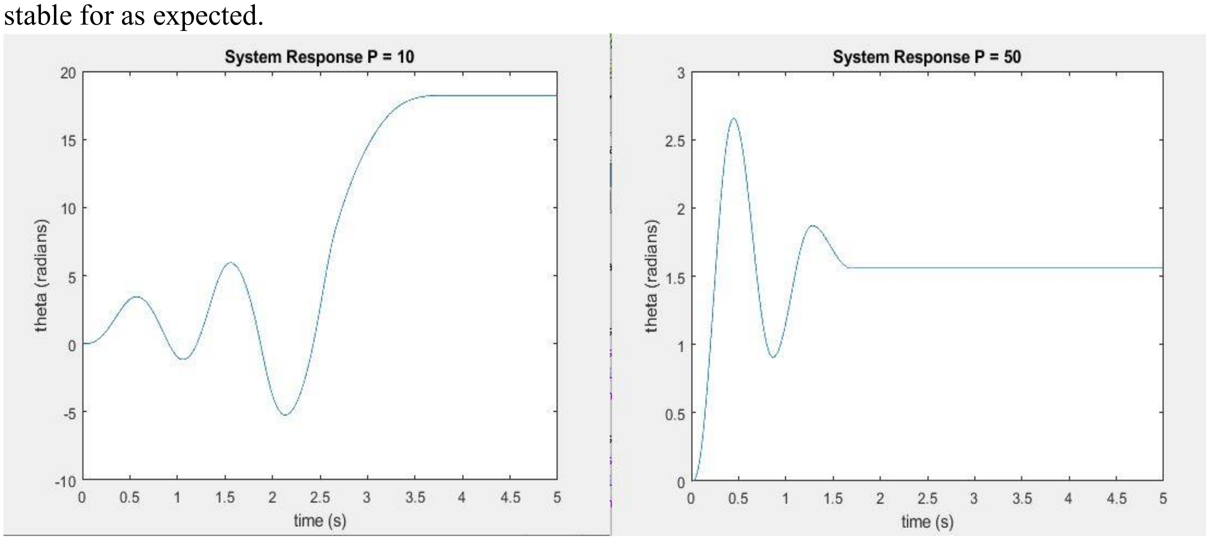

Figure 5: Time domain response of the system with varying filter coefficients. A lower filter coefficient as seen in the left image causes the system to become unstable. A higher filter coefficient as seen in the right causes the system to become underdamped.

Comparison of Results

As predicted, the analytical calculation of the range of stability for the system is insufficient to characterize the real-world system. Had the system been set with an initial gain of 0.483, as determined analytically, it would have immediately become unstable; the true value of the gain that makes the system unstable is much closer to 0.4. The discrepancy between theory and experiment can likely be attributed to unforeseen system dynamics in the electronics, mechanics, and sensors that were not accounted for in theory.

Additionally, the true results of the system response at marginal stability are not the same as theoretically predicted. For the maximum gain value that causes the system to become marginally stable, it is predicted that the system has two imaginary poles on the imaginary axis with a magnitude of 5.78, causing the system to have an oscillation of 5.78 $\frac{rad}{sec}$. The true natural frequency exists at approximately 6.98 $\frac{rad}{sec}$. Ultimately, theory is valuable for determining a general range of stability, but experiments must be conducted to ensure a system does not become unstable.