Homework #6
ME EN 5210/6210 & CH EN 5203/6203 & ECE 5652/6652
Linear Systems & State-Space Control

Use this page as the cover page on your assignment, submitted as a single pdf.

Problem 1

Consider a system with state-space equations

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$$
$$\boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u}$$

for state vector $\boldsymbol{x}$, and a change of coordinates defined by

$$\boldsymbol{z} = M\boldsymbol{x}$$

Write the state-space equations for the state vector $\boldsymbol{z}$, with the same inputs and outputs as the original system.

Problem 2

Find the companion-form equivalent equations of

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \boldsymbol{x}$$

Problem 3

For the same system from Problem 2, perform an equivalence transformation such that the new A matrix is in Jordan form. Provide the equivalent equations.

Problem 4

Discretize the following state-space equations for $T = 1$ and $T = \pi$.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \boldsymbol{x}$$

Problem 5

Solve for the analytic solution of $\boldsymbol{x}(t)$ for the unforced system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -3 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -6 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \end{bmatrix} u \qquad \text{with} \quad \boldsymbol{x}(0) = \begin{bmatrix} 10 \\ -8 \\ -4 \\ 5 \end{bmatrix}$$

(a) Solve for the analytic solution of $\boldsymbol{x}(t)$ for the unforced system (i.e., when $u(t) = 0$). Fully simplify your answer.

(b) What is the approximate amount of time it will take for this system to reach a steady-state value for any constant input?

(c) Use the lsim function in MATLAB to plot the zero-input response and the unit-step response. Choose a time duration that lets you see the states reach their steady-state values, but is not so long that the transients are hard to see. Include all of the states for a given input type on a single plot. Make sure your plots are clearly labeled, and include a legend. In addition to turning in your plots, turn in a printout of the .m file that you used to make them.