ME 5200/6200 and ECE 5615/6615 Exam 01 Practice Problems

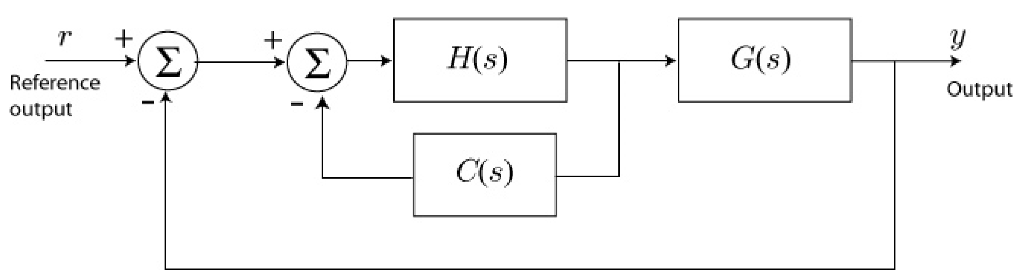
What's covered on Exam 1:

- Homeworks 1-5
- Lectures notes from Week 1 through Week 6, up to and includes Stability. No steady-state error problems will be covered in Exam 1.
- Covers reading from Chapters 1 through 6 of the text.
- Exam 1 will be 50-minutes long, approximately 4-5 problems. Exam 1 will be closed notes and book, no electronic devices of any kind. Exam will be **in-person** on Thursday, October 5, starting at 3:40 -4:40 pm (50-minute exam). Please come prepared with something to write with.
- Laplace table below will be provided.

Practice Problems – Note, some problems below have solutions while other do not, so it is encouraged that you do the problems and study with classmates and compare solutions.

Problem

Consider the following system:



$$H(s) = \frac{1}{s^2(s+1)};\quad G(s) = \frac{1}{s^2(s+3)}\quad C(s) = \frac{1}{s}$$

(a) Find the closed-loop transfer function
(b) What is the order of the closed-loop system?

Solution:

a. For the inner loop:

$$G_1(s) = \frac{\frac{1}{s^2(s+1)}}{1+\frac{1}{s^3(s+1)}} = \frac{s}{s^4+s^3+1}$$

$$G_e(s) = \frac{1}{s^2(s+3)}\ G_1(s) = \frac{1}{s(s^5+4s^4+3s^3+s+3)}$$

$$T(s) = \frac{G_e(s)}{1+G_e(s)} = \frac{1}{s^6+4s^5+3s^4+s^2+3s+1}$$

b. System is 6^{th} order.

Problem

If an open-loop system has oscillatory behavior, some of its poles must be where?

Solution: Poles must have imaginary component, so above/below the real axis.

Problem

Answer the following questions:

(a) Define transfer function
(b) What assumption is made concerning initial conditions when dealing with transfer functions?
(c) The imaginary part of a pole generates what part of a response?
(d) What is the difference between the natural frequency and damped natural frequency of oscillation?

Problem

For the following transfer function, write the corresponding differential equation:

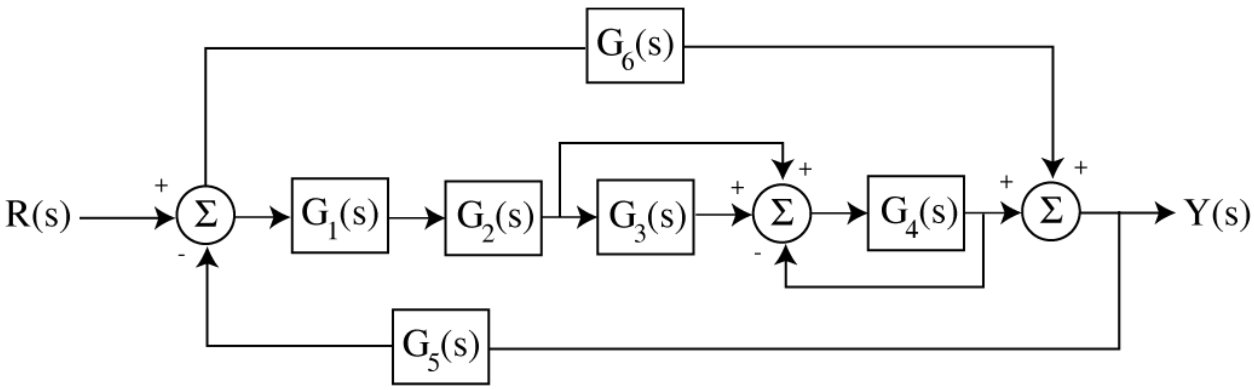
$$\frac{X(s)}{F(s)} = \frac{s}{(s+7)(s+8)}$$

Solution:

$$\ddot{x}(t) + 15\dot{x}(t) + 56 = \dot{f}(t)$$

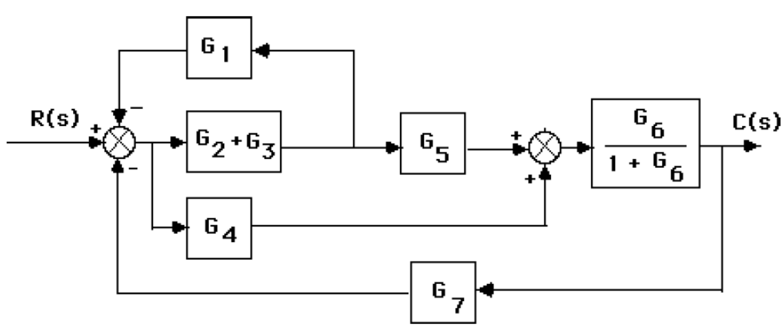
Problem

Use block-diagram algebra to find the transfer function between R(s) and Y(s) for the following block diagram. Repeat the problem using Mason's Rule.



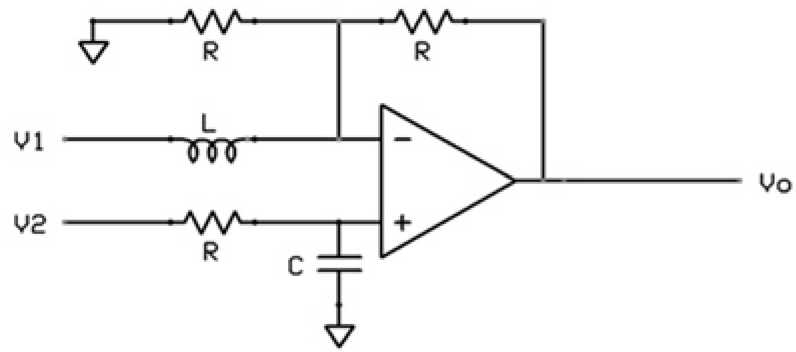
Problem

Determine the transfer function C(s)/R(s) using block diagram reduction and Mason's Rule:



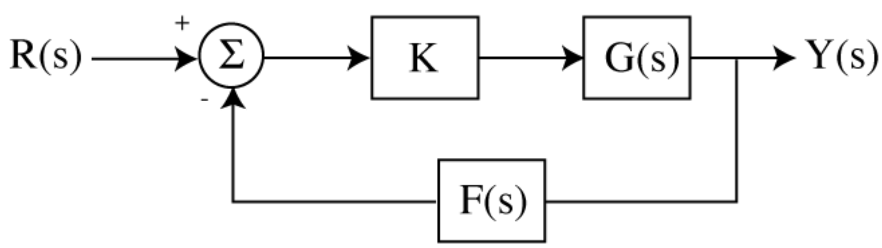
Problem

Find the output voltage Vo(s) in terms of the components and V1(s) and V2(s).



Problem

Consider the system shown below:



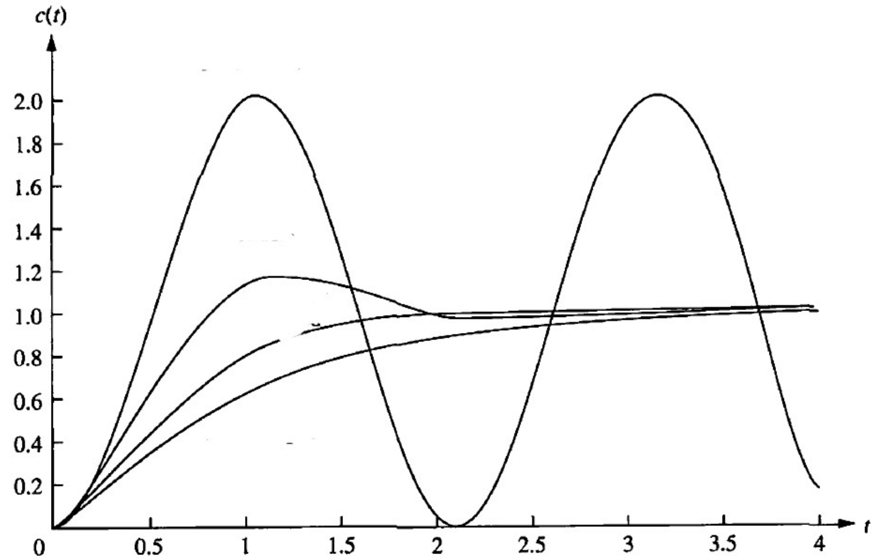
$$G(s) = \frac{1}{(s+1)(s+2)}; \quad F(s) = 1$$

(a) What is the order of the closed-loop feedback system?

(b) What gain K would give the closed loop system a damping ratio of 0.5? Remember how the damping ratio gets mapped into the s-plane.

Problem

The step responses for a second-order system are shown below. For each response, label the corresponding damping characteristics.

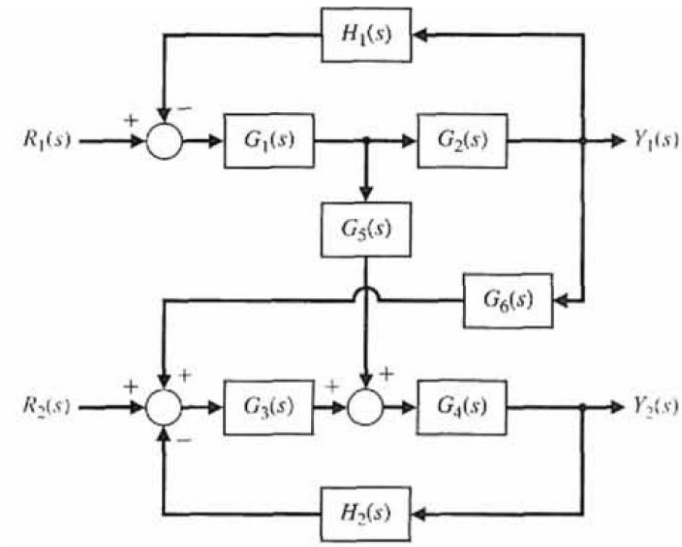


Problem

Suppose that a closed-loop control system has a damping ratio of 0.4 for the dominant poles, and the settling time for the closed-loop system is 0.5 seconds. What are the coordinates of the dominant poles? In other words, where exactly are they in the s-plane? Show all your work!

Problem

What's the transfer function from R2(s) to Y1(s)?



Problem

Suppose the characteristic equation for a closed-loop system is:

$$1 + K\frac{s(s+4)}{s^2+2s+2} = 0$$

(a) What is the open-loop transfer function?
(b) Determine the range of K for stability.

Problem

Consider the system shown in Fig. 4.39 with PI control.

(a) Determine the transfer function from R to Y.

(b) Determine the transfer function from W to Y.

(c) Use Routh's criteria to find the range of (k_p, k_I) for which the system is stable.

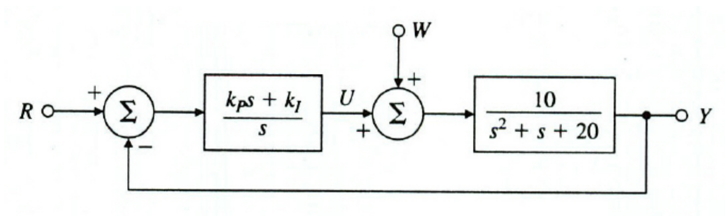


Figure 4.39: Control system

Solution:

(a)

$$\frac{Y(s)}{R(s)} = \frac{10(k_I + k_p s)}{s[s(s+1)+20] + 10(k_I + k_p s)}.$$

(b)

$$\frac{Y(s)}{W(s)} = \frac{10s}{s[s(s+1)+20] + 10(k_I + k_p s)}.$$

(c) The characteristic equation is $s^3 + s^2 + (10k_p + 20)s + 10k_I = 0$. The Routh's array is

$$\begin{array}{lcc} s^3: & 1 & 10k_p + 20 \\ s^2: & 1 & 10k_I \\ s^1: & 10k_p + 20 - 10k_I & \\ s^0: & 10k_I & \end{array}$$

For stability we must have $k_I > 0$ and $k_p > k_I - 2$.

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$	2.	$\mathbf{e}^{at}$	$\dfrac{1}{s-a}$
3.	$t^n, \quad n=1,2,3,\ldots$	$\dfrac{n!}{s^{n+1}}$	4.	$t^p, p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\dfrac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}}, \quad n=1,2,3,\ldots$	$\dfrac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\dfrac{a}{s^2+a^2}$	8.	$\cos(at)$	$\dfrac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\dfrac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\dfrac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\dfrac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\dfrac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\dfrac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\dfrac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\dfrac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\dfrac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\dfrac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at}, \quad n=1,2,3,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\dfrac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t), \quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\dfrac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\dfrac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\dfrac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			