Aerospace Propulsion

Lecture 20

Rocket Propulsion III

IMAGINE U

Rocket Propulsion: Part II

- Ideal Rocket Flight
- Less Ideal Rocket Flight
- Multiple Stage Rocket

Ideal Rocket Flight

- Previously analyzed how much thrust a rocket produces
- What can a rocket achieve with that thrust?
- <u>Mass varies dramatically with time</u>
- Assumptions
  - Gravity-free
  - Drag-free
  - Flight direction and thrust aligned
  - Constant mass flow rate and thrust
  - Valid for deep space environments far from massive bodies

*; p: momentum

Ideal Rocket Flight

- Ideal Rocket Equation

Newton's 2nd Law: $\sum F = \lim_{\Delta t \to 0} \frac{P_2 - P_1}{\Delta t}$

$P_1 = (m + \Delta m) V$

$P_2 = m(V + \Delta V) - \Delta m (V_e - V)$

$P_2 - P_1 = m(V + \Delta V) - \Delta m (V_e - V) - (m + \Delta m) V$

$= \boxed{mV} + m\Delta V - \Delta m V_e \boxed{+ \Delta m V} \boxed{- mV} \boxed{- \Delta m V}$

$P_2 - P_1 = m \Delta V - \Delta m V_e$

$\sum F = \lim_{\Delta t \to 0} \left[\frac{m \Delta V}{\Delta t} - \frac{\Delta m V_e}{\Delta t} \right]$

$\sum F = m \frac{dV}{dt} + V_e \frac{dm}{dt}$

Generally:

$D + F_g = m \frac{dV}{dt} + V_e \frac{dm}{dt}$

Assume $D = 0$, $F_g = 0$

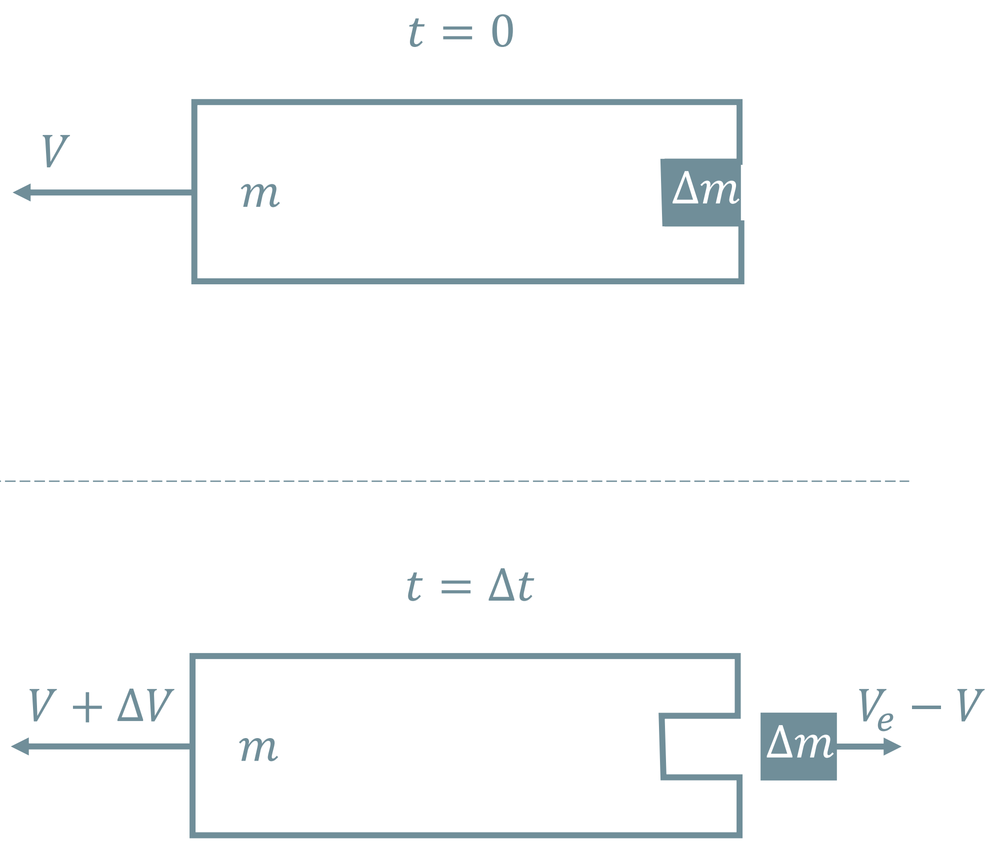
$0 = m \frac{dV}{dt} + V_e \frac{dm}{dt}$

$\frac{dV}{dt} = -V_e \frac{1}{m} \frac{dm}{dt}$

Assume constant V_e

$\int_V^{V+\Delta V} dV = -V_e \int_{m_0}^{m_{end}} \frac{dm}{m}$

$\hookrightarrow V + \Delta V - V = -V_e \ln\left(\frac{m_{end}}{m_0}\right) \rightarrow \Delta V = -V_e \ln\left(\frac{m_{end}}{m_0}\right)$



- $\Delta V = V_e \ln \frac{m_0}{m_{end}}$

Ideal Rocket Flight

- Ideal Rocket Equation
  - $\Delta V = V_e \ln \frac{m_0}{m_{end}}$

- ΔV represents the maximum velocity increment that could be obtained in an ideal rocket
  - Gravity free
  - Drag free
  - (Deep space away from masses)

Ideal Rocket Flight

- A few forms of the Ideal Rocket Equation
  - $\Delta V = V_e \ln \frac{m_0}{m_{end}}$
- Previously defined $MR = \frac{m_{end}}{m_0}$
  - $\Delta V = V_e \ln \frac{1}{MR} = -V_e \ln MR$
- Showed that $V_e = c$ $(P_e = P_a)$
  - $\Delta V = -c \ln MR$
- Taking the exponential
  - $e^{\frac{\Delta V}{V_e}} = \frac{1}{MR}$

Ideal Rocket Flight

- A few forms of the Ideal Rocket Equation
  - $\Delta V = V_e \ln \frac{m_0}{m_{end}}$
- Recall that for $p_e = p_a$, we can show
  - $V_e = I_s g_0$
- A common form of the equation is
  - $\Delta V = I_s g_0 \ln \frac{m_0}{m_{end}}$

$$\boxed{\Delta V = -V_e \ln MR}$$

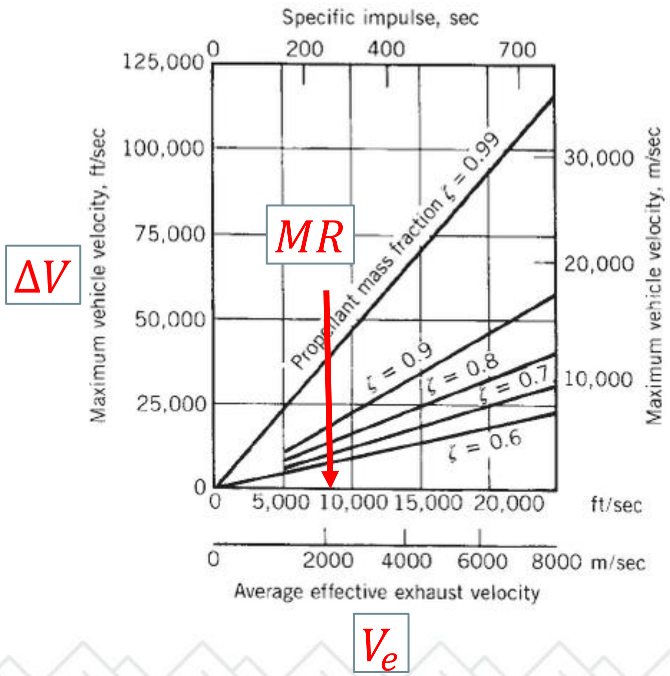
$$\boxed{\Delta V = -I_s g_0 \ln MR}$$

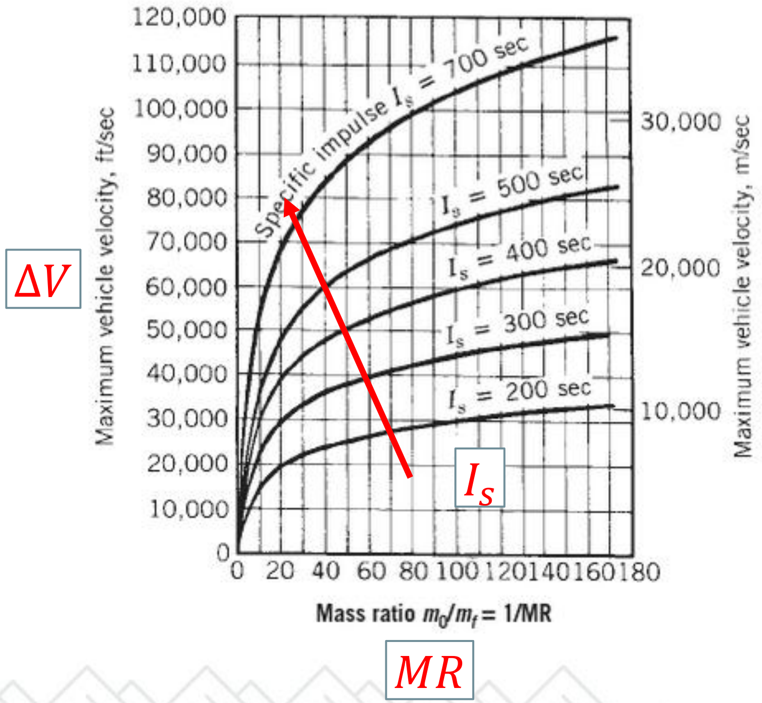
Ideal Rocket Flight

- Delta V is useful even when drag/gravity are present
  - Common way to understand how changing rocket parameters will affect its capabilities

- MR has a logarithmic effect on velocity

- I_s has a linear effect on velocity





Ideal Rocket Flight

- Rocket equation is useful for outlining mission requirements
  - $\Delta V = V_e \ln \frac{m_0}{m_{end}}$

$m_0 = m_{end} + m_p$

$\Delta V = V_e \ln\left(\frac{m_{end} + m_p}{m_{end}}\right)$

$\frac{\Delta V}{V_e} = \ln\left(\frac{m_{end} + m_p}{m_{end}}\right)$

$\exp\left(\frac{\Delta V}{V_e}\right) = \frac{m_{end} + m_p}{m_{end}}$

$\exp\left(\frac{\Delta V}{V_e}\right) = 1 + \frac{m_p}{m_{end}}$

$\left[\exp\left(\frac{\Delta V}{V_e}\right) - 1\right] m_{end} = m_p$

how much acceleration do you need?

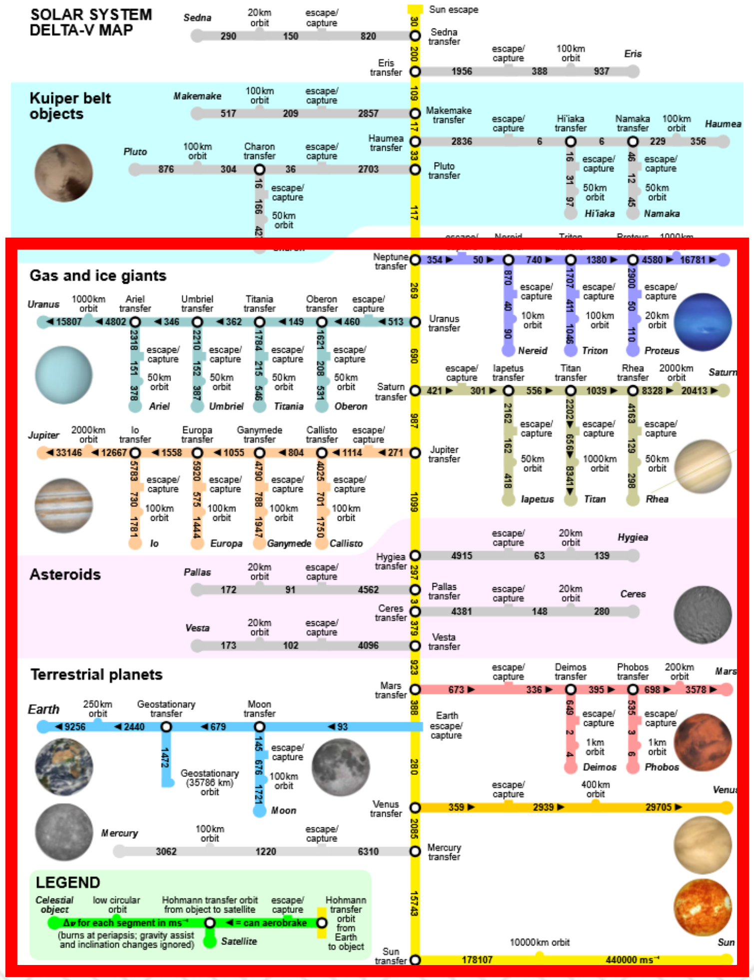
how much propellant

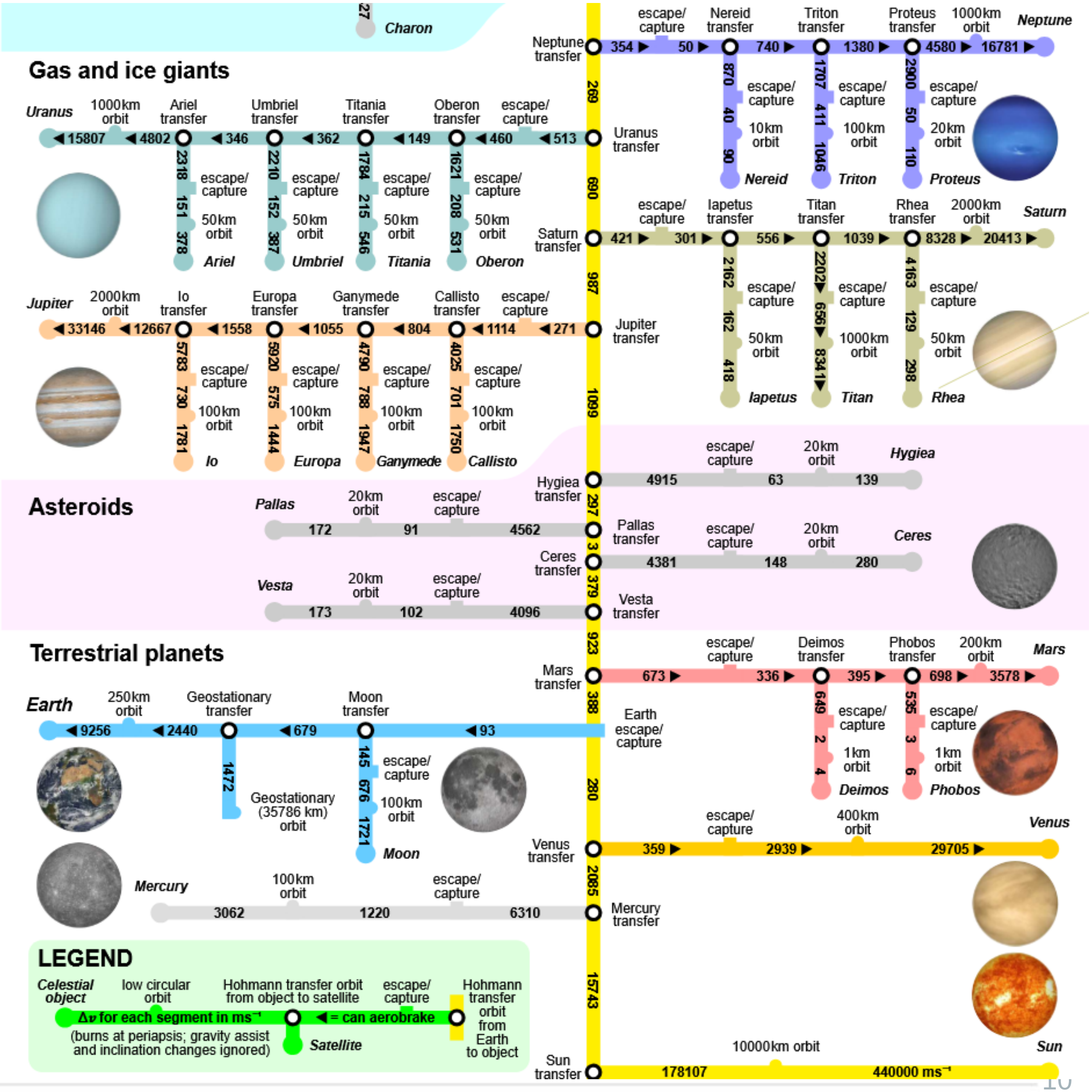
empty rocket mass

  - $m_p = m_{end}\left[\exp\frac{\Delta V}{V_e} - 1\right] = m_0\left[1 - \exp\frac{\Delta V}{V_e}\right]$

how much thrust

Ideal Rocket Flight





Less Ideal Rocket Flight

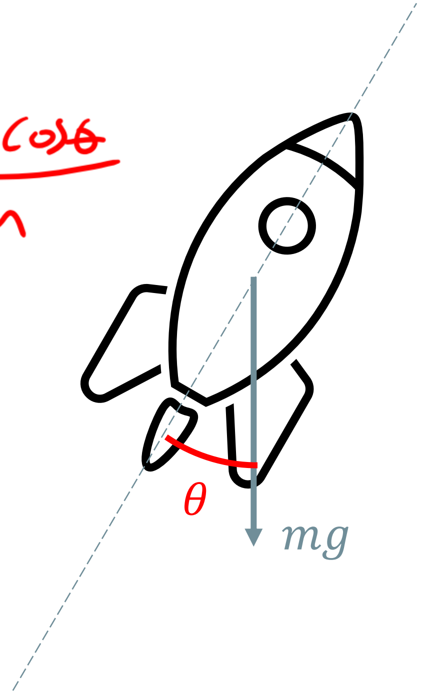
- Gravity
  - $g(h) = g_0 \left(\frac{R}{R+h}\right)^2$
    - $g(h)$: acceleration due to gravity
    - R: radius of earth (or other planet)
    - h: distance from earth
  - Assume gravity is constant (or use average gravity)
  - $\Delta V = V_e \ln \frac{m_0}{m_{end}} - g \cos\theta \, \Delta t$
    - Average (or constant) value of g
    - Average (or constant) angle θ
    - Burn time of Δt

$$F_g = m \frac{dV}{dt} + V_e \frac{dm}{dt}$$

$$\frac{dV}{dt} = V_e \frac{1}{m} \frac{dm}{dt} - \frac{F_g}{m}$$

$$\frac{dV}{dt} = V_e \frac{1}{m} \frac{dm}{dt} - \frac{mg\cos\theta}{m}$$

$$\int_0^{\Delta t} g \cos\theta \, dt$$



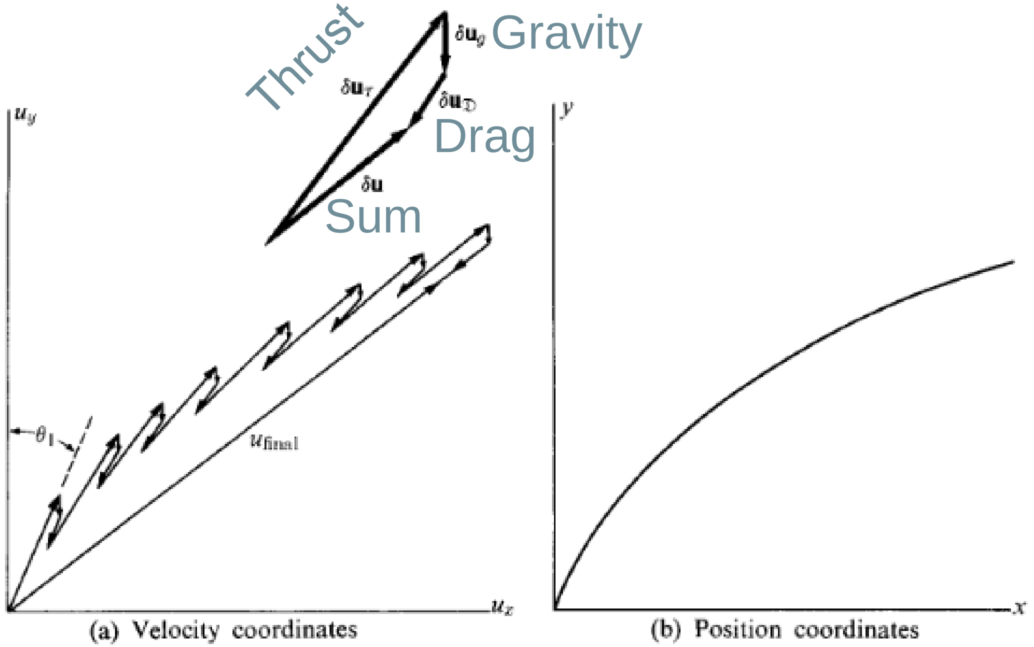
Less Ideal Rocket Flight

- Drag
  - $D = \frac{1}{2} C_D \rho V^2 A_f$
    - D: Drag Force
    - C_D: Drag coefficient
    - A_f: Frontal cross-sectional area

  - Given expressions for how the various components vary as a function of elevation and velocity, can compute drag force in ΔV equation
  - For example:
    - $\rho(h) = a \exp\left(-bh^{1.15}\right)$
      - $a = 1.2$
      - $b = 2.9 \times 10^{-5}$

Less Ideal Rocket Flight

- Simple Numerical integration techniques
  - Step 1: For a small timestep Δt, compute $\Delta V = V_e \ln \frac{m_t}{m_{t+\Delta t}}$
  - Step 2: For Δt, compute the gravity term
  - Step 3: For Δt, compute the drag term
  - Step 4: Repeat for any other forces worth considering
  - Step 5: Sum all velocity changes
- Allows for variation in θ over flight



$_b$ = end of burn
$_{end}$ = very top

(Assume $V_0 = 0$)

Less Ideal Rocket Flight

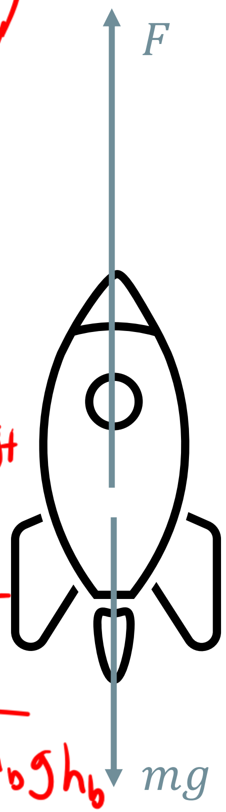
linearly exhausting mass

$V = -V_e \ln\left(\frac{m}{m_0}\right) - gt$

$m(t) = m_0 - (m_0 - m_b)\frac{t}{t_b}$

$V = -V_e \ln\left(\frac{m_0}{m_0} - \frac{m_0 - m_b}{m_0}\frac{t}{t_b}\right) - gt$

$V(t) = -V_e \ln\left[1 - (1 - MR)\frac{t}{t_b}\right] - gt$



- Elevation calculations
  - Assume a rocket firing directly upward
  - Constant exhaust velocity
  - Neglect drag

$h_b = \int_0^{t_b} V(t)\,dt$

$h_b = \int_0^{t_b} \left(-V_e \ln\left[1 - (1 - MR)\frac{t}{t_b}\right] - gt\right) dt$

$h_b = -V_e t_b \ln\left(\frac{MR}{MR-1}\right) + V_e t_b - \frac{1}{2} g t_b^2$

At burnout: $KE_b = \frac{1}{2} m_b V_b^2$, $PE_b = m_b g h_b$

At max height, $KE_{end} = 0$

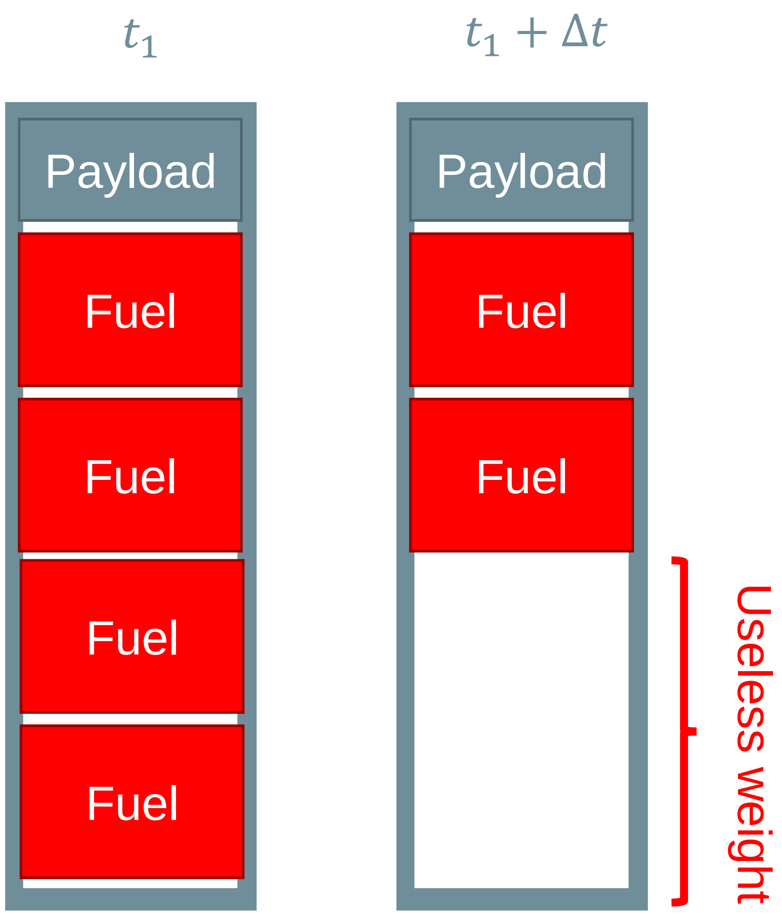
$PE_{end} = KE_b + PE_b = m_{end}\, g\, h_{max}$

Solve for h_{max}

- $h_{max} = \frac{V_e^2 (\ln MR)^2}{2 g_0} - V_e t_{\cancel{end}\; b} \left(\frac{MR}{MR-1} \ln MR - 1\right)$

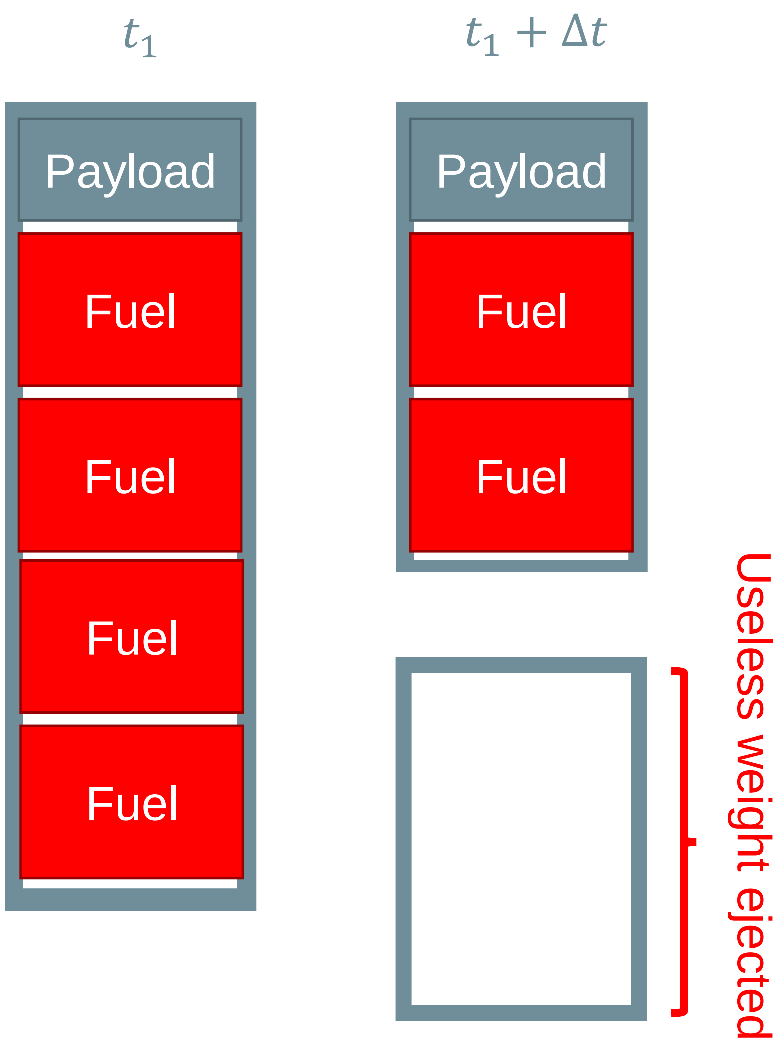
Multiple Stage Rockets

- Motivation
  - Consider a single stage rocket
  - This rocket carries a lot of fuel, but also a lot of structural mass
  - While fuel is expelled as its burned, the structural mass that held that fuel remains
  - The structure held fuel for Δt, but needs to be accelerated for the entire burn time
    - $\Delta V = V_e \ln \frac{m_0}{m_{end}}$ $\rightarrow$ higher m_{end} $=$ lower ΔV
  - Ideally, we would shed structural weight when it is no longer necessary



Multiple Stage Rockets

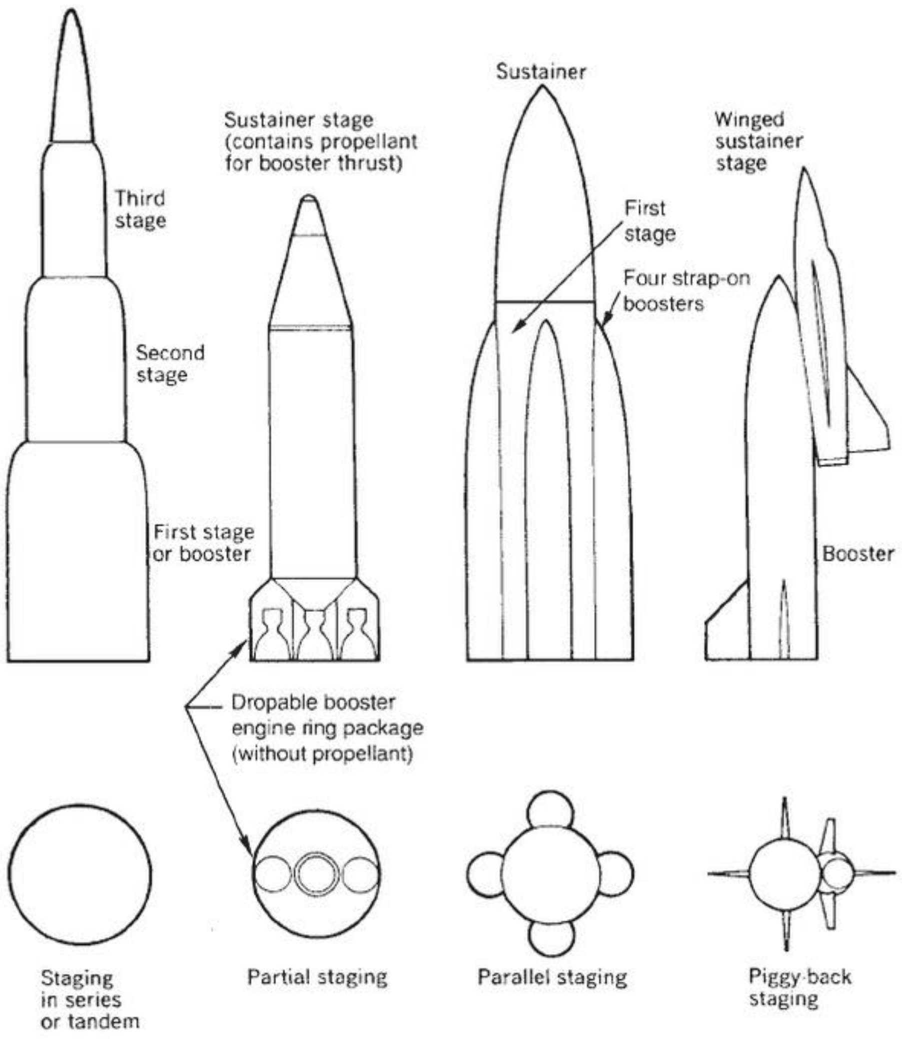
- Solution
  - Create a rocket with multiple stages
  - Each stage has its own structure, propellant, and engine that separates when finished
    - Each stage runs serially
- Multi-stage rockets are very common, especially when it comes to reaching orbit



Multiple Stage Rockets

- Various designs exist with same goal
  - Remove wasted mass when possible
- Payload usually held on last stage
  - Generally, on top of rocket
- Mass that i^{th} stage carries is the sum of its mass with all stages above it

- Launching from an airplane technically counts as multi-staging*



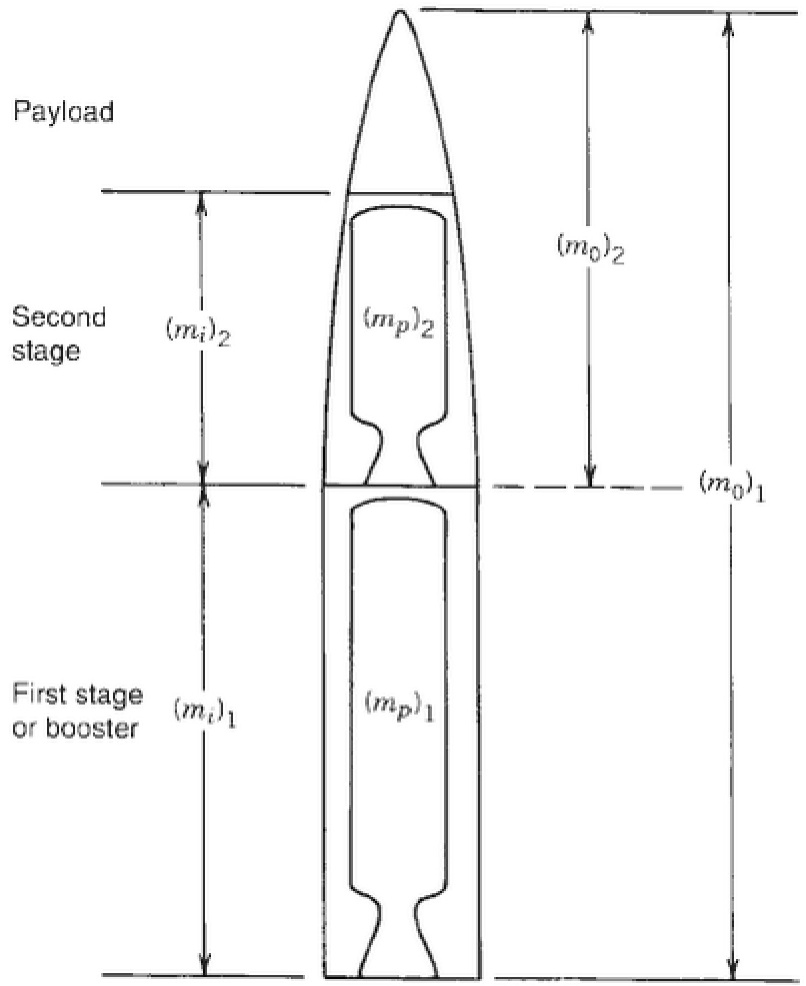
Multiple Stage Rockets

- Total change in velocity is the sum of each stages velocity change
  - $\Delta V_{tot} = \sum_{1}^{n} \Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots$

- Ideal rocket equation for staged flight
  - $\Delta V_{tot} = V_{e,1} \ln \frac{1}{MR_1} + V_{e,2} \ln \frac{1}{MR_2} + V_{e,3} \ln \frac{1}{MR_3} + \cdots$
  - Assuming no gravity or drag
  - Assuming end of one stage and beginning of next instantaneous
    - Note that there are usually a few seconds between stages
      - Separation followed by safety buffer

Multiple Stage Rockets

- Mass ratios for staged rocket
  - $MR_1 = \frac{(m_0)_1 - (m_p)_1}{(m_0)_1}$
  - $MR_2 = \frac{(m_0)_2 - (m_p)_2}{(m_0)_2}$



Multiple Stage Rockets

- Example: Saturn V
  - "Saturn V remains the only launch vehicle to carry humans beyond low Earth orbit (LEO)"
  - Dramatic decrease in required thrust due to dramatic decrease in mass between stages
  - $\zeta_1 = 0.912$
  - $\zeta_2 = 0.795$
  - $\zeta_3 = 0.507$

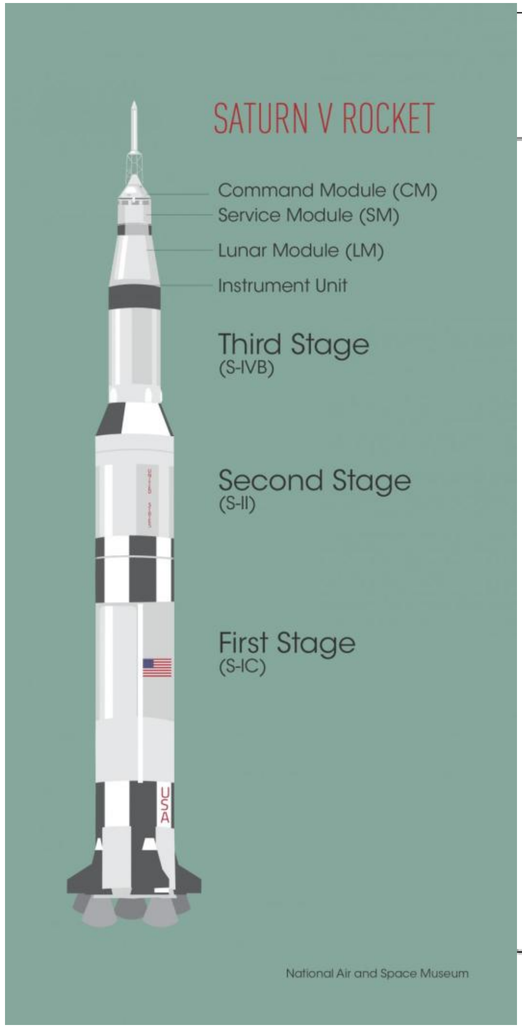


TABLE 10.3 *Saturn V Apollo* flight configuration

Mass and thrust features	Stage 1	Stage 2	Stage 3
Engine	F-1	J-2	J-2
Fuel	RP1 (hydrocarbon)	LH_2	LH_2
Oxidant	LO_2	LO_2	LO_2
Number of engines	5	5	1
Total thrust			
lb_f	7,500,000	1,000,000	200,000
kN	33,400	4,450	890
Total initial mass			
lb	6,115,000	1,488,000	473,000
kg	2,780,000	677,000	215,000
Mass of propellant			
lb	4,393,000	943,000	239,000
kg	1,997,000	429,000	109,000
Mass of structure and engines			
lb	234,000	71,600	56,500
kg	106,000	32,600	25,700
ϵ_i	0.050	0.071	0.191
Payload			
lb			178,000
kg			81,100
λ_i	0.321	0.466	0.603