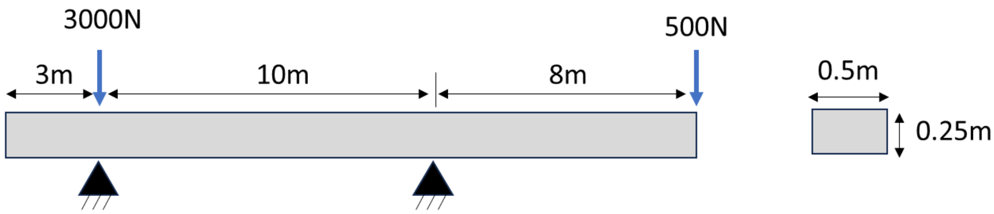
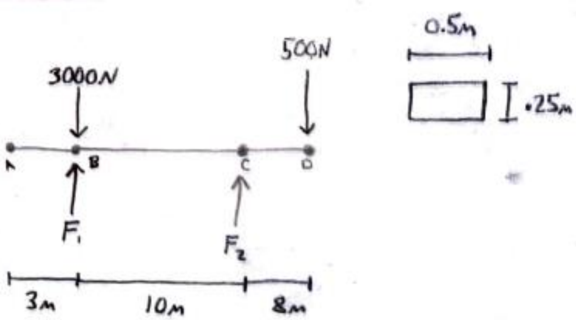
Homework # 5 – Design for Static Strength - SOLUTIONS

Problem 1 (10 points)

Find the locations and magnitudes of the maximum tensile bending stress (σ_{max}) due to moment (M) and the maximum shear stress (τ_{max}) due to shear force (V).



Problem One



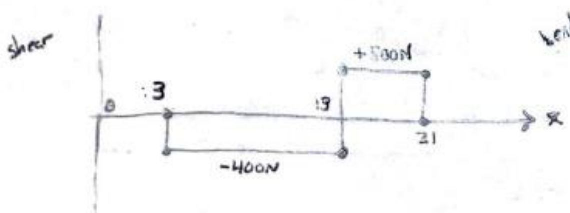
$\sum F_y = 0: \; F_1 + F_2 - 3000 - 500N = 0$

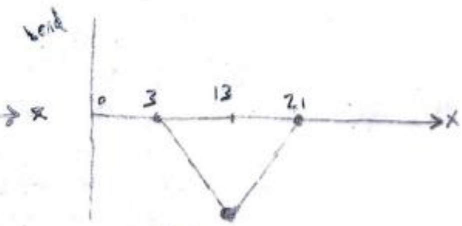
$F_1 + 900 - 3000 - 500 = 0$

$\underline{F_1 = 2600}$

$\sum M_B = 0: \; F_2(10) - 500(18) = 0$

$\underline{F_2 = 900}$





$M = -4000 \text{ N·m}$

$\tau_{max} = \dfrac{3V}{2A}$

$= \dfrac{3(500)}{2(0.5)(0.25)}$

$\tau_{max} = 6000 \text{ Pa}$

$I = \frac{1}{12} b h^3$

$\sigma_{max} = \dfrac{My}{I_{zz}}$

$\sigma_{max} = \dfrac{4000\left(\frac{.25}{2}\right)}{\left(\frac{1}{12}\right)(.5)(.25)^3}$

$\underline{\sigma_{max} = 768000}$

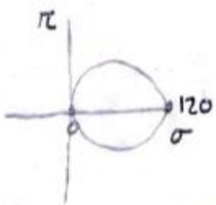
Problem 2 (20 points, 5 points each part)

A ductile material has a yield strength of $S_y = 200\ MPa$. Calculate the implied factor of safety against yielding for each of the failure theories for ductile materials (maximum shear stress, distortion energy), for each of the following cases of plane stress:

a) $\sigma_1 = 120\ MPa, \sigma_2 = 120\ MPa, \sigma_3 = 0$
b) $\sigma_1 = 0, \sigma_2 = -40\ MPa, \sigma_3 = -60\ MPa$
c) $\sigma_1 = 100\ MPa, \sigma_2 = 0, \sigma_3 = 0$
d) $\sigma_1 = 120\ MPa, \sigma_2 = 20\ MPa, \sigma_3 = 0$
(note: values given are all principal stresses)

Problem Two $S_y = 200 MPa$

(a.) $\sigma_1 = 120$, $\sigma_2 = 120$, $\sigma_3 = 0$



$\tau_{max} = 60$
$2\tau_{max} = 120$

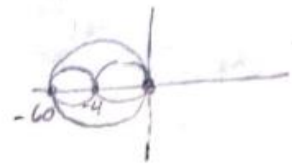
$\sigma_{vm} = \sqrt{\frac{(120-120)^2+(120-0)^2+(120-0)^2}{2}}$

$\sigma_{vm} = 120$

$n_{MSS} = \frac{200}{120}$ $n_{DE} = \frac{200}{120}$

$n_{MSS} = 1.67$ $n_{DE} = 1.67$

(b) $\sigma_1 = 0$, $\sigma_2 = -40$, $\sigma_3 = -60$



$\tau_{max} = 30$
$2\tau_{max} = 60$

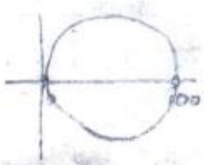
$\sigma_{vm} = \sqrt{\frac{(0+40)^2+(0-60)^2+(-40-(-60))^2}{2}}$

$\sigma_{vm} = 52.91$

$n_{MSS} = \frac{200}{60}$

$n_{MSS} = 3.33$ $n_{DE} = 3.78$

(c.) $\sigma_1 = 100$, $\sigma_2 = 0$, $\sigma_3 = 0$



$\tau_{max} = 50$
$2\tau_{max} = 100$

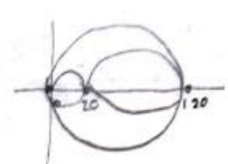
$\sigma_{vm} = \sqrt{\frac{(100-0)^2+(100-0)^2+0^2}{2}}$

$\sigma_{vm} = 100$

$n_{MSS} = \frac{200}{100}$

$n_{MSS} = 2$ $n_{DE} = 2$

(d.) $\sigma_1 = 120$, $\sigma_2 = 20$, $\sigma_3 = 0$



$\tau_{max} = 60$
$2\tau_{max} = 120$

$\sigma_{vm} = \sqrt{\frac{(120-20)^2+(120-0)^2+(20-0)^2}{2}}$

$\sigma_{vm} = 111.35$

$n_{MSS} = \frac{200}{120}$

$n_{MSS} = 1.67$ $n_{DE} = 1.80$

Problem 3 (25 points, 5 points for each of a-c, 10 points for d)

A brittle material has the following properties: $S_{ut} = 300\ MPa$, and $S_{uc} = 600\ MPa$. Calculate the implied factor of safety for the Brittle Coulomb-Mohr failure theory for the following cases of plane stress:

a) $\sigma_x = 150\ MPa, \sigma_y = 150\ MPa$
b) $\sigma_x = 80\ MPa, \tau_{xy} = 40\ MPa$
c) $\sigma_x = 150\ MPa, \sigma_y = -50\ MPa, \tau_{xy} = 50\ MPa$

(note: the values given are not principal stresses)

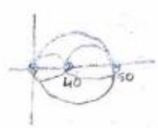
d) Use some computing platform (i.e., Matlab, Excel, Python, etc.) to plot the contour for the Brittle Mohr-Coulomb theory in the σ_B vs. σ_A plane. Make sure to scale your plot the same in the x- and y-direction and indicate and identify cases a) through c) with black square markers on this plot. Add a grid and axes labels to the plot.

Problem Three

$S_{UT} = 300$ MPa
$S_{UC} = 600$ MPa

(a.) $\sigma_x = 150$ MPa
$\sigma_y = 40$ MPa

$$\text{eig}\begin{bmatrix} 150 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sigma_1 = 0,\ \sigma_2 = 40,\ \sigma_3 = 150$$



$$n = \frac{S_{UT}}{\sigma_3} = \frac{300}{150}$$

$n = 2$

(b.) $\sigma_x = 80$ MPa
$\tau_{xy} = 40$ MPa

$$\text{eig}\begin{bmatrix} 80 & 40 & 0 \\ 40 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sigma_1 = -16.57,\ \sigma_2 = 0,\ \sigma_3 = 96.57$$

$$\frac{\sigma_3}{S_{UT}} - \frac{\sigma_1}{S_{UC}} = \frac{1}{n}$$

$$n = \left(\frac{\sigma_3}{S_{UT}} - \frac{\sigma_1}{S_{UC}}\right)^{-1} = \left(\frac{96.57}{300} - \frac{-16.57}{600}\right)^{-1}$$

$n = 2.86$

(c.) $\sigma_x = 150$
$\sigma_y = -50$
$\tau_{xy} = 50$

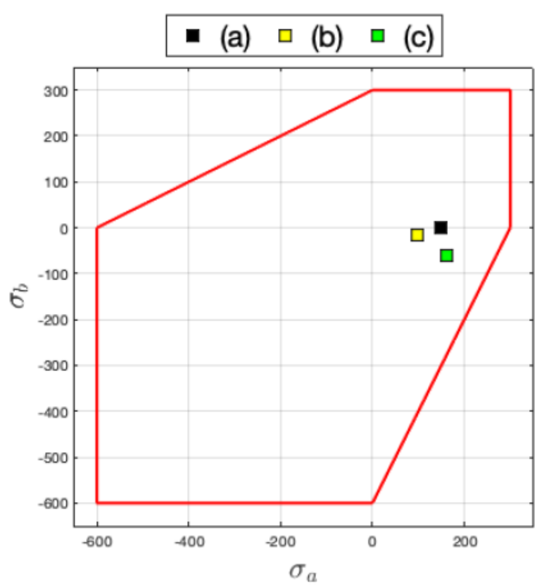
$$\text{eig}\begin{bmatrix} 150 & 50 & 0 \\ 50 & -50 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sigma_1 = -61.8,\ \sigma_2 = 0,\ \sigma_3 = 161.8$$

$$n = \left(\frac{161.8}{300} + \frac{(61.8)}{600}\right)^{-1}$$

$n = 1.56$

(d.) See attached

```
clc
close all
clear all

% Failure conditions
Sut = 300;
Suc = 600;

% Quad 2 and 4
sa4 = linspace(0,Sut,1000);
sb4 = (sa4/Sut-1)*Suc;

% Quad 1
sa1 = [0 Sut Sut];
sb1 = [Sut Sut 0];

% Quad 3
sa3 = [-Suc -Suc 0];
sb3 = [0 -Suc -Suc];

a = [150 0];
b = [96.57 -16.57];
c = [161.8 -61.8];

% Plot
plot(a(1),a(2),'sk','MarkerFaceColor','k','markersize',10); hold on;
plot(b(1),b(2),'sk','MarkerFaceColor','y','markersize',10); hold on;
plot(c(1),c(2),'sk','MarkerFaceColor','g','markersize',10); hold on;
plot(sa4,sb4,'r','linewidth',2); hold on;
plot(sb4,sa4,'r','linewidth',2); hold on;
plot(sa1,sb1,'r','linewidth',2); hold on;
plot(sa3,sb3,'r','linewidth',2); hold on;
pbaspect([1 1 1])
xlim([-650 350])
ylim([-650 350])
grid on
xlabel('$\sigma_a$','interpreter','latex','fontsize',20)
ylabel('$\sigma_b$','interpreter','latex','fontsize',20)
legend('(a)','(b)','(c)','location','northoutside','numcolumns',3,'fontsize',20)
```

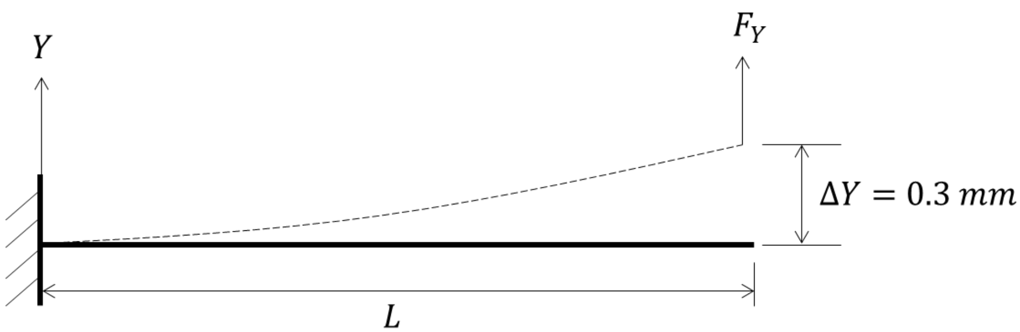


Problem 4 (20 points)

The structure you are designing contains a cantilever beam of rectangular cross-section. The beam will be deflected $\Delta Y = 0.3\ mm$ until it hits a limit stop as shown in the figure below. The length of the beam, L, is $2.5\ mm$. The width of the beam is b and the thickness of the beam is h. b and h are free design variables, but must be kept within the following ranges: $b = [0.1\ mm, 0.3\ mm]$ $h = [0.01\ mm, 0.1\ mm]$. Select the values of b and h that maximize the factor of safety of the beam for a deflection of $\Delta Y = 0.3\ mm$. What is the maximum factor of safety?

The beam is made of polysilicon (a brittle material) and has the following properties: ($E = 160\ GPa, S_{ut} = 1.4\ GPa, S_{uc} = 20\ GPa$). Use the Brittle Coulomb-Mohr failure theory.

(Hint: F_Y could be any value. It just must deflect the beam $\Delta Y = 0.3\ mm$. The stiffness of a cantilever beam is defined by the ratio of tip displacement to end load (i.e. $k = \frac{F_Y}{\Delta Y}$) For a cantilever beam, is $k = 3EI/L^3$ where E is Young's modulus, I is the second moment of area, and L is the length of the beam.)



Hand in

- All calculations to find the optimal b, and h and the factor of safety.
- If you use Matlab, Excel or some other tool (you don't have to), turn in your code or a screen shot of the spreadsheet.

Maximum stress will be the normal bending stress at the base of the beam on the top and bottom surface. The top surface will be on compression and the bottom surface will be in tension. So, failure will happen in tension (i.e., $S_{ut} \ll S_{uc}$). Find the maximum bending stress in tension.

$$\sigma_{max} = \frac{M\left(\frac{h}{2}\right)}{I} = \frac{F_Y L h}{2I} = \frac{6F_Y L h}{bh^3}$$

where M is the bending moment.

Now, find F_Y as a function of other parameters.

$$F_Y = k\Delta Y = \frac{3EI\Delta Y}{L^3} = \frac{Ebh^3\Delta Y}{4L^3}$$

Substitute in and simplify.

$$\sigma_{max} = \frac{6F_Y Lh}{bh^3} = \frac{6Lh}{bh^3}\left(\frac{Ebh^3\Delta Y}{4L^3}\right) = \frac{3Eh\Delta Y}{2L^2}$$

Noting that σ_{max} is not a function of b, it doesn't really matter what b is. σ_{max} is a linear function of h, so I want the smallest h possible.

$$\boxed{\begin{array}{c} h = 0.01\ mm \\ b = [0.1\ mm, 0.3\ mm] \end{array}}$$

The point of maximum stress is uniaxial and has no shear stress, so we are operation in quadrant I of the principle stresses domain, and the safety factor is:

$$n = \frac{S_{ut}}{\sigma_{max}}$$

$$\sigma_{max} = \frac{3Eh\Delta Y}{2L^2} = 115\ MPa$$

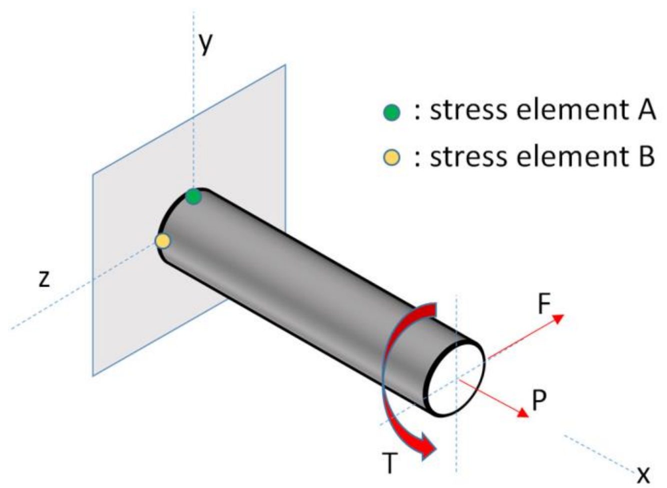
$$\boxed{n = \frac{1400\ MPa}{115\ MPa} = 12.2}$$

Note, I could have written Matlab code to calculate this stress over all b and h. As F_Y is not constrained by the problem, any combination of b and h could result in $\Delta Y = 0.3\ mm$. But, it is much simpler to just look at the equation.

<u>Problem 5</u> (20 points, 10 each part)

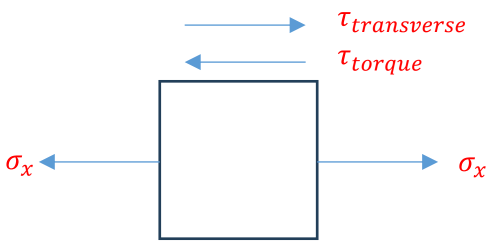
Consider a beam made of AISI 1006 Cold-drawn steel as shown in the figure ($S_y = 280\ MPa$). The relevant geometry and loading are:

- Length of beam (l): 100 mm
- Diameter of beam (d): 15 mm
- $F = 0.55\ kN$
- $P = 4\ kN$
- $T = 25\ Nm$



a) Determine the factor of safety based on the Distortion-energy theory for stress elements A and B.
b) Assume that the length is exact, but the diameter is $d = 15 \pm 1.2\ mm$ where the tolerance range is $\pm 3\sigma$. Considering only the stresses at points A and B, what proportion of parts will have factor of safety less than 1.3?

a) Stress element A



$$\sigma_x = \frac{P}{A} = \frac{4000\text{N}}{\frac{\pi}{4}(0.015\text{m})^2} = 22.6\text{ MPa}$$

as stress due to bending is zero at point A

$$\tau_{xy} = \frac{T}{I_p} - \frac{4F}{3A} = \frac{16T}{\pi d^3} - \frac{16F}{3\pi d^2}$$

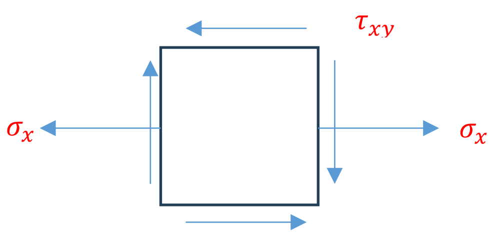
$$= \frac{16(25\text{ Nm})}{\pi(0.015)^3} - \frac{16(550\text{N})}{\pi(0.015)^3} = 33.6\text{ MPa}$$

$$\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sqrt{(22.6MPa)^2 + 3(33.6MPa)^2} = 62.4\ MPa$$

$$\boxed{n = \frac{S_y}{\sigma_{eq}} = \frac{280\ MPa}{62.4\ MPa} = 4.48}$$

(Note, you could use a different method to get σ_{eq}, such as calculating the principal stresses and then using a different equation with the principal stresses.)

Stress element B



$$\sigma_x = \frac{P}{A} + \frac{Fl\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = 22.6\text{ MPa} + \frac{32(550N)(0.1m)}{\pi(0.015m)^3} =$$

$$189\ MPa$$

$$\tau_{xy} = \frac{T}{I_p} = \frac{16T}{\pi d^3} = \frac{16(25\text{ Nm})}{\pi(0.015)^2} = 37.7\text{ MPa}$$

$$\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sqrt{(189MPa)^2 + 3(37.7MPa)^2} = 200\ MPa$$

$$\boxed{n = \frac{S_y}{\sigma_{eq}} = \frac{280\ MPa}{200\ MPa} = 1.4}$$

b) First, given that the safety factor for point B is smaller, we can just focus on point B. Any part that fails at point A will also fail at point B.

There are two approaches to this problem.

1) Find the critical d, d', that results in an $n = 1.3$. Convert d' to an equivalent z value and then use the z table to get the proportion of parts below $n = 1.3$.

Using the equations from part a) I calculate that $d' = 14.6\ mm$ results in $n = 1.3$.

$$z = \frac{d' - \mu_d}{\sigma_d} = \frac{14.6 - 15}{\frac{1.2}{3}} = -1.0$$

From the z table, a 16% of the distribution falls below $z = -1.0$.

$\boxed{\text{Point B: } P(d < 14.6\ mm) \approx 0.16 \quad or \quad 16\%}$

2) Find a distribution for the factor of safety, n, and calculate the proportion of that distribution below $n = 1.3$. Stress is a nonlinear function of the diameter. Thus, I cannot use a linear combination of variables to directly calculate a distribution on n.

So, use a Monte Carlo simulation in Matlab. The results of this simulation are:

$\boxed{\text{Point B: } P(n < 1.3) \approx 0.16 \quad or \quad 16\%}$

Matlab Code

```
Sy = 280e6;      % Yield strength [Pa]
F = 550;         % Bending load [N]
P = 4000;        % Axial load [N]
T = 25;           % Torque [Nm]
l = 0.1;         % length [m]
d_mean = 0.015;       % mean diameter [m]
d_sigma = 0.0012/3;      % standard deviation of diameter [m]


% Calucate equivalent stresses
n = 10000;       % Number of iterations to simulate
countA = 0;
countB = 0;

for i = 1:n
    d = norminv(rand,d_mean,d_sigma);

    % Point A
    sigma_x_A = 4*P/(pi*d^2);
    tau_xy_A = 16*T/(pi*d^3) - 16*F/(3*pi*d^2);
    sigma_eq_A = sqrt(sigma_x_A^2 + 3*tau_xy_A^2);
    n_A(i) = Sy/sigma_eq_A;
    if n_A(i) < 1.3
        countA = countA+1;
    end

    % Point B
    sigma_x_B = 4*P/(pi*d^2) + 32*F*l/(pi*d^3);
    tau_xy_B = 16*T/(pi*d^3);
    sigma_eq_B = sqrt(sigma_x_B^2 + 3*tau_xy_B^2);
    n_B(i) = Sy/sigma_eq_B;
    if n_B(i) < 1.3
        countB = countB+1;
    end
end
```

```
ratioA = countA/n       % proportion below 1.3 for point A
ratioB = countB/n       % proportion below 1.3 for point B

figure()
hist(n_A)
title('FoS point A')

figure()
hist(n_B)
title('FoS point B')
```

Problem 6 (15 points)

The figure below shows a cantilever beam with a circular cross-section of diameter d, made from 1035 hot rolled steel ($S_y = 270\ MPa$). The cantilever beam is end loaded with a force $F = 2000\ N$ resulting in a bending moment, an axial force $P = 15{,}000\ N$, and a torsional moment $M_t = 150\ Nm$ which acts in the yz-plane. The length of the beam is $L = 0.1\ m$. Using the Distortion-energy failure criterion, plot the factor of safety as a function of d in the range $25\ mm \leq d \leq 45\ mm$.

(Hint: Normal stress due to bending will contribute more than transverse shear stress.)

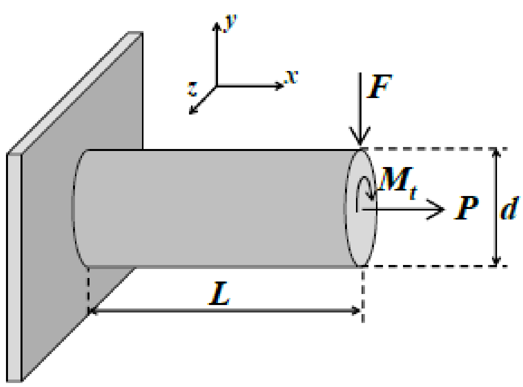


Figure : Cantilever beam

Hand in

- All calculations / equations used to find the relationship between d and the factor of safety.
- A graph showing the factor of safety as a function of d. (Label axes.)
- Any Matlab or python code you used, or a screen shot of your spreadsheet.

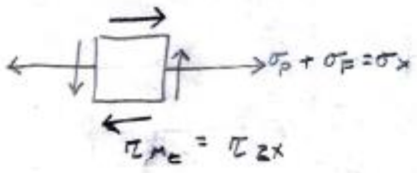
Problem Six

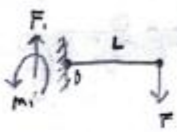
$S_y = 270 P_a$
$E = 196\ GPa$

$F = 2000 N$
$P = 15000 N$
$M_t = 150 N{\cdot}m$
$L = 0.1 m$
$25mm \leq d \leq 45mm$
$n\ 2.$

Most stress occurs at top and bottom of the beam, normal stress contribution highest here

top view draw plane

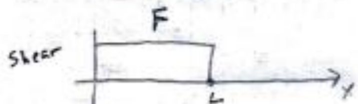


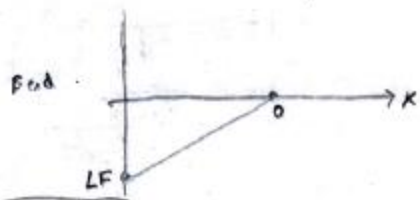


$\Sigma F_y = 0 :\ F_1 - F = 0 \quad F_1 = F$

$\Sigma M_D = 0 :\ M_1 - L \cdot F = 0$

$M_1 = L \cdot F$





$M_{max\ bend} = L \cdot F$

$$\sigma_F = \frac{(L \cdot F)(\frac{d}{2})}{(\pi \cdot d^4/64)}$$

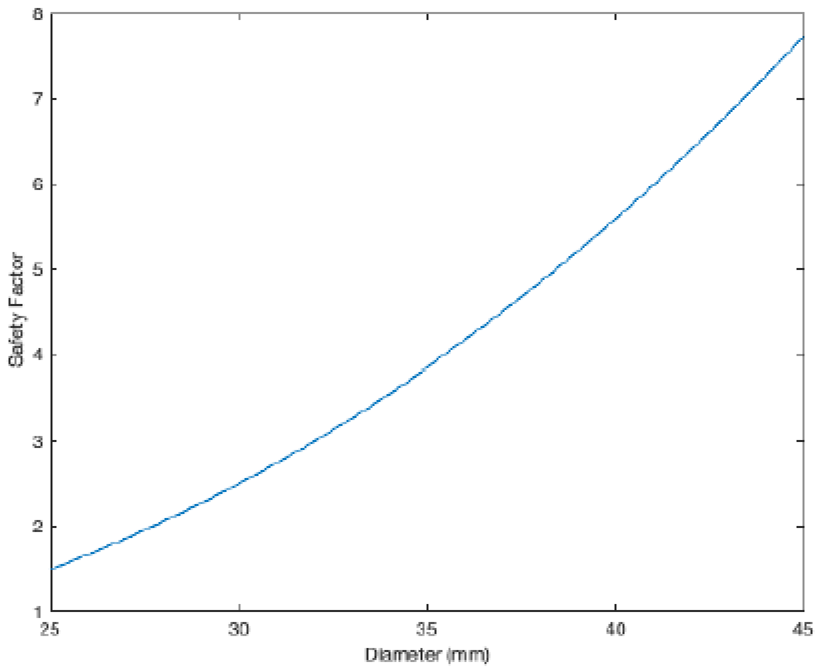
$$\sigma_P = P \Big/ \left(\pi \cdot \frac{d^2}{4}\right)$$

$$\tau_{Mt} = \frac{16 \cdot M_t}{\pi\, d^3}$$

$$\sigma_x = \sigma_F + \sigma_P$$

$$\tau_{zx} = \tau_{Mt}$$

See MatLab

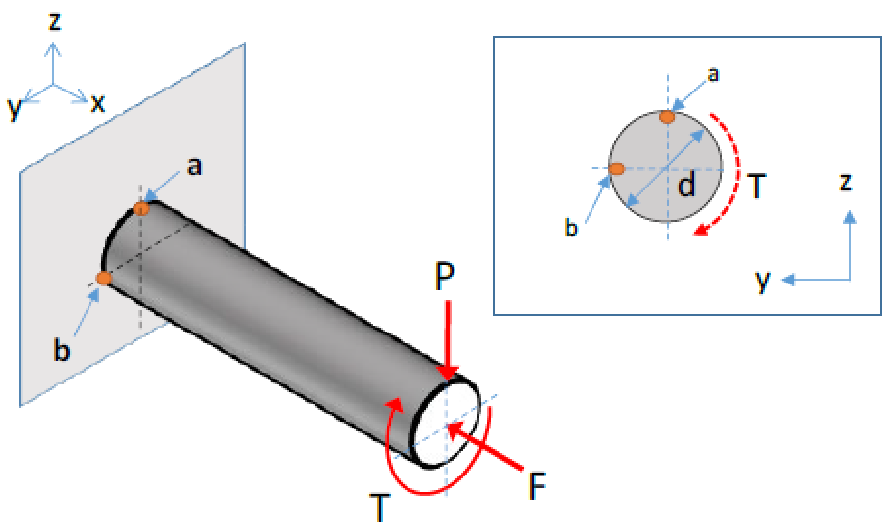


Matlab code

```
clc
clear all
close all

Sy = 270*10^6;

F = 2000;
P = 15000;
Mt = 150;
L = 0.1;

d = linspace(0.025,0.045,1000);

sf = (L*F*d/2) ./ (pi*d.^4/64);

sp = P ./ (pi*d.^2/4);

tmt = (16*Mt) ./ (pi*d.^3);

sx = sf + sp;
txz = tmt;

for i = 1:1:length(d)
    A(:,:,i) = [sx(i) 0 txz(i);0 0 0;txz(i) 0 0];
    B(:,:,i) = eig(A(:,:,i));
    C(i) = sqrt((B(1,1,i)-B(2,1,i))^2+(B(1,1,i)-B(3,1,i))^2+(B(2,1,i)-
B(3,1,i))^2)/sqrt(2);
    D(i) = Sy / C(i);
end

plot(d*1000,D)
xlabel('Diameter (mm)')
ylabel('Safety Factor')
```

Problem 7 (20 points, 10 each part)

Consider a solid round bar made of brittle material that has $S_{ut} = 30\ MPa, S_{uc} = 100\ MPa$. The bar is firmly fixed to the wall and is subject to: 1) an axial load of $F = 100\ kN$, 2) a bending load of $P = 5\ kN$, and 3) a torque of $T = 3\ kNm$. The length of the bar is $L = 500\ mm$ and the diameter is $d = 100\ mm$.



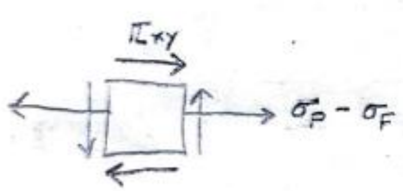
a) Determine the factor of safety at point “a” using Brittle Coulomb-Mohr theory.
b) Determine the factor of safety at point “b” using the Modified Mohr theory.

Problem Seven

$S_{uT} = 30 MPa$
$S_{uc} = 100 MPa$

$F = 100 kN = 100,000 N$
$P = 5 kN = 5000 N$
$T = 3000 N$
$L = 0.5 m$
$d = 0.1 m$

(a) Contribution of bending normal F in tension ~~tension~~, Compression of F and torsion of T



$$\sigma_F = \frac{-(100,000)}{\pi/4\,(0.1m)^2} = -12,732,395\ Pa$$

$$\tau_{xy} = \frac{16 \cdot T}{\pi d^3} = \frac{16\,(3000)}{\pi\,(.1)^3} = 15,278,874\ Pa$$

$$\sigma_P = \frac{L \cdot P\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{64}\right)} = \frac{(0.5 \times 5000)\left(\frac{.1}{2}\right)}{\left(\frac{\pi \cdot 0.1^4}{64}\right)} = 25{,}464{,}790 \text{ Pa}$$

$$\sigma_x = \sigma_P + \sigma_F = 12{,}732{,}395 \text{ Pa} = 12.73 \text{ MPa}$$

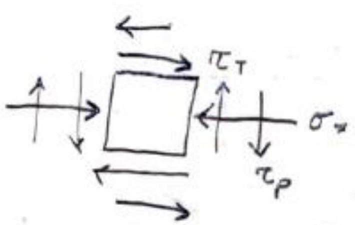
$$\tau_{xy} = 15.28 \text{ MPa}$$

$$\text{eig}\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} \sigma_1 &= -10.1877 \text{ MPa} \\ \sigma_2 &= 0 \\ \sigma_3 &= 22.92 \text{ MPa} \end{aligned}$$

$$\frac{1}{n} = \frac{\sigma_3}{S_{UT}} - \frac{\sigma_1}{S_{UC}}$$

$$n = 1.15$$

(b) Contribution of compression F, shear twist T, shear from P



$$\sigma_x = -12.73 \text{ MPa}$$

$$\tau_T = 15.28 \text{ MPa}$$

$$\tau_P = \frac{4P}{3A} = \frac{4 \cdot 5000}{3\left(\pi/4 \cdot 0.1^2\right)} = 0.849 \text{ MPa}$$

$$\tau_{xy} = 14.431 \text{ MPa}$$

$$\text{eig}\begin{bmatrix} -12.73 & 0 & 14.431 \\ 0 & 0 & 0 \\ 14.431 & 0 & \end{bmatrix} = \begin{aligned} \sigma_1 &= 9.41 \text{ MPa} \\ \sigma_2 &= 0 \text{ MPa} \\ \sigma_3 &= -22.1 \text{ MPa} \end{aligned}$$

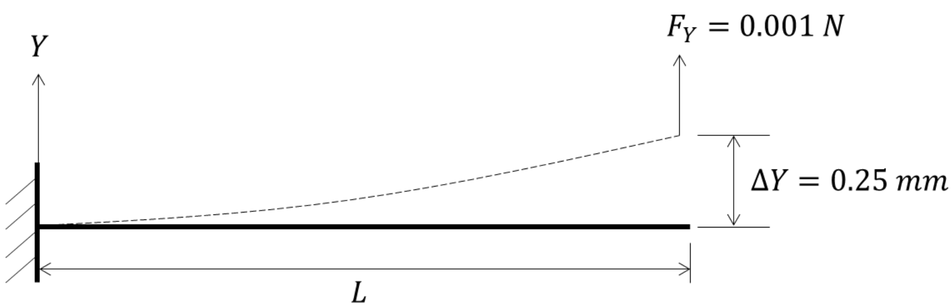
$$\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_1}{S_{uc} \cdot S_{ut}} - \frac{\sigma_3}{S_{uc}}$$

$$n = 2.27$$

Extra Credit Problem (15 points)

The structure you are designing contains a cantilever beam of rectangular cross-section similar to Problem 4, but there is no limit stop. Under the load $F = 0.001\ N$, the free end needs to deflect $0.25\ mm$. b and h are free design variables, but must be kept within the following ranges: $b = [0.1\ mm, 0.3\ mm]$ $h = [0.01\ mm, 0.1\ mm]$. Select the values of b and h that maximize the safety of factor of the beam.

The beam is made of polysilicon (a brittle material) and has the following properties: ($E = 160\ GPa, S_{ut} = 1.4\ GPa, S_{uc} = 20\ GPa$). Use the Brittle Coulomb-Mohr failure theory.



Hand in

- All calculations to find the optimal b, and h and the factor of safety.
- If you use Matlab, Excel or some other tool, turn in your code or a screen shot of the spreadsheet.

We know that $k = \frac{Ebh^3}{4L^3} = \frac{F}{\Delta Y} = \frac{0.001N}{0.00025m} = 4\ N/m$.

$$bh^3 = \frac{4L^3}{E}\left(4\frac{N}{m}\right) = \frac{4(0.0025\ m)^3}{160 \times 10^9\ N/m^2}\left(4\frac{N}{m}\right) = 1.56 \times 10^{-18}\ m^4$$

From problem 4 we have

$$\sigma_{max} = \frac{3Eh\Delta Y}{2L^2}$$

And

$$n = \frac{S_{ut}}{\sigma_{max}} = \frac{2L^2 S_{ut}}{3Eh\Delta Y}$$

This equation tells me that n is not a function of b and that I want the smallest h allowable.

So, select $h = 0.01\ mm$.

Then $b = \frac{(1.56\times10^{-18})}{h^3} = \frac{(1.56\times10^{-18})}{0.00001^3} = 0.00156\ m = 1.56\ mm$.

But, this b is outside the range of possibilities. It is too large. So I have to reduce b to the maximum possible value of $b = 0.3\ mm$ and calculate the corresponding h.

$$h = \left(\frac{1.56 \times 10^{-18}}{0.0003}\right)^{1/3} = 1.73 \times 10^{-5} m = 0.0173\ mm$$

So:

$$\boxed{\begin{gathered} b = 1.56\ mm \\ h = 0.017\ mm \end{gathered}}$$

Now calculate the resulting safety of factor.

$$n = \frac{S_{ut}}{\sigma_{max}} = \frac{2L^2 S_{ut}}{3Eh\Delta Y} = \frac{2(0.0025)^2(1.4 \times 10^9)}{3(160 \times 10^9)(1.73 \times 10^{-5})(0.00025)} = 8.4$$

Again, I could have written Matlab code to get the b and h combinations that give me the right stiffness and then calculate safety factor for each of those combinations. But I think this method was simpler.