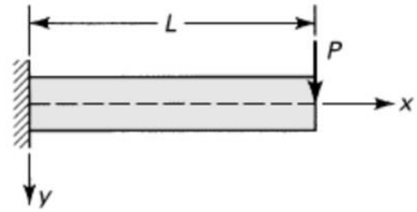
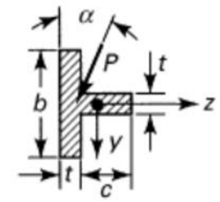
Practice Problem 2

A concentrated load $P = 4\ kN$ acts on the cantilever at an angle of 15°, as shown. The beam is constructed of a 2024-T4 aluminum alloy ($\sigma_{yp} = 290\ MPa$) with known dimensions ($L = 1.5\ m$; $t = 20\ mm$; $c = 60\ mm$; $b = 80\ mm$). Determine (a) the <u>stress of greatest magnitude</u> in the beam and (b) <u>whether it would be expected to yield</u> under these conditions (answer yes or no). Also report (c) the <u>angle of the neutral axis</u> (in deg). Neglect the effect of shear in bending and assume that the load acts through the shear center. The centroid of the section is located 27.14 mm from the far left of the section. Take $I_y = 15.11x10^{-7}\ m^4$ and $I_z = 8.93x10^{-7}\ m^4$.

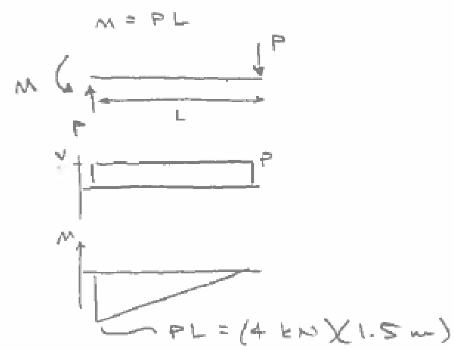




a) σ_{max}:	______________	MPa
b) yield?	______________	
c) ϕ:		deg

Bending eq'n : $\sigma_x = \dfrac{(M_y I_z + M_z \cancel{I_{yz}}^{\,0}) z - (M_y \cancel{I_{yz}}^{\,0} + M_z I_y) y}{I_y I_z - \cancel{I_{yz}^2}^{\,0}}$

- $I_{yz} = 0$ due to symmetry about z



- Moment is highest at wall
- $M_y = (P \sin\alpha) L = PL \sin\alpha$

  $= P(1.5\ m)\sin(15°) = 1.55\ kN\cdot m$
- $M_z = (P\cos\alpha)L = PL\cos\alpha$

  $= P(1.5\ m)\cos(15°) = 5.80\ kN\cdot m$

- Need angle of neutral axis to determine point for evaluation. From above, $\sigma_x = \dfrac{M_y I_z z - M_z I_y y}{I_y I_z}$
  - For $\sigma_x = 0$, $M_y I_z z = M_z I_y y \Rightarrow \tan\phi = \dfrac{y}{z} = \dfrac{M_y I_z}{M_z I_y}$

  $\Rightarrow \tan\phi = \dfrac{(0.388\ \cancel{P})(8.93)}{(1.45\ \cancel{P})(15.105)} = 0.158 \Rightarrow \underline{\phi = 8.9^0}$

- $\sigma_x = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z}$

  $= \dfrac{(1.55\ kN\cdot m)(-0.02714\ m)}{15.11 \times 10^{-7}\ m^4} - \dfrac{(5.8\ kN\cdot m)(0.04\ m)}{8.93 \times 10^{-7}\ m^4}$

  $= \underline{-288\ MPa}$

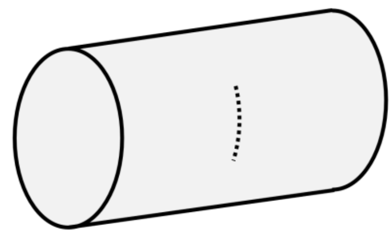
$y = b/2 = 40\ mm$

$z = -27.14\ mm$

- $|\sigma_x| < 290\ MPa \Rightarrow$ <u>no failure</u>

Practice Problem 3

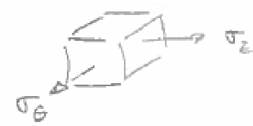
A hollow cylinder with outer and inner radii of 20 and 10 mm, respectively, is capped at its ends and pressurized. Measurements show that a circumferentially oriented, dashed line drawn on the outside surface changes in length from 10.0000 to 10.0004 mm when internal pressure p_i is applied. Determine the <u>internal pressure</u> (in kPa) given $E = 200\ MPa$ and $\nu = 0.28$. (*Hint*: Consider Hooke's Law.)



p_i:	kPa

- $\varepsilon_\theta = 0.00004$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{bmatrix} = \begin{bmatrix} 1/E & -\frac{\nu}{E} & -\frac{\nu}{E} \\ & 1/E & -\frac{\nu}{E} \\ & & 1/E \end{bmatrix} \begin{bmatrix} \overset{0}{\cancel{\sigma_r}} \\ \sigma_\theta \\ \sigma_z \end{bmatrix}$$



$$\Rightarrow 0.00004 = \frac{1}{E}\sigma_\theta - \frac{\nu}{E}\sigma_z$$

- $\sigma_z = \dfrac{p_i a^2}{b^2 - a^2} = \dfrac{1}{3} p_i$

- $\sigma_\theta = \dfrac{a^2 p_i}{b^2 - a^2} + \dfrac{p_i a^2 \cancel{b^2}}{(b^2 - a^2)\cancel{b^2}} = 2\dfrac{a^2}{b^2 - a^2} p_i = 2\sigma_z = \dfrac{2}{3} p_i$

$$\Rightarrow E\varepsilon_\theta = 2\frac{a^2}{b^2 - a^2} p_i - \nu \frac{a^2}{b^2 - a^2} p_i = (2 - \nu)\frac{a^2}{b^2 - a^2} p_i$$

$$\Rightarrow p_i = \frac{E\varepsilon_\theta}{2 - \nu}\,\frac{b^2 - a^2}{a^2} = \frac{(200MPa)(0.00004)}{2 - 0.28}\,\frac{(0.02)^2 - (0.01)^2}{(0.01)^2} = \underline{14\ kPa}$$