Lab 10: DC Motor Torque/Speed Characteristics

1. Introduction

The goal of this exercise is to measure and plot the torque-speed curve of a DC electric motor. To do this you will use a force/torque measurement setup as a Prony Brake dynamometer. This will require that you calibrate the force sensor and then use the calibration coefficient along with output from an optical tachometer to obtain the torque-speed curve for the DC motor used in the lab setup.

1.1 Lab Objectives

- Understand how to calibrate a force sensor
- Understand the torque/speed characteristics for a DC motor

1.2 Project Objectives

- Motor selection based on torque and speed requirements

1.3 Lab Hardware

- Digital multimeter
- Solderless breadboard and jumper wires
- One INA126 instrumentation amplifier
- One dynamometer lab setup

1.4 Project hardware

- Students should buy parts for the lab as needed
- Students should finalize their motors for their robot

2. Laboratory Concepts

2.1 Force Sensor

Neither force nor torque are measured directly. Typically, engineers measure the deformation or strain of some material and then calculate the force or torque required to produce the deformation in that material using. Deformation can be measured in many ways. If the displacement is large, as with a spring, the displacement can be read directly on a scale or linear potentiometer. If the displacement is smaller, a linear variable differential transformer (LVDT), linear encoder, or other sensitive displacement measuring transducer can be used. If the deformation is very small, strain gauges can be applied to the surface of the deformed material.

The torque applied by the motor will be calculated using the electrical output of strain gauges placed on a thin beam that is undergoing transverse deflection from applied forces. A bonded metal foil strain gauge is a variable resistor whose change in resistance is proportional to the strain in the material upon which it is mounted. For this lab, we will be using Full Bridge Thin Beam Load Cells, which is essentially a complete package of strain gauges mounted on a thin beam Figure 1.

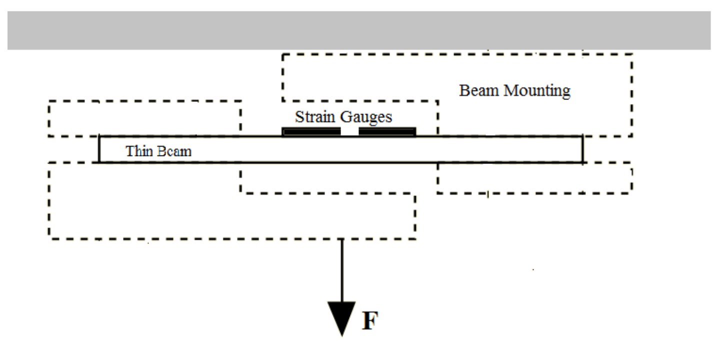


Figure 1. Full bridge thin beam load cell setup.

In this setup four gauges are mounted on the top of the beam and are wired together to form a Wheatstone bridge circuit. This arrangement of four gauges provides higher sensitivity than using only one or two gauges and it is insensitive to off-axis forces and temperature changes. You will deform the beam by applying a point force of magnitude F by hanging measured weights at the center of the beam mounting as shown in Figure 1, thus deflecting the beam and creating strain. The strain along the beam is proportional to the force exerted by the weights. The strain will be measured using strain gauges in a Wheatstone bridge configuration. The Wheatstone bridge circuit used in this lab is depicted in the Figure 2. You will calibrate the output of the strain gauges as a function of the applied force using a linear approximation obtained via a least-squares fit. From this calibration, the measurements taken from the strain gauges can be directly converted to the applied force.

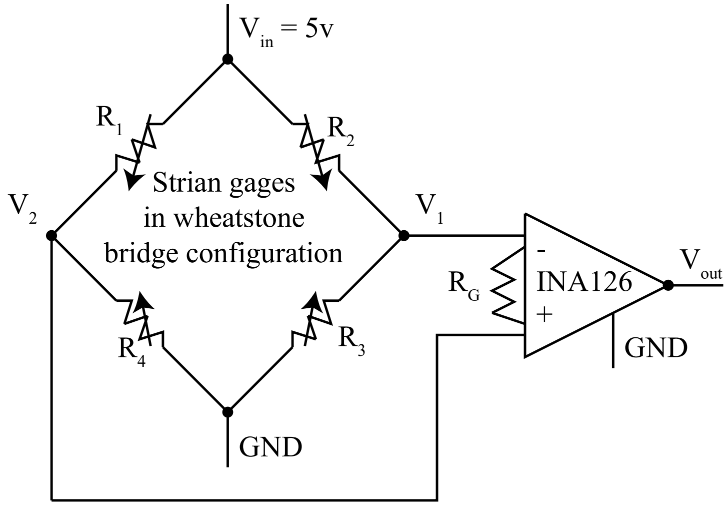


Figure 2. Wheatstone bridge amplification circuit

Note that the Wheatstone bridge creates a pair of voltage dividers. In this case, the voltage V_1 can be determined using the voltage divider law as follows:

$$V_1 = V_{in} \frac{R_3}{R_2 + R_3} \tag{1}$$

Similarly, the voltage V_2 is:

$$V_2 = V_{in} \frac{R_4}{R_1 + R_4} \tag{2}$$

The difference between V_2 and V_1 gives a highly sensitive and accurate measurement of the strain experienced by the beam at the location of the strain gauges. The voltage that corresponds to this strain, V_ε is:

$$V_\varepsilon = V_2 - V_1 = V_{in} \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) \tag{3}$$

If the strain gauges are all similar, when the bridge is balanced and there is no load applied to the end of the beam, $R_1 = R_2 = R_3 = R_4 = R$. When a load is applied to the beam in the $-y$ direction, strain gauges 1 and 3 experience tension and gauges 2 and 4 experience compression. The resistance of any strain gauge changes by ΔR as its surface area increases ($+\Delta R$, tension) or decreases ($-\Delta R$, compression): i.e. $\Delta R_1 = -\Delta R_2 = \Delta R_3 = -\Delta R_4 = \Delta R$. After the force is applied to the beam, (3) becomes:

$$\begin{aligned} V_\varepsilon &= V_{in}\left(\frac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)} - \frac{R-\Delta R}{(R-\Delta R)+(R+\Delta R)}\right) \\ &= V_{in}\left(\frac{\Delta R}{R}\right) \end{aligned} \tag{4}$$

The simplified relationship of the equation above reveals a linear relationship between V_ε and ΔR.

The op-amp-like symbol on the right of the figure above is actually an instrumentation amplifier, INA 126. The instrumentation amplifier has two jobs: it subtracts one signal from another, and it amplifies this difference according to the following relationship:

$$V_o = G\left(V_{in}^+ - V_{in}^-\right) \tag{5}$$

This relationship looks very like the output equation for an ideal op-amp. The difference is that for this instrumentation amplifier, the gain, G can be set anywhere in the range of 1 to 10,000 using a single resistor that is selected by the user. Noting that $V_{in}^- = V_1$ and $V_{in}^+ = V_2$, combining the equations above yields the following relation:

$$V_o = GV_\varepsilon \tag{6}$$

It should be noted that the same functionality provided by the instrumentation amplifier could be obtained using normal op-amps, however, it is much easier to configure the gain and the results are more reliable than a circuit made from discrete op-amp and large number of resistors.

2.2 INA126 Instrumentation Amplifier

Below are the INA126 chip diagram (Figure 3), the connection table, and equation for setting the gain on the INA126.

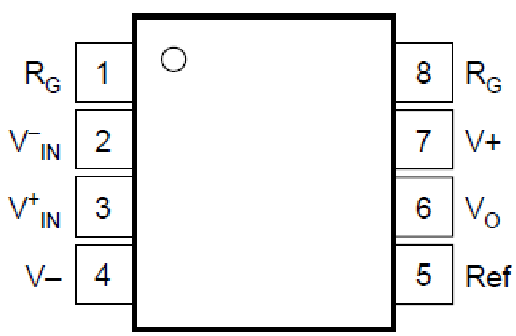


Figure 3. Pinout diagram of INA126 Instrumentation Amp

Table 1. Pin connections for INA126.

INA126	Connection	INA126	Connection
1 – RG	One side of the gain resistor	8- RG	Other side for the gain resistor
2 – V_{IN}^{-}	Negative side of input	7 – V+	+12V supply
3 – V_{IN}^{+}	Positive side of input	6 - VO	Output voltage
4 – V-	-12V supply	5 - Ref	Voltage supply ground

$$G = 5 + \frac{80k\Omega}{R_G} \tag{7}$$

2.3 Torque/Speed Curves

A permanent magnet DC motor can be characterized by its torque-speed curve. The torque, T, provided by the motor is proportional to the rotational velocity, Ω, at which it is rotating. The relationship between torque and rotational velocity can be stated in slope-intercept form as follows:

$$T = -\frac{T_s}{\Omega_{nl}}\Omega + T_s \tag{8}$$

Where the intercept, T_s is the stall torque of the motor and the slope is the ratio of the stall torque and the no-load speed, Ω_{nl}. The negative sign indicates that the torque output will decrease as the rotational speed increases. Note that when the motor is rotating at the no-load speed, there is still a finite amount of current flowing through the armature and torque is being generated inside the motor. However, all of the torque is used to overcome internal friction.

The stall torque and no-load speed of a DC motor are both directly proportional to the input voltage. As a result, the terms in the equation above will vary with the input voltage as will the torque-speed curve. This is demonstrated in the figure below, which shows the relationship between torque and speed, as well as the fact that this relationship is actually a family of curves that depend on the input voltage. Note that the torque-speed curve moves up and to the right with increasing input voltage.

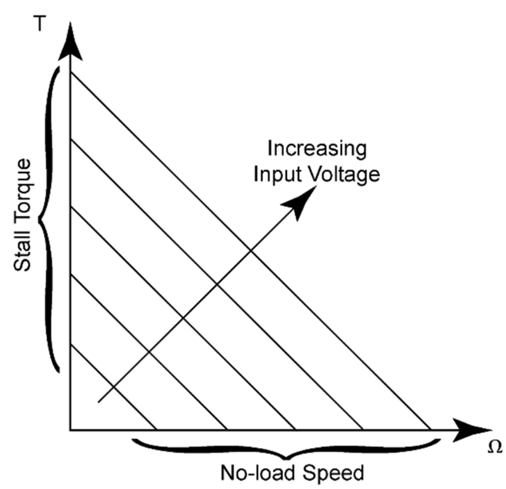


Figure 4. A family of torque-speed curves for a DC motor.

The torque-speed curve of a DC motor also indicates the total amount of power that the motor can provide given a specific input voltage. For rotational motion, the mechanical power P is defined as the product of the torque and the rotational velocity, as shown below.

$$P = T\Omega \tag{9}$$

Substituting the two previous equations yields the following quadratic equation.

$$P = -\frac{T_s}{\Omega_{nl}}\Omega^2 + T_s\Omega \tag{10}$$

The motor converts electrical power applied to the system ($e.g. P_{in} = V * i$) to mechanical power output ($e.g. P_{out} = T * \Omega$ where rad/sec must be used for Ω). However, some of the electrical power is dissipated by the armature resistance. Furthermore, some of the mechanical power is dissipated by friction in the bearings. The *efficiency* of the motor, η, defined as the ratio of shaft power to electrical input power, will thus be less than one:

$$\eta = \frac{mechanical\ output}{electrical\ input} = \frac{T\Omega}{Vi} < 1 \tag{11}$$

Note that the efficiency is zero at no load operation ($T = 0$) and at stall ($\Omega = 0$). Here V is the input voltage to the motor. Figure 5 below shows how efficiency and torque vary with angular velocity.

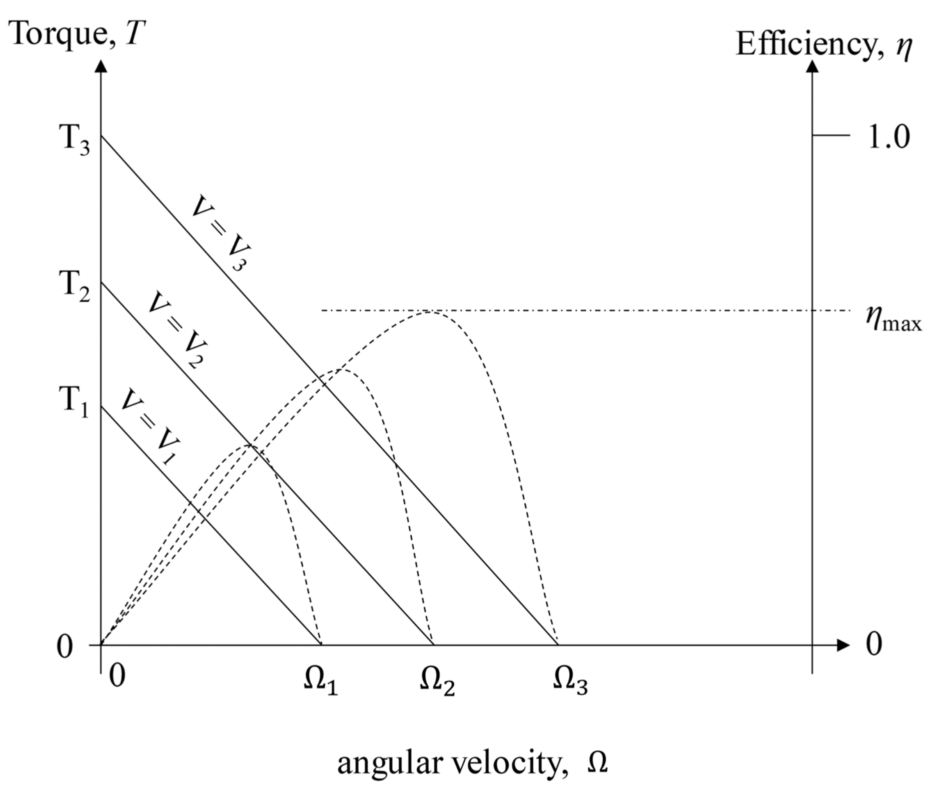


Figure 5. Typical motor performance.

2.4 Dynamometer

The Prony Brake, invented by the French engineer G.C.F.M. Riche (Baron de Prony) in 1822, is a mechanical dynamometer that measures the torque exerted by a rotating shaft. The operating principle of the Prony Brake is illustrated in the figure below. A cord is passed once or twice around the shaft whose torque is to be measured. The net torque T acting on the shaft from the friction is proportional to the difference in tensions τ_1 and τ_2 in either end of the cord:

$$T = (\tau_1 - \tau_2) \cdot r \tag{12}$$

Where r is the radius of the pulley which is mounted on the shaft. Note that the details of the frictional force on the pulley (coefficient of friction, etc.) are irrelevant. Changes in the amount of friction will change the tensions in the cord; however as long as the pulley is not accelerating, the friction-induced tensions must always be balanced by the shaft torque in accordance with the equation above. Thus, in order to measure the shaft torque, you simply need to measure the tensions at both ends of the cord.

In this experiment, you will connect the upper end of the cord to a force sensor and attach a known weight to the lower end. The tension in the lower part of the cord is equal to the weight, mg, while the tension in the upper part of the cord is measured by the force sensor.

$$T = (F - mg) \cdot r$$

Since a DC motor operating at constant voltage changes speed when the load is changed, the torque/speed curve may be mapped out by systematically varying the torque on the shaft. The torque may be varied either by changing the weight attached to the cord or by changing the number of loops of the cord about the shaft.

In this experiment, the dynamometer consists of a DC motor, a Prony Brake, an optical tachometer, a strain-gauge-based load cell, an amplifier circuit, an ammeter and a data acquisition system. A simplified schematic of the experimental setup is provided in the figure below.

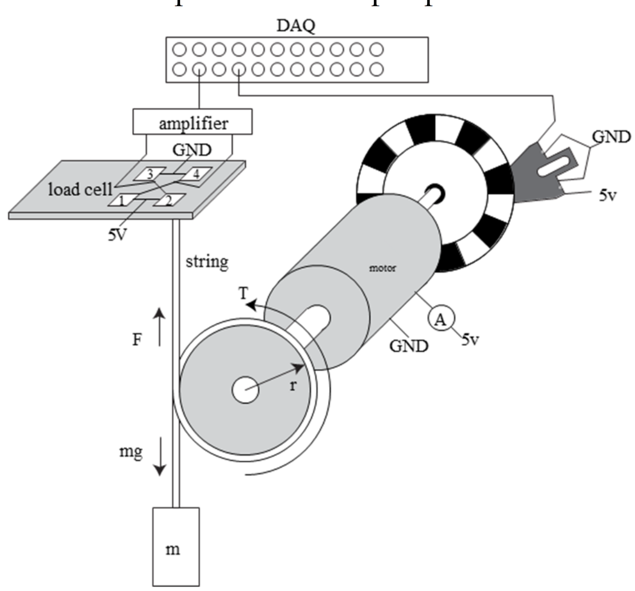


Figure 6. Experimental setup with Prony Brake dynamometer and strain gauge tension measurement.

3. Pre-Lab Exercises

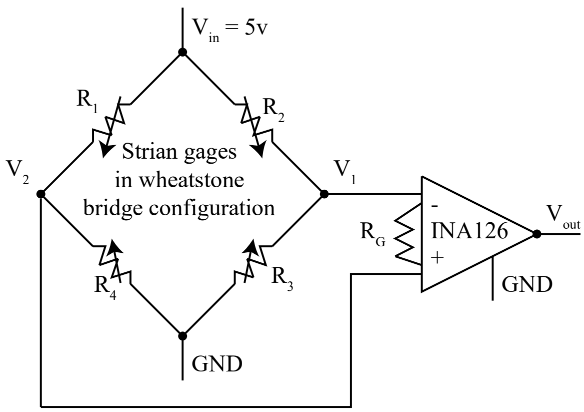
1. Read the lab handout and answer the following questions.
   a. What quantity is typically measured to obtain an indirect measurement of force or torque?
   b. What are two reasons for using 4 strain gauges in the lab setup, as shown in Fig. 2?
2. Referring to Fig. 2, what is the resistance value of R_G that will give a gain of 1005.
3. What are the stall torque and no-load speed of a motor both directly proportional to?
4. Given the stripe pattern shown in Figure 6, how would you convert from the frequency of the signal measured by the optical pickup to the rotational speed of the motor shaft in RPM?
5. Using equation (10), find the maximum power output for a DC motor in terms of the stall torque and no-load speed. What is the rotational speed at which max powers occurs?
6. The ***Lab10_motordata.mat*** file contains rotational speed (rad/s), measured torque (N-m), motor current (A), and applied voltage (V) from a dynamometer test. Using this data do the following. Be careful of units:
   a. Generate a Torque (N-m) vs Speed (RPM) curve, with proper labels. Display the stall torque and no-load speed values on the plot. Attach your plot to the pre-lab.
   b. Generate an Efficiency vs Speed (RPM) curve with proper labels. Attach your plot to the pre-lab.

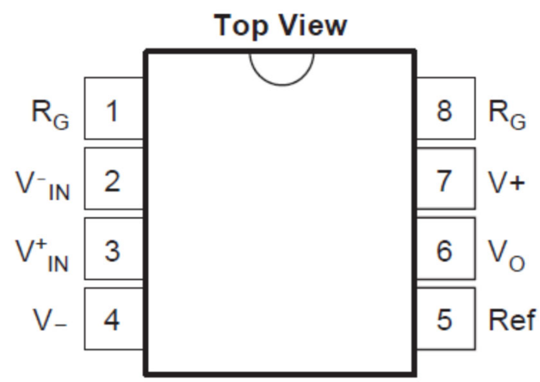
4. Laboratory Exercises

4.1 Connect Force Sensor Circuit

4.1.1 Create the Wheatstone bridge amplification circuit shown in the laboratory concepts.

- The strain gages are already in the Wheatstone bridge configuration and connected in the black box.
- Use the R_G value you calculated in the pre-lab exercise to set the gain near 1000.
- The figures below have the circuit, INA126 pin out diagram, and a pinout table.
- DO NOT turn on power until the TA checks your circuit.
- Use +/-12v from the power supply





Pin Number	Label	Connection
1	R_G	One lead of R_G
2	V_{in}^{-}	V_1 node of Wheatstone bridge circuit.
3	V_{in}^{+}	V_2 node of Wheatstone bridge circuit.
4	V-	-12 V from power supply, and through a 100nF capacitor to GND
5	Ref	Power supply ground
6	V_o	CH0 of the data acquisition board
7	V+	+12 V from power supply, and through a 100nF capacitor to GND
8	R_G	The other lead of R_G

Wheatstone bridge amplifier circuit and INA126 pinout

4.1.2 Connect the V_{out} from the circuit to ACH0 on the DAQ. Make sure to **common ground** the DAQ.

4.1.3 Open and run your **Lab3_GUI**. Your signal should increase when you put more tension on the sensor. If not switch the V_1 and V_2 nodes of the Wheatstone bridge.

Have a TA check your progress

4.2 Calibrate Force Sensor

4.2.1 With the thread unwrapped, measure the output voltage of the amplifier circuit while you hang different masses.

- In the table in section 4.4 Student Work and Tables, record the applied force from the weights and the voltages from the **Lab3_GUI** for different applied masses.
- Aim for a maximum applied mass of about 1000 g and minimum of 12 data points.
- Start with a no load (no string wrapped)

4.2.2 Plot your collected data and fit a line to the data. The x-axis will be Volts and y-axis will be force (N). Record the sensitivity, *m*, and bias, *b* in section 4.4 Student Work and Tables.

Have a TA check your progress

4.3 Measure Torque/Speed Characteristics

In this exercise you will use a constant voltage to drive the motor. The load (and thus torque) applied on the Prony Brake will be slowly increased by hanging incrementally larger weights on the hanger. At each increment of weight readings will be taken from the load cell and the optical sensor via your **Lab3_GUI**, and then translated into motor torque and speed, respectively.

4.3.1 Measure the radius for the pulley mounted on the shaft on the motor.

4.3.2 Determine the number of black or white encoder stripes.

4.3.3 Make sure the optical encoder +5V and GND are connected.

4.3.4 Make sure the encoder output is connected to ACH1 on the DAQ. Remember to check that it has a **common ground.**

4.3.5 Check that one of the motor's terminals is connected to ground and the other to +5V. There should be a multimeter in series with the +5V input (measuring DC amperage).

4.3.6 Double check connections and turn on the power supply. The motor should begin to spin.

4.3.7 Hang a small weight on the thread that is wrapped around the pulley. The motor should begin to slow down. If not switch the motor leads.

4.3.8 Record the mass of the hanging weights, frequency of optical encoder, the current from the ammeter, and the force sensor's output voltage in the table below. You will use this data to calculate other values. Record values in section 4.4 Student Work and Tables.

- Increase the weights until the motor stalls. Do not let the motor stall for long.
- Also take a measurement without the thread wrapped (hanging mass = 0) and with the thread wrapped with no hanging weight. (hanging mass = 5g).

4.3.9 When you are finished taking data turn off all power supplies, DO NOT disconnect your setup.

4.3.10 Calculate the following values for each hanging weight. Save this data. You will need it for the post lab.

- Calculate Force (N) sensor force using the force sensor voltage and the calibration equation developed in 4.2.
- Calculate the motor speed in (RPM) using the frequency and encoder counts.
- Calculate Motor Torque (N*m) using the force F, the hanging mass, and the radius of the pulley.
- Calculate the efficiency using the Torque, motor speed, motor voltage (5V), and motor current (Be sure to use correct units).

4.3.11 Plot Motor Torque vs angular speed to create the 5V torque-speed curve.

4.3.12 Plot Motor Efficiency vs angular speed.

Have a TA check your progress

4.4 Student Work and Tables

Name:______________________________________

Table 4.2.1

Mass (g)	Force (N)	Voltage

Calibration 4.2.2

m = ______________________N/V

b = ______________________V

4.3.1 Radius of pulley r = ______________ mm = ________________ m

4.3.2 Number of Black or White counts N = ___________________

Table 4.3.9

Hanging weight (kg)	Encoder Frequency (Hz)	Motor Current (A)	Force Sensor Voltage (V)

5. Post-Lab Exercises

1. Attach your calibration plot from 4.2.2 with the proper labels, fit line, and equation displayed.

2. Attach your 5V Torque (N*m) vs Speed (RPM) plot with appropriate labels from 4.3. Fit a line to this data. Show the line and equation on the plot. What are the stall torque (in N*m) and no-load speed (in RPM)?

3. Using your data from 4.3. Plot the Efficiency vs Speed (RPM). Attach your properly labeled plot to the post lab.

4. Calculate the maximum power output for the motor with the 5 V supply. Show your work.

5. The Mechatronics Lab stocks 37 mm diameter Pololu gear-motors with four different gear ratios: 50:1, 70:1, 100:1, and 131:1. Each motor has two important performance characteristics: the stall torque and the no-load speed. Generally, these two parameters are dependent on the input voltage. Given the stall torque and no-load speed at 12 V for each gear ratio, calculate the stall torque and no-load speed at 10 V (about how much your battery will output). Using the radius of the drive wheels on your robot, and estimating the mass of your robot, calculate the maximum acceleration and maximum speed that your robot could achieve using the different gear ratios.

Pololu Gear ratios	**T_{stall} (kg-cm) @ 12V**	**Ω_{nl} (RPM) @ 12V**	**T_{stall} (kg-cm) @ 10V**	**Ω_{nl} (RPM) @ 10V**	**Max robot acceleration (cm/s^2) @ 10V**	**Max robot speed (cm/s) @ 10V**
50:1	21	200				
70:1	27	150				
100:1	34	100				
131:1	45	76				

6. Project Milestone 10

The purpose of PM 10 is to finalize and test your mechanisms/manipulators. You should also be working towards PM12, which will involve a full trial competition run of your robot).

During Lab 11, you will present a final analysis of your mechanisms and progress in a 5-10 minute PowerPoint. You will submit your presentation with your team number in the filename on Canvas.

Mechanism/Manipulator Analysis

For the competition this semester, you are required to acquire blocks from the block dispenser and fuse (magnetically attach) them to the chassis. Each team has designed various mechanisms to manipulate the block, such as rotating arms, linkages, belts, lead screws, rack and pinions, etc. For any such manipulators on your robot, analyze the leverage of your mechanism and use the stall torque and no-load speed of your motor/gearbox to estimate the maximum force and speed with which your mechanism can manipulate the block (This will be similar to the calculations you did for problem 5 of Postlab 10).

Presentation

- Discuss each team members contributions for this milestone
- Drive motor analysis
  - Based on the computations you did in Problem 5 of Postlab 10, present the maximum acceleration and speed of your robot, given your choice of motor/gearbox.
- Mechanism/Manipulator analysis
  - Use an appropriately labeled diagram of your mechanism to show how the stall torque and no-load speed of your motor(s)/gearbox(es) can be related to the force/speed at the output of your mechanism(s)
  - Present the maximum force that your manipulators can apply to the block
  - Present the maximum speed that your manipulators can move the block
  - Discuss whether the force and/or speed are sufficient to manipulate the block
- Your current progress for performing the competition objectives
  - Discuss where your robot needs to be for PM 12 and compare to where it is currently. What are your major problems?
  - What is your plan for reaching your final goal?

Demonstration

During Lab 11 you will demonstrate your robot's ability to load/unload blocks. You will first position your robot at the block dispenser, and demonstrate the ability of your robot to acquire/load a block (if your design involves manipulating the block into a certain spot on your robot, then you should demonstrate that functionality as well). Second, you will position your robot along one side of the chassis, and demonstrate the ability of your robot to fuse/unload a block onto the chassis (if your robot is designed to fuse blocks to both the side and top of the chassis, you should demonstrate both in turn). Since not all teams have designed their robots to acquire and/or fuse multiple blocks at once, you will only be required to demo the loading/unloading of one block at a time. We won't be evaluating your sensing per se, however your robot should autonomously load/manipulate/unload the block, which may require some sensing.