ME EN 7210 Optimal Control
Homework #1 **Due Tuesday, Jan. 20 at the beginning of class**

Homework should be typeset in either LaTex or Word, printed, and turned in at the beginning of class. Solutions should be neatly presented and show all your work for full credit.

Problem 1

What do you expect to get out of this class? Please be specific about your expectations.

Problem 2

In class, Dr. Leang described open and closed-loop control and showed the basic block diagrams for the two structures. Prove the benefits of closed-loop control over open-loop control through the sensitivity of each structure with respect to a change in a system parameter. Hint: You may recall discussions in your prior classical control course about sensitivity analysis and how it can be used to show the benefits of closed-loop control.

Problem 3

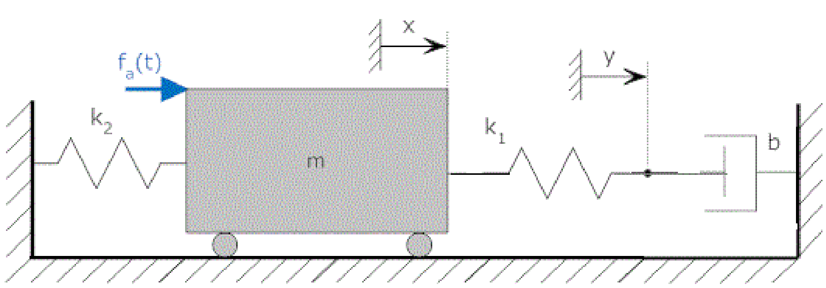
In class, the transfer function for a transformed state $\bar{x} = Px$ from a state-space representation was by:

$$Y(s) = \left[\hat{C}\left(SI - \hat{A}\right)^{-1}\hat{B} + D\right]U(s) = \hat{G}(s)U(s)$$

where $\hat{A} = PAP^{-1}$ and $\hat{B} = PB$. Note that the matrix P is invertible. Show that $\hat{G}(s) = C(SI - A)^{-1}B + D$. What does this mean exactly in terms of the poles of the transformed and original system?

Problem 4

Consider the following mechanical system where the input is $f_a(t)$ and let the states be x, dx/dt, and y. Derive the state-space equation for this system. Show all your work, including relevant free body diagrams.



Problem 5

(a) Determine whether the system given by the following state-space representation is controllable:

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b) Verify this result using MATLAB and include your Matlab results.

Problem 6

Consider the following system with state-space representation:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(a) What are the poles of the system?
(b) Design a full-state feedback regulator for the system such that the closed-loop poles are at s = -0.5 $\pm$ i. Hint: use pole placement technique, such as Ackermann's approach.
(c) Simulate the response of the closed-loop system using Matlab and show plots of the evolution of the states, x_1 and x_2, vs. time for initial conditions $x_1(0)=1$ and $x_2(0)=0$. Label all plots and include all Matlab code, etc.

Problem 7

(a) Do this problem by hand:

Calculate the 1-norm of $B = \begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}$

(b) Do this problem by hand:

Calculate the infinity-norm of $B = \begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}$

Problem 8

Determine the definiteness of the form $Q(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$.