Dynamic Response

Transfer Function
-Ratio of the Laplace Transform of the system output to its input assuming zero initial conditions

Frequency Response
- Response to a sinusoidal input
- A system with a transfer function, H(s), with sinusoidal input of magnitude A, will have an output that will be sinusoidal with the same frequency of the input with magnitude AM and phase shifted by angle ψ

Final Value Theorem
- Necessary Condition: All poles of the transfer function must have negative real parts and one pole at s = 0 is allowed

$$\lim_{t \longrightarrow \infty} y(t) = \lim_{s \longrightarrow 0} sY(s) = \lim_{s \longrightarrow 0} sG(s)U(S)$$

DC Gain
-Ratio of the systems output to its input after all transients have decayed
-Assume a unit step input

$$\text{DC Gain} = \lim_{s \longrightarrow 0} G(s)$$

First Order System

$$T(s) = k\frac{a_o}{s + a_o}, \tau = \frac{1}{a_o}$$

Second Order System

$$T(s) = \overline{k}(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}), \overline{k} = \text{DC Gain (s= o)}$$

Poles:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} = -\sigma \pm j\omega_d$$

Time Domain Specifications

Specification	1st Order	2nd Order
t_r	$\frac{2.2}{a}$	$\frac{1.8}{\omega_n}$
t_s	$4\tau = \frac{4}{a}$	$\frac{4}{\zeta\omega_n} = \frac{4}{\sigma}$
t_p	N/A	$\frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
M_p	N/A	$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$

These time domain specifications for the 2nd order system are approximations based on no finite zeros and a DC gain of 1.

Damping Ratio from percent overshoot

$$\zeta = \frac{-ln(\%OS/100)}{\sqrt{\pi^2 + ln^2(\%OS/100)}}$$

Log Decrement

$$\delta = ln\left(\frac{y(t)}{y(t+p)}\right)$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$