Homework 5 Solutions

1) The state of stress at a point in a cast-iron structure ($\sigma_u = 290\ MPa$, $\sigma_u' = 650\ MPa$) is described by $\sigma_x = 0$, $\sigma_y = -180\ MPa$, and $\tau_{xy} = 200\ MPa$. Determine whether failure occurs at the point according to the Coulomb–Mohr criterion.

Principal stresses are

$$\sigma_{1,2} = \tfrac{-180}{2} \pm [(\tfrac{180}{2})^2 + 200^2]^{\frac{1}{2}}$$

Or $\sigma_1 = 129.3\ MPa, \qquad \sigma_2 = -309.3\ MPa$

Equation (4.12a):

$$\tfrac{129.3}{290} - \tfrac{-309.3}{650} = 1$$

gives $0.446 + 0.476 = 0.922 < 1$

Thus, no fracture ◀

Note that Coulomb-Mohr theory is the most reliable when $\sigma_u' >> \sigma_u$, as in this example.

2) The ultimate strengths in tension and compression of a material are 420 and 900 MPa, respectively. If the stress at a point within a member made of this material is

$$\begin{bmatrix} 200 & 150 \\ 150 & 20 \end{bmatrix} \text{MPa}$$

determine the factor of safety according to the Coulomb–Mohr criterion.

Principal stresses are

$$\sigma_{1,2} = \tfrac{1}{2}(200 + 20) \pm [(90)^2 + (150^2)]^{\frac{1}{2}}$$

or

$$\sigma_1 = 284.9\ MPa \qquad \sigma_2 = -64.9\ MPa$$

Equation (4.12a): $\frac{284.9}{420} - \frac{-64.9}{900} = \frac{1}{n}$

Solving, $n = 1.33$ ◀

3) A long Ti-6Al-6V alloy plate of 130-mm width is loaded by a 200-kN tensile force in longitudinal direction with a safety factor of 2.2. Determine the thickness t required to prevent a central crack to grow to a length of 20 mm (Case A, Table 4.2).

By Table 4.3: $K_c = 66\sqrt{1000}\ MPa\sqrt{mm}$ and $\sigma_{yp} = 1149\ MPa$. Table 4.2:

$$\frac{a}{w} = \frac{10}{65} = 0.15 \qquad \lambda = 1.02$$

We have

$$\sigma = \frac{K_c}{\lambda n\sqrt{\pi a}} = \frac{66\sqrt{1000}}{(1.02)(2.2)\sqrt{\pi(10)}} = 165.9\ MPa$$

Thus

$$t_{req} = \frac{P}{2w\sigma} = \frac{200(10^3)}{2(65)(165.9)} = 9.27\ mm \qquad \blacktriangleleft$$

A thickness of 9.3 mm should be used. Note that both values of a and t satisfy Table 4.3.

4) An AISI-4340 steel pressure vessel (having closed ends) of 60-mm diameter and 5-mm wall thickness contains a 12-mm-long crack. Using the thin-wall assumption, calculate the pressure that will cause fracture when (a) the crack is longitudinal; (b) the crack is circumferential. Assumption: Use a factor of safety $n = 2$ and geometry factor $\lambda = 1.01$ (Table 4.2).

Case A of Table 4.2 and Table 4.3:

$$K_c = 59\sqrt{1000}\ MPa\sqrt{mm} \qquad \sigma_{yp} = 1503\ MPa$$

$$\lambda = 1.01 \quad \text{(assumed)}$$

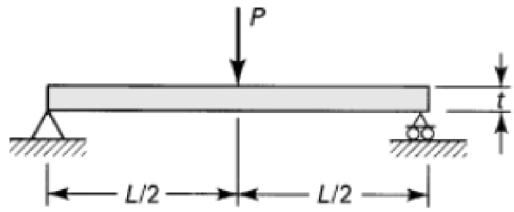
By Eq.(4.18):

$$\sigma = \frac{K_c}{n\lambda\sqrt{\pi a}} = \frac{59\sqrt{1000}}{(2)(1.01)\sqrt{\pi(6)}} = 213\ MPa < \sigma_{yp}$$

(a) $\sigma = \frac{p_f r}{t}$, $\quad p_f = \frac{\sigma t}{r} = \frac{213(\cancel{1}0)}{30} = \cancel{7}1\ MPa$ 35.5 $\blacktriangleleft$

(b) $\sigma = \frac{p_f r}{2t}$, $\quad p_f = \frac{1}{2}(\cancel{7}1) = 3\cancel{5}.5\ MPa$ 71 $\blacktriangleleft$

5) A small leaf spring $b = 10\ mm$ wide by $L = 125\ mm$ long by $t\ mm$ thick is simply supported at its ends and subjected to a center load P that varies continuously from 0 to 20 N. Using the Modified Goodman criterion, determine the value of t, given a fatigue strength $\sigma_{cr} = 740\ MPa$, ultimate tensile strength $\sigma_u = 1500\ MPa$, and safety factor of $n = 2.5$.



We have $\sigma_a = \sigma_m$. From Table 4.4:

$$\frac{\sigma_m}{740/2.5} + \frac{\sigma_m}{1500/2.5} = 1$$

or

$$\sigma_m = 198.2\ MPa.$$

At the center of the beam:

$$M_{\max} = PL/4 = (0.25)(20)(0.125) = 0.625\ N \cdot m$$

Hence,

$$M_{a,m} = \frac{(M_{\max} \pm M_{\min})}{2} = 0.3125\ N \cdot m$$

and

$$\sigma_a = \sigma_m = \frac{6M_m}{bt^2} = \frac{6(0.3125)}{0.01t^2} = \frac{187.5}{t^2} = 198.2(10^6)$$

Solving,

$$t = 0.973\ mm$$ ◄

6) Determine the fatigue life of a machine element subjected to the following respective maximum and minimum stresses (in megapascals):

$$\begin{bmatrix} 800 & 200 \\ 200 & 500 \end{bmatrix}, \qquad \begin{bmatrix} -600 & -150 \\ -150 & -300 \end{bmatrix}$$

Use the maximum energy of distortion theory of failure together with the (a) modified Goodman criterion and (b) Soderberg criterion. Let $\sigma_u = 1600\ MPa$, $\sigma_{yp} = 1000\ MPa$, and $K = 1$.

$$\sigma_{xa} = (800 + 600)/2 = 700\ MPa$$

$$\sigma_{xm} = (800 - 600)/2 = 100\ MPa$$

$$\sigma_{ya} = (500 + 300)/2 = 400\ MPa$$

$$\sigma_{ym} = (500 - 300)/2 = 100\ MPa$$

$$\tau_{xya} = (200 + 150)/2 = 175\ MPa$$

$$\tau_{xym} = (200 - 150)/2 = 25\ MPa$$

Equations (4.21) give then

$$2\sigma_{ea}^2 = (700 - 400)^2 + 400^2 + 700^2 + 6(175)^2$$

$$2\sigma_{em}^2 = (200 - 100)^2 + 100^2 + 100^2 + 6(25)^2$$

or $\qquad \sigma_{ea} = 679.61\ MPa, \qquad \sigma_{em} = 108.97\ MPa$

(a) Modified Goodman criterion:

$$\sigma_{cr} = \frac{679.61}{1-(108.97/1600)} = 729.27\ MPa$$

$$b = \frac{\ln(0.9\times1600/0.5\times1600)}{\ln(10^3/10^8)} = -0.08509$$

$$N_{cr} = 10^3\left(\frac{729.27}{0.9\times1600}\right)^{-11.752} = 2.97(10^6)\ \text{cycles} \quad ◀$$

(b) Soderberg criterion:

$$\sigma_{cr} = \frac{679.61}{1-(108.97/1000)} = 762.72\ MPa$$

$$N_{cr} = 10^3\left(\frac{762.72}{0.9\times1600}\right)^{-11.752} = 1.75(10^6)\ \text{cycles} \quad ◀$$