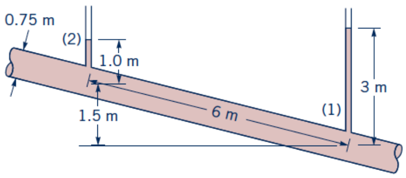
ME 3710
Homework 7
Due Tuesday March 29 at 11:59pm – upload to Gradescope
Integral Conservation of Energy and Bernoulli's equation
[9 problems – 27 pts]

Solution 5.98



Assume flow from (1) to (2) and use the energy equation $\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_s-h_L$ to get for the contents of the control volume show:

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+\cancelto{0}{h_s}-h_\ell$$

Thus

$$h_\ell=\frac{p_1}{\gamma}-\frac{p_2}{\gamma}+z_1-z_2=3\,\text{m}-1.0\,\text{m}-1.5\,\text{m}=\underline{\underline{0.5\,\text{m}}}$$

and since $h_\ell>0$, the assumed direction of flow is correct.
The flow is uphill.

Solution 5.101

Apply the mechanical energy equation from point 1 to point 2. Assume constant water density, steady flow in pipe, zero elevation change, and uniform velocity over each flow area.

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}=\frac{p_2}{\rho}+\frac{V_2^2}{2}+gh_L+\text{loss}$$

Using $p_2 = 0\,\text{kPa}$ gage,

$$K\frac{V_1^2}{2}=\frac{p_1}{\rho}+\frac{1}{2}(V_1^2-V_2^2)$$

and

$$\text{K}=\frac{2p_1}{\rho V_1^2}+\left(1-\frac{V_2^2}{V_1^2}\right).$$

Using $V=\dfrac{4Q}{\pi D^2}$ gives

$$K=\frac{\pi^2 D_1^4 p_1}{8Q^2\rho}+\left(1-\frac{D_1^4}{D_2^4}\right).$$

The numerical value give

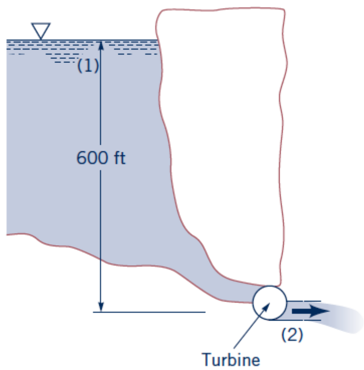
$$Q=\frac{1.5\,\text{m}^3}{175\,\text{s}}=0.00857\frac{\text{m}^3}{\text{s}},$$

$$K=\frac{\pi^2(6\,\text{cm})^4\left(40\times10^3\,\dfrac{\text{N}}{\text{m}^2}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)}{8\left(0.00857\,\dfrac{\text{m}^3}{\text{s}}\right)^2\left(998\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(100\,\dfrac{\text{cm}}{\text{m}}\right)^4}+\left(1-\frac{(6\,\text{cm})^4}{(4\,\text{cm})^4}\right),$$

or

$$\boxed{K=4.66}$$

Solution 5.108



From the energy equation,

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_s-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

where $p_1=0$, $p_2=0$, and $V_1=0$.
Thus,

$$h_s=(z_2-z_1)+h_L+\frac{V_2^2}{2g}$$

And, the power is given by

$$\dot{W}_{\text{turb}}=\gamma Qh=\gamma Q\left[(z_2-z_1)+h_L+\frac{V_2^2}{2g}\right]$$

The maximum power would occur if there were no losses $(h_L=0)$ and negligible kinetic energy at the exit $(V_2\approx 0$; large diameter outlet$)$.

Thus,

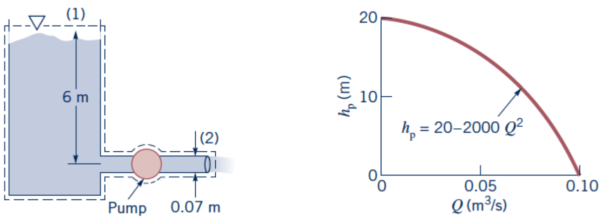
$$\dot{W}_{\text{turb}}=\gamma Q(z_2-z_1)=62.4\frac{\text{lb}}{\text{ft}^3}\left(8\times10^6\ \frac{\text{gal}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1\,\text{ft}^3}{7.48\,\text{gal}}\right)(-600\,\text{ft})$$

$$-6.67\times10^8\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\,\text{hp}}{550\frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right)=\underline{\underline{-1.21\times10^6\ \text{hp}}}$$

The minus sign is associated with power out.

The actual power will be less due to internal frictional losses and the exit kinetic energy of the flow.

Solution 5.110



(*a*) (*b*)

We want to know the flowrate Q. For the control volume shown, application of the energy equation $\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_s-h_L$ yields.

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_1}{\gamma}+\frac{\cancel{V_1^2}^{\,0}}{2g}+z_1+h_s-h_\ell \qquad (1)$$

However

$$h_\ell=1.2\frac{V^2}{2g} \qquad (2)$$

and

$$h_s=h_p=\ 20-2000Q^2 \qquad (3)$$

Since $Q=V_2A_2$, we have from Eq. (2)

$$h_\ell=\frac{1.2}{2g}\left(\frac{Q}{A}\right)^2 \qquad (4)$$

and combining Eqs. (1), (3) and (4), we get:

$$\frac{1}{2g}\left(\frac{Q}{A_2}\right)^2+z_2=z_1+20-2000Q^2-\frac{1.2}{2g}\left(\frac{Q}{A_2}\right)^2 \qquad (5)$$

or

$$Q^2\left(\frac{1}{2gA_2^2}+\frac{1.2}{2gA_2^2}+2000\right)=z_1-z_2+20$$

so

$$Q=\left[\frac{z_1-z_2+20}{\frac{1}{2g\left(\frac{\pi d_2^2}{4}\right)^2}+\frac{1.2}{2g\left(\frac{\pi d_2^2}{4}\right)^2}+2000}\right]^{\frac{1}{2}}$$

$$=\left[\frac{6\,\text{m}+20\,\text{m}}{\frac{1}{(2)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left[\frac{\pi(0.07\,\text{m})^2}{4}\right]^2}+\frac{1.2}{(2)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left[\frac{\pi(0.07\,\text{m})^2}{4}\right]^2}+2000}\right]^{\frac{1}{2}}$$

$$Q=\underline{\underline{0.052}}\frac{\text{m}^3}{\text{s}}$$

Solution 5.118

The efficiency of the pump, η, is

$$\eta = \frac{\text{ideal work required}}{\text{actual work required}} = \frac{\text{actual work required} - \text{loss}}{\text{actual work required}} = \frac{w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}}{w_{\substack{\text{shaft}\\ \text{net in}}}}$$

To determine $w_{\substack{\text{shaft}\\ \text{net in}}}$, we use the equation $\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}$

to obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{p_{out} - p_{in}}{\rho} + \frac{V_{out}^2 - V_{in}^2}{2} + \text{loss} \qquad (1)$$

From the volume flowrate, we obtain

$$V_{out} = \frac{Q}{A_{out}} = \frac{Q}{\frac{\pi D_{out}^2}{4}} = \frac{\left(0.02\,\frac{\text{m}^3}{\text{s}}\right)}{\pi\frac{(0.030\,\text{m})^2}{4}} = 28.29\,\frac{\text{m}}{\text{s}}$$

Also, from mass conservation

$$V_{in} = V_{out}\frac{D_{out}^2}{D_{in}^2} = \left(28.29\,\frac{\text{m}}{\text{s}}\right)\frac{(0.030)^2}{(0.090)^2} = 3.143\,\frac{\text{m}}{\text{s}}$$

Thus from Eq. (1), we obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\left(400000\,\frac{\text{N}}{\text{m}^2} - 120000\,\frac{\text{N}}{\text{m}^2}\right)}{\left(999\,\frac{\text{kg}}{\text{m}^3}\right)} + \frac{\left[\left(28.29\,\frac{\text{m}}{\text{s}}\right)^2 - \left(3.143\,\frac{\text{m}}{\text{s}}\right)^2\right]}{2}\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right) + 170\,\frac{\text{N}\cdot\text{m}}{\text{kg}}$$

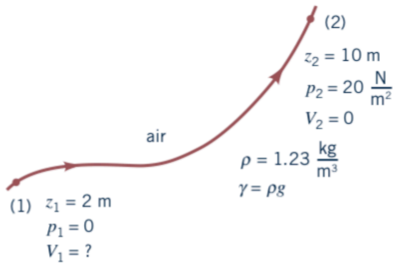
or

$$w_{\substack{\text{shaft}\\ \text{net in}}} = 846\,\frac{\text{N}\cdot\text{m}}{\text{kg}}$$

Then

$$\eta = \frac{846\,\frac{\text{N}\cdot\text{m}}{\text{kg}} - 170\,\frac{\text{N}\cdot\text{m}}{\text{kg}}}{846\,\frac{\text{N}\cdot\text{m}}{\text{kg}}} = \underline{\underline{0.799}}$$

Solution 3.2



$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Thus, with $p_1 = 0$ and $V_2 = 0$,

$$\frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \gamma z_2$$

or

$$\frac{1}{2}\left[1.23\frac{\text{kg}}{\text{m}^3}\right]V_1^2 = 20\frac{\text{N}}{\text{m}^2} + \left[1.23\frac{\text{kg}}{\text{m}^3}\right]9.81\frac{\text{m}}{\text{s}^2}(10\text{ m} - 2\text{ m})$$

or

$$V_1^2 = \frac{2(20)}{1.23}\frac{\text{N}\cdot\text{m}}{\text{kg}} + 2\left[9.81\frac{\text{m}}{\text{s}^2}\right](8\text{m}) = 189\frac{\text{m}^2}{\text{s}^2} \qquad (\text{Note: } \frac{\text{N}\cdot\text{m}}{\text{kg}} = \frac{\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right]\text{m}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2})$$

Thus,

$$\underline{\underline{V_1 = 13.7\frac{\text{m}}{\text{s}}}}$$

Solution 3.8

(a) $\dfrac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \dfrac{\partial V}{\partial s}$ but $\theta = 0$ and $\dfrac{\partial V}{\partial s} = \dfrac{\partial V}{\partial x}\dfrac{\partial x}{\partial s} = \dfrac{\partial V}{\partial x} = V_0[-a^2]\left[\dfrac{-2}{x^3}\right] = \dfrac{2a^2V_0}{x^3}$

Thus,

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial x} = \underline{\underline{-2\rho a^2 V_0^2 \left[1-\left(\frac{a}{x}\right)^2\right]\frac{1}{x^3}}}$$

(b) $\displaystyle\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}dx$

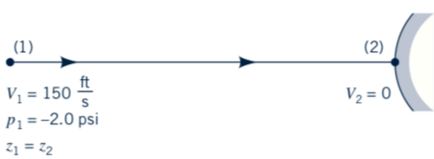
Or

$$p - p_0 = -2\rho a^2 V_0^2 \int_{-\infty}^{x} \left[1-\left(\frac{a}{x}\right)^2\right]\frac{dx}{x^3} = -2\rho a^2 V_0^2 \int_{-\infty}^{x} [x^{-3} - a^2 x^{-5}]dx$$

Thus,

$$\underline{\underline{p = p_0 + \rho V_0^2 \left[\left(\frac{a}{x}\right)^2 - \frac{1}{2}\left(\frac{a}{x}\right)^4\right]}} \text{ for } (-\infty \le x \le -a)$$

Solution 3.17



$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

where $z_1 = z_2$ and $V_2 = 0$

Thus,

$$p_2 = p_1 + \frac{1}{2}\rho V_1^2 = \left(-2.0 \frac{\text{lb}}{\text{in.}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right) + \frac{1}{2}\left(0.00238 \frac{\text{slugs}}{\text{ft}^3}\right)\left(150 \frac{\text{ft}}{\text{s}}\right)^2$$

$$= -2.88 \frac{\text{lb}}{\text{ft}^2} + 26.8 \frac{\text{slug}\cdot\text{ft}}{\text{ft}^2\cdot\text{s}^2}\left(\frac{1\text{lb}}{\frac{\text{slug}\cdot\text{ft}}{\text{s}^2}}\right)$$

$$= -261 \frac{\text{lb}}{\text{ft}^2} = \underline{\underline{-1.81\text{psi}}}$$

Solution 3.47

Apply Bernoulli's equation between the oil surface (0) and the outlet (T).

$$\frac{p_0}{\rho}+\frac{V_0^2}{2}+gz_0=\frac{p_T}{\rho}+\frac{V_T^2}{2}+gz_T$$

Since $D^2 \gg d_T^2$, assume $V_0=0$. Also $p_0 = p_T = p_{atm}$ and set $z_0=0$. This gives

$$V_T=\sqrt{2gz_0}=\sqrt{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)(10.5\,\text{cm})\left(100\frac{\text{cm}}{\text{m}}\right)}=144\frac{\text{cm}}{\text{s}}.$$

Now apply Bernoulli's equation between the oil surface (0) and the outlet (B).

$$\frac{p_0}{\rho}+\frac{V_0^2}{2}+gz_0=\frac{p_B}{\rho}+\frac{V_B^2}{2}+gz_B.$$

Again note $V_0=0$ and $p_0 = p_B = p_{atm}$ and set $z_B=0$.

Then

$$V_B=\sqrt{2gz_B}=\sqrt{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)(13.5\,\text{cm})\left(100\frac{\text{cm}}{\text{m}}\right)}=163\frac{\text{cm}}{\text{s}}.$$

Since $A_T=A_B$, the total flow rate is

$$Q=V_TA_T+V_BA_B=A_T\left(V_T+V_B\right)$$
$$=\frac{\pi}{4}(3\,\text{cm})^2\,(144+163)\frac{\text{cm}}{\text{s}}=\underline{\underline{Q=2170\frac{\text{cm}^3}{\text{s}}}}$$