5000

Solutions Manual: Chapter 5

8th Edition

Feedback Control of Dynamic Systems

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Chapter 5

The Root-Locus Design Method

Problems and solutions for Section 5.1

1. Set up the listed characteristic equations in the form suited to Evans's root-locus method. Give $L(s)$, $a(s)$, and $b(s)$ and the parameter K in terms of the original parameters in each case. Be sure to select K so that $a(s)$ and $b(s)$ are monic in each case and the degree of $b(s)$ is not greater than that of $a(s)$.

   (a) $s + (1/\tau) = 0$ versus parameter τ

   (b) $s^2 + cs + c + 1 = 0$ versus parameter c

   (c) $(s + c)^3 + A(Ts + 1) = 0$

      i. versus parameter A,

      ii. versus parameter T,

      iii. versus the parameter c, if possible. Say why you can or can not. Can a plot of the roots be drawn versus c for given constant values of A and T by any means at all?

   (d) $1 + \left[k_p + \dfrac{k_I}{s} + \dfrac{k_D s}{\tau s + 1}\right] G(s) = 0$. Assume that $G(s) = A\dfrac{c(s)}{d(s)}$, where $c(s)$ and $d(s)$ are monic polynomials with the degree of $d(s)$ greater than that of $c(s)$.

      i. versus k_p

      ii. versus k_I

      iii. versus k_D

      iv. versus τ

Solution:

(a) $K = 1/\tau$;
$a = s$;
$b = 1$

(b) $K = c$;
$a = s^2 + 1$;
$b = s + 1$

(c) i. $K = AT$;
$a = (s + c)^3$;
$b = s + 1/T$

ii. $K = AT$;
$a = (s + c)^3 + A$;
$b = s$

iii. The parameter c enters the equation in a nonlinear way and a standard root locus does not apply. However, using a polynomial solver, the roots can be plotted versus c.

(d) i. $K = k_p A$;
$a = s(s + 1/\tau)d(s) + Ak_I(s + 1/\tau)c(s) + \dfrac{k_D}{\tau}s^2 Ac(s)$;
$b = s(s + 1/\tau)c(s)$

ii. $K = Ak_I$;
$a = s(s + 1/\tau)d(s) + Ak_p s(s + 1/\tau)c(s) + \dfrac{k_D}{\tau}s^2 Ac(s)$;
$b = (s + 1/\tau)c(s)$

iii. $K = \dfrac{Ak_D}{\tau}$;
$a = s(s + 1/\tau)d(s) + Ak_p s(s + 1/\tau)c(s) + Ak_I(s + 1/\tau)c(s)$;
$b = s^2 c(s)$

iv. $K = 1/\tau$;
$a = s^2 d(s) + Ak_p s^2 c(s) + Ak_I sc(s)$;
$b = sd(s) + Ak_p sc(s) + Ak_I c(s) + Ak_D s^2 c(s)$

Problems and solutions for Section 5.2

2. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 5.50 without the aid of a computer. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles o and the roots of the denominator $a(s)$ are shown as $\times's$ on the s-plane. Note that in Fig. 5.50(c) there are two poles at the origin.

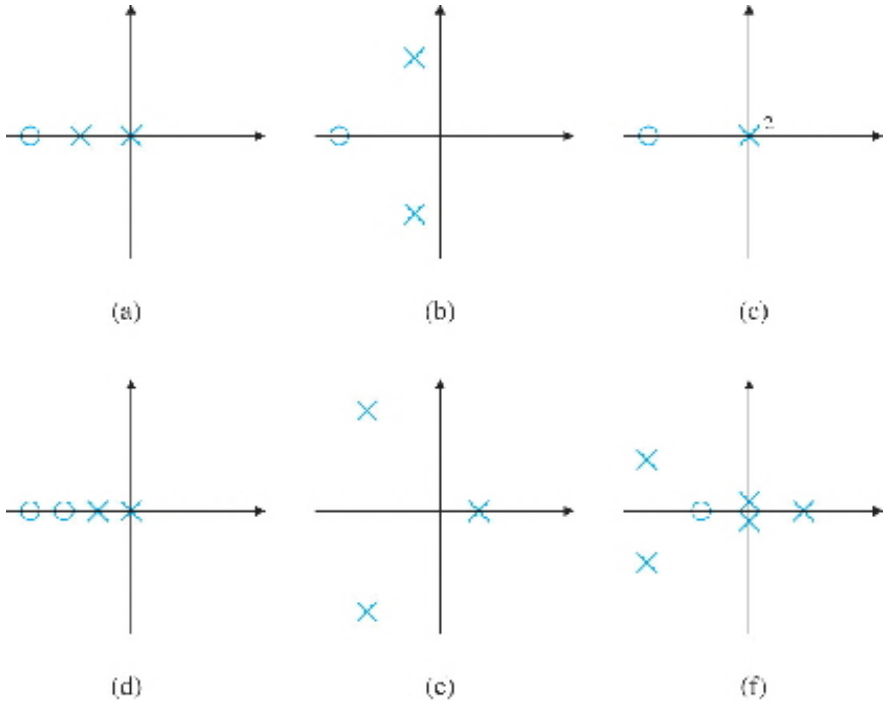


Fig. 5.50 Pole-zero maps for Problem 5.2

Solution:

We had to make up some numbers to do it on Matlab, so the results partly depend on what was dreamed up, but the idea here is just get the basic rules right.

(a) $a(s) = s^2 + s$; $b(s) = s + 2$
Breakin(s): -3.414
Breakaway(s): -0.586

(b) $a(s) = s^2 + 0.2s + 1$; $b(s) = s + 1$
Angle of departure: 137.9°
Breakin(s): -2.342

(c) $a(s) = s^2$; $b(s) = (s + 1)$
Breakin(s): -2

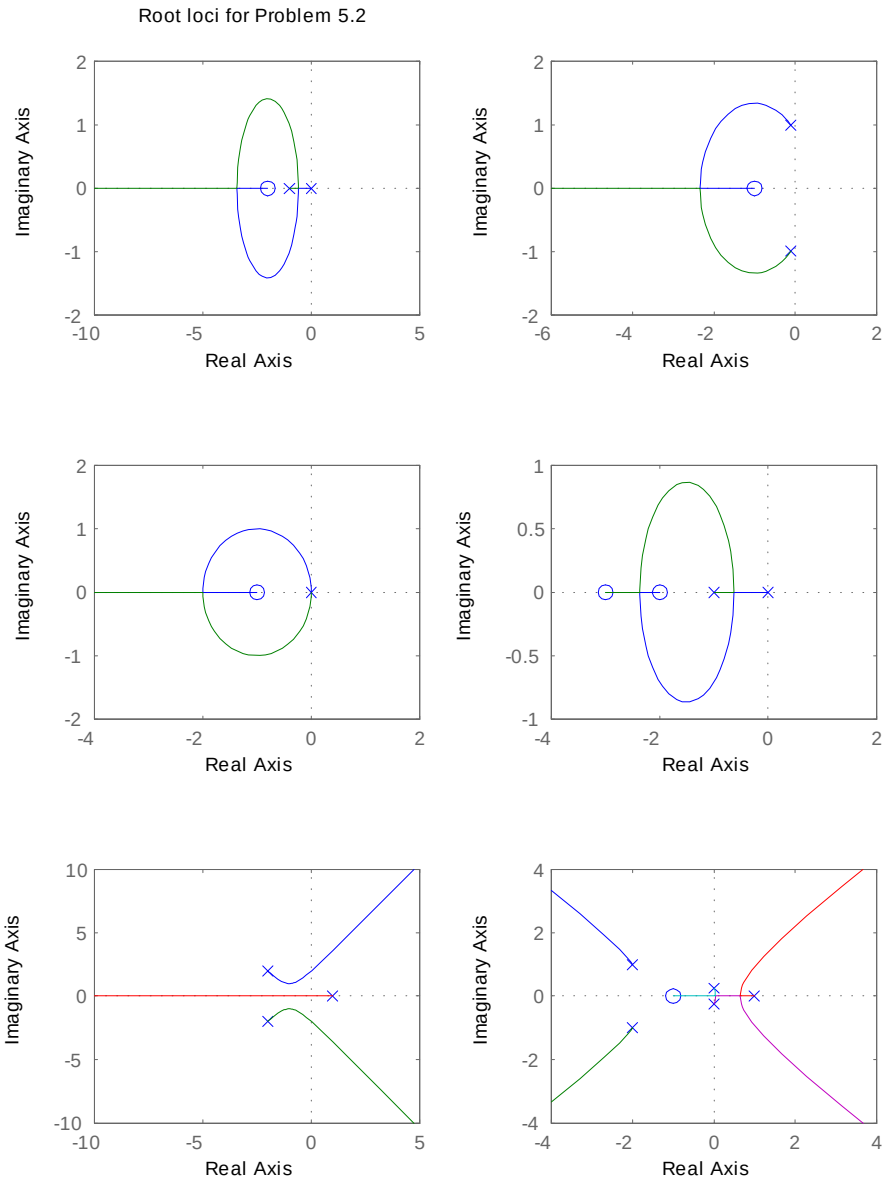
(d) $a(s) = s^2 + s$; $b(s) = s^2 + 5s + 6$
Breakin(s): -2.366
Breakaway(s): -0.634

(e) $a(s) = s^3 + 3s^2 + 4s - 8$
Center of asymptotes: -1
Angles of asymptotes: $\pm 60^\circ$, 180°
Angle of departure: -56.3°

(f) $a(s) = s^5 + 3s^4 + s^3 - 5s^2 - 0.5$; $b(s) = s + 1$
Center of asymptotes: -0.5
Angles of asymptotes: $\pm 45^\circ$, $\pm 135^\circ$
Angle of departure:

- 115.8° at $s = -2.01 + 1.01j$
- -70.5° at $s = -0.01 + 0.31j$

Breakin(s): 0.05
Breakaway(s): 0.652


Root loci for Problem 5.2
Imaginary Axis
Real Axis
Imaginary Axis
Real Axis
Imaginary Axis
Real Axis
Imaginary Axis
Real Axis
Imaginary Axis
Real Axis
Imaginary Axis
Real Axis

3. For the characteristic equation

$$1 + \frac{K}{s^2(s+1)(s+5)} = 0,$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) Sketch the locus.

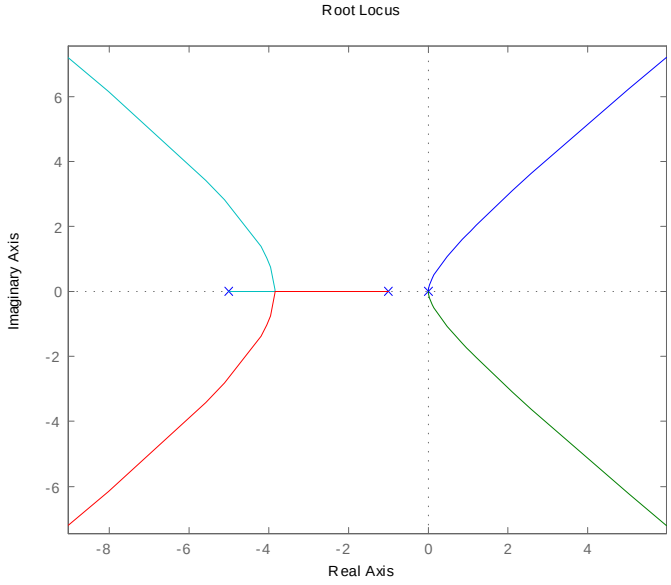
(d) Verify your sketch with a Matlab plot.

Solution:

(a) The real axis segment is $-1 > \sigma > -5$.

(b) $\alpha = -6/4 = -1.5$; $\phi_i = \pm 45°, \pm 135°$

(c) The plot is shown below.



Solution for Problem 5.3

4. *Real poles and zeros.* Sketch the root locus with respect to K for the equation $1+KL(s)=0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

(a) $L(s) = \dfrac{2}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

(c) $L(s) = \dfrac{(s+2)(s+20)}{s(s+1)(s+5)(s+10)}$

(d) $L(s) = \dfrac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$

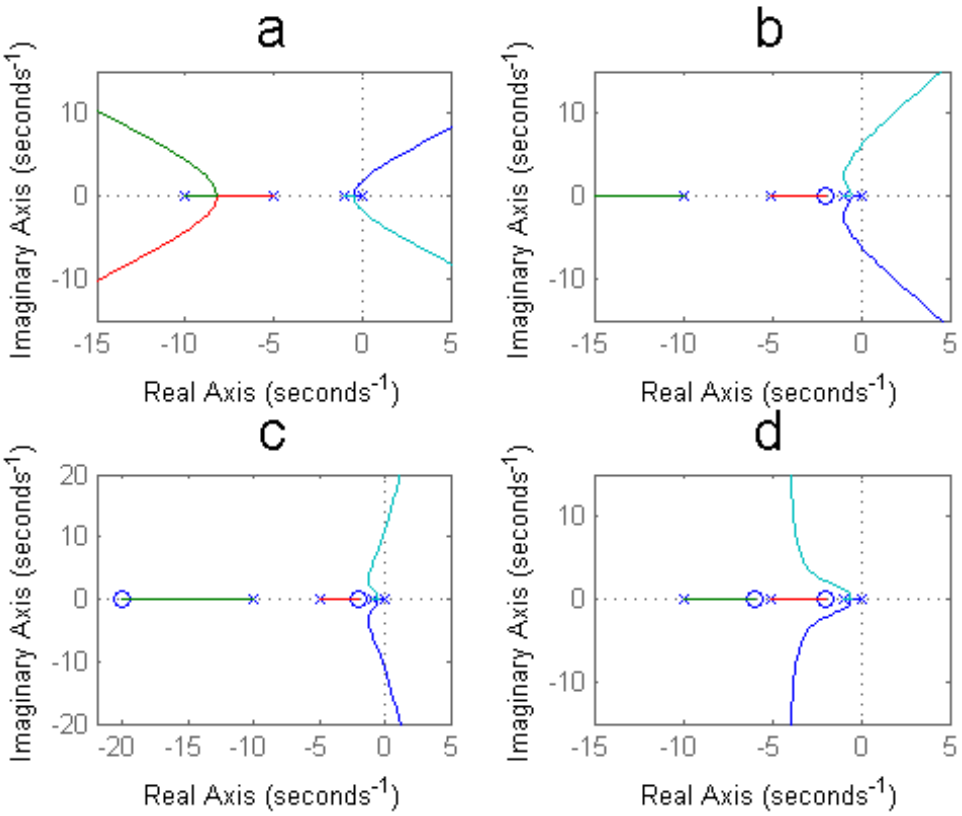
Solution:
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -4$; $\phi_i = \pm 45^\circ, \pm 135^\circ$

(b) $\alpha = -4.67$; $\phi_i = \pm 60^\circ, 180^\circ$

(c) $\alpha = +3$; $\phi_i = \pm 90^\circ$

(d) $\alpha = -4$; $\phi_i = \pm 90^\circ$



Root loci for Problem 5.4

5. *Complex poles and zeros.* Sketch the root locus with respect to K for the equation $1+KL(s)=0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

(a) $L(s) = \dfrac{1}{s^2+3s+10}$

(b) $L(s) = \dfrac{1}{s(s^2+3s+10)}$

(c) $L(s) = \dfrac{(s^2+2s+8)}{s(s^2+2s+10)}$

(d) $L(s) = \dfrac{(s^2+2s+12)}{s(s^2+2s+10)}$

(e) $L(s) = \dfrac{(s^2+1)}{s(s^2+4)}$

(f) $L(s) = \dfrac{(s^2+4)}{s(s^2+1)}$

Solution:
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -1.5$; $\phi_i = \pm 90°$; $\theta_d = 90°$ at $s = -1.5 + 2.78j$

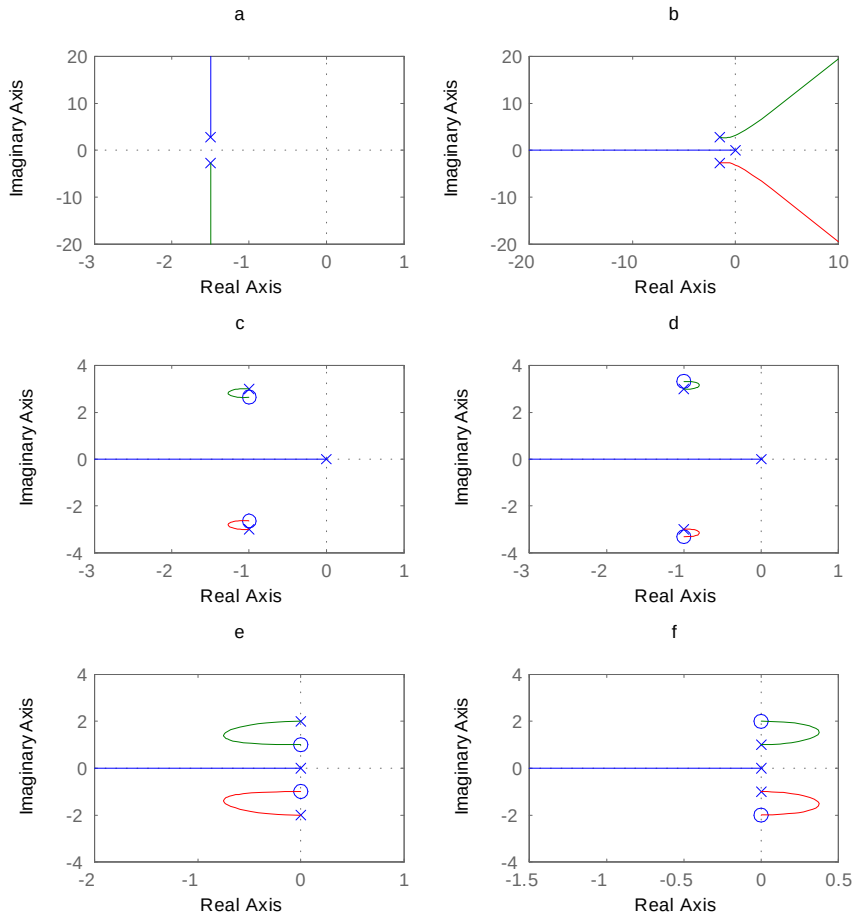
(b) $\alpha = -1$; $\phi_i = \pm 60°, 180°$; $\theta_d = -28.3°$ at $s = -1.5 + 2.78j$

(c) $\alpha = 0$; $\phi_i = 180°$; $\theta_d = 161.6°$ at $s = -1 + 3j$;
$\theta_a = 200.7°$ at $s = -1 + 2.65j$

(d) $\alpha = 0$; $\phi_i = 180°$; $\theta_d = -18.4°$ at $s = -1 + 3j$;
$\theta_a = 16.8°$ at $s = -1 + 3.32j$

(e) $\alpha = 0$; $\phi_i = 180°$; $\theta_d = 180°$ at $s = 2j$; $\theta_a = 180°$ at $s = j$

(f) $\alpha = 0$; $\phi_i = 180°$; $\theta_d = 0°$ at $s = j$; $\theta_a = 0°$ at $s = 2j$



Root loci for Problem 5.5

6. *Multiple poles at the origin.* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+10)}$

(b) $L(s) = \dfrac{1}{s^3(s+10)}$

(c) $L(s) = \dfrac{1}{s^4(s+10)}$

(d) $L(s) = \dfrac{(s+3)}{s^2(s+10)}$

(e) $L(s) = \dfrac{(s+3)}{s^3(s+4)}$

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$

Solution:
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -3.33$; $\phi_i = \pm 60^\circ, 180^\circ$

(b) $\alpha = -2.5$; $\phi_i = \pm 45^\circ, \pm 135^\circ$

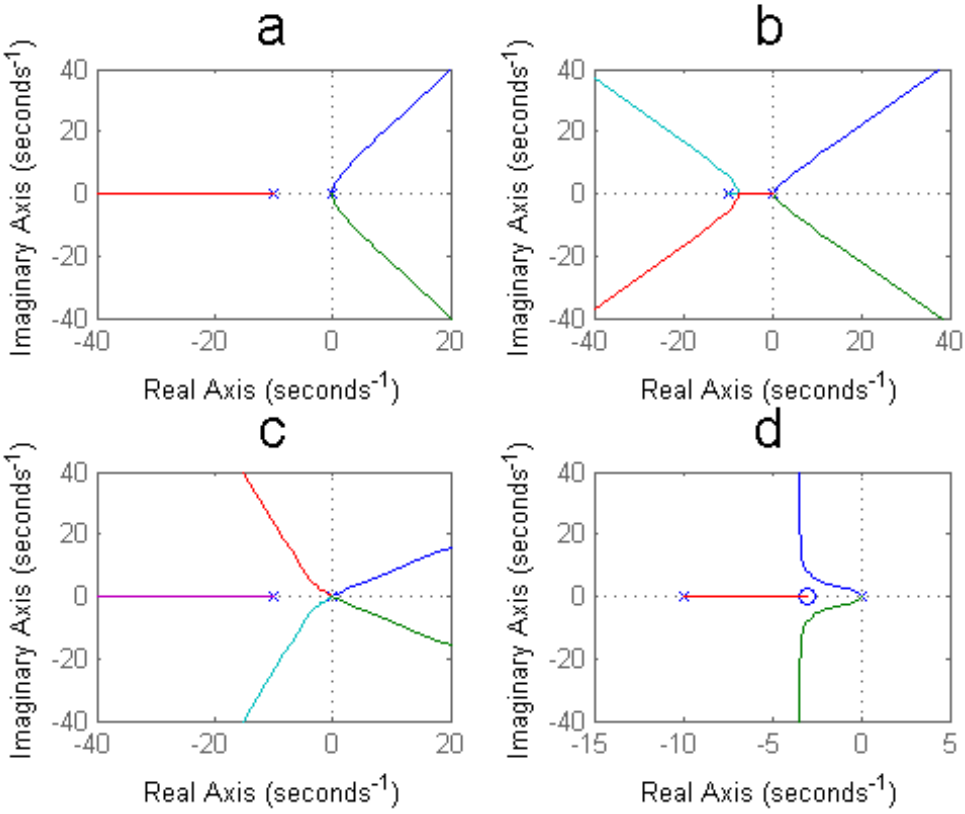
(c) $\alpha = -2$; $\phi_i = \pm 36^\circ, \pm 108^\circ, 180^\circ$

(d) $\alpha = -3.5$; $\phi_i = \pm 90^\circ$

(e) $\alpha = -0.33$; $\phi_i = \pm 60^\circ, 180^\circ$

(f) $\alpha = -1$; $\phi_i = \pm 90^\circ$

(g) $\alpha = -4$; $\phi_i = \pm 60^\circ, 180^\circ$



Solution for Problem 5.6abcd

7. *Mixed real and complex poles.* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

(a) $L(s) = \dfrac{(s+3)}{s(s+10)(s^2+2s+2)}$

(b) $L(s) = \dfrac{(s+3)}{s^2(s+10)(s^2+6s+25)}$

(c) $L(s) = \dfrac{(s+3)^2}{s^2(s+10)(s^2+6s+25)}$

(d) $L(s) = \dfrac{(s+3)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$

(e) $L(s) = \dfrac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$

Solution:
All the plots are attached at the end of the solution set.

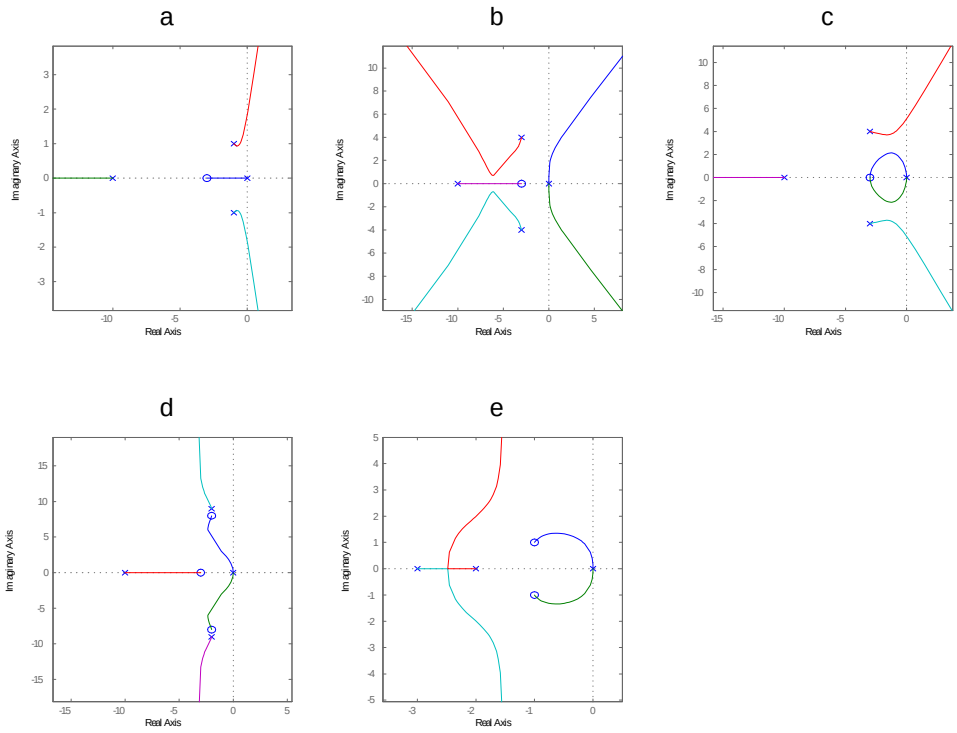
(a) $\alpha = -3$; $\phi_i = \pm 60°$, $180°$; $\theta_d = -24.8°$ at $s = -1 + j$

(b) $\alpha = -3.25$; $\phi_i = \pm 45°$, $\pm 135°$; $\theta_d = -103.5°$ at $s = -3 + 4j$

(c) $\alpha = -3.33$; $\phi_i = \pm 60°$, $180°$; $\theta_d = -13.5°$ at $s = -3 + 4j$

(d) $\alpha = -3.5$; $\phi_i = \pm 90°$; $\theta_d = 100.2°$ at $s = -2 + 9j$;
$\theta_a = -99.8°$ at $s = -2 + 8j$

(e) $\alpha = -1.5$; $\phi_i = \pm 90°$; $\theta_a = 71.6°$ at $s = -1 + j$



Solution for Problem 5.7

8. *RHP and zeros.* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

   (a) $L(s) = \dfrac{s+2}{s+10}\dfrac{1}{s^2-1}$; the model for a case of magnetic levitation with lead compensation.

   (b) $L(s) = \dfrac{s+2}{s(s+10)}\dfrac{1}{(s^2-1)}$; the magnetic levitation system with integral control and lead compensation.

   (c) $L(s) = \dfrac{s-1}{s^2}$

   (d) $L(s) = \dfrac{s^2+2s+1}{s(s+20)^2(s^2-2s+2)}$. What is the largest value that can be obtained for the damping ratio of the stable complex roots on this locus?

   (e) $L(s) = \dfrac{(s+2)}{s(s-1)(s+6)^2}$,

   (f) $L(s) = \dfrac{1}{(s-1)[(s+2)^2+3]}$

Solution:

   (a) $\alpha = -4$; $\phi_i = \pm 90^\circ$

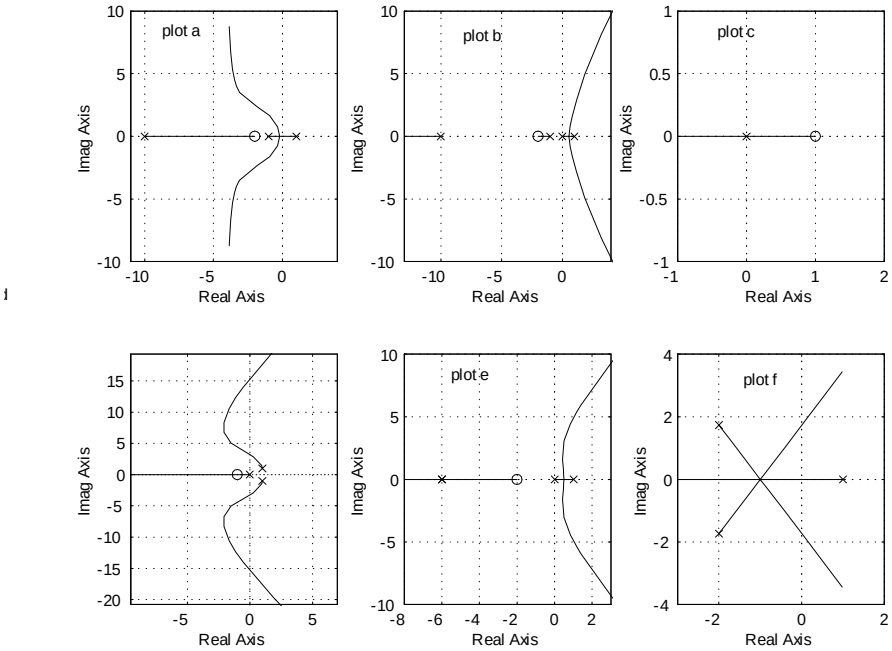
   (b) $\alpha = -2.67$; $\phi_i = \pm 60^\circ, 180^\circ$

   (c) $\alpha = -1$; $\phi_i = 180^\circ$

   (d) $\alpha = -12$; $\phi_i = \pm 60^\circ, 180^\circ$; $\theta_d = 92.7^\circ$ at $s = 1 + j$
   The maximum damping ratio is obtained at a point at the smallest angle off the horizontal. From the locus, the maximum damping is 0.31 when $K \approx 2600$.

   (e) $\alpha = -3$; $\phi_i = \pm 60^\circ, 180^\circ$

   (f) $\alpha = -1$; $\phi_i = \pm 60^\circ, 180^\circ$; $\theta_d = -60.0^\circ$ at $s = -2 + 1.73j$



Solution for Problem 5.8

9. Put the characteristic equation of the system shown in Fig.5.51 in root locus form with respect to the parameter α, and identify the corresponding $L(s)$, $a(s)$, and $b(s)$. Sketch the root locus with respect to the parameter α, estimate the closed-loop pole locations, and sketch the corresponding step responses when $\alpha = 0$, 0.5, and 2. Use Matlab to check the accuracy of your approximate step responses.

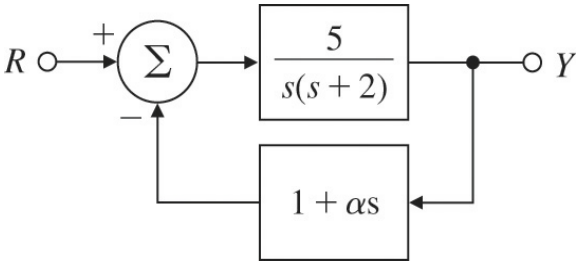
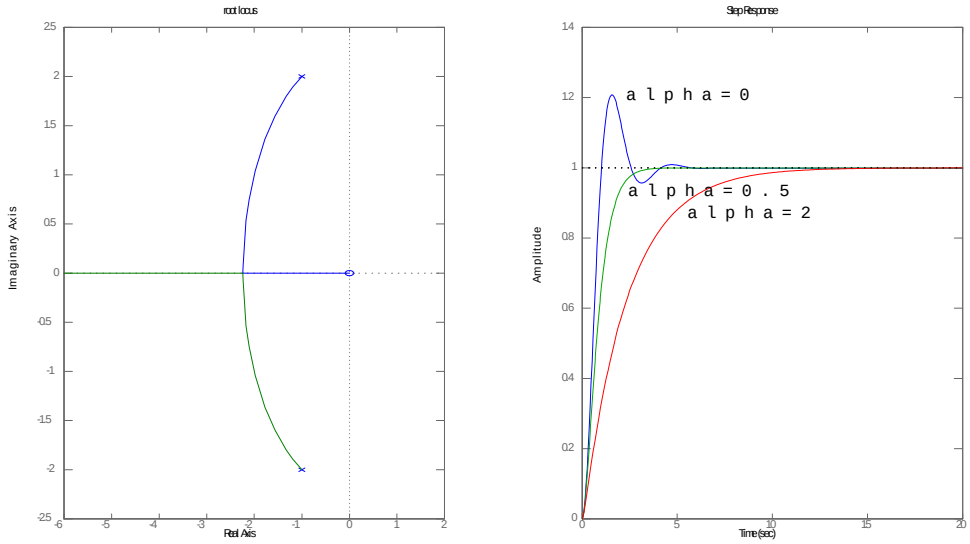


Fig. 5.51 Control system for Problem 5.9

Solution:

The characteristic equation is $s^2+2s+5+5\alpha s = 0$ and $L(s) = \dfrac{s}{s^2+2s+5}$. the root locus and step responses are plotted below.



Problem 5.9 RL and step responses

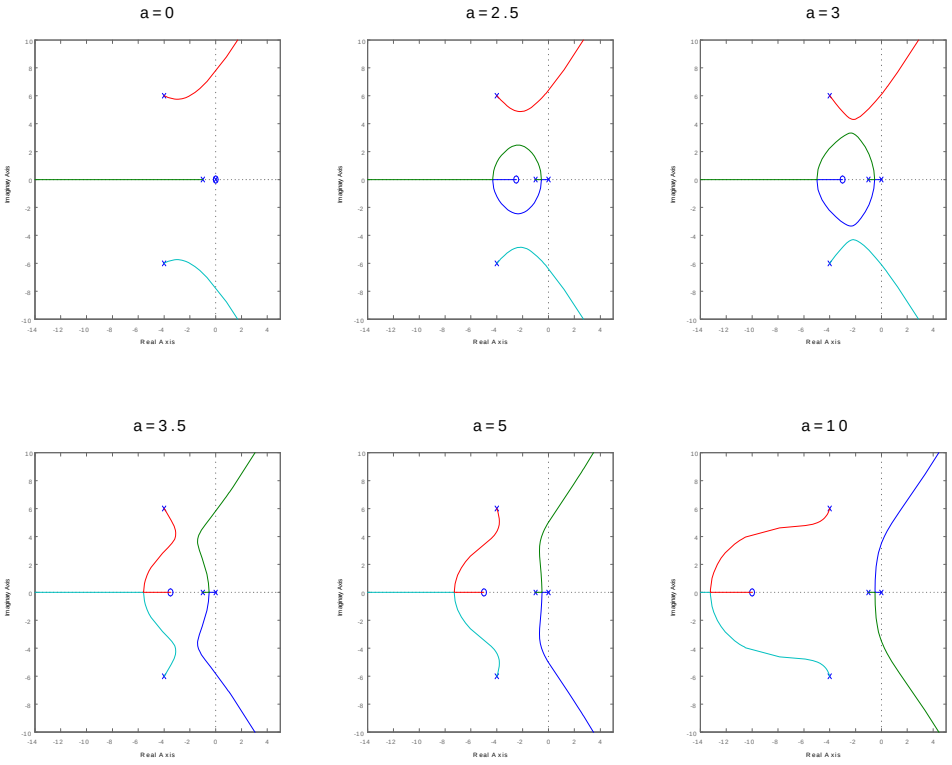
10. Use the Matlab function sisotool to study the behavior of the root locus of $1 + KL(s)$ for

$$L(s) = \frac{(s+a)}{s(s+1)(s^2+8s+52)}$$

as the parameter a is varied from 0 to 10, paying particular attention to the region between 2.5 and 3.5. Verify that a multiple root occurs at a complex value of s for some value of a in this range.

Solution:

For small values of α, the locus branch from $0, -1$ makes a circular path around the zero and the branches from the complex roots curve off toward the asymptotes. For large values of α the branches from the complex roots break into the real axis and those from 0, -1 curve off toward the asymptotes. At about $\alpha = 3.11$ these loci touch corresponding to complex multiple roots.



Solution for Problem 5.10

11. Use Routh's criterion to find the range of the gain K for which the systems in Fig. 5.52 are stable, and use the root locus to confirm your calculations.

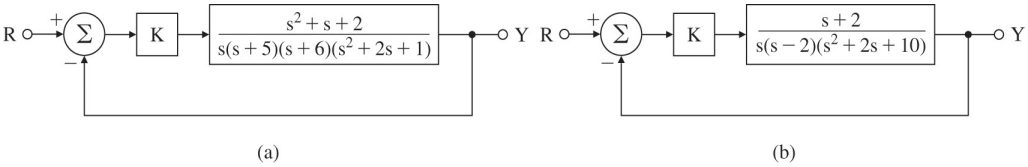
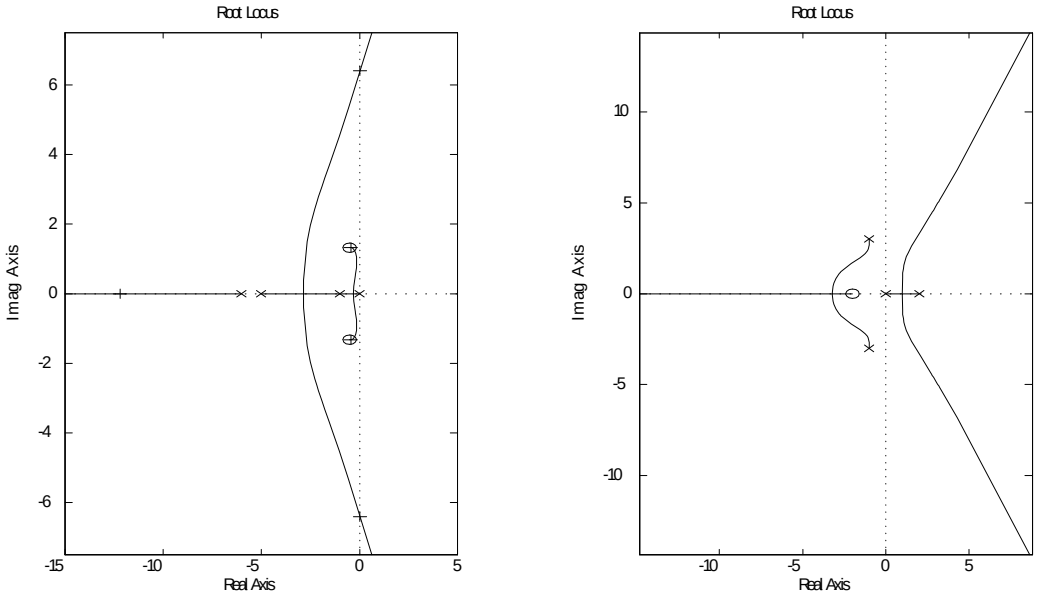


Fig. 5.52 Feedback systems for Problem 5.11

Solution:

(a) The system is stable for $0 \leq K \leq 478.234$. The root locus of the system and the location of the roots at the crossover points are shown in the left plot.

(b) There is a pole in the right hand plane thus the system is unstable for all values of K as shown in the right plot.



Solution for Problem 5.11

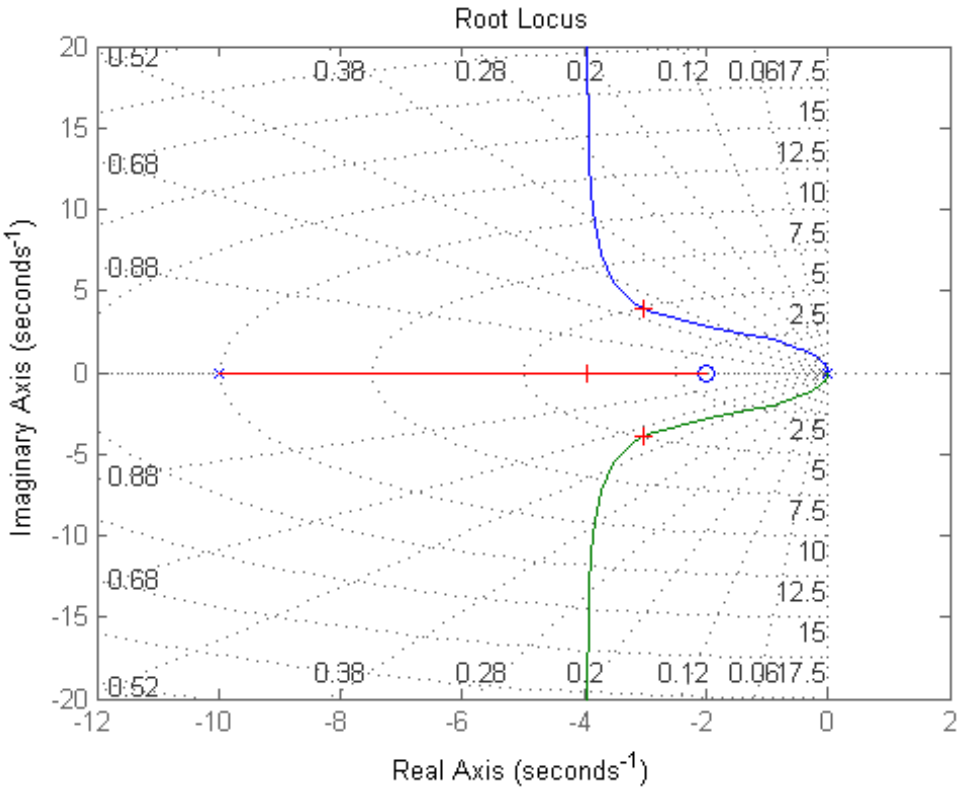
12. Sketch the root locus for the characteristic equation of the system for which

$$L(s) = \frac{(s+2)}{s^2(s+10)},$$

and determine the value of the root-locus gain for which the complex conjugate poles have the maximum damping ratio. What is the approximate value of the damping?

Solution:

Plot the system on Matlab using rlocus(sys), and use [K]= rlocfind(sys) to pick the gain for the maximum damping. We find that the maximum damping occurs at approximately $\zeta = 0.6$ where the + signs appear on the RL plot. Matlab shows that $K \approx 47.3$. at that point, and exact damping can be found using feedback with $K = 47.3$ to form the CLoop system, and then using damp to find the exact damping of $\zeta = 0.614$ for that closed-loop system.



Prob 5.12 Root locus with point of max damping marked with + (where damping approx = 0.6)

13. For the system in Fig. 5.53,

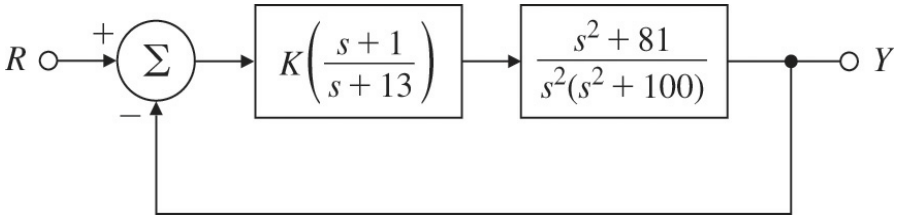


Fig. 5.53 Feedback system for Problem 5.13

(a) Find the locus of closed-loop roots with respect to K.

(b) Is there a value of K that will cause all roots to have a damping ratio greater than 0.5?

(c) Find the values of K that yield closed-loop poles with the damping ratio $\zeta = 0.707$.

(d) Use Matlab to plot the response of the resulting design to a reference step.

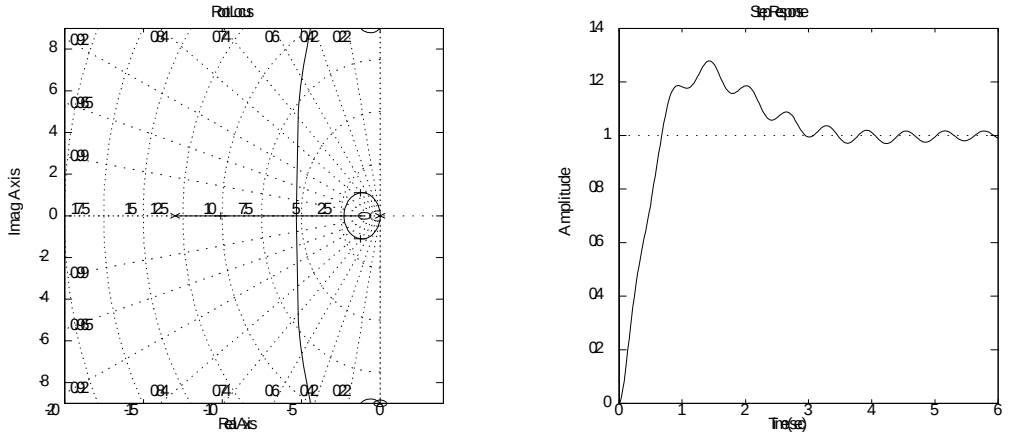
Solution:

(a) The locus is plotted below

(b) There is a K which will make the 'dominant' poles have damping 0.5 but none that will make the poles from the resonance have that much damping.

(c) Using rlocfind, the gain is about 35.

(d) The step response shows the basic form of a well damped response with the vibration of the response element added.



Root locus and step response for Problem 5.13

14. For the feedback system shown in Fig. 5.54, find the value of the gain K that results in dominant closed-loop poles with a damping ratio $\zeta = 0.5$.

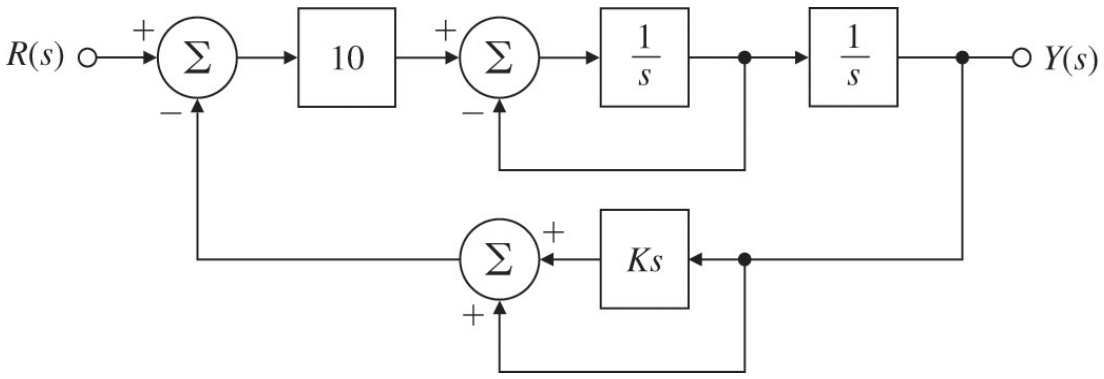
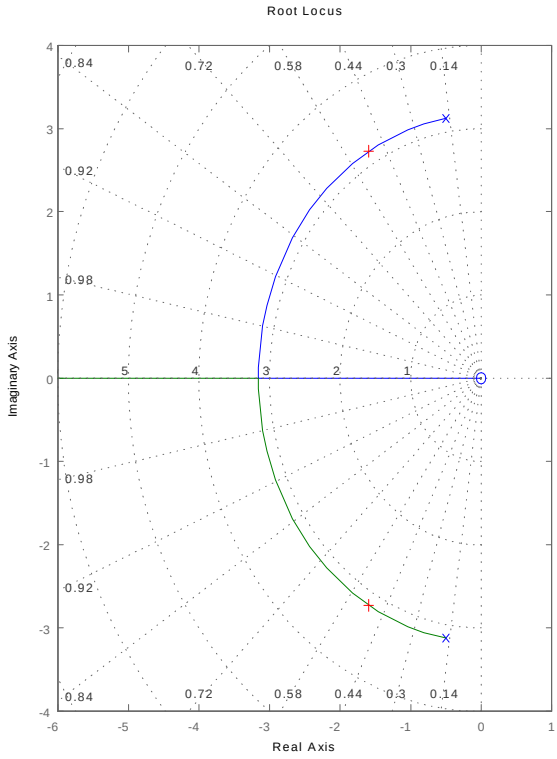


Fig. 5.54 Feedback system for Problem 5.14

Solution:

Use block diagram reduction to find the characteristic equation of the closed loop system, then divide that up into terms with and without K to find the root locus form, where $L(s) = \dfrac{10s}{s^2 + s + 10}$. Plugging into Matlab and using rlocfind produces the required gain to be $K = 0.22$.The locus is



Root locus with 0.5 damping marked

Problems and solutions for Section 5.3

15. A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}.$$

and the characteristic equation $1 + D_c(s)G(s) = 0$. Let $D_c(s) = k_p$ at first.

(a) Compute the departure and arrival angles at the complex poles and zeros.

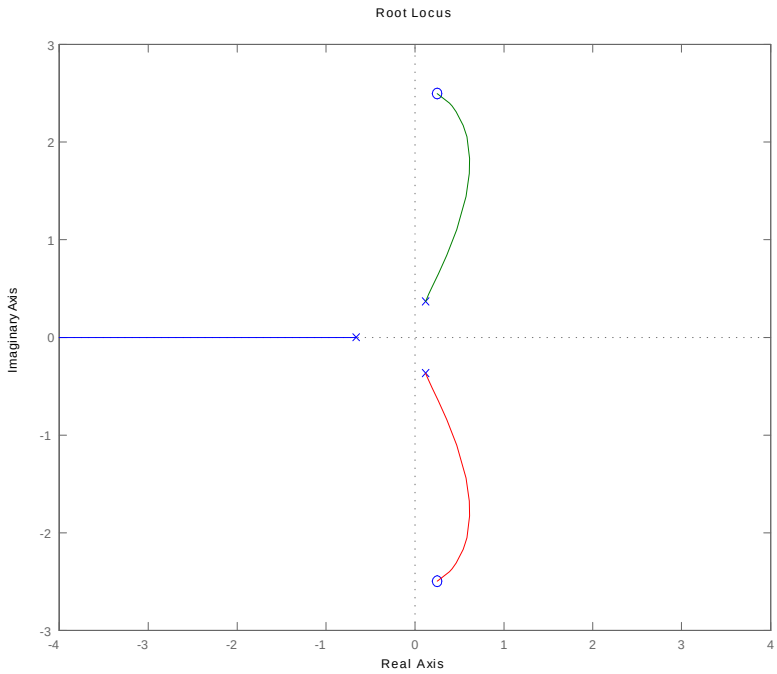
(b) Sketch the root locus for this system for parameter $K = 9.8k_p$. Use axes -4 $\leq x \leq$ 4; $-3 \leq y \leq 3$;

(c) Verify your answer using Matlab. Use the command axis([-4 4 -3 3]) to get the right scales.

(d) Suggest a practical (at least as many poles as zeros) alternative compensation $D_c(s)$ which will at least result in a stable system.

Solution:

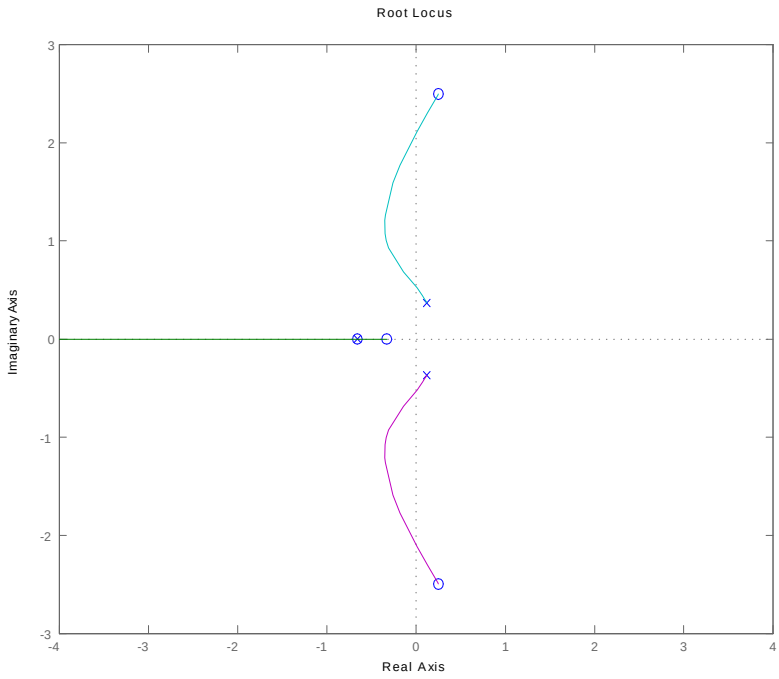
(a) $\theta_d = -180° - 25.26° - 90° + 266.5° + 92.6° = 63.83°$;
$\theta_a = -90° + 86.5° + 69.9° + 87.4° - 180° = -26.11°$



Problem 5.15(b)(c)

(b) and **c**.

(d) For this problem a double lead is needed to bring the roots into the left half-plane. The plot shows the rootlocus for control for. Let $D_c(s) = \dfrac{(s+.66)(s+.33)}{(s+5)^2}$.



Problem 5.15(d)

16. For the system given in Fig. 5.55

    (a) Plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

    (b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

    (c) Repeat part (a) for fixed $K_1 = 2$ with the parameter $K = \lambda$ varying from 0 to ∞.

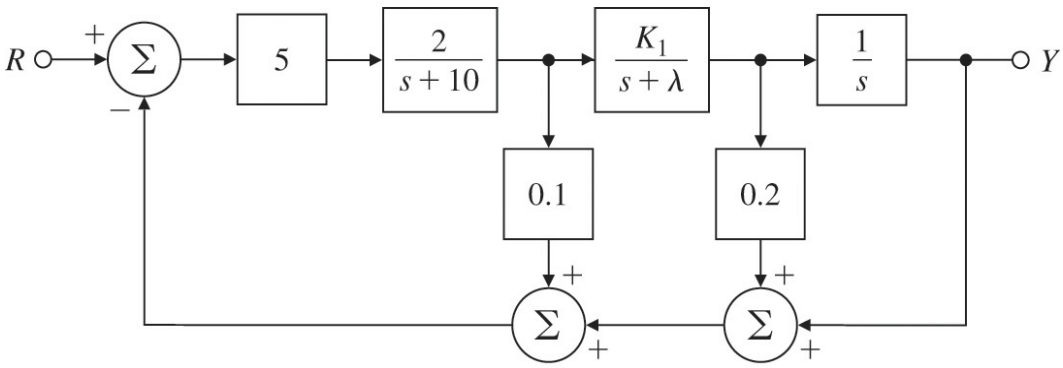


Fig. 5.55 Control system for Problem 5.16

Solution:
Use block diagram reduction to find the characteristic equation of the closed-loop system:

$$1 + \left\{\frac{10K_1}{s(s+10)(s+\lambda)}\right\}\left\{0.1\frac{s(s+\lambda)}{K_1} + 0.2s + 1\right\} = 0$$
$$\text{or} \qquad s(s+\lambda)(s+11) + 2K_1(s+5) = 0$$

The root locus for each part is attached at the end.

(a) Substituting $\lambda = 2$ and divide the equation above up into terms with and without K_1 to find Evans form:

$$1 + K_1\frac{2(s+5)}{s(s+2)(s+11)} = 0 \quad \Rightarrow \quad L(s) = \frac{2(s+5)}{s(s+2)(s+11)}$$

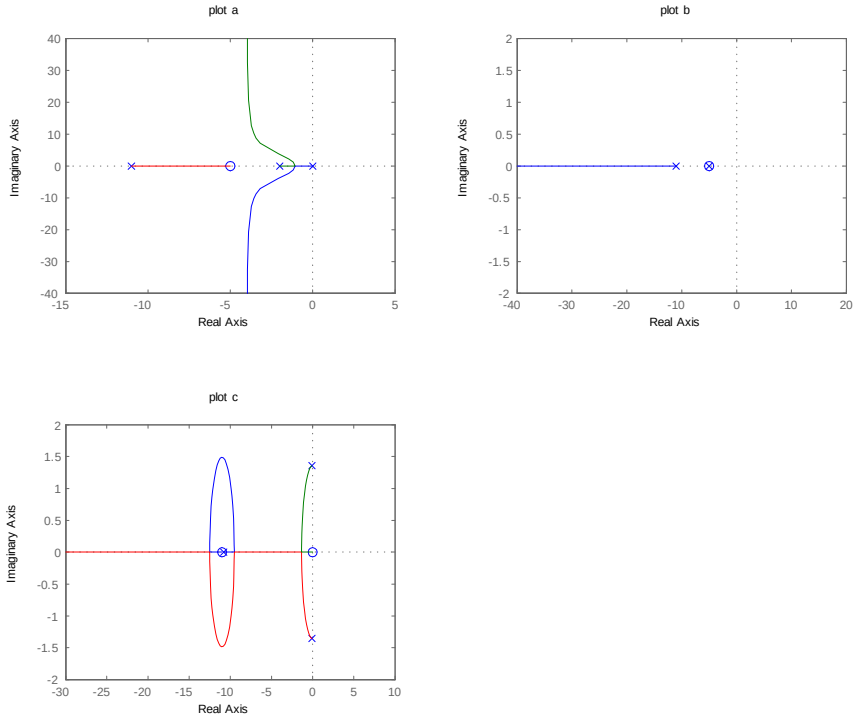
(b) Substituting $\lambda = 5$ and rewrite the equation in Evans form with respect to K_1:

$$1 + K_1\frac{2(s+5)}{s(s+5)(s+11)} = 0 \quad \Rightarrow \quad L(s) = \frac{2}{s(s+11)}$$

We have a pole-zero cancellation at $s = -5$.

(c) Substituting $K_1 = 2$ and divide the characteristic equation up into terms with and without λ to find Evans form:

$$1 + \lambda \frac{s(s+11)}{s^2(s+11)+4(s+5)} = 0 \quad \Rightarrow \quad L(s) = \frac{s(s+11)}{s^3+11s^2+4s+20}$$



Solution for Problem 5.16

17. For the system shown in Fig. 5.56, determine the characteristic equation and sketch the root locus of it with respect to positive values of the parameter c. Give $L(s)$, $a(s)$, and $b(s)$ and be sure to show with arrows the direction in which c increases on the locus.

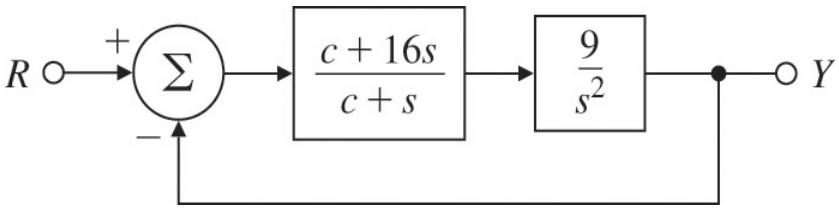
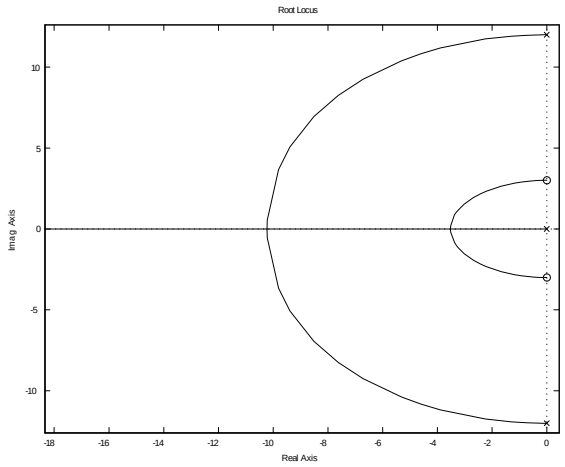


Fig. 5.56 Control system for Problem 5.17

Solution:

$$L(s) = \frac{s^2 + 9}{s^3 + 144s} = \frac{a(s)}{b(s)}$$

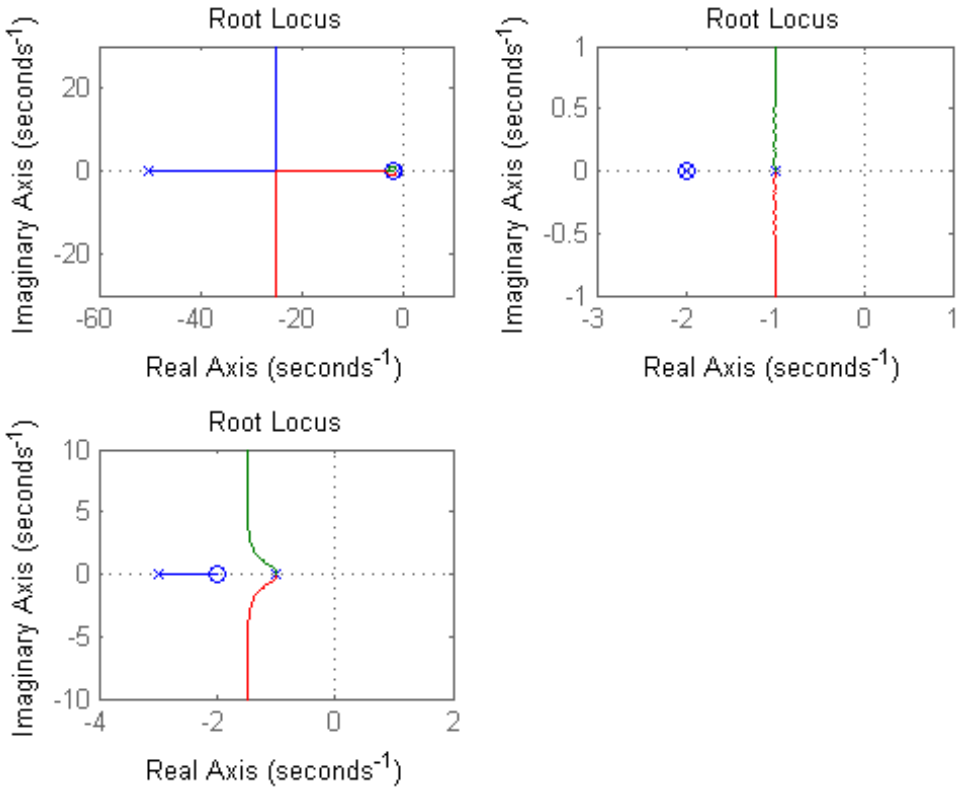


Solution for Problem 5.17

18. The loop transmission of a system has two poles at $s = -1$ and a zero at $s = -2$. There is a third real-axis pole p located somewhere to the *left* of the zero. Several different root loci are possible, depending on the exact location of the third pole. The extreme cases occur when the pole is located at infinity or when it is located at $s = -2$. Give values for p and sketch the three distinct types of loci.

Solution:

To give examples, we've picked a pole at s=-50, -2, and -3. For the 1st case, there is a tiny circle around the zero at s=-2 with two loci heading north and south with the centroid of the asymptotes at s=-25. In the 2nd case, the pole and zero at s=-2 cancel each other, and the loci depart north and south from the two poles at s=-1, ie their asymptote centroid is at s=-1.. For the 3rd, intermediate case, the pole and zero at -3 and -2 draw the locus from the two poles at s=-1 over to the left, but they still end up going north and south with the centroid of the asymptotes at s=-1.5, ie they have moved to the left by 0.5.



Solution for Problem 5.18

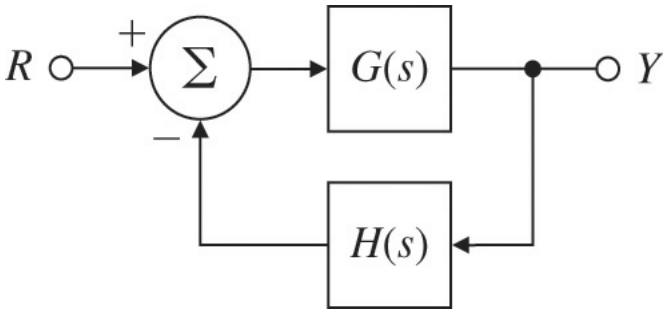
19. For the feedback configuration of Fig. 5.57, use asymptotes, center of asymptotes, angles of departure and arrival, and the Routh array to sketch root loci for the characteristic equations of the listed feedback control systems versus the parameter K. Use Matlab to verify your results.

(a) $G(s) = \dfrac{K}{s(s+1+3j)(s+1-3j)}$, $\quad H(s) = \dfrac{s+2}{s+8}$

(b) $G(s) = \dfrac{K}{s^2}$, $\quad H(s) = \dfrac{s+1}{s+3}$

(c) $G(s) = \dfrac{K(s+5)}{(s+1)}$, $\quad H(s) = \dfrac{s+7}{s+3}$

(d) $G(s) = \dfrac{K(s+3+4j)(s+3-4j)}{s(s+1+2j)(s+1-2j)}$, $\quad H(s) = 1+3s$



Feedback system for Problem 5.19

Solution:
The root locus for each part is attached at the end.

(a)

$$L(s) = \frac{(s+2)}{s(s+1+3j)(s+1-3j)(s+8)}$$

- Asymptotes: $4-1=3$
- Center of asymptotes: $\alpha = -2.67$
- Angle of asymptotoes: $\phi = \pm 60^\circ, 180^\circ$
- Angle of departure: $\theta_d = 29.93^\circ$ at $s = -1+3j$
- Imaginary-axis crossings:

$$\begin{array}{rcccc} \Delta(s) & = & s^4+10s^3+26s^2+(80+K)\,s+2K & & \\ & s^4: & 1 & 26 & 2K \\ & s^3: & 10 & 80+K & \\ & s^2: & 18-\frac{K}{10} & 2K & \\ & s: & \frac{-K^2-100K+14400}{180-K} & & \\ & s^0: & 2K & & \end{array}$$

Routh's test gives $0 < K < 80$ for stability. Solving $\Delta(s)$ with $K = 80$, the crossings are $s = \pm 4j$.

(b)

$$L(s) = \frac{(s+1)}{s^2(s+3)}$$

- Asymptotes: $3 - 1 = 2$
- Center of asymptotes: $\alpha = -1$
- Angle of asymptotoes: $\phi = \pm 90^\circ$
- Imaginary-axis crossings:

$$\begin{array}{rcl} \Delta(s) & = & s^3 + 3s^2 + Ks + K \\ & & \begin{array}{rcc} s^3: & 1 & K \\ s^2: & 3 & K \\ s: & \frac{2K}{3} & \\ s^0: & K & \end{array} \end{array}$$

Routh's test gives $K > 0$ for stability. Solving $\Delta(s)$ with $K = 0$, the crossings are $s = 0$.

(c)

$$L(s) = \frac{(s+5)(s+7)}{(s+1)(s+3)}$$

- Asymptotes: $2 - 2 = 0$
- Breakin/Breakaway:

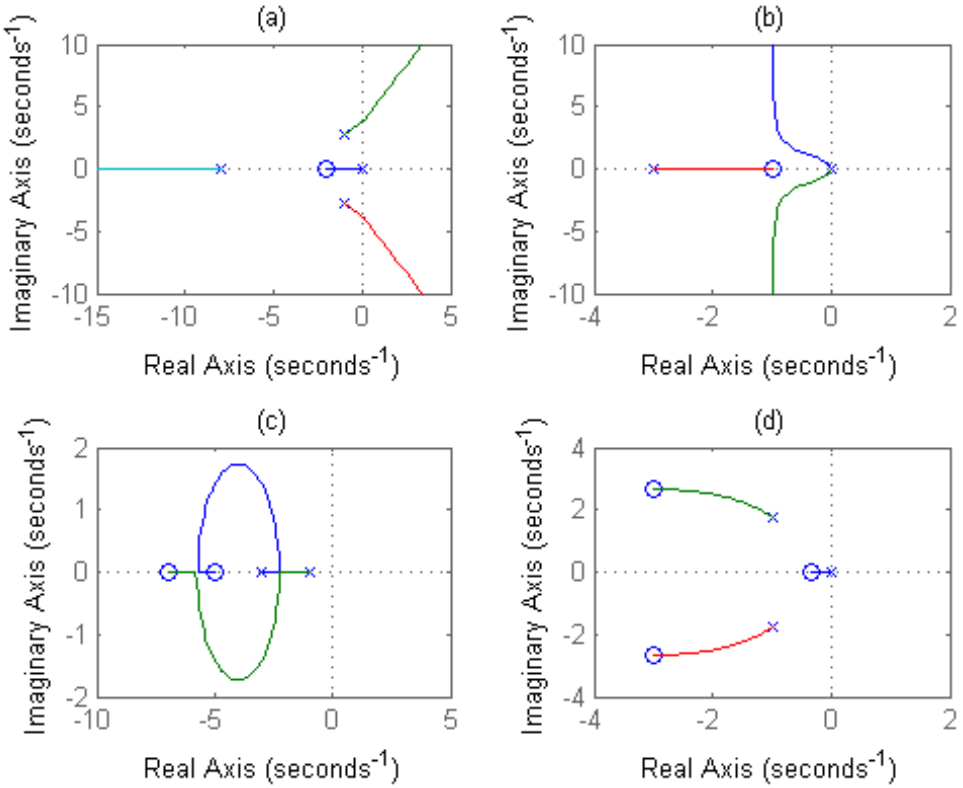
$$\frac{dL(s)}{ds} = 0 \Longrightarrow 8s^3 + 64s + 104 = 0$$

Therefore the breakin/breakaway points are at $s = -2.27, -5.73$.

(d)

$$L(s) = \frac{(1+3s)\,(s+3+4j)\,(s+3-4j)}{s(s+1+2j)(s+1-2j)}$$

- Asymptotes: $3 - 3 = 0$
- Angle of departure: $\theta_d = 108.4^\circ$ at $s = -1 + 2j$
- Angle of arrival: $\theta_a = -23.4^\circ$ at $s = -3 + 4j$



Solution for Problem 5.19

20. Consider the system in Fig. 5.58.

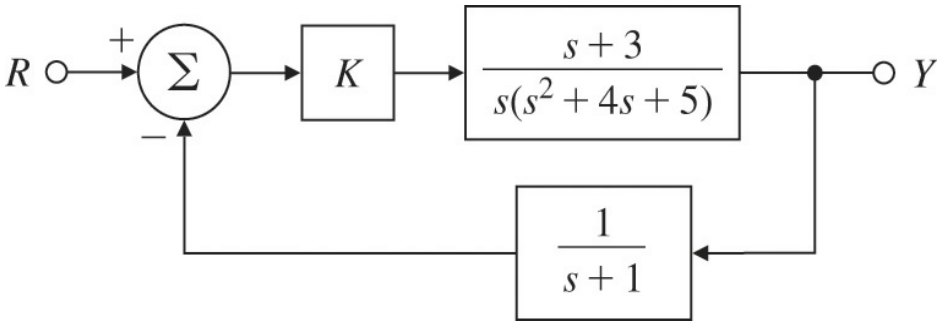


Fig. 5.58 Feedback system for Problem 5.20

(a) Using Routh's stability criterion, determine all values of K for which the system is stable.

(b) Use Matlab to find the root locus versus K. Find the values for K at imaginary-axis crossings.

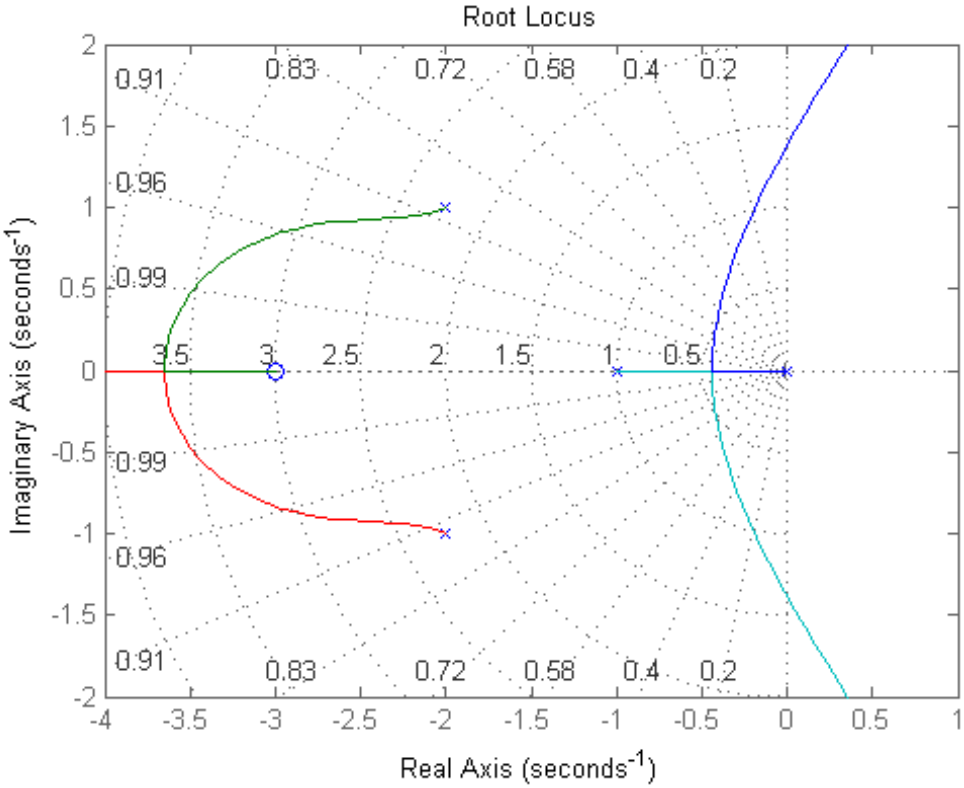
Solution:

(a)

$$\begin{aligned}\Delta(s) &= s^4+5s^3+9s^2+(5+K)s+3K\\ s^4:&\quad 1 \quad 9 \quad 3K\\ s^3:&\quad 5 \quad 5+K\\ s^2:&\quad 8-\tfrac{K}{5} \quad 3K\\ s:&\quad \tfrac{-K^2-40K+200}{40-K}\\ s^0:&\quad 3K\end{aligned}$$

For the system to be stable, $0 < K < 4.49$.

(b) The root locus is shown below and verifies that the system goes unstable as K increases, and the imaginary axis crossing is at $s \simeq \pm 1.4j$.



Root locus for Problem 5.20

Problems and solutions for Section 5.4

21. Let

$$G(s) = \frac{1}{(s+2)(s+3)} \quad \text{and} \quad D_c(s) = K\frac{s+a}{s+b}.$$

Using root-locus techniques, find values for the parameters a, b, and K of the compensation $D_c(s)$ that will produce closed-loop poles at $s = -1 \pm j$ for the system shown in Fig.5.59.

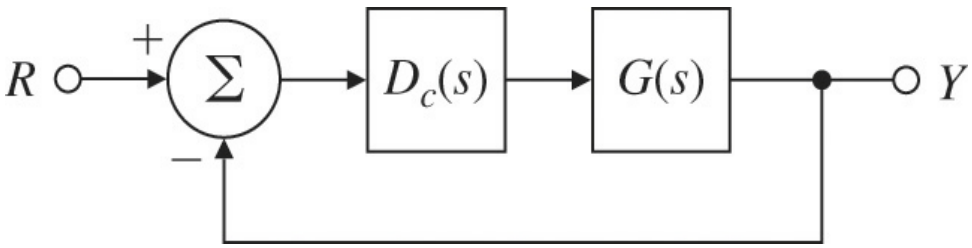
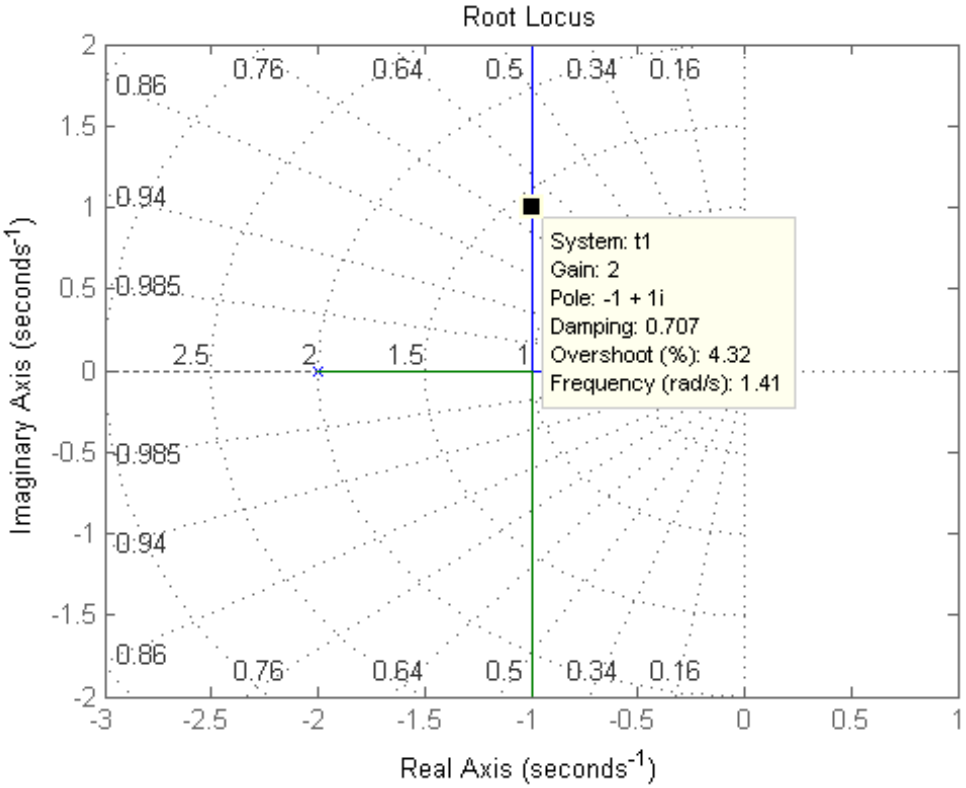


Fig. 5.59 Unity feedback system for Problems 5.21 - 5.27, & 5.32

Solution:

Generally, it's not a good idea to cancel a pole in the plant with a zero in the compensator, since you can't ever do this perfectly and a small residual may continue. But since the desired CL pole location is slower than the plant poles, this actually would make sense in real life. The expedient solution is to cancel the pole at -3 with the zero, place the compensator pole at $s = 0$. This results in the loci between $s = -2$ and $= 0$ to meet at $s = -1$, then splitting and heading north and south, which clearly will go through the desired point, where $K = 2$. Thus, $a = 3$, $b = 0$, and $K = 2$ is the desired answer. Sketch it out... you don't really need Matlab to check it.



RL for Problem 5.21 showing K = 2 for root at s=-1 +1j

22. Suppose that in Fig.5.59,

$$G(s) = \frac{1}{s(s^2+2s+5)} \quad \text{and} \quad D_c(s) = \frac{K}{s+2}.$$

Without using Matlab, sketch the root locus with respect to K of the characteristic equation for the closed-loop system, paying particular attention to points that generate multiple roots. Find the value of K at that point, state what the location of the multiple roots is, and how many multiple roots there are.

Solution:
The root locus for the system is attached at the end.

$$(s) = \frac{1}{s(s+2)(s^2+2s+5)}$$

- Asymptotes: $4 - 0 = 4$
- Center of asymptotes: $\alpha = -1$
- Angle of asymptotoes: $\phi = \pm 45^\circ, \pm 135^\circ$
- Angle of departure: $\theta_d = -90^\circ$ at $s = -1 + 2j$
- Imaginary-axis crossings:

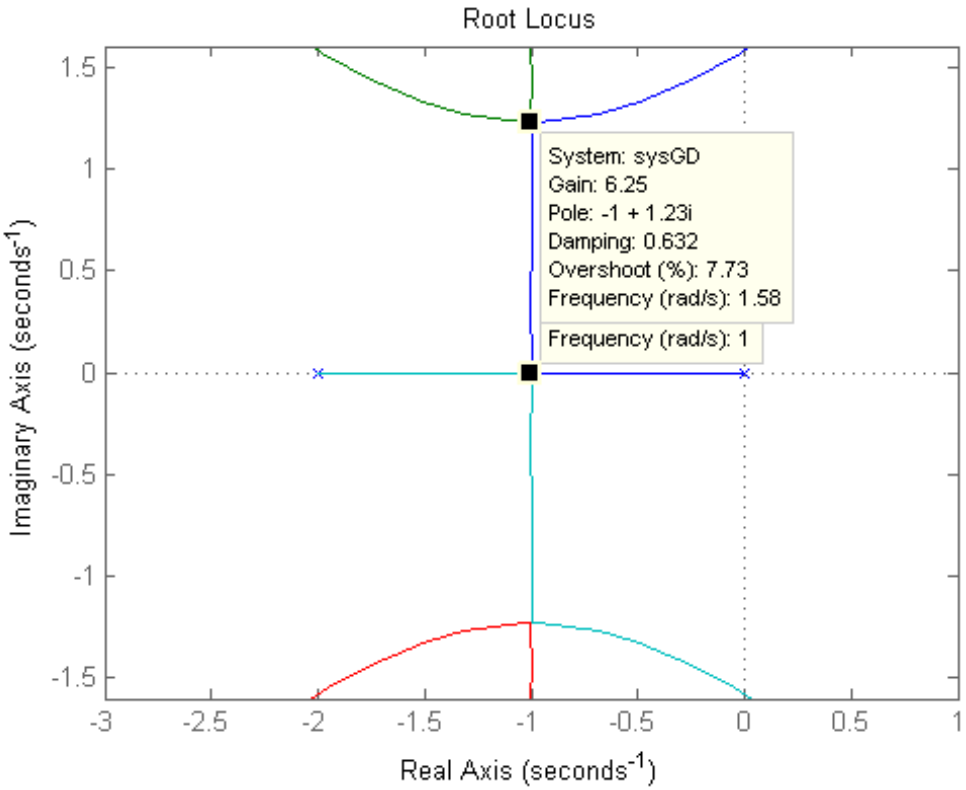
$$\begin{array}{rcccc} \Delta(s) & = & s^4 + 4s^3 + 9s^2 + 10s + K & & \\ & & s^4: & 1 & 9 \quad K \\ & & s^3: & 4 & 10 \\ & & s^2: & 6.5 & K \\ & & s: & 10 - \frac{8K}{13} & \\ & & s^0: & K & \end{array}$$

Routh's test gives $0 < K < 16.25$ for stability. Solving $\Delta(s)$ with $K = 16.25$, the crossings are $s = \pm 1.58j$, which can be seen to be true from the RL below.

- Location of multiple roots:
If a polynomial has repeated roots, its derivative is equal to zero at the multiple roots. Therefore

$$\frac{d\Delta(s)}{ds} = 4s^3 + 12s^2 + 18s + 10 = 0$$

Thus the repeated roots are at $s = -1,\ -1 \pm 1.225j$. Plugging the roots into the characteristic equation, the corresponding value of K is $K = 4$ and.6.25, respectively, which can also be verified from putting your cursor on the repeated roots spots on a RL from Matlab.

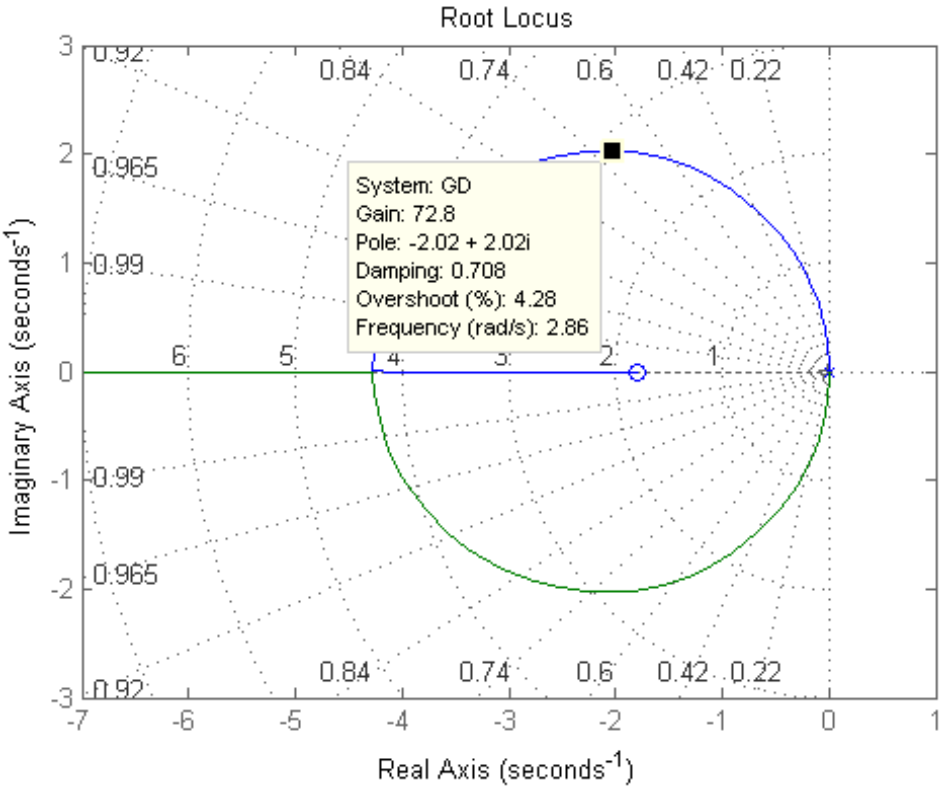


Root locus for Problem 5.22

23. Suppose the unity feedback system of Fig. 5.59 has an open-loop plant given by $G(s) = 1/s^2$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in series with the plant so that the dominant poles of the closed-loop system are located at $s = -2 \pm 2j$.

Solution:

Setting the pole of the lead to be at $p = -20$, and the zero is at $z = -2$ produces a locus with a circle that goes a bit too high and mises the desired 2 +2i. So move the zero a bit to the East, ie let $z = -1.8$. It does the job, so put your cursor on the spot and find that with a gain of $K = 72.8$.gives the desired roots. The locus is plotted below.



RL for Problem 5.23.

24. Assume that the unity feedback system of Fig. 5.59 has the open-loop plant

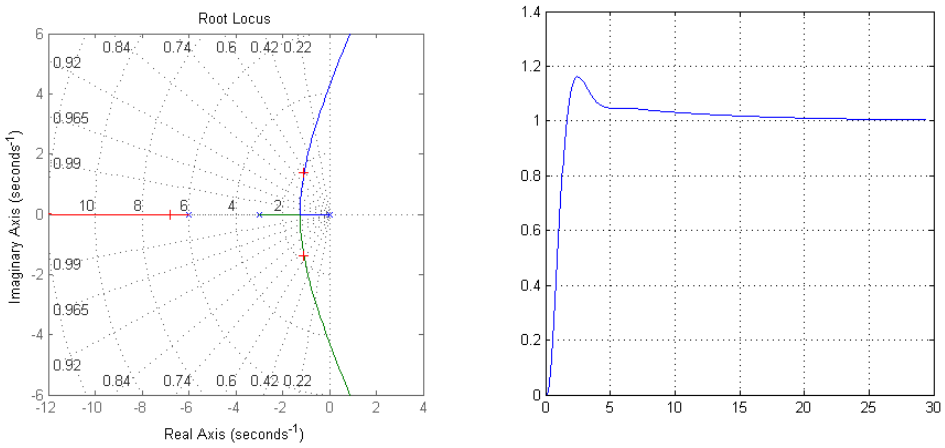
$$G(s) = \frac{1}{s(s+3)(s+6)}.$$

Design a lag compensation to meet the following specifications:

- The step response settling time is to be less than 5 sec.
- The step response overshoot is to be less than 17%.
- The steady-state error to a unit ramp input must not exceed 10%.

Solution:

The overshoot specification requires that damping be $\gtrsim 0.5$ and the settling time requires that $\omega_n > 1.8$. From the root locus plotted below, these can be met at $K = 21$ where the $\omega_n \simeq 1.8$. With this gain, the $K_v = 21/18 = 1.17$. To get a $K_v = 10$, we need a lag gain of about 8.6. Selecting the lag zero to be at 0.1 requires the pole to be at $0.1/8.6 = 0.012$. To round it out, we'll set $p = 0.01$. Other choices are of course possible. The step response of this design is plotted below. Note that the settling time is not quite met. So to meet all the specs, it would be advisable to also add a little bit of lead so that the gain could be raised enough to meet the settling time.



Root locus and step response for Problem 5.24

25. A numerically controlled machine tool positioning servomechanism has a normalized and scaled transfer function given by

$$G(s) = \frac{1}{s(s+1)}.$$

Performance specifications of the system in the unity feedback configuration of Fig. 5.59 are satisfied if the closed-loop poles are located at $s = -1 \pm j\sqrt{3}$.

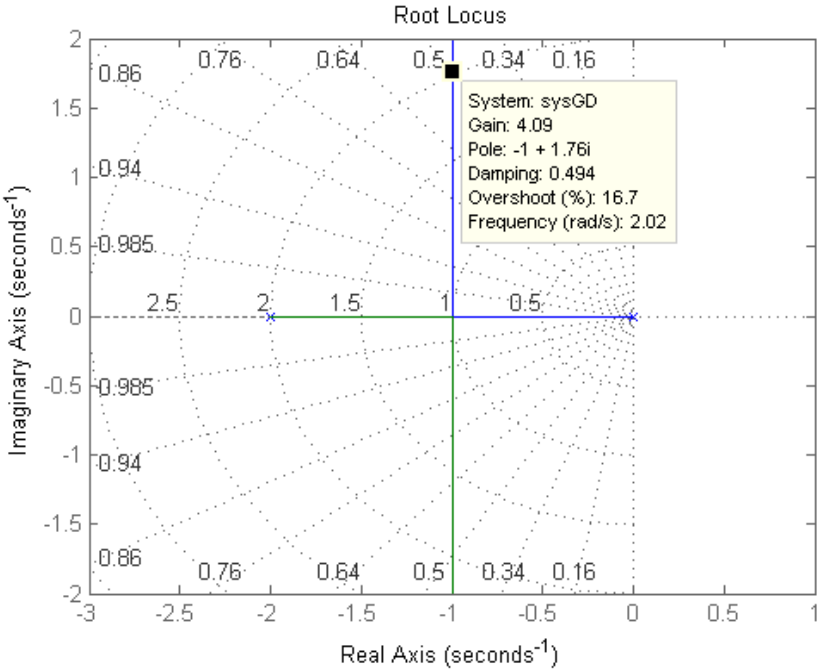
(a) Show that this specification cannot be achieved by choosing proportional control alone, $D_c(s) = k_p$.

(b) Design a lead compensator $D_c(s) = K\frac{s+z}{s+p}$ that will meet the specification.

Solution:

(a) With proportional control, the complex poles have real part at $s = -0.5$ no matter what the value of the the gain. Therefore, there is no proportional gain that will cause the roots to have a real componant of $s = -1$.

(b) There are many satisfactory lead designs that can easily be found by playing with a RL. One possibility is to put a pole at $p = -10$ and find the zero and gain to be $z = -3$, $K = 12$. Another very simple design is to cancel the plant pole at $s = -1$ with a lead zero at $s = -1$, and put the lead pole at $s = -2$. This will create a locus with vertical lines going north and south along $s = -1$. For $s = \pm j\sqrt{3}$ requires that $K = 4$.



RL for Probelm 5.25

26. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D_c(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.
- The steady-state error to a unit ramp at the reference input must be less than 0.05.

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications, ignoring the error requirement.

(b) What is the velocity constant K_v for your design? Does it meet the error specification?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the Matlab plot of the root locus of your final design.

(e) Give the Matlab response of your final design to a reference step.

Solution:

(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245. With the lead compensator, the overshoot is reduced to 3.64% and the rise time is 0.35 sec.

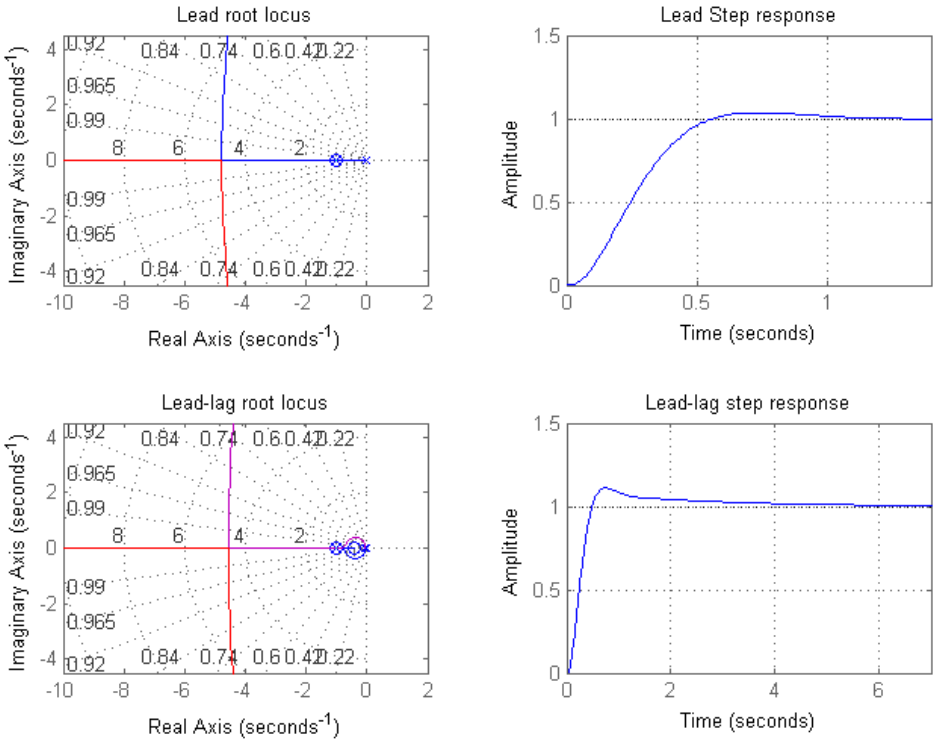
(b)

$$K_v = \lim_{s\to 0} sGD_c = \lim_{s\to 0} s\frac{10}{s(s+1)(s+10)}\frac{245(s+1)}{(s+6)} = 4.083$$

(c) To meet the steady-state requirement, we need a new $K_v = 20$, which is an increase of a factor of 5. If we set the lag zero at $z = -0.4$, the pole needs to be at $p = -0.08$.

(d) The root locus is plotted below.

(e) The step response is plotted below.



Solution to Problem 5.26

27. Assume the closed-loop system of Fig. 5.59 has a feed forward transfer function $G(s)$ given by

$$G(s) = \frac{1}{s(s+2)}.$$

Design a lag compensation so that the dominant poles of the closed-loop system are located at $s = -1 \pm j$ and the steady-state error to a unit ramp input is less than 0.2.

Solution:

The poles can be put in the desired location with proportional control alone, with a gain of $k_p = 2$ resulting in a $K_v = 1$. To get a $K_v = 5$, we add a compensation with zero at -0.1 and a pole at -0.02. $D_c(s) = 2\dfrac{s+0.1}{s+0.02}$. The lag compensator will not make a significant difference to the pole locations.

28. An elementary magnetic suspension scheme is depicted in Fig. 5.60. For small motions near the reference position, the voltage e on the photo detector is related to the ball displacement x(in meters) by $e = 100x$. The upward force (in newtons) on the ball caused by the current i (in amperes) may be approximated by $f = 0.5i + 20x$. The mass of the ball is 20 g, and the gravitational force is 9.8 N/kg. The power amplifier is a voltage-to-current device with an output (in amperes) of $i = u + V_0$.

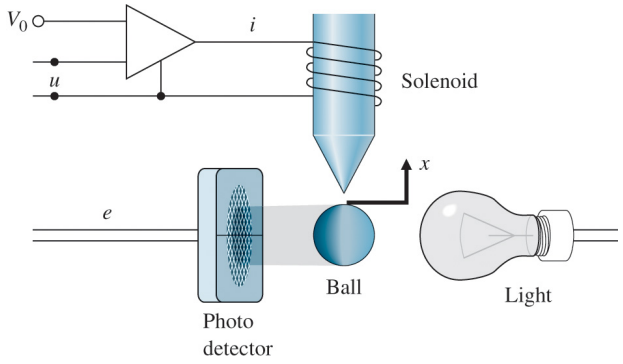


Fig. 5.60 Elementary magnetic suspension

(a) Write the equations of motion for this setup.

(b) Give the value of the bias V_0 that results in the ball being in equilibrium at $x = 0$.

(c) What is the transfer function from u to e?

(d) Suppose the control input u is given by $u = -Ke$. Sketch the root locus of the closed-loop system as a function of K.

(e) Assume that a lead compensation is available in the form $\frac{U}{E} = D_c(s) = K\frac{s+z}{s+p}$. Give values of K, z, and p that yields improved performance over the one proposed in part (d).

Solution:

(a) The equations of motion can be written as

$$\begin{aligned} m\ddot{x} &= \sum forces \\ &= 0.5i + 20x - mg = 0.5(u + V_o) + 20x - mg \end{aligned}$$

Substituting numbers, we have

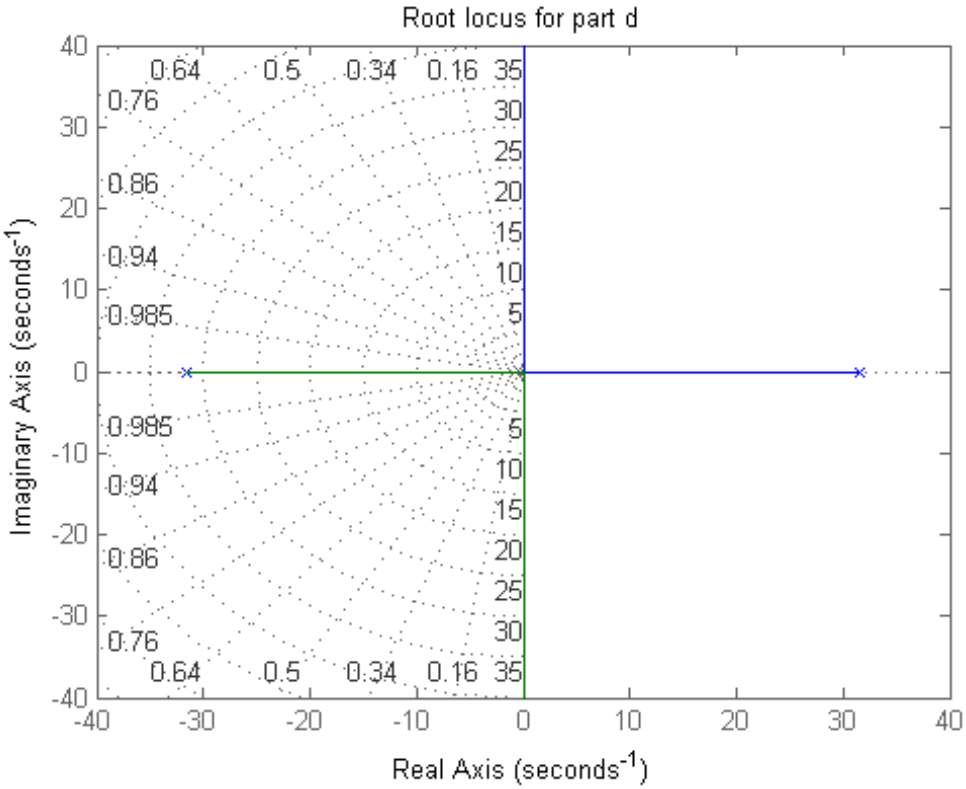
$$0.02\ddot{x} = 0.5(u + V_o) + 20x - 0.196.$$

(b) In equilibrium at $x = 0$, $\ddot{x} = 0$ and $u = 0$. Therefore to have the bias cancel gravity, $0.5V_o - 0.196 = 0$ or $V_o = 0.392$.

(c) Taking Laplace transforms of the equation and substituting $e = 100x$,

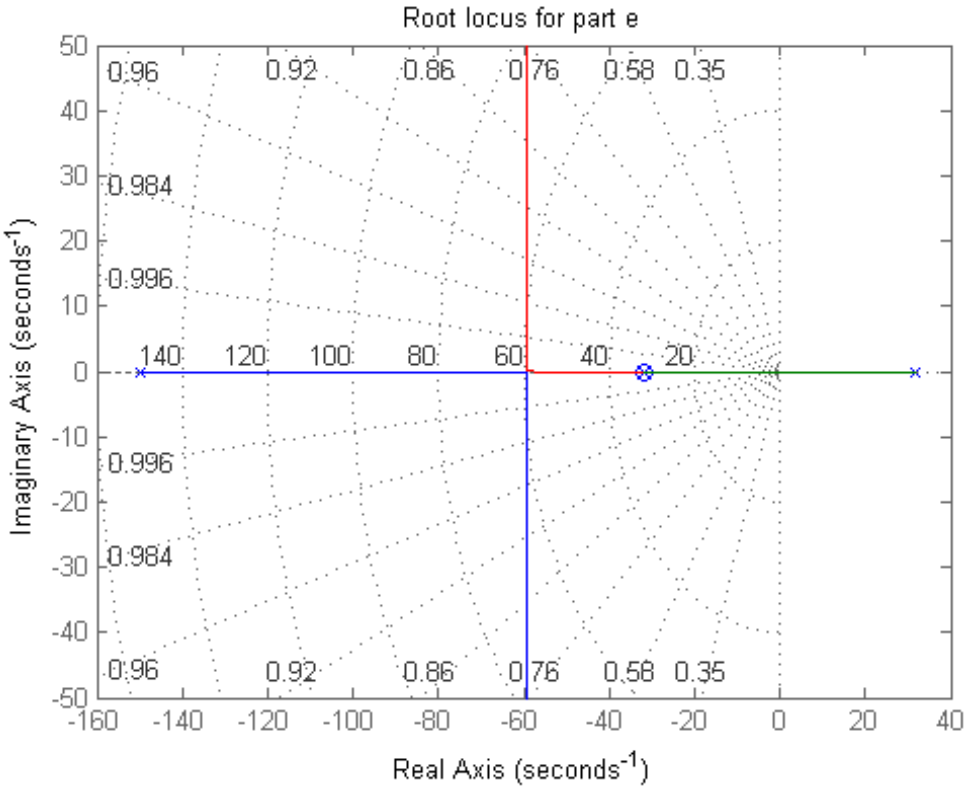
$$\frac{E}{U} = \frac{2500}{s^2 - 1000}$$

(d) The locus starts at the two poles symmetric to the imaginary axis, meet at the origin and cover the imaginary axis. The locus is plotted below.



Root loci for Problem 5.28(d)

(e) Since the system with a proportional gain is on the stability boundary, any lead will improve its performance. For example, we can pick $z = \sqrt{1000}$ to cancel one of the open-loop plant poles, and pick $p = 150$ to pull the locus into the left-hand plane. K can be selected to give a desired amount of damping, say 0.7. $K = 4.75$ gives a damping of 0.7. See the plot below.



Root loci for Problem 5.28(e)

29. A certain plant with the non minimum phase transfer function

$$G(s) = \frac{4 - 2s}{s^2 + s + 9},$$

is in a unity positive feedback system with the controller transfer function $D_c(s)$.

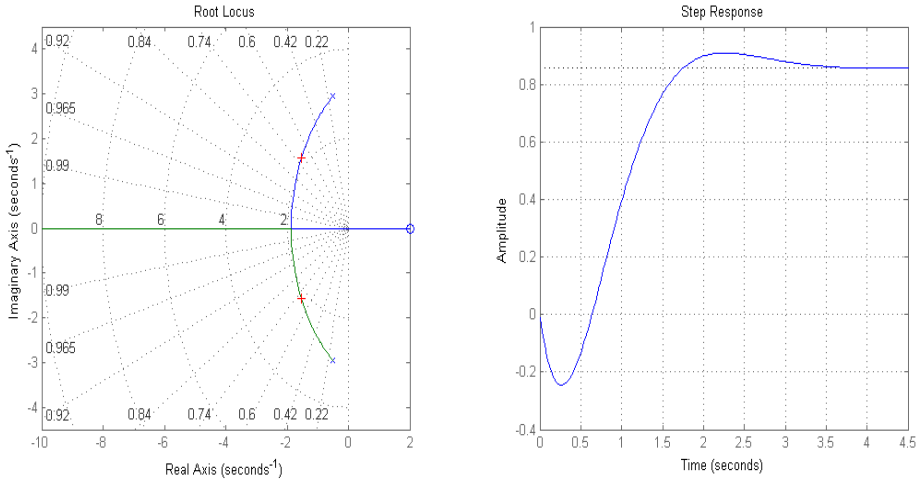
(a) Use Matlab to determine a (negative) value for $D_c(s) = K$ so that the closed-loop system with negative feedback has a damping ratio $\zeta = 0.707$.

(b) Use Matlab to plot the system's response to a reference step.

Solution:

(a) With all the negatives, the problem statement might be confusing. With the $G(s)$ as given, Matlab needs to plot the negative locus, which is the regular positive locus for $-G$. The locus is plotted below. The value of gain for closed loop roots at damping of 0.7 is approx $K = -1.04$

(b) The final value of the step response plotted below is approximately 0.85.



Solution to Prob. 5.29

30. Consider the rocket-positioning system shown in Fig. 5.61.

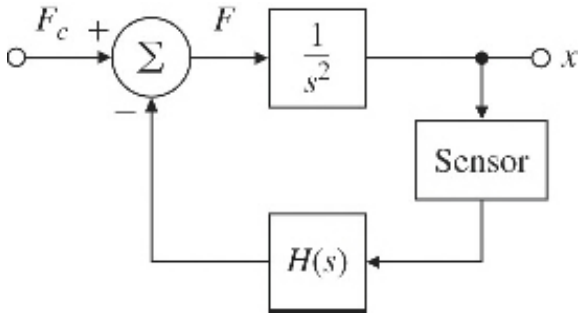


Fig. 5.61 Block diagram for rocket-positioning control system

(a) Show that if the sensor that measures x has a unity transfer function, the lead compensator

$$H(s) = K\frac{s+2}{s+4}$$

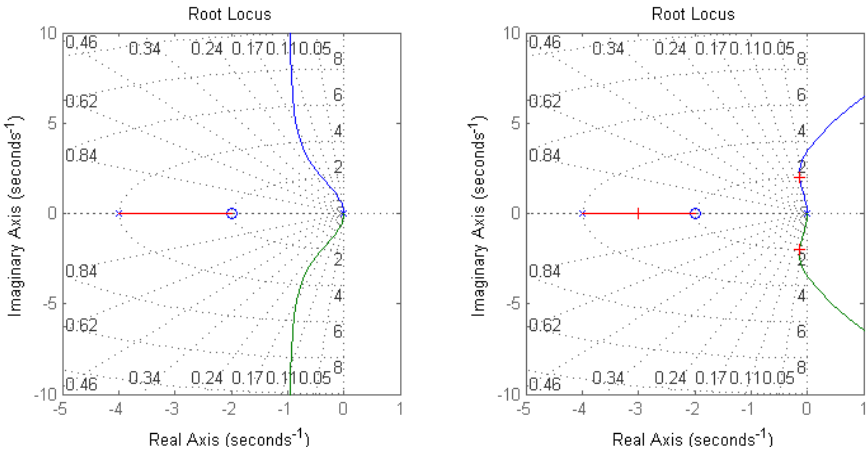
stabilizes the system.

(b) Assume that the sensor transfer function is modeled by a single pole with a 0.1 sec time constant and unity DC gain. Using the root-locus procedure, find a value for the gain K that will provide the maximum damping ratio.

Solution:

(a) The root locus is plotted below and lies entirely in the left-half plane. So the system is stable for all K.

(b) At maximum damping, the gain is $K = 6.3$ but the damping of the complex poles is only 0.07. A practical design would require much more lead. Note the drastic effect that the sensor lag had on the RL, although the original system had very poor damping to start with.



Loci for Problem 5.30

θ

31. For the system in Fig. 5.62:

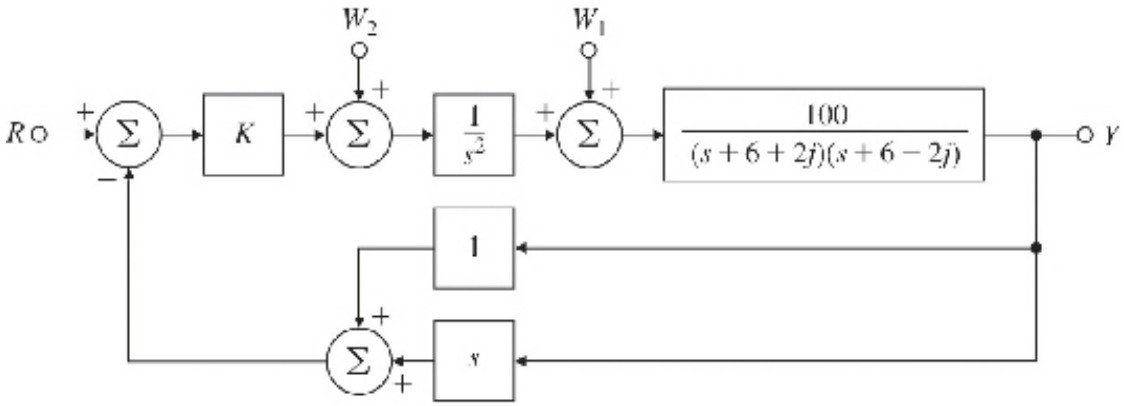


Fig. 5.62 Control system for Problem 5.31

(a) Find the locus of closed-loop roots with respect to K.

(b) Find the maximum value of K for which the system is stable. Assume $K = 2$ for the remaining parts of this problem.

(c) What is the steady-state error ($e = r - y$) for a step change in r?

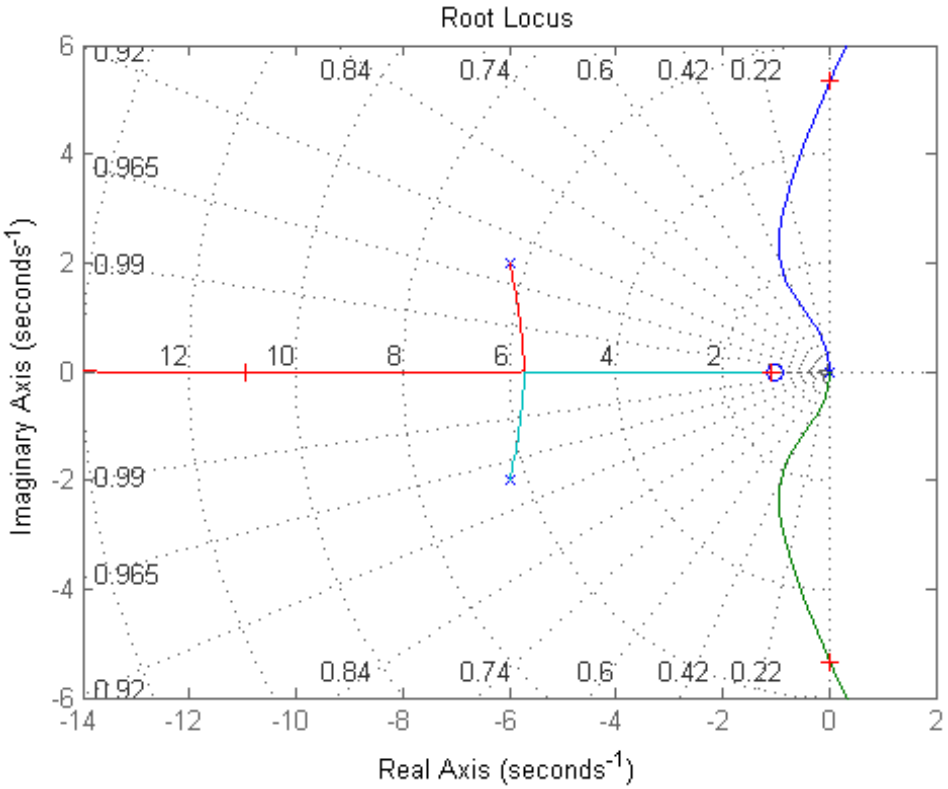
(d) What is the steady-state error in y for a constant disturbance w_1?

(e) What is the steady-state error in y for a constant disturbance w_2?

(f) If you wished to have more damping, what changes would you make to the system?

Solution:

(a) For the locus, $L(s) = \dfrac{100(s+1)}{s^2(s^2+12s+40)}$. The locus is plotted below.



Locus for Problem 5.31

(b) The maximum value of K for stability is $K = 3.43$.

(c) The transfer function from R to Y assuming $K = 2$ is

$$\frac{Y}{R} = \frac{200}{s^2(s^2 + 12s + 40) + 200(s + 1)}.$$

Therefore the steady-state error for a step change in r is

$$\begin{aligned} e_{step}(\infty) &= \lim_{s \to 0} s \left(1 - \frac{Y}{R}\right) \frac{1}{s} \\ &= \lim_{s \to 0} \frac{s^2(s^2 + 12s + 40) + 200s}{s^2(s^2 + 12s + 40) + 200(s + 1)} = 0 \end{aligned}$$

(d) The transfer function from W_1 to Y is:

$$\frac{Y}{W_1} = \frac{100s^2}{s^2(s^2 + 12s + 40) + 200(s + 1)}$$

Therefore the steady-state error for a constant disturbance w_1 is

$$e_{step}(\infty) = \lim_{s \to 0} s \left(-\frac{Y}{W_1} \right) \frac{1}{s} = 0$$

(e) The transfer function from W_2 to Y is:

$$\frac{Y}{W_2} = \frac{100}{s^2(s^2 + 12s + 40) + 200(s + 1)}$$

Therefore the steady-state error for a constant disturbance w_1 is

$$e_{step}(\infty) = \lim_{s \to 0} s \left(-\frac{Y}{W_2} \right) \frac{1}{s} = 0.5$$

(f) To get more damping in the closed-loop response, the controller needs to have a lead compensation.

32. Consider the plant transfer function

$$G(s) = \frac{bs + k}{s^2[mMs^2 + (M + m)bs + (M + m)k]}$$

to be put in the unity feedback loop of Fig. 5.59. This is the transfer function relating the input force $u(t)$ and the position $y(t)$ of mass M in the non-collocated sensor and actuator problem. In this problem, we will use root-locus techniques to design a controller $D_c(s)$ so that the closed-loop step response has a rise time of less than 0.1 sec and an overshoot of less than 10%. You may use Matlab for any of the following questions:

(a) Approximate $G(s)$ by assuming that $m \cong 0$, and let $M = 1$, $k = 1$, $b = 0.1$, and $D_c(s) = K$. Can K be chosen to satisfy the performance specifications? Why or why not?

(b) Repeat part (a) assuming $D_c(s) = K(s + z)$, and show that K and z can be chosen to meet the specifications.

(c) Repeat part (b) but with a practical controller given by the transfer function

$$D_c(s) = K\,\frac{p(s + z)}{s + p},$$

and pick p so that the values for K and z computed in part (b) remain more or less valid.

(d) Now suppose that the small mass m is not negligible, but is given by $m = M/10$. Check to see if the controller you designed in part (c) still meets the given specifications. If not, adjust the controller parameters so that the specifications are met.

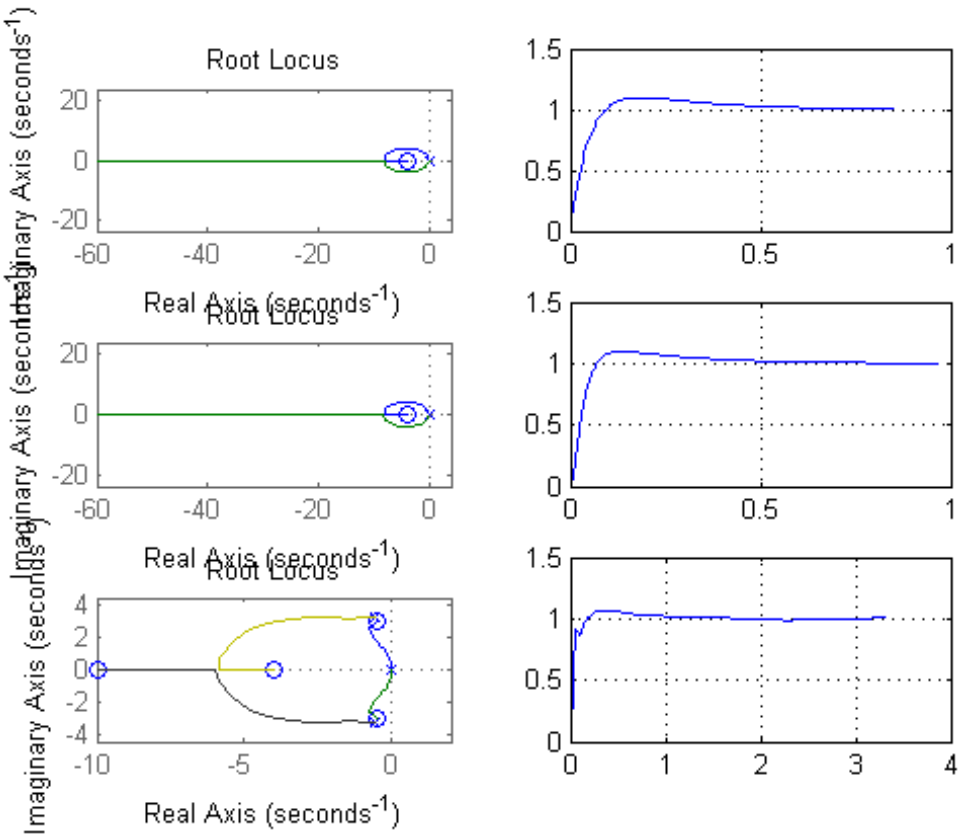
Solution:

(a) The approximate plant transfer function is $G(s) \cong \frac{1}{s^2}$. The locus in this case is the imaginary axis and cannot meet the specs for any K.

(b) The specs require that $\zeta > 0.6$, $\omega_n > 18$. Select $z = 15$ for a start. The locus will be a circle with radius 15. Because of the zero, the overshoot will be increased and Figure 3.29 indicates that we'd better make the damping greater than 0.7. As a matter of fact, experimentation shows that we can lower the overshoot of less than 10% only by setting the zero at a low value and putting the poles on the real axis. The plot shows the result when $D_c = 25(s + 4)$. The resulting overshoot is 9.9% and the rise time is 0.06 sec.

(c) In this case, we pick $p = 150$, $z = 4$, and $K = 30$. Then the resulting overshoot is 9.8% and the rise time is 0.05 sec.

(d) With the resonance present, the only chance we have is to introduce a notch as well as a lead. The compensation resulting in the plots shown is $D_c(s) = 12\dfrac{s + 4}{(.01s + 1)}\dfrac{s^2/9.25 + s/9.25 + 1}{s^2/3600 + s/30 + 1}$. The overshoot is 7% and the rise time is 0.04 sec .



Root loci and step responses for Problem 5.32

33. Consider the Type 1 system drawn in Fig. 5.63. We would like to design the compensation $D_c(s)$ to meet the following requirements: (1) The steady-state value of y due to a constant unit disturbance w should be less than $\frac{4}{5}$, and (2) the damping ratio $\zeta = 0.7$. Using root-locus techniques,

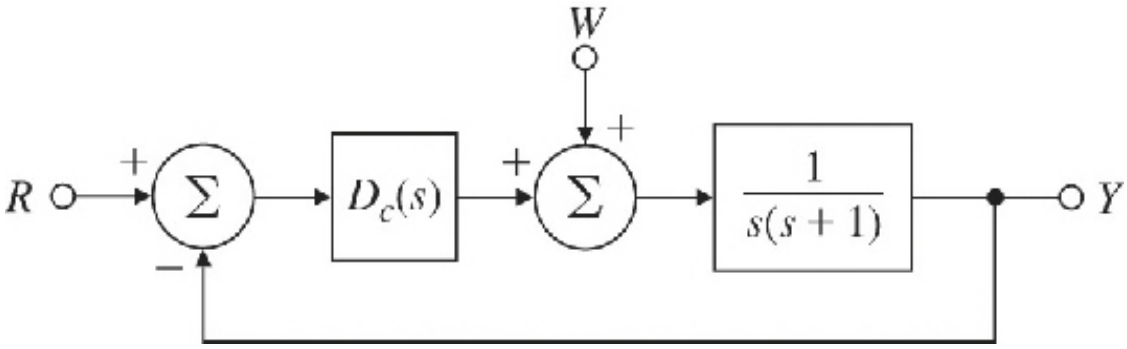


Fig. 5.63 Control system for Problem 5.33

(a) Show that proportional control alone is not adequate.

(b) Show that proportional-derivative control will work.

(c) Find values of the gains k_p and k_D for $D_c(s) = k_p + k_D\, s$ that meet the design specifications with at least a 10% margin.

Solution:

(a) To meet the error requirement, we need

$$y_{step}(\infty) = \lim_{s\to 0} s\frac{Y}{W}\frac{1}{s} = \lim_{s\to 0} \frac{1}{s^2 + s + K} \leq 0.8.$$

Thus K must be at least $K \geq 1.25$. With this K, the damping ratio will be

$$\zeta = \frac{1}{2\sqrt{K}} \quad \Rightarrow \quad \zeta \leq 0.45$$

So we can't meet both requirements with proportional control.

(b) With PD control,

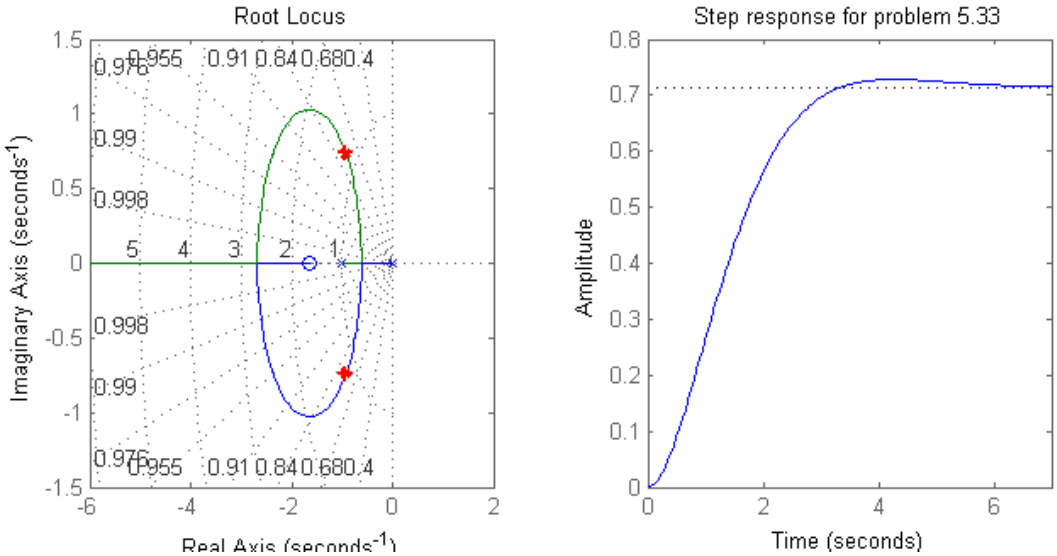
$$y_{step}(\infty) = \lim_{s\to 0} s\frac{Y}{W}\frac{1}{s} = \lim_{s\to 0} \frac{1}{s^2 + (1 + k_D)s + k_p} \leq 0.8.$$

So the error requirement can be met by setting $k_p \geq 1.25$. The damping ratio requirement can be written as

$$\zeta = \frac{1 + k_D}{2\sqrt{k_p}} \geq 0.7$$

By choosing $k_D \geq 1.4\sqrt{k_p} - 1$, we can satisfy both specifications.

(c) Setting $k_p = 1.4$ and $k_D = 0.85$, we get $y_{step}(\infty) = 0.714$ and $\zeta = 0.782$. The root loci and disturbance step response are plotted below.



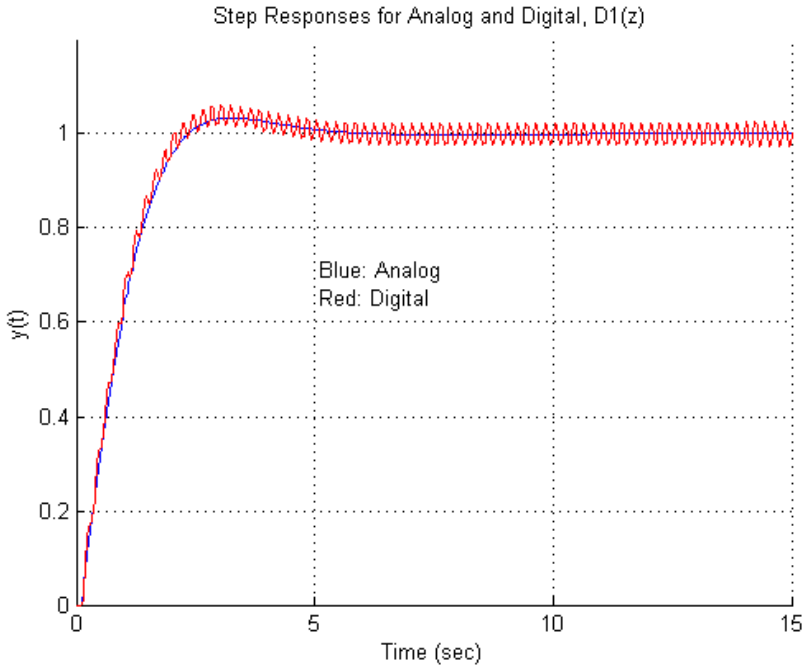
Solution for Problem 5.33

34. Using a sample rate of 10 Hz, find the $D_c(z)$ that is the discrete equivalent to your $D_c(s)$ from Problem 5.33 using the trapezoid rule. Evaluate the time reponse using Simulink, and determine whether the damping ratio requirement is met with the digital implementation. (*Note: The material to do this problem is covered in the Appendix W4.5 at www.FPE8e.com or in Chapter 8.)* (Note: The first printing of the 8th edition had an error in the problem statement. It said to use the $D_c(s)$ from Problem 5.7 rather than the correct one, Problem 5.33)

Solution:
From Problem 5.33, we have $D_c(s) = 0.85s + 1.4$. The discrete equivalent for $T_s = 0.1\,\text{sec}$ is given by substituting $s = \dfrac{2}{0.1} \cdot \dfrac{z-1}{z+1}$ in the $D_c(s)$:

$$D_d(z) = 0.85 \cdot \frac{2}{0.1} \cdot \frac{z-1}{z+1} + 1.4 = \frac{18.4z - 15.6}{z+1}.$$

To evaluate this discrete controller, we use Simulink to compare the two implementations. The results of the step responses are shown below.



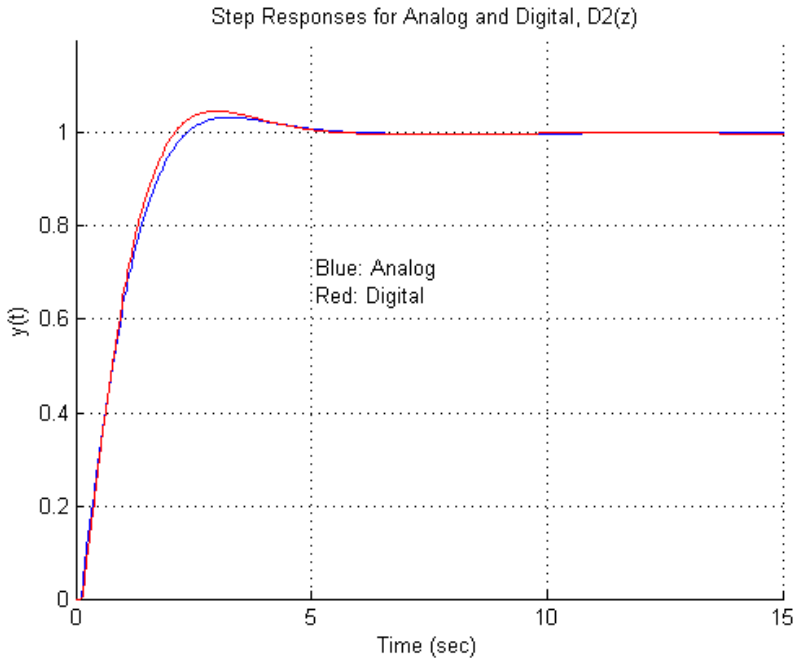
Solution for Problem 5.34 with the first $D(z)$

Note that there is slightly greater overshoot in the digital system, which suggests a decrease in the damping due to the digital implementation.

However, the damping ratio requirement of 0.7 is still met with the digital control. It is interesting to note, however, that there is an oscillation of the output at 5 Hz, which is half the sample rate. This arises because the use of the trapezoidal equivalent places a root of the compensation at z = -1, which has created an extraneous oscillatory root in the closed-loop system. .This is also a result of having a pure differentiation in the D(s), i.e., a zero in the numerater with no pole. Using the Matched Pole-Zero discrete equivalent will help this situation since it does not create the oscillatory pole in the compensator. Using Matlab's C2D function for this discrete equivalent produces

$$D_{d2}(z) = \frac{9.219z - 7.819}{z},$$

which eliminates the oscillation at 5 hz. The step responses with $D_2(z)$ are shown:



Solution for Problem 5.34 with the first second $D(z)$

Problems and solutions for Section 5.5

35. Consider the positioning servomechanism system shown in Fig. 5.64, where

$$
\begin{aligned}
e_i = K_o\theta_i, \quad e_o = K_o\theta_o, \quad K_o &= 10\text{V/rad}, \\
\text{(note: 1st printing of book had } e_o = K_{pot}\theta_o, &\text{ which is not correct)} \\
T &= \text{motor torque} = K_t i_a, \\
k_m = K_t &= \text{torque constant} = 0.1 \text{ N} \cdot \text{m/A}, \\
K_e &= \text{back emf constant} = 0.1\text{V} \cdot \text{sec} \\
R_a &= \text{armature resistance} = 10\Omega, \\
\text{Gear ratio} &= 1:1, \\
J_L + J_m &= \text{total inertia} = 10^{-3} \text{ kg} \cdot \text{m}^2, \\
v_a &= K_A(e_i - e_f).
\end{aligned}
$$

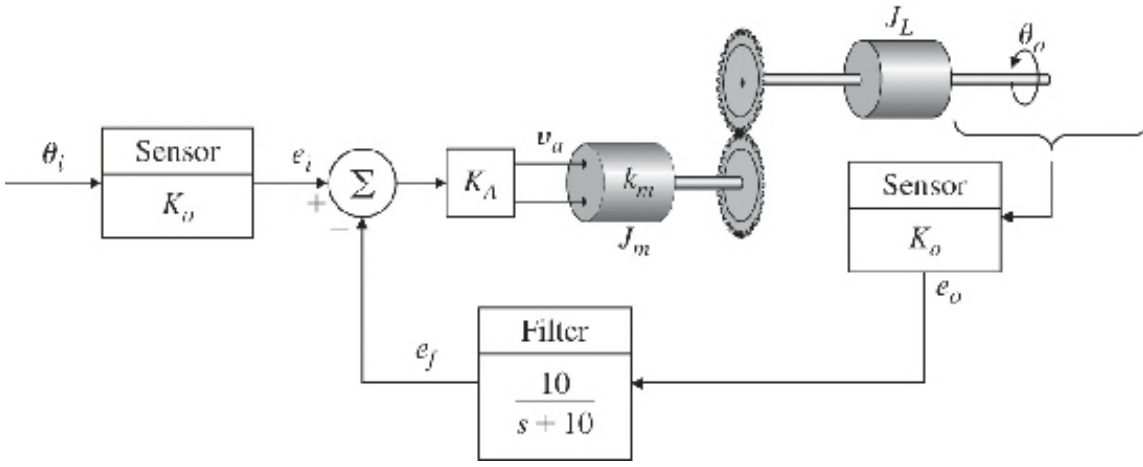


Fig. 5.64 Positioning servomechanism

(a) What is the range of the amplifier gain K_A for which the system is stable? Estimate the upper limit graphically using a root-locus plot.

(b) Choose a gain K_A that gives roots at $\zeta = 0.7$. Where are all three closed-loop root locations for this value of K_A?

Solution:

(a) Neglecting viscous friction and the effect of inductance, we see from Eq. (2.66) on page 58 and the given parameter values, that the

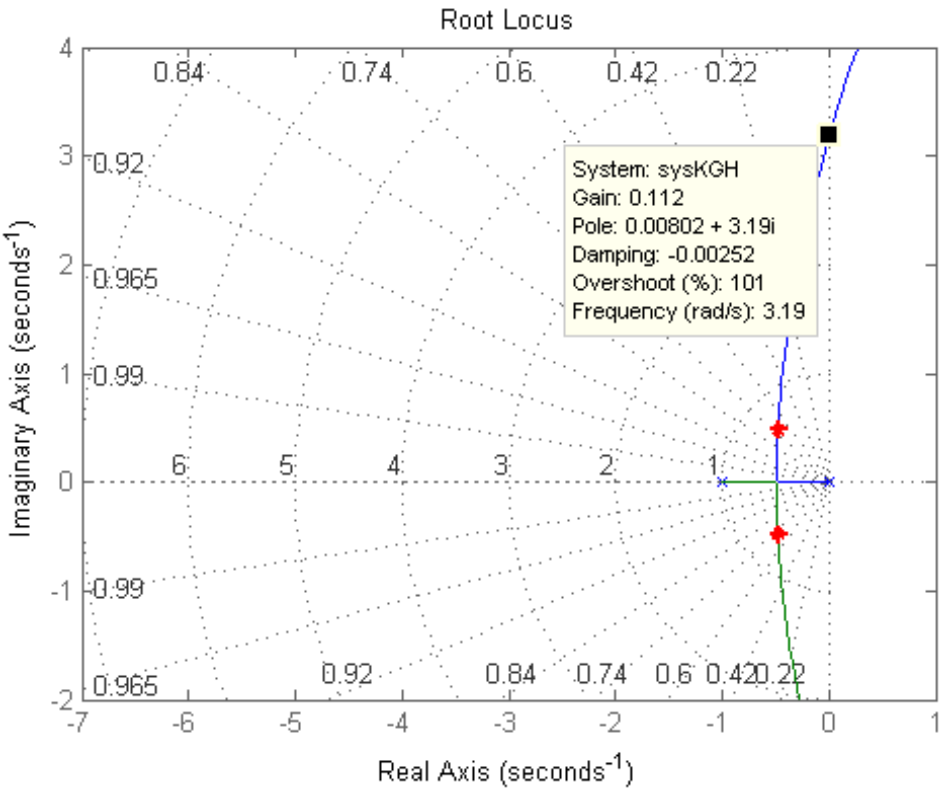


Figure 5.45: Root locus for Problem 5.35

transfer function of the DC motor is

$$\frac{\Theta_o}{V_a} = \frac{\frac{K_t}{R_a}}{(J_L + J_m)s^2 + \frac{K_t K_e}{R_a}s} = \frac{10}{s^2 + s}$$

From the root locus plotted below, the upper limit of K_A for stability is 0.11.

(b) The damping is 0.7 when $K = 0.0046$. For that value of K_A, poles are at $s = -10.05, -0.475 \pm 0.482j$, as shown on the root locus above at the red marks.

36. We wish to design a velocity control for a tape-drive servomechanism. The transfer function from current $I(s)$ to tape velocity $\Omega(s)$ (in millimeters per millisecond per ampere) is

$$\frac{\Omega(s)}{I(s)} = \frac{15(s^2 + 0.9s + 0.8)}{(s+1)(s^2 + 1.1s + 1)}.$$

We wish to design a Type 1 feedback system so that the response to a reference step satisfies

$$t_r \leq 4\text{msec}, \quad t_s \leq 15\text{msec}, \quad M_p \leq 0.05$$

(a) Use the integral compensator k_I/s to achieve Type 1 behavior, and sketch the root-locus with respect to k_I. Show on the same plot the region of acceptable pole locations corresponding to the specifications.

(b) Assume a proportional-integral compensator of the form $k_p(s+\alpha)/s$, and select the best possible values of k_p and α you can find. Sketch the root-locus plot of your design, giving values for k_p and α, and the velocity constant K_v your design achieves. On your plot, indicate the closed-loop poles with a dot ($\bullet$) and include the boundary of the region of acceptable root locations.

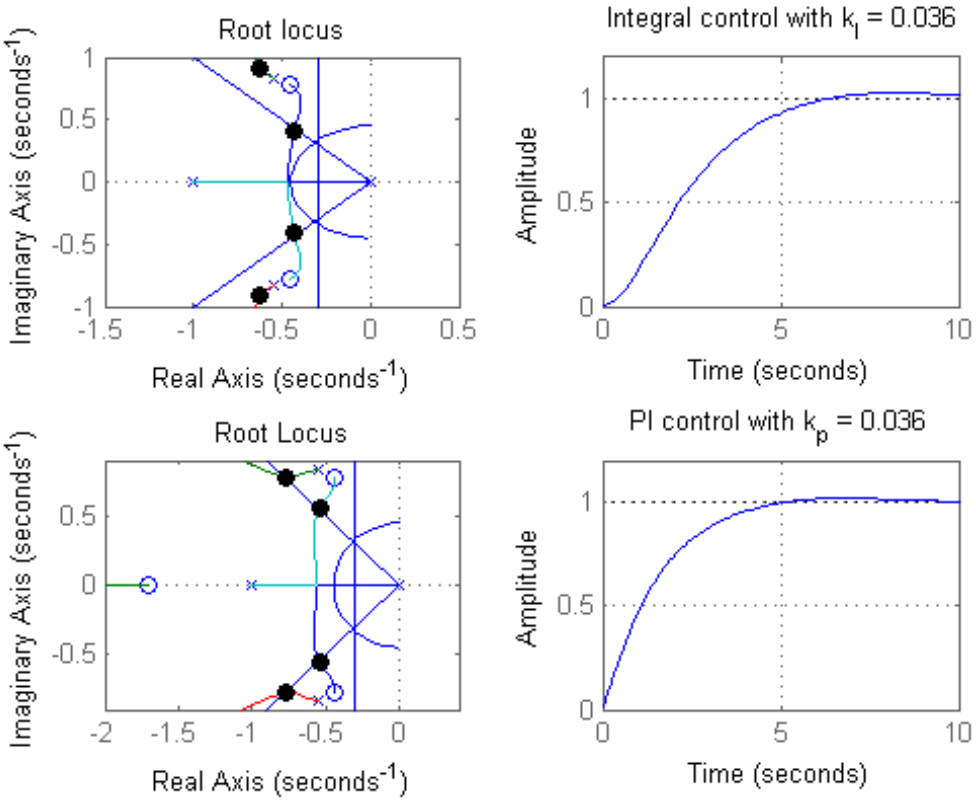
Solution:

(a) The root locus with respect to k_I and the step response with $k_I = 0.036$ are plotted in the first row below.

(b) Using *rltool*, we can choose the location of the zero to pull the locus to the left-hand plane. This will improve the transient response. Here, the zero was put at $s = -1.7$, and k_p was set to 0.036. The root locus and the step response are plotted in the second row below.

With the PI compensator, the closed-loop poles are at $s = -0.7749 \pm 0.7774j$, $-0.5451 \pm 0.5589j$, and the closed-loop zeros are at $s = -1.7$, $-0.45 \pm 0.773j$. Thus the velocity constant can be calculated from Truxal's formula.

$$\frac{1}{K_v} = \sum_{i=1}^{n} -\frac{1}{p_i} + \sum_{i=1}^{m} \frac{1}{z_i} \qquad \Rightarrow \qquad K_v = 0.7344$$



Solution for Problem 5.36

37. The normalized, scaled equations of a cart as drawn in Fig. 5.65 of mass m_c holding an inverted uniform pendulum of mass m_p and length ℓ with no friction are (note that the 2nd θ term in the 2nd equation should be $\ddot{\theta}$) Also note that the sign of this term is reversed compared to Eq (2.30) because of the reversed definition of θ in Figures 2.22 vs Fig. 5.65.

$$\begin{aligned} \ddot{\theta} - \theta &= -v \\ \ddot{y} + \beta\ddot{\theta} &= v \end{aligned} \tag{5.88}$$

where $\beta = \frac{3m_p}{4(m_c+m_p)}$ is a mass ratio bounded by $0 < \beta < 0.75$. Time is measured in terms of $\tau = \omega_o t$ where $\omega_o^2 = \frac{3g(m_c+m_p)}{\ell(4m_c+m_p)}$. The cart motion, y, is measured in units of pendulum length as $y = \frac{3x}{4\ell}$ and the input is force normalized by the system weight, $v = \frac{u}{g(m_c+m_p)}$. These equations can be used to compute the transfer functions

$$\frac{\Theta}{V} = -\frac{1}{s^2-1} \tag{5.89}$$

$$\frac{Y}{V} = \frac{s^2-1+\beta}{s^2(s^2-1)} \tag{5.90}$$

In this problem you are to design a control for the system by first closing a loop around the pendulum, Eq.(5.89) and then, with this loop closed, closing a second loop around the cart plus pendulum, Eq.(5.90). For this problem, let the mass ratio be $m_c = 5m_p$.

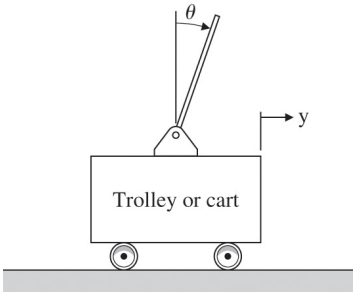


Fig. 5.65 Figure of cart-pendulum for Problem 5.37

(a) Draw a block diagram for the system with V input and both Y and Θ as outputs.

(b) Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ for the Θ loop to cancel the pole at $s = -1$ and place the two remaining poles at $-4 \pm j4$. The new control is $U(s)$, where the force is $V(s) = U(s) + D_c(s)\Theta(s)$. Draw the root locus of the angle loop.

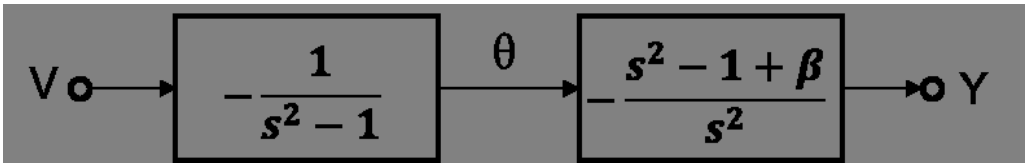
(c) Compute the transfer function of the new plant from U to Y with $D_c(s)$ in place.

(d) Design a controller $D_c(s)$ for the cart position with the pendulum loop closed. Draw the root locus with respect to the gain of $D_c(s)$

(e) Use Matlab to plot the control, cart position, and pendulum position for a unit step change in cart position.

Solution:

(a)



Block diagram for Problem 5.37

(b) To cancel the pole at $s = -1$, we set $z = 1$. Then the closed loop transfer function for the Θ loop becomes

$$\frac{\Theta}{V} = \frac{K}{(s-1)(s+p)+K} = \frac{K}{s^2+(p-1)s+K-p}.$$

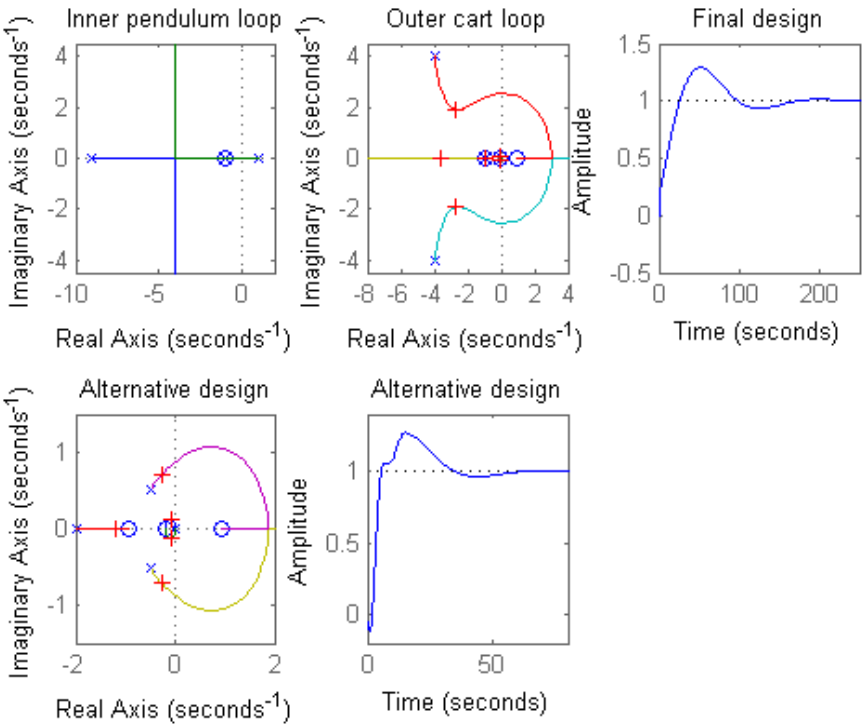
Setting $p = 9$ and $K = 41$, the remaining poles can be placed at $s = -4 \pm 4j$. Therefore $D_c(s) = 41\dfrac{s+1}{s+9}$ The root locus is shown below.

(c) Since $m_c = 5m_p$, $\beta = 0.125$. Therefore the transfer function from U to Y with $D_c(s)$ is

$$\frac{Y}{U} = \frac{-41}{s^2+8s+32}\frac{s^2-0.875}{s^2}$$

(d) $D_c = k_c\dfrac{s^2+0.2s+0.01}{s^2+2s+1}$. The root locus is shown below.

(e) The step responses are shown below. The pendulum position control is rather fast for this problem. A more reasonable alternative choice would be to place the pendulum roots at $s = -0.5 \pm j0.5$.



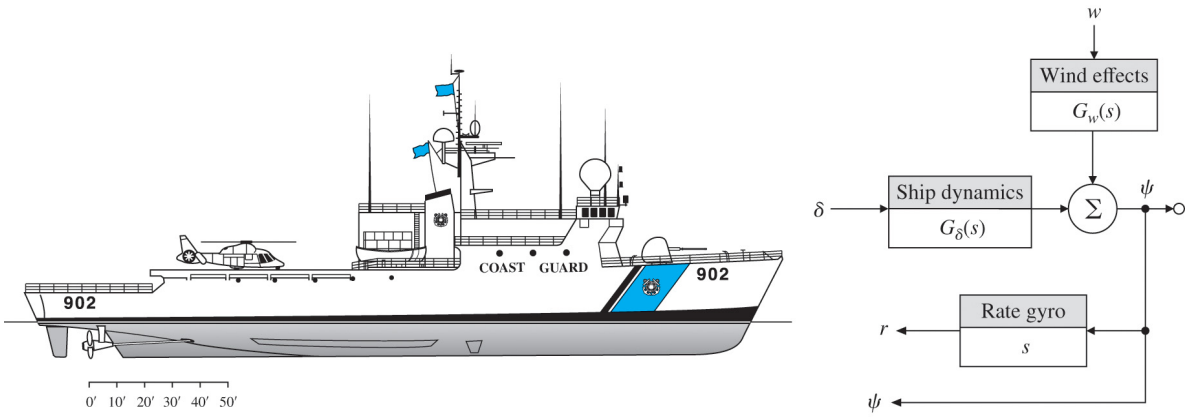
Root loci and step responses for Problem 5.37

38. Consider the 270-ft U.S. Coast Guard cutter *Tampa* (902) shown in Fig. 5.66(a). Parameter identification based on sea-trials data (Trankle, 1987) was used to estimate the hydrodynamic coefficients in the equations of motion. The result is that the response of the heading angle of the ship ψ to rudder angle δ and wind changes w can be described by the block diagram in Fig. 5.66(b) and the second-order transfer functions

$$G_\delta(s) = \frac{\psi(s)}{\delta(s)} = \frac{-0.0184(s+0.0068)}{s(s+0.2647)(s+0.0063)},$$
$$G_w(s) = \frac{\psi(s)}{w(s)} = \frac{0.0000064}{s(s+0.2647)(s+0.0063)},$$

where

$$\begin{aligned}
\psi &= \text{ heading angle, rad} \\
\psi_r &= \text{reference heading angle, rad.} \\
r &= \text{yaw rate, } \dot{\psi}\text{, rad/sec,} \\
\delta &= \text{rudder angle, rad,} \\
w &= \text{wind speed, m/sec.}
\end{aligned}$$



(a) (b)

Fig. 5.66 (a) USCG Tampa for Problem 5.38, (b) partial block diagram for the system

(a) Determine the open-loop settling time of r for a step change in δ.

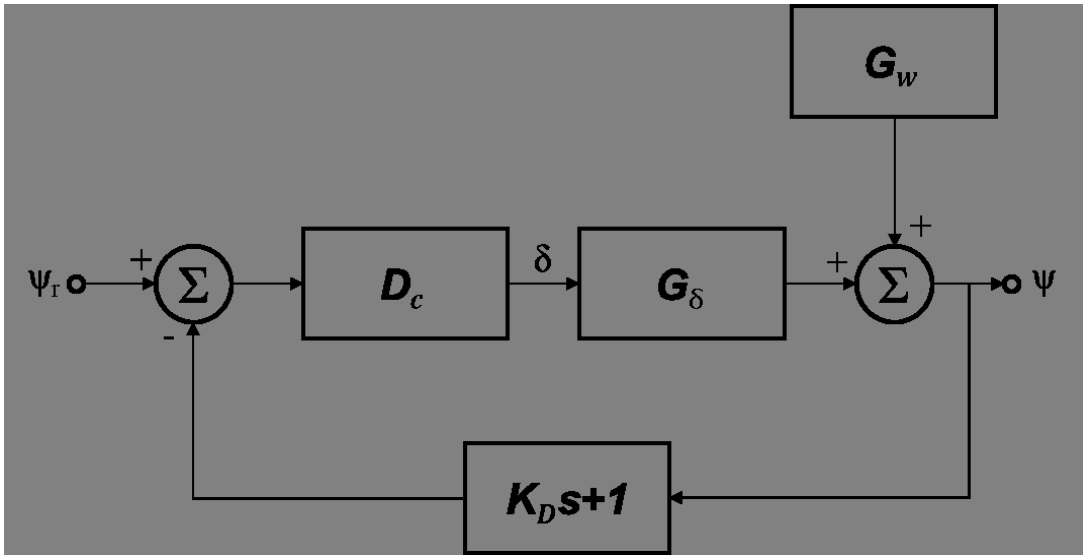
(b) In order to regulate the heading angle ψ , design a compensator that uses ψ and the measurement provided by a yaw-rate gyroscope (that is, by $\dot{\psi} = r$). The settling time of ψ to a step change in ψ_r is specified to be less than 50 *sec*, and, for a 5° change in heading the maximum allowable rudder angle deflection is specified to be less than 10°.

(c) Check the response of the closed-loop system you designed in part (b) to a wind gust disturbance of 10 $m/\sec$ (Model the disturbance as a step input.) If the *steady-state* value of the heading due to this wind gust is more than 0.5°, modify your design so that it meets this specification as well.

Solution:

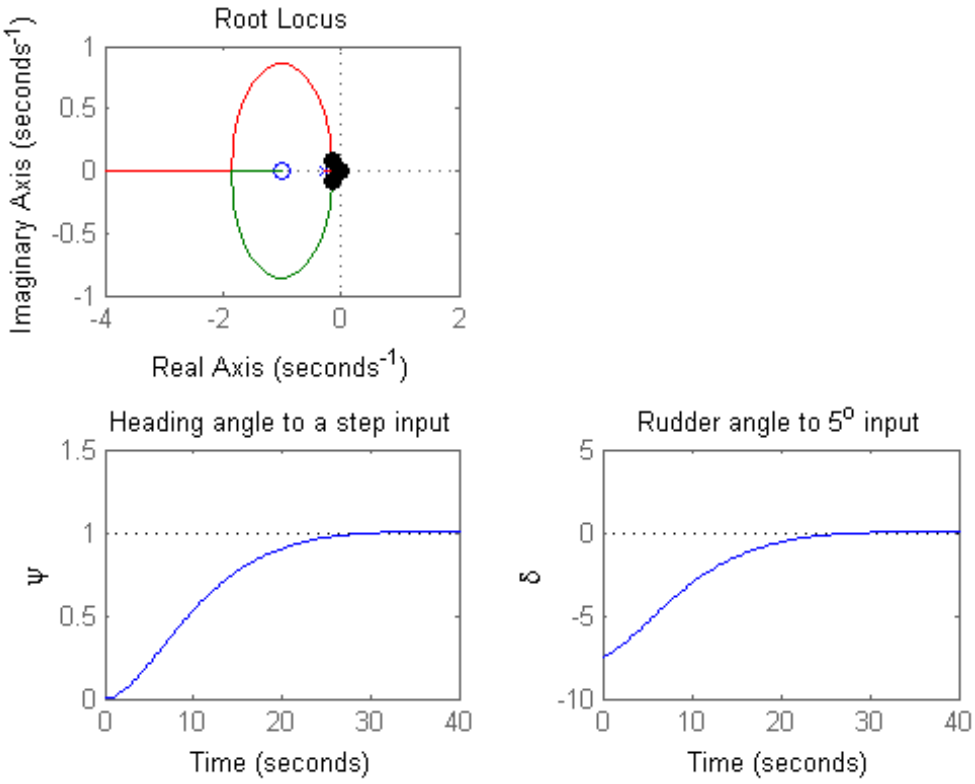
(a) To determine the open-loop settling time to 1% of the final value, we can use the stepinfo command in Matlab. The value from Matlab is $t_s = 314.64\,\text{sec}$.

(b) The rate feedback from a yaw-rate gyroscope is giving us a derivative control for free. Thus the block diagram of the system will look like shown below.



Block diagram for Problem 5.38

Let's set $K_D = 1$ and see what we can do with a simple proportional controller, i.e. $D_c(s) = K_p$. (Since the plant has a negative sign, the controller gain will also be negative.) With the proportional control, the maximum deflection of the rudder is almost surely at the initial instant, when it is $\delta(0) = K_p\psi_r(0)$. Thus to keep δ below 10° for a step of 5°, we need $|K_p| < 2$. And for a settling time less than 50 sec, we need $\sigma > 4.6/50 = 0.092$ from the design relations for the standard second order system. Using root-locus technique, $K_p = -1.5$ was picked. Checking the step response with this proportional control using Matlab, we find $t_s = 28.2\,\text{sec}$, which meets the settling time requirement. And the rudder angle deflection for a 5° input is less than -7.5°. Therefore $D_c(s) = -1.5$ is adequate for the problem.



Root locus and Step response for Problem 5.38b

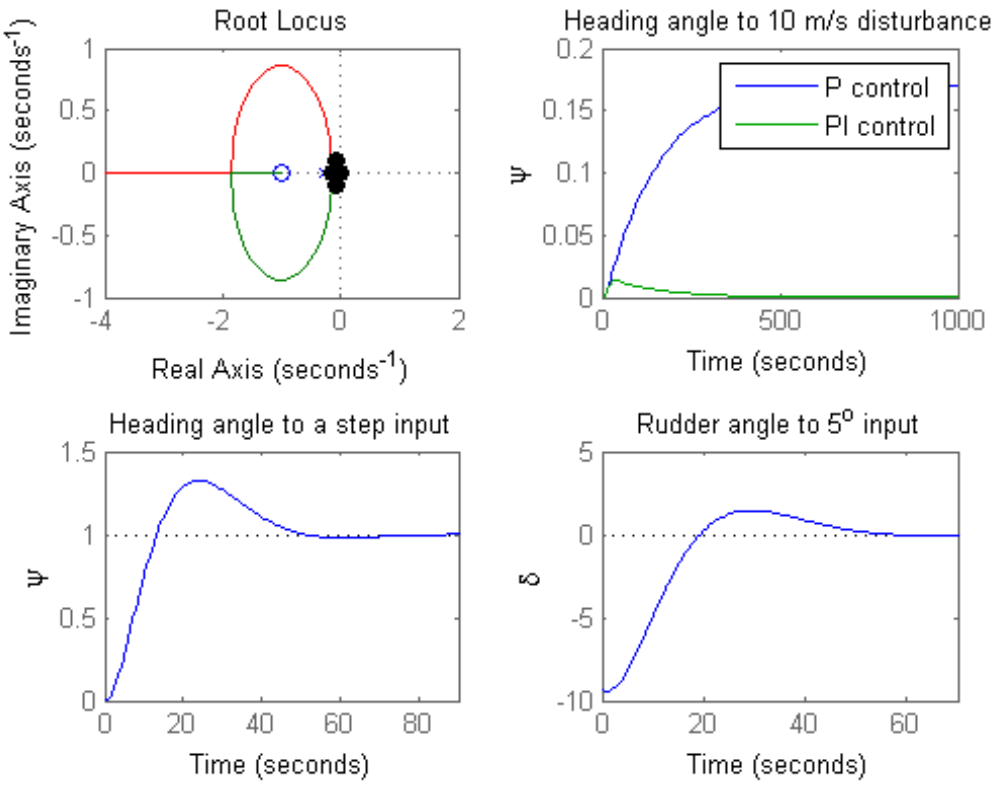
(c) With the compensator from part (b), the closed-loop transfer function from w to ψ is

$$\frac{\psi(s)}{w(s)} = \frac{G_w}{1 + K_p G_\delta (1 + K_D s)}$$
$$= \frac{0.0000064}{s(s + 0.2647)(s + 0.0063) + 0.0278(s + 0.0068)(1 + s)}$$

Using the Final Value Theorem, the steady-state value of the heading angle due to a disturbance of $10m/\sec$ is

$$\psi(\infty) = \lim_{s \to 0} s \frac{\psi(s)}{w(s)} \frac{10}{s} = 0.3386 \text{ rad} = 19.4^\circ > 0.5^\circ$$

So we need to modify our design. To reject the disturbance completely, let's add an integral term to the controller, $D_c(s) = K_p + K_I/s$. Using rltool, we find that when $D_c(s) = -1.87 - 0.11/s$, all specifications are met as shown below.



Root locus and Step response for Problem 5.38c

39. Golden Nugget Airlines has opened a free bar in the tail of their airplanes in an attempt to lure customers. In order to automatically adjust for the sudden weight shift due to passengers rushing to the bar when it first opens, the airline is mechanizing a pitch-attitude auto pilot. Figure 5.67 shows the block diagram of the proposed arrangement. We will model the passenger moment as a step disturbance $M_p(s) = M_0/s$, with a maximum expected value for M_0 of 0.6.

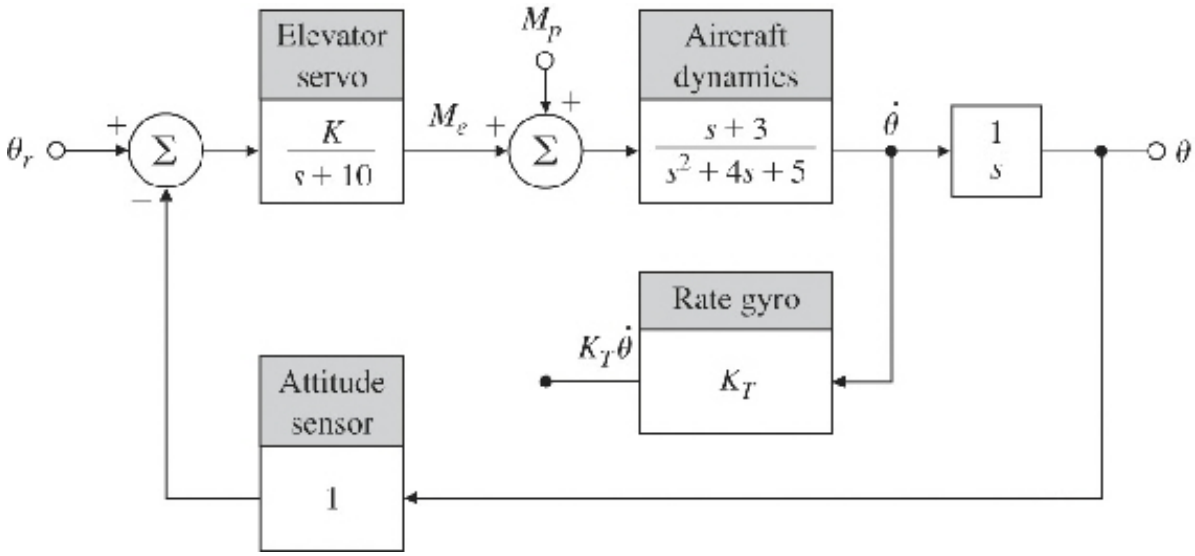


Fig. 5.67 Golden Nugget Airlines Autopilot

(a) Assuming the bar has opened, and the passengers have rushed to it, what value of K is required to keep the steady-state error in θ to less than 0.02 rad($\cong 1°$)? (Assume the system is stable.)

(b) Draw a root locus with respect to K.

(c) Based on your root locus, what is the value of K when the system becomes unstable?

(d) Suppose the value of K required for acceptable steady-state behavior is 600. Show that this value yields an unstable system with roots at

$$s = -2.9, -13.5, +1.2 \pm 6.6j.$$

(e) You are given a black box with *rate gyro* written on the side and told that when installed, it provides a perfect measure of $\dot{\theta}$, with output $K_T\dot{\theta}$. Assume $K = 600$ as in part (d) and draw a block diagram indicating how you would incorporate the rate gyro into the auto pilot. (Include transfer functions in boxes.)

(f) For the rate gyro in part (e), sketch a root locus with respect to K_T.

(g) What is the maximum damping factor of the complex roots obtainable with the configuration in part (e)?

(h) What is the value of K_T for part (g)?

(i) Suppose you are not satisfied with the steady-state errors and damping ratio of the system with a rate gyro in parts (e) through (h). Discuss the advantages and disadvantages of adding an integral term and extra lead networks in the control law. Support your comments using Matlab or with rough root-locus sketches.

Solution:

(a) Since any error is due to the disturbance M_p, we define the transfer function from M_p to θ:

$$\frac{\theta(s)}{M_p(s)} = \frac{(s+3)(s+10)}{s(s+10)(s^2+4s+5)+K(s+3)}$$

Using the Final Value Theorem, the steady-state error is

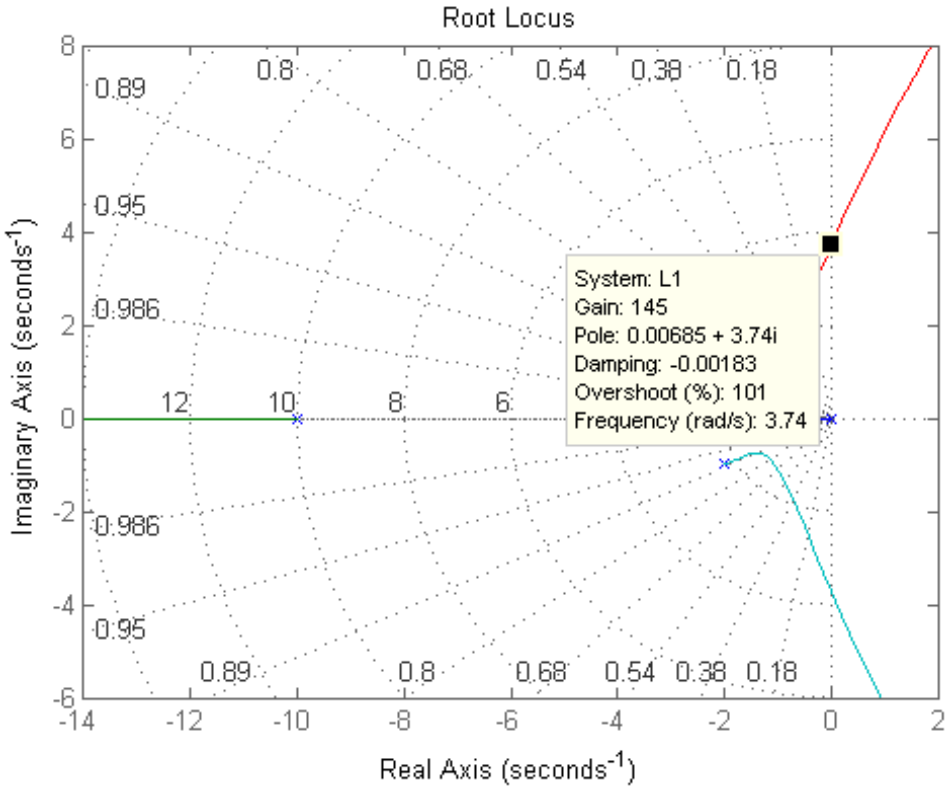
$$e(\infty) = \lim_{s\to 0} s\frac{\theta(s)}{M_p(s)}\frac{M_o}{s} = \frac{10M_o}{K} \leq \frac{(10)(0.6)}{K} < 0.02$$

Therefore we need $K > 300$.

(b) The characteristic equation of the system in Evans form is

$$1 + K\frac{(s+3)}{s(s+10)(s^2+4s+5)} = 0$$

The root locus is plotted below.



Problem 5.39 RL showing gain at stability boundary. (K=145)

(c) To find the stability boundary, we can click on the imaginary axis on the RL plot for part (b), and doing that yields $K = 145$.
We could also do the Routh test or solve the characteristic equation for the $j\omega$ crossings. Here, the latter method is used. The characteristic equation of the system is

$$\begin{aligned}\Delta(s) &= 1 + K\frac{(s+3)}{s(s+10)(s^2+4s+5)} \\ &= s^4 + 14s^3 + 45s^2 + (50+K)s + 3K = 0\end{aligned}$$

Plugging $s = j\omega$, we can write

$$\Delta(j\omega) = \left(\omega^4 - 45\omega^2 + 3K\right) + j\left(-14\omega^3 + (50+K)\omega\right) = 0$$

From the imaginary part, we have $\omega = 0, \pm\sqrt{\frac{50+K}{14}}$. Substituting these into the real part, we see that the roots are on the imaginary axis when $K = 143.7$. So the system goes unstable if $K > 143.7$. This agrees with the root locus, but is a lot more work!.

(d) When $K = 600$, the characteristic equation is

$$s^4 + 14s^3 + 45s^2 + 650s + 1800 = 0$$

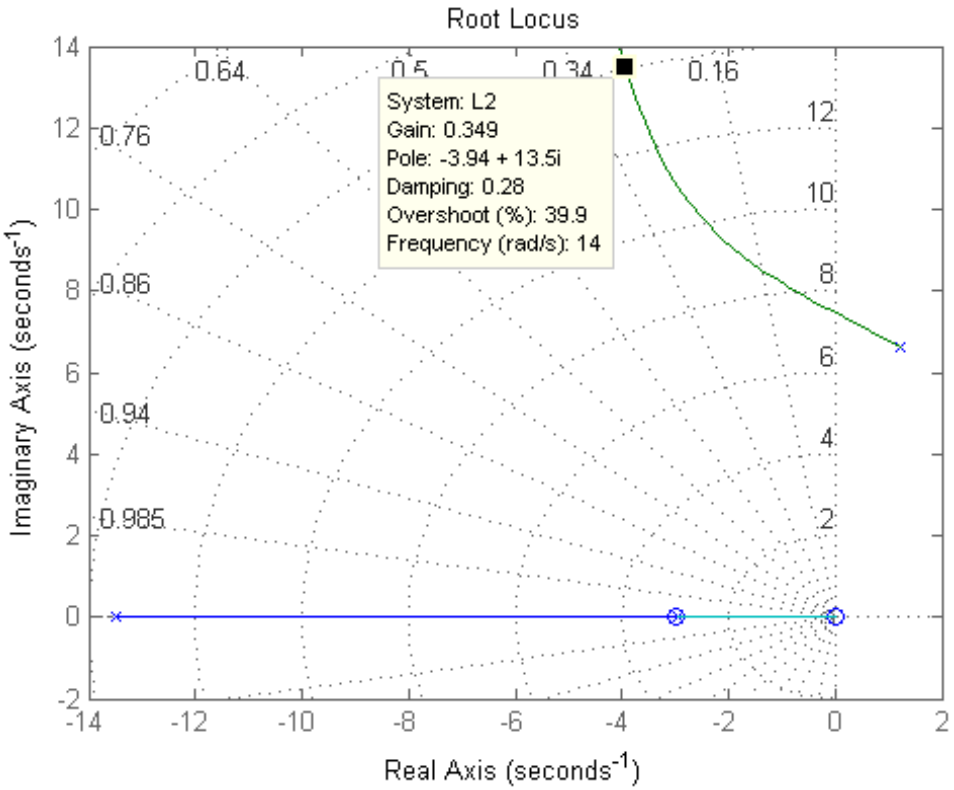
The roots of the equation are $s = -13.5, \ -2.94, \ +1.22 \pm 6.63j$.

(e) The output of the rate gyro box would be added at the same spot as the attitude sensor output.

(f) With the rate feedback, the characteristic equation in Evans form is

$$1 + K_T \frac{600s\,(s+3)}{s\,(s+10)\,(s^2+4s+5) + 600\,(s+3)} = 0$$

The root locus is shown below.



Root locus for Problem 5.39 showing gain selection for Kt given that K=600, (best damping is for Kt=0.35)

(g) From the root locus in Matlab, we can draw a point around and find that the maximum damping ratio occurs at $s = -3.94 \pm 13.5j$ with a max damping of $\zeta = 0.28$.

(h) From the root locus in Matlab, we can immediately get the gain value at the maximum damping; it is $K_T = 0.35$.

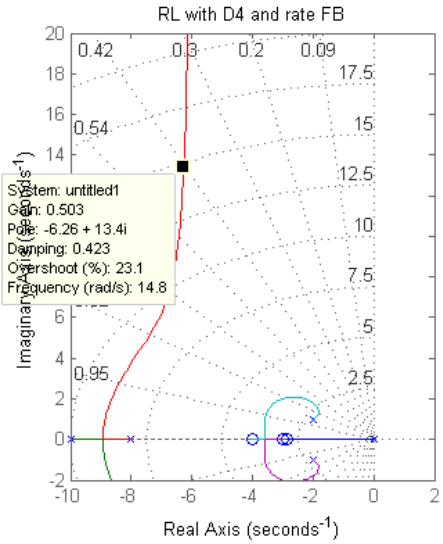
(i) Integral (PI) control would reduce the steady-state error to the moment to zero but would make the damping less and the settling time longer. But the attitude error due to a disturbing torque, Mp, is miniscule with K=600, as we can see from the step response We do see the effect of the low damping, however; ie the wiggly response of the step in Mp. So let's try a mild lead,

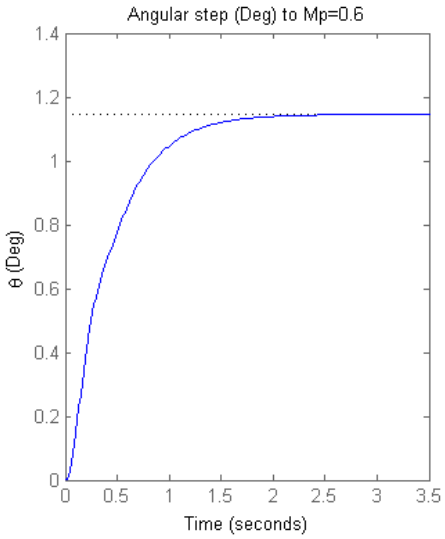
$$D_3(s) = 2\,\frac{(s+4)}{(s+8)},$$

in serieswith the rest of the feedback. So the entire feedback path will then be:

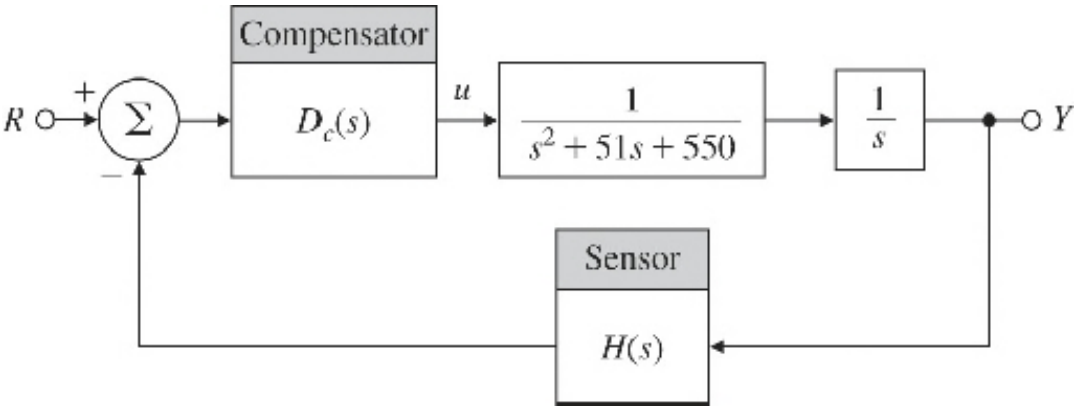
$$D_4(s) = K_4 * 2\,\frac{(s+4)}{(s+8)} * (s * Kt + 1) * \frac{K}{s+10},$$

As seen in the RL below, if $K_4 = 1$, there is very little increase in damping. However, we can achieve better damping by lowering the gain. A good compromise seems to be around $K_4 = 0.5$, which will increase ss errors, but will improve the system damping to approx $\zeta = 0.42$ from the previous $\zeta = 0.28$.. This is borne out by the step response. This compensation will almost meet the original goal in part (a) to keep the ss pitch errors to about 1^o when all the passengers rushed to the rear to load up on booze. Furthermore, the step response has smoothed out considerably with the additon of a little lead. If the pitch error was unacceptable, one could add some integral control, but it doesn't seem worth the trouble and degradation of stability to do that.


RL with D4 and rate FB
System: untitled1
Gain: 0.503
Pole: -6.26 + 13.4i
Damping: 0.423
Overshoot (%): 23.1
Frequency (rad/s): 14.8
0.42
0.3
0.2
0.09
0.54
0.95
17.5
15
12.5
10
7.5
5
2.5
Imaginary Axis (seconds-1)
Real Axis (seconds-1)


Angular step (Deg) to Mp=0.6
θ (Deg)
Time (seconds)

40. Consider the instrument servomechanism with the parameters given in Fig.5.68. For each of the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your final design.



Control system for Problem 5.40

(a) *Lead network* : Let

$$H(s) = 1, \quad D_c(s) = K\frac{s+z}{s+p}, \quad \frac{p}{z} = 6.$$

Select z and K so that the roots nearest the origin (the dominant roots) yield

$$\zeta \geq 0.4, \quad -\sigma \leq -7, \quad K_v \geq 16\frac{2}{3}\text{sec}^{-1}.$$

(b) *Output-velocity (tachometer) feedback*: Let

$$H(s) = 1 + K_T s \quad \text{and} \quad D_c(s) = K.$$

Select K_T and K so that the dominant roots are in the same location as those of part (a). Compute K_v. If you can, give a physical reason explaining the reduction in K_v when output derivative feedback is used.

(c) *Lag network* : Let

$$H(s) = 1 \quad \text{and} \quad D(s) = K\frac{s+1}{s+p}.$$

Using proportional control, is it possible to obtain a $K_v = 12$ at $\zeta = 0.4$? Select K and p so that the dominant roots correspond to the proportional-control case but with $K_v = 100$ rather than $K_v = 12$.

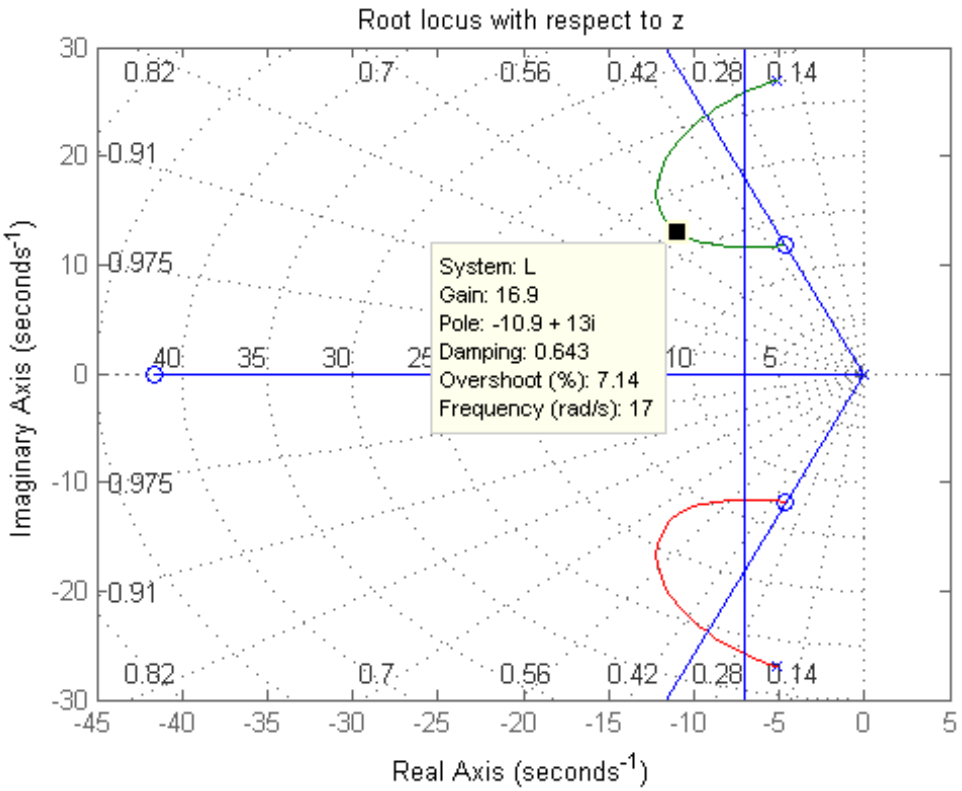
Solution:

(a) Setting $p = 6z$, the velocity constant is

$$K_v = \lim_{s\to 0} s\frac{K(s+z)}{s+6z}\frac{1}{s(s^2+51s+550)} = \frac{K}{3300}$$

Thus the K_v requirement leads to $K \geq 35200$. With $K = 40000$, a root locus can be drawn with respect to z.

$$1 + z\frac{6s(s^2+51s+550)+40000}{s^2(s^2+51s+550)+40000s} = 0$$



Root locus for Problem 5.40(a)

At the point of maximum damping, the values are $z = 16.8$ and the dominant roots are at $s = -11 \pm 13j$. So the compensator is $D_c(s) = 40000\dfrac{s+16.8}{s+100.8}$.

(b) With $H(s) = 1 + K_T s$ and $D_c(s) = K$, the closed-loop transfer function is

$$\frac{Y}{R} = \frac{K}{s^3 + 51s^2 + (550 + KK_T)s + K}$$

For this system to have poles at $s = -11 \pm 13j.$, the characteristic polynomial should be in the form of

$$(s + p)(s^2 + 22s + 290) = s^3 + (p + 22)s^2 + (22p + 290)s + 290p$$

Equating the coefficients leads to $p = 29$, $K = 8410$, and $K_T = 0.045$. With these value, the velocity constant is

$$\frac{1}{K_v} = \lim_{s\to 0} s\left(1 - \frac{Y}{R}\right)\frac{1}{s^2} = \frac{550 + KK_T}{K} \quad \Rightarrow \quad K_v = 9.058$$

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.

(c) Using proportional control ($D_c(s) = K$), the velocity constant is

$$K_v = \lim_{s\to 0} sK\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550}$$

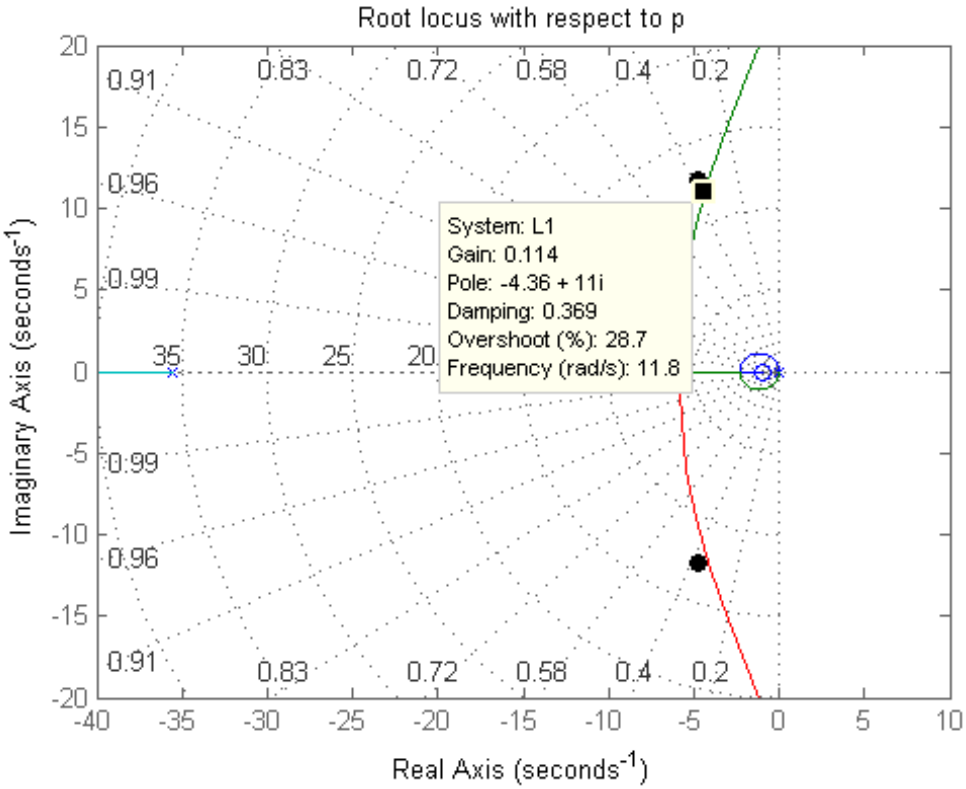
Therefore $K_v = 12$ can be obtained by setting $K = 6600$. With this value, the dominant roots are at $s = -4.7 \pm 11.69j$, and $\zeta = 0.37$.

With $D_c(s) = K\frac{s+1}{s+p}$, the velocity constant is

$$K_v = \lim_{s\to 0} s\frac{K(s + 1)}{s + p}\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550p}$$

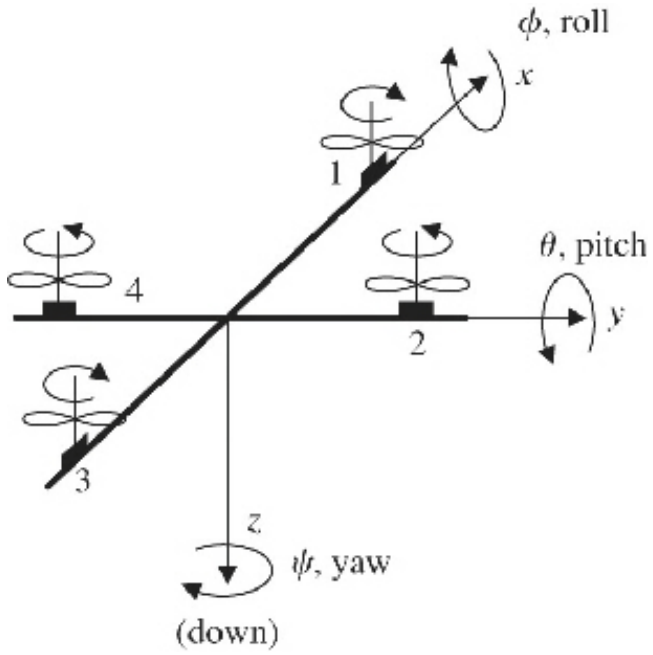
So $K_v = 100$ can be obtained by setting $\frac{K}{p} = 55000$. Setting $K = 55000p$, a root locus can be drawn with the parameter p

$$1 + p\frac{s(s^2 + 51s + 550) + 55000(s + 1)}{s^2(s^2 + 51s + 550)} = 0$$



Root locus for Problem 5.40(c)

In the plot, the desired pole locations are marked with a dot (•). Thus we can choose $p = 0.11$ to place the poles near the desired locations. Thus the compensator is $D_c(s) = 6050\dfrac{s+1}{s+0.11}$.



41. For the quadrotor shown in Figs. 2.13 and 2.14 (Example 2.5),

    (a) Describe what the commands should be to rotors 1, 2, 3, & 4 in order to produce a yaw torque, T_ψ, that has no effect on pitch or roll, and will not produce any net vertical thrust of the 4 rotors. In other words, find the relation between δT_1, δT_2, δT_3, δT_4 so that T_ψ produces the desired response.

    (b) The system dynamics for the yaw motion of a quadrotor are given in Eq. (2.17). Assuming the value of $I_z = 200$, find a compensation that gives a rise time less 0.2 seconds with an overshoot less than 20%.

Solution:

(a) The solution is described in Example 2.5, and the desired result is that all the rotors need to be torqued in the same direction, ie apply torque to the rotors in the CW direction (a positive torque) so that rotors 1 and 3 will increase their speed, while rotors 2 and 4 will decrease their speed. Thus there will be no net increase in lift, no pitching moment, nor a rolling moment. However, the positive torque applied to all rotors will exert a negative reaction torque back on the quadrotor vehicle, thus producing a negative yaw angle rotation.

So the result is that

$$\delta T_1 = \delta T_2 = \delta T_3 = \delta T_4 = -T_\psi$$

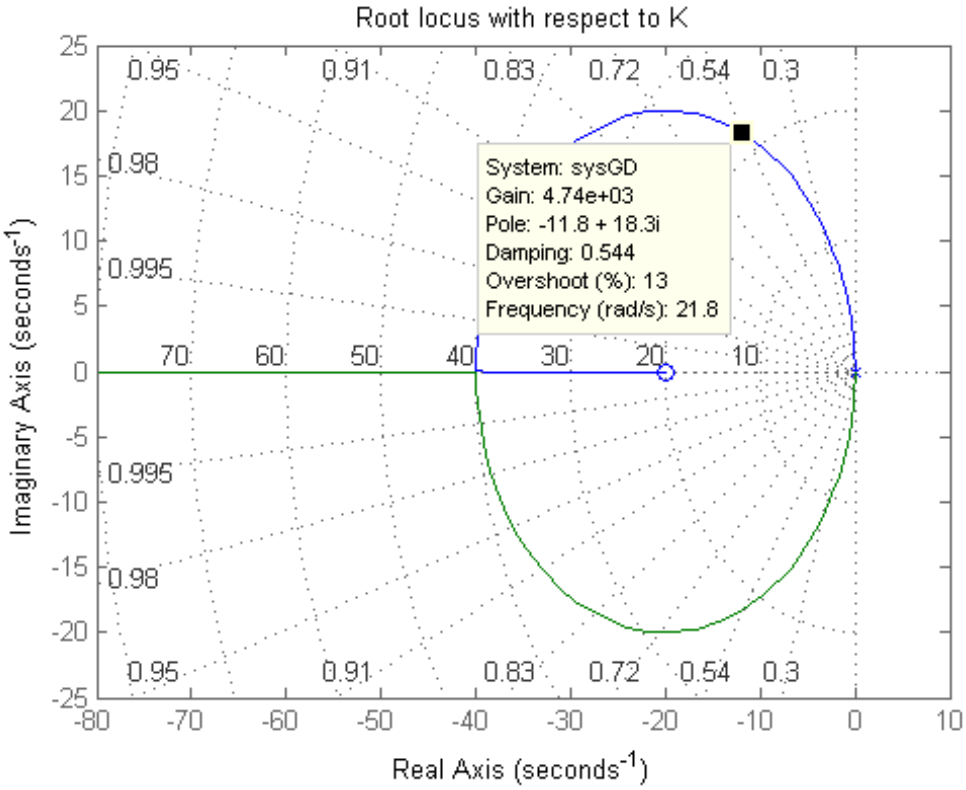
(b) For $I_z = 200$, the transfer function from Eq.(2.17) becomes

$$\frac{\Psi(s)}{T_\psi(s)} = \frac{0.005}{s^2}$$

Assume we have tach feedback from a gyro, so the compensation that will do the trick is

$$D(s) = K + K_t s = K(1 + \frac{Kt}{K}s) = K_t(s + \frac{K}{Kt})$$

For a rise time of 0.2 seconds, the Design Relations inside the back cover of the book show that we'll need an $\omega_n \simeq 20$ rad/sec. Also, for an overshoot less than 20%, we'll want $\zeta \simeq 0.5$ or better. So let's pick the zero at $s = -20$, (thus $\frac{K}{K_t} = 20$) The RL for this is:



RLocus for Problem 5.41

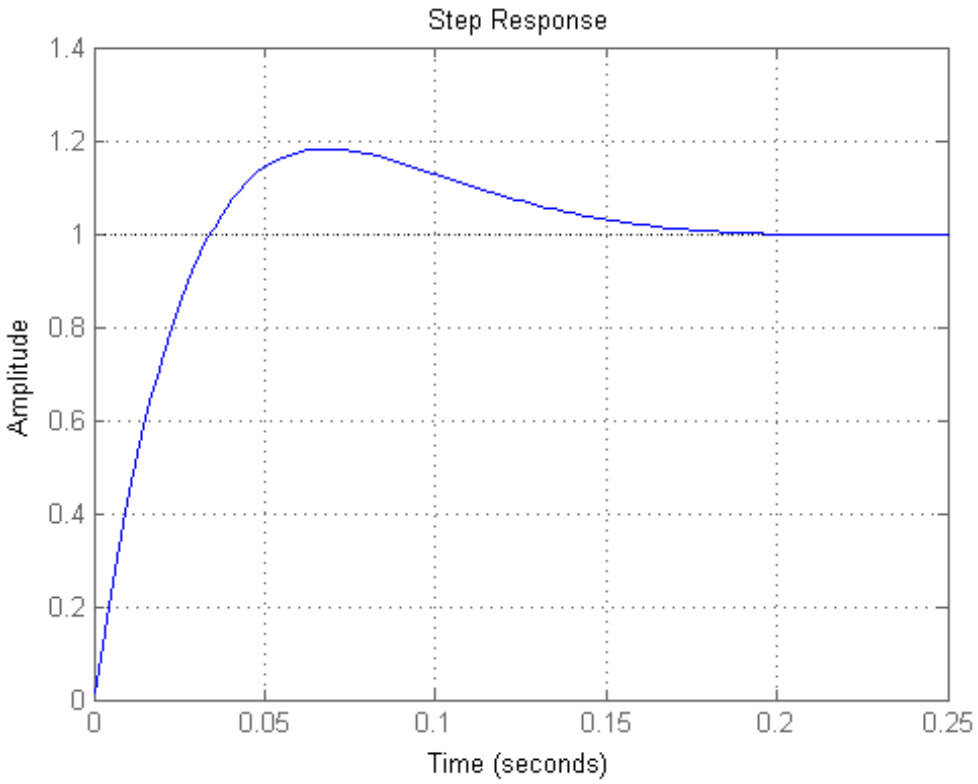


Figure 5.46: Step response for Prob 5.41 showing specs are met.

The RL shows that $K_t = 4700$ should satisfy the required conditions. However, it didn't work. The first pass showed an overshoot of about 30%. So we go back to the RL, and see that a Kt of 10,000 gives us a damping of 0.83 and natural freq of 33, so that certainly should do it. The result is confirmed: In fact, the rise time is more like 0.03 sed and the settling time is about 0.2 seconds.

Problems and solutions for Section 5.6

42. Plot the loci for the 0° locus or negative K for each of the following:

    (a) The examples given in Problem 5.3

    (b) The examples given in Problem 5.4

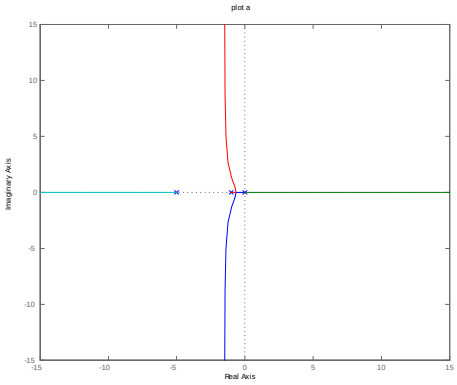
    (c) The examples given in Problem 5.5

(d) The examples given in Problem 5.6

(e) The examples given in Problem 5.7

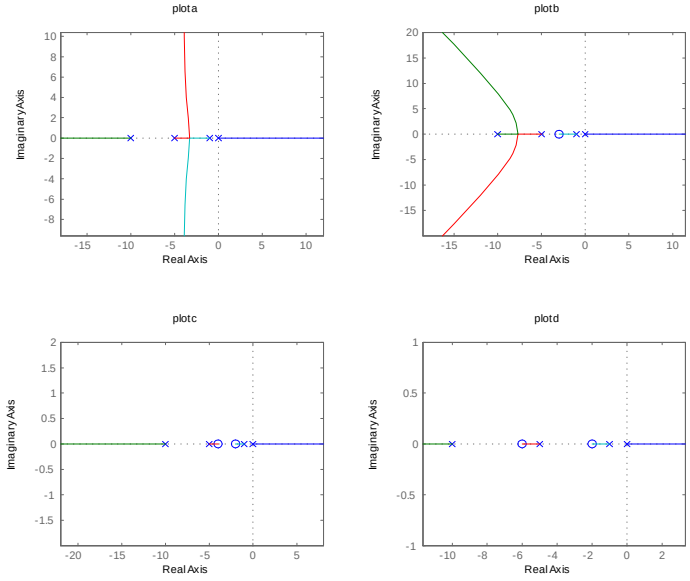
(f) The examples given in Problem 5.8

Solution:

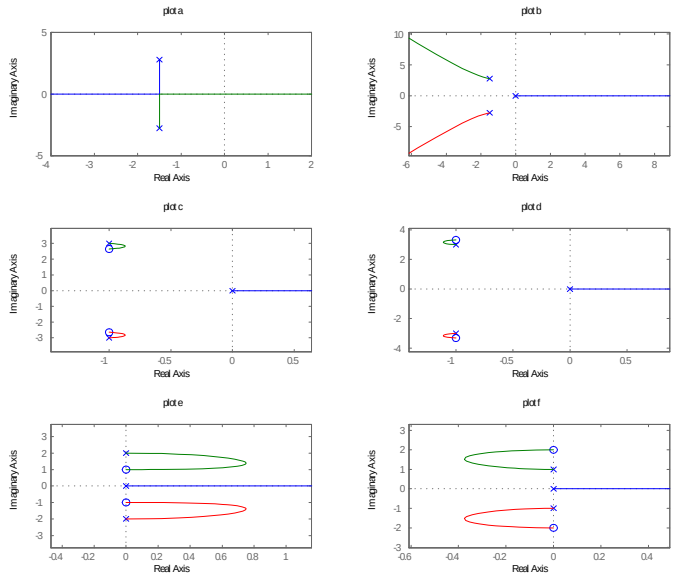


(a)

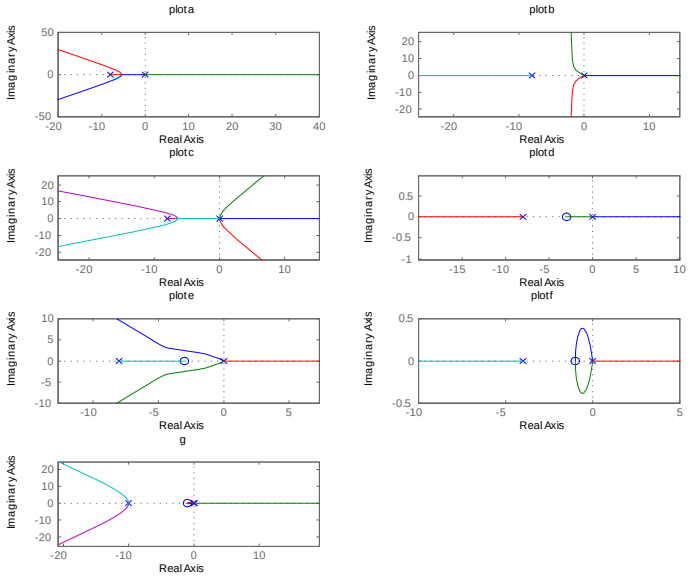
Problem 5.42(a)



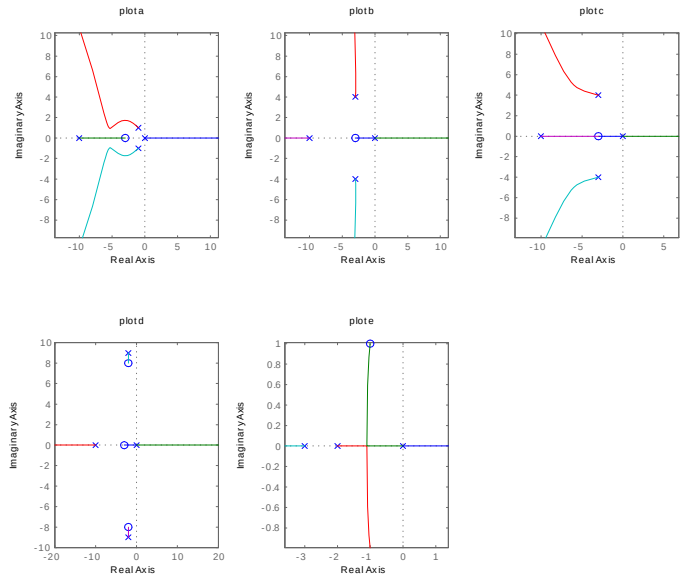
Problem 5.42(b)



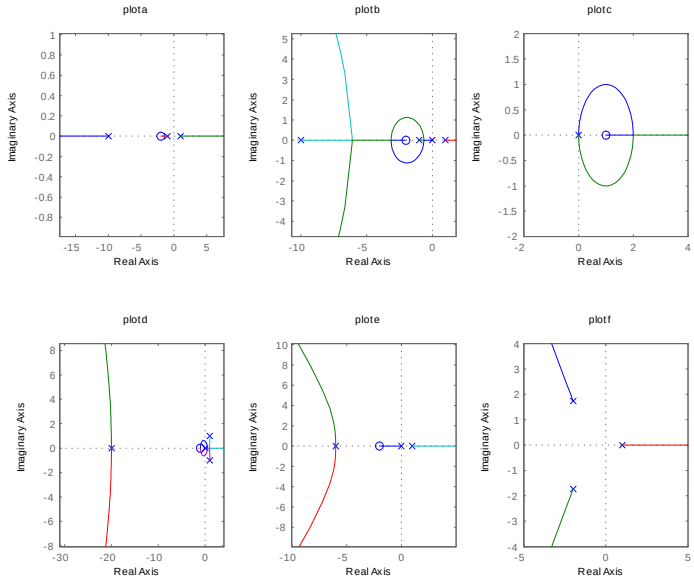
Problem 5.42(c)



Problem 5.42(d)



Problem 5.42(e)



Problem 5.42(f)

43. Suppose you are given the plant

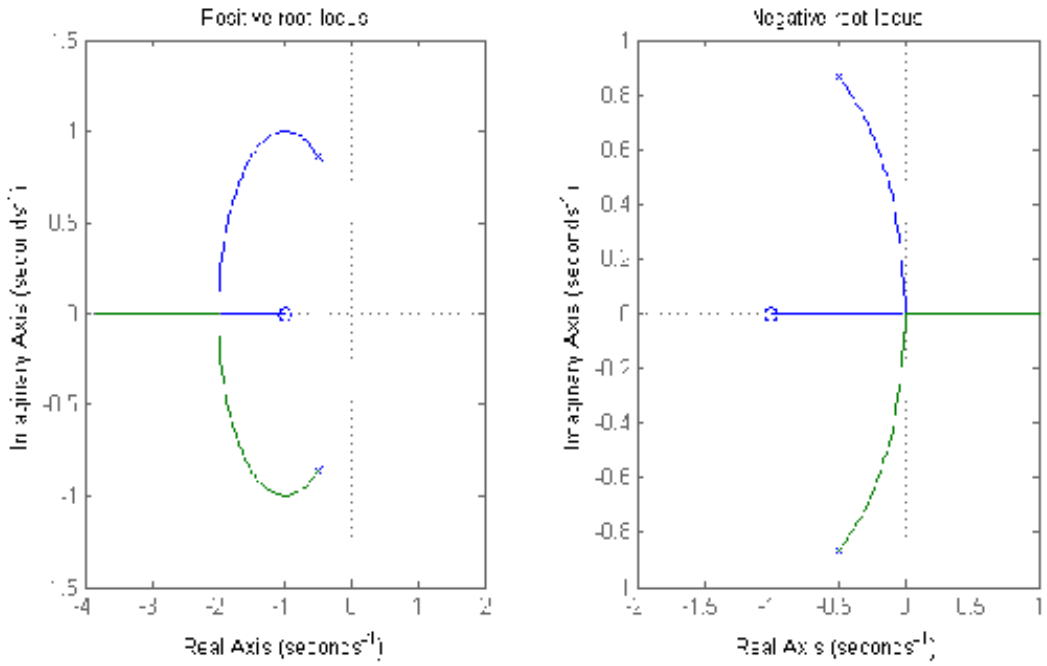
$$L(s) = \frac{1}{s^2 + (1+\alpha)s + (1+\alpha)},$$

where α is a system parameter that is subject to variations. Use both positive and negative root-locus methods to determine what variations in α can be tolerated before instability occurs.

Solution:
The characteristic polynomial in Evans form with respect to α is

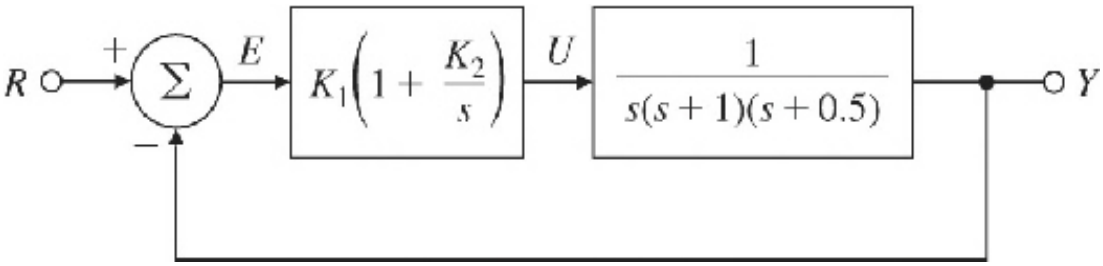
$$1 + \alpha \frac{s+1}{s^2+s+1} = 0$$

The positive and negative root locus are shown below. From the root locus, we see that the system is stable for all $\alpha > -1$.



Positive(left) and Negative(right) root locus for Problem 5.43

44. Consider the system in Fig.5.69.



Feedback system for Problem 5.44

(a) Use Routh's criterion to determine the regions in the (K_1, K_2) plane for which the system is stable.

(b) Use rltool to verify your answer to part (a).

Solution:

(a) Define $k_p = K_1$ and $k_I = K_1 K_2$ and the characteristic polynomial is

$$a(s) = s^4 + 1.5s^3 + 0.5s^2 + k_p s + k_I$$

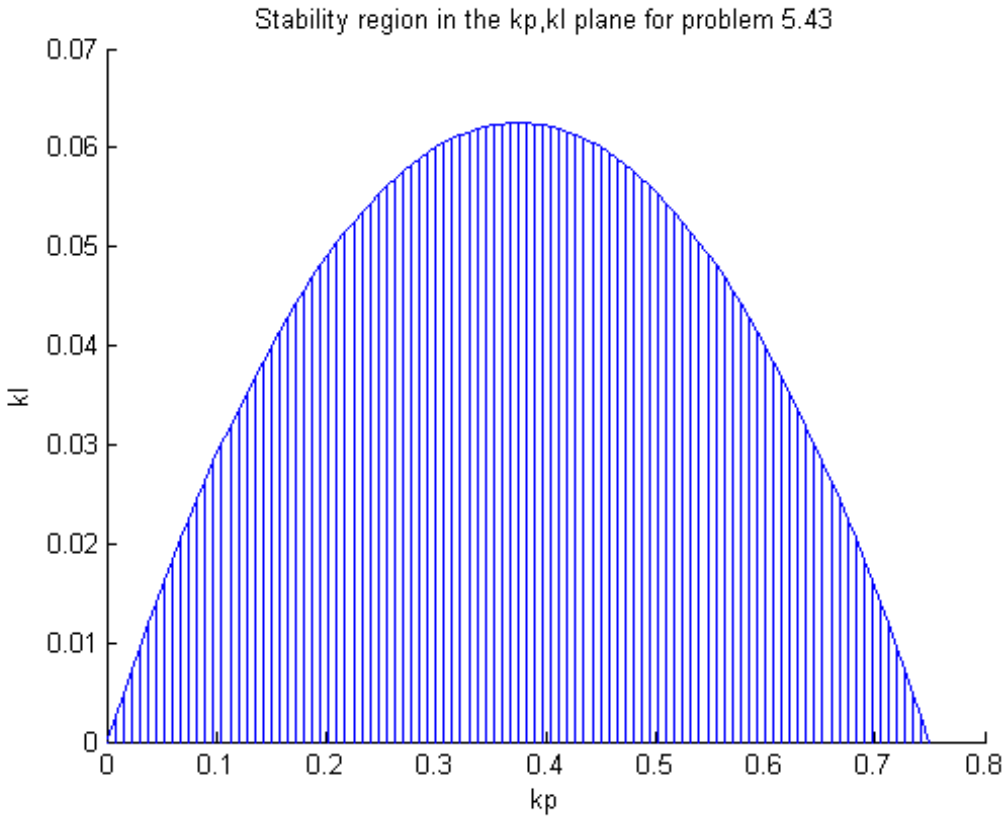
The Routh array for this polynomial is

$$\begin{array}{rcccc} \Delta(s) & = & s^4 + 1.5s^3 + 0.5s^2 + k_p s + k_I & & \\ & s^4: & 1 & 0.5 & k_I \\ & s^3: & 1.5 & k_p & \\ & s^2: & \frac{3-4k_p}{6} & k_I & \\ & s: & k_p - \frac{9k_I}{3-4k_p} & & \\ & s^0: & k_I & & \end{array}$$

For the system to be stable, it is necessary that

$$k_I > 0,\ k_p < 0.75, \text{ and } 4k_p^2 - 3k_p + 9k_I < 0$$

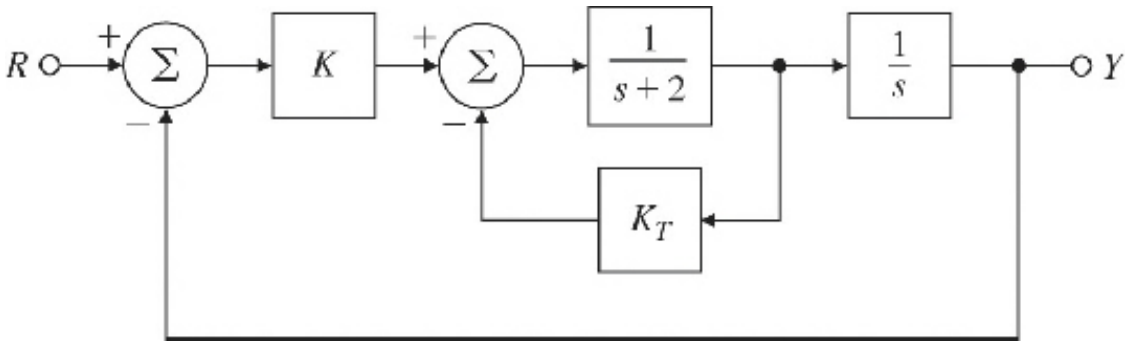
The third of these represents a parabola in the $[k_p,\ k_I]$ plane plotted below. The region of stability is the area under the parabola and above the k_p axis.



Plots for Problem 5.45

(b) When $k_I = 0$, there is obviously a pole at the origin. For points on the parabola, consider $k_p = 3/8$ and $k_I = 1/16$. The roots of the characteristic equation are -1.309, -0.191, and $\pm 0.5j$.

45. The block diagram of a positioning servomechanism is shown in Fig. 5.70.



Control system for Problem 5.45

(a) Sketch the root locus with respect to K when no tachometer feedback is present ($K_T = 0$).

(b) Indicate the root locations corresponding to $K = 16$ on the locus of part (a). For these locations, estimate the transient-response parameters t_r, M_p, and t_s. Compare your estimates to measurements obtained using the step command in Matlab.

(c) For $K = 16$, draw the root locus with respect to K_T.

(d) For $K = 16$ and with K_T set so that $M_p = 0.05(\zeta = 0.707)$, estimate t_r and t_s. Compare your estimates to the actual values of t_r and t_s obtained using Matlab.

(e) For the values of K and K_T in part (d), what is the velocity constant K_v of this system?

Solution:

(a) When $K_T = 0$, the characteristic equation of the system is

$$1 + K\frac{1}{s(s+2)} = 0$$

The root locus is plotted at the end.

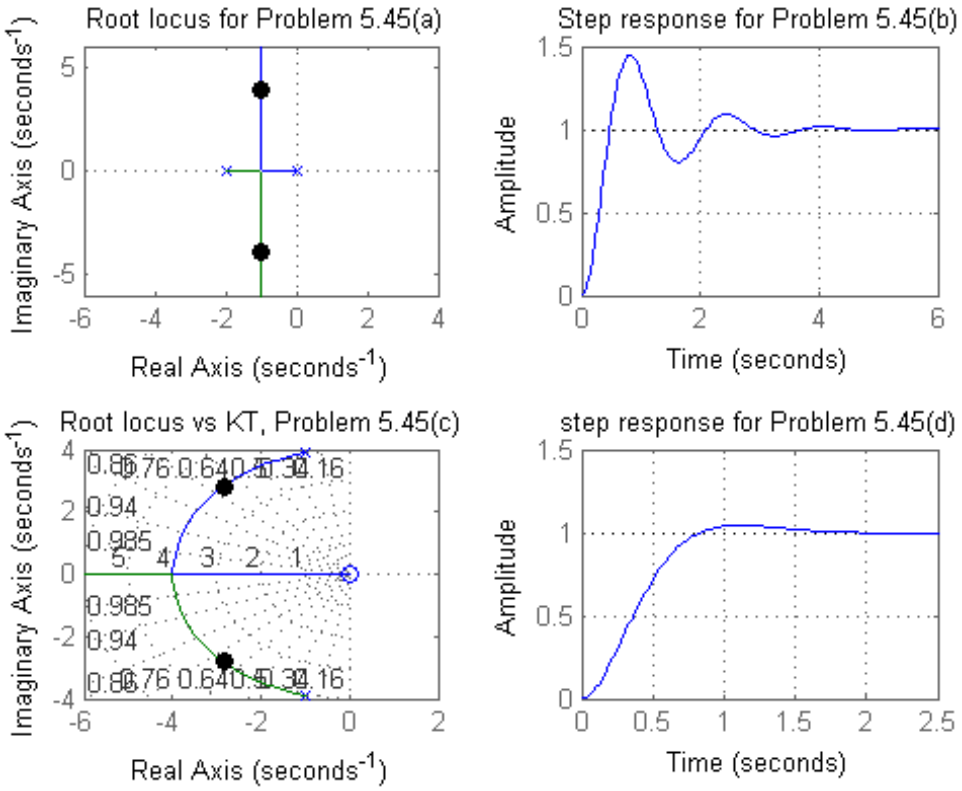
(b) With $K = 16$, $\omega_n = 4$ and $\zeta = 0.25$. Using the design relations, we'd estimate the overshoot to be $M_p = 45\%$ and a rise time of 0.45 sec, and a settling time of 4.6 sec. The values from the step command in Matlab are $M_p = 44.2\%$, $t_r = 0.32$ sec, and $t_s = 4.32$ sec . The pole locations are indicated with (•) on the plot for (a).

(c) The characteristic equation of the system is $s^2 + (2 + K_T)s + K = 0$. With $K = 16$, it can be written in Evans form as

$$1 + K_T\frac{s}{s^2 + s + 16} = 0$$

The root locus is shown below.

(d) Use rlocfind on the locus vs K_T to find the K_T value that yields 0.7 damping; the locations are marked with (•). This shows that $K_T = 3.66$. Using the formulas inside the back cover yields $M_p = 5\%$, $t_r = 0.45\,\text{sec}$, and $t_s = 1.62\,\text{sec}$. The values from the step command in Matlab are $M_p = 4.3\%$, $t_r = 0.54\,\text{sec}$, and $t_s = 1.65\,\text{sec}$..

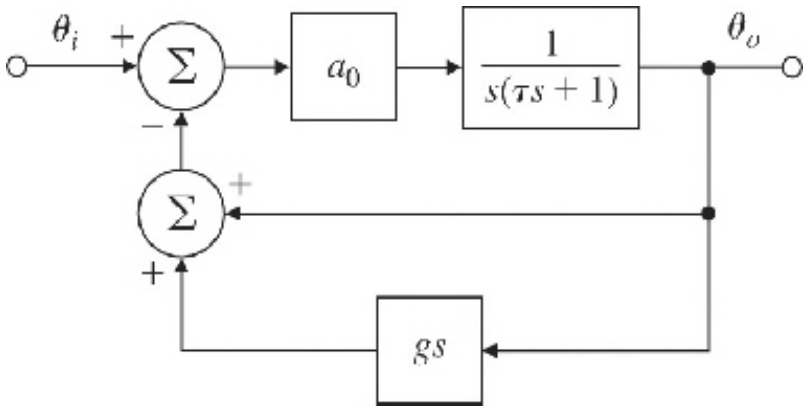


Plots for Problem 5.45

(e) The velocity constant is

$$K_v = \lim_{s\to 0} s\frac{K}{s(s+2+K_T)} = \frac{K}{2+K_T} = 2.83$$

46. Consider the mechanical system shown in Fig. 5.71, where g and a_0 are gains. The feedback path containing gs controls the amount of rate feedback. For a fixed value of a_0, adjusting g corresponds to varying the location of a zero in the s-plane.



Control system for Problem 5.46

(a) With $g = 0$ and $\tau = 1$, find a value for a_0 such that the poles are complex.

(b) Fix a_0 at this value, and construct a root locus that demonstrates the effect of varying g.

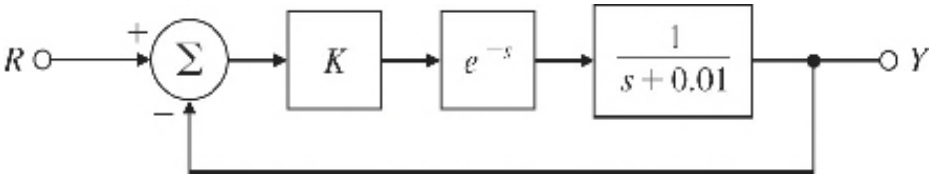
Solution:

(a) The roots are complex for $a_0 > 0.25$. We select $a_0 = 1$ and the roots are at $s = -0.5 \pm 0.866j$.

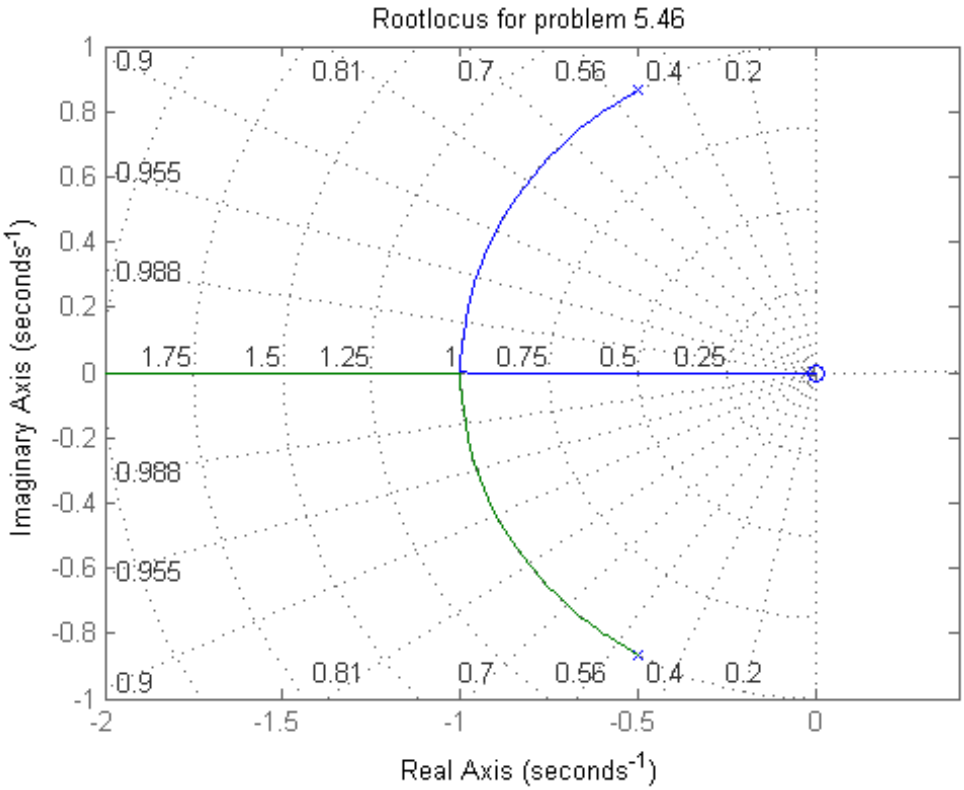
(b) With respect to g, the root-locus form of the characteristic equation is $1 + g\dfrac{s}{s^2 + s + 1} = 0$. The locus is plotted below.

47. Sketch the root locus with respect to K for the system in Fig.5.72 using the Padé(1,1) approximation and the first-order lag approximation. For both approximations, what is the range of values of K for which the system is unstable?

(Note: The material to answer this question is contained in Appendix W5.6.3 discussed in www.FPE8e.com.)



Control system for Problem 5.47


Rootlocus for problem 5.46
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.9
0.81
0.7
0.56
0.4
0.2
0.955
0.988
1.75
1.5
1.25
1
0.75
0.5
0.25
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0

Solution:

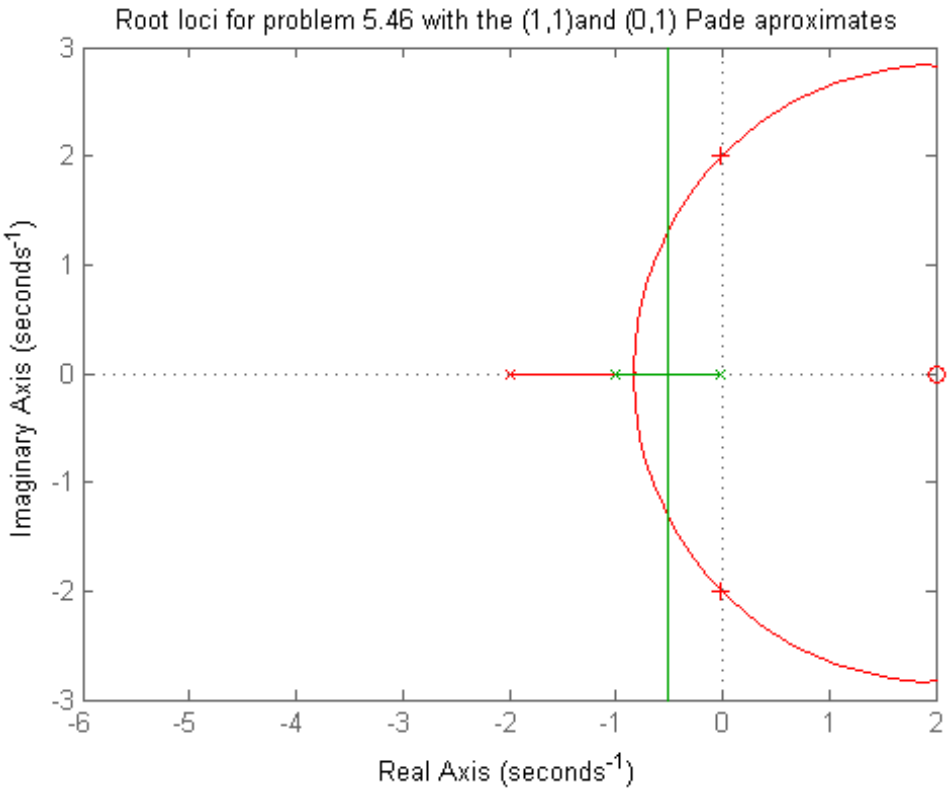
Matlab cannot directly plot a root locus for a transcendental function. From the Appendix W5.6.3, we see that the Padé(1,1) approximation for e^{-s} is

$$e^{-s} \cong \frac{1-(s/2)}{1+(s/2)} = -\frac{s-2}{s+2},$$

and the first-order lag approximation (Padé(0,1)) is

$$e^{-s} \cong \frac{1}{s+1}.$$

With the Padé(1,1) approximation, a locus valid for small values of s can be plotted, as shown below by the red curve. The rlocfind routine is used by placing the cursor on the $j\omega$ axis to find the maximum value of K at the instability boundary. This yields $K_{max} = 2$ for the Padé(1,1). The locus for the first-order lag is also shown in green; however, it produces a locus with two branches going to infinity at $s = -1/2$. thus there is no value of K that produces instability for this approximation for a first order system. This demonstrates a limitation for this approximation of the delay. However, for higher order systems, the first order lag can be useful.



Solutions for Problem 5.47

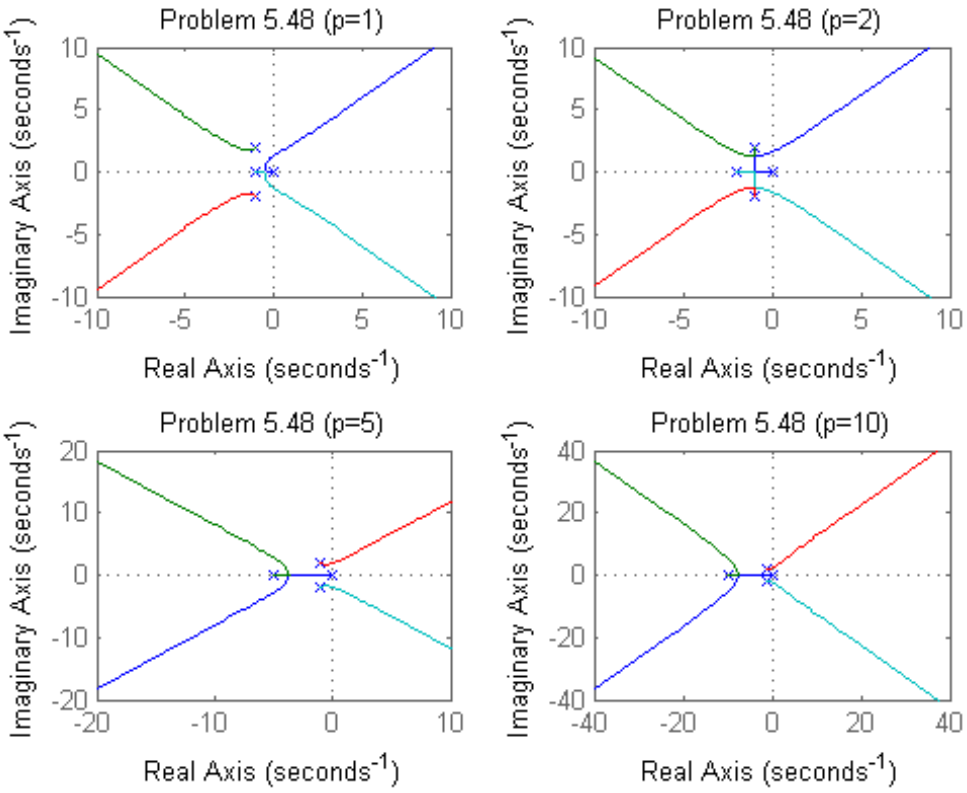


Figure 5.47: Problem 5.48 Root Loci

48. For the equation $1 + KG(s)$ where,

$$G(s) = \frac{1}{s(s+p)[(s+1)^2+4]},$$

use Matlab to examine the root locus as a function of K for p in the range from $p = 1$ to $p = 10$, making sure to include the point $p = 2$.

Solution:

The root loci for four values are given in the figure. The point is that the locus for $p = 2$ has multiple roots at a complex value of s.