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3. Pre-Lab Exercises

1. Starting from equations (3) and (4) derive the transfer function (6).

$$V = iR + L\frac{di}{dt} + k_\tau \Omega$$

$$J\frac{d\Omega}{dt} + B_v\Omega = k_\tau i$$

$$V = iR + k_\tau \Omega$$

$$Js\Omega + B_v\Omega = k_\tau i$$

$$i = \frac{s\Omega J}{k_\tau} + \frac{B_v\Omega}{k_\tau}$$

$$V = \left(\frac{s\Omega J}{k_\tau} + \frac{B_v\Omega}{k_\tau}\right)R + k_\tau\Omega$$

$$V = \frac{s\Omega JR}{k_\tau} + \frac{B_v\Omega R}{k_\tau} + k_\tau\Omega$$

$$V = \Omega\left(\frac{sJR}{k_\tau} + \frac{B_vR}{k_\tau} + k_\tau\right)$$

$$\frac{\Omega}{V} = \frac{1}{\frac{sJR}{k_\tau} + \frac{B_vR}{k_\tau} + k_\tau}$$

$$\boxed{\frac{\Omega}{V} = \frac{\frac{k_\tau}{RJ}}{s + \frac{B_vR + k_\tau^2}{JR}}}$$

2. By equating the transfer functions in equations (6) and (7), derive algebraic expressions for K and tau in terms of the individual motor parameters (K_t, R, B_v, J).

$$G(s) = \frac{k}{\tau s + 1} = \frac{\frac{k_\tau}{RJ}\left(\frac{JR}{B_vR + k_\tau^2}\right)}{\frac{JR}{B_vR + k_\tau^2}s + 1} = \frac{\left(\frac{k_\tau}{B_vR + k_\tau^2}\right)}{\frac{JR}{B_vR + k_\tau^2}s + 1}$$

$$k = \frac{k_\tau}{B_vR + k_\tau^2}$$

$$\tau = \frac{JR}{B_vR + k_\tau^2}$$

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3. Suppose a motor has the following parameter values:

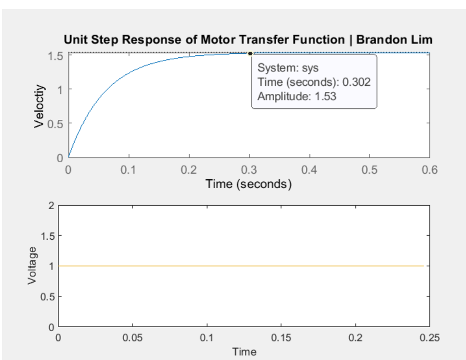
K_t	0.05 N·m
R	2 Ω
B_v	0.015 N·m/rad/s
J	0.001 N·m/rad/s^2

a. Use the parameter values in the table to compute the numerical values of K and tau for this motor.

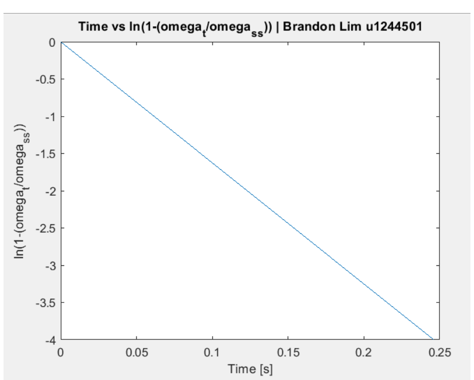
$$K = \frac{K_t}{B_v R + K_t^2} = \frac{0.05\ Nm}{(0.015\ Nms)(2\Omega) + 0.05^2\ Nm} = 1.538 \frac{1}{s\Omega + Vm}$$

$$\tau = \frac{JR}{B_v R + K_t^2} = \frac{0.001 \cdot 2}{(0.015)(2) + 0.05^2} = 0.0615$$

b. Use the *tf()* and *step()* commands in MATLAB to simulate the theoretical step response of this motor. Plot the voltage vs. time and velocity vs. time for a unit step of 1 Volt (*d=1*). Verify that the steady-state speed $\Omega_{ss} = K/d$.



c. Plot ln(1-$\Omega(t)/\Omega_{ss}$) vs. time for t < 4tau and verify that the slope = -1/tau



slope = -16 = -1/tau

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4. A PWM signal with a frequency of 20kHz is used to command a motor driver that is powered by a 9 Volt battery. The *setSpeed()* command in the motor driver library accepts a number from 0 to 400, where 400 represents a 100% duty cycle. If you send a *setSpeed()* command of 300, what duty cycle does this correspond to, and what will be average motor voltage? What is the corresponding pulse width (in seconds) of the PWM signal?

$f = 20kHz$

$v = 9$ Volts

set speed = 300

a) Duty Cycle $= \frac{300}{400} \times 100\% = 75\%$

b) $V_{avg} = (0.75)(9v) = 6.75v$

c) $f = 20000\ \frac{1}{s}$

$20000 \frac{1}{s}\ (0.75) = 15000 \frac{1}{s} \Rightarrow 0.00006$ sec

```
%Brandon Lim
clear, clc, close all

sys = tf(1.538,[0.0615, 1]);
figure
subplot(2,1,1)
step(sys)
title("Unit Step Response of Motor Transfer Function | Brandon Lim")
ylabel("Veloctiy")
xlabel("Time")


tau = 0.0615;
tauss = 4*tau;
omegass = 1.538;
subplot(2,1,2)
plot(linspace(0,tauss,10),ones(10))
ylabel("Voltage")
xlabel("Time")

t = linspace(0,tauss,100);
omega = omegass.*(1-exp(-t./tau));

figure
plot(t,log(1-(omega./omegass)))
xlabel("Time [s]");
ylabel("ln(1-(omega_t/omega_s_s))")
title("Time vs ln(1-(omega_t/omega_s_s)) | Brandon Lim u1244501")
```