Homework 2

(due Thurs, Jan 25)

1) The state of stress at a point relative to an x, y, z coordinate system is given by

$$\begin{bmatrix} 12 & 4 & 2 \\ 4 & -8 & -1 \\ 2 & -1 & 6 \end{bmatrix} \text{MPa}$$

Calculate the maximum shearing stress at the point.

2) At a point in a loaded member, the stresses relative to an x, y, z coordinate system are given by

$$\begin{bmatrix} 60 & 20 & 10 \\ 20 & -40 & -5 \\ 10 & -5 & 30 \end{bmatrix} \text{MPa}$$

Calculate the magnitude and direction of maximum principal stress. Give the direction as the components of a unit vector.

3) The stresses (in MPa) with respect to an x, y, z coordinate system are described by

$$\sigma_x = x^2 + y, \qquad \sigma_z = -x + 6y + z$$
$$\sigma_y = y^2 - 5, \qquad \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

At point $(3, 1, 5)$, determine (a) the stress components with respect to x', y', z' if

$$l_1 = 1, \qquad m_2 = \frac{1}{2}, \qquad n_2 = \frac{\sqrt{3}}{2}, \qquad n_3 = \frac{1}{2}, \qquad m_3 = -\frac{\sqrt{3}}{2}$$

and (b) the stress components with respect to x'', y'', z'' if

$$l_1 = \frac{2}{\sqrt{5}}, \qquad m_1 = -\frac{1}{\sqrt{5}}, \qquad n_3 = 1$$

(c) Also show that the three invariants of the stress tensor, defined by Eq. (1.34), are, indeed, invariant under the transformations described for (a) and (b).

4) At a point in a loaded body, the stresses relative to an x, y, z coordinate system are

$$\begin{bmatrix} 40 & 40 & 30 \\ 40 & 20 & 0 \\ 30 & 0 & 20 \end{bmatrix} \text{MPa}$$

Determine the normal stress σ and the shearing stress τ on a plane whose outward normal is oriented at angles of $40°$, $75°$, and $54°$ with the x, y, and z axes, respectively.

5) A displacement field in a body is given by

$$u = c(x^2 + 10)$$
$$v = 2cyz$$
$$w = c(-xy + z^2)$$

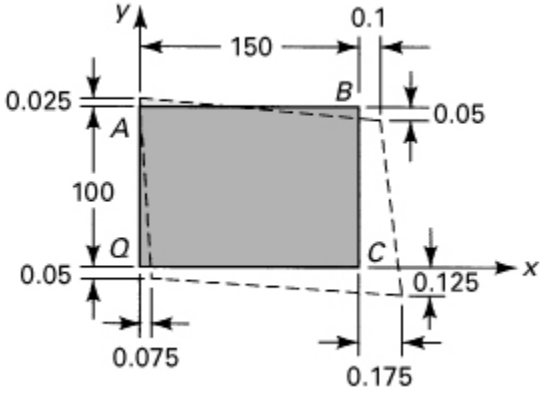
where $c = 10^{-4}$. Determine the state of strain on an element positioned at $(0, 2, 1)$.

6) The plane displacement field and shear strain in a member have the form

$$u = a_0x^2y^2 + a_1xy^2 + a_2x^2y$$
$$v = b_0x^2y + b_1xy$$
$$\gamma_{xy} = c_0x^2y + c_1xy + c_2x^2 + c_3y^2$$

Determine the expressions for c_i (in terms of a_i and b_i) that must be satisfied for the given displacements and strain to be compatible.

7) A $100\ mm$ by $150\ mm$ rectangular plate $QABC$ is deformed into the shape shown by the dashed lines. All dimensions shown in the figure are in millimeters. Determine at point Q (a) the strain components ε_x, ε_y, γ_{xy}, and (b) the principal strains and the direction of the principal axes.



8) At a point in a stressed body, the tensorial strains, related to the coordinate set xyz, are given by

$$\begin{bmatrix} 200 & 300 & 200 \\ 300 & -100 & 500 \\ 200 & 500 & -400 \end{bmatrix} \mu$$

Determine (a) the strain invariants; (b) the normal strain in the x' direction, which is directed at an angle $\theta = 30°$ from the x axis (in the x-y plane); (c) the principal strains ε_1, ε_2, and ε_3; and (d) the maximum shear strain.