TFES Lab (ME EN 4650)
Two-Dimensional Conduction

Textbook Reference: Section 2.3 (pp. 70–73), Section 4.4 (pp. 212–222), Section 4.5 (pp. 222–230) from Incropera, DeWitt, Bergman, Lavine, 6th ed.

Objectives

(i) investigate two-dimensional conduction through a flat plate driven by heating/cooling along the boundaries (edges of the plate),

(ii) utilize a thermal IR camera for temperature measurement and imaging,

(iii) calculate the heat flux across the boundaries using the temperature data from the IR camera,

(iv) perform a numerical simulation of the heat diffusion equation by writing a computer code that implements the finite difference method in a rectangular domain, and

(v) validate the numerical results with the experimental data by comparing the two via contour plots.

Introduction

Steady-state heat conduction in a two-dimensional geometry, as shown in Figure 1, is governed by conservation of energy, which requires that the heat flux q into a differential control volume in the domain must equal the heat flux out of the differential control volume. Analytical solutions may be performed in some simple geometries, such as rectangular or cylindrical surfaces, when boundary conditions are completely defined on each surface. The

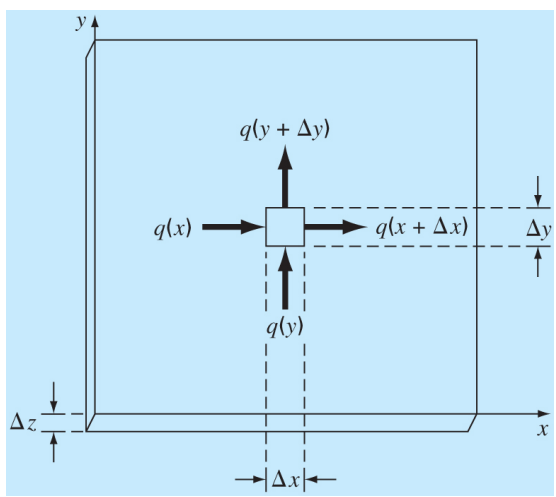


Figure 1. Schematic of the two-dimensional domain examined in the present lab experiment.

boundary conditions may be combinations of boundary conditions of the first kind (constant temperature), second kind (constant heat flux), and third kind (convection condition). Unfortunately, analytical solutions are difficult to perform and only simple geometries may be used. As an alternative to analytical solutions, numerical techniques, such as the finite-difference technique or the finite element method, may be employed to approximate the temperature field in the 2D geometry, given information at the boundaries. Regardless of the numerical method, appropriate boundary conditions must be known in order to solve for the temperature distribution within the solid.

Validation is an important step when utilizing a numerical simulation to study the scientific behavior of a system or aid in the design of a device. Validation refers to the process of quantifying uncertainty bounds on a numerical result by direct comparison with experimental data. In order to validate a numerical model of two-dimensional heat conduction, direct measurements of the temperature distribution over the surface of interest are required. This may be achieved using an array of thermocouples or a thermal imaging camera (TIC).

In the present experiment, you will used an infrared TIC (FLIR T-420), which is a non-contact device that detects infrared energy (heat) within a wavelength band of 7.5–13 μm. The intensity of the energy detected at each pixel on the sensing unit can be converted to temperature using the manufacturer's calibration. In this manner, a matrix of temperature values over the surface of the object is obtained. The distribution of temperatures is visualized using an isothermal color contour plot that maps temperature values to a spectrum of colors, which is then displayed on the screen. In this manner, a thermal image is produced, showing the complete surface temperature field of the object of interest. Heat sensed by an infrared camera can be precisely quantified, which facilitates monitoring of thermal performance and the identification of potential heat-related problems in industrial settings. A photograph of the FLIR T-420 camera is shown in Figure 2.

In the present lab assignment, a simple geometry consisting of a flat rectangular plate is used as the object of interest. The temperature distribution on the front face is measured with the TIC. The measured temperature data along the perimeter of three sides of the plate, in addition to an adiabatic condition on the remaining side, is used as the input to a

Figure 2. FLIR T-420 infrared thermal imaging camera.

finite-difference based numerical model. The temperature field produced by the numerical model is then compared to the measured temperature field at each point on the front face of the flat plate. The average difference between the model and data serves as a measure of the accuracy of the model. If the difference remains below an acceptable tolerance, then the numerical model is claimed to be validated, and may henceforth be confidently used as an appropriate and accurate tool for predicting thermal performance in two-dimensional structures in similar geometries to which it was validated.

Experimental Setup

The experimental setup in this lab utilizes a unique rectangular flat plate, designed to produce heat transfer that is very nearly two-dimensional. In other words, the predominant temperature gradients driving the heat flux occur in a single plane (coincident with the front face). Heat transfer in the third (out-of-plane) dimension is minimized. Thus, the temperature field in the principal plane is sufficient to understand the thermal behavior of the system.

The schematic of the flat plate apparatus is shown in Figure 3. Heat transfer in the x-y plane (coincident with the front face) is of primary interest; while, heat transfer in the z (out-of-plane) direction is designed to be negligible. The flat plate is machined aluminum

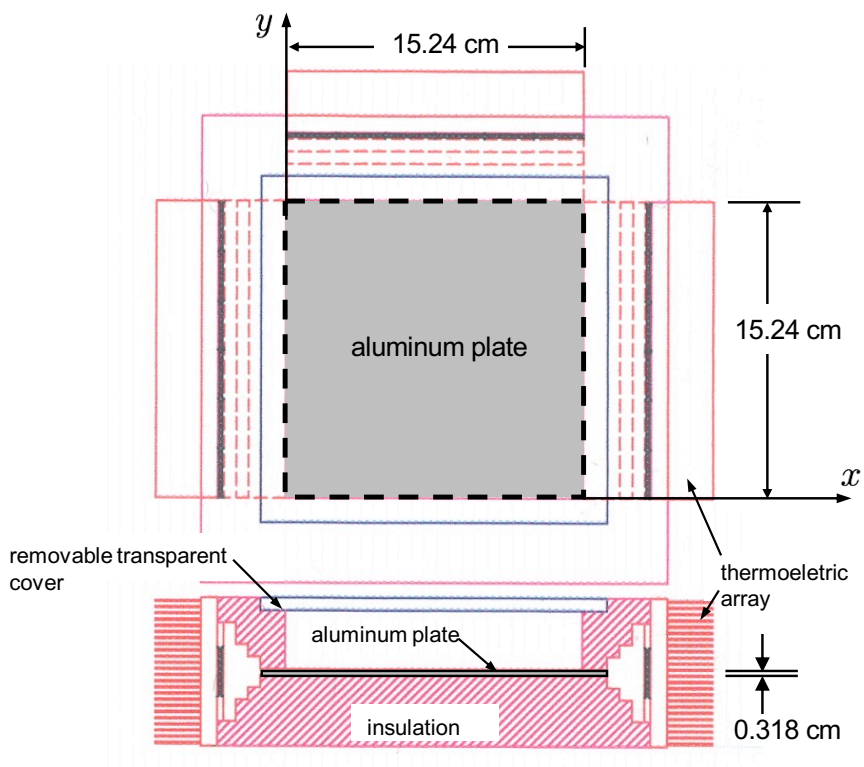


Figure 3. Schematic of the experimental setup, showing the flat plate outlined by the black dashed lines with dimensions 15.24 cm $\times$ 15.25 cm $\times$ 0.318 cm. The left, top, and right edges can be heated or cooled using an array of thermoelectric coolers. The bottom edge is insulated.

with dimensions of 15.24 cm × 15.24 cm × 0.318 cm ($6'' \times 6'' \times 0.125''$). The plate is supported by a frame that has a thickness of 7.62 cm ($3''$) at the edges. Foam insulation with a thickness of 3.81 cm ($1.5''$) is attached to the rear of the plate to reduce heat loss. All four edges of the plate are also insulated with 3.81 cm ($1.5''$) of foam insulation. The bottom edge is completely insulated and may be assumed to be adiabatic. Six small thermoelectric coolers (TEC) are attached to each of the left, right, and top edges. The TEC arrays are powered by three separate voltage supplies (one for each edge), which include switches to control polarity (and thus set the TEC array to either heat or cool). The wide variety of thermal boundary combinations makes it possible to produce a diverse set of heat flux patterns in the plate, yielding a wide range of temperature distributions.

The flat plate assembly is mounted vertically; and, the FLIR camera is oriented such that the viewing angle is perpendicular to the front face (x-y plane). Note, the camera sensing element has a resolution of 320 (columns) by 240 (rows) pixels. This means that with any given thermal image, we can obtain, at most, a temperature field consisting of 320×240 points (or, equivalently, 7.6800×10^4 temperature measurements). The complete experimental setup is illustrated in Figure 4. A list of measurements acquired in the experiment along with their native units are provided in Table 1. Note, in addition to measuring the temperature distribution over the plate, the electrical power supplied to each bank of TECs will be measured using a power meter that outputs both the voltage and current.

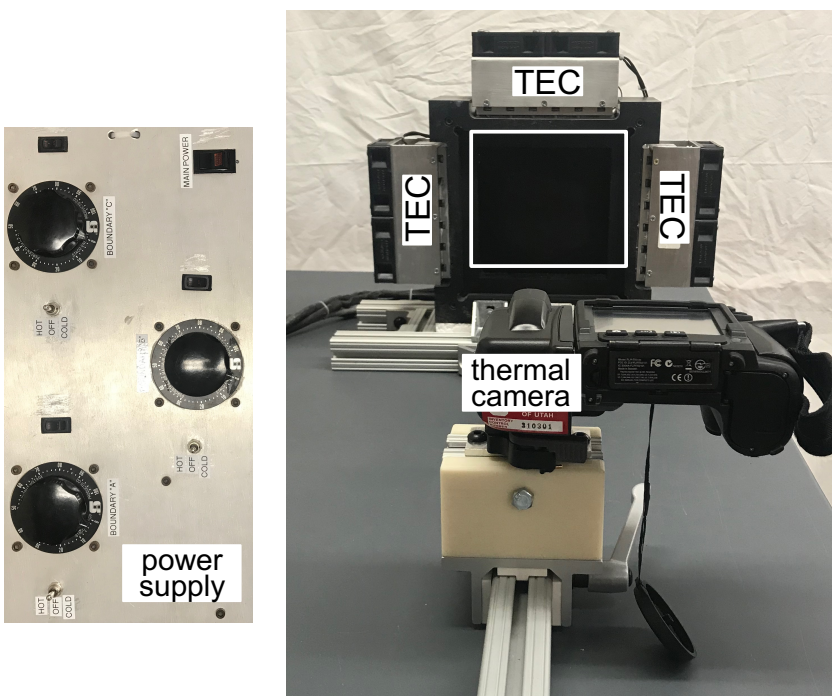


Figure 4. Photograph of test setup in the lab looking from behind the camera toward the flat plate. The white box outlines the boundary of the plate that will be interrogated. The power supply to control the TECs is shown in the left image.

Table 1. List of measurements acquired in the experiment with their native units.

Quantity	Symbol	Units	Instrument
Plate temperature distribution	$T(x, y)$	°C	IR camera
Electrical power to TECs	P_E	W	power meter

Thermoelectric devices (shown in Figure 5) are solid-state heat pumps used in applications where temperature stabilization, temperature cycling, or cooling below ambient may be required. There are many commercial products using thermoelectric coolers, including CCD cameras, laser diodes, microprocessors, blood analyzers, portable picnic coolers, and the thermal imaging camera used in this experiment. Thermoelectric devices are based on the Peltier Effect, and consist of an array of p-type and n-type semiconductor columns, sandwiched between two copper plate conductors. As the electrons enter the p-type semiconductor column, their energy state drops causing them to reject heat. When the electrons leave the p-type material, their energy state increases causing them to absorb heat. Recall that electronics move in a direction opposite to the current. A similar phenomenon occurs in the n-type semiconductor, but in reverse. Thermoelectric devices can be used to both heat and cool, depending on the direction of the current. Using a thermoelectric device in the heating mode is very efficient because all of the electric resistance heating (Joule heating) as well as the load from the cold side are pumped to the hot side. This reduces the power needed to achieve the desired heating. In contrast, using a thermoelectric device in the cooling mode is not especially efficient, with typical efficiencies reaching about 15% for TECs, depending on the supply voltage.

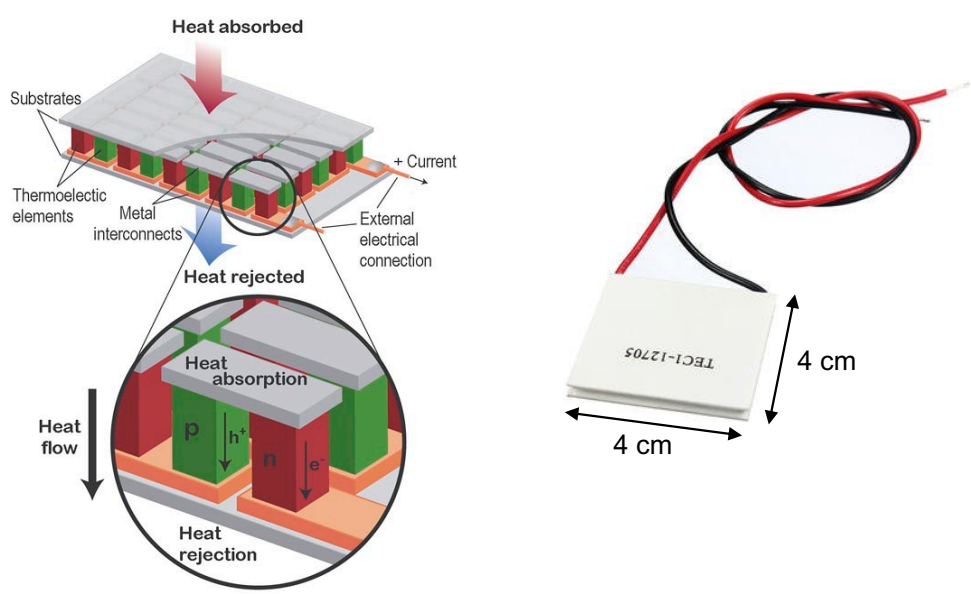


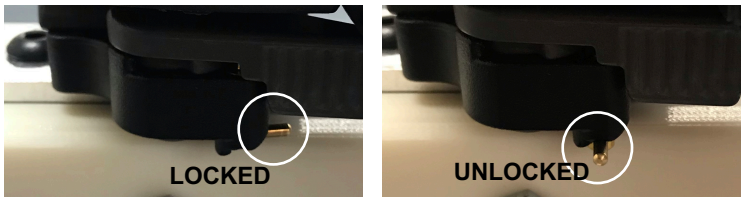
Figure 5. (left) Principle of operation of a solid-state thermoelectric device. (right) Image of a thermoelectric device (4×4×0.4 cm), rated at 12V and 4.5A.

Laboratory Procedures

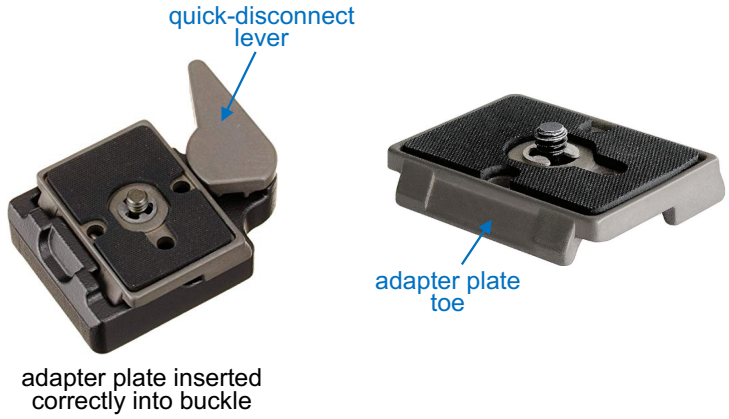
(1) Extra TECs will be set out on a workbench in the lab. Students can plug-in the provided power supplies and monitor the temperature difference across the two sides of the TEC with a voltmeter and thermocouple probe. Observe how the temperature changes with applied voltage. Note, after a couple minutes of being powered-on, you will notice that the temperature difference across the two sides of the TEC starts to equilibrate. This is due to conduction of the heat from the hot side to the cold side through the semiconductor material.

(2) The FLIR IR camera can be removed from the mounting stand via the quick-disconnect tripod latch, and used to take thermal images of objects of interest in the lab (for fun). When removing the camera from the stand, be sure to disconnect the USB cable first.

Note that the camera is attached to the teststand via a quick-disconnect tripod buckle and adapter plate. The latch on the buckle has a safety pin. This pin must be rotated into the "unlock" position to remove the camera as shown below. Be sure to "lock" the camera back into place when securing back into the tripod buckle. Press down slightly on the safety pin with your finger to rotate it into its unlocked and locked positions.



After the safety pin has been unlocked, rotate the quick-disconnect lever about 60° toward the back of the camera to release the adapter plate, as shown below. You can now CAREFULLY lift the back of camera up and slide the front toe of the adapter plate backwards, out of the buckle.



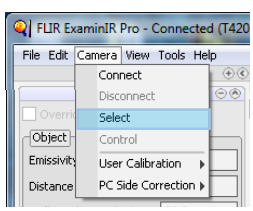
(3) Turn on the FLIR IR camera using the power button on the lower right side of the back of the camera, as shown below. If the power does not come on, or stay on, then the battery needs to be replaced. To do so, open the battery hatch lid and GENTLY

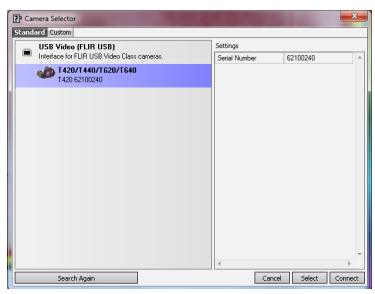
press the red lever towards the back of the camera to release the battery pack. Insert the drained battery pack into the recharging station. Then, insert the fully-charged battery pack into the battery hatch. Push down GENTLY on the battery pack to snap it into place and close the battery hatch lid.

(4) When ready to start the experiment, secure the camera back into the tripod buckle on the mounting stand, so that the view is exactly perpendicular to the front face of the aluminum test specimen.

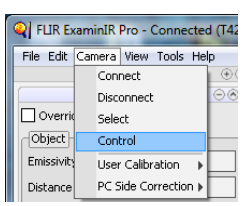
(5) Use the provided USB cable to connect the FLIR camera to the computer. Open the FLIR ExaminIR software on the computer.

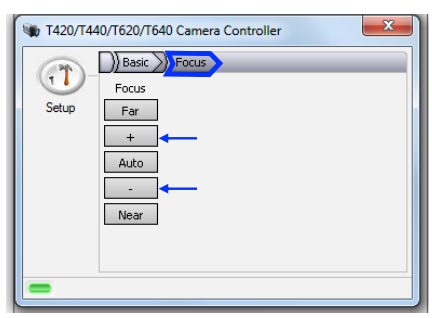
(6) First you must connect the software with the camera. From the top menubar, click Camera > Select. In the Camera Select popup window, select "T420/T440/T620/T640". Now the image in the main image window of the software should be a live thermal image as seen through the camera.





(7) Focus the camera on the front face of the flat plate test specimen by selecting Camera > Control from the top menubar. In the Camera Controller popup window, select the Foucs tab, as shown below. Then, click the + and - buttons to focus the image. Placing a small object (such as a washer) against the test specimen will aid in focusing on the front face of the flat plate.





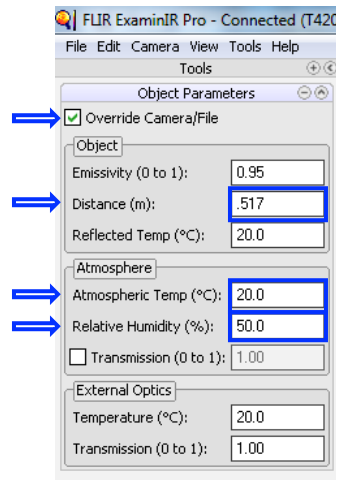
(8) Calibrate the spatial dimension of the camera.

(i) Expand the Tools Pane on the leftside of the environment window, if it is not already expanded. Expand the toolbox entitled "Object Parameters", and click the checkbox labeled Override Camera/File, as shown below.

(ii) Measure the approximate distance from the front face of the test specimen to the front of the IR lens on the camera. DO NOT TOUCH THE FRONT SURFACE OF THE IR LENS.

(iii) Enter the correct value in the Distance (m) textbox.

(iv) Enter the correct atmospheric properties (atmospheric temperature and relative humidity), if known, in the textboxes indicated.

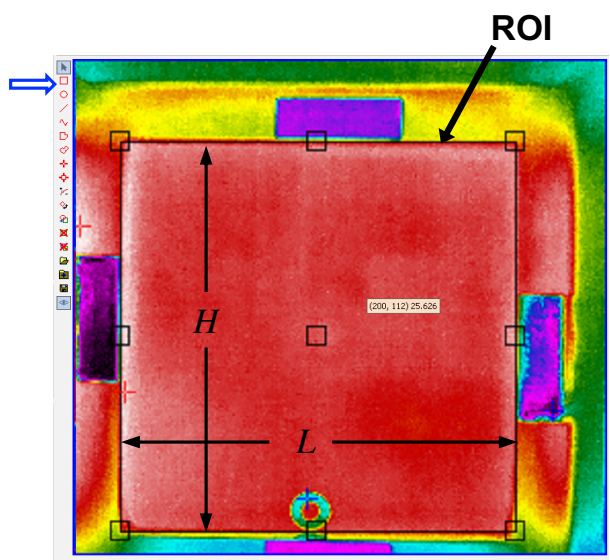


(9) Select the Region of Interest (ROI).

(i) Select the Rectange tool from the Analysis Palette located along the leftside of the image window, as indicated by the blue arrow in the adjacent figure.

(ii) Draw a box around the front face of the test specimen in the Image Window as illustrated. You can click and drag the handles of the ROI to adjust the placement of the edges.

Importantly, the edges of the ROI should be coincident to the edges of the flat plate test specimen, or slightly INSIDE the actual edges of the test specimen. This will help ensure good agreement between the numerical simulation and experimental data. It is not necessary for the length L and height H of the ROI to be the same.



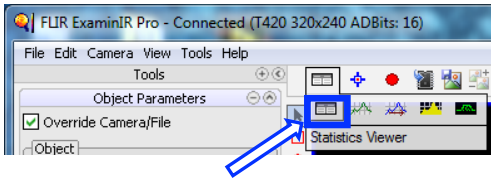
(10) Open the Statistics Window.

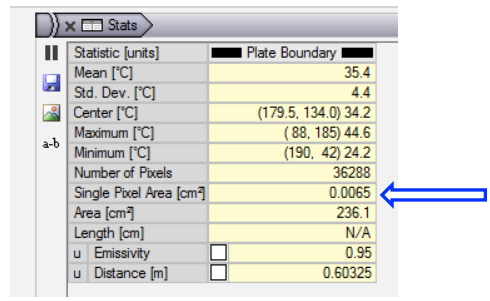
a) On the toolbar along the top of the environment window, click the Statistics Viewer icon, as indicated by the blue box in the figure below.

b) Relevant information about your ROI will be displayed in the Stats window, including dimensions.

c) Of particular importance is the "Single Pixel Area" dimension, as this is the value you will use to obtain the size (L and H) of your ROI. This also sets the resolution of the mesh in your numerical simulation.

d) By monitoring the "Center" temperature of your ROI listed in the Statistics Widow, you will be able to determine when the system has reached steady state.



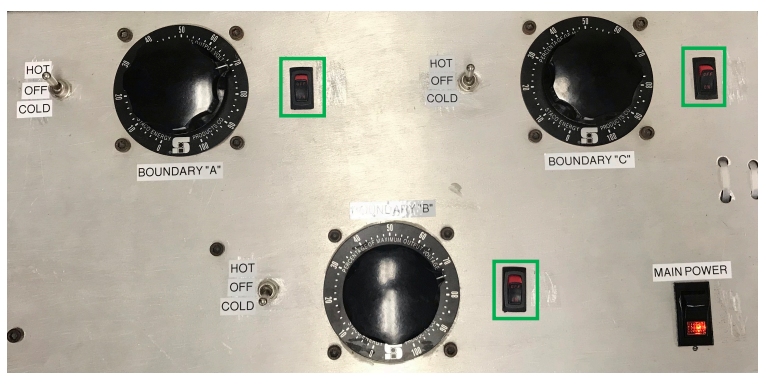


(11) Adjust the temperature of the test specimen boundaries.

(i) Turn the Main Power Switch on the TEC power supply box to ON. A red light on the switch will illuminate when power is on.

(ii) Toggle the individual switches to the TEC arrays for the three boundaries to ON, as highlighted in green in the figure below.

(iii) Arbitrarily adjust the power supply voltage using the black dials, and polarity settings using the toggle switches for each TEC array. Importantly, make sure the dials are turned to 50% or greater.



(12) Wait for the center temperature of the ROI to reach steady-state. You can monitor the center temperature from the Statistics Window, or via the cursor. In case of the latter, place the cursor near the center of the ROI; a tag will appear next to the cursor showing the temperature at the cursor location. It should take about 5 minutes for the system to reach steady state.

(13) Capture and export the data.

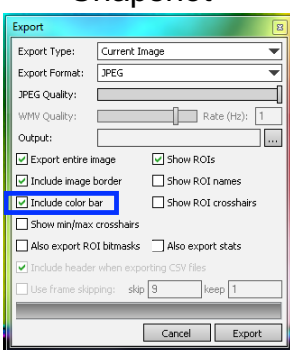
(i) Capture the image by pressing the keyboard Pause/Break key (above the end key).

(ii) Export the snapshot of the captured image by selecting File > Export from top menubar. In the Export popup window, select the JPEG file format. And, check the include colorbar option, as indicated in the figure below.

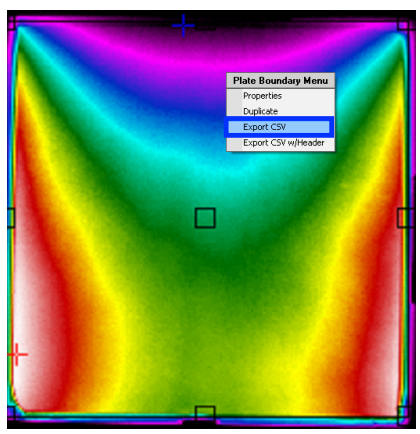
(iii) Export the statistics for the ROI by selecting the Save File icon on the Stats Window, as shown in the figure below.

(iv) Export the numeric temperature data from the captured image by right-clicking inside the ROI. In the Plate Boundary Menu popup window, select the "Export CSV" option as shown below. Note, the CSV file contains the temperature measurements in °C in a comma-separated file format. Note, the first row of measurements in the file corresponds to the temperature along the TOP edge of the plate; while, the bottom row of measurements corresponds to the temperature along the BOTTOM edge of the plate.

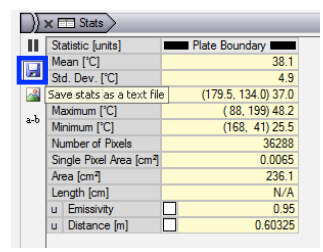
Export Image Snapshot



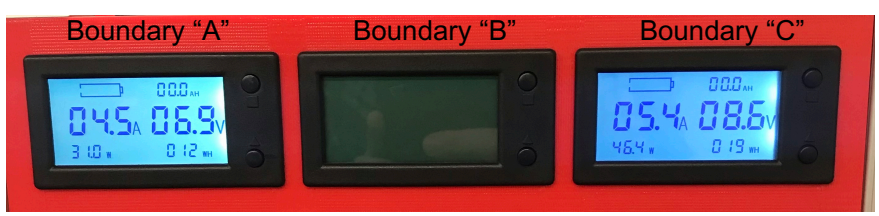
Export Temperature Measurements



Export Statistics of ROI



(v) Using your personal phone, take a photo of the Power Meters located at the backside of the test specimen. Note, if any of the boundaries are operated in the "cooling" mode, then the power meter for that boundary will be blank. You should obtain an image similar to the one below.



*Boundary "B" was cooled in this case; while boundaries "A" and "C" were heated.

(14) When the experiment is complete: (i) turn off the IR camera via the power button, and (ii) turn off the power supply to the TECs by flipping all individual switches on the power box to OFF and then flipping the main switch to OFF.

Data Analysis

1. Loading and Plotting the Temperature Data in Matlab

   It is important to recognize that the ExaminIR software saves the temperature data starting from the upper-left corner of the image. This means that the first row in your data file contains the temperatures along the top edge of the plate; and, the last row in your data file contains the temperatures along the bottom edge of the plate. If you use the load or readmatrix command to read your data file into Matlab, the temperatures in the first row (corresponding to the top edge of the plate) will be assigned an index of $i = 1$. Matlab assumes that an index of $i = 1$ corresponds physically to $y = 0$. This means that when you generate a contour plot of your data, the first row will be plotted at the bottom along $y = 0$; and your measured temperature matrix will look upside down compared to the snapshot of the thermal image from the camera. To remedy this, you will need to flip the rows of your measured temperature matrix as follows:

```
Tdata = readmatrix('MyFileName.csv');
Tdata = flipud(Tdata);
```

   In Matlab, you can use the following commands of code to generate a filled-color contour plot with 20 isotherms (lines of constant temperature):

```
[M,N]=size(Tdata);
x=linspace(0,L,N);
y=linspace(0,H,M)';
contourf(x,y,Tdata,20);
colorbar;
```

   Here, `L` denotes the horizontal length of the plate in the x-direction, `H` denotes the vertical height of the plate in the y-direction, `N` and `M` denote the number of pixels in your image in the x and y-directions, respectively. Note, L and H should be determined based on the size (number of pixels) in your temperature matrix, along with the Single Pixel Area value output by the software. Note, each pixel is square with the same area. However, L and H may not be square, depending on how you selected your ROI.

2. Calculating the Heat Transfer Rate across the Boundaries

   In many instances, you might be interested in calculating the rate of heat transfer across the boundaries of your object. For example, at any y-location along the left boundary, the heat flux would be

$$q''_L(y) = -k \left.\frac{\partial T}{\partial x}\right|_{x=0} , \tag{1}$$

   where k denotes the thermal conductivity of the test specimen. To find the rate of heat transfer across the entire left boundary, we would integrate with respect to y and multiply by the thickness t of the test specimen,

$$q_L = t \int_{y=0}^{H} q''_L(y)\, dy , \tag{2}$$

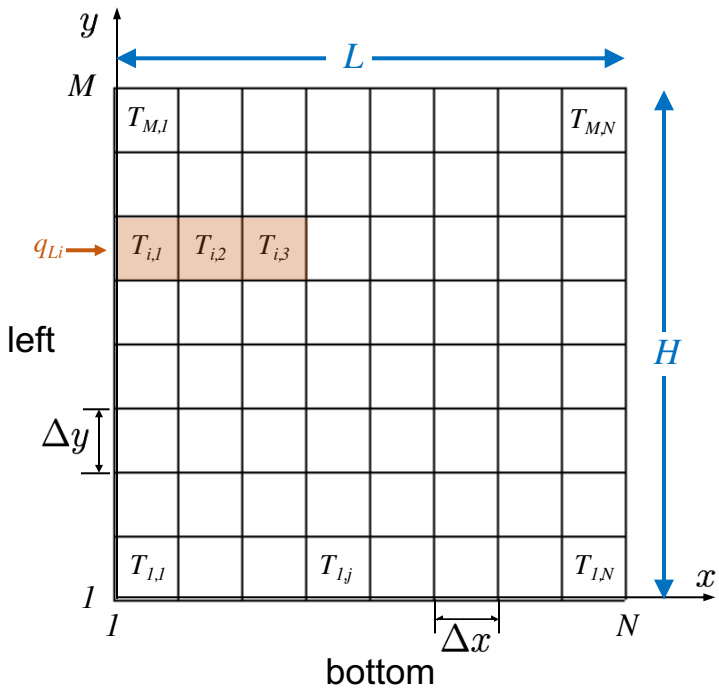


Figure 6. Temperature matrix with indices used to calculate the rate of heat transfer across the boundaries of the test specimen. Three pixels are required in order to obtain a second-order accurate derivative.

where the units of q_L are Watts. We will use the measured temperatures and perform the derivatives in (1) numerically using the second-order forward finite difference method, and the integral in (2) numerically using the second-order trapezoidal method.

Figure 6 illustrates the temperature matrix where each square represents a pixel. If we want to find the heat transfer rate from the left boundary at the ith row, as highlighted in the figure, then the second-order finite forward difference equation yields

$$q''_{Li} = -k \left[\frac{-T_{i,3} + 4\,T_{i,2} - 3\,T_{i,1}}{2\,\Delta x} \right] , \tag{3}$$

where Δx denotes the pixel size. Forward finite difference operations would be performed to find the heat flux across the bottom boundary as well. To calculate the heat flux across the top and right boundaries, a second-order backward difference method would be used. For example, to calculate the heat transfer rate from the right boundary at the ith row, one would use the following,

$$q''_{Ri} = -k \left[\frac{3\,T_{i,N} - 4\,T_{i,N-1} + T_{i,N-2}}{2\,\Delta x} \right] . \tag{4}$$

One can then use the `trapz` command in Matlab to integrate the heat flux values across each of the four boundary edges to find the heat transfer rate according to (2).

Numerical Simulation

Numerical simulation is an important design tool in mechanical engineering. In this lab, you will write a computer code to solve the heat diffusion equation in a two-dimensional domain using boundary conditions that are consistent with those from the experiment. The numerical simulation is based on the First Law of Thermodynamics, which can be simplified for the control volume drawn in Figure 3 as

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = 0\,. \tag{5}$$

Note, the above equation assumes constant material properties and no heat generation inside the domain. Furthermore, the validity of (5) relies on the assumption that no heat leaves the front/back faces of the domain, i.e., that the system is truly two-dimensional. In order to solve (5), four boundary conditions are required, two each in the x- and y-directions. For consistency with the experiment performed in the lab, the left $(x=0,y)$, right $(x=L,y)$, and top $(x,y=H)$ edges are taken as constant temperature boundaries; while, the bottom $(x,y=0)$ edge is taken as an insulated boundary (i.e., zero heat-flux). Here, L and H denote the horizontal and vertical side lengths of the test specimen, respectively. The boundary conditions can be expressed mathematically as

$$T(x=0,y) = T_a(y)\,, \quad T(x=L,y) = T_b(y)\,, \quad T(x,y=H) = T_c(x)\,, \quad \left.\frac{\partial T}{\partial y}\right|_{(x,y=0)} = 0\,, \tag{6}$$

where T_a, T_b, and T_c are the prescribed temperatures along the left, right, and top edges of the plate, respectively. Note, since the temperatures are not entirely uniform across the boundaries, T_a, T_b, and T_c are not constant, but vary with x and y as indicated in (6).

This problem can be solved numerically for the temperature distribution $T(x,y)$ everywhere in the domain $(0 \le x \le L,\ 0 \le y \le H)$ using the finite-difference formulas for the derivatives. The finite-difference method requires that the physical space is discretized into a mesh (or grid) of $N \cdot M$ nodes with a spacing between nodes of $\Delta x = \Delta y$, as illustrated in Figure 7. Note, the product $\Delta x \cdot \Delta y$ is equal to the Single Pixel Area output by the FLIR ExaminIR software. Importantly, the number of nodes in your numerical mesh should equal the number of pixels in your experimental data set. Applying a second-order central difference formula for the derivatives in (5) yields an algebraic equation for the unknown temperature at the (i,j) node relative to its nearest neighbors,

$$T_{i,j} = \tfrac{1}{4}\left(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}\right)\,. \tag{7}$$

Note, the above equation must be satisfied at every (i,j) node in the domain, for $1 \le i \le M$ and $1 \le j \le N$. The boundary conditions appropriate for the experiment are as follows:

$$\text{west:}\ \ T_{i,j=1} = \tilde{T}_{i,j=1} \tag{8}$$

$$\text{north:}\ \ T_{i=N,j} = \tilde{T}_{i=N,j} \tag{9}$$

$$\text{east:}\ \ T_{i,j=N} = \tilde{T}_{i,j=N} \tag{10}$$

$$\text{south:}\ \ T_{i=1,j} = \tfrac{1}{4}\left(T_{i=1,j-1} + T_{i=1,j+1} + 2\,T_{i=2,j}\right) \tag{11}$$

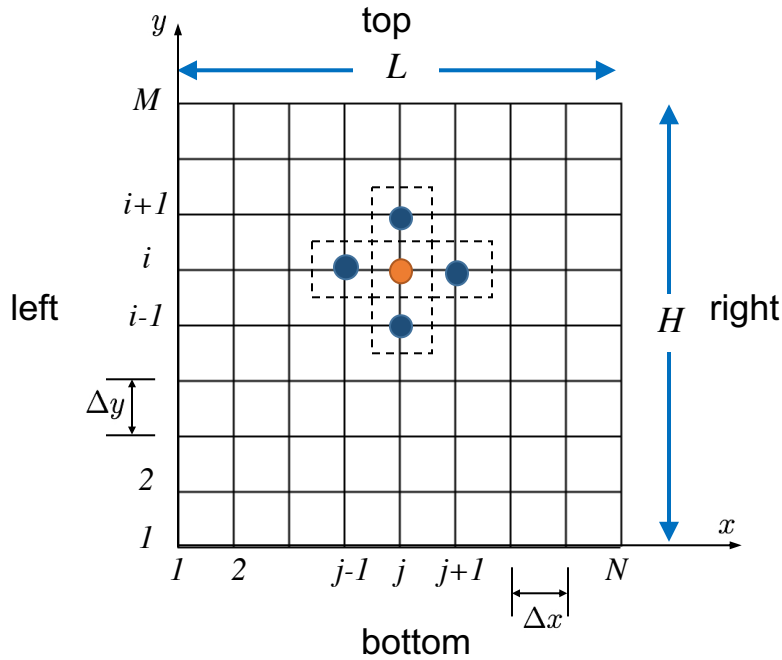


Figure 7. Grid notation for the finite-difference formulation. The intersections of the grid lines represent the nodes at which the numerical solution is obtained. The orange node represents the (i, j) node; while, the blue nodes represent its nearest neighbors. For comparison with the data from the IR camera, the location of the nodes correspond to the center of the pixels on the IR camera (drawn as dashed boxes).

where $\tilde{T}$ denotes the measured temperature at the corresponding pixels from the experimental data. The condition along the south boundary (11) represents that of an adiabatic, insulated boundary (i.e., zero heat flux along the bottom edge of the plate). Note, singularities at the corners of the mesh should be modeled as constant temperature boundaries (i.e., with prescribed temperature values based on the experimental data).

The code to solve your numerical model must be written in Matlab or Python (or other similar computer language). The numerical model will result in a very large system of linear equations of the form

$$[A]\,\vec{x} = \vec{b}, \tag{12}$$

where $\vec{x}$ denotes the unknown temperatures at each interior node in the domain plus the nodes along the south boundary. In other words, there is one unknown for each pixel in the image, or approximately 7.6800×10^5 unknowns. You may solve this problem by either (i) the Gauss Elimination method, using the backslash operator \ in Matlab, or (ii) the Gauss-Seidel method WITH over-relaxation. If attempting the first method, you will need to utilize the sparse command in Matlab when initializing the system matrix $[A]$; otherwise, your computer will likely not be able to allocate enough memory. If attempting the Gauss-Seidel method, over-relaxation must be implemented to speed up convergence, because $[A]$ ends up being diagonally dominant for this problem. When using the Gauss-Seidel method, a convergence

tolerance equal to 1×10^{-5} should be applied to ALL grid points in your mesh,

$$\frac{T_{i,j}^{\text{new}} - T_{i,j}^{\text{old}}}{T_{i,j}^{\text{new}}} \leq 1\times10^{-5}, \quad \text{for} \quad 1 \leq i \leq M \;\text{ and }\; 1 \leq j \leq N \tag{13}$$

where $T_{i,j}^{\text{new}}$ is the value of the temperature at the current iteration, and $T_{i,j}^{\text{old}}$ is the value of the temperature at the previous iteration for the same grid point. Once the temperature field is converged, the numerical result can be visualized using a contour plot of the isotherms.

Figures

Captions

A meaningful and comprehensive figure caption must accompany all figures (plots and tables). The figure caption must include the following label: Figure 1x. where x denotes the letter a – d according to the plot order listed below or Table 1e. for the last figure. Use the following guidelines when writing captions:

→ Ideally, the figure caption should provide a "standalone" description of the plot, and contain enough information that the reader can understand what is being shown without having to refer back to the main text of the document (report, memo, etc.).

→ The caption should start with a statement of the quantities plotted on both axes. Typically the axis labels will utilize mathematical symbols (with appropriate units) for the quantities being plotted; however, in the caption, these quantities are described in words.

→ Include relevant contextual information about the plot. For example, you should state that the measurements were acquired under steady-state conditions. You can also state the camera make/model used for the measurements. For the temperature distribution from the numerical simulation, you should state the type of numerical method used.

→ When your plot includes errorbars, it is helpful to provide information in the caption about the errorbars. For example, you should include the fact that the errorbars represent a 95% confidence interval.

→ When comparing with published data, it is appropriate to provide a reference for the publication.

→ If in doubt about what information to include in the caption, one should err on the side of providing more information, rather than less.

Plots and Tables

1a. Provide two figures, displayed side-by-side: (i) a snapshot of the thermal image displayed by the IR camera, and (ii) a filled-color contour plot of the measured temperatures from the IR camera. Use 20 isotherms when generating the contour plot, and include a colorbar for your contour plot. The horizontal and vertical axes of the contour plot must be in units of cm (NOT pixels). Provide labels with appropriate units for the axes. Either provide a label for the colorbar on the plot, or include a description of the colorbar with correct units in the figure caption.

1b. Display the computed temperatures from your numerical solution in a filled-color contour plot with 20 isotherms. Be sure to include a colorbar. The horizontal and vertical axes must be in units of cm (NOT pixels). Provide labels with appropriate units for the axes. Either provide a label for the colorbar on the plot, or include a description of the colorbar with correct units in the figure caption.

1c. Display the percent difference between the measured and computed temperature fields in a filled-color contour plot with 20 isotherms,

$$\epsilon_{i,j} = \frac{|\tilde{T}_{i,j} - T_{i,j}|}{\tilde{T}_{i,j}} \cdot 100\% \,,$$

where $\tilde{T}_{i,j}$ denotes the measured temperature field from the IR camera, and $T_{i,j}$ denotes the corresponding temperature field from your numerical simulation. Be sure to include a colorbar. The horizontal and vertical axes must be in units of cm (NOT pixels). Provide labels with appropriate units for the axes. Either provide a label for the colorbar on the plot, or include a description of the colorbar with correct units in the figure caption.

1d. Create two subplots that show the temperature distribution along each of the edges as a function of distance along the edge, based on the measurements from the IR camera. The two subplots should be presented as follows:

(i) Plot the measured temperatures along the top and bottom edges. Plot temperature in °C along the vertical axis, and x-distance across the edge in cm along the horizontal axis.

(ii) Plot the measured temperatures along the left and right edges. Plot temperature in °C along the horizontal axis, and y-distance across the edge in cm along the vertical axis.

Use a solid line with a different color for each of the four edges. Indicate the mean value for each edge using a dashed line with the same colors. Supply a legend in each subplot (only for the solid lines NOT the dashed lines). Label the axes in each subplot with appropriate units.

1e. Calculate the net heat transfer rate, q, in Watts across each boundary based on the measured temperatures from the IR camera, and complete the table below. Note, these calculations will involve taking both numerical derivatives and numerical integrals. You MUST use second-order or higher methods, such as the forward/backward finite difference formulas or a Lagrange interpolating polynomial for the derivaties, and the trapezoidal rule or Simpson rule for the integrals. Note, a second-order forward/backward finite difference method will utilize three data points (pixels) in order to calculate the derivative.

Edge	q (W)	P_E (W)
Left		
Top		
Right		
Bottom		

Short-Answer Questions

2a. Quantify (in terms of a percentage) the average difference between the measured and computed temperatures, i.e., average $\epsilon_{i,j}$ spatially over the entire domain. Are there regions in the domain where the differences are greater than the average? Explicitly state the maximum percent difference, and indicate where that occurs in the domain. Are there any trends observed in the difference contour (e.g., are there larger differences in the center of the domain compared to the edges)? If so, explain why differences are not uniform across the entire domain. Based on your engineering judgment, is your numerical solution accurate (explain why or why not)? What can be done to improve the accuracy of your numerical solution? [4-6 sentences]

2b. How uniform are the temperatures along the left, top, and right edges? Quantify your answer in terms of a percentage based on the ratio of the standard deviation over the mean, i.e.,

$$2\,\frac{\sigma_T}{\overline{T}} \cdot 100\% \tag{14}$$

for each edge, where σ_T denotes the standard deviation along the edge and $\overline{T}$ denotes the average value along the edge. Note, the factor of 2 is used, because a spread of 2 standard deviations in a normal (Gaussian) probability density distribution captures 95% of the entire distribution, and thus yields a good measure of how far the measurements deviate from the average. Based on your engineering judgment, would it have been appropriate to assume a uniform temperature distribution along the left, top, and right boundaries in your numerical model (explain why or why not)? Does the measured temperature data along the bottom edge indicate that an adiabatic condition is appropriate for the numerical model (justify your answer)? [4 sentences]

2c. Based on your q calculations from Table 1e, estimate how much total power was required to heat/cool the boundaries of the plate in Watts. Assume that the TECs are 15% efficient in cooling mode and 95% efficient in heating mode, which are typical values specified for these types of devices. Compare these calculations to the actual measured electrical power supplied to the TECs. Explain any discrepancies between the two. [3-4 sentences, plus equations as appropriate]