**ME 5200/6200 Classical Controls**

**Homework 09 Solutions**

**Problem 1**

Calculate the magnitude and phase of

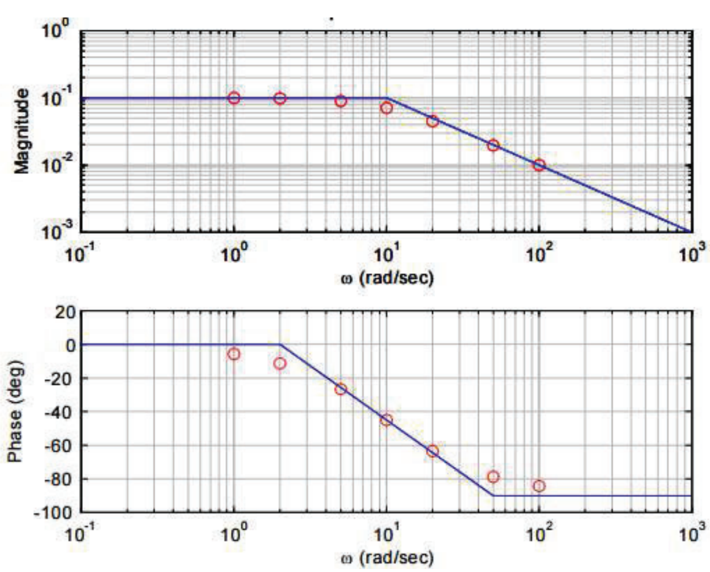
$$G(s) = \frac{1}{s+10}$$

by hand for $\omega = 1, 2, 5, 10, 20, 50,$ and $100$ rad/sec.

**Solution:**

$$\begin{aligned} G(s) &= \frac{1}{s+10},\ G(j\omega) = \frac{1}{10+j\omega} = \frac{10-j\omega}{100+\omega^2} \\ |G(j\omega)| &= \frac{1}{\sqrt{100+\omega^2}},\ \angle G(j\omega) = -\tan^{-1}\frac{\omega}{10} \end{aligned}$$

| $\omega$ | $\lvert G(j\omega)\rvert$ | $\angle G(j\omega)$ |
|---|---|---|
| 1 | 0.0995 | −5.71 |
| 2 | 0.0981 | −11.3 |
| 5 | 0.0894 | −26.6 |
| 10 | 0.0707 | −45.0 |
| 20 | 0.0447 | −63.4 |
| 50 | 0.0196 | −78.7 |
| 100 | 0.00995 | −84.3 |



**Problem 2**

**Sketch by hand the Bode plots for L(s) for part (a), (b) and (c). Use the 'bode(SYS)" plot function in Matlab to create Bode plots for part (d). Note the units for the frequency when using the Matlab "bode" function.**

(a) $L(s) = \dfrac{2000}{s(s+200)}$

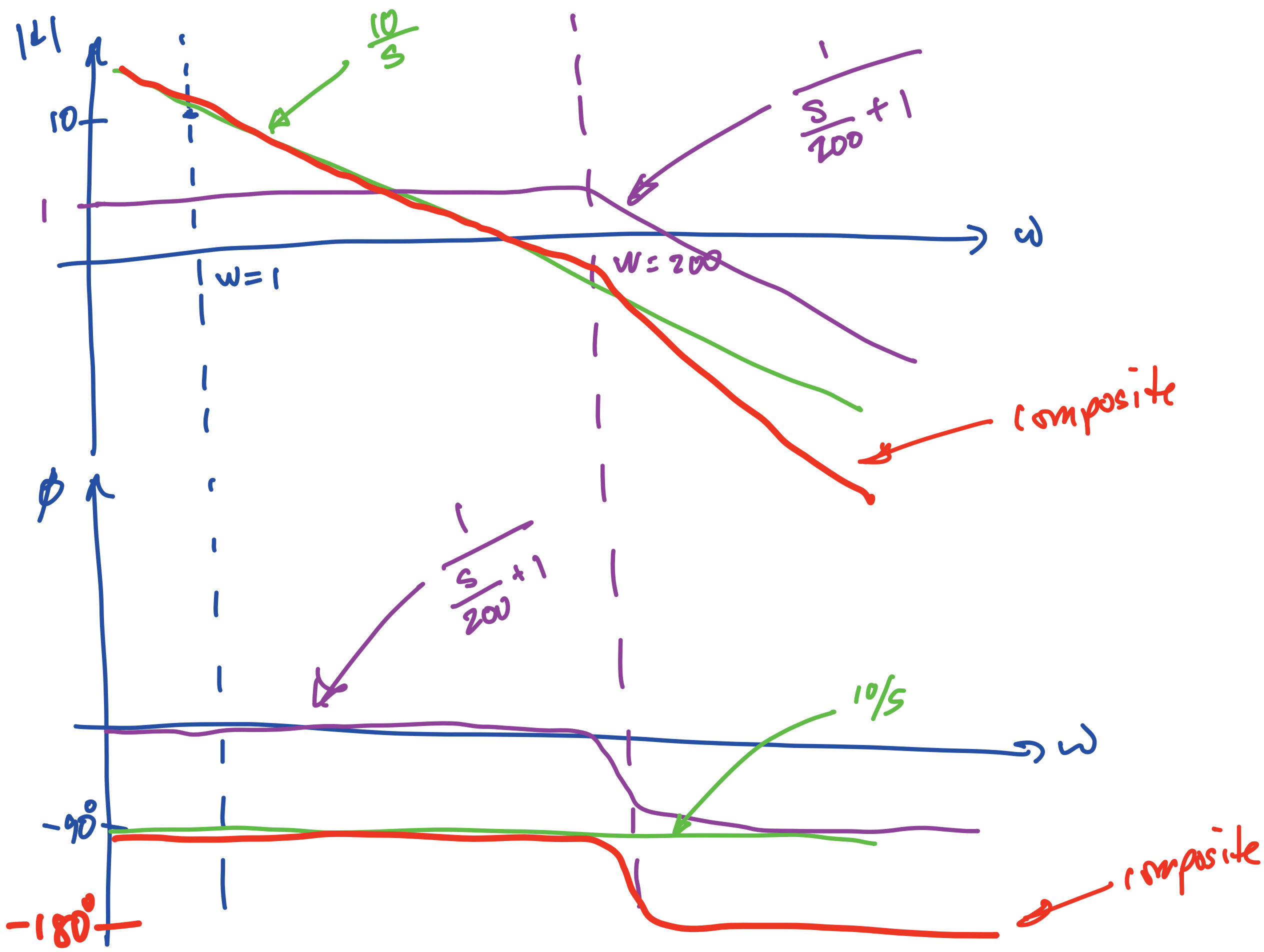
(b) $L(s) = \dfrac{100}{s(0.1s+1)(0.5s+1)}$

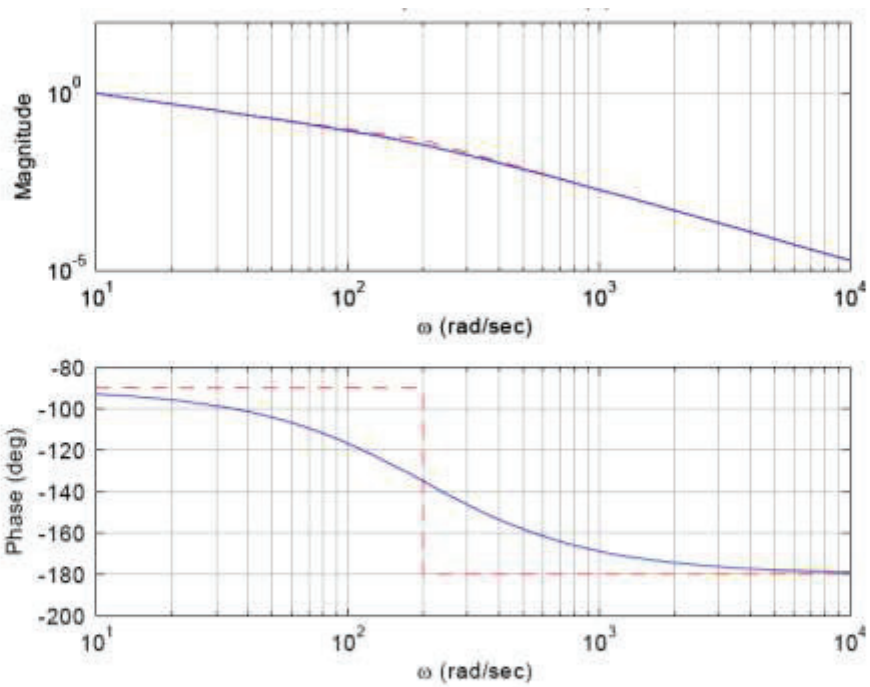
(c) $L(s) = \dfrac{(s+2)}{s(s+10)(s^2+2s+2)}$

(d) $L(s) = \dfrac{(s+2)}{s^2(s+10)(s^2+6s+25)}$

(a) $L(s) = \dfrac{10}{s\left[\frac{s}{200}+1\right]}$

The asymptote will have magnitude = 10 at $\omega = 1$ and have a -1 slope at the low frequencies. So the low frequency asymptote will pass through magnitude = 1, at $\omega = 10$. At $\omega = 200$, the slope changes to -2, and that slope continues forever. The phase asymptotes are even simpler and shown in the plot below along with the Matlab generated exact curve.



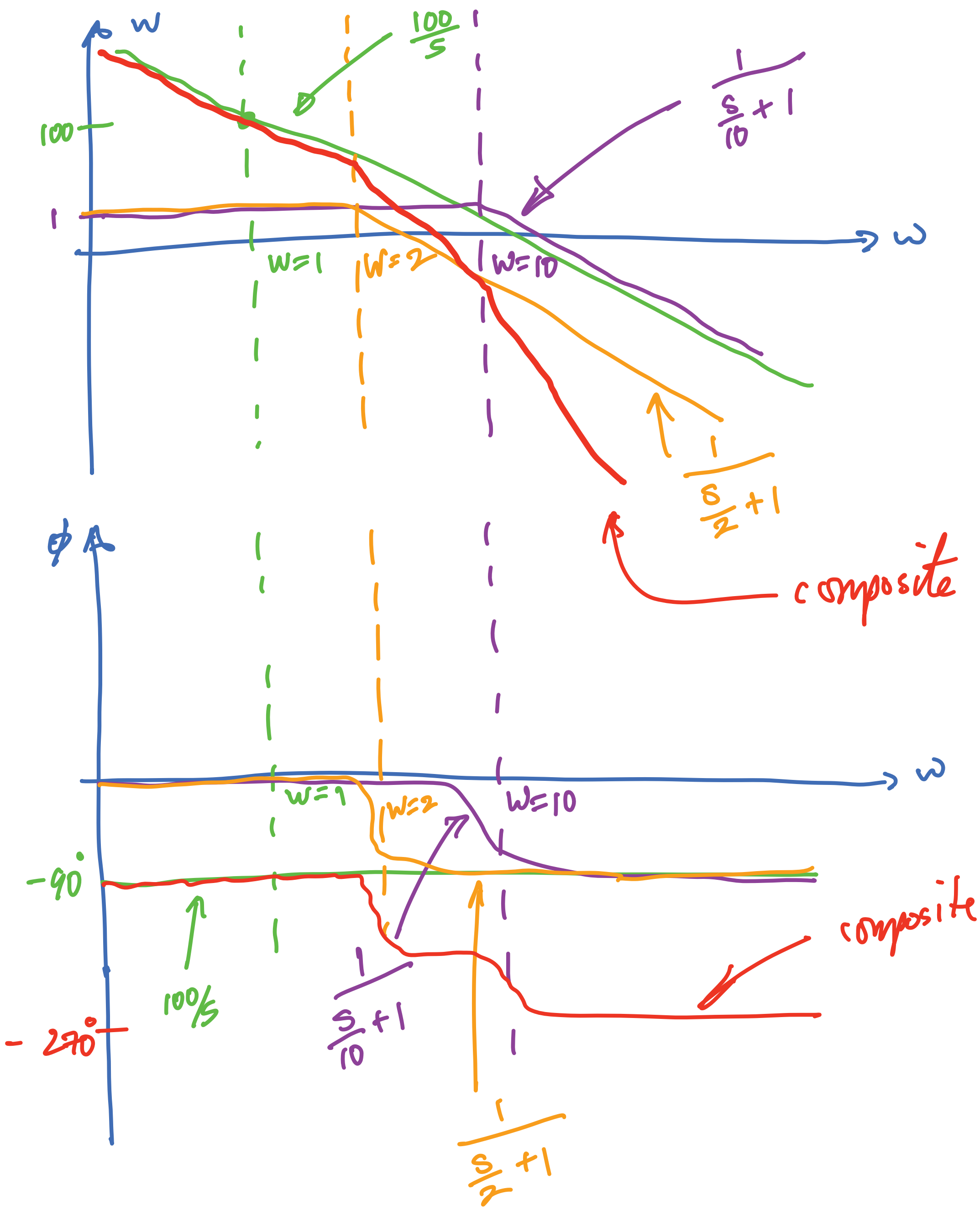


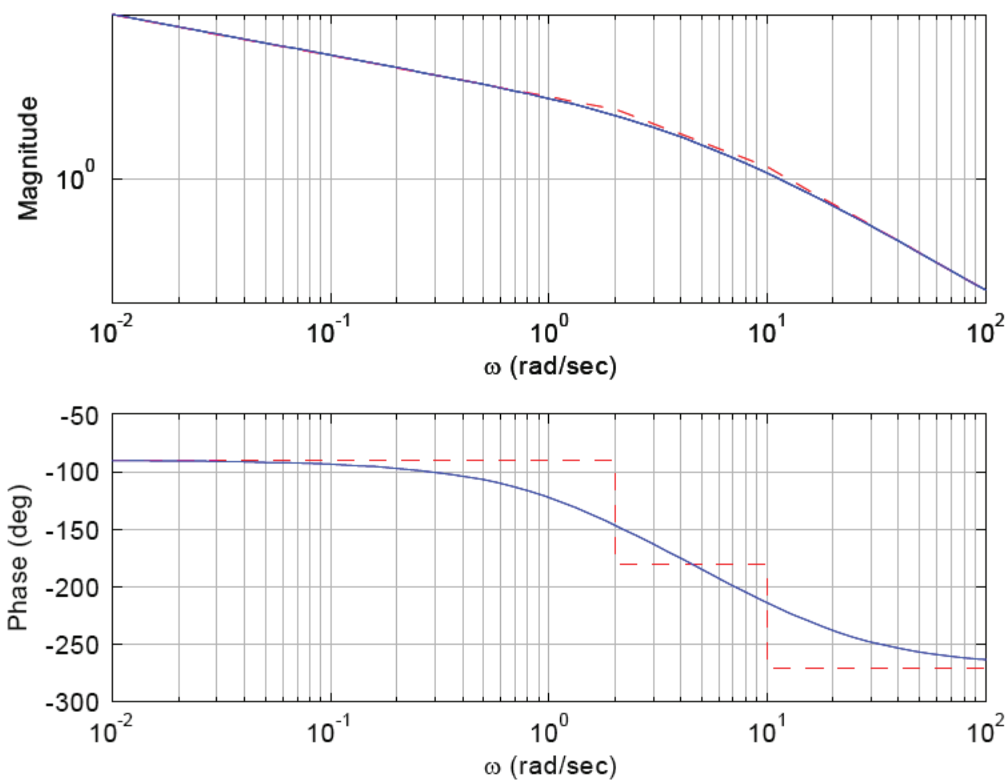
Magnitude
$10^0$
$10^{-5}$
$10^1$
$10^2$
$10^3$
$10^4$
ω (rad/sec)
Phase (deg)
-80
-100
-120
-140
-160
-180
-200
$10^1$
$10^2$
$10^3$
$10^4$
ω (rad/sec)

(b)

$$L(s) = \frac{100}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{2}+1\right)}$$

Breakpoints are at 2 and 10. The slope starts at n = -1, the changes to -2 at $\omega = 2$, then changes to -3 at $\omega = 10$. The magnitude on the -1 slope goes through 100 at $\omega = 1$.Thus, will be 10,000 at $\omega = 0.01$

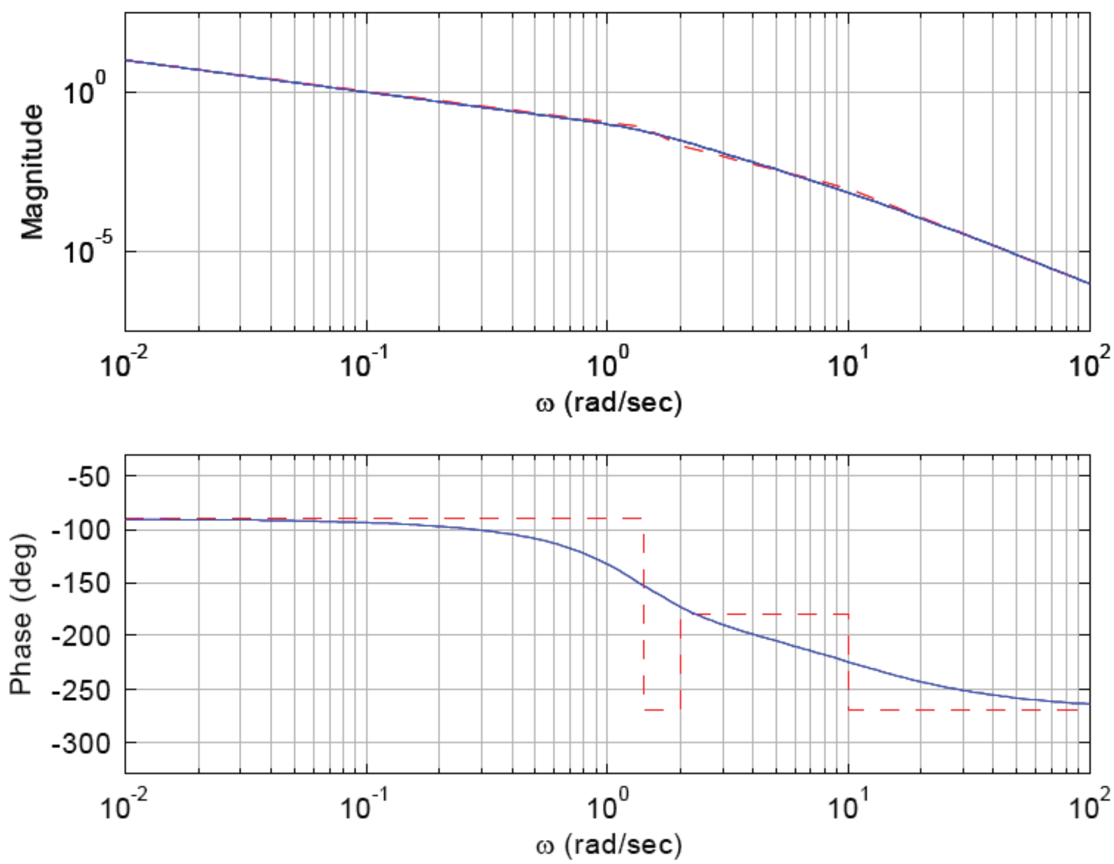




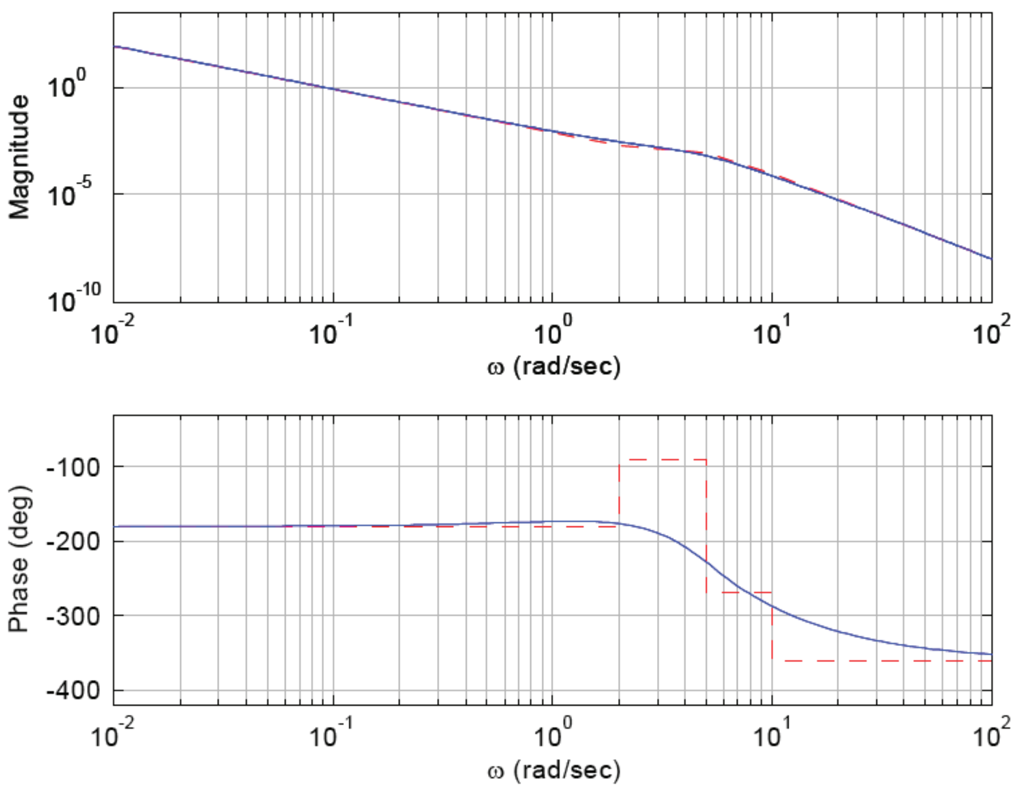
Magnitude
$10^0$
$10^{-2}$
$10^{-1}$
$10^0$
$10^1$
$10^2$
ω (rad/sec)
-50
-100
-150
-200
-250
-300
Phase (deg)
$10^{-2}$
$10^{-1}$
$10^0$
$10^1$
$10^2$
ω (rad/sec)

(c) and (d)

$$L(s) = \frac{\frac{1}{10}\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{10}+1\right)\left[\left(\frac{s}{\sqrt{2}}\right)^2 + s + 1\right]}$$



$$L(s) = \frac{\frac{1}{125}\left(\frac{s}{2}+1\right)}{s^2\left(\frac{s}{10}+1\right)\left[\left(\frac{s}{5}\right)^2+\frac{6}{25}s+1\right]}$$



**Problem 3**

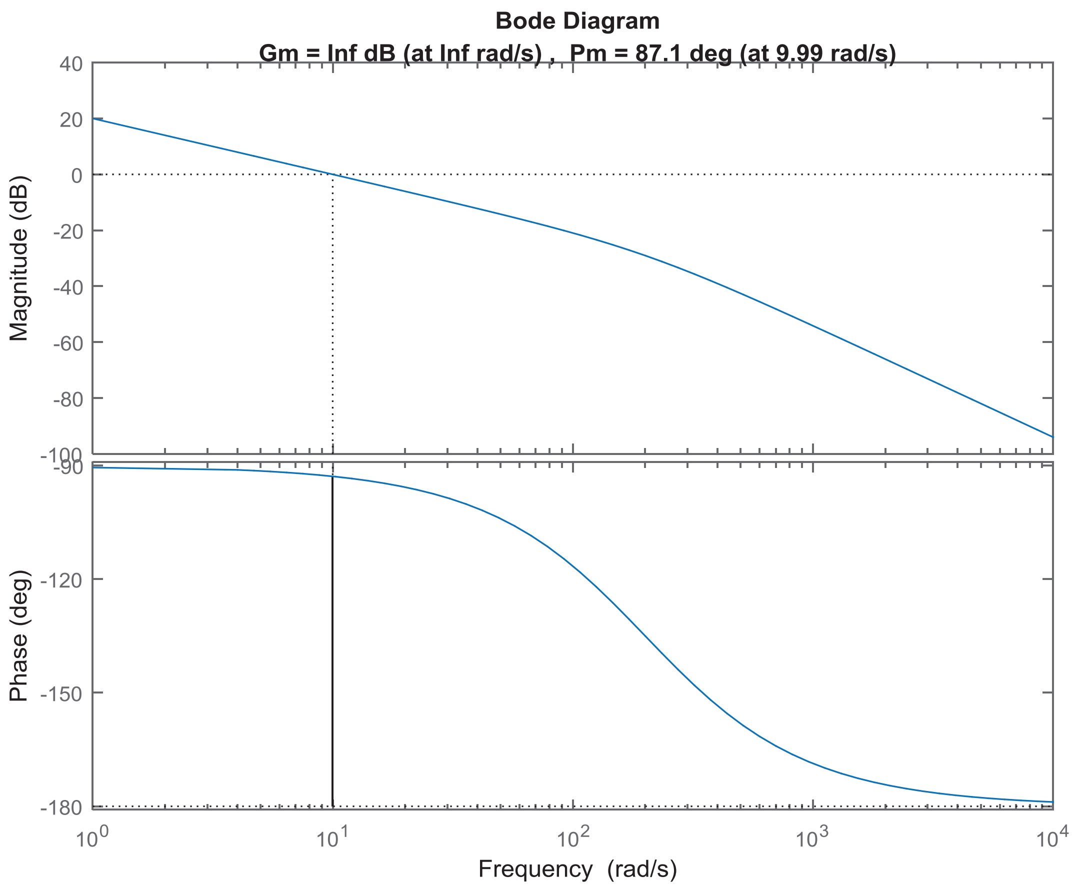
**Use Matlab to make the Bode plots for each below and determine the gain and phase margins. Submit our Matlab plots and sketch on the plots how you found the gain and phase margins.**

(a) $L(s) = \dfrac{2000}{s(s+200)}$

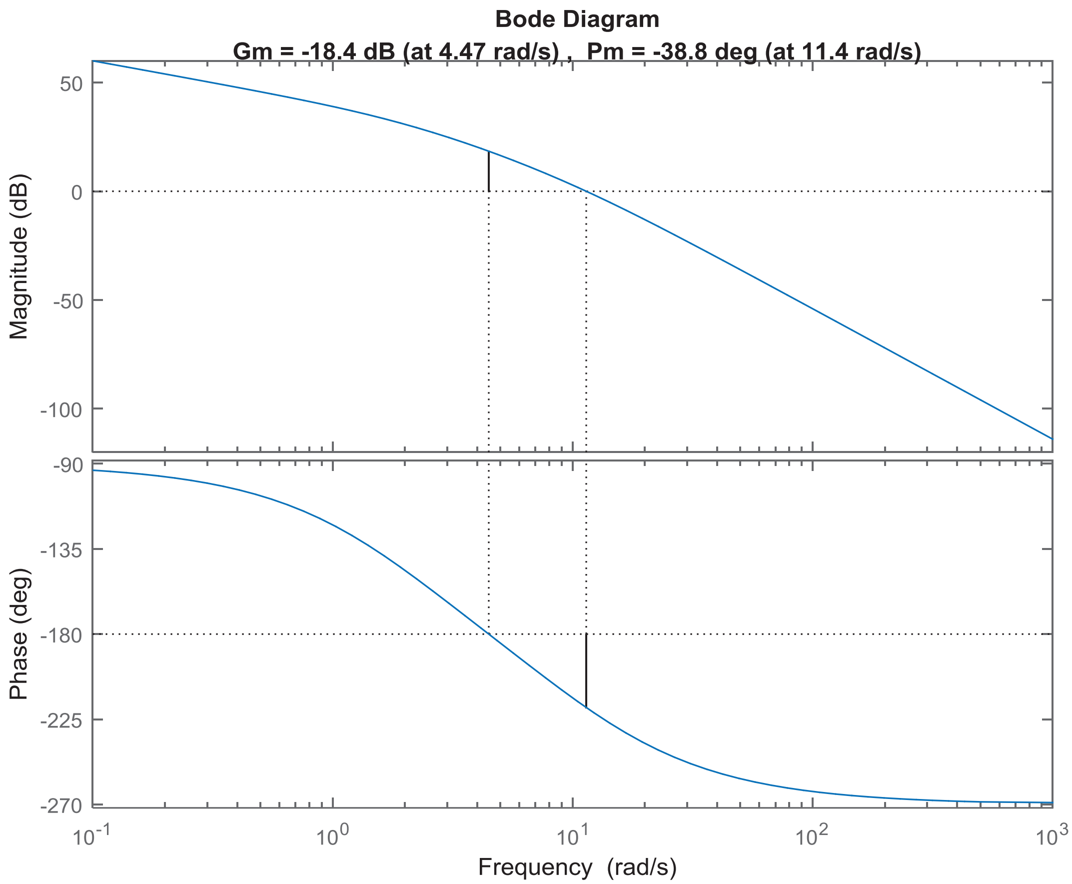
(b) $L(s) = \dfrac{100}{s(0.1s+1)(0.5s+1)}$

(c) $L(s) = \dfrac{1}{s(s+1)(0.02s+1)}$

**(a)**

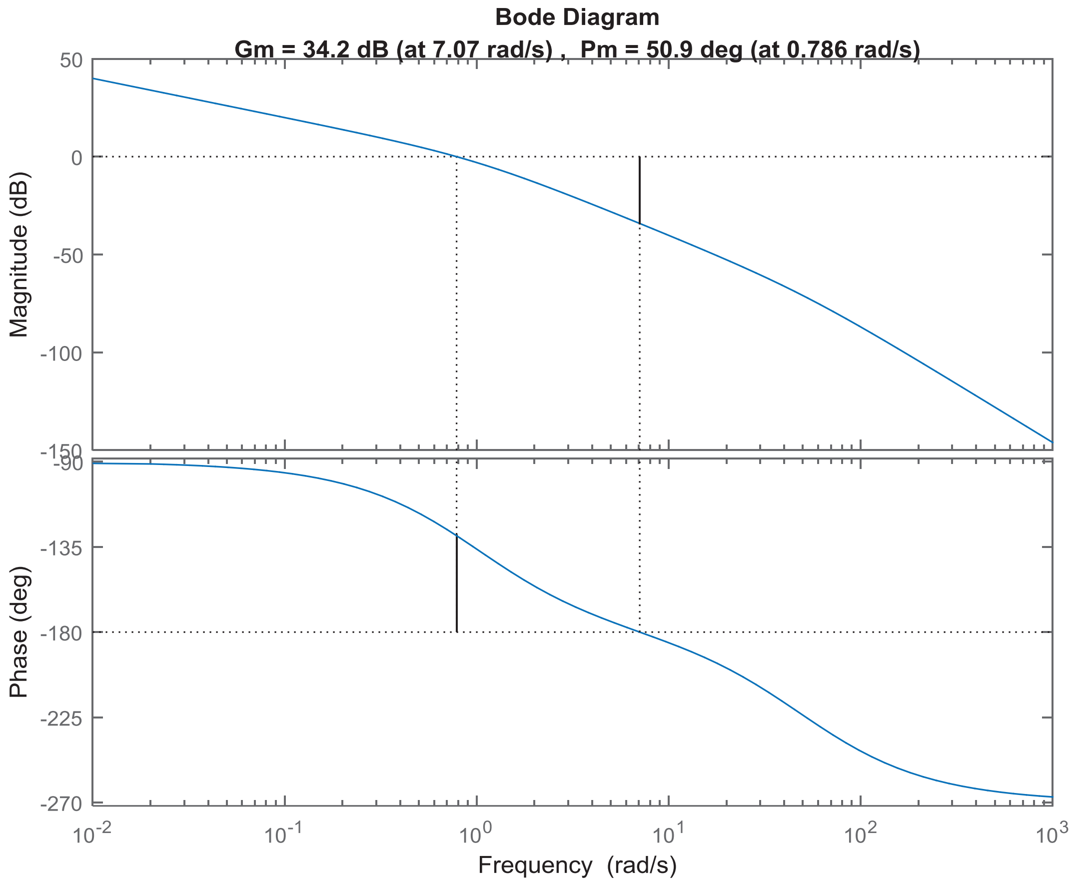


**(b)**



Bode Diagram
Gm = -18.4 dB (at 4.47 rad/s) , Pm = -38.8 deg (at 11.4 rad/s)
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-135
-180
-225
-270
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
Frequency (rad/s)

**(c)**


Bode Diagram
Gm = 34.2 dB (at 7.07 rad/s) , Pm = 50.9 deg (at 0.786 rad/s)
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
-90
-135
-180
-225
-270
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/s)

**Problem 4**

Consider the unity feedback system with the open-loop transfer function

$$G(s) = \frac{K}{s(s+1)[(s^2/25) + 0.4(s/5) + 1]}.$$

(a) Use MATLAB to draw the Bode plots for $G(j\omega)$ assuming $K = 1$.

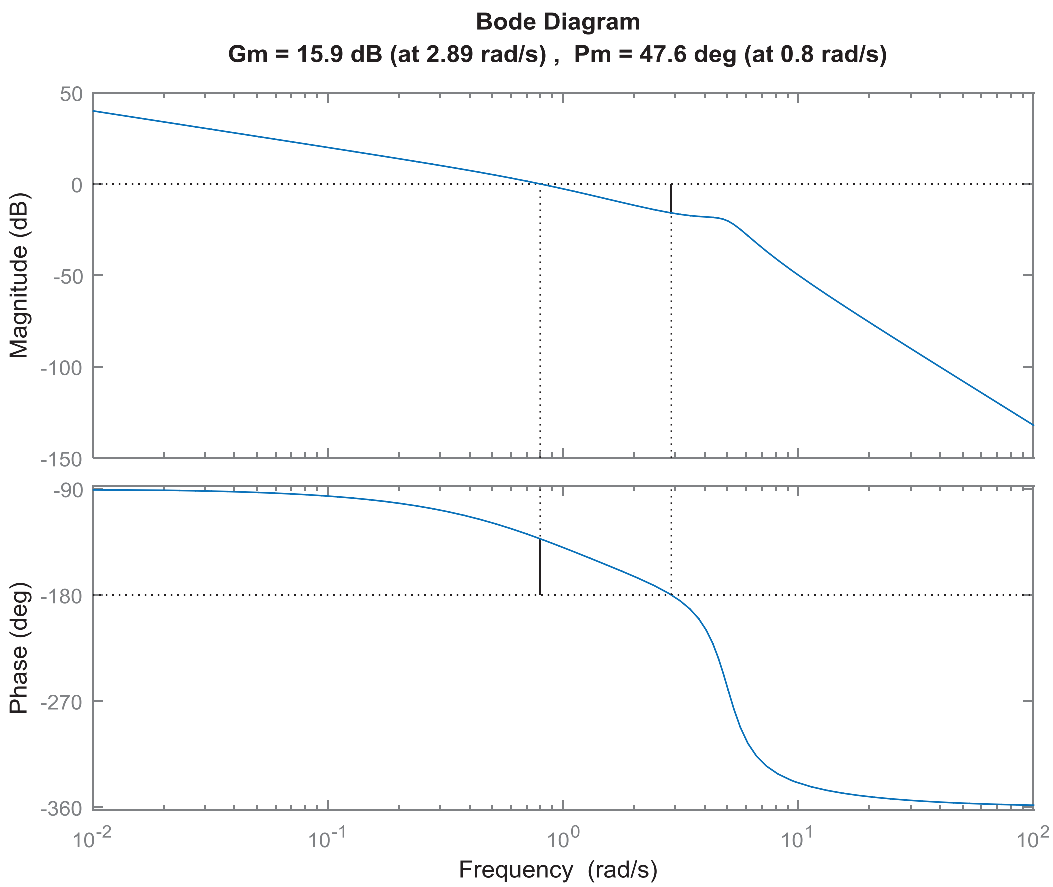
(b) What gain $K$ is required for a PM of 45°? What is the GM for this value of $K$?

(c) What is $K_v$ when the gain $K$ is set for PM = 45°?

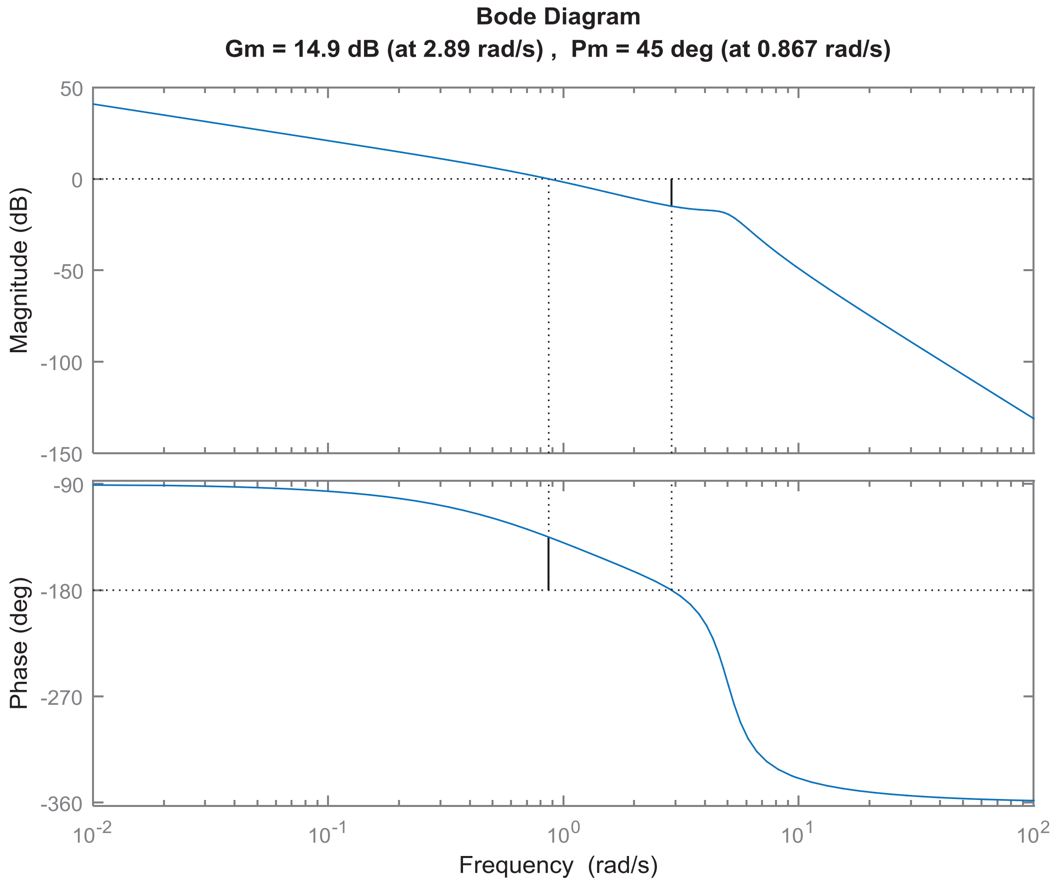
(d) Create a root locus with respect to $K$, and indicate the roots for a PM of 45°.

Note for Part (c), Kv is the velocity constant from the steady-state error analysis lecture.

(a)



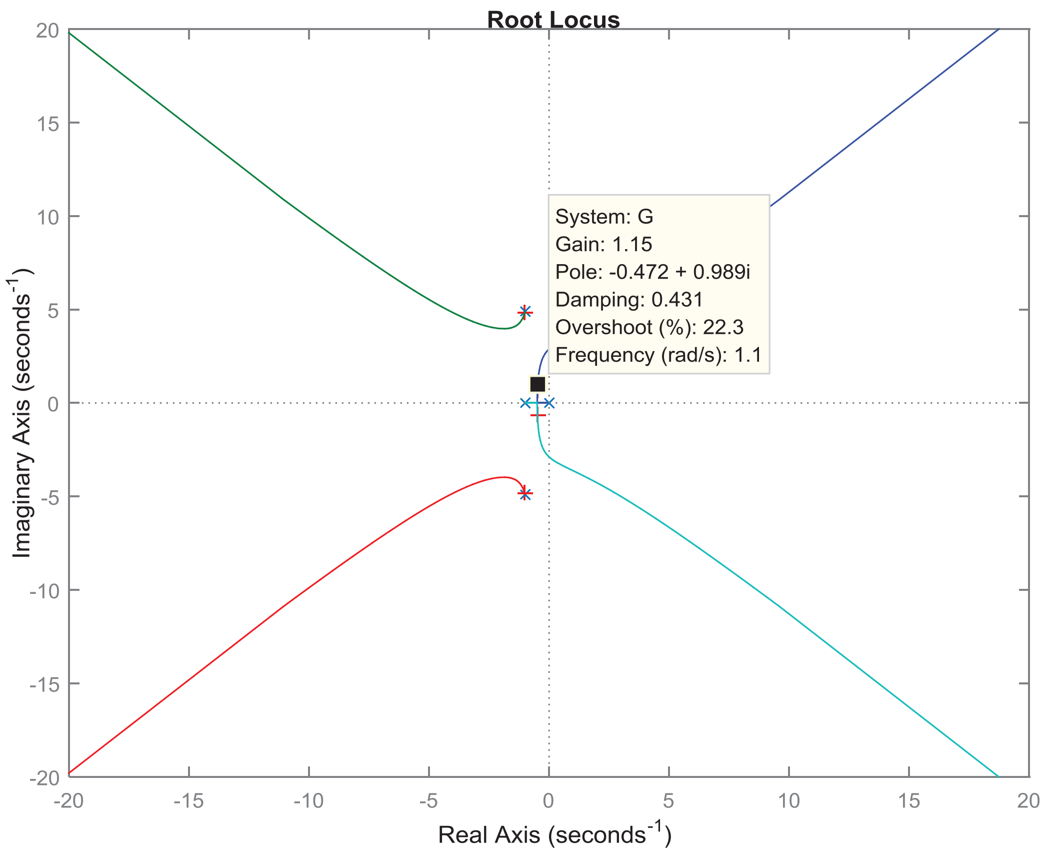
(b) From the plot above, PM is 47.6 degrees for K=1. To get PM of 45 degrees, we need to increase K to about K=1.115, found from trial by error. See plot below for bode of K=1.115:




(c)

Kv = lim s→0 [sG(s)] = 1.115

(d) See below, location is approximately as shown using rlocfind function in Matlab.



Root Locus
System: G
Gain: 1.15
Pole: -0.472 + 0.989i
Damping: 0.431
Overshoot (%): 22.3
Frequency (rad/s): 1.1
20
15
10
5
0
-5
-10
-15
-20
Imaginary Axis (seconds-1)
-20
-15
-10
-5
0
5
10
15
20
Real Axis (seconds-1)