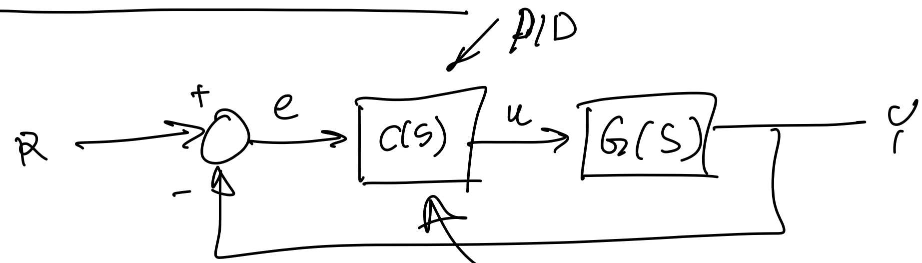
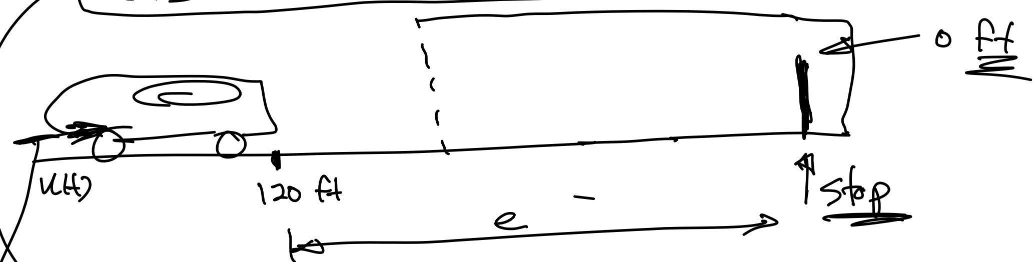
Classical PID control



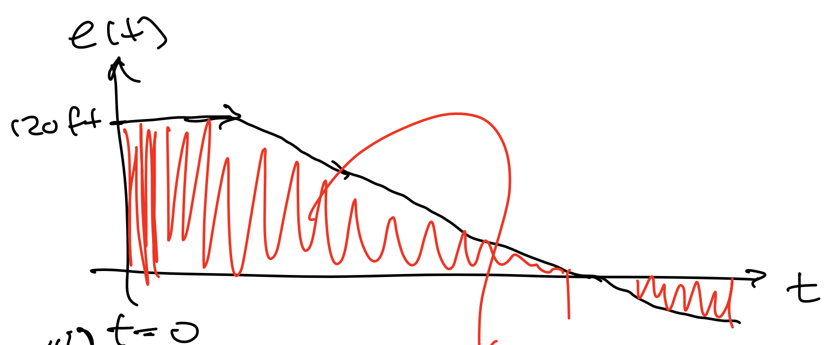
PID: proportional + Integral + derivative.

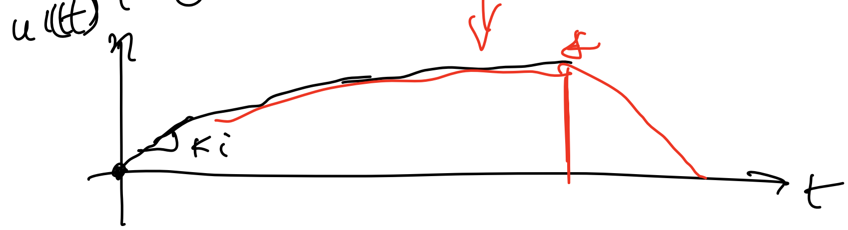
$$\boxed{\frac{U(s)}{e(s)} = K_p + K_i/s + K_d s}$$



$$U(s) = K_p e(s) + \frac{K_i}{s} e(s) + K_d s\, e(s)$$

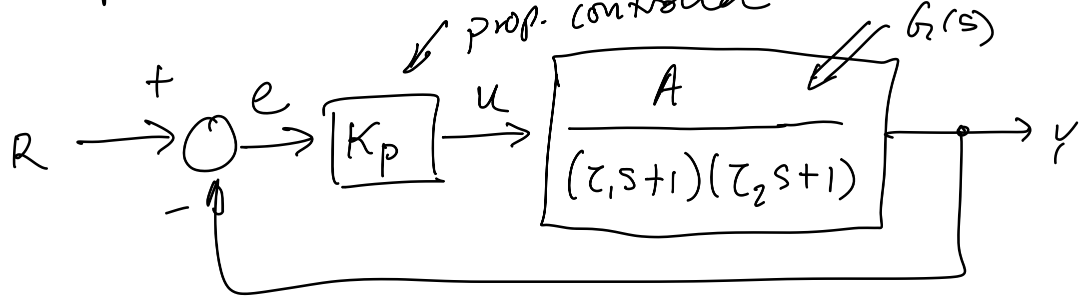
$$\boxed{u(t) = K_p e(t) + K_i \int e(t)dt + K_d \dot{e}(t)}$$





PID : Closer look

1) Proportional controller



$$\frac{u(s)}{e(s)} = K_p \quad \text{prop. controller}$$

$$\frac{Y}{R} = \frac{K_p G(s)}{1+K_p G(s)} = \frac{K_p\left(\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}\right)}{\underbrace{1+K_p\left(\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}\right)}_{\text{poles of Cloop}}}$$

characteristic eq:

$$1+K_p G(s) = 1 + K_p \frac{A}{(\tau_1 s+1)(\tau_2 s+1)} = 0$$

$$(\tau_1 s+1)(\tau_2 s+1) + K_p A = 0$$

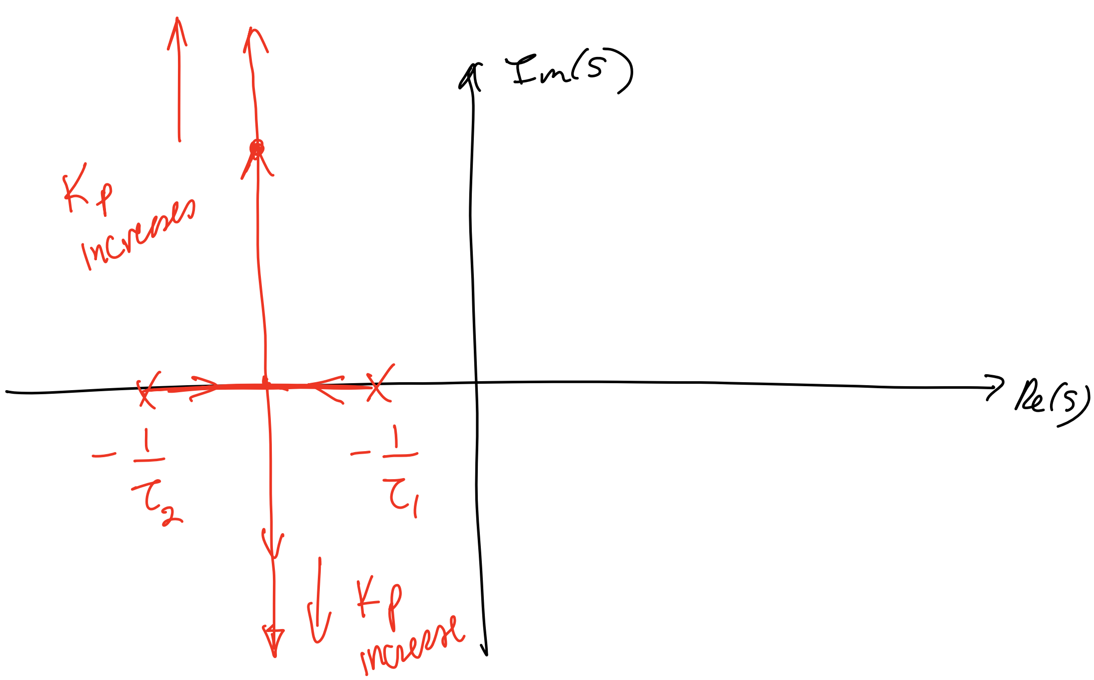
$$\underbrace{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + AK_p + 1 = 0}_{\text{quadratic eq} \Rightarrow 2 \text{ poles!}}$$

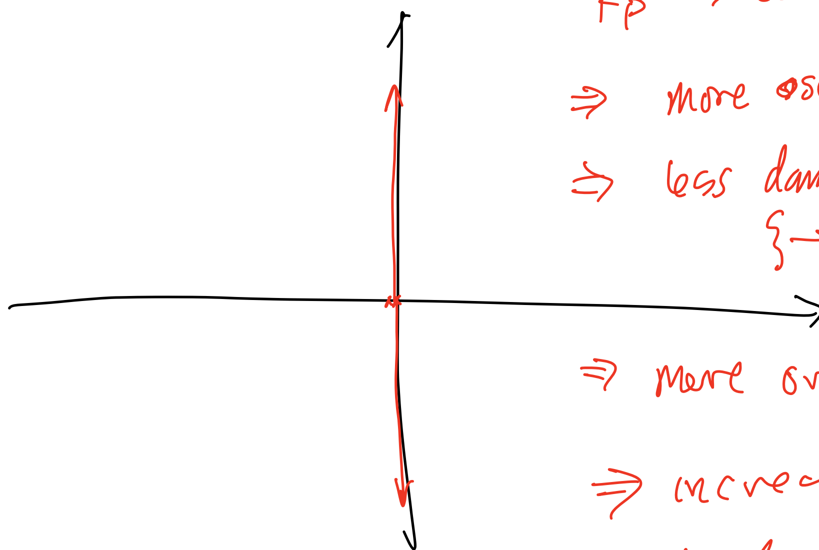
<u>$K_p = 0$</u>

$$s = -\frac{1}{\tau_1} \quad \text{and} \quad s = -\frac{1}{\tau_2}$$

<u>when $K_p > 0$</u>

$$s_{1,2} = \frac{-(\tau_1+\tau_2) \pm \sqrt{(\tau_1+\tau_2)^2 - 4\tau_1\tau_2(1+AK_p)}}{2\tau_1\tau_2}$$





$K_p \to \infty$

$\Rightarrow$ more oscillations

$\Rightarrow$ less damping $\zeta \to 0$

$\Rightarrow$ more overshoot

$\Rightarrow$ increase ω_n leads to faster response.

Integral Control

Primarily used to deal w/ steady-state error, but can result in poor transient behavior and worse, can destabilize the system.

$$C(s) = \frac{u(s)}{e(s)} = \frac{K_i}{s}$$

Take back to time domain.

$$\Rightarrow u(t) = K_i \int e(t)\, dt$$

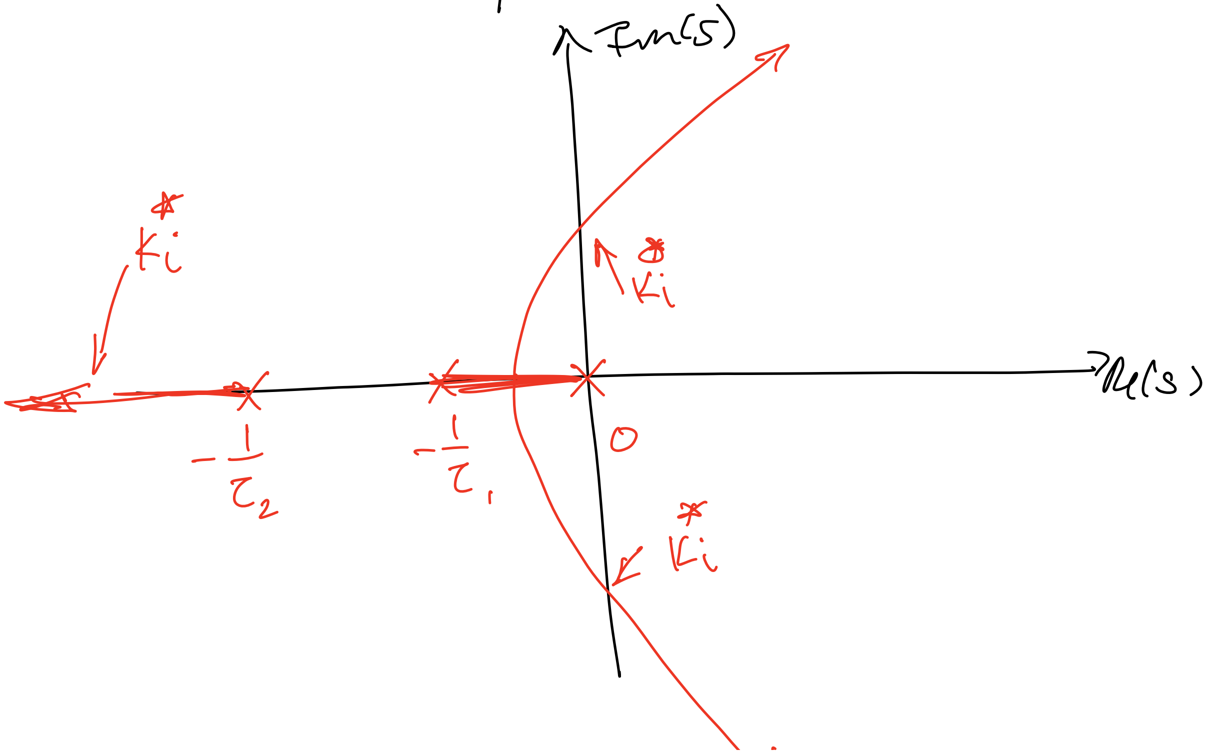
$$\boxed{G(s) = \frac{A}{(\tau_1 s+1)(\tau_2 s+1)} \qquad C(s) = \frac{K_i}{s}}$$

<u>Closed-loop poles:</u>

$$1 + C(s)\,G(s) = 1 + \frac{K_i}{s}\left(\frac{A}{(s\tau_1+1)(s\tau_2+1)}\right) = 0$$

$$\Rightarrow s(\tau_1 s+1)(\tau_2 s+1) + K_i A = 0$$

$$\Rightarrow \underbrace{\tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + s + K_i A}_{\text{poles}} = 0$$

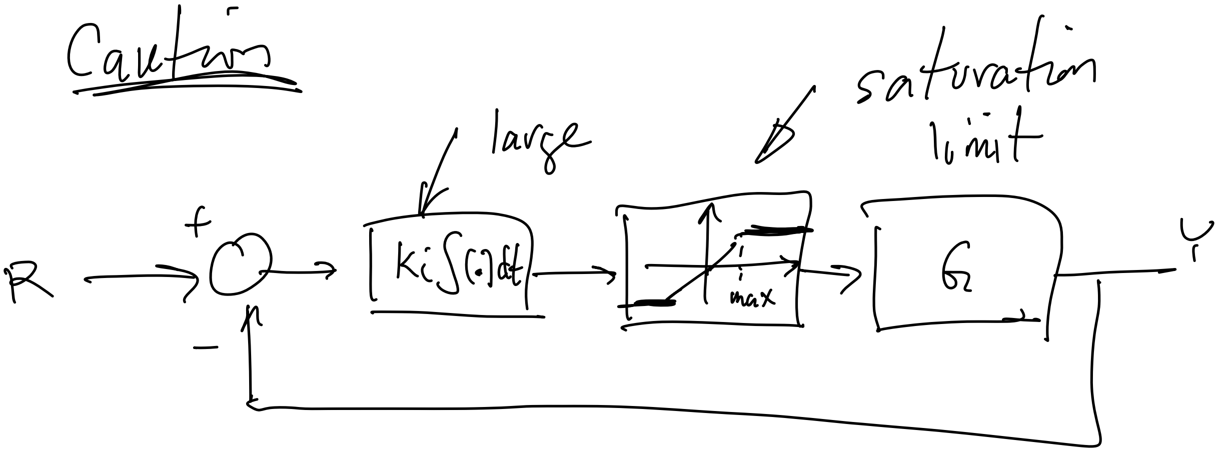


Integral control:

$K_i \rightarrow \infty$

- ζ decrease
- Slower response
- Can go unstable!
- Decrease s.s. error

Caution



Integrator wind-up — use saturation limit to cap the max control effort so you don't cause additional problems/instability.

Derivative

$C(s) = K_d s + K_p$ (prop. + derivative)

$$G(s) = \frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$\frac{Y(s)}{R(s)} = \frac{A(K + K_d s)}{\underbrace{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1 + AK_p + AK_d s}}$$

charcteristic eq. for cloop system:

$$\underbrace{\tau_1\tau_2 s^2 + (\tau_1+\tau_2+AK_d)s + (1+AK_p)}_{\text{2nd order}}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\boxed{s^2 + \left(\frac{\tau_1+\tau_2+AK_d}{\tau_1\tau_2}\right)s + \frac{1+K_pA}{\tau_1\tau_2}}$$

Knobs: K_p , K_d

if I adjust

$K_d \longrightarrow$ damping

$K_p \longrightarrow \omega_n$

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root locus

$$G(s) = \frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$C(s) = K_p + K_d s$$

$$s = -\frac{K_p}{K_d}$$