# Lab 5: Passive Filters and Frequency Response

## 1. Introduction

In this lab, you will investigate the behavior of first order passive RC filters. The filters used in this lab are high and low pass filters. You will compare the theoretical frequency response with the experimentally measured frequency response for each of these filters. The effect of these filters on non-sinusoid waveforms will also be investigated.

### 1.1 Lab Objectives

- To understand the steady-state sinusoidal response of R-C circuits
- To compare theoretical and measured frequency response of first-order low-pass and high-pass filters

### 1.2 Project Objectives

- Prototype refinement

### 1.3 Lab Hardware

- Resistors
- Capacitors
- Jumper wire kit
- Solderless breadboard

### 1.4 Project hardware

- Students should purchase parts from TAs as necessary

# 2. Laboratory Concepts

## 2.1 Theoretical Frequency Response of RC circuits

The term filter is used to describe a dynamic system that is used to accentuate (amplify) or attenuate (suppress) regions of the frequency spectrum of a signal or waveform. Consider, for example, a very small (microVolts) output from a thermocouple. The temperature signal varies very slowly (low frequencies) but the sensor output may be contaminated by 60 Hz AC interference through electromagnetic induction in the wires. A *low-pass* filter may be used to *attenuate* the 60 Hz component while *passing* the low frequency temperature signal.

Similarly, the output of a sensor might be represented by the amplitude of a high frequency AC signal (for example 1 kHz) that is superimposed on a slowly wandering baseline voltage. A *high-pass* filter can be used to eliminate the DC component while passing the 1 kHz signal.

Dynamic systems (both mechanical and electrical) are inherently filters and are described by their *frequency response functions*, usually expressed in Bode plot form. The ability to attenuate a signal of a specific frequency varies as a function of frequency. In this lab we are interested in determining the frequency response characteristics of a first-order passive RC filter, which as will be seen, can be configured as a high or low pass filter. These filters are termed "passive" because they use passive components only (resistors and capacitors), as opposed to active filters that use op-amps (We will take a closer look at active filters in lab 6).

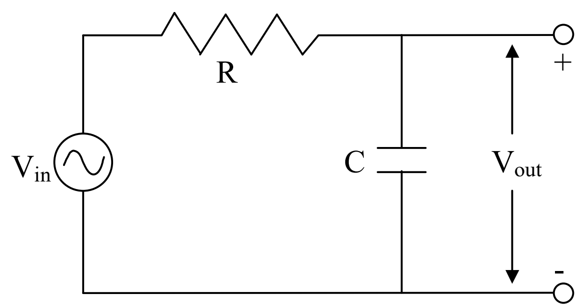



Figure 1: First-Order RC Low-Pass Filter Circuit

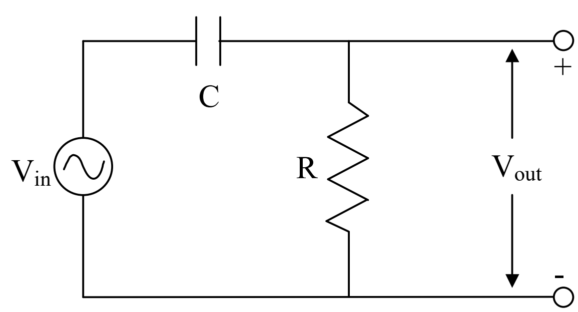



Figure 2: First-Order RC High-Pass Filter Circuit

Consider the simple passive RC filters, using a resistor and capacitor in series, as shown in Figs. 1 and 2. Notice that for the low-pass filter, the output is the voltage across the capacitor, whereas for the high-pass filter it is the voltage across the resistor. The differential equations relating the output voltage $V_{out}(t)$ to the input $V_{in}(t)$ are:

Low Pass: $$RC\frac{dv_{out}}{dt} + V_{out} = V_{in} \tag{1}$$

High Pass: $$RC\frac{dV_{out}}{dt} + V_{out} = RC\frac{dV_{in}}{dt} \tag{2}$$

If $V_{in}(t)$ is a sinusoidal signal with a single frequency $f$:

$$V_{in}(t) = A\sin(2\pi f t) \tag{3}$$

then the *steady-state* response, $V_{out}(t)$ is also a sinusoidal signal with the same frequency, but different amplitude and phase:

$$V_{out}(t) = B\sin(2\pi f t + \phi) \tag{4}$$

Solving the differential equations (e.g. using Laplace Transforms), we can obtain the transfer functions for the filters:

Low Pass: $$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j(\omega / \omega_c)} = \frac{1}{1 + j(f / f_c)} \tag{5}$$

High Pass: $$\frac{V_{out}}{V_{in}} = \frac{j(\omega / \omega_c)}{1 + j(\omega / \omega_c)} = \frac{j(f / f_c)}{1 + j(f / f_c)} \tag{6}$$

where the cut-off frequency ($f_c$ in Hz, or $\omega_c$ in rad/s) is given by:

$$2\pi f_c = \omega_c = \frac{1}{RC} \tag{7}$$

The ratio $V_{out}/V_{in}$ is a function of frequency and uniquely characterizes the circuit. For any frequency value, $V_{out}/V_{in}$ is a complex number with a magnitude and phase. The magnitude $|V_{out}/V_{in}|$ is the circuit *gain* (or attenuation), and the angle $\phi$ is the *phase shift*. For $0^\circ < \phi < 180^\circ$, $V_{out}(t)$ is said to lead $V_{in}(t)$; for $-180^\circ < \phi < 0^\circ$, $V_{out}(t)$ lags $V_{in}(t)$. The magnitude and phase for the low and high pass filters as a function of frequency can be shown to be given by the following equations:

Low Pass: $$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (f / f_c)^2}}, \qquad \phi = -\tan^{-1}\left(\frac{f}{f_c}\right) \tag{8}$$

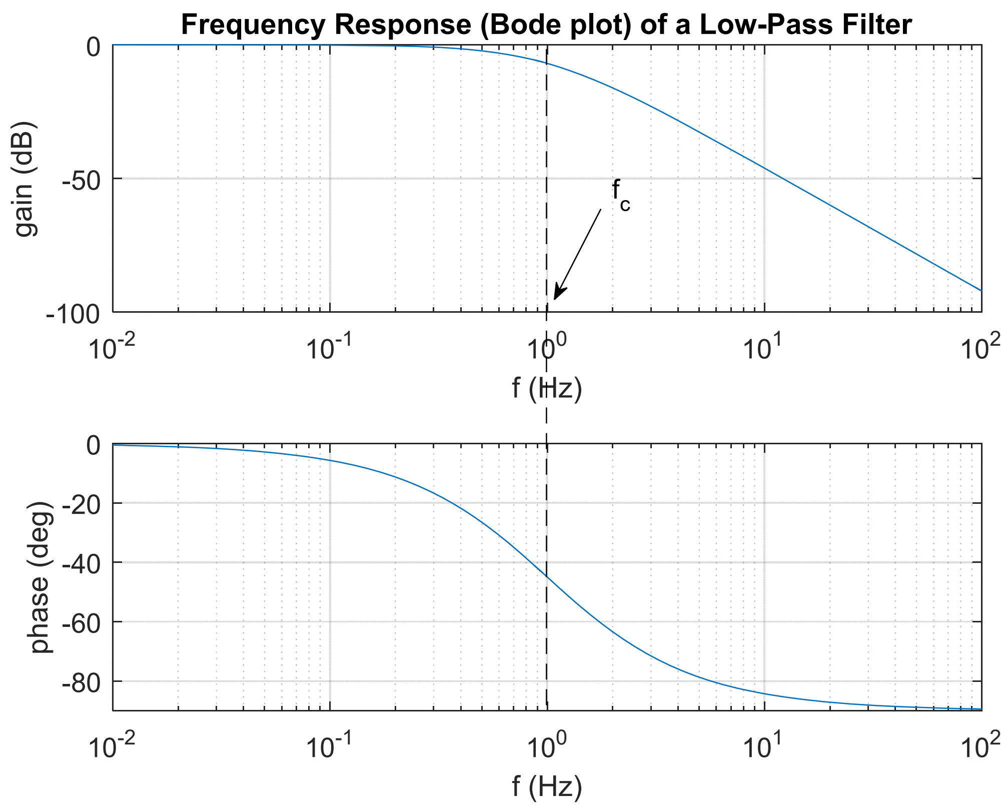
High Pass: $$\left|\frac{V_{out}}{V_{in}}\right| = \frac{f / f_c}{\sqrt{1 + (f / f_c)^2}}, \qquad \phi = \tan^{-1}\left(\frac{f_c}{f}\right) \tag{9}$$

In practice, the gain is often expressed in the dimensionless unit of the decibel (dB), defined as:

$$\left(\frac{V_{out}}{V_{in}}\right)_{dB} = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{10}$$

For both filters, the gain is $|V_{out}/V_{in}| = 1/\sqrt{2}$ (≈-3 dB) at $f_c$.

The frequency response is typically illustrated using a Bode plot, in which the gain and phase are plotted as a function of frequency, as shown below. The frequency is plotted on a logarithmic scale. The gain is either plotted on a logarithmic scale, or converted to dB, in which case it can be plotted on a linear scale.




The experimental frequency response can also be plotted by measuring the gain and phase angles of $V_{in}$ and $V_{out}$ for a range of frequencies. Note that for a purely AC signal, the voltage amplitude and the peak-to-peak and RMS voltages are all linearly related, so that the gain can be calculated from any of these measurements.

# 3. Pre-Lab Exercises

1. In lecture, we showed how to derive the transfer function of an RC circuit using impedances. Use this method to derive equations (5) and (6).

2. Show how equations (8) and (9) can be derived from equations (5) and (6). Hint: a complex number $a + bj$ has magnitude $\sqrt{a^2 + b^2}$ and phase angle of $\tan^{-1}\left(\frac{b}{a}\right)$.

3. Using equations (8), (9) and (10), plot the frequency response of a low and high pass filter using the parameters indicated below. Calculate the Magnitude (dB) and phase (deg) for a range of logarithmically-spaced frequencies spanning 2 orders of magnitude above and below cut-off frequency (hint: use *logspace* function in MATLAB). Be sure to plot the frequencies on a log scale (i.e. *semilogx* in MATLAB) and properly label your axes. Also label the cut-off frequency on your plot. Include screenshots of your code and plots in line with this question with your name indicated. Also submit your .m files to canvas.

   a. <u>Low-Pass filter</u> with R = 4.7 kΩ and C = 0.1 μF

   b. <u>High-Pass filter</u> with R = 4.7 kΩ and C = 0.1 μF

# 4. Laboratory Exercises

## 4.1 Low Pass RC Circuit Frequency response

In this exercise, you are asked to determine the gain and phase for a low-pass RC filter, to compare measured responses with theory, to identify asymptotic behavior at high and low frequencies, and to determine the cut-off frequency.

4.1.1 Construct the low-pass RC circuit using R = 4.7 $k\Omega$ and C = 0.1 μF
- Measure the resistor with a DMM.
- Record R and calculate $f_c$ in the "4.4 Student Work" section.

4.1.2 Open the Lab3_GUI. Set the sample rate to 80000 Hz.
4.1.3 Connect the output Function generator to Channel 0 and to the input of the low-pass filter
4.1.4 Connect the output of the low-pass filter to Channel 1
4.1.5 Set Vin to 1 V peak-to-peak and remove any DC offset
4.1.6 Collect amplitude and phase measurements at the frequencies indicated in the table provided in Section 4.4. Recording and saving data in Excel is recommended. Remember to copy your file when done for the post lab.

**DO NOT DISASSEMBLE THE LOW PASS FILTER**

**Have a TA check your progress**

## 4.2 High Pass RC Circuit Frequency response

In this exercise, you are asked to determine the gain and phase for a high-pass RC filter, to compare measured responses with theory, to identify asymptotic behavior at high and low frequencies, and to determine the cut-off frequency.

4.2.1 Construct a high-pass RC circuit using R = 4.7 kΩ and C = 0.1 μF
- Measure the resistance of the resistor.
  - Record R and calculate $f_c$ in the "4.4 Student Work" section.

4.2.2 Open the Lab3_GUI. Set the sample rate to 80000 Hz.
4.2.3 Connect the output Function generator to Channel 0 and to the input of the high-pass filter
4.2.4 Connect the output of the high-pass filter to Channel 1
4.2.5 Set Vin to 1 V peak-to-peak and remove any DC offset
4.2.6 Collect amplitude and phase measurements at the frequencies indicated in the table provided in Section 4.4. Recording and saving data in Excel is recommended. Remember to copy your file when done for the post lab.

**Have a TA check your progress**

## 4.3 Response of Square and Triangular Waves

4.3.1 40 Hz square wave response to low-pass filter

- Connect the FG to the low-pass filter
- Select a 40 Hz, 1 V peak-to-peak square-wave signal for $V_{in}$.
- Connect the signals to the filter and A/D channels.
- Open the Lab3_GUI. Set the sampling rate to 1600 Hz. Note that this is 40 times the actual frequency.
- Adjust the number of samples to fit 3 periods of the signal.
- Measure and record the peak-to-peak amplitude of $V_{out}$ in the "4.4 Student Work" section.
- Take a screenshot of the shape of the response vs the shape of the input. Organize it as shown in "4.4 Student Work".
- You will discuss the response in the post lab.

4.3.2 5 kHz square wave response to low-pass filter

- Select a 5 kHz, 1 V peak-to-peak square-wave signal for $V_{in}$.
- Set the sampling rate to 100 kHz. Note that this NOT 40 times the actual frequency. Our equipment has limitations, otherwise we would sample at 200 kHz.
- Adjust the number of samples to fit 3 periods of the signal.
- Measure and record the peak-to-peak amplitude of $V_{out}$ in the "4.4 Student Work" section.
- Take a screenshot of the shape of the response vs the shape of the input. Organize it as shown in "4.4 Student Work".

4.3.3 40 Hz triangular wave response to high-pass filter

- Connect the FG to the high-pass filter
- Select a 40 Hz, 1 V peak-to-peak triangular-wave signal for $V_{in}$.
- Connect the signals to the filter and A/D channels.
- Open the Lab3_GUI. Set the sampling rate to 1600 Hz. Note that this is 40 times the actual frequency.
- Adjust the number of samples to fit 3 periods of the signal.
- Measure and record the peak-to-peak amplitude of $V_{out}$ in the "4.4 Student Work" section.
- Take a screenshot of the shape of the response vs the shape of the input. Organize it as shown in "4.4 Student Work".

4.3.4 5 kHz triangular wave response to high-pass filter

- Select a 5 kHz, 1 V peak-to-peak triangular-wave signal for $V_{in}$.
- Set the sampling rate to 100 kHz. Note that this NOT 40 times the actual frequency. Our equipment has limitations, otherwise we would sample at 200 kHz.
- Adjust the number of samples to fit 3 periods of the signal.
- Measure and record the peak-to-peak amplitude of $V_{out}$ in the "4.4 Student Work" section.
- Take a screenshot of the shape of the response vs the shape of the input. Organize it as shown in "4.4 Student Work".

**Have a TA check your progress**

## 4.4 Student work – Tables, calculations, and sketches of responses

**Name(s):** ____________________________ **Lab Section:**_________________________

- Student work for "4.1.1 Record R and calculate $f_c$."
  - R = ___________
  - $f_c$ = ____________

**Table for 4.1.6, Low-Pass Filter**

| $f$ (Hz) | $V_{in}$ (V) | $V_{out}$ (V) | $V_{out}/V_{in}$ | $V_{out}/V_{in}$ (dB) | $\phi$ (º) |
|---|---|---|---|---|---|
| 10 | | | | | |
| 20 | | | | | |
| 40 | | | | | |
| 150 | | | | | |
| ~$f_c$ | | | | | |
| 500 | | | | | |
| 700 | | | | | |
| 1000 | | | | | |
| 2000 | | | | | |

- Student work for "4.2.1 Record R and calculate $f_c$."
  - R = ___________
  - $f_c$ = ____________

**Table for 4.2.6, High-Pass Filter**

| $f$ (Hz) | $V_{in}$ (V) | $V_{out}$ (V) | $V_{out}/V_{in}$ | $V_{out}/V_{in}$ (dB) | $\phi$ (º) |
|---|---|---|---|---|---|
| 10 | | | | | |
| 20 | | | | | |
| 40 | | | | | |
| 150 | | | | | |
| ~$f_c$ | | | | | |
| 500 | | | | | |
| 700 | | | | | |
| 1000 | | | | | |
| 2000 | | | | | |

Student work for 4.3.1 - “40 Hz square wave response to low-pass filter”

- Measure and record the peak-to-peak amplitude of $V_{out}$.______________
- Screenshot of the shape of the response vs the shape of the input.

Student work for 4.3.2 - “5 kHz square wave response to low-pass filter”

- Measure and record the peak-to-peak amplitude of $V_{out}$.______________
- Screenshot of the shape of the response vs the shape of the input.

Student work for 4.3.3 - “40 Hz triangular wave response to high-pass filter”

- Measure and record the peak-to-peak amplitude of $V_{out}$.______________
- Screenshot of the shape of the response vs the shape of the input.

Student work for 4.3.4 - “5 kHz triangular wave response to high-pass filter”

- Measure and record the peak-to-peak amplitude of $V_{out}$.______________
- Screenshot of the shape of the response vs the shape of the input.

# 5 Post-Lab Exercises

1. Using your m-file and data from the prelab (3.a and 3.b), plot the gain and phase of the theoretical frequency response and the experimental response collected in lab. Don't forget to change the theoretical R and C values from the prelab to the actual measured values when making the plot. Use a line for the theoretical data and markers for the experimental data. Include screenshots of your code and plots in line with this question with your name indicated. Also submit your .m files to canvas.

2. Comment on the agreement of the theoretical and experimental response of the low-pass filter. What could be the causes for any disagreement?

3. Comment on the agreement of the theoretical and experimental response of the high-pass filter. What could be the causes for any disagreement?

4. Can you interpret the reason for the shape of the resulting signals from the square-wave response to the low-pass filter?

5. Can you interpret the reason for the shape of the resulting signals from the triangular-wave response to the high-pass filter?

# 6 Project Milestone 5

In preparation for PM 6, make refinements on your mechanical design and continue work on wireless communications. During Lab 6 you will present your current progress and refinements to your prototypes. Your presentation will be 5-10 minute PowerPoint presentation. You will submit the presentation before lab with your team number in the filename on Canvas.

Have at least one slide on each of the following:

- Discuss each team members contributions for this week.
- Summary of your robot design and any changes made
  - Include CAD model from PM 4
  - Updated CAD with any changes made from PM 4
  - Briefly summaries how your robot will complete each obstacle
- Each mechanism prototype progress/changes
- Mobile platform prototype progress/changes
- Progress on wireless communication
- Your goals toward completing PM 6