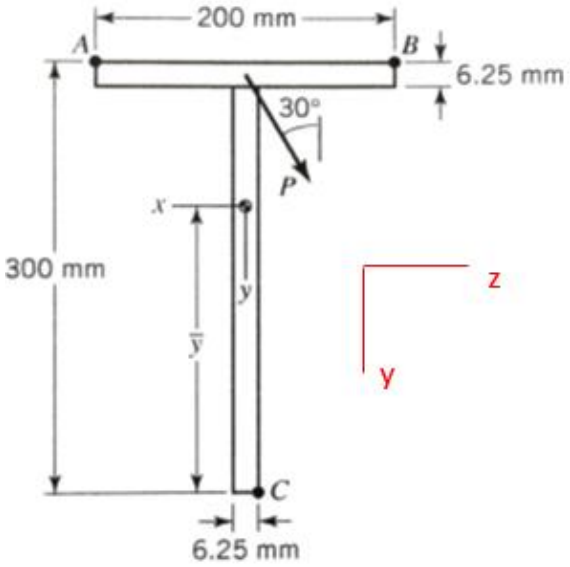
Exam 3 Practice Problems

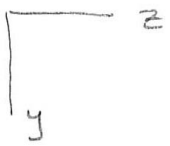
Problem 1

The T-shaped cantilever beam of structural steel is subjected to a transverse load P at its free end, acting through the shear center. The beam is $6.1\ m$ long. According to the Tresca yield criterion, the material yields when the maximum shear stress reaches $165\ MPa$. Determine the maximum load P. Note that the coordinate system shown at the centroid is different than what we've typically used for these problems in class (shown in red). Use $\bar{y} = 207.64\ mm$, $I_y = 4.167 * 10^{-6}\ m^4$, and $I_z = 29.94 * 10^{-6}\ m^4$. You must determine I_{yz} yourself.

Also draw the orientation of the neutral axis on the diagram and provide the angle.



Find max stress, then P.



$$\sigma_x = \frac{(M_y I_z + M_z \cancel{I_{yz}})z - (M_y \cancel{I_{yz}} + M_z I_y)y}{I_y I_z - \cancel{I_{yz}^2}} \quad \text{(0 (symmetry))}$$

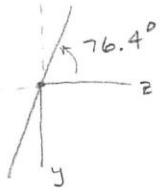
$$= \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

- $M_y = -PL\sin 30 = -\frac{1}{2}PL$
- $M_z = PL\cos 30 = \frac{\sqrt{3}}{2}PL$
- Worst-case location $\Rightarrow$ find ϕ of neutral axis

-Angle of neutral axis, ϕ

$$\sigma_x = 0 \Rightarrow \frac{M_y z}{I_y} = \frac{M_z y}{I_z} \Rightarrow \tan\phi = \frac{y}{z} = \frac{M_y}{M_z}\frac{I_z}{I_y}$$

$$\Rightarrow \tan\phi = \frac{-1/2}{\sqrt{3}/2}\frac{29.94}{4.167} = -4.148 \Rightarrow \underline{\phi = -76.4^\circ}$$



- A: $y = 207.64 - 300 = -92.36$ mm ; $z = -100$ mm

$$\Rightarrow \sigma_{x,A} = 84.49 \times 10^3 P \Rightarrow \sigma_1 > \sigma_2 > \sigma_3 \Rightarrow \sigma_{x,A} > 0 > 0$$

$$\Rightarrow \tau_{max} = \frac{\sigma_{x,A} - 0}{2} = \frac{\sigma_{x,A}}{2}$$

$$\Rightarrow \sigma_{x,A} = 84.49 \times 10^3 P = 2(165\ MPa)$$

$$\Rightarrow \underline{P = 3688\ N}$$