Fluid Mechanics (ME EN 5700/6700)
Final exam, Fall 2015

(Open Book, Open Notes, Closed neighbor)

0. [1 pt] What is your name?

1. [11 pt] Short answer questions.

a. Define the no-slip condition in words.

b. Consider a boundary layer developing over a flat plat starting from the plate's leading edge. At what point does this boundary layer become fully developed?

c. List all the assumptions required to derive the following form of the boundary layer equations from the Navier-Stokes equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu\frac{\partial^2 u}{\partial y^2}$$

d. In words describe the following:

1. Favorable pressure gradient –

2. Adverse pressure gradient –

2. [14 pt] Vorticity transport.

a. Write down the full vorticity transport equation using index notation for an incompressible flow.

b. Assume you have a flow with a velocity field that only varies in the vertical direction ($\vec{V} = f(y)$) and is constant in the x and z directions and for which the density ρ is constant everywhere. Simplify your equation from a.

c. Are their any cases of turbulent flow for which your simplified equation from part b. is valid? Why or why not?

d. List four rules about vortex lines that can be deduced based on the solenoidal condition.

1.

2.

3.

4.

e. Does vorticity exist at a solid wall? Why or why not [support your position with equations or words]

3.[13 pt] Turbulent boundary layers.

a. What is the "closure problem" for turbulent flow in the context of Reynolds averaging?

b. State Kolmogorov's 1^{st} hypothesis in words.

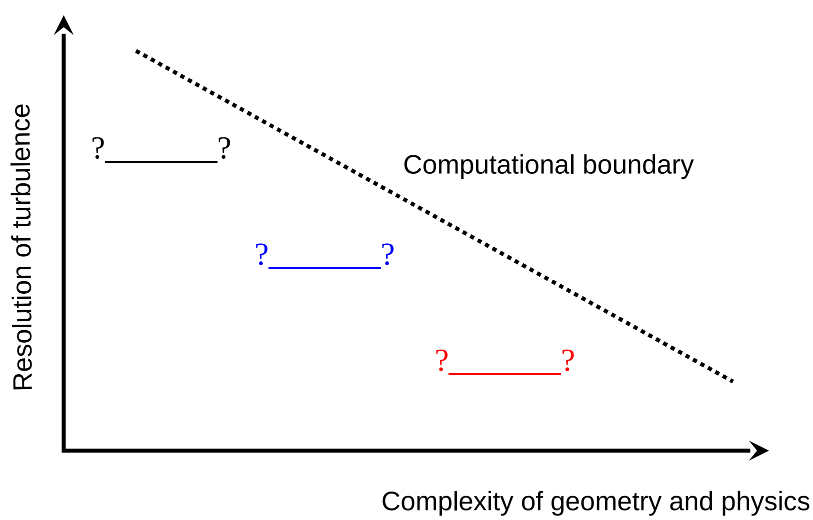
c. List three different techniques used to analyze/numerically investigate the Navier-Stokes equations in turbulent flows:

i. ______________________

ii. ______________________

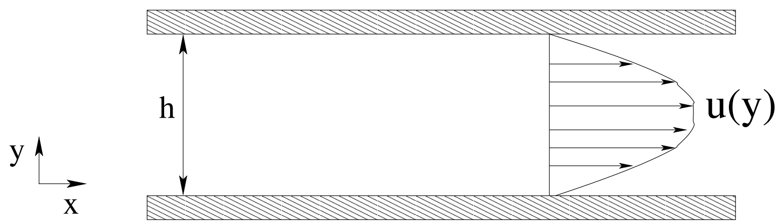
iii. ______________________

d. On the graph below place each of the three simulation techniques from part c. in its appropriate place on the graph based on its level of representation.



4. [13 pt] The stream wise velocity profile for fully-developed, steady-state, incompressible flow between two infinite parallel plates driven by a constant stream wise pressure gradient dP/dx is given by:

$$u = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - hy\right) \text{ where } h \text{ is the distance between the plates}$$



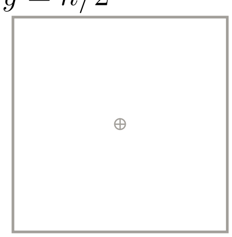
a. Calculate the equation for the streamlines in the channel

b. Determine the components in the strain-rate tensor, e_{ij} (show your work).

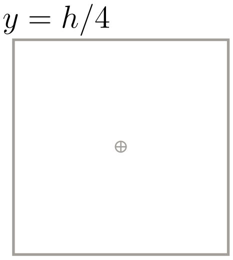
c. Determine the components in the vorticity vector, ω_i (show your work).

d. Consider two initially square particles with centroid located at vertical positions $y = h/2$ and $y = h/4$. How will the fluid particles look after a infinitesmial time Δt later in each case? [Support your reasoning with calculations]

$y = h/2$



$y = h/4$

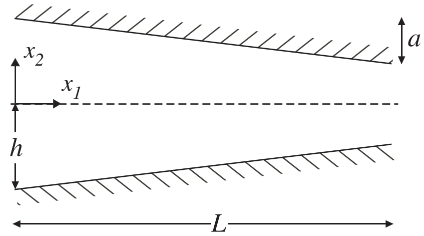


5.[15 pt] As part of a research project, we are interested in turbulent deposition of fungal spores onto different susceptible components of plants (e.g. leaves, flowers). Our strongest interest is in inertial impaction of spherical particles which happens when particles approaching a surface have enough momentum to leave their streamline and intersect the surface. We deduce that inertial impaction is at most a function of particle diameter d_p, particle velocity U_p, particle density ρ_p, fluid dynamic viscosity μ, fluid density ρ, and the size of the object the particle will deposit onto (a length L).

a. Find the relevant non dimensional parameter groups for this problem.

b. You want to study the this process for airborne fungal diseases of grapes. A colleague suggests you build a scale model of a grape vineyard in your wind tunnel (wind tunnel height of 0.4 m, max velocity 40 $\mathrm{ms^{-1}}$) to examine fungal spore (diameter $\sim 30\mu$m) deposition onto leaves. In the vineyard, the canopy height is approximately 2 m and leaves have a length scale of about 0.1 m with typical winds of 2 $\mathrm{ms^{-1}}$. Discuss the merit of the proposed experiment and suggest how (and if) it should be carried out (assume you can measure all relevant parameters).

6. [15 pt] Consider steady, incompressible flow through the plane converging channel shown below. The velocity in the channel can be approximated by $u_1 = V_1\left(1 + \frac{x_1}{L}\right)$ and $u_2 = -V_1\frac{x_2}{L}$.



Consider the case where the top and bottom of the channel are both at the same constant temperature T_0.

a. Write down and then simplify (don't solve) the conservation of <u>internal</u> energy equation (written for temperature) for this problem [clearly state all assumptions].

b. Solve for the viscous dissipation rate ϕ in the channel.

c. Write down and then simplify (don't solve) the conservation of <u>mechanical</u> energy for this problem [clearly state all assumptions].

d. Calculate the change in kinetic energy that results from advection [hint should be a term from part c.].

e. Describe (don't calculate) in words and/or equations how you would calculate the work rate required to drive the fluid through this channel.

7. [18 pt] Laminar boundary layers:

a. Write the von Karman integral equation for a boundary layer over a solid wall with no suction/blowing.

b. Under what assumptions is the above equation valid?

c. Define mathematically (and in words) the displacement thickness.

d. Define mathematically (and in words) the momentum thickness.

e. Simplify the von Karman integral for flow over a flat plate (don't solve just simplify).

f. Assume a laminar boundary layer stream wise velocity profile given by:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right) = \eta \text{ where } \eta = y/\delta.$$

Using this and your reduced form of the von Karman integral from part [e.], solve for $\delta(x)$ [Hint: change the integration limits in the momentum thickness definition from $0 \rightarrow \delta$ to $0 \rightarrow 1$].

f. How does this compare to the $\delta(x)$ you would obtain using the Blasius profile?