ME 3710 – Spring 2024
Homework 1
Due January 18 at 11:59pm – upload to files to Gradescope
26 points

Solultion to Problem 1.2.13

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\frac{\left[L^4\right]\left[FL^{-2}\right]}{\left[FL^{-2}T\right]\left[L\right]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

The constant is $\frac{\pi}{8}$ is $\underline{\underline{\text{dimensionless}}}$.

Yes. This is a general homogeneous equation because it is valid in any consistent units system.

Solution to Problem 1.4.9

$$SG = \frac{\rho}{\rho_{H_2O\,@4^\circ\mathrm{C}}}$$

$$1.15 = \frac{\rho}{1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}}$$

$$\rho = (1.15)\left(1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right) = \underline{\underline{1150\ \frac{\mathrm{kg}}{\mathrm{m}^3}}}$$

$$\gamma = \rho g = \left(1150\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{1\ \mathrm{N}\cdot\mathrm{s}^2}{1\ \mathrm{kg}\cdot\mathrm{m}}\right) = \underline{\underline{11.3\ \frac{\mathrm{kN}}{\mathrm{m}^3}}}$$

Solution to Problem 1.6.4

$$\nu = KR^4 t$$

For glycerin @ 20 °C $\nu = 1.19\times10^{-3}\ \frac{\mathrm{m}^2}{\mathrm{s}^2} = \left(KR^4\right)(1430\ \mathrm{s})$

$$KR^4 = 8.32\times10^{-7}\ \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

For unknown liquid with $t = 900$ s

$$\nu = \left(8.32\times10^{-7}\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right)(900\ \mathrm{s}) = 7.49\times10^{-4}\ \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

By definition: $\nu \equiv \frac{\mu}{\rho} \rightarrow \quad \mu = \left(970\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(7.49\times10^{-4}\ \frac{\mathrm{m}^2}{s}\right) = 0.727\ \frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}} \times \frac{1\ \mathrm{N}\cdot\mathrm{s}^2}{1\ \mathrm{kg}\cdot\mathrm{m}}$

$$\mu = \underline{\underline{0.727\ \frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}^2}}}$$

Solution to Problem 1.6.9

$$\nu \equiv \frac{\mu}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{150 \times 10^3 \ \frac{\text{N}}{\text{m}^2}}{\left(259.8 \ \frac{\text{J}}{\text{kg} \cdot \text{K}}\right)\left(\frac{1\,\text{N} \cdot \text{m}}{1\,\text{J}}\right)\left[(20\ ^\circ\text{C} + 273)\,\text{K}\right]} = 1.97 \ \frac{\text{kg}}{\text{m}^3}$$

$$\nu = 0.104 \text{ stokes} = 0.104 \ \frac{\text{cm}^2}{\text{s}}$$

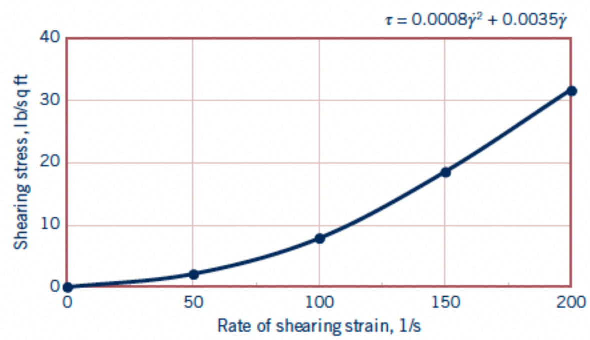
$$\mu = \nu\rho = \left(0.104 \ \frac{\text{cm}^2}{\text{s}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^2\left(1.97 \ \frac{\text{kg}}{\text{m}^3}\right)$$

$$= 2.05 \times 10^{-5} \ \frac{\text{kg}}{\text{m} \cdot \text{s}} \times \frac{1\,\text{N} \cdot \text{s}^2}{1\,\text{kg} \cdot \text{m}} = \underline{\underline{2.05 \times 10^{-5} \ \frac{\text{N} \cdot \text{s}}{\text{m}^2}}}$$

Solution to Problem 1.6.10

Note that you should submit your code as well.

Rate of shearing strain, 1/s	Shearing stress, lb/sq ft
0	0
50	2.11
100	7.82
150	18.5
200	31.7



From the graph $\tau = 0.0008\dot{\gamma}^2 + 0.0035\dot{\gamma}$ where τ is the shearing stress in $\frac{\text{lb}}{\text{ft}^2}$ and $\dot{\gamma}$ is the rate of shearing strain in s^{-1}. Fitting a second-order polynomial to the data yields:

$$\mu_{apparent} = \frac{d\tau}{d\dot{\gamma}} = (2)(0.0008)\dot{\gamma} + 0.0035$$

At $\dot{\gamma} = 70 \text{ s}^{-1}$

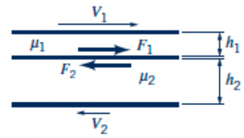
$$\mu_{apparent} = (2)\left(0.0008 \ \frac{\text{lb} \cdot \text{s}^2}{\text{ft}^2}\right)\left(70 \text{ s}^{-1}\right) + 0.0035 \ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2} = \underline{\underline{0.116 \ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}}}$$

From Table B.1 Physical Properties of Water (BG/EE Units)

$\mu_{H_2O \ @ \ 80\ ^\circ\text{F}} = 1.791 \times 10^{-5} \ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}$. Water is a Newtonian fluid so this value is independent of $\dot{\gamma}$. Thus, the viscosity of the non-Newtonian fluid when sheared at a rate of 70 s^{-1} is much <u>larger</u> than the viscosity of water at 80 °F.

Problem 1.6.21

The center plate is stationary if $F_1 = F_2$ (see image). Assuming Newtonian fluids and thin layers,



$$F = \mu\left(\frac{du}{dy}\right)_{\substack{\text{center}\\ \text{plate}}} \cong \mu\frac{V}{h}$$

so

$$\mu_1\frac{V_1}{h_1} = \mu_2\frac{V_2}{h_2}$$

or

$$V_2 = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{h_2}{h_1}\right)V_1 = \left(\frac{\mu_w}{\mu_{eg}}\right)\left(\frac{h_{eg}}{h_w}\right)V_1.$$

From the liquid properties table: $\mu_{eg} = 1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ and $\mu_w = 1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$.

$$V_2 = \left(\frac{1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\right)\left(\frac{0.2\,\text{cm}}{0.1\,\text{cm}}\right)\left(2\,\frac{\text{m}}{\text{s}}\right)$$

$$\boxed{V_2 = 0.201\,\frac{\text{m}}{\text{s}},\ \text{left.}}$$

Problem 1.6.28

Enforcing the no-slip boundary condition at the solid surface:

$$\tau = \mu\frac{du}{dy} = \mu\frac{d}{dy}\left[U\left(2\frac{y}{h}-\frac{y^2}{h^2}\right)\right] = \mu U\left(\frac{2}{h}-\frac{2y}{h^2}\right)$$

Thus, at the fixed surface $(y=0)$

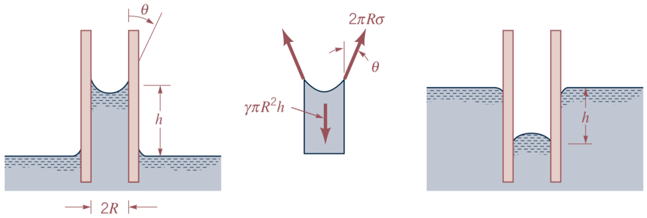
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2D}{h}$$

Thus

$$\tau_{y=0} = \mu U\frac{2}{h} = \left(1.12\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(2\,\frac{\text{m}}{\text{s}}\right)\frac{2}{0.1\,\text{m}}$$

$$= 4.48\times10^{-2}\,\frac{\text{N}}{\text{m}^2}\ \text{acting in direction of flow}$$

Problem 1.9.2



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified contact angle, $\theta = 0$:

$$\sigma = \frac{\gamma h R}{2\cos\theta} = \frac{1.2\times10^{4}\,\dfrac{\text{N}}{\text{m}^3}\left(10\times10^{-3}\ \text{m}\right)\left(\dfrac{2\times10^{-3}\ \text{m}}{2}\right)}{2\cos 0} = \underline{\underline{0.060\,\frac{\text{N}}{\text{m}}}}$$