Advanced Mechanics of Materials Vocabulary

Stress and Strain

- Vector Transformation: $\boldsymbol{a}' = \boldsymbol{M}\boldsymbol{a}$, where $\boldsymbol{M}$ is the transformation, or rotation or direction cosine matrix
  - $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{x}'\cdot\boldsymbol{x} & \boldsymbol{x}'\cdot\boldsymbol{y} & \boldsymbol{x}'\cdot\boldsymbol{z} \\ \boldsymbol{y}'\cdot\boldsymbol{x} & \boldsymbol{y}'\cdot\boldsymbol{y} & \boldsymbol{y}'\cdot\boldsymbol{z} \\ \boldsymbol{z}'\cdot\boldsymbol{x} & \boldsymbol{z}'\cdot\boldsymbol{y} & \boldsymbol{z}'\cdot\boldsymbol{z} \end{bmatrix}$
- Tensor (2nd order) Transformation: $\boldsymbol{T}' = \boldsymbol{M}\boldsymbol{T}\boldsymbol{M}^{\boldsymbol{T}}$
- Stress vector on arbitrary surface with normal unit vector $\boldsymbol{n}$: $\boldsymbol{t} = \boldsymbol{T}\boldsymbol{n} = \boldsymbol{\tau} + \boldsymbol{\sigma}$

Material Response

- Hooke's Law (Voigt Notation)

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

Failure Theory

- Max distortion energy (Von Mises): $(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) = 2\sigma_{yp}^2$
- Coulomb-Mohr: $\frac{\sigma_1}{\sigma_u} - \frac{\sigma_3}{\sigma_u'} = 1$

Fracture Mechanics

- Stress intensity factor: $K = \lambda\sigma\sqrt{\pi a}$

Fatigue

- $N_{cr} = N_f\left(\frac{\sigma_{cf}}{\sigma_f}\right)^{1/b}$, where $b = \frac{ln(\sigma_f/\sigma_e)}{ln(N_f/N_e)}$
- Modified Goodman: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_u} = 1$
- Soderberg: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_{yp}} = 1$
- Gerber: $\frac{\sigma_a}{\sigma_{cr}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$
- SAE: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_f} = 1$

Dynamic Loading

- Impact factor (vertical drop): $K = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$

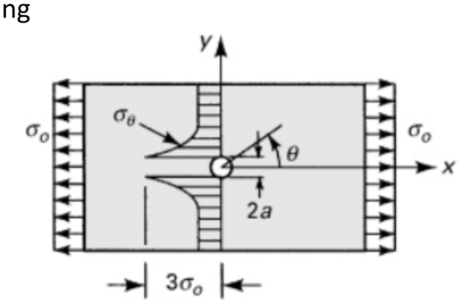
Stress Concentrations

- Stress distribution around a hole in a flat plate under uniaxial loading

$$\sigma_r = \tfrac{1}{2}\sigma_o\left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

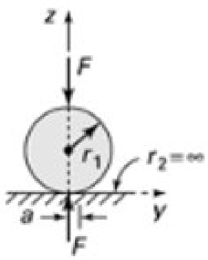
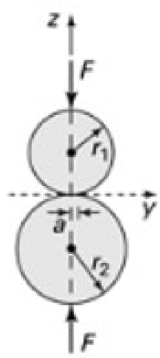
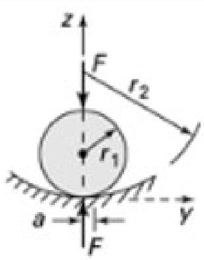
$$\sigma_\theta = \tfrac{1}{2}\sigma_o\left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = -\tfrac{1}{2}\sigma_o\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$



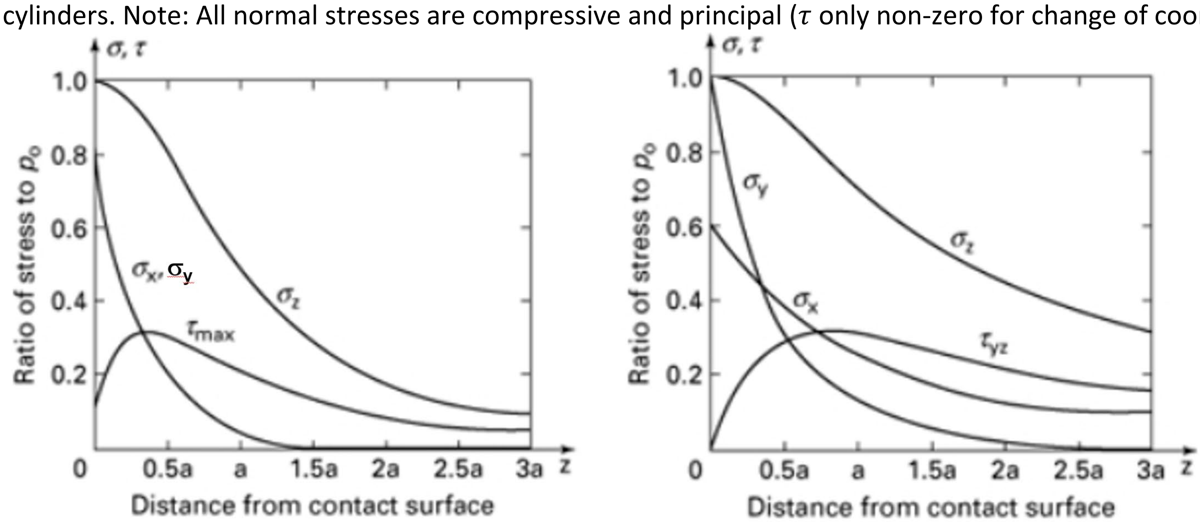
Contact

Table 3.2. Maximum Pressure P_o and Deflection δ of Two Bodies in Contact

Configuration	*Spheres:* $p_o = 1.5\dfrac{F}{\pi a^2}$	*Cylinders:* $p_o = \dfrac{2}{\pi}\dfrac{F}{aL}$
A 	Sphere on a Flat Surface $a = 0.880\sqrt[3]{Fr_1\Delta}$ $\delta = 0.775\sqrt[3]{F^2\dfrac{\Delta^2}{r_1}}$	Cylinder on a Flat Surface $a = 1.076\sqrt{\dfrac{F}{L}r_l\Delta}$ For $E_1 = E_2 = E$: $\delta = \dfrac{0.579F}{EL}\left(\dfrac{1}{3} + \ln\dfrac{2r_1}{a}\right)$
B 	Two Spherical Balls $a = 0.880\sqrt[3]{F\dfrac{\Delta}{m}}$ $\delta = 0.775\sqrt[3]{F^2\Delta^2\cancel{n}}$ m	Two Cylindrical Rollers $a = 1.076\sqrt{\dfrac{F\Delta}{Lm}}$
C 	Sphere on a Spherical Seat $a = 0.880\sqrt[3]{F\dfrac{\Delta}{n}}$ $\delta = 0.775\sqrt[3]{F^2\Delta^2 n}$	Cylinder on a Cylindrical Seat $a = 1.076\sqrt{\dfrac{F\Delta}{Ln}}$

Note: $\Delta = \dfrac{1}{E_1} + \dfrac{1}{E_2}$, $m = \dfrac{1}{r_1} + \dfrac{1}{r_2}$, $n = \dfrac{1}{r_1} - \dfrac{1}{r_2}$ * all formulae in this table assume $\nu = 0.3$

Stress distributions: Stresses as a function of the load axis, z (for $\nu = 0.3$): (left) two spheres; (right) two parallel cylinders. Note: All normal stresses are compressive and principal (τ only non-zero for change of coordinates)

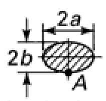
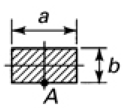
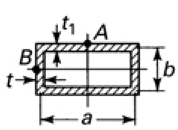
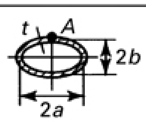
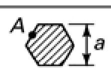


Beam Bending

$$\sigma_x = \frac{(M_y I_z + M_z I_{yz})z - (M_y I_{yz} + M_z I_y)y}{I_y I_z - I_{yz}^2}$$

Torsion

Table 6.2. Shear Stress and Angle of Twist of Various Members in Torsion

Cross section	*Maximum shearing stress*	*Angle of twist per unit length*
 For circular bar: $a=b$	$\tau_A = \frac{2T}{\pi a b^2}$	$\theta = \frac{(a^2+b^2)T}{\pi a^3 b^3 G}$
 Equilateral triangle	$\tau_A = \frac{20T}{a^3}$	$\theta = \frac{46.2T}{a^4 G}$
	$\tau_A = \frac{T}{\alpha a b^2}$	$\theta = \frac{T}{\beta a b^3 G}$
a/b	β	α
1.0	0.141	0.208
1.5	0.196	0.231
2.0	0.229	0.246
2.5	0.249	0.256
3.0	0.263	0.267
4.0	0.281	0.282
5.0	0.291	0.292
10.0	0.312	0.312
∞	0.333	0.333
	$\tau_A = \frac{T}{2abt_1}$ $\tau_B = \frac{T}{2abt}$	$\theta = \frac{(at+bt_1)T}{2tt_1 a^2 b^2 G}$
 For circular tube: $a=b$	$\tau_A = \frac{T}{2\pi abt}$	$\theta = \frac{\sqrt{2(a^2+b^2)}T}{4\pi a^2 b^2 tG}$
 Hexagon	$\tau_A = \frac{5.7T}{a^3}$	$\theta = \frac{8.8T}{a^4 G}$

Narrow, rectangular cross section

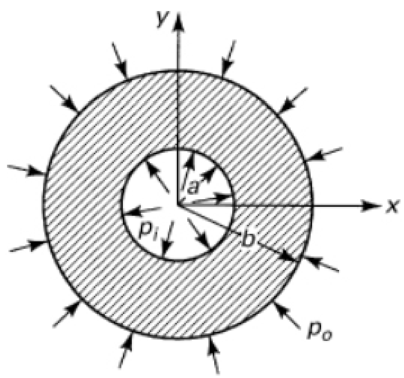
$$\tau_{max} = \frac{3T}{bt^2} \qquad \theta = \frac{3T}{bt^3 G}$$

Thin-walled, closed cross sections

$$\tau = \frac{T}{2At} \qquad \theta = \frac{1}{2AG}\oint \tau ds$$

Thick-Walled Pressure Vessels

- Stress equations



$$\sigma_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o)a^2 b^2}{(b^2 - a^2)r^2}$$

$$\sigma_\theta = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o)a^2 b^2}{(b^2 - a^2)r^2}$$

$$\sigma_z = \frac{p_i a^2 - p_o b^2}{b^2 - a^2}$$

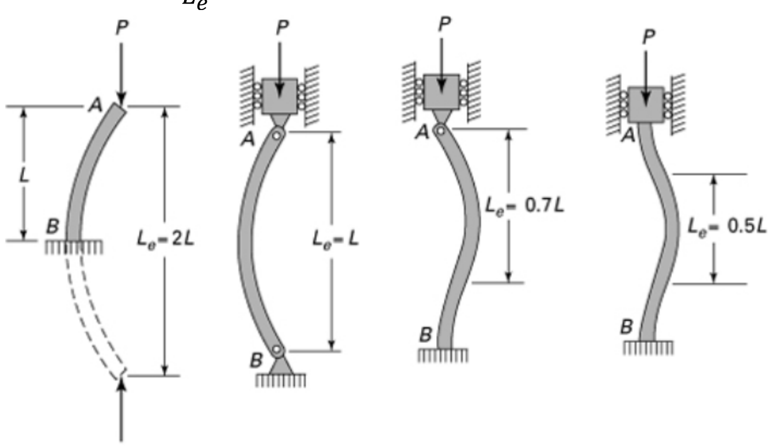
Castigliano's Method Equations

$$\delta_i = \frac{1}{AE}\int N\frac{\partial N}{\partial P_i}dx + \frac{1}{EI}\int M\frac{\partial M}{\partial P_i}dx + \frac{1}{AG}\int \alpha V\frac{\partial V}{\partial P_i}dx + \frac{1}{JG}\int T\frac{\partial T}{\partial P}dx$$

$$\theta_i = \frac{1}{AE}\int N\frac{\partial N}{\partial C_i}dx + \frac{1}{EI}\int M\frac{\partial M}{\partial C_i}dx + \frac{1}{AG}\int \alpha V\frac{\partial V}{\partial C_i}dx + \frac{1}{JG}\int T\frac{\partial T}{\partial C_i}dx$$

Buckling

Given $P_{cr} = \frac{\pi^2 EI}{L_e^2}$



Plasticity

$$M = \tfrac{3}{2} M_{\text{yp}}\left[1 - \frac{1}{3}\left(\frac{e}{h}\right)^2\right]$$

$$T = \frac{\pi c^3}{6}\left(4 - \frac{\rho_0^3}{c^3}\right)\tau_{\text{yp}} = \frac{4}{3} T_{\text{yp}}\left(1 - \frac{1}{4}\frac{\rho_0^3}{c^3}\right)$$