Response Versus Pole Location

From the Laplace transform of ODE, we get a transfer function:

$$H(s) = \frac{n(s)}{d(s)} \quad \begin{matrix} \vdots \text{ zeros} \\ \vdots \text{ poles} \end{matrix} \quad \frac{\text{output}}{\text{input}}$$

to find them:

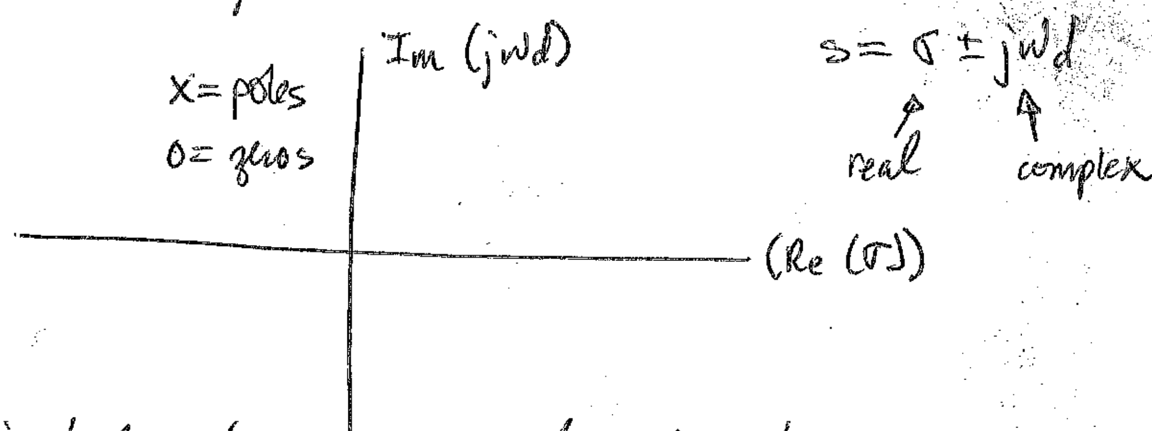
$$n(s) = 0$$

$$d(s) = 0 .$$

These poles and zeros describe the system.

Recall that impulse response $\Rightarrow$ natural response thus we can identify the time histories with pole location in the s-plane.

<u>s-plane</u>: A graphical tool for plotting the zeros and poles thus giving us a pictorial view of the systems response:



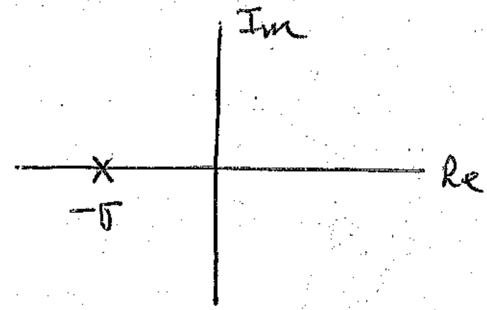
Now let's look at some examples of system response versus pole location

Ex: (Real Poles)

$$H(s) = \frac{1}{s+\sigma} = \text{Laplace T.F. of impulse Response} \quad (1^{st} \text{ order})$$

from Table A.2 (#7)

$$h(t) = e^{-\sigma t}$$



when $\sigma > 0$, $s < 0$ $\Rightarrow$ stable system

when $\sigma < 0$, $s > 0$ $\Rightarrow$ unstable system

$\sigma = 0$, $s = 0$ $\Rightarrow$ marginally stable system (small perturbation can make it unstable).

Since this is a first order response:

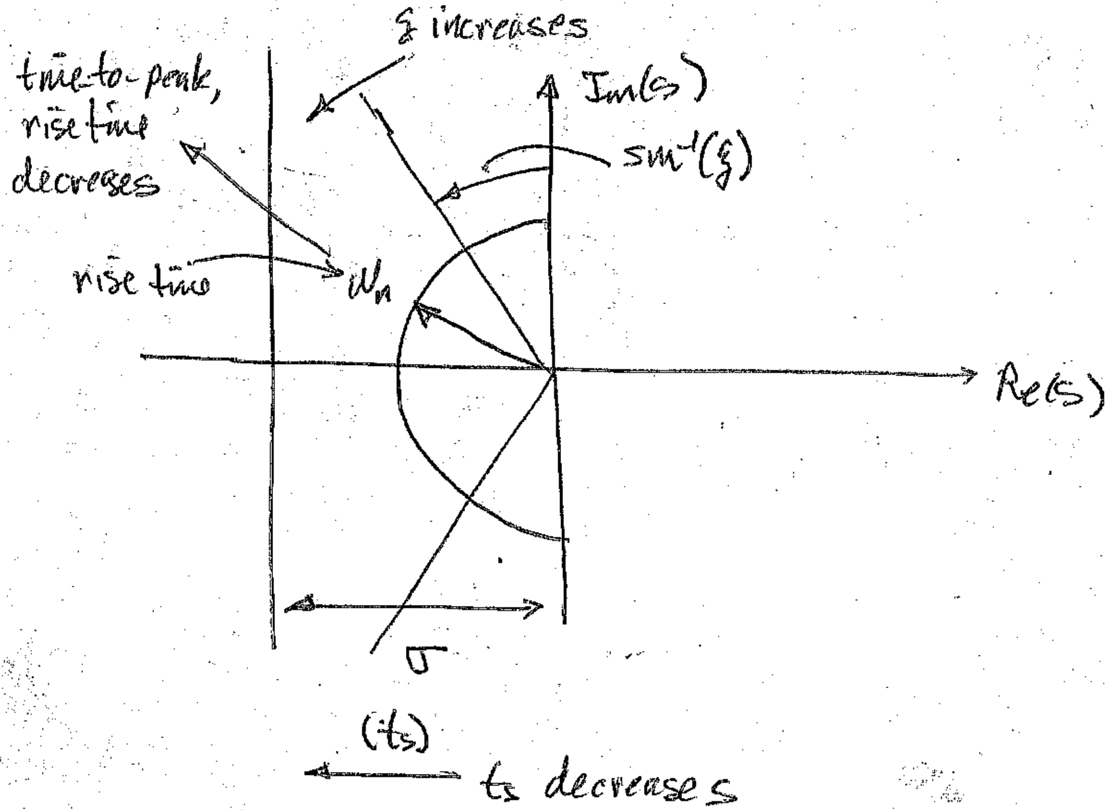
$$\tau = 1/\sigma \quad \text{(time constant)}$$

- Time it takes to reach 63% of final value. τ characterizes the exponential behavior of system.

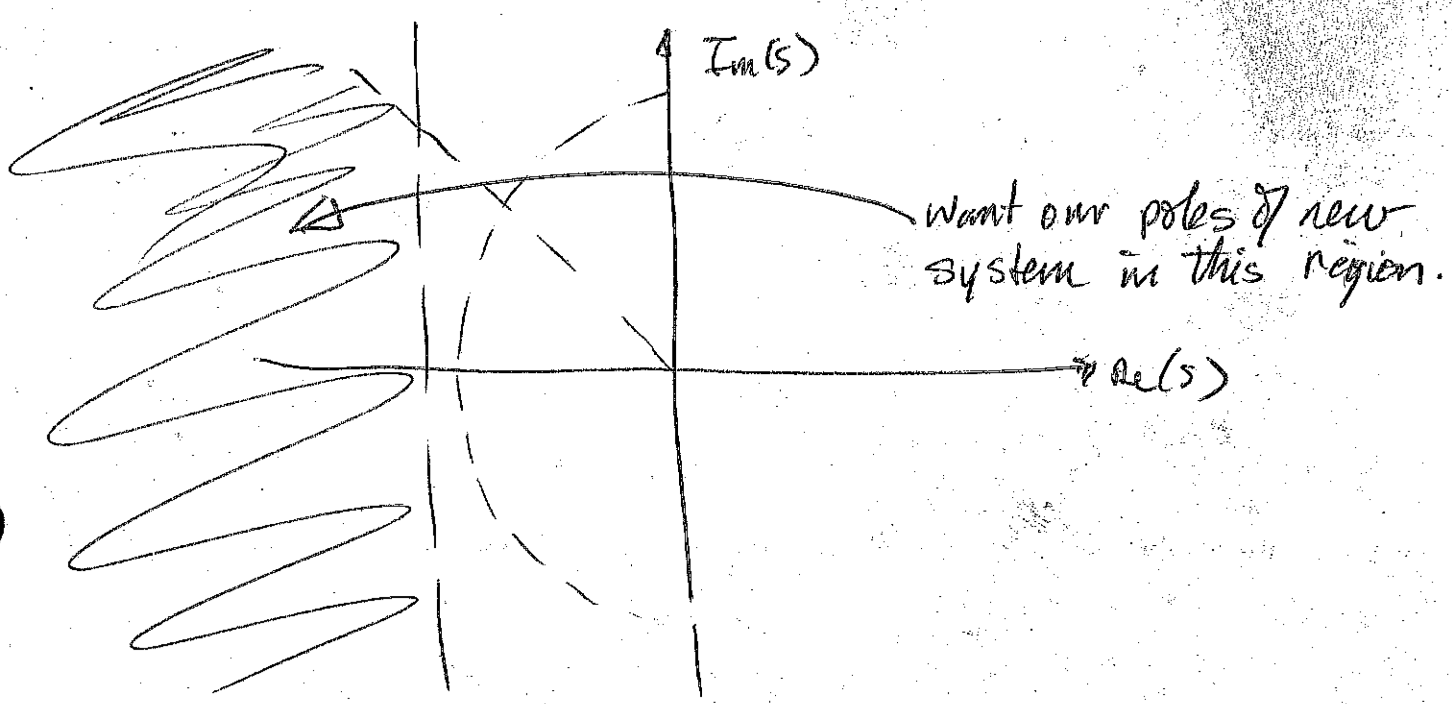
Larger σ means the decay is faster $\Rightarrow$ faster pole

smaller σ mean slower decay and is usually the dominate behavior of a system.

How do they map into the s-plane?



Therefore, we can use these mapping to design a controller, ex:



Effects of zeros and additional Poles on 2nd order Systems

In general, for a 2nd order system in the form:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

the transient response parameters are approximated by

Rise time: $t_r \cong \frac{1.8}{\omega_n}$

settling time: $t_s \cong \frac{4}{\sigma} = \frac{4}{\zeta\omega_n}$ (2%)

time-to-peak: $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$$\%OS = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

But these approximations only apply to 2nd order systems w/ no finite zeros.

But what happens if we add more poles and zeros to the system?

* For some cases, the systems with more poles/zeros can be approximated as a 2nd order system if it has two dominant complex poles.

Adding Poles:

Take for example adding a pole (real) to a 2nd order system:

$$\frac{Y(s)}{U(s)} = \frac{B(s+\xi\omega_n) + C\omega_d}{(s+\xi\omega_n)^2 + \omega_d^2} + \frac{D}{(s+\alpha)}$$

Now consider a step input $U(s) = \frac{1}{s}$.

$$\Rightarrow Y(s) = \frac{A}{s} + \frac{B(s+\xi\omega_n) + C\omega_d}{(s+\xi\omega_n)^2 + \omega_d^2} + \frac{D}{s+\alpha}$$

$$\Rightarrow Y(t) = Au(t) + e^{-\xi\omega_n t}(B\cos\omega_d t + C\sin\omega_d t) + De^{-\alpha t}$$

Then if α is far away, the contribution of $De^{-\alpha t}$ term is small became rate of decay for large α is fast.

But if α is close to poles of the second order system i.e.

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

In general, if poles are at least 5-10 times away from 2nd order poles, then we can approximate the response as 2nd order.

Adding zeros

Consider adding an zero to a 2nd order system:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{as+b}{s^2+2\xi\omega_n+\omega_n^2}$$

$$Y(s) = \frac{as}{s^2+2\xi\omega_n+\omega_n^2} + \frac{b}{s^2+2\xi\omega_n+\omega_n^2} + \frac{c}{s}$$

$$Y(s) = saH_1(s) + bH_1(s) + H_2(s)$$

step input response

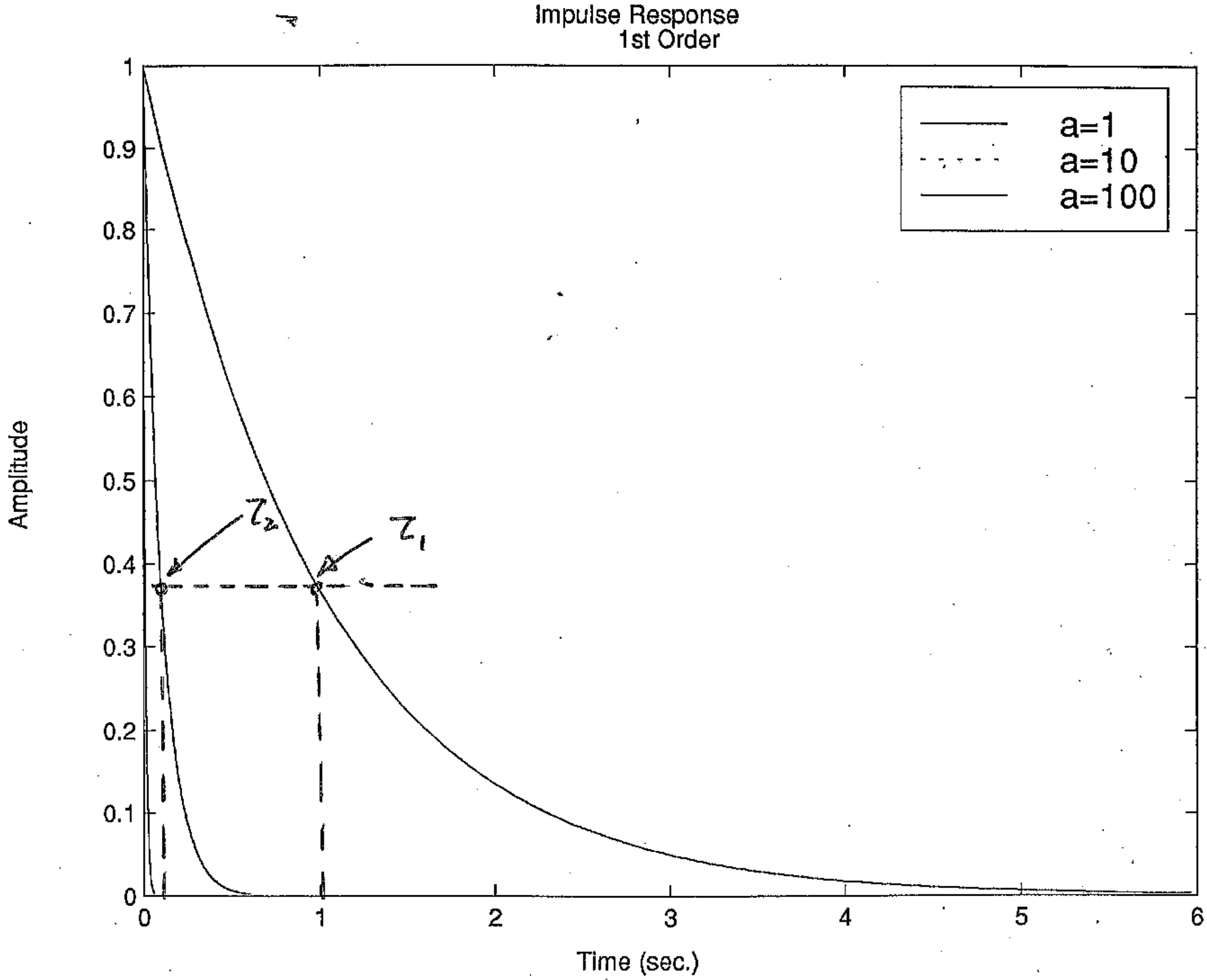
if $b>0 \Rightarrow$ LHP zero, therfore the response of $H_1(s) > 0 \Rightarrow saH_1(s) > 0$

so, depending upon the magnitude of b, the response of a zero will add to the system

if $b<0 \Rightarrow$ RHP zero, therfore $H_1(s) < 0$ and the system with respond in negative direction. Non minimphase systems.

$$H(s) = \frac{1}{s + a}$$

a=1, 10, 100



Matlab Impulse Function

```
>>n=[1];
>>d=[1  a];
>>impulse(n,d);
```

Complex Roots

- Come in complex conjugate pairs:

$$s = \sigma \pm j\omega_d$$

$$\Rightarrow d(s) = (s+\sigma+j\omega_d)(s+\sigma-j\omega_d)$$

$$d(s) = (s+\sigma)^2 + \omega_d^2$$

For a second order system, general form is

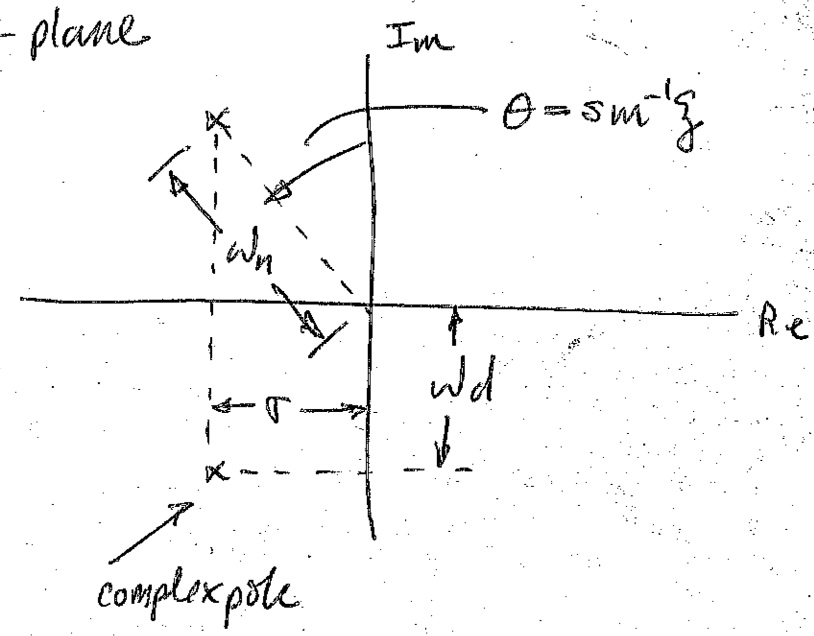
$$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Comparing $d(s)$ w/ $H(s)$'s denomenator, we ~~sed~~ see that

$$\sigma = \zeta\omega_n \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

ζ = Dampening ratio $\qquad \omega_d$ = damped natural frequency

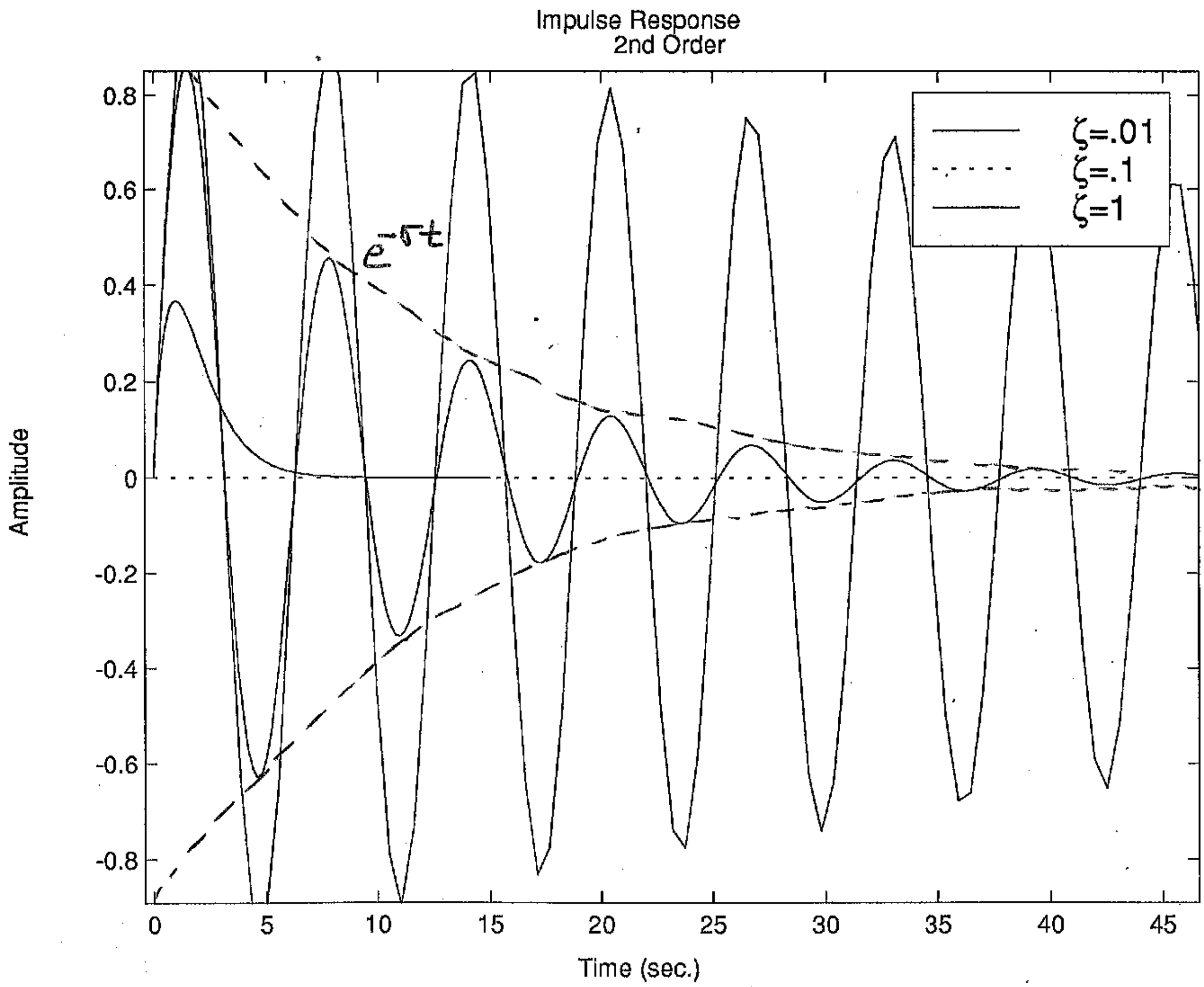
then in s-plane



&

$$H(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}$$

$\omega_n = 1.0$



Matlab Impulse Function

```
>>n=[wn^2];
>>d=[1  2*z*wn  wn^2];
>>impulse(n,d);
```

stop!!
6/3

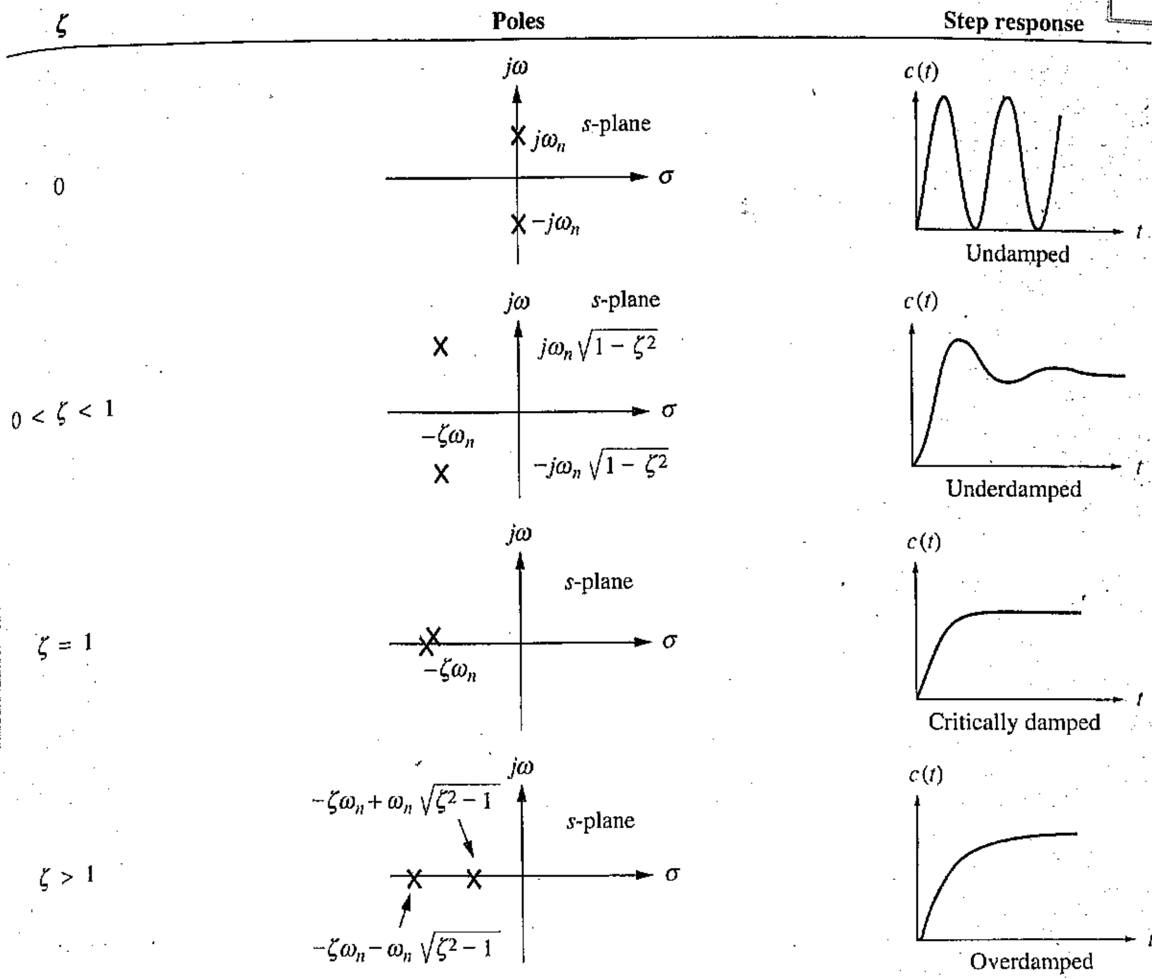


Figure 4.11 Second-order response as a function of damping ratio

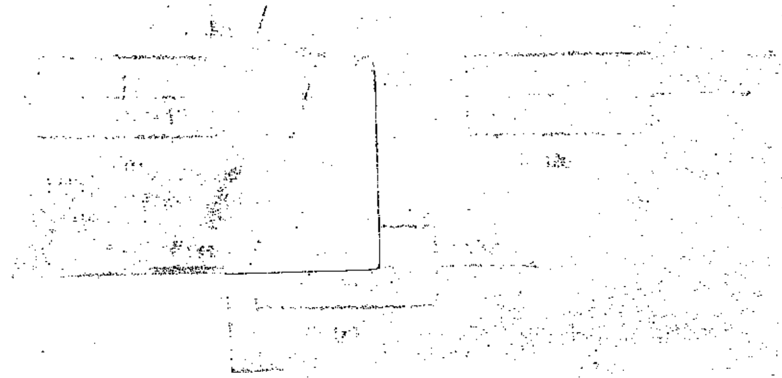


Fig
Systems for Example 4.4