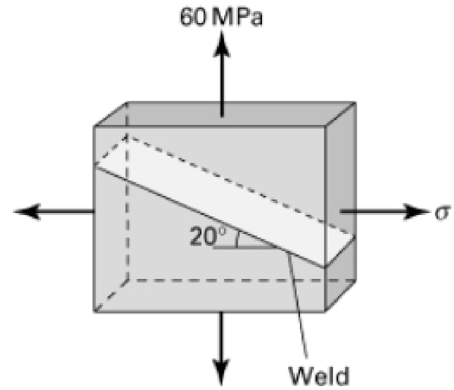
Problem 1 (20 pts)
A welded plate carries uniform biaxial tension. Determine the maximum stress σ if the weld has an allowable normal stress of $80\ MPa$.



$\mapsto \sigma = \sigma_x l^2 + \sigma_y m^2 + \sigma_z n^2 + 2(\tau_{xy} lm + \tau_{yz} mn + \tau_{xz} ln)$

Use $\underline{t} = \underline{T}\,\underline{n}$ (or equation for oblique surface from text)

- Find $\underline{n}$

$\underline{n} = \sin 20\, \underline{i} + \cos 20\, \underline{j}$

$= [\sin 20, \cos 20]^T$

$$\Rightarrow [\underline{t}] = \begin{bmatrix} \sigma & 0 \\ 0 & 60 \end{bmatrix} \begin{bmatrix} \sin 20 \\ \cos 20 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma \sin 20 \\ 60 \cos 20 \end{bmatrix}$$

- $$\hat{\sigma} = \underline{t} \cdot \underline{n} = \begin{bmatrix} \sigma \sin 20 \\ 60 \cos 20 \end{bmatrix} \cdot \begin{bmatrix} \sin 20 \\ \cos 20 \end{bmatrix} = \sigma \sin^2 20 + 60 \cos^2 20$$

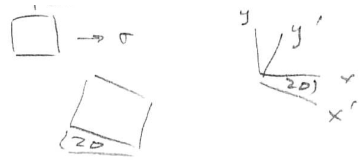
(Note $\hat{\sigma}$ is normal stress on surface, while σ is unknown applied stress)

$$\Rightarrow \hat{\sigma} = \sigma \sin^2 20 + 60 \cos^2 20 \Rightarrow \sigma \sin^2 20 = \hat{\sigma} - 60 \cos^2 20$$

$$\Rightarrow \sigma = \frac{1}{\sin^2 20}(\hat{\sigma} - 60 \cos^2 20)$$

For $\hat{\sigma} = 80$ MPa, $\underline{\sigma = 231 \text{ MPa}}$

OR ...



$$\sigma_{y'} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$$

$$80 = \frac{1}{2}(\sigma + 60) - \frac{1}{2}(\sigma - 60)\cos(40)$$

$$80 = \frac{1}{2}\sigma + 30 - \frac{1}{2}\sigma\cos 40 + 30\cos 40 \quad (*)$$

$$= \frac{1}{2}\sigma(1 - \cos 40) + 30 + 30\cos 40$$

$$\frac{2(27.02)}{1-\cos 40} = \sigma = \underline{231}$$

OR

$$\begin{bmatrix} 80 & \tau_{x'y'} & \tau_{x'z'} \\ & \sigma_{y'} & \tau_{y'z'} \\ & & \sigma_{z'} \end{bmatrix} = \begin{bmatrix} c70 & s70 & 0 \\ -s70 & c70 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sigma & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} c70 & -s70 & 0 \\ s70 & c70 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

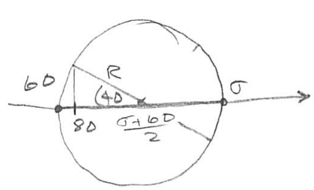
$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \sigma c70 & -\sigma s70 & 0 \\ 60s70 & 60c70 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma c^2 70 + 60 s^2 70 & \sim \\ & \\ & \end{bmatrix}$$

$$\Rightarrow 80 = \sigma c^2 70 + 60 s^2 70$$

$$\sigma = \frac{80 - 60 s^2 70}{c^2 70} = \underline{231 \text{ MPa}}$$

OR



$$80 = \frac{\sigma + 60}{2} - R\cos 40$$

$$\cdot\ R = \frac{\sigma - 60}{2}$$

$$\Rightarrow 80 = \frac{\sigma}{2} + 30 - \frac{\sigma}{2}\cos 40 + 30\cos 40$$

(same as above *)

Problem 2 (30 pts)
A steel plate ($E = 200\ GPa$ and $\nu = 0.29$) is subjected to a state of plane stress ($\sigma_x = -80\ MPa$, $\sigma_y = 100\ MPa$, and $\sigma_{xy} = 50\ MPa$). Report the associated tensorial strains and determine the dilatation.

Find strains using Hooke's Law

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \\ \sigma_{xy} \\ 0 \\ 0 \end{bmatrix}$$

* steel is isotropic

* $G = \frac{E}{2(1+\nu)}$

- $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200\,GPa}\left[-80 - (0.29)(100)\right] MPa = \underline{-545\ \mu}$
- $\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200\,GPa}\left[100 - (0.29)(-80)\right] MPa = \underline{616\ \mu}$
- $\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{0.29}{200\,GPa}(-80 + 100)\,MPa = \underline{-29\ \mu}$
- $\gamma_{xy} = \frac{1}{G}\sigma_{xy} = \frac{2(1+\nu)}{E}\sigma_{xy} = \frac{2(1.29)}{200\,GPa}(50\,MPa) = \underline{645\ \mu}$

$$\Rightarrow [\underline{\varepsilon}] = \begin{bmatrix} -545 & 322.5 & 0 \\ 322.5 & 616 & 0 \\ 0 & 0 & -29 \end{bmatrix} \mu$$

Dilatation: $e = \varepsilon_x + \varepsilon_y + \varepsilon_z = (-545 + 616 - 29)\mu = \underline{42\ \mu}$

Problem 3 (20 pts)
The stress state in a component of the roof rack on an automobile as it passes over a speed bump is defined by the following stress tensor (relative to an x, y, z coordinate system):

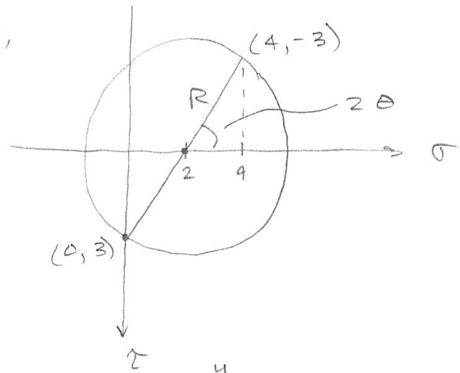
$$\begin{bmatrix} 0 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix} MPa$$

Determine

(a) the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$

(b) the eigenvector, or direction cosines (l, m, n), associated with the minimum principal stress.

* 7 MPa is one of the princ. stresses, since no shear stresses are present on z-planes

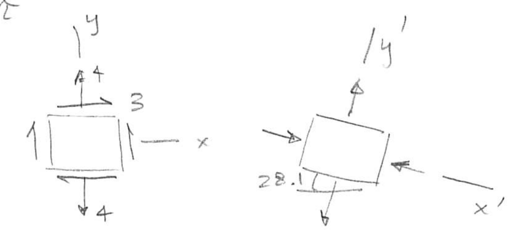


• Mohr's Circle →

• $R = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\Rightarrow \quad \sigma_{2,3} = 2 \pm \sqrt{13} = 5.6, -1.6$

$\Rightarrow \quad \underline{\sigma_{1,2,3} = 7, 5.6, -1.6 \ MPa}$



• $\tan 2\theta = \dfrac{3}{2} \Rightarrow 2\theta = 56.3°$

$\theta = 28.1°$

• min. princ stress is in x' direction

$\Rightarrow \quad [\cos 28.1°, -\sin 28.1°, 0]^T = \underline{[0.88, -0.47, 0]^T}$

<u>OR</u> $[\underline{x}' \cdot \underline{x}, \ \underline{x}' \cdot \underline{y}, \ \underline{x}' \cdot \underline{z}]$

<u>OR</u> $(\underline{T} - \lambda \underline{I})\underline{v} = \underline{0}$

$$\Rightarrow \begin{bmatrix} 0+1.6 & 3 & 0 \\ 3 & 4+1.6 & 0 \\ 0 & 0 & 7+1.6 \end{bmatrix} = \begin{bmatrix} 1.6 & 3 & 0 \\ 3 & 5.6 & 0 \\ 0 & 0 & 8.6 \end{bmatrix}$$

$\Rightarrow \quad 1.6 v_1 + 3 v_2 = 0 \Rightarrow v_1 = -\frac{3}{1.6} v_2 = -1.875 v_2$

$3 v_1 + 5.6 v_2 = 0 \Rightarrow v_1 = -\frac{5.6}{3} v_2 = -1.87 v_2$ same

$8.6 v_3 = 0 \Rightarrow v_3 = 0$

$\Rightarrow \underline{v} = \begin{bmatrix} -1.875 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \quad \hat{\underline{v}} = \dfrac{1}{2.125} \underline{v} = \begin{bmatrix} -0.88 \\ 0.45 \\ 0 \end{bmatrix}$ * signs can flip here

4