Mason's Rule

Transfer function from an input signal to an output signal of a signal flow graph:

$$\frac{\text{output signal}}{\text{Input signal}} = \frac{Y(s)}{U(s)} = G(s) = \frac{1}{\Delta} \sum_i F_i \Delta_i$$

F_i = path gain of the i^{th} forward path from

$U(s) \longrightarrow Y(s)$

Δ = system determinant

= $1 - \Sigma$ (all individual loop gains)

+ Σ (gain products of all possible combinations of two-loops that do not touch)

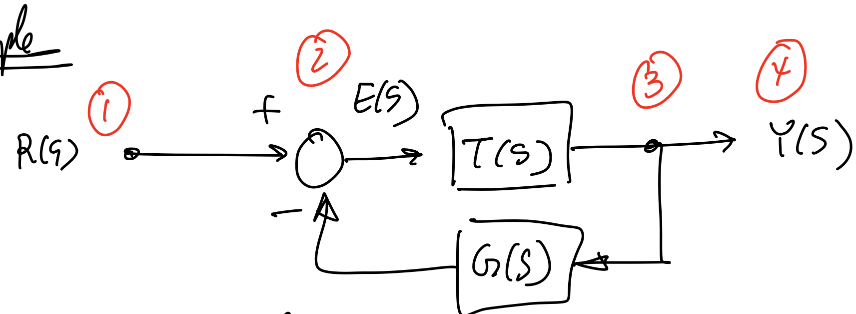
$-\Sigma$ (gain products of all possible combinations of three-loops that do not touch)

+ Σ (gain products of all possible four-loops .....)

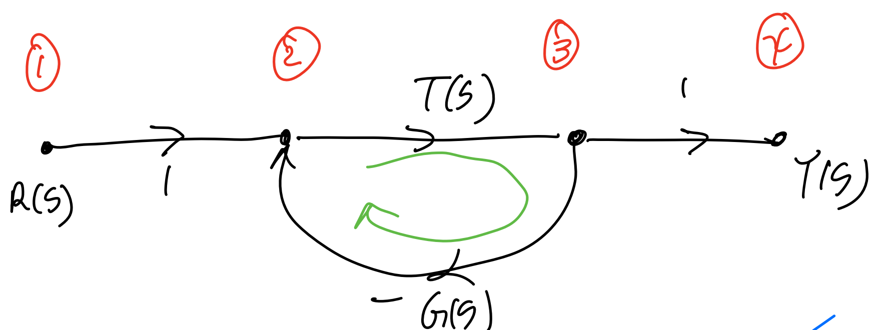
$-\Sigma$ (gain products of all possible... five-loops . . . . . + Σ . . . .

Δ_i = ith forward path determinant
= value of Δ for that part of the block diagram that does not touch the ith forward path F_i.

Example



Step 1: Signal flow graph



Step 2:

Mason's Rule: $\dfrac{Y(s)}{R(s)} = \dfrac{1}{\Delta} \sum_i F_i \Delta_i$

Forward paths: $F_1 = (1)\,T(s)\,(1) = T(s) \qquad i = 1$

loop gains :

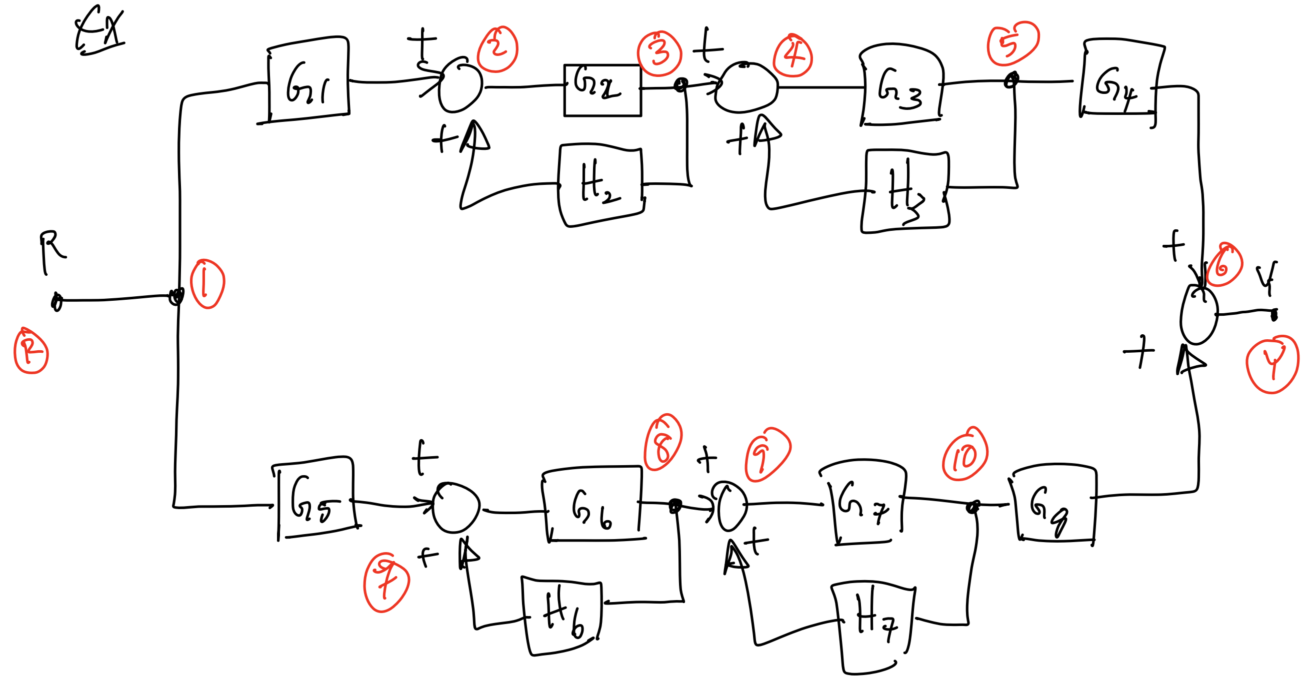
$$L_1 = T(s)[-G(s)] = -T(s)G(s)$$

$$\Delta = 1 - (-T(s)G(s)) = \underline{1 + G(s)T(s)}$$

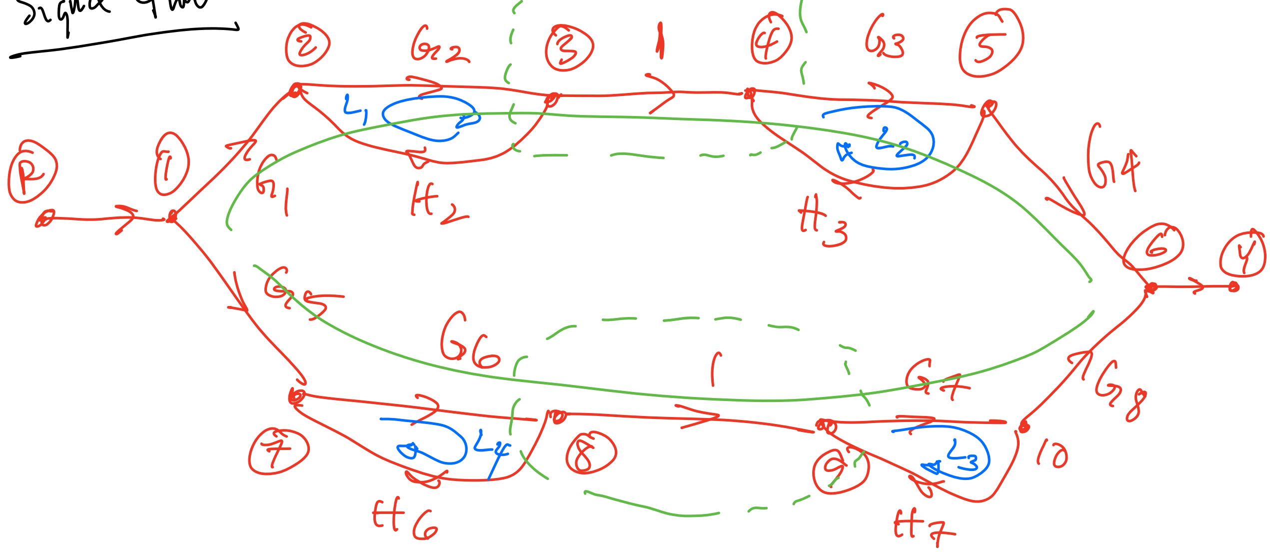
$$\Delta_1 = 1$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{1}{1+G(s)T(s)} \cdot T(s) = \boxed{\frac{T(s)}{1+G(s)T(s)}}$$

Ex



Signal flow



Find F_i, Δ, Δ_i, L_j

$L_1 = G_2 H_2$ $\quad L_2 = G_3 H_3$ $\quad L_3 = G_7 H_7$ $\quad L_4 = G_6 H_6$

$F_1 = G_1 G_2 G_3 G_4$ $\quad F_2 = G_5 G_6 G_7 G_8$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_2 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4 + L_3L_4) - (L_1L_2L_3 + L_1L_2L_4 + L_1L_3L_4 + L_2L_3L_4) + L_1L_2L_3L_4$$