Fluid Mechanics (ME EN 5700/6700)
Final exam, Fall 2013

(Open Book, Open Notes, Closed neighbor)

0. [1 pt] What is your name?

1. [14 pt] Short answer questions.

a. Write the following vector quantity in index notation: $\left(\vec{\nabla} \times \vec{a}\right) \cdot \left(\vec{c} \times \vec{b}\right)$

b. Consider a boundary layer developing over a flat plat starting from the plate's leading edge. At what point does this boundary layer become fully developed?

c. Under what conditions can a two-dimensional turbulent flow exist?

d. In the following equations a is a constant, u and v are random variables (e.g., velocity components), $u' = u - \langle u \rangle$ and $v' = v - \langle v \rangle$ are the deviations of u and v from their mean values, and the angle brackets $\langle\ \rangle$ denote an appropriate averaging operator. Reducing the following averaging operations to their simplest form.

1. $\langle\langle a(u+v)\rangle\rangle$

2. $\langle a\left(\langle u\rangle^2 + \langle u'\rangle^2\right)\rangle$

3. $\langle\left(\langle u\rangle + u'\right)\left(\langle v\rangle + v'\right)\rangle$

e. In words describe the following:

1. Favorable pressure gradient –

2. Adverse pressure gradient –

2. [8 pt] Vorticity transport.

a. Consider the vorticity transport equation shown below.

$$\underbrace{\frac{\partial \vec{\omega}}{\partial t}}_{I} + \underbrace{\left(\vec{u} \cdot \vec{\nabla}\right) \vec{\omega}}_{II} = \underbrace{\left(\vec{\omega} \cdot \vec{\nabla}\right) \vec{u}}_{III} + \underbrace{\frac{\vec{\nabla}\rho \times \vec{\nabla}P}{\rho^2}}_{IV} + \underbrace{\nu \vec{\nabla}^2 \vec{\omega}}_{V}$$

1. Give the name and a short description of the physical meaning of each term I–V.

   I –

   II –

   III –

   IV –

   V –

2. Which term in the vorticity transport equation is linked to the turbulent energy "cascade" process?

b. Define (in words) what a vortex line is.

c. What is the solenoidal condition for vorticity?

3. [6 pt] Navier-Stokes equations.

a. List all the assumptions required to derive the following form of the Navier-Stokes equations

$$\rho \frac{Du_i}{Dt} = \rho g_i - \frac{\partial P}{\partial x_i} + \mu \frac{\partial}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j} + \frac{1}{3}\frac{\partial u_m}{\partial x_m}\right] + F_i$$

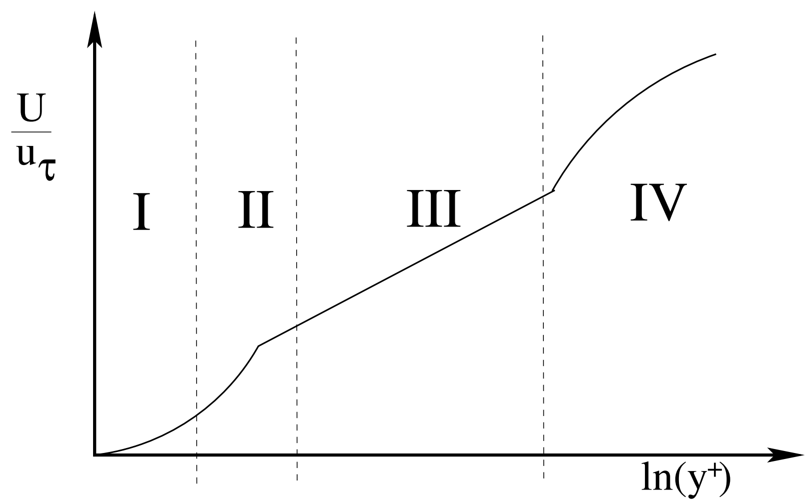
a. Write the equation from part [a.] with the additional assumption that the flow is incompressible.

4.[10 pt] Turbulent boundary layers.

a. What is the "law of the wall" for a turbulent boundary layer over a smooth flat plate?

b. What is the "log law" for a turbulent boundary layer over a smooth flat plate?

c. The following graph illustrates the typical velocity profile over a smooth flat plate



Label regions I–IV:

I –

II –

III –

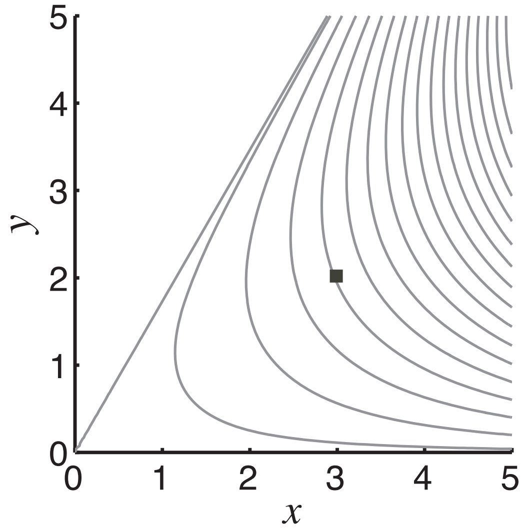
IV –

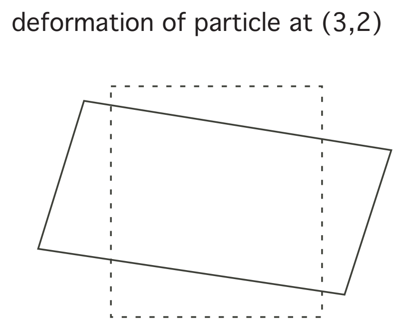
d. Which layers belong to:

1 – The inner region:

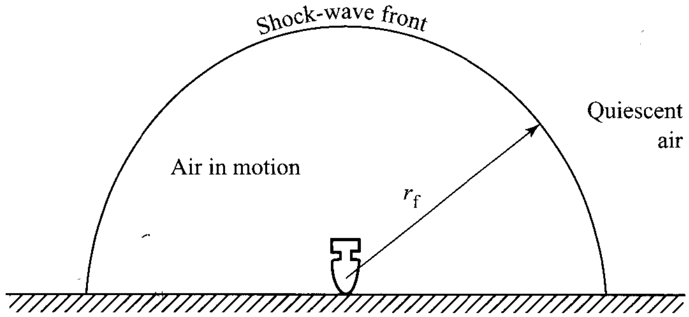
2 – The outer region:

5. [12 pt] Consider incompressible flow in a corner, the streamlines of which are drawn in the left figure below. The corresponding streamfunction is $\psi = x^2\, y - y^3/3$. The fluid particle located at $x\!=\!3$, $y\!=\!2$ deforms as shown in the right figure. Based on this write the corresponding strain-rate tensor and vorticity at this instant in space-time.





6.[14 pt] In 1940, G. I. Taylor developed a simple yet accurate model for the shock wave propagation from a nuclear blast. The figure shows a basic schematic of the problem.



Taylor assumed that the only variables that were important to model the shock wave radius r_f where:

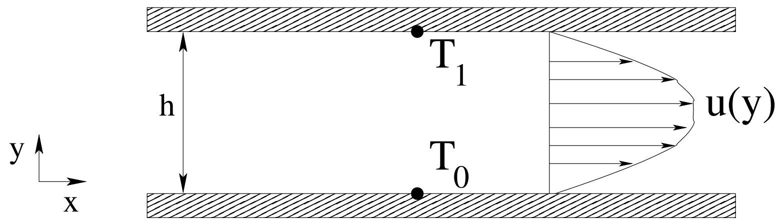
- the total explosion energy energy of the blast E in Joules [Kg m^{2}s^{-2}]
- the initial density of the ambient air ρ_o
- the time from the moment of explosion t
- and the ratio of specific heats γ

Using dimensional analysis and these variables, develop a functional relationship for r_f

7. [18 pt] The stream wise velocity profile for fully-developed, steady-state, incompressible flow between two infinite parallel plates driven by a constant stream wise pressure gradient dP/dx is given by:

$$u = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - hy\right) \text{ where } h \text{ is the distance between the plates}$$

Consider the case where the top and bottom plates are heated to constant temperatures T_0 and T_1 as shown in the figure.



a. Write down and then simplify (don't solve) the conservation of energy equation for this problem [clearly state all assumptions].

b. Using the velocity profile given above, solve for the temperature distribution in the channel.

c. Non dimensionalize your answer to part [b.] using the temperature difference $\Delta T = T_1 - T_0$ and then sketch what the non dimensional profile should look like and how it will change with $\chi = \frac{h^4}{144 k \Delta T} \left(\frac{dP}{dx}\right)^2$ for $T_1 > T_0$.

8. [18 pt] Laminar boundary layers:

a. Write the von Karman integral equation for a boundary layer over a solid wall with no suction/blowing.

b. Under what assumptions is the above equation valid?

c. Define mathematically (and in words) the displacement thickness.

d. Define mathematically (and in words) the momentum thickness.

e. Simplify the von Karman integral for flow over a flat plate (don't solve just simplify).

f. Assume a laminar boundary layer stream wise velocity profile given by:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right) = \eta \text{ where } \eta = y/\delta.$$

Using this and your reduced form of the von Karman integerl from part [e.], solve for $\delta(x)$ [Hint: change the integration limits in the momentum thickness definition from $0 \to \delta$ to $0 \to 1$].

f. How does this compare to the $\delta(x)$ you would obtain using the Blasius profile?