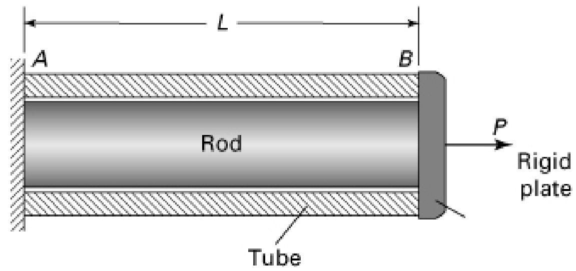
Homework 14 Solutions

1) The figure depicts a cylindrical rod of cross-sectional area A_r inserted into a tube of the same length L and of cross-sectional area A_t. The left ends of the members are attached to a rigid support and the right ends to a rigid plate. When an axial load P is applied as shown, determine the deflection at which both components begin to plastically deform and draw the load-deflection diagram of the rod–tube assembly. Given: $L = 1.2\ m$, $A_r = 45\ mm^2$, $A_t = 60\ mm^2$, $E_r = 200\ GPa$, $E_t = 100\ GPa$, $(\sigma_r)_{yp} = 250\ MPa$, and $(\sigma_t)_{yp} = 310\ MPa$. *Assume*: The rod and tube are both made of elastic-perfectly plastic materials, and they have no lateral interactions with each other.



Rod begins to yield at:

$$(P_r)_{yp} = (\sigma_r)_{yp} A_r = (250)(45) = 11.25\ kN$$

$$(\delta_r) = (\varepsilon_r)_{yp} L = \frac{(\sigma_r)_{yp}}{E_r} L = \frac{250 \times 10^6}{200 \times 10^9}(1.2) = 1.5\ mm$$

The result is shown in Fig. (a). Here Y_r corresponds to the onset of yield in the rod.

Tube begins to yield at:

$$(P_t)_{yp} = (\sigma_t)_{yp} A_t = (310)(60) = 18.6\ kN$$

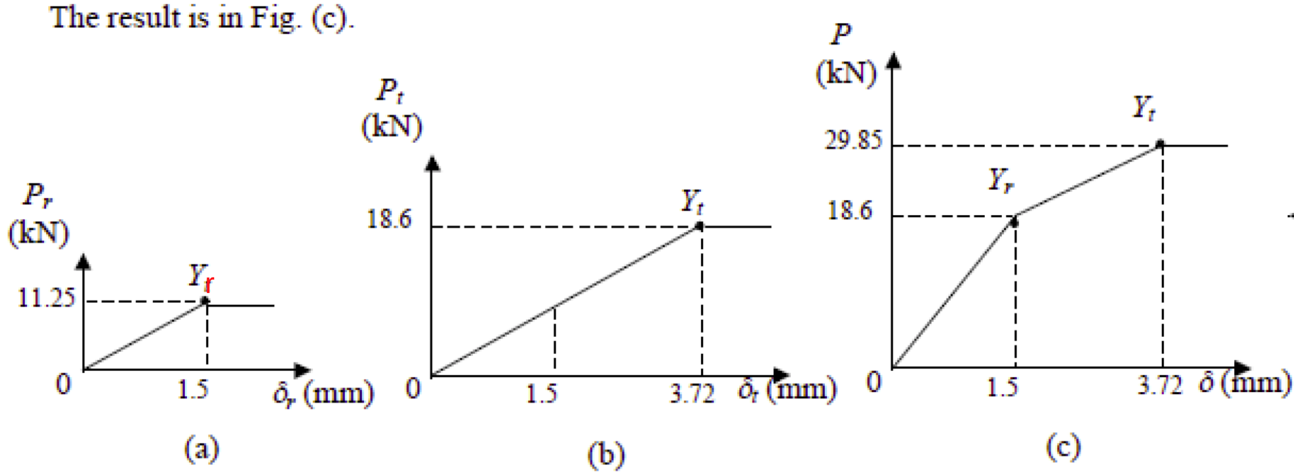
$$(\delta_t)_{yp} = \frac{(\sigma_t)_{yp}}{E_t} L = \frac{310 \times 10^6}{100 \times 10^9}(1.2) = 3.72\ mm$$ ◀

The result is shown in Fig. (b), where Y_t represents the onset of yield in the tube.

Total P-δ of the rod-tube combination:

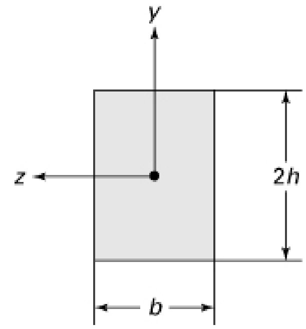
$$P = P_r + P_t \qquad \delta = \delta_r = \delta_t$$

The result is in Fig. (c).



◀

2) The figure shows the cross section of a rectangular beam made of mild steel with $\sigma_{yp} = 240\ MPa$. For bending about the z-axis, find (a) the yield moment; (b) the moment producing a $e = 20\text{-}mm$-thick plastic zone at the top and bottom of the beam. Given: $b = 60\ mm$ and $h = 40\ mm$.



Equation (12.9):

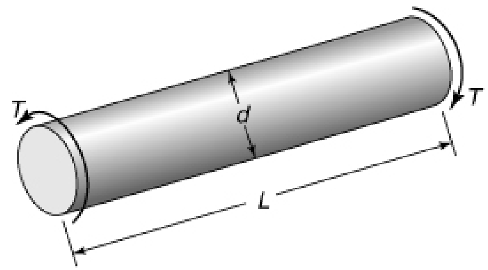
(a) $$M_{yp} = \frac{2}{3} b h^2 \sigma_{yp}$$

$$= \frac{2 \times 0.06(0.04)^2}{3}(240 \times 10^6) = 15.36\ kN \cdot m \quad ◀$$

(b) Equation (12.10b):

$$M = \frac{3}{2}(15.36 \times 10^3)[1 - \frac{1}{3}(\frac{20}{40})^2] = 21.12\ kN \cdot m \quad ◀$$

3) A circular shaft of diameter d and length L is subjected to a torque of T, as shown. The shaft is made of 6061-$T6$ aluminum alloy (see Table $D.1$), which is assumed to be elastoplastic. Find (a) the radius of the elastic core ρ_o; (b) the angle of twist φ. Given: $d = 50\ mm$, $L = 1.2\ m$, and $T = 4.5\ kN \cdot m$.



$G = 26 \cancel{\times 10^6}\ GPa, \qquad \tau_{yp} = 140\ MPa$ (Table D.1)

(a) For partially plastic shaft, using Eq.(12.19):

$$\left(\frac{\rho_0}{c}\right)^3 = 4 - \frac{3T}{T_{yp}} = 4 - \frac{6T}{\pi c^3 \tau_{yp}}$$

Substituting the given values

$$\left(\frac{\rho_0}{0.025}\right)^3 = 4 - \frac{6(4.5\times 10^3)}{\pi (0.025)^3 (140\times 10^6)} = 0.0711$$

$\rho_0 = \cancel{12.4}\ mm$ 10.4

(b) $\gamma_y = \dfrac{\rho_0 \phi}{L} = \dfrac{\tau_{yp}}{G}, \qquad \phi = \dfrac{\tau_{yp}}{G}\dfrac{L}{\rho_0}$

$\phi = \dfrac{140\times 10^6 (1.2)}{26\times 10^9 (0.\cancel{0124})} = 0.\cancel{5211}\ rad = \cancel{29.9}^{o}$ 0.6213 rad = 35.6 deg