Fluid Mechanics Qualifier Exam Equation Sheet

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial_x & \partial_y & \partial_z \\ v_x & v_y & v_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\theta}} & \hat{\mathbf{z}} \\ \partial_x & \partial_y & \partial_z \\ v_r & rv_\theta & v_z \end{vmatrix} \tag{1}$$

$$\nabla\phi = \hat{\mathbf{x}}\partial_x\phi + \hat{\mathbf{y}}\partial_y\phi + \hat{\mathbf{z}}\partial_z\phi = \hat{\mathbf{r}}\partial_r\phi + \hat{\theta}\frac{1}{r}\partial_\theta\phi + \hat{\mathbf{z}}\partial_z\phi \tag{2}$$

$$\nabla \cdot v = \partial_x v_x + \partial_y v_y + \partial_z v_z = \frac{1}{r}\partial_r(rv_r) + \frac{1}{r}\partial_\theta v_\theta + \partial_z v_z \tag{3}$$

$$\nu = \mu/\rho \tag{4}$$

$$Q = \iint_A \mathbf{v} \cdot \mathrm{d}\mathbf{A} \tag{5}$$

$$p = \rho RT \tag{6}$$

$$\frac{1}{V}\frac{DV}{Dt} = \nabla \cdot \mathbf{v} \tag{7}$$

$$e_{ij} = (\partial_i v_j + \partial_j v_i)/2 \tag{8}$$

$$v_i(\mathbf{r}_0 + \mathbf{dr}) = v_i(\mathbf{r}_0) + \sum_j e_{ij} dr_j + \frac{1}{2}(\boldsymbol{\omega} \times \mathbf{dr})_i \tag{9}$$

$$\Gamma = -\int_C \mathbf{v} \cdot \mathrm{d}\mathbf{s} = -\int_A (\nabla \times \mathbf{v}) \cdot \mathrm{d}\mathbf{A} \tag{10}$$

$$\tau_{ij} = -\delta_{ij}\left(p + \frac{2}{3}\mu\nabla \cdot \mathbf{v}\right) + \mu\left(\partial_i v_j + \partial_j v_i\right) \tag{11}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left(\frac{1}{r}\partial_\theta v_r + \partial_r v_\theta - \frac{v_\theta}{r}\right) \tag{12}$$

$$\mathbf{F} = \int_A \boldsymbol{\tau} \cdot \mathrm{d}\mathbf{A} \tag{13}$$

$$\mathbf{f} = \nabla \cdot \boldsymbol{\tau} \tag{14}$$

$$\partial_t\rho + \nabla \cdot (\rho\mathbf{v}) = 0 \tag{15}$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \mathbf{v} = 0 \tag{16}$$

$$\frac{D}{Dt}\mathbf{v} = \partial_t\mathbf{v} + \mathbf{v} \cdot \nabla\mathbf{v} \tag{17}$$

$$\rho\frac{\mathrm{D}}{\mathrm{D}t}(\vec{\mathbf{v}}) = \frac{\partial}{\partial t}(\rho\vec{\mathbf{v}}) + \nabla \cdot (\rho\vec{\mathbf{v}}\vec{\mathbf{v}}) = \rho\vec{\mathbf{g}} + \nabla \cdot \overleftrightarrow{\boldsymbol{\tau}} \tag{18}$$

$$
\begin{aligned}
\sum_{ij} \partial_j(v_i\tau_{ij}) &= \sum_{ij}(\tau_{ij}\partial_j v_i + v_i\partial_j\tau_{ij}) && (19)\\
\sum_{ij} \tau_{ij}\partial_j v_i &= -p(\nabla\cdot\mathbf{v}) + \phi && (20)\\
\phi &= 2\mu\sum_{ij} e_{ij}e_{ij} - \frac{2\mu}{3}(\nabla\cdot\mathbf{v})^2 && (21)
\end{aligned}
$$

$$
\sum \mathbf{F} = \frac{\mathrm{d}\,\mathrm{Mom}_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}\,\mathrm{Mom}_{CV}}{\mathrm{d}t} + \sum \mathrm{Mom}_{\mathrm{in}} - \sum \mathrm{Mom}_{\mathrm{out}} \tag{22}
$$

$$
\begin{aligned}
\rho\partial_t\mathbf{v} + \rho\mathbf{v}\cdot\nabla\mathbf{v} &= \rho\mathbf{g} - \nabla p + \mu\nabla^2\mathbf{v} && (23)\\
\rho\partial_t v_i + \rho v_j\partial_j v_i &= \rho g_i - \partial_i p + \mu\partial^2 v_i && (24)\\
\nabla\cdot\mathbf{v} &= \partial_i v_i = 0 && (25)\\
\partial_t v_x + \left[v_r\partial_r + \frac{v_\theta}{r}\partial_\theta + v_x\partial_x\right] v_x &= -\frac{1}{\rho}\partial_x p + \nu\left[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\theta^2 + \partial_x^2\right] v_x && (26)\\
\partial_t v_r + \left[v_r\partial_r + \frac{v_\theta}{r}\partial_\theta + v_x\partial_x\right] v_r - \frac{v_\theta^2}{r} &= -\frac{1}{\rho}\partial_r p + \nu\left(\left[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\theta^2 + \partial_x^2\right] v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\partial_\theta v_\theta\right) && (27)\\
\partial_t v_\theta + \left[v_r\partial_r + \frac{v_\theta}{r}\partial_\theta + v_x\partial_x\right] v_\theta + \frac{v_r v_\theta}{r} &= -\frac{1}{\rho r}\partial_\theta p + \nu\left(\left[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\theta^2 + \partial_x^2\right] v_\theta - \frac{2}{r^2}\partial_\theta v_r - \frac{v_\theta}{r^2}\right) && (28)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Re} &= \rho UL/\mu && (29)\\
\mathrm{Fr} &= U/\sqrt{gL} && (30)\\
\mathrm{Ma} &= U/a && (31)\\
\mathrm{Bo} &= \rho g L^2/\sigma && (32)\\
\mathrm{Ca} &= \mu U/\sigma && (33)\\
\frac{1}{2}\rho\mathbf{v}^2 + p + \rho g z &= const. && (34)\\
\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{\omega} &= (\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u} + \nu\nabla^2\boldsymbol{\omega} && (35)\\
C_D &= F_D/(A\rho U^2/2) && (36)\\
C_p &= \frac{p - p_\infty}{\frac{1}{2}\rho U^2} && (37)\\
\mathbf{v} &= \nabla\phi = \nabla\times(\psi\hat{\mathbf{z}}) && (38)
\end{aligned}
$$

water:

$$
\rho = 1000kg/m^3 \qquad \mu = 10^{-3}kg/(m\cdot s) \qquad \nu = 10^{-6}m^2/s \tag{39}
$$

air:

$$
\rho = 1.2kg/m^3 \qquad \mu = 1.8\times10^{-5}kg/(m\cdot s) \qquad \nu = 1.5\times10^{-5}m^2/s \tag{40}
$$