

Quiz 1: Flow Visualization

August 27, 2025

Q1.- A steady-state two dimensional velocity field is described by:

$$\vec{v} = (3x\hat{i} + 2y\hat{j}) \text{ m/s} \quad (1)$$

where x and y are the coordinates in meters, and $x, y \geq 0$. What is the equation of the streamlines in the flow domain?

- (a) $y = Cx^{(2/3)}$
- (b) $y = Ax + B$
- (c) $y = Cx^{(-2/3)} + D$

Solution:

To solve this problem one needs to start from the definition of **Streamlines** for a 2D flow field; hence:

$$\frac{dx}{u} = \frac{dy}{v} \quad (2)$$

which upon substitution leads to,

$$\frac{dx}{u} = \frac{dy}{v} \quad (3)$$

$$\frac{dx}{3x} = \frac{dy}{2y} \quad (4)$$

$$\frac{1}{3} \ln x = \frac{1}{2} \ln y \quad (5)$$

$$\implies y = Cx^{2/3} \quad (6)$$

Q2.- How is the pathline for a particle in this flow?

- (a) equal to the streamline, and different than the streakline.
- (b) equal to the streamline and the streakline.
- (c) different from both, the streamline and streakline.

Solution:

In this case, since the flow is steady, the pathline will be equal to the streamline and the streakline. One can solve the equations for a generic pathline equation, but to specifically determine the

pathline associated to a given particle we would need to know their corresponding position at $t = 0$.

Q3.- Given the flow field from Q1,

- (a) What is the local acceleration?
- (b) What is the advective acceleration?
- (c) What is the material acceleration of a fluid particle?

Solution:

[a] The local acceleration of a flow field is given by:

$$\frac{\partial \vec{v}}{\partial t}, \quad \text{which in this case is equal to} \quad \frac{\partial \vec{v}}{\partial t} = \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} = 0 \text{ m/s}^2. \quad (7)$$

[b] The advective acceleration of a flow field is given by:

$$\vec{v} \cdot \vec{\nabla} \vec{v} = \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \hat{i} + \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \hat{j} = \quad (8)$$

$$(9x + 0)\hat{i} + (0 + 4y)\hat{j} = 9x\hat{i} + 4y\hat{j} = (9x, 4y) \text{ m/s}^2 \quad (9)$$

[c] The material acceleration is,

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = (0, 0) + (9x, 4y) = (9x, 4y) \text{ m/s}^2 \quad (10)$$