1) a) $\underbrace{\rho \frac{Du}{Dt}}_{\mathrm{I}} = \rho\left(\underbrace{\frac{\partial u}{\partial t}}_{\mathrm{II}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\mathrm{III}}\right)$

b) It is important because it describes time rate of change of a fluid property in both Lagrangian and Eulerian view points, and provides a link between the two.

c) I: Acceleration of individual fluid particles
II: "local" acceleration of fluid at fixed points
III: Apparent acceleration in the x, y and z directions

2) a) streamline: $u\,dy - v\,dx = 0$
streamfunction: $\partial\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$
also: $u \equiv \frac{\partial\psi}{\partial y}$, $v \equiv -\frac{\partial\psi}{\partial x}$

so we have:

$$\partial\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy \Rightarrow \underbrace{-v\,dx + u\,dy}_{\text{streamline definition}} = 0$$

$$\Rightarrow \partial\psi = 0 \Rightarrow \int\partial\psi = \int 0 \Rightarrow \boxed{\psi = C}$$

c) Points 1 and 3 are about the same magnitude, and point 1 is less than points 2 and 4. You can see the flow field picking up speed as it moves in the +x direction. Also, no flow between streamlines so $A_1V_1 = A_2V_2 \Rightarrow$ V_2 must have bigger magnitude

3)

a) $u = x^2 + 2xy \qquad v = -y^2 - 2xy$

$$a_x = \frac{Du}{Dt} = \cancelto{0}{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancelto{0}{w\frac{\partial u}{\partial z}}$$

$$(x^2+2xy)(2x+2y) + (-y^2-2xy)(2x)$$

$$\boxed{a_x = 2x^3 + 2x^2y + 2xy^2}$$

$$a_y = \frac{Dv}{Dt} = \cancelto{0}{\frac{\partial v}{\partial t}} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \cancelto{0}{w\frac{\partial v}{\partial z}}$$

$$\boxed{a_y = 2y^3 + 2x^2y + 2xy^3}$$

b) } see attached MATLAB
c) }

4) a) $u = y, \; v = x$

$u\,dy - v\,dx = 0 \Rightarrow y\,dy = x\,dx \Rightarrow \int y\,dy = \int x\,dx$

$$\Rightarrow \frac{y^2}{2} + C_1 = \frac{x^2}{2} + C_2 \Rightarrow \boxed{x^2 - y^2 = C_3}$$

b) see attached MATLAB

5) a) $u = \frac{x}{1+t}$, $v = \frac{2y}{2+t}$

$u\,dy - v\,dx = 0 \Rightarrow \frac{x}{1+t}\,dy = \frac{2y}{2+t}\,dx \Rightarrow \frac{2+t}{2y}\,dy = \frac{1+t}{x}\,dx$

$$\int_{y_0}^{y} \frac{2+t}{2y}\,dy = \int_{x_0}^{x} \frac{1+t}{x}\,dx \Rightarrow \frac{(2+t)}{2}\ln\left(\frac{y}{y_0}\right) = (1+t)\ln\left(\frac{x}{x_0}\right)$$

$$\ln\left(\frac{y}{y_0}\right) = \frac{(2+2t)}{(2+t)}\ln\left(\frac{x}{x_0}\right) \Rightarrow \boxed{y = y_0\left(\frac{x}{x_0}\right)^{\left[\frac{(2+2t)}{(2+t)}\right]}}$$

b) $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$

for x:

$$\frac{dx}{dt} = \frac{x}{1+t} \Rightarrow \int \frac{dx}{x} = \int \frac{dt}{1+t} \Rightarrow x = C_1(1+t)$$

ICs: $x(t=0) = x_0 \Rightarrow x_0 = C_1(1+0) \Rightarrow C_1 = x_0$

$$x = x_0(1+t) \Rightarrow t = \left(\frac{x}{x_0} - 1\right)$$

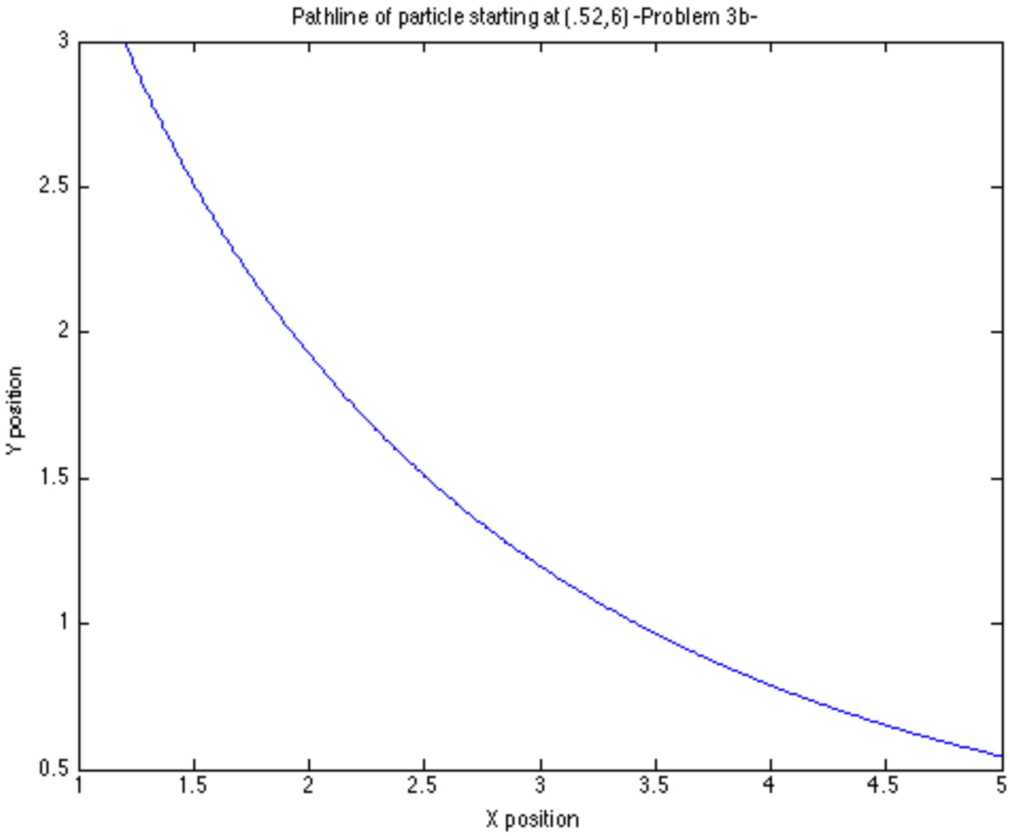
for y: $\frac{dy}{dt} = \frac{2y}{2+t} \Rightarrow \int \frac{dy}{2y} = \int \frac{dt}{2+t} \Rightarrow \frac{1}{2}\ln(y) = \ln(2+t) + C_2$

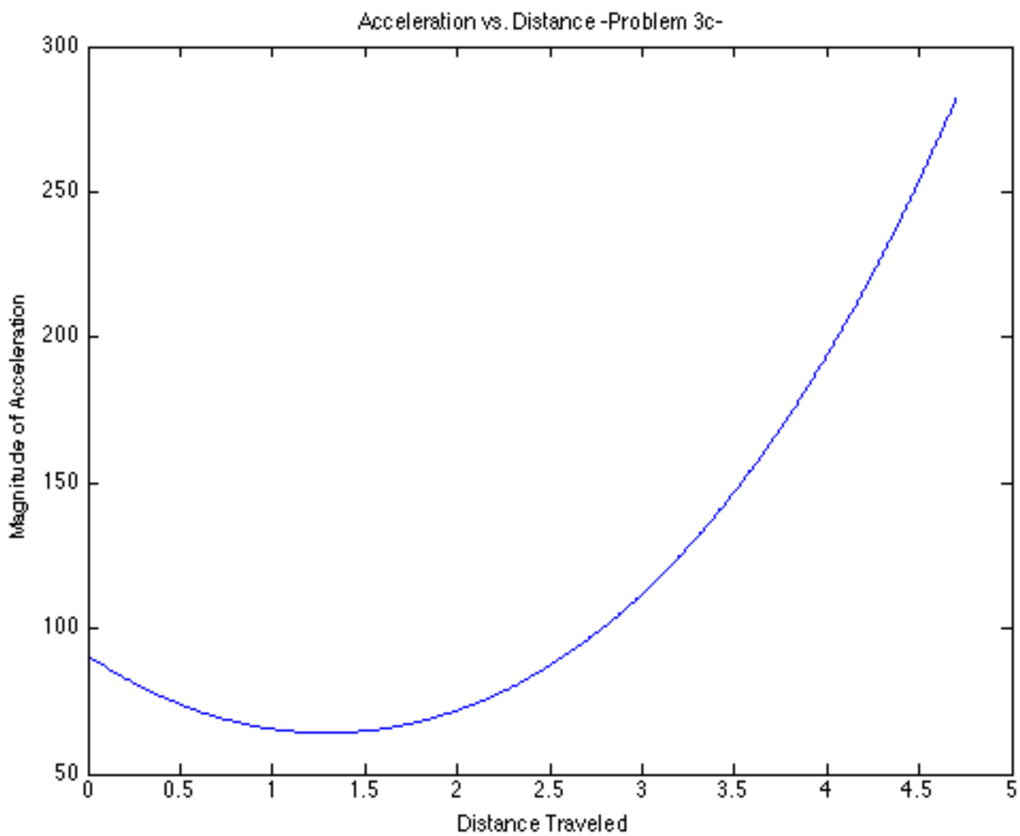
$y = C_2(t+2)^2 \Rightarrow$ ICs: $y(t=0) = y_0 \Rightarrow y_0 = C_2(0+2)^2 = C_2 = \frac{y_0}{4}$

$$y = \frac{y_0}{4}(t+2)^2 \Rightarrow \boxed{y = \frac{y_0}{4}\left[\left(\frac{x}{x_0}\right) + 1\right]^2}$$

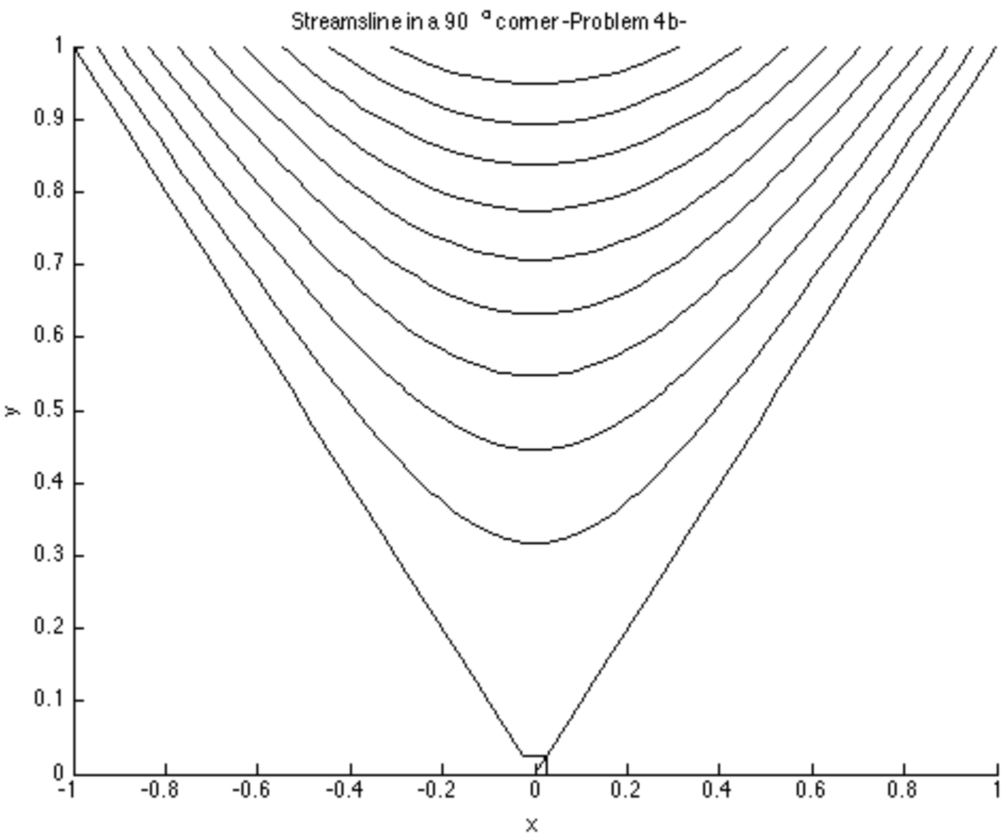
c) see attached MATLAB

```
%
% Class: ME 5700/6700
% Assignment: HW#2 Matlab Solutions
%
%
clc; close all;
% set array length for the x position of the particle
x_pos = linspace(1.2,5,500);
% set initial position for particle
x_0 = x_pos(1);
y_0 = 3;
% compute y position of the particle
y_pos = (-x_pos.^2 + sqrt(x_pos.^4 + 4.*x_pos*(x_0^2*y_0+y_0^2*x_0)))./(2.*x_pos);
% plot the x vs. y position of the particle
plot(x_pos,y_pos)
% label plot of x vs. y position
xlabel('X position')
ylabel('Y position')
title('Pathline of particle starting at (.52,6) -Problem 3b-')
% calculate acceleration in the x direction
acc_x = 2*x_pos.^3+2*x_pos.^2.*y_pos+2*x_pos.*y_pos.^2;
% calculate acceleration in the y direction
acc_y = 2*y_pos.^3+2*x_pos.^2.*y_pos+2*x_pos.*y_pos.^2;
% calculate the magnitude of acceleration
acc_tot = sqrt(acc_x.^2+acc_y.^2);
% set array length of distance traveled between each point
dist = zeros(1,500);
% set array length for the total distance traveled
dist_tot = zeros(1,500);
% calculate the distance traveled between two points and sums them to find
% the total distance travelled
for j = 1:499
    dist(j+1) = sqrt((x_pos(j+1)-x_pos(j))^2+(y_pos(j+1)-y_pos(j))^2);
    dist_tot(j+1) = sum(dist);
end
figure;
plot(dist_tot,acc_tot)
xlabel('Distance Traveled')
ylabel('Magnitude of Acceleration')
title('Acceleration vs. Distance -Problem 3c-')
```


Pathline of particle starting at (.52,6) -Problem 3b-
Y position
X position
3
2.5
2
1.5
1
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

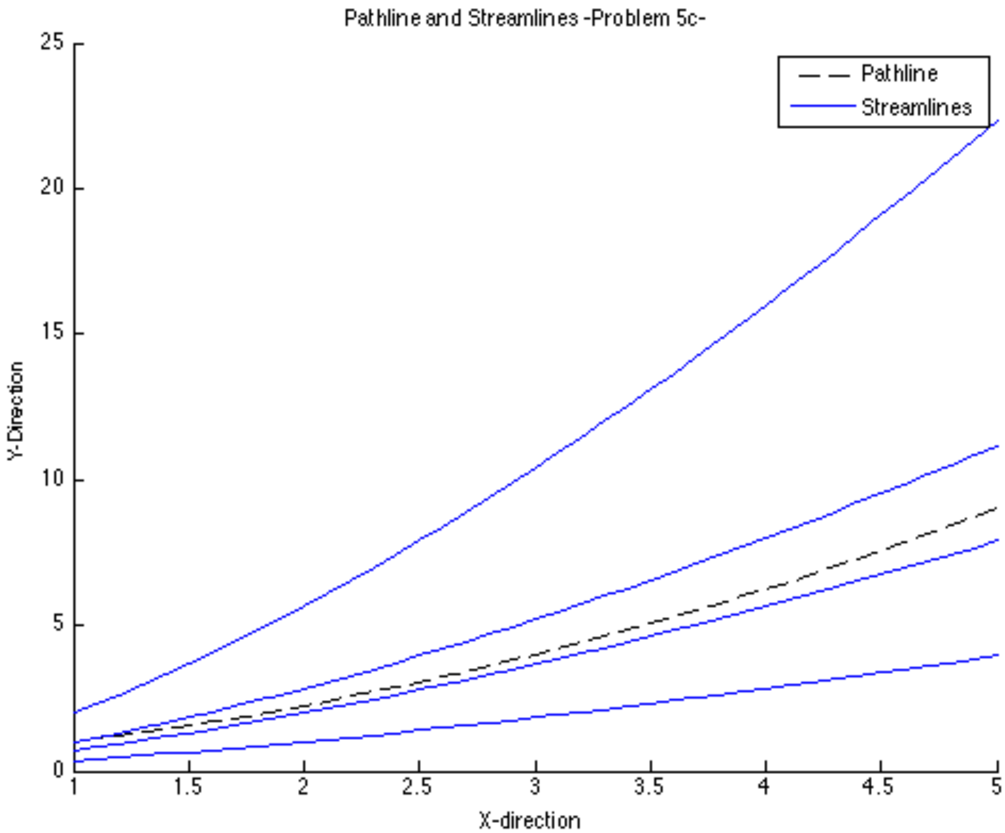



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
syms x syms y
% plots streamlines
figure;
hold on
ezplot('x^2-y^2+.1')
ezplot('x^2-y^2+.2')
ezplot('x^2-y^2+.3')
ezplot('x^2-y^2+.4')
ezplot('x^2-y^2+.5')
ezplot('x^2-y^2+.6')
ezplot('x^2-y^2+.7')
ezplot('x^2-y^2+.8')
ezplot('x^2-y^2+.9')
ezplot('x^2-y^2')
axis([-1, 1, 0, 1])
title('Streamsline in a 90^o corner -Problem 4b-')
xlabel('x')
ylabel('y')
% sets line color to black
colormap([0 0 0 ])
```




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% sets arrary for x values
x_sm = 1:.1:5;
% sets time as described in HW#1 Problem 5
time = 2;
% sets different (x0,y0) values, including (1,1) from Problem 5
%x_01 = [1,2];
%y_01 = [1,2];
x_01 = [1,2];
y_01 = [1,2];
% streamline equation for different (x0,y0)
y_sm = y_01(1)*(x_sm./x_01(1)).^((2+2*time)/(2+time));
y_sm1 = y_01(1)*(x_sm./x_01(2)).^((2+2*time)/(2+time));
y_sm2 = y_01(2)*(x_sm./x_01(1)).^((2+2*time)/(2+time));
y_sm3 = y_01(2)*(x_sm./x_01(2)).^((2+2*time)/(2+time));
% pathline equation for (x0,y0) at t = 2
y_path = (y_01(1)/4)*((x_sm./x_01(1)+1)).^2;
figure;
hold on
% plots pathline as dashed line
plot(x_sm,y_path,'--k')
% plots streamlines as solid lines
plot(x_sm,y_sm)
plot(x_sm,y_sm1)
plot(x_sm,y_sm2)
```

```
plot(x_sm,y_sm3)
xlabel('X-direction')
ylabel('Y-Direction')
title('Pathline and Streamlines -Problem 5c-')
legend('Pathline','Streamlines')
```



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