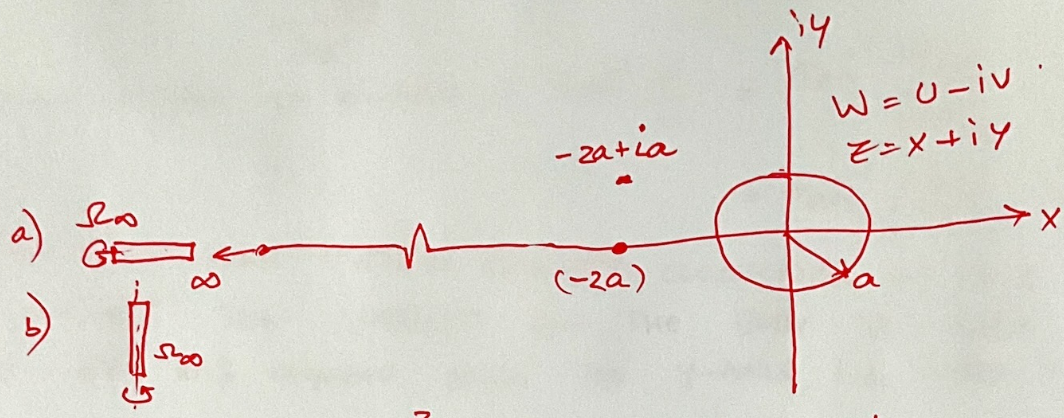
1) ASSUME THE VARIOUS TORQUES ARE NEGLIBLE, ~~AND~~ THE VORTEX TUBES OR EDDY WILL CONVECT WITH FLUID TUBE AND VORTICITY $\propto \frac{1}{\text{AREA OF EDDY}}$



~~NOTE~~ SINCE $W = V_\infty - \frac{V_\infty a^2}{z^2}$, at $z \to \infty$, $\underline{W = V_\infty}$

a) $z = -2a$, ~~X~~EDDY ALONG X−AXIS.

$$W = V_\infty - \frac{V_\infty a^2}{z^2} = V_\infty - \frac{V_\infty a^2}{4a^2} = \frac{3V_\infty}{4}$$

FOR THE EDDY, THE ~~R EDDY~~ X−EXTENT DECREASES BY 25% (CONS. OF MASS), AND THEREFORE THE Y−EXTENT OR THE C-S AREA INCREASES BY 4/3.

Since $\Omega \propto \frac{1}{\text{Area}}$, the $\Omega_0 = \frac{3}{4}\Omega_\infty$

b) $z = -2a$ EDDY ALONG Y−AXIS.

$$W = V_\infty - \frac{V_\infty a^2}{4a^2} = \frac{3}{4}V_\infty$$

FOR THE VERTICAL EDDY, THE CROSS−SECTIONAL AREA DECREASES BY 25% ie Area at $z = -2a$ $= \frac{3}{4}$(Area at ∞), so THE VORTICITY $\Omega = \frac{4}{3}\Omega_\infty$.

c) POINT $z = -2a + ia$

$$w(\equiv u - iv) = V_\infty - \frac{V_\infty a^2}{(-2a+ia)^2} = V_\infty - \frac{V_\infty a^2}{(3-4i)a^2}$$

$$= V_\infty - \frac{V_\infty(3+4i)}{25}$$

SO $\cancel{W}$ $u = V_\infty - V_\infty \frac{3}{25}$

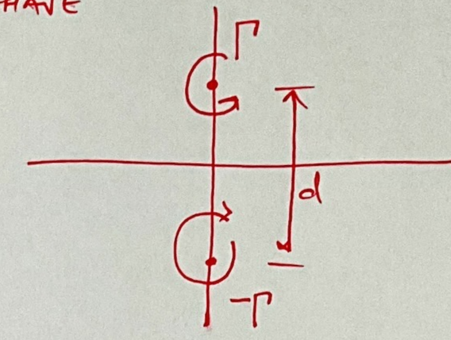
$v = + \frac{4 V_\infty}{25}$

SO FLOW ANGLE TO X-AXIS $= \tan^{-1} \frac{v}{u} = \tan^{-1} \frac{4/25}{1-3/25}$

$= \tan^{-1}\left(\frac{2}{11}\right) = \underline{\underline{10.3^\circ}}$

SINCE THE FLOW HAS TURNED ~~C~~COUNTER CLOCKWISE BY 10.3° WE WOULD EXPECT THE VORTICITY AND THE EDDY TO TURN CLOCKWISE BY 10.3 DEGREES FROM THE Y-AXIS, i.e ANGLE $= \underline{\underline{-10.3^\circ}}$

2) a) IN TERMS OF THE COMPLEX VELOCITY; WE HAVE



$$W = \frac{-i\Gamma}{2\pi(z - id/2)} + \frac{i\Gamma}{2\pi(z + id/2)}$$

$$= \frac{i\Gamma}{2\pi}\left[\frac{(z - id/2) - (z + id/2)}{z^2 + d^2/4}\right]$$

$$= \frac{i\Gamma}{2\pi}\left[\frac{-id}{z^2 + d^2/4}\right] = \frac{\Gamma d}{2\pi}\frac{1}{z^2 + d^2/4}$$

WE ARE INTERESTED IN $\displaystyle\lim_{\substack{d \to 0 \\ \Gamma d \to \text{CONST}}} W(z) = \frac{\Gamma d}{2\pi z^2}$

THIS IS THE VELOCITY FIELD OF A DOUBLET WITH STRENGTH Γd AND DIRECTION PERPENDICULAR TO THE LINE INITIALLY JOINING THE TWO VORTICES.

b) $\log_e(z) = \log_e(re^{i\theta}) = \log_e r + i\theta$

SINCE θ IS MULTI-VALUED, INCREASING BY 2π EVERYTIME YOU GO AROUND THE ORIGIN, SO DOES $\log_e(z)$ INCREASING BY $2\pi i$ ON ANY COUNTER CLOCK WISE LOOP THAT ENCLOSES THE ORIGIN.

LIKEWISE $\log_e(z - A)$, CAN BE WRITTEN AS $\log_e(r_1 e^{i\theta_1})$ WHERE r_1 and θ_1 ARE MEASURED FROM POINT A INSTEAD OF THE ORIGIN. THUS THIS FUNCTION INCREASES BY $2\pi i$ ON ANY COUNTERCLOCKWISE loop that ENCLOSES A.

3) INVESTIGATING $W(z) = Az^2$, $A = ce^{i\beta}$, $z = re^{i\theta}$

$$V_r - iV_\theta = W(z)\,e^{i\theta} = c e^{i\beta} r^2 e^{2i\theta} e^{i\theta} = cr^2 e^{i(3\theta+\beta)}$$

$$= cr^2\left[\cos(3\theta+\beta) + i\sin(3\theta+\beta)\right]$$

So, $V_r = cr^2\cos(3\theta+\beta)$; $V_\theta = -cr^2\sin(3\theta+\beta)$

$$\phi + i\psi = F(z) = \int W\,dz = \frac{Az^3}{3} = \frac{cr^3}{3}e^{i(3\theta+\beta)}$$

So,

$$\underline{\underline{\phi = \frac{cr^3}{3}\cos(3\theta+\beta)}} \quad ; \quad \underline{\underline{\psi = \frac{cr^3}{3}\sin(3\theta+\beta)}}$$

WE CAN PLOT STREAMLINES BY GRAPHING lines of $\psi = \text{const}$

ie

$$r = \sqrt[3]{\frac{\text{CONST.}}{\sin(3\theta+\beta)}}$$

FOR A = real ($\beta = 0$), PLOT IS SHOWN BELOW. THIS IS A 6-WAY STAGNATION ~~STREAMLINE~~ POINT AT THE ORIGIN. SETTING A = COMPLEX THE STREAMLINE THAT APPEARS HORIZONTAL IN THE FIGURE WILL APPEAR AT AN ANGLE 'β' TO THE X-AXIS, THE WHOLE FLOW BEING ROTATED BY THIS ANGLE.