Homework 5

2D Ideal Flow

Your answers to these questions, and what you learn from them, will be greatly enhanced through collaboration and discussion amongst your discussion group and in the recitation. This is actively encouraged. However, once you have decided how to answer these, the final solutions must be prepared individually.

1. Consider a free stream flow of velocity V_∞ at an angle α to the x axis. A vortex of strength Γ is placed in this flow at the origin. This flow combination is the simplest possible model of a lifting airfoil. The airfoil of chordlength c is thought of as extending from the point $(-c/4{,}0)$ to $(3c/4{,}0)$. Thus, the vortex is at the quarter-chord point. The vortex strength is set so that, at the 3/4-chord point (at $x = c/2$) there is no vertical component of velocity (i.e. no flow through the chord line). This is a manifestation of what we will come to call the Kutta condition.
   a) Determine the complex potential and velocity of this flow expressing Γ in terms of V_∞, α and c
   b) Express the lift coefficient on the airfoil as a function of α. Note that the lift coefficient Cl is defined as the lift per unit span normalized on $\frac{1}{2}\rho V_\infty^2 c$.
   c) The actual flow is far from that over the imagined airfoil. Find equation(s) for the location(s) of any stagnation point(s) in the flow. Obtain an equation for the streamlines of the flow.
   d) Plot the body shape and flow streamlines using the Ideal Flow Machine. Consider an airfoil of chord 4 units, in a free stream of velocity 1 unit, at an angle of attack of 10 degrees. (Note that you will need to use 'Print Screen' to record your Ideal Flow Machine results.)

2. The point singularities we have introduced may be generalized by distributing their effects along lines (usually called 'sheets' or 'panels') rather than concentrating their effects at a point. The strength of such singularities is expressed per unit length - e.g. volumetric outflow per unit length, in the case of a source panel, or circulation per unit length of a vortex panel. Thus, each differential element of the sheet may be thought of as behaving as the equivalent point singularity with a strength given by the local sheet strength times the differential length of the element.
   a. Find an integral expression for the complex velocity field produced by a vortex sheet of length l lying on the x axis, allowing the sheet strength $\gamma(x)$ to be a function of distance along it.
   b. Reduce this to an algebraic expression in which the strength varies linearly with distance along the sheet from γ_1 to γ_2.
   c. Visualize the flow produced by this sheet with $\gamma_1 = 1$ and $\gamma_2 = 2$ and unit sheet length using Ideal Flow Machine. (*Hint*: Choose the vortex panel from the dropdown and in the input dialog box enter "1 2" i.e 1*space*2.)

d. Plot the velocity vectors for this panel using Matlab. Hint: use the quiver function or the code snippet below.

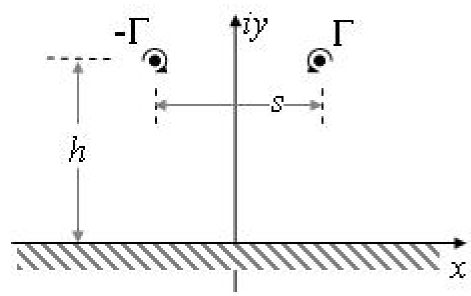
```
clear all
Gamma1 = 1; Gamma2 = 2; l=1;
[X, Y] = meshgrid([-2:0.15:3] ,[-2:.15:3]);
Z = X + i*Y;

W = 'Put your estimate here'

u = real(W); v = -imag(W);

figure
quiver(X,Y,u,v,0)
axis image
axis tight
title('Vortex sheet between (0,0) and (1,0)')
```

3. The two wing tip vortices produced by an aircraft tend to convect each other towards the ground. The figure shows a model of this flow, consisting of two ideal point vortices, separated by distance s a distance h above the ground at a particular time instant. Determine the complex potential and velocity of this flow at this instant. Draw the flow field at the instant shown (taking s=h) using the Ideal Flow Machine.



Hint: Remember to consider the mirror images accounting for the change in sign.