

Differential Forms of the Curl

$$\text{curl} \vec{A} \equiv -\lim_{\delta\tau \rightarrow 0} \frac{1}{\delta\tau} \oint_{\Delta S} \vec{A} \times \vec{n} dS$$

$$\text{curl} \vec{A} = \nabla \times A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \vec{e}_r & r \vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & r A_\theta & A_z \end{vmatrix} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \vec{e}_r & r \vec{e}_\theta & r \sin \theta \vec{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

Cartesian
Cylindrical
Spherical