

**ME EN 5830/6830: Aerospace Propulsion**  
**Problem Set #8: Airbreathing Propulsion VI and VII**  
**Due date: 03/18/2023 by 11:59pm**

**Submission**

Assignments can only be submitted on Gradescope, which can be accessed through Canvas. If you have any questions about submission, please email the class TA, John Gardner at [john.w.gardner@utah.edu](mailto:john.w.gardner@utah.edu). Submissions will be automatically locked at the due date given above.

**Introduction**

This problem set primarily covers the material from Lectures 16 and 17. The goal is for students to perform analysis of supersonic engines and consider how temperature limitations affect such engines. After completion of the assignment, students should be able to:

- Analyze a supersonic turbojet engine.
- Compute and understand the effects of an afterburner.
- Analyze a ramjet.
- Understand how properties like thrust and fuel consumption vary with flight speed.

## Assignment

**Problem #1:** Consider the Rolls Royce Olympus 593 turbojet engine used in the Concorde, traveling at Mach 2.0 at 51,000 ft. At this elevation, the static temperature and pressure are  $T_0 = 216.65$  K and  $p_0 = 11,000$  Pa. Because this turbojet is traveling supersonically, two oblique shocks followed by a normal shock are present at the inlet. In addition, this engine utilizes an afterburner between the turbine exit and nozzle inlet. You may assume:

The pressure ratio across the compressor:  $r_p = p_{t3}/p_{t2} = 15.5$

The efficiency of the compressor:  $\eta_c = 0.85$

The efficiency of the turbine:  $\eta_t = 0.90$

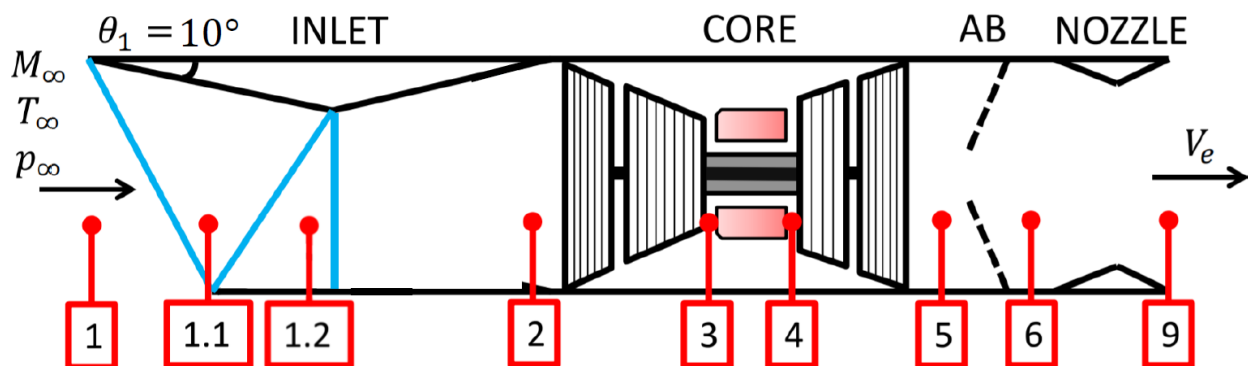
All other devices have isentropic efficiencies of 1.0

The fuel being used: n-dodecane,  $LHV = 43$  MJ/kg,  $(F/A)_{st} = 0.067$  (Same as Problem Set #6)

An overall equivalence ratio of  $\phi = 0.387$  (counting fuel used in combustor + afterburner)

The working fluid is air throughout the engine ( $\gamma = 1.4$ ,  $R = 287$  J/kg-K)

Throughout the problem set, you will analyze variations of this engine. **Please use Matlab or Excel to solve your equations, but as always, provide the equations and final calculations in your submission when explicitly asked. You are welcome to adapt your code/spreadsheets from Problem Set #6. A Matlab code is provided for a standard turbojet analysis on Canvas.**



- Compute the inlet velocity, stagnation temperature, and stagnation pressure ( $U_0, T_{t0}, p_{t0}$ ). Show your work.
- Assume the engine has a circular inlet with diameter  $D = 1.212$  m. Calculate the mass flow rate through the engine.

- c) Assume the first oblique shockwave deflects the flow  $\theta_1 = 10^\circ$ . At a Mach number of  $M_0 = 2$ , this corresponds to a shock angle of  $\beta_1 = 39.32^\circ$ . Compute the Mach number, stagnation temperature, and stagnation pressure after the first shock ( $M_{1.1}, T_{t1.1}, p_{t1.1}$ ). Show your work.
- d) Assume the second oblique shockwave deflects the flow  $\theta_2 = 10^\circ$ . At a Mach number of  $M_{1.1}$  (computed in part a), this corresponds to a shock angle of  $\beta_2 = 49.40^\circ$ . Compute the Mach number, stagnation temperature, and stagnation pressure after this shock ( $M_{1.2}, T_{t1.2}, p_{t1.2}$ ). Show your work.
- e) Compute the Mach number, stagnation temperature, and stagnation pressure after the normal shock ( $M_2, T_{t2}, p_{t2}$ ). Show your work.
- f) The analysis between states 2 and 5 follows the same procedure as in Problem Set #6 (excluding the fan). Calculate the stagnation temperature and pressure after the turbine ( $T_{t5}, p_{t5}$ ). You do not need to show your work.
- g) Assume that no fuel is fed into the afterburner. Then,  $T_{t6} = T_{t5}$  and  $p_{t6} = p_{t5}$ . Compute the thrust (not specific thrust), propulsive efficiency, and thrust specific fuel consumption. Show your work.
- h) Perform a second analysis of the engine where, rather than 100% of fuel being used in the combustor, there is instead a 50%/50% split of fuel between the combustor and the afterburner. Compute the thrust (not specific thrust), propulsive efficiency, and thrust specific fuel consumption. You do not need to show your work. *Hint: Splitting the fuel is equivalent to an equivalence ratio of  $\phi/2 = 0.387/2$  being used in each the combustor and the afterburner (for a total equivalence ratio of  $\phi = 0.387$  used throughout).*
- i) Perform a third analysis of the engine where the percentage of fuel going to the afterburner is varied from 0% to 100%. Plot the thrust (not specific thrust), propulsive efficiency, and thrust specific fuel consumption as a function of the percentage of fuel to the afterburner. *Note: It is not typical to split the fuel between combustor and afterburner. Rather, additional fuel is typically added to the afterburner. However, it is instructive to look at some of the trends.*
- j) Based on the plots from the previous question, explain why thrust decreases and thrust specific fuel consumption increases with increased afterburner percentage.

- k) Plot the stagnation temperature at stations 4 and 6 ( $T_{t4}, T_{t6}$ ) as a function of afterburner fuel percentage. Describe the trends you see and why they occur.
- l) Assume our system cannot handle a turbine inlet stagnation temperature above  $T_{t4} = 1750$  K. Approximate from your plots the minimum afterburner fuel percentage that maintains this temperature.
- m) Our system is characterized by a Pattern Factor of  $PF = 0.25$ . Compute the maximum stagnation temperature we can expect for this system at the turbine inlet. Show your work.
- n) Plot the maximum stagnation temperature at station 4 based on the Pattern Factor in the previous part as a function of afterburner fuel percentage.
- o) Rather than assume our system cannot handle a turbine inlet stagnation temperature above  $T_{t4} = 1750$  K as in part l), instead assume our system cannot handle a turbine inlet maximum stagnation temperature (based on a Pattern Factor of 0.25) above  $T_{t_{max}} = 1750$  K. Approximate from your plots the minimum afterburner fuel percentage that maintains this temperature.

**Problem #2:** Working with the same system and assumptions as in Problem #1, we will now analyze the engine as a function of flight speed. Assume fuel only goes to the combustor. In addition, assume that for an oblique shock at  $\theta = 10^\circ$ , we can approximate the relationship between the Mach number before the oblique shock and the shock angle as  $\beta = -1.556M^3 + 18.66M^2 - 77.21M + 132.5$  (in degrees).

- a) Re-using the analysis from Problem #1, but dynamically computing  $\beta$ , plot the thrust and thrust specific fuel consumption as a function of the aircraft travel Mach number ( $M_0$ ) from  $2 < M_0 < 4.5$ . Describe and explain the trends you see in both plots.
- b) Compute and plot the ratio of the static pressure at state 2 to state 1 ( $p_2/p_1$ ). This is the compression performed purely by the shocks.
- c) At what Mach number can we expect to get a pressure ratio of approximately  $p_2/p_1 = 45$ ? *Note: Recall that this pressure ratio is reminiscent of what compressors achieve in subsonic gas turbines.*

**Problem #3:** Assume that the engine described throughout this problem transitions from a turbojet to a ramjet at high velocities. (*Note: The Concorde does NOT do this, but it is an interesting problem and reminiscent of the SR-71*). To analyze this ramjet, you can take your previous turbojet analysis, remove the compressor, combustor, and turbine, and divert all fuel to the afterburner. Use the same equation for the shock angle as in Problem #2.

- a) Compute and plot the thrust and thrust specific fuel consumption as a function of the aircraft travel Mach number ( $M_0$ ) from  $2 < M_0 < 4.5$ . Describe and explain the trends you see in both plots. How do these trends compare to the trends from Problem #2?
- b) How does the order of magnitude of ramjet thrust and thrust specific fuel consumption compare to that of the turbojet from Problem #2?
- c) How much thrust does the ramjet produce at zero flight velocity?