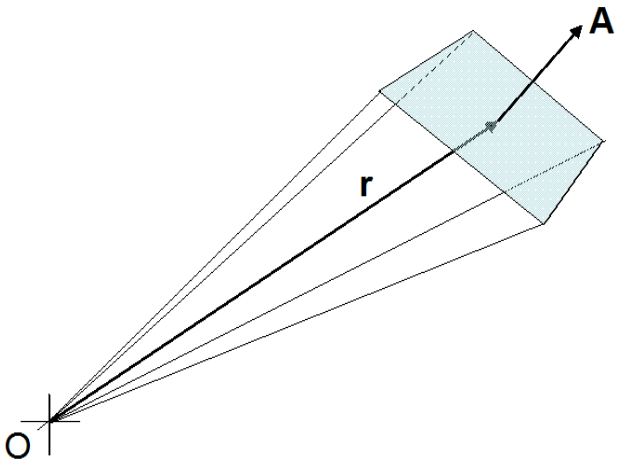
Homework 2

Math, fluids and statics

Your answers to these questions, and what you learn from them, will be greatly enhanced through collaboration and discussion amongst your discussion group and in the recitation. This is actively encouraged. However, once you have decided how to answer these, the final solutions must be prepared individually.

1. (a) Consider a flat panel (e.g. a polygon) in 3D space. The panel has an area defined by the vector A and a centroid at a position relative to the origin given by the position vector r. The panel and the lines joining its vertices to the origin define a pyramidal shape. What is the volume of this shape in terms of r and A?



(b) Consider a closed 3D body with a surface formed by a series of flat panels. The centroid of the ith panel is given, relative to a fixed origin, by the position vector ri. The area of the ith panel is given by the outward-pointing vector Ai. Find an expression for the volume of the body in terms of Ai and ri. Discuss whether there are restrictions on the applicability of your result, i.e. does it work if there are voids in the body, does it work if the body is separated into more than one piece, or if the origin sits outside the body.

2. Working in terms of cylindrical coordinates and components make up a velocity field of a flow with zero divergence but non-zero curl. (You are expected to use a documented deterministic process, rather than trial and error to obtain a divergenceless field.) Your flow must involve all three velocity components. (a) Sketch your flow. (b) Find functions describing the average angular velocity and the proportionate rate of change of volume of fluid particles in the flow.

3. Determine the distribution of hydrostatic pressure in a compressible fluid subject to constant gravity. Assume a pressure density relation $p \propto \rho^{\gamma}$. Assuming the fluid is a perfect gas, with $p = \rho R T$, determine the temperature distribution.