Homework 3 Solutions

1) A 12-mm-diameter specimen is subjected to tensile loading. The increase in length resulting from a load of $9\ kN$ is $0.025\ mm$ for an original length L_0 of $75\ mm$. Report the true and conventional (i.e., engineering or nominal) strains and stresses? Also determine the modulus of elasticity.

Nominal strain

$$\varepsilon_0 = \frac{0.025}{75} = 333\ \mu$$

Nominal stress

$$\sigma_0 = \frac{9(10^3)}{\pi(0.012)^2/4} = 79.577\ MPa$$

Modulus of elasticity

$$E = \frac{79.577(10^6)}{333(10^{-6})} = 238.97\ GPa \quad \blacktriangleleft$$

True strain

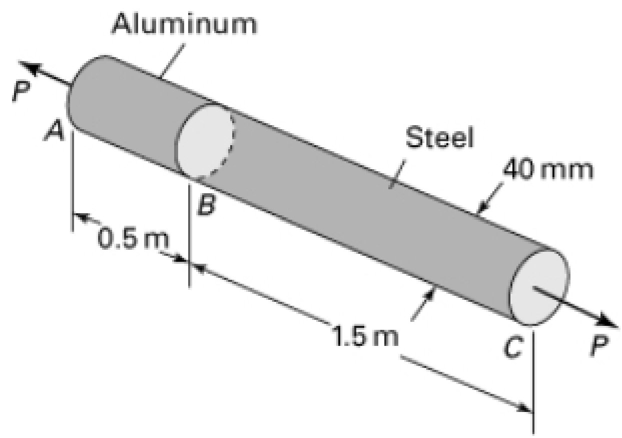
$$\varepsilon = \ln(1 + 0.000333) = 333\ \mu \quad \blacktriangleleft$$

True stress

$$\sigma = 79.577(1 + 0.000333) = 79.603\ MPa \quad \blacktriangleleft$$

* Use of this equation is strictly not correct (though close) since it is derived by assuming constant volume, which only applies to the plastic range. Instead, we could just say that the true stress is equal to the engineering stress, since we are in the elastic range.

2) A 40-mm-diameter bar ABC is composed of an aluminum part AB and a steel part BC. After axial force P is applied, a strain gage attached to the steel measures normal strain at the longitudinal direction as $\varepsilon_s = 600\ \mu$. Determine (a) the magnitude of the applied force P and (b) the total elongation of the bar if each material behaves elastically. Take $E_a = 70\ GPa$ and $E_s = 210\ GPa$.



(a) Axial stress in the bar $\sigma_s = \sigma_a = \sigma$ is

$$\sigma = \varepsilon_s E_s = 600(210) = 126\ MPa$$

Hence

$$P = \sigma A = 126[\frac{\pi}{4}(40^2)] = 158.3\ kN$$

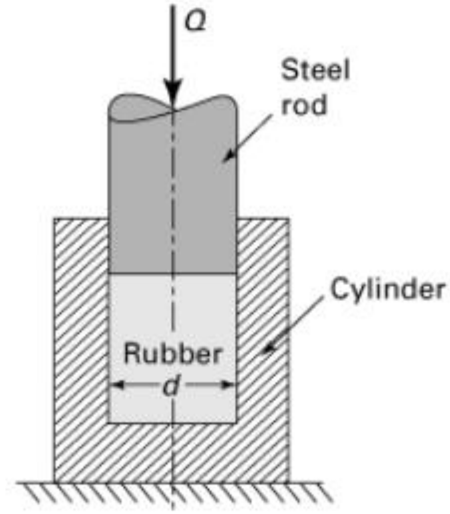
(b) Axial strain in aluminum equals

$$\varepsilon_a = \frac{\sigma_a}{E_a} = \frac{126(10^6)}{70(10^9)} = 1{,}800\ \mu$$

Therefore

$$\delta = \varepsilon_a L_a + \varepsilon_s L_s$$
$$= [1{,}800(0.5) + 600(1.5)]10^{-6} = 1.8\ mm$$

3) A typical vibration isolation device consists of rubber cylinder of diameter d compressed inside of a steel cylinder by a force Q applied to a steel rod. Do the following:



a. Find, in terms of d, Q, and Poisson's ratio ν for the rubber, an expression for the lateral pressure p between the rubber and the steel cylinder.
b. Determine the value of the lateral pressure p between the rubber and the steel cylinder for $d = 50\ mm$, $\nu = 0.3$, and $Q = 5\ kN$.

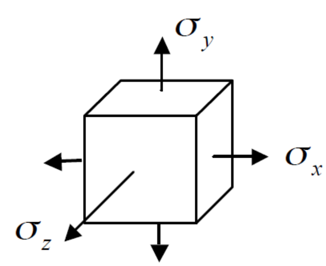
Assumptions:
1) Friction between the rubber and steel can be neglected.
2) The steel cylinder and rod are both rigid.

(a) According to assumption 1, the rubber is in triaxial stress:

$$\sigma_x = \sigma_z = -p, \qquad \sigma_y = -\frac{Q}{\frac{\pi}{4}d^2} = -\frac{4Q}{\pi d^2}$$

Strains are: $\varepsilon_x = \varepsilon_z = 0$. The first of Eqs. (2.34) gives

$$\varepsilon_x = 0 = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$



or

$$0 = p - \nu(-\frac{4Q}{\pi d^2} - p)$$

Solving,

$$p = \frac{4\nu Q}{\pi d^2(1-\nu)}$$ ◀

(b) Substituting the data,

$$p = \frac{4(0.3)(5\times10^3)}{\pi(0.05)^2(1-0.3)} = 1.091\ MPa\ (C)$$ ◀

4) For a given steel, $E = 200\ GPa$ and $G = 80\ GPa$. If the state of strain at a point within this material is given as shown, determine the corresponding components of the stress tensor.

$$\begin{bmatrix} 200 & 100 & 0 \\ 100 & 300 & 400 \\ 0 & 400 & 0 \end{bmatrix} \mu$$

From Eqs. (2.35), (2.37), and (2.38), we have

$$\nu = \frac{200(10^9)}{2(80\times10^9)} - 1 = 0.25$$

$$\lambda = \frac{0.25\times200(10^9)}{1.25\times0.5} = 80(10^9)$$

and

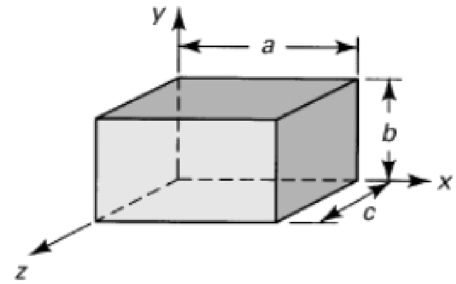
$$e = 200 + 300 = 500\ \mu$$

Then, Eqs. (2.36) lead to the following stress components, $[\tau_{ij}]$:

$$\begin{bmatrix} 72 & 16 & 0 \\ 16 & 88 & 64 \\ 0 & 64 & 40 \end{bmatrix} MPa$$

◀

5) The steel ($E = 200\ GPa$ and $\nu = 0.3$), rectangular parallelepiped has dimensions $a = 250\ mm$, $b = 200\ mm$, and $c = 150\ mm$. It is subjected to triaxial stresses $\sigma_x = -60\ MPa$, $\sigma_y = -50\ MPa$, and $\sigma_z = -40\ MPa$ acting on the x, y, and z faces, respectively. Determine (a) the changes Δa, Δb, and Δc in the dimensions of the block, and (b) the change ΔV in the volume.



(a) Using generalized Hooke's law,

$$\varepsilon_x = \frac{10^6}{200(10^9)}[-60 - 0.3(-50 - 40)] = -165\ \mu$$

$$\varepsilon_y = \frac{1}{200(10^3)}[-50 - 0.3(-60 - 40)] = -100\ \mu$$

$$\varepsilon_z = \frac{1}{200(10^3)}[-40 - 0.3(-60 - 50)] = -35\ \mu$$

Thus,

$$\Delta a = a\varepsilon_x = -0.04125\ mm$$

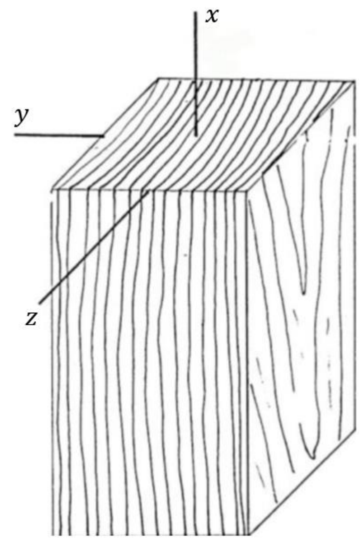
$$\Delta b = b\varepsilon_y = -0.02\ mm \quad \blacktriangleleft$$

$$\Delta c = c\varepsilon_z = -0.00525\ mm$$

(b)
$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -300\ \mu$$

and
$$\Delta V = e(abc) = -2250\ mm^3 \quad \blacktriangleleft$$

6) A board cut from a birch tree has the following elastic constants (FPS, 1999) relative to orthotropic axes (x, y, z):



$$E_x = 15{,}290\ MPa \qquad E_y = 1195\ MPa \qquad E_z = 765\ MPa$$
$$G_{xy} = 1130\ MPa \qquad G_{xz} = 1040\ MPa \qquad G_{yz} = 260\ MPa$$
$$\nu_{xy} = 0.426 \qquad \nu_{xz} = 0.451 \qquad \nu_{yz} = 0.697$$

where the x, y, and z axes aligned with, perpendicular to, and tangent to the grain of the wood. At a point in the board, the components of stress are determined to be $\sigma_{xx} = 7\ MPa$, $\sigma_{yy} = 2.1\ MPa$, $\sigma_{zz} = -2.8\ MPa$, $\sigma_{xy} = 1.4\ MPa$, and $\sigma_{xz} = \sigma_{yz} = 0$.

a. Determine the orientation of the principal axes of stress.
b. Determine the strain components.
c. Determine the orientation of the principal axes of strain.

3.22 As in Problem 3.21, the z axis is a principal axis for both stress and strain.

(a) In the (x, y) plane, for stress, the angle of rotation from (x, y) axes to principal axes is, with $\sigma_{xx} = 7\ MPa$, $\sigma_{yy} = 2.1\ MPa$, $\sigma_{xy} = 1.4\ MPa$

$$\theta_{stress} = \frac{1}{2}\arctan\frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = \frac{1}{2}\arctan\frac{2(1.4)}{7-2.1} = 14.87^\circ \quad (a)$$

Counterclockwise

(Cont.)

3.22 cont. (b). With $E_x = 15{,}920$ MPa, $E_y = 1195$ MPa, $E_z = 765$ MPa, $\nu_{xy} = 0.426$, $\nu_{xz} = 0.451$, $\nu_{yz} = 0.697$, Eqs. (3.52) yield (b)

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x} = \frac{0.426}{15{,}290} = 2.786\times10^{-5}, \quad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} = \frac{0.451}{15{,}290} = 2.950\times10^{-5}$$

$$\frac{\nu_{zy}}{E_z} = \frac{\nu_{yz}}{E_y} = \frac{0.697}{1195} = 5.833\times10^{-4} \qquad (c)$$

With Eqs(b) and (c) and $\sigma_{xx} = 7$ MPa, $\sigma_{yy} = 2.1$ MPa, $\sigma_{zz} = 2.8$ MPa, $\sigma_{xy} = 1.4$ MPa, $G_{xy} = 1130$ MPa, $G_{xz} = 1040$ MPa, $G_{yz} = 260$ MPa and $\sigma_{xz} = \sigma_{yz} = 0$, Eqs. (3.51) yield

$$\epsilon_{xx} = \frac{1}{E_x}\sigma_{xx} - \frac{\nu_{yx}}{E_y}\sigma_{yy} - \frac{\nu_{zx}}{E_z}\sigma_{zz} = \frac{7}{15290} - (2.786\times10^{-5})(2.1) - (2.95\times10^{-5})(-2.8) = 0.0004819$$

$$\epsilon_{yy} = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_{yy} - \frac{\nu_{zy}}{E_z}\sigma_{zz} = -(2.786\times10^{-5})(7) + \frac{2.1}{1195} - (5.833\times10^{-4})(-2.8) = 0.003195$$

$$\epsilon_{zz} = -\frac{\nu_{xz}}{E_x}\sigma_x - \frac{\nu_{yz}}{E_y}\sigma_{yy} + \frac{1}{E_z}\sigma_{zz} = -(2.95\times10^{-5})(7) - (5.833\times10^{-4})(2.1) + \frac{(-2.1)}{765} = -0.005092 \qquad (d)$$

$$\gamma_{xy} = \frac{1}{G_{xy}}\sigma_{xy} = \frac{1.4}{1130} = 0.001239$$

$$\gamma_{xz} = \gamma_{yz} = 0$$

(c) With Eqs. (d), the angle of rotation in the (x, y) plane from the (x, y) axes to the principal axes is

$$\theta_{strain} = \frac{1}{2}\arctan\frac{\gamma_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{1}{2}\arctan\frac{0.001239}{0.0004819 - 0.003195}$$

or

$$\theta_{strain} = -12.27^\circ \text{ (}12.27^\circ\text{ clockwise)} \qquad (e)$$

Comparison of Eqs. (a) and (e) shows that

$$\theta_{stress} \neq \theta_{strain}$$