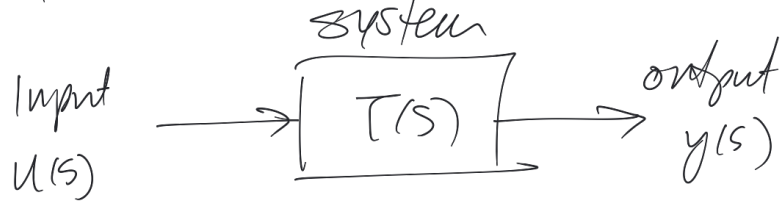
# Transfer functions and system response



Transfer function is a ratio of the output to the input:




$$T(s) = \frac{y(s)}{U(s)}$$

Get from the O.D.E

⇒ Taking the L.T. of O.D.E w/ zero I.C.

General form:

$$T(s) = \frac{y(s)}{U(s)} = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

Conditions:

$m \leq n$ and $a_n \neq 0$

Note:

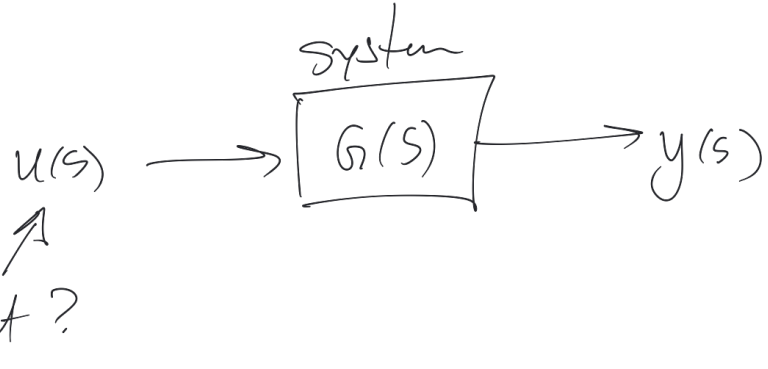
$m \triangleq$ order of the numerator

$b_m$'s are constants

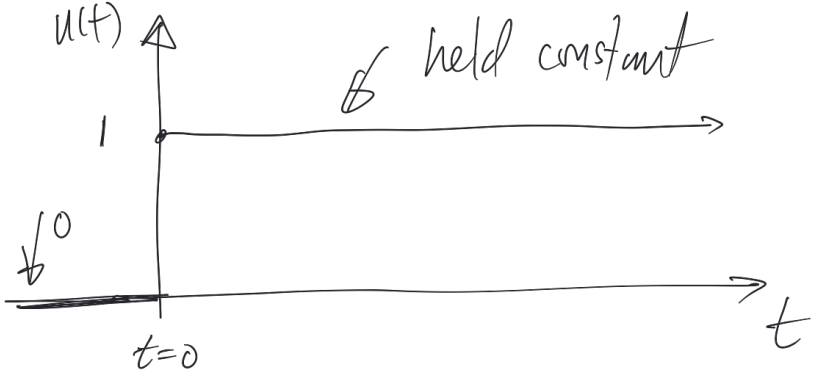
$n \triangleq$ order of the denominator
(order of the system)

$a_n$'s are constants.

Input - output behavior:

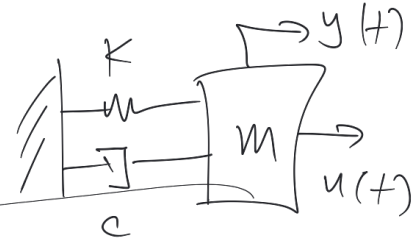


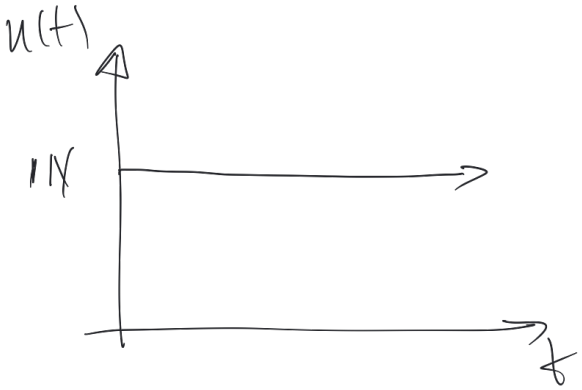

Input can be just about anything, most general and widely applied is the unit step input:

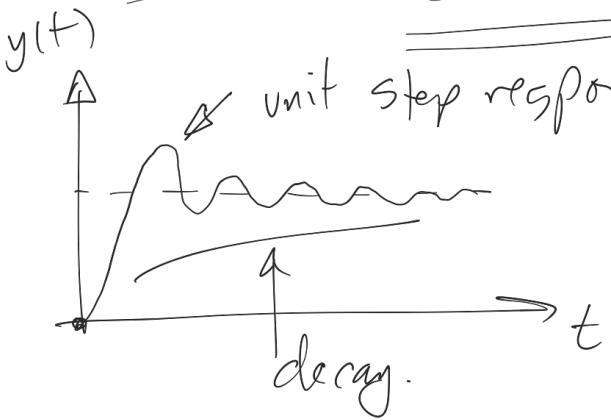



$$u(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases}$$

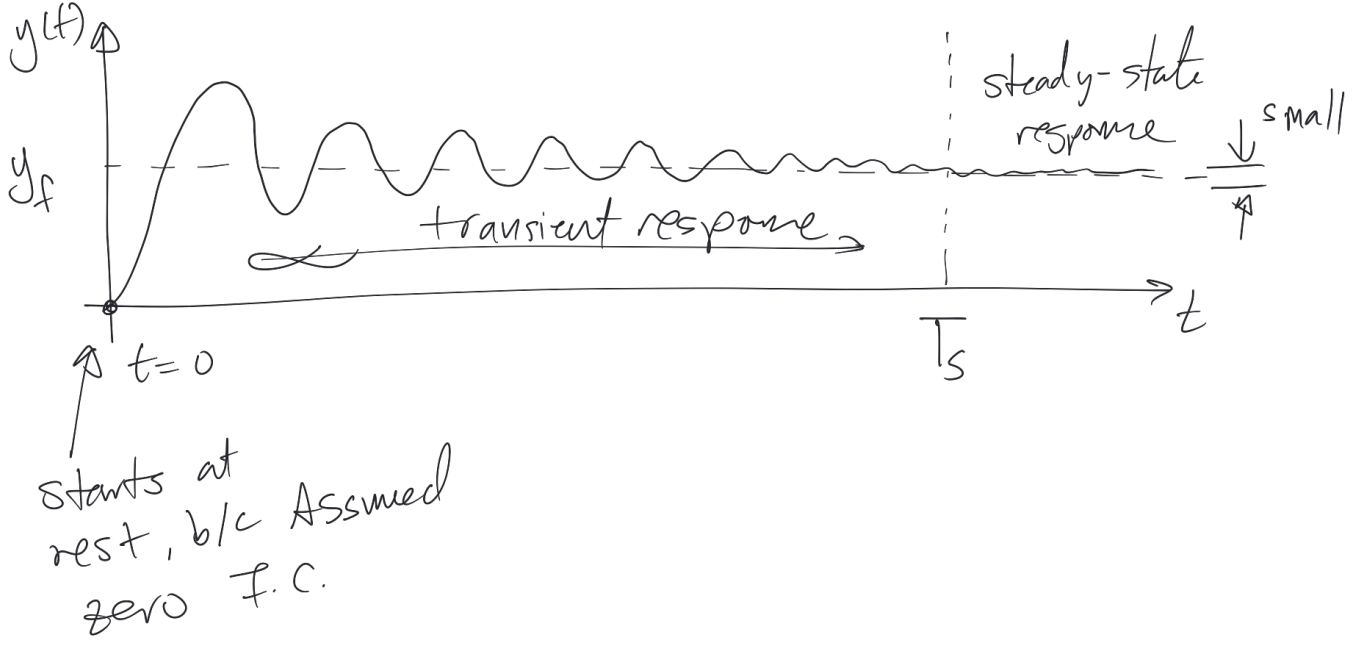
Unit step input is important because we apply input to a system to see how the system responds. The resulting response is a unit step response.

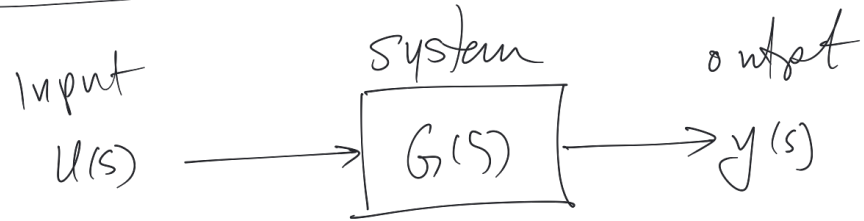



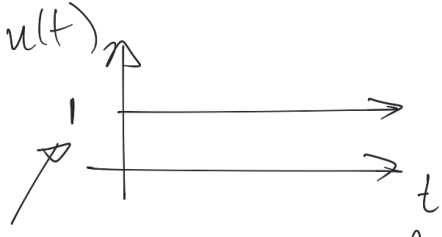
Time response : unit step response

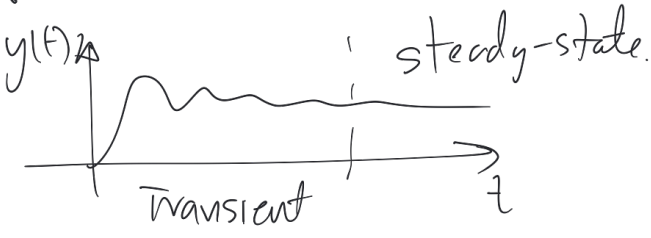



Total response = transient response + steady-state response.

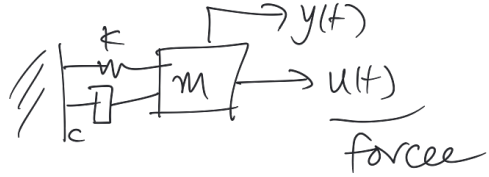
# Free and forced responses




can be another value, for example 3.5.



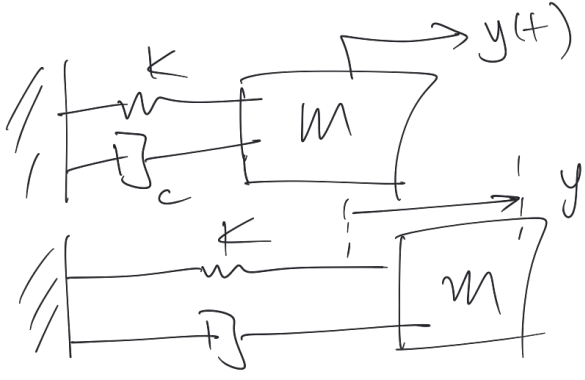

Go back to differential equation:

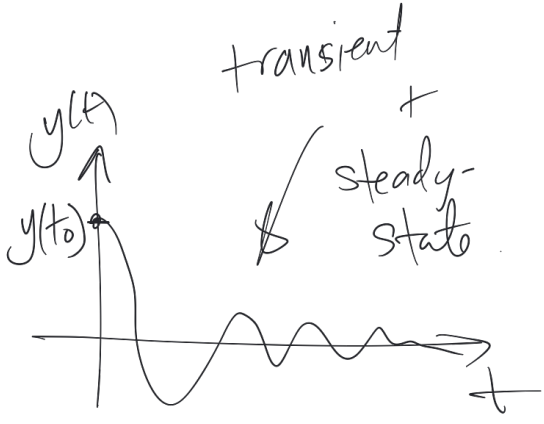
$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = u(t)$$

Free response: response due purely to initial conditions

forced response: response due to an applied input.

1) Free respone is the response due to initial conditions:




2) Forced respone is caused by inputs.

A unit step respone is a forced response: