# HW 2: Index Notation

**1.** Which of the following are valid expressions in index notation? If an expression is not valid, explain why.

**(a)** $a = b_i c_{ij} d_j$

**(b)** $a = b_i c_i + d_j$

**(c)** $a_i = \delta_{ij} b_i + c_i$

**(d)** $a_k = b_i c_{ki}$

**(e)** $a_k = b_k c + d_i e_{ik}$

**(f)** $a_i = b_i + c_{ij} d_{ji} e_i$

**(g)** $a_\ell = \epsilon_{ijk} b_j c_k$

**(h)** $a_{ij} = b_{ji}$

**(i)** $a_{ij} = b_i c_j + e_{jk}$

**(j)** $a_{k\ell} = b_i c_{ki} d_\ell + e_{ki}$

**2.** Prove the following by writing out the implicit summations:

**(a)** $\delta_{ij}\,\delta_{ij} = 3$

**(b)** $\epsilon_{pqr}\,\epsilon_{pqr} = 6$

**3.** Prove the following by using the epsilon-delta relation (equation 2.19 in the textbook):

$$\epsilon_{pqi}\,\epsilon_{pqj} = 2\,\delta_{ij}$$

**4.** Using index notation, show that

**(a)** $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$

**(b)** **[6700 only]** $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

**(c)** **[6700 only]** $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{a}) \cdot \vec{b}$

**5.** Using index notation, show that for *any* vector $\vec{u}$, the following is always true:

$$\vec{\nabla} \cdot \left( \vec{\nabla} \times \vec{u} \right) = 0.$$