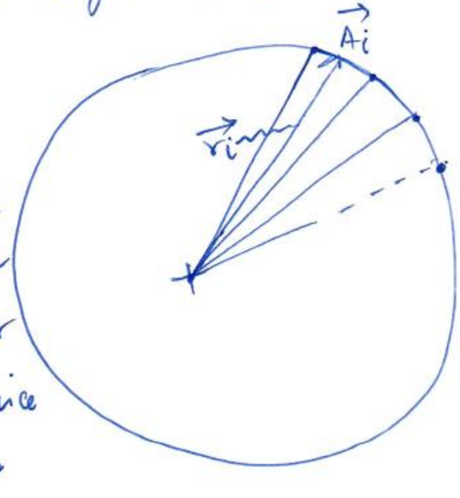
1.

1) a) Volume of pyramid is 1/3 Base area x height, so in this case. it will be $\frac{1}{3}\vec{r}\cdot\vec{A}$

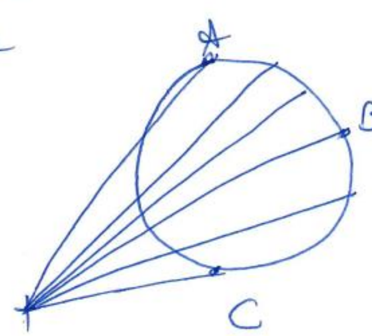
since by taking the dot product of $\vec{r}$ we will get its length perpendicular to $\vec{A}$

b) Assume an ~~ar~~ initially convex body surface with an origin inside that surface. In this case the volume of the body will be just the sum of the volumes of the pyramids subtended by ~~the~~ each panel at the origin. i.e

$$\text{Volume} = \frac{1}{3}\sum_i \vec{r_i}\vec{A_i}$$



This also works for the case when the origin is outside the body, or the body contains concave portions, voids or is split into several pieces, since the extra panels created by these complications subtend the additional positive or negative volumes that cancel to leave only the volume of the body. Eg. volume OAC from the nearside panels subtracts ~~the~~ from volume OABC from the farside panels to give actual body volume.



2.

2) cylindrical co-ordinates.

Zero-divergence but non zero curl.

i.e $\vec{\nabla}\cdot\vec{V} = 0$ but $\vec{\nabla}\times\vec{V} \neq 0$.

Noting that $\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} \equiv 0$ (where $\vec{V}$ can be $= \vec{\nabla}\times\vec{A}$ and $\vec{A}$ is any vector potential)

Now
$$\vec{\nabla}\times\vec{A} = \frac{1}{r}\begin{vmatrix} \vec{e_r} & r\vec{e_\theta} & \vec{e_z} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial z \\ A_r & rA_\theta & A_z \end{vmatrix}$$

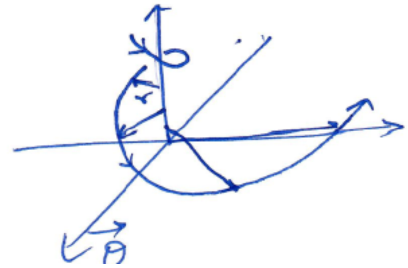
so choosing $A_r = z$, $A_\theta = 1$ and $A_z = 0$ we should end up with a fairly simple 3-component velocity field

so
$$\vec{V} = \frac{1}{r}\begin{vmatrix} \vec{e_r} & r\vec{e_\theta} & \vec{e_z} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial z \\ z & r & \theta \end{vmatrix}$$

$$= \frac{1}{r}\left[\vec{e_r}\times 1 + r\vec{e_\theta}\times 1 - \vec{e_z}\times 1\right]$$

$$= \vec{e_r}/r + \vec{e_\theta} - \vec{e_z}/r$$

(a) This flow will be like a rotating stagnation point except with infinite velocities at origin



2) b) Now the ROC of volume of the fluid particles $= \vec{\nabla} \cdot \vec{V} = 0$

Average angular velocity of fluid particles $= \dfrac{\vec{\Omega}}{2} = \dfrac{1}{2r} \begin{vmatrix} \vec{e_r} & r\vec{e_\theta} & \vec{e_z} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial z \\ \frac{1}{r} & r & -\frac{1}{r} \end{vmatrix}$

$$= \frac{1}{2r}\left[\vec{e_r}(0) - r\vec{e_\theta}\left(\frac{1}{r^2}\right) + e_z \times 1\right] = \frac{-\vec{e_\theta}}{2r^2} + \frac{\vec{e_z}}{2r}$$

3.

3) for a static fluid $\nabla p = \rho \vec{f}$

Given $p \propto \rho^{\gamma}$, $p = \rho R T$

$\vec{f} = -g\vec{k}$ for gravity.

$p = k\rho^{\gamma}$ where k is a constant

so
$$\frac{dp}{dz}\vec{k} = -\rho g \vec{k}$$

$$\Rightarrow \frac{dp}{dz} = -\left(\frac{p}{k}\right)^{1/\gamma} g$$

$$\Rightarrow \int p^{-1/\gamma} dp = -\frac{g}{k^{1/\gamma}} \int dz$$

$$\Rightarrow \frac{p^{1-1/\gamma}}{1-1/\gamma} = \frac{-gz}{k^{1/\gamma}} + C$$

$$\Rightarrow p = \left(-\frac{(1-1/\gamma)gz}{k^{1/\gamma}} + (1-1/\gamma)C\right)^{\frac{\gamma}{\gamma-1}}$$

or $p = \left(-Agz + B\right)^{\gamma/\gamma-1}$ where $A = \frac{1-1/\gamma}{k^{1/\gamma}}$

$B = (1-1/\gamma)C$

For perfect gas $T = \frac{p}{\rho R} = \frac{p k^{1/\gamma}}{p^{1/\gamma} R} = \frac{k^{1/\gamma}}{R} \cdot p^{\gamma-1/\gamma}$

$$\Rightarrow T = -\frac{Ak^{1/\gamma} g z}{R} + \frac{Bk^{1/\gamma}}{R}$$

We see that temperature decreases linearly; as you go up it gets colder!!