ME EN 5700/6700 – Intermediate Fluid Dynamics
Sample Final Exam – Fall 2023

1. When does the energy equation for temperature become completely decoupled from the incompressible N.S equations?

2. What is the concept of displacement thickness in boundary layers?

3. Describe the turbulence energy cascade.

4. What is the solenoidal condition for vorticity?

5. Consider a velocity field in the form:

$$u = 16x^2 + y$$
$$v = 10$$
$$w = yz^2$$

a. Compute the circulation, Γ, for the flow field around the contour given below by integrating the velocity around the contour counterclockwise

$0 \leq x \leq 10$	$y = 0$
$0 \leq y \leq 5$	$x = 10$
$0 \leq x \leq 10$	$y = 5$
$0 \leq y \leq 5$	$x = 0$

b. Also perform the calculation using vorticity flux and confirm similar results.

6. Obtain an expression for the boundary layer thickness in terms of viscosity, free-stream velocity, and distance from leading edge of a plate x. Derive the expression using two approaches:
   a. The scaling of governing equations
   b. Using the advection and diffusion time-scales and setting them equal

7. Consider a polynomial BL approximation in the form

$$\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

   a. Does this equation satisfy the boundary conditions applicable to a laminar BL velocity profile?

   b. Derive an expression for δ^*/δ

8. Start with the NS equation and project it onto a no-slip wall. Subsequently, use the vector identity $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ to write the viscous diffusion term in your wall-NS equation in terms of vorticity. Use this equation to find the components of the pressure gradient at the wall in terms of vorticity at the wall.

9. Turbulence
   a. List 4 properties of turbulence

   b. Explain the procedure for deriving the RANS equations starting from Reynolds decomposition (in words).

   c. What is the closure problem in RANS modeling?

   d. Consider the turbulence energy budget equations. We have a term $\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$ that appears in both the TKE and MKE equations. Explain the net effect of this term in each equation. That is, does it reduce or increase energy for turbulent kinetic energy? What about mean kinetic energy? Why?