ME 3710 – Spring 2024
Homework 3
Due February 1 at 11:59pm – upload to files to Gradescope
18 points

Solution 2.68

A force of 30 lb applied to the level results in a plunger force, F_1, of $F_1 = (8)(30) = 240\,\text{lb}$.

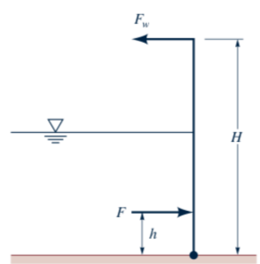
Since $F_1 = pA_1$ and $F_2 = pA_2$ where p is the pressure and A_1 and A_2 are the areas of the plunger and piston, respectively. Since p is constant throughout the chamber,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

so that $F_2 = \frac{A_2}{A_1} F_1 = \left(\frac{150\,\text{in.}^2}{1\,\text{in.}^2}\right)(240\,\text{lb}) \rightarrow \boxed{F_2 = 36{,}000\,\text{lb}}$

Solution 2.76

The hydrostatic force F on the wall is found from



$$\begin{aligned} F &= \rho g h_c A \\ &= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2\text{m})\left(4\times 10\text{m}^2\right) \\ &= 78500\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)\left(\frac{\text{kN}}{1000\,\text{N}}\right) \\ &= 785\,\text{kN} \end{aligned}$$

The force F is located one-third of the water depth from the bottom of the water.

$$h = \frac{1}{3}(4\text{m}) = 1.33\,\text{m}$$

Summing moments about the pinned joint,

$$F_W = \frac{h}{H}F = \frac{(1.33\,\text{m})}{(7\,\text{m})}(785\,\text{kN}) = 149\,\text{kN}$$

Assuming no friction between the rope and the pulley,

$$W = F_W \rightarrow \boxed{W = 149\,\text{kN}}$$

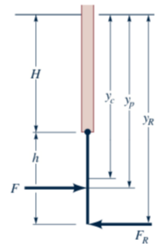
DISCUSSION

Note that the atmospheric pressure acts on both sides of the wall.

Therefore, the forces due to atmospheric pressure are equal and opposite, and cancel.

Solution 2.77

The hydrostatic force on the gate is



$$F = \gamma y_c A$$
$$= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(1.3\,\text{m}+0.4\,\text{m})(2\,\text{m}\times 0.8\,\text{m})$$
$$= 26700\,\text{N}$$

The location of the force F is

$$y_p = y_c + \frac{I_{xc}}{12 y_c A}$$

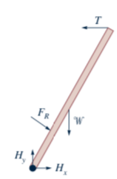
Using Appendix,

$$y_p = y_c + \frac{bh^3}{12 y_c A} = y_c + \frac{h^2}{12 y_c}$$
$$= (1.3+0.4)\,\text{m} + \frac{(0.8\,\text{m})^2}{12(1.3+0.4)\,\text{m}} = 1.73\,\text{m}$$

Summing moments about the hinge,

$$\sum M_{\text{hinge}} = F_R h - F\left(y_p - H\right) = 0$$
$$F_R = \frac{F\left(y_p - H\right)}{h} = \frac{(26700\text{N})(1.73-1.3)\,\text{m}}{0.8\,\text{m}} \quad \rightarrow \quad \boxed{F_R = 14,400\text{ N}}$$

Solution 2.87



$F_R = \gamma h_c A$ where $h_c = \left(\frac{6\,\text{ft}}{2}\right)\sin 60°$

Thus,

$$F_R = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6\,\text{ft}}{2}\right)(\sin 60°)(6\,\text{ft}\times 4\,\text{ft}) = 3890\,\text{lb}$$

To locate F_R,

$y_R = \frac{I_{xc}}{y_c A} + y_c$ where $y_c = 3\,\text{ft}$

so that

$$y_R = \frac{\frac{1}{12}(4\,\text{ft})(6\,\text{ft})^3}{(3\,\text{ft})(6\,\text{ft}\times 4\,\text{ft})} + 3\,\text{ft} = 4.0\,\text{ft}$$

For equilibrium,

$$\sum M_H = 0$$

and

$$T(8\,\text{ft})(\sin 60°) = W(4\,\text{ft})(\cos 60°) + F_R(2\,\text{ft})$$

$$T = \frac{(800\,\text{lb})(4\,\text{ft})(\cos 60°) + (3890\,\text{lb})(2\,\text{ft})}{(8\,\text{ft})(\sin 60°)} \quad \rightarrow \quad \boxed{T = 1350\text{ lb}}$$

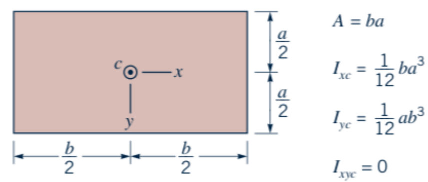
Solution 2.88

(a) For rectangular portion,

$(F_R)_r = \gamma h_c A$ where $h_c = 3\,\text{m}$

$$(F_R)_r = \left(9800\,\frac{\text{N}}{\text{m}^3}\right)(3\,\text{m})(6\,\text{m}\times 6\,\text{m})$$

$$\boxed{(F_R)_r = 1060\,\text{kN}}$$



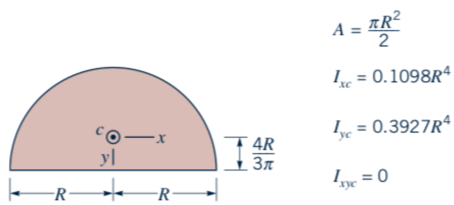
For semi-circular portion,

$(F_R)_{sc} = \gamma h_c A$ where

$$h_c = 6\,\text{m} + \frac{4\,\text{R}}{3\pi} = 6\,\text{m} + \frac{4(3\,\text{m})}{3\pi} = 7.27\,\text{m}$$

$$(F_R)_{sc} = \left(9800\,\frac{\text{N}}{\text{m}^3}\right)(7.27\,\text{m})\left(\frac{\pi}{2}(3\,\text{m})^2\right)$$

$$\boxed{(F_R)_{sc} = 1010\,\text{kN}}$$

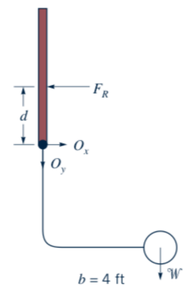


(b) For semi-circular portion $y_R = \dfrac{I_{xc}}{y_c A} + y_c = \dfrac{0.1098\,R^4}{(7.27\,\text{m})\left(\frac{\pi}{2}\right)R^2} + 7.27\,\text{m} = 7.36\,\text{m}$

Thus, moment with respect to shaft, M:

$$M = (F_R)_{sc} \times (7.36\,\text{m} - 6.00\,\text{m}) = \left(1010\times 10^3\,\text{N}\right)(1.36\,\text{m}) \rightarrow \boxed{M = 1.37\times 10^6\,\text{N}\cdot\text{m}}$$

Solution 2.104



$F_R = \gamma h_c A$ where $h_c = \dfrac{h}{2}$

Thus,

$$F_R = \gamma_{\mathrm{H_2O}} \frac{h}{2}(h \times b) = \gamma_{\mathrm{H_2O}} \frac{h^2}{2}\,(4\,\mathrm{ft})$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,\mathrm{ft})\left(h^3\right)}{\frac{h}{2}(4\,\mathrm{ft} \times h)} + \frac{h}{2} = \frac{2}{3}h$$

For equilibrium, $\sum M_0 = 0$

$$F_R d = W(3\,\mathrm{ft}) \text{ where } d = h - y_R = \frac{h}{3}$$

so that
$$\frac{h}{3} = \frac{(2000\,\mathrm{lb})(3\,\mathrm{ft})}{\left(\gamma_{\mathrm{H_2O}}\right)\left(\dfrac{h^2}{2}\right)(4\,\mathrm{ft})}$$

Thus,
$$h^3 = \frac{(3)(2000\,\mathrm{lb})(3\,\mathrm{ft})}{\left(62.4\,\dfrac{\mathrm{lb}}{\mathrm{ft}^3}\right)\left(\dfrac{1}{2}\right)(4\,\mathrm{ft})}$$

$$h = \underline{\underline{5.24\,\mathrm{ft}}}$$