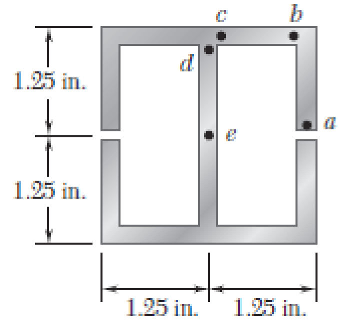
**Homework 9 Solutions**

1) The extruded aluminum beam has a uniform wall thickness of $0.125\ in$. Knowing that the vertical shear in the beam is $2\ kips$, determine the corresponding shearing stress at each of the five points indicated. Assume the gaps in the outer webs are small.




$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.23812\ \text{in}^4$$

$t = 0.125$ in. at all sections.

$V = 2$ kips

$$Q_a = 0 \qquad \tau_a = \frac{VQ_a}{It} \qquad \tau_a = 0 \blacktriangleleft$$

$$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.097656\ \text{in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.097656)}{(1.23812)(0.125)} \qquad \tau_b = 1.262\ \text{ksi} \blacktriangleleft$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537\ \text{in.}^2$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.23812)(0.125)} \qquad \tau_c = 3.30\ \text{ksi} \blacktriangleleft$$

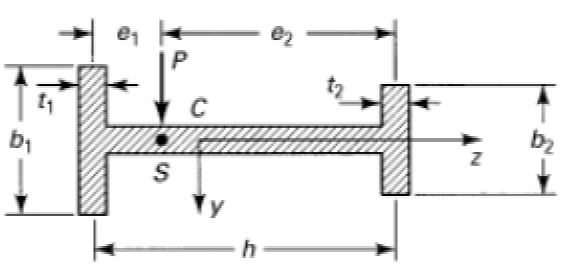
$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.23812)(0.125)} \qquad \tau_d = 6.84\ \text{ksi} \blacktriangleleft$$

$$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$$

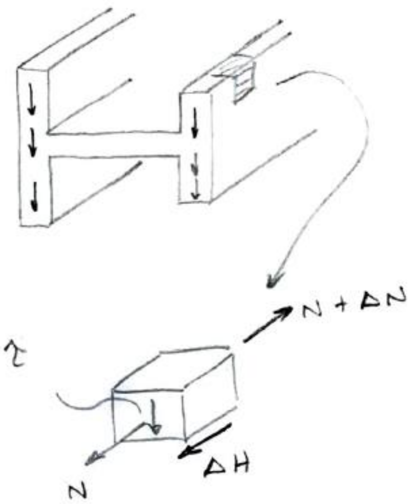
$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.23812)(0.125)} \qquad \tau_e = 7.86\ \text{ksi} \blacktriangleleft$$

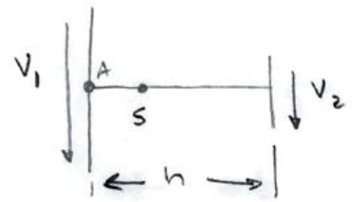
2) An H-section cantilever beam with unequal flanges is subjected to a vertical load $P$. The following assumptions are applicable:
   a. The total resisting shear occurs in the flanges.
   b. The rotation of a plane section during bending occurs about the symmetry axis so that the radii of curvature of the flanges are equal.




Determine the location of the shear center $S$.

- Analysis of shear stress direction shows direction is downward in both flanges

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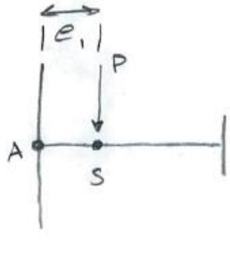
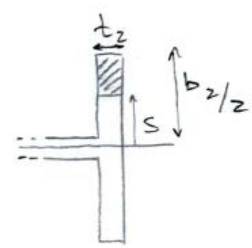



$$\Rightarrow \sum M_A = -V_2 h = -P e_1$$

$$\Rightarrow e_1 = \frac{V_2}{P} h$$

- Find $V_2$ : $V_2 = \iint \tau \, dA = \int \tau t \, ds = \int q \, ds$




- Taking advantage of symmetry,

$$V_2 = 2 \int_0^{b_2/2} q \, ds$$

- $q = \frac{VQ}{I} = \frac{PQ}{I}$
- $Q = A^* \bar{y} = \left[\left(\frac{b_2}{2} - s\right) t_2\right]\left(\frac{1}{2}\right)\left(\frac{b_2}{2} + s\right)$

$$= \frac{t_2}{2}\left(\frac{b_2^2}{4} - s^2\right)$$

$$\Rightarrow V_2 = 2\int q \, ds = 2\int \frac{PQ}{I} ds = \frac{2P}{I}\int_0^{b_2/2} \frac{t_2}{2}\left(\frac{b_2^2}{4} - s^2\right) ds$$

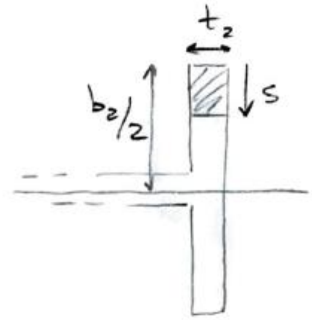
$$= \frac{\cancel{2} P t_2}{\cancel{2} I}\left(\frac{b_2^2}{4} s - \frac{s^3}{3}\right)\Bigg|_0^{b_2/2} = \frac{P t_2}{I}\left(\underbrace{\frac{b_2^2}{4}\frac{b_2}{2}}_{8} - \frac{b_2^3}{24}\right)$$

$$= \frac{P t_2}{I} b_2^3 \left(\frac{\overset{3}{1}}{\underset{24}{8}} - \frac{1}{24}\right) = \frac{1}{12}\frac{P t_2 b_2^3}{I} = P \frac{I_2}{I_1 + I_2}$$

- $I = I_1 + I_2 = \frac{1}{12} t_1 b_1^3 + \frac{1}{12} t_2 b_2^3$

$$\Rightarrow e_1 = \frac{P\left(\frac{I_2}{I_1 + I_2}\right)}{P} h = \underline{\frac{I_2}{I_1 + I_2} h}$$

• OR let $V_2 = \int_0^{b_2} q \, ds$




• $Q = A^* \bar{y} = (s t_2) \frac{1}{2}\left(\frac{b_2}{2} + \frac{b_2}{2} - s\right)$

$$= \frac{1}{2} s t_2 (b_2 - s)$$

$$= \frac{t_2}{2}(b_2 s - s^2)$$

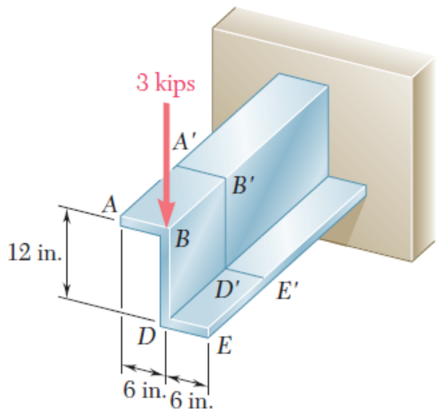
$$\Rightarrow V_2 = \int \frac{PQ}{I} ds = \frac{P}{I}\int_0^{b_2} \frac{t_2}{2}(b_2 s - s^2)$$

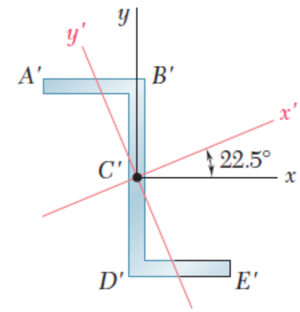
$$= \frac{P t_2}{2I}\left(b_2 \frac{s^2}{2} - \frac{s^3}{3}\right)\Big|_0^{b_2} = \frac{P t_2}{2I}\underbrace{\left(\frac{b_2^3}{2} - \frac{b_2^3}{3}\right)}_{b_2^3/6}$$

$$= \frac{P t_2 b_2^3}{12 I} = \frac{P}{I} I_2 = P \frac{I_2}{I_1 + I_2}$$

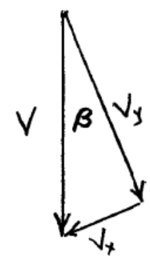
$$\Rightarrow e_1 = \frac{P \frac{I_2}{I_1 + I_2} h}{P} = \underline{\frac{I_2}{I_1 + I_2} h}$$

3) The cantilever beam shown consists of a $Z$ shape of $1/4$-$in$ thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the $Z$ shape. The $x'$ and $y'$ axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ in^4$ and $I_{y'} = 13.61\ in^4$.




$$V = 3\text{ kips} \qquad \beta = 22.5^\circ$$
$$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$$

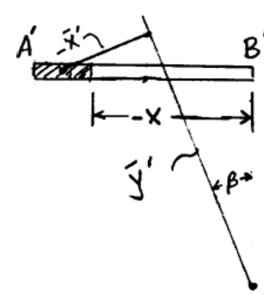



In upper horizontal leg, use coordinate $x$: $(-6\text{ in} \leq x \leq 0)$

$$A = \frac{1}{4}(6+x)\text{ in.}$$
$$\bar{x} = \frac{1}{2}(-6+x)\text{ in.}$$
$$\bar{y} = 6\text{ in.}$$
$$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$$
$$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'}A\bar{x}'}{I_{y'}t}$$




$$\tau_1 = \frac{(V\sin\beta)\left(\frac{1}{4}\right)(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)\left(\frac{1}{4}\right)}$$
$$= 0.084353(6+x)(-0.47554 + 0.46194x)$$

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'}A\bar{y}'}{I_{x'}t} = \frac{(V\cos\beta)\left(\frac{1}{4}\right)(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)\left(\frac{1}{4}\right)}$$
$$= 0.0166665(6+x)[6.69132 - 0.19134x]$$

Total: $\tau_1 + \tau_2 = (6+x)[-0.07141 + 0.035396x]$

| $x$ (in.) | $-6$ | $-5$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|---|---|---|---|
| $\tau$ (ksi) | 0 | $-0.105$ | $-0.140$ | $-0.104$ | 0.003 | 0.180 | 0.428 |

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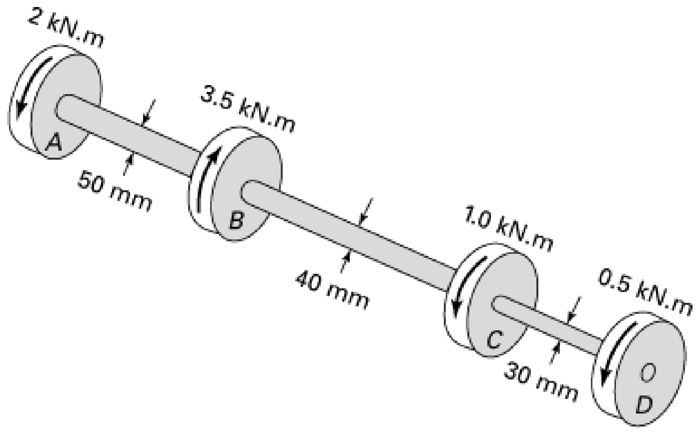
4) A tubular, stepped shaft with a 16 $mm$ inner diameter is attached to four pulleys that transmit the torques shown. Find the maximum shear stress for each shaft segment.




We have, applying the method of sections:

$$T_{CD} = 0.5\ kN \cdot m \qquad T_{BC} = 1.5\ kN \cdot m$$

$$T_{AB} = 2\ kN \cdot m$$

Hence,

$$\tau_{\max} = \frac{2Tc}{\pi(c^4 - b^4)}$$

gives,

$$\tau_{AB} = \frac{2(2\times10^3)(0.025)}{\pi[(0.025)^4 - (0.008)^4]} = 82.4\ MPa$$

$$\tau_{BC} = \frac{2(1.5\times10^3)(0.02)}{\pi[(0.02)^4 - (0.008)^4]} = 122.5\ MPa$$

$$\tau_{CD} = \frac{2(0.5\times10^3)(0.015)}{\pi[(0.015)^4 - (0.008)^4]} = 102.6\ MPa$$