ME 3000 Design of Mechanical Elements **Prof. Shad Roundy**

Midterm #1 **Formulas and Tables** **Spring 2024**

Statistical Considerations

- Probability Density Function (PDF): $P(a \le X \le b) = \int_a^b f(x)dx$ = area under $f(x)$ from a to b
- Cumulative Density Function (CDF): $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$
- Metrics to describe random variables

Metric	Continuous Random Var.	Discrete Random Variable	Sample data
Mean	$\mu_x = \int_{-\infty}^{\infty} x f(x)dx$	$\mu_x = \sum_{i=1}^{n} x_i f(x_i)$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Variance	$\sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu_x)^2 f(x)dx$	$\sigma_x^2 = \sum_{i=1}^{n} (x_i-\mu_x)^2 f(x_i)$	$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i-\mu_x)^2$
Covariance	$\sigma_{xy} = \mu_{XY} - \mu_X\mu_Y$	NA	$s_{xy} = \frac{1}{n-1}\sum_{i}^{n} (x_i-\bar{x})(y_i-\bar{y})$

  - Correlation coefficient: $C_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y} = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y}^2)}}$
- Linear combinations of random variables for random variable $Z = aX + bY$
  - $\mu_z = a\mu_x + b\mu_y$
  - $\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_{xy}$
- Regression
  - $y = b_0 + b_1 x$
  - $b_1 = \frac{s_y}{s_x} C_{xy}$
  - $b_0 = \bar{y} - b_1\bar{x}$
- Distributions

Distribution	Probability Density Func. (PDF)	Cumulative Density Func. (CDF)
Uniform	$f(x) = c$	$F(x) = \int_a^{\infty} c\,dx$
Normal (Gaussian)	$f(x) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	Use Z-table where $Z = \frac{X-\mu_x}{\sigma_x}$
Weibull	$f(x) = \frac{k}{\lambda}\left(\frac{x-x_0}{\lambda}\right)^{k-1} e^{-\left(\frac{x-x_0}{\lambda}\right)^k}$	$F(x) = 1 - e^{-\left(\frac{x-x_0}{\lambda}\right)^k}$

Tolerances

No new formulas. Know basics of GD&T. Tolerance stackup using worst case and root mean squared (RSS). Understand basics of tolerance stackup using Monte Carlo simulation.

Material Considerations

- Modulus of Resilience: $u_R = \int_0^{\epsilon_y} \sigma d\epsilon = \frac{1}{2}\sigma_y \epsilon_y$
- Modulus of Toughness: $u_T = \int_0^{f} \sigma d\epsilon \approx \frac{\sigma_{ut}+\sigma_y}{2}\epsilon_f$
- Reduction in in cross-sectional area at fracture: $R = \frac{A_0 - A_f}{A_0}$ (A_0: original cross-sectional area, A_f:cross-sectional area after fracture)
- Cold-work factor: $W = \frac{A_0 - A_i}{A_0}$
- New yield strength after cold work:
  - $S_y' = \sigma_0 \epsilon_i^m$
  - $\epsilon_i = \ln\left[\frac{1}{1-W}\right]$
  - $S_y' = \sigma_0 \ln\left[\frac{1}{1-W}\right]^m$
- New ultimate strength after cold work:
  - $S_{ut}' = \frac{S_{ut}}{1-W}$ (when $\epsilon_i < m$), m = strain strengthening exponent or ultimate strain
  - $S_{ut}' \approx S_y'$ (when $\epsilon_i > m$)
- True strain: $\epsilon = \ln\left(\frac{l_i}{l_0}\right)$, $A_0 l_0 = A_i l_i$ (l_0 = original length, l_i = length at certain state i in plastic region, A_0 = original cross-sectional area, A_i = cross-sectional area at certain state i in plastic region)
- True stress-strain in plastic region: $\sigma = \sigma_0 \epsilon^m$ (m = strain strengthening exponent, σ_0 = strain strengthening coefficient).
- Safety factor or Factor of safety: $n = \frac{S}{\sigma}$ (S = loss-of-function strength, σ = allowable stress)

Standard Normal Probabilities

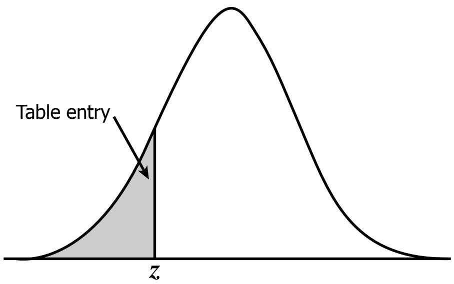

Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

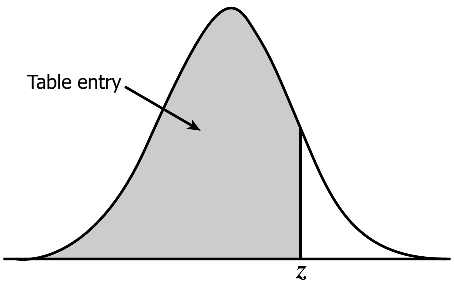

Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998