

## Advanced Mechanics of Materials Vocabulary

### Stress and Strain

- Vector Transformation:  $\mathbf{a}' = \mathbf{M}\mathbf{a}$ , where  $\mathbf{M}$  is the transformation, or rotation or direction cosine matrix
  - $\mathbf{M} = \begin{bmatrix} \mathbf{x}' \cdot \mathbf{x} & \mathbf{x}' \cdot \mathbf{y} & \mathbf{x}' \cdot \mathbf{z} \\ \mathbf{y}' \cdot \mathbf{x} & \mathbf{y}' \cdot \mathbf{y} & \mathbf{y}' \cdot \mathbf{z} \\ \mathbf{z}' \cdot \mathbf{x} & \mathbf{z}' \cdot \mathbf{y} & \mathbf{z}' \cdot \mathbf{z} \end{bmatrix}$
- Tensor (2<sup>nd</sup> order) Transformation:  $\mathbf{T}' = \mathbf{M}\mathbf{T}\mathbf{M}^T$
- Stress vector on arbitrary surface with normal unit vector  $\mathbf{n}$ :  $\mathbf{t} = \mathbf{T}\mathbf{n} = \boldsymbol{\tau} + \boldsymbol{\sigma}$

### Material Response

- Hooke's Law (Voigt Notation)

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

### Failure Theory

- Max distortion energy (Von Mises):  $(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) = 2\sigma_{yp}^2$
- Coulomb-Mohr:  $\frac{\sigma_1}{\sigma_u} - \frac{\sigma_3}{\sigma'_u} = 1$

### Fracture Mechanics

- Stress intensity factor:  $K = \lambda\sigma\sqrt{\pi a}$

### Fatigue

- $N_{cr} = N_f \left( \frac{\sigma_{cf}}{\sigma_f} \right)^{1/b}$ , where  $b = \frac{\ln(\sigma_f/\sigma_e)}{\ln(N_f/N_e)}$
- Modified Goodman:  $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_u} = 1$
- Soderberg:  $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_{yp}} = 1$
- Gerber:  $\frac{\sigma_a}{\sigma_{cr}} + \left( \frac{\sigma_m}{\sigma_u} \right)^2 = 1$
- SAE:  $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_f} = 1$

### Dynamic Loading

- Impact factor (vertical drop):  $K = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$

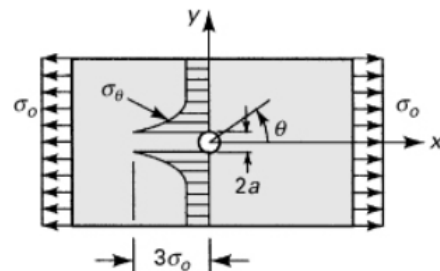
### Stress Concentrations

- Stress distribution around a hole in a flat plate under uniaxial loading

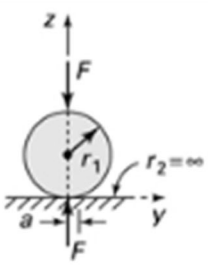
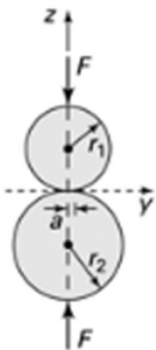
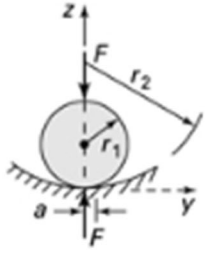
$$\sigma_r = \frac{1}{2}\sigma_o \left[ \left( 1 - \frac{a^2}{r^2} \right) + \left( 1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2} \right) \cos 2\theta \right]$$

$$\sigma_\theta = \frac{1}{2}\sigma_o \left[ \left( 1 + \frac{a^2}{r^2} \right) - \left( 1 + \frac{3a^4}{r^4} \right) \cos 2\theta \right]$$

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o \left( 1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2} \right) \sin 2\theta$$



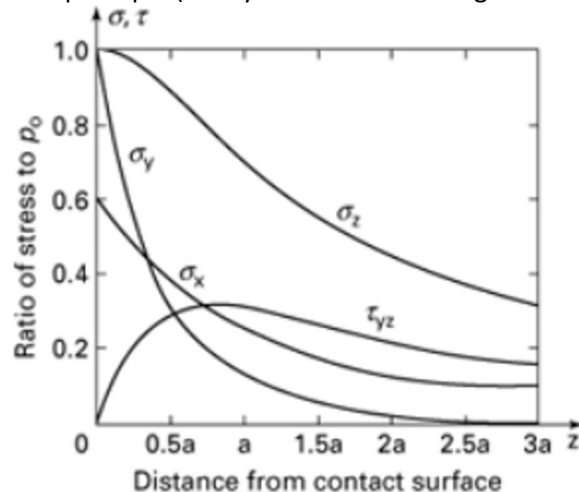
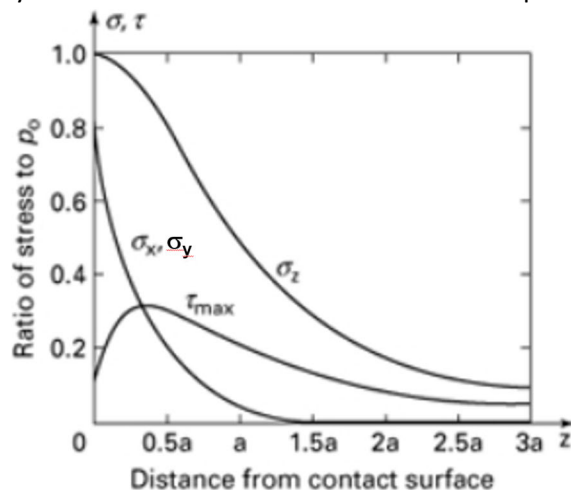
**Table 3.2. Maximum Pressure  $P_o$  and Deflection  $\delta$  of Two Bodies in Contact**

Configuration	Spheres: $p_o = 1.5 \frac{F}{\pi a^2}$	Cylinders: $p_o = \frac{2}{\pi} \frac{F}{aL}$
<b>A</b> 	<b>Sphere on a Flat Surface</b> $a = 0.880 \sqrt[3]{F r_1 \Delta}$ $\delta = 0.775 \sqrt[3]{\frac{F^2 \Delta^2}{r_1}}$	<b>Cylinder on a Flat Surface</b> $a = 1.076 \sqrt{\frac{F}{L} r_1 \Delta}$ For $E_1 = E_2 = E$ : $\delta = \frac{0.579 F}{EL} \left( \frac{1}{3} + \ln \frac{2 r_1}{a} \right)$
<b>B</b> 	<b>Two Spherical Balls</b> $a = 0.880 \sqrt[3]{F \frac{\Delta}{m}}$ $\delta = 0.775 \sqrt[3]{\frac{F^2 \Delta^2}{m}}$	<b>Two Cylindrical Rollers</b> $a = 1.076 \sqrt{\frac{F \Delta}{L m}}$
<b>C</b> 	<b>Sphere on a Spherical Seat</b> $a = 0.880 \sqrt[3]{F \frac{\Delta}{n}}$ $\delta = 0.775 \sqrt[3]{\frac{F^2 \Delta^2}{n}}$	<b>Cylinder on a Cylindrical Seat</b> $a = 1.076 \sqrt{\frac{F \Delta}{L n}}$

Note:  $\Delta = \frac{1}{E_1} + \frac{1}{E_2}$ ,  $m = \frac{1}{r_1} + \frac{1}{r_2}$ ,  $n = \frac{1}{r_1} - \frac{1}{r_2}$

\* all formulae in this table assume  $\nu = 0.3$

Stress distributions: Stresses as a function of the load axis,  $z$  (for  $\nu = 0.3$ ): (left) two spheres; (right) two parallel cylinders. Note: All normal stresses are compressive and principal ( $\tau$  only non-zero for change of coordinates)

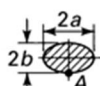

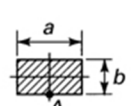
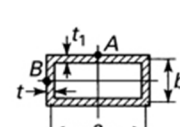
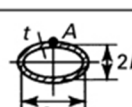



## Beam Bending

$$\sigma_x = \frac{(M_y I_z + M_z I_{yz})z - (M_y I_{yz} + M_z I_y)y}{I_y I_z - I_{yz}^2}$$

## Torsion

Table 6.2. Shear Stress and Angle of Twist of Various Members in Torsion

Cross section	Maximum shearing stress	Angle of twist per unit length	
 For circular bar: $a=b$	$\tau_A = \frac{2T}{\pi ab^2}$	$\theta = \frac{(a^2 + b^2)T}{\pi a^3 b^3 G}$	
 Equilateral triangle	$\tau_A = \frac{20T}{a^3}$	$\theta = \frac{46.2T}{a^4 G}$	
	$\tau_A = \frac{T}{\alpha ab^2}$	$\theta = \frac{T}{\beta ab^3 G}$	
	$a/b$	$\beta$	$\alpha$
	1.0	0.141	0.208
	1.5	0.196	0.231
	2.0	0.229	0.246
	2.5	0.249	0.256
	3.0	0.263	0.267
	4.0	0.281	0.282
	5.0	0.291	0.292
	10.0	0.312	0.312
$\infty$	0.333	0.333	
	$\tau_A = \frac{T}{2abt_1}$ $\tau_B = \frac{T}{2abt}$	$\theta = \frac{(at + bt_1)T}{2t_1 a^2 b^2 G}$	
 For circular tube: $a=b$	$\tau_A = \frac{T}{2\pi abt}$	$\theta = \frac{\sqrt{2(a^2 + b^2)}T}{4\pi a^2 b^2 t G}$	
 Hexagon	$\tau_A = \frac{5.7T}{a^3}$	$\theta = \frac{8.8T}{a^4 G}$	

Narrow, rectangular cross section

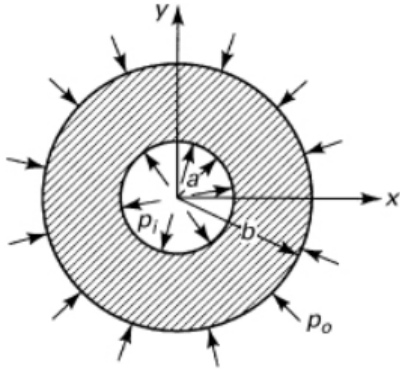
$$\tau_{max} = \frac{3T}{bt^2} \quad \theta = \frac{3T}{bt^3 G}$$

Thin-walled, closed cross sections

$$\tau = \frac{T}{2At} \quad \theta = \frac{1}{2AG} \oint \tau ds$$

## Thick-Walled Pressure Vessels

- Stress equations



$$\begin{aligned}\sigma_r &= \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r^2} \\ \sigma_\theta &= \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o) a^2 b^2}{(b^2 - a^2) r^2} \\ \sigma_z &= \frac{p_i a^2 - p_o b^2}{b^2 - a^2}\end{aligned}$$