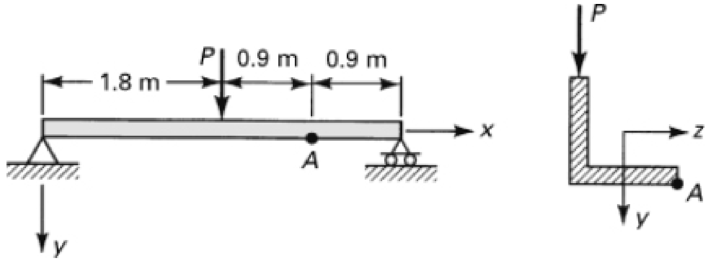
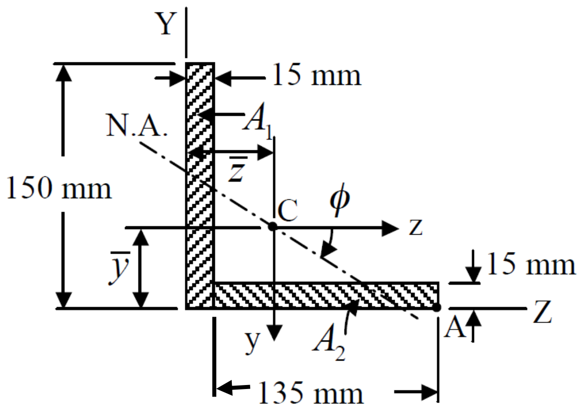
Homework 8 Solutions

1) A simply supported beam constructed of a $0.15 \times 0.15 \times 0.015\ m$ angle is loaded by concentrated force $P = 22.5\ kN$ at its midspan. Calculate stress σ_x at A and the orientation of the neutral axis. Neglect the effect of shear in bending and assume that beam twisting is prevented.





$$\bar{z} = \frac{A_1 z_1 + A_2 z_2}{A_1 + A_2} = \frac{(150\times15)7.5+(135\times15)[15+(135/2)}{150\times15+135\times15}$$

or $\quad \bar{z} = \bar{y} = 43\ mm$

Then,

$$I_y = \tfrac{1}{12}(150)(15)^3 + (150\times15)(35.5)^2 + \tfrac{1}{12}(15)(135)^3 + (135\times15)(39.5)^2$$

or $\quad I_y = I_z = 9.11(10^6)\ mm^4$

$$I_{yz} = (150\times15)(-32)(-35.5) + (135\times15)(35.5)(39.5) = 5.4(10^6)\ mm^4$$

We have the moment components:

$$M_y = 0, \qquad M_z = -11.25(0.9) = -10.125\ kN\cdot m$$

Thus,

$$(\sigma_x)_A = \frac{-10125(5.4)(0.107)+10125(9.11)(0.043)}{[(9.11)^2-(5.4)^2](10^{-6})} = -35\ MPa \quad ◀$$

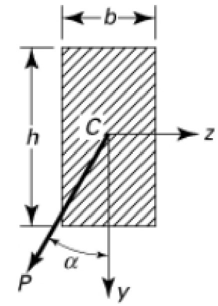
Equation (5.15) gives

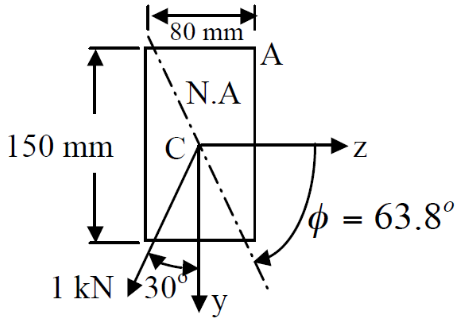
$$\tan\phi = \tfrac{5.4}{9.11} = 0.593$$

or

$$\phi = 30.66^o \quad ◀$$

2) A wood cantilever beam with the cross section shown is subjected to an angled (as shown) load P at its free end. Determine (a) the orientation of the neutral axis; (b) the maximum bending stress. Given: $P = 1\ kN$, $\alpha = 30°$, $b = 80\ mm$, $h = 150\ mm$, and length $L = 1.2\ m$.





$$I_y = \tfrac{1}{12} hb^3 = \tfrac{1}{12}(150)(80)^3 = 6.4 \times 10^6\ mm^4$$

$$I_z = \tfrac{1}{12} bh^3 = \tfrac{1}{12}(80)(150)^3 = 22.5 \times 10^6\ mm^4$$

$$M_y = (P \sin\alpha)L = 600\ N \cdot m$$

$$M_z = (P \cos\alpha)L = 1{,}039.2\ N \cdot m$$

$$y_d = -75\ mm \qquad z_d = 40\ mm$$

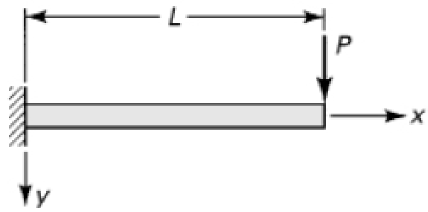
(a) Equation (5.15) with $I_{yz} = 0$:

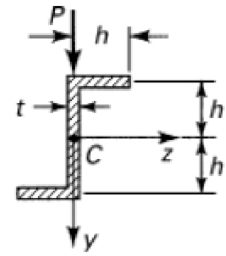
$$\tan\phi = \tfrac{I_z}{I_y}\tfrac{M_y}{M_z} = \tfrac{I_z}{I_y}\tan\alpha \qquad \therefore \phi = 63.8^o$$ ◀

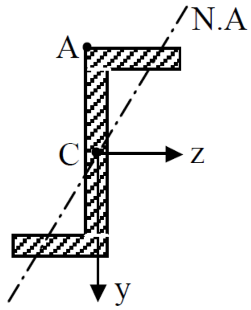
(b) Thus, maximum tensile stress is at point A.
Equation (5.16) gives

$$\sigma_{\max} = \frac{600(0.04)}{6.4x10^{-6}} - \frac{1{,}039.2(-0.075)}{22.5x10^{-6}} = 7.21\ MPa$$ ◀

3) A cantilever beam has a Z section of uniform thickness for which $I_y = \frac{2}{3}th^3$, $I_z = \frac{8}{3}th^3$, and $I_{yz} = -th^3$. Determine the maximum bending stress in the beam subjected to a load P at its free end.







We have $M_y = 0$ and $M_z = PL$

Equation (5.14) becomes

$$I_{yz}z = I_y y \quad \text{or} \quad -th^3 z = \tfrac{2}{3}th^3 y$$

or

$$y = -\tfrac{3z}{2}$$

Point A is the farthest from the N.A. Thus, with

$$y_A = -h - \tfrac{t}{2} \quad \text{and} \quad z_A = -\tfrac{t}{2}$$

(In order to get the maximum stress, We consider point A, which is the farthest from the N.A.)

Equation (5.13) yields

$$(\sigma_x)_A = \frac{PL[-th^3(-t/2)-(2th^3/3)h^3(-h-t/2)}{(2th^3/3)(8th^3/3)-(-th^3)^2}$$

or

$$(\sigma_x)_A = \frac{3PL(2.5t+2h)}{7th^3} = \sigma_{max}$$ ◀