ME EN 5830/6830 Aerospace Propulsion Equation Sheet

Thermodynamics

$R = \bar{R}/\bar{m}$

$R = c_p - c_v$

$\gamma = c_p/c_v$

$c_p = R\frac{\gamma}{\gamma-1}$

$pv = RT, p = \rho RT$ (Ideal gas law)

$de = \delta q - \delta w$ (First law)

$\delta w = pdv$ (Reversible work definition)

$h = u + pv$ (Enthalpy definition)

For an isentropic, constant specific heat gas:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}, \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}, \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$$

For a single (non-reacting) gas:

$$u = \int_{T_{ref}}^{T} c_v dT' + u(T_{ref})$$

$$h = \int_{T_{ref}}^{T} c_p dT' + h(T_{ref})$$

$$s = s^0(T_{ref}, P_{ref}) + \int_{T_{ref}}^{T} c_p \frac{dT'}{T'} - R\, ln \frac{P}{P_{ref}}$$

$\eta = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$ (Efficiency of power cycle)

Ideal Gas Mixtures

$M = \sum M_i,\ N = \sum N_i$ (Total mass/mole)

$\rho = \frac{M}{V},\ \rho_i = \frac{M_i}{V}$ (Total/species density)

$C = \frac{N}{V},\ C_i = \frac{N_i}{V}$ (Total/species concentration)

$Y_i = \frac{M_i}{M} = \frac{\rho_i}{\rho},\ X_i = \frac{N_i}{N} = \frac{C_i}{C}$ (Mass/Mole fracs)

For a mixture of gases:

$u = \sum Y_i u_i \qquad \bar{u} = \sum X_i \bar{u}_i$

$h = \sum Y_i h_i \qquad \bar{h} = \sum X_i \bar{h}_i$

$s = \sum Y_i s_i \qquad \bar{s} = \sum X_i \bar{s}_i$

$\bar{M} = \sum X_i \bar{M}_i = \frac{1}{\sum \frac{Y_i}{\bar{M}_i}}$ (Mixture molar mass)

$Y_i = X_i\left(\frac{\bar{M}_i}{\bar{M}}\right)$ (Mole to mass fraction conversion)

$p_i = X_i p$ (Partial pressure)

Combustion

For a single reacting gaseous species:

$$h_i(T) = \int_{T_{ref}}^{T} c_{p,i} dT' + h_{fi}(T_{ref})$$

$$s_i(T) = \int_{0K}^{T} c_{p,i}\frac{dT'}{T'} - R_i\, ln\frac{p_i}{p_{ref}}$$

$\phi = \left(\frac{M_f}{M_a}\right)_{actual} / \left(\frac{M_f}{M_a}\right)_{stoich}$ (Equivalence ratio)

$\Delta h_R = h_{prod} - h_{reac}$ (Heat of reaction)

Compute Adiabatic Flame temperature:

$du = 0$ (Constant volume first law)

$dh = 0$ (Constant pressure first law)

For a general reaction given by:

$aA + bB + \ \ldots \leftrightarrow zZ + yY + \ \ldots$

Compute equilibrium constant (pressure):

$$K_p(T) = \frac{\left(\frac{p_Z}{p_{ref}}\right)^z \left(\frac{p_Y}{p_{ref}}\right)^y \ldots}{\left(\frac{p_A}{p_{ref}}\right)^a \left(\frac{p_B}{p_{ref}}\right)^b \ldots} = exp\left(-\frac{\Delta\bar{h}}{\bar{R}T}\right) exp\left(\frac{\Delta\bar{s}^0}{\bar{R}}\right)$$

$$K_C(T) \equiv \frac{C_Z^z C_Y^y \ldots}{C_A^a C_B^b \ldots} = K_p(T)\left(\frac{p_{ref}}{\bar{R}T}\right)^{z+y+\cdots-a-b-\cdots} = \frac{k_f}{k_b}$$

Compute reaction rate:

$w_f = A_f(T)\, exp\left(-\frac{E_f}{\bar{R}T}\right) C_A^a C_B^b \ldots$ (forward)

$w_b = A_b(T)\, \exp\left(-\frac{E_b}{\bar{R}T}\right) C_Z^z C_Y^y \ldots$ (backwards)

Compressible Flows

$a = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma R T}$ (Speed of sound, ideal gas)

$M = V/a$ (Mach number)

Stagnation quantities:

$h_t = h + \frac{1}{2}V^2$ (Enthalpy)

$T_t = T\left(1 + \frac{\gamma-1}{2}M^2\right)$ (Temperature)

$p_t = p\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$ (Pressure)

$dh_t = \delta q - \delta w$ (First law)

Isentropic Flow (Static):

$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)}$ (Temperature)

$\frac{p_2}{p_1} = \left[\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)}\right]^{\frac{\gamma}{\gamma-1}}$ (Pressure)

$\frac{\rho_2}{\rho_1} = \left[\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)}\right]^{\frac{1}{\gamma-1}}$ (Density)

$\frac{A_2}{A_1} = \frac{M_1}{M_2}\left[\frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)}{\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$ (Area)

Isentropic Flow (Stagnation):

$h_{t1} = h_{t2}$ (Enthalpy)

$T_{t1} = T_{t2}$ (Temperature)

$p_{t1} = p_{t2}$ (Pressure)

Normal Shock (Static):

$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2 - 1}$ (Mach number)

$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)}{\left[\frac{(\gamma+1)^2}{2(\gamma-1)}\right]M_1^2}$ (Temperature)

$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}$ (Pressure)

$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$ (Density)

Oblique shock angle decomposition:

$M_{n1} = M_1 \sin\beta$ (Before shock)

$M_{n2} = M_2 \sin(\beta - \theta)$ (After Shock)

Airbreathing propulsion

$T = \dot{m}(V_e - V)$ (Net thrust)

$\eta_p = \frac{2V}{Ve+V}$ (Propulsive efficiency)

$tsfc = \dot{m}/T$ (Thrust specific fuel consumption)

For a Brayton Cycle:

$r_p = p_2/p_1$ (Pressure ratio)

$\eta = 1 - r_p^{\frac{1-\gamma}{\gamma}}$ (Efficiency)

Turbojet:

Adiabatic Inlet (0-2)

$\frac{p_{t2}}{p_{t0}} = \frac{\left(1+\eta_d\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}$

Adiabatic Compressor (2-3)

$r_p = p_{t3}/p_{t2}$ (Compressor pressure ratio)

$\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$

Constant pressure combustor (3-4)

$T_{t4} = T_{t3} + \frac{\phi\left(\frac{F}{A}\right)_{st}LHV}{c_p} = T_{t3} + \frac{\left(\frac{F}{A}\right)LHV}{c_p}$

Adiabatic Turbine (4-5)

$T_{t5} = T_{t4} - \frac{T_{t2}}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$

$p_{t5} = p_{t4}\left[1 - \frac{1}{\eta_c\eta_t}\frac{T_{t2}}{T_{t4}}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$

Perfectly expanded exhaust (5-9)

$V_e = \sqrt{2\frac{\gamma}{\gamma-1}\eta_n R T_{t5}\left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

Constant pressure afterburner (5-6)

$T_{t6} = T_{t5} + \frac{\phi\left(\frac{F}{A}\right)_{st}LHV}{c_p} = T_{t5} + \frac{\left(\frac{F}{A}\right)LHV}{c_p}$

(Compute V_e with T_{t6} instead of T_{t5})

$BPR = \dot{m}_{bp}/\dot{m}_{core}$ (Bypass ratio, turbofan)

Adiabatic Fan (2-13)

$r_f = p_{t13}/p_{t2}$ (Fan pressure ratio)

$\frac{T_{t13}}{T_{t2}} = 1 + \frac{1}{\eta_f}\left(r_f^{\frac{\gamma-1}{\gamma}} - 1\right)$

Perfectly expanded exhaust (13-19)

$V_{e,bp} = \sqrt{2\frac{\gamma}{\gamma-1}\eta_{n,bp} R T_{t13}\left[1 - \left(\frac{p_a}{p_{t13}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

Airbreathing propulsion (continued)

Adiabatic Compressor (13-3)

$r_p = p_{t3}/p_{t13}$ (Compressor pressure ratio)

$$\frac{T_{t3}}{T_{t13}} = 1 + \frac{1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$$

Constant pressure combustor (3-4) (see above)

Adiabatic HP Turbine (4-4.5)

$$T_{t4.5} = T_{t4} - \frac{T_{t13}}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$p_{t4.5} = p_{t4}\left[1 - \frac{1}{\eta_c \eta_{HPT}} \frac{T_{t13}}{T_{t4}}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Adiabatic LP Turbine (4.5-5)

$$T_{t5} = T_{t4.5} - (1 + BPR)\frac{T_{t2}}{\eta_f}\left(r_f^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$p_{t5} = p_{t4.5}\left[1 - \frac{1+BPR}{\eta_f \eta_{LPT}} \frac{T_{t2}}{T_{t4}.5}\left(r_f^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Perfectly expanded exhaust (5-9)

$$V_{e,c} = \sqrt{2\frac{\gamma}{\gamma-1}\eta_{n,c} R T_{t5}\left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$T = \dot{m}_{bp}\left(V_{e,bp} - V\right) + \dot{m}_c\left(V_{e,c} - V\right)$$

$$\eta_p = \frac{TV}{\dot{m}_{bp}\left[\frac{V_{e,bp}^2}{2} - \frac{V^2}{2}\right] + \dot{m}_c\left[\frac{V_{e,c}^2}{2} - \frac{V^2}{2}\right]}$$

Turbomachinery

$U = \omega r$ (Rotational velocity)

$C_{zi} = C_i \cos \alpha_i$ (Axial)

$C_{\theta i} = C_i \sin \alpha_i$ (Tangential)

Across a compressor:

$$\frac{p_{t3}}{p_{t1}} = \left[1 + \eta_{cs}\frac{U}{c_p T_{t1}}\left(C_{\theta 2} - C_{\theta 1}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Across a turbine:

$$\frac{p_{t3}}{p_{t1}} = \left[1 - \frac{1}{\eta_{ts}}\frac{U}{c_p T_{t1}}\left(C_{\theta 2} - C_{\theta 3}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Ramjet/Scramjet

Follow analysis of turbojet skipping core:

$p_{t5} = p_{t2}$

$T_{t5} = T_{t2}$

Rocket propulsion

$I_t = \int_0^t F dt = F\Delta t$ (Total impulse)

$I_s = \frac{\int_0^t F dt}{g_0 \int_0^t \dot{m} dt} = \frac{I_t}{m_p g_0}$ (Specific impulse)

$V_e = I_s g_0$ (Perfectly expanded exhaust)

$MR = \frac{m_{end}}{m_0}$ (Mass Ratio)

$\zeta = \frac{m_p}{m_0}$ (Propellant Mass Fraction)

$F = \dot{m} V_e + (p_e - p_a) A_e$ (Rocket Thrust)

Basic Rocket Analysis:

$T_{t0} = T_0$

$p_{t0} = p_0$

Constant Pressure Combustor:

$$T_{t1} = T_{t0} + \frac{\phi\left(\frac{F}{A}\right)_{st} LHV}{c_p} = T_{t0} + \frac{\left(\frac{F}{A}\right) LHV}{c_p}$$

Perfectly Expanded Nozzle:

$$V_e = \sqrt{2\frac{\gamma}{\gamma-1} R T_{t1}\left[1 - \left(\frac{p_a}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$\Delta V = V_e \ln \frac{m_0}{m_{end}}$ (Rocket equation)

$\Delta V = V_e \ln \frac{m_0}{m_{end}} - g\,\Delta t$ (With gravity, straight)

$\Delta V = V_e \ln \frac{m_0}{m_{end}} - g \cos\theta\,\Delta t$ (at angle θ)

$\Delta V_{tot} = \sum_1^n \Delta V$ (Multi-stage rocket)