HW 5: Laminar Boundary Layers
Due: 12/05/2025

1. Consider laminar, steady, two-dimensional flow inside the boundary layer that develops over a infinitely long flat plate, where the x direction is parallel to the plate and the y direction is normal to the plate.

- **(a)** Write the governing equations that describe the fluid dynamics for this particular flow (i.e., two momentum equations, and conservation of mass). Label and identify each term in the equations.
- **(b)** Is the boundary layer flow fully developed? Why or why not?
- **(c)** State the Reynolds number range for which the equations in part (a) are valid.
- **(d)** Plot the nondimensional horizontal velocity profile (u/U versus y/δ) corresponding to the Blasius solution, the data for which may be downloaded from Canvas (same module where the HWs are posted). Place y/δ along the ordinate and u/U along the abscissa.
- **(e)** Calculate δ^*/δ, θ/δ, and $C_f \, Re_\delta$ from the data provided for the Blasius solution. Note, because this is a numerical solution to a nonlinear ODE, you will need to perform *numerical* integration and differentiation of the data provided to calculate the required quantities.

2. A flat plate 4 m wide and 1 m long (in the direction of the flow) is immersed in kerosene at 20 °C ($\nu = 2.29 \times 10^{-6}$ m^2/s, $\rho = 800$ kg/m^3). The freestream velocity above the plate is $U_\infty = 0.5$ m/s. The critical Reynolds number (based on distance x along the plate) at which the flow over a flat plate transitions to turbulence is $Re_x \approx 5 \times 10^5$.

- **(a)** What is the critical Reynolds number based on the momentum thickness θ for a flat plate?
- **(b)** Verify that the Reynolds number based on x is less than critical everywhere so that the flow remains laminar along the entire plate.
- **(c)** Show that the boundary layer thickness and wall shear stress at the *center* of the plate are $\delta = 0.74$ cm and $\tau_w = 0.2$ N/m^2.
- **(d)** Show that the boundary layer thickness and wall shear stress at the *trailing edge* of the plate are $\delta = 1.05$ cm and $\tau_w = 0.14$ N/m^2.
- **(e)** Show that the *total* skin friction drag along one side of the plate is 1.14 N.

3. Of course, the Blasius profile is an exact solution for laminar flow over a flat plate, and therefore should be used in all such applications that require the utmost accuracy. However, because the Blasius profile derives from a numerical solution (i.e., the data are tabulated), no function exists that represents the exact profile. In many engineering applications, however, accuracy may be sacrificed in favor of efficiency. In these instances, one prefers to estimate the exact numerical solution with an approximate function, which can be easily integrated or manipulated in other ways. Consider the following approximate profiles, all of which satisfy the boundary conditions at the wall and edge of the boundary layer:

$$\begin{aligned}
\text{Linear:} \quad & \frac{u}{U} = \frac{y}{\delta} \\
\text{Quadratic:} \quad & \frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \\
\text{Cubic:} \quad & \frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \\
\text{Quartic:} \quad & \frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4 \\
\text{Sinusoidal:} \quad & \frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)
\end{aligned}$$

(a) Plot u/U versus y/δ for all of the approximate solutions, including the Blasius solution. Place all profiles on a single plot, using a different linestyle or grayscale for each profile. Supply a legend.

(b) Calculate δ^*/δ, θ/δ, and $C_f\, Re_\delta$ for each profile and tabulate the results. Include a column that lists the percent error relative to the Blasius solution.

Profile	δ^*/δ	θ/δ	$C_f\, Re_\delta$	% error
Blasius				—
Linear				
Quadratic				
Cubic				
Quartic				
Sinusoidal				

(c) In your opinion, which of the profiles, besides the Blasius solution, provides the *best* approximation for engineering purposes. Why?