Homework #11
ME EN 5210/6210 & CH EN 5203/6203 & ECE 5652/6652
Linear Systems & State-Space Control

Use this page as the cover page on your assignment, submitted as a single pdf.

Problem 1

Read this entire problem before beginning. In this problem, you will implement an open-loop observer and a closed-loop observer to estimate the states of your system. You will find that the closed-loop observer can do a better job of estimating the states with a controlled time constant. Use MATLAB for as much of this problem as desired. Let's consider the system below, for which we have a perfect model:

$$\dot{\vec{x}}(t) = \begin{bmatrix} -3 & 5 \\ 0 & -2 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x}(t)$$

Verify that it is observable. The system is stable, so an open-loop observer is an option (but not a good one). Implement an open-loop observer. Simulate both the system and the observer using the *lsim* function. Use a unit step input for *u(t)*. Plot the four states (two real states and two estimated states) on a single plot, with the four curves clearly labeled.

Repeat this process for the closed-loop observer. Choose the values of your observer-gain matrix L such that the observer eigenvalues are both at -1; calculate the two gain values in L explicitly. Note that these eigenvalues are slower than those of the original plant. In this case, you will first simulate the real system, and then use its recorded output *y(t)* as an input when you simulate the observer (this is not how you would do things in a real control system, but it is possible in simulation). To do this, you can modify the B matrix of the closed-loop observer as [B L], and then treat *y(t)* as a second input. Again, plot the four states (two real states and two estimated states) on a single plot, with the four curves clearly labeled. Repeat for closed-loop observer eigenvalues both at -6. Note that these eigenvalues are faster than the original plant.

For both the open-loop observer and closed-loop observer problems above, initialize the observer assuming that the initial state is

$$\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

as is typically done. However, the actual initial state of the real system should be

$$\vec{x}(0) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Verify which (if any) of the observers is able to reject the initial error in the estimate. Include a summary of what you observed along with your plots and printouts of your MATLAB m-files.

Problem 2

In Problem 1, we assumed that we had a perfect model of our system to implement in the observer. In this problem, use the same idealized model of the system from last time in your observer, but for the physical system use the model:

$$\dot{\vec{x}}(t) = \begin{bmatrix} -3.4 & 4.6 \\ 0 & -1.7 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1.2 \\ -0.8 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1.1 & 0 \end{bmatrix} \vec{x}(t)$$

Repeat each part of Problem 1, given the above information. Verify which (if any) of the observers is able to reject the initial error in the estimate. Include a summary of what you observed compared to Problem 1, along with your plots and printouts of your MATLAB m-files.

Problem 3

Use a linear-quadratic regulator (using the LQR function in MATLAB) to design a closed-loop position controller for a simple forced mass.

$$\begin{bmatrix} \dot{v}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} 1/m \\ 0 \end{bmatrix} f(t)$$

where the two states are the position and velocity of the mass. Assume we control the applied force directly, and that we have full state feedback. The mass is 5 kg, and the maximum force that we can apply is 20 N. Design a closed-loop controller such that our system, when started from rest, servos to a desired position as quickly as possible, but with no oscillations. The largest step in position that we will ever give our system is 0.1 m.

Include your MATLAB script and well-labeled plots of the states and input (three separate plots) for your final design. Include a summary that includes your final gain matrix K, and the settling time in position that you were able to achieve.