ME 3710
Homework 5
Due Tuesday February 22 at 11:59pm – upload to canvas
[6 problems –18 pts]

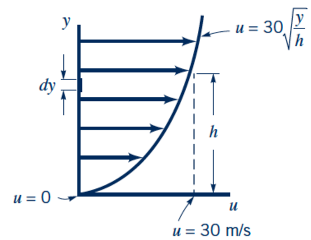
Note that for chapter 5 problems, the solution manual doesn't always go through all of the steps that we have gone through in class: 1) draw CV, 2)state Assumptions, 3) list Givens, 4) write down fundamental equations, 5) Simplify and solve. Please do this!

Solution 5.2

From the equation

$$\overline{V} = \frac{\int_A \rho \mathbf{V} \cdot \mathbf{n} dA}{\rho A}$$ or with $\rho = \text{constant}$,

$$\overline{V} = \frac{\int_A \mathbf{V} \cdot \mathbf{n} dA}{A}$$



Consider a unit depth normal to the x–y plane so that

$A = 1 \times h = h$ and $dA = 1 \times dy = dy$

Thus,

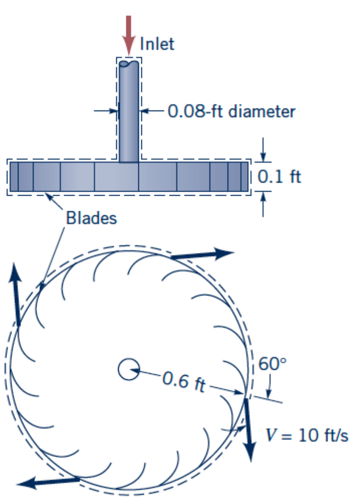
$$\overline{V} = \frac{\int u dA}{A} = \frac{\int_0^h u dy}{h} = \frac{\int_0^h 30\sqrt{\frac{y}{h}} dy}{h} = \frac{30}{h^{\frac{3}{2}}} \int_0^h y^{\frac{1}{2}} dy = \frac{30}{h^{\frac{3}{2}}} y^{\frac{3}{2}} \left(\frac{2}{3}\right) \Big|_{y=0}^{y=h} = \frac{2}{3}(30)\frac{\text{m}}{\text{s}}$$

or

$$\underline{\underline{\overline{V} = 20\frac{\text{m}}{\text{s}}}}$$

Solution 5.4

Use the control volume container within the broken lines as shown in the sketch below.



From the conservation of mass principle $\dot{m}_{\text{inlet}} = \dot{m}_{\text{outlet}}$

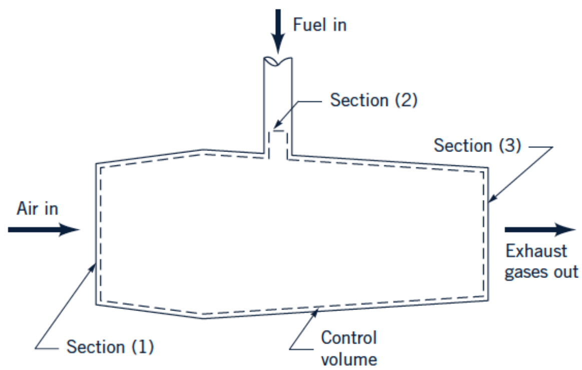
Also $\dot{m}_{\text{outlet}} = \rho A_{\text{outlet}} V_{\text{outlet}} \cos 60° = \rho 2\pi r_{\text{outlet}} h V_{\text{outlet}} \cos 60°$

$$= \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) 2\pi (0.6\,\text{ft})(0.1\,\text{ft})\left(10\frac{\text{ft}}{\text{s}}\right)\cos 60°$$

$$= \underline{\underline{3.66}}\,\frac{\text{slugs}}{\text{s}}$$

Solution 5.13

For steady flow



$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

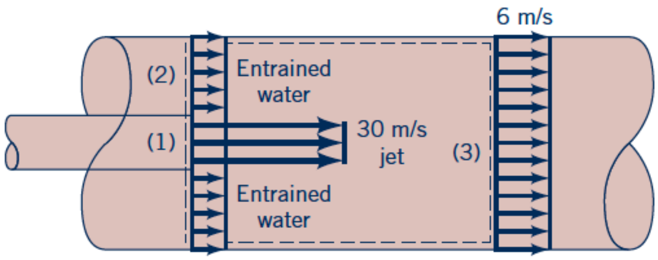
$$\rho_3 A_3 \overline{V}_3 = \dot{m}_1 + \dot{m}_2$$

Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \overline{V}_3} = \frac{65\frac{\text{lbm}}{\text{s}} + 0.60\frac{\text{lbm}}{\text{s}}}{(3.5\,\text{ft}^2)\left(1500\frac{\text{ft}}{\text{s}}\right)}$$

$$\rho_3 = \underline{\underline{0.0125}}\,\frac{\text{lbm}}{\text{ft}^3}$$

Solution 5.17



For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\overline{V_1}A_1 + Q_2 = \overline{V_3}A_3$$

Thus

$$Q_2 = \overline{V_3}A_3 - \overline{V_1}A_1 = \left[\left(6\frac{\text{m}}{\text{s}}\right)\left(0.075\,\text{m}^2\right) - \left(30\frac{\text{m}}{\text{s}}\right)\left(0.01\,\text{m}^2\right)\right]\left(1000\frac{\text{l}}{\text{m}^3}\right)$$

$$Q_2 = \underline{\underline{150\frac{\text{l}}{\text{s}}}}$$

Solution 5.18

The mass flowrate is calculated with

$$\dot{m} = \int_0^R \rho u 2\pi r dr = 2\pi\rho\int_0^R u r dr$$

where

$R = 3\,\text{in.}$

$\rho = 0.00238\dfrac{\text{slug}}{\text{ft}^3}$

$u =$ local axial velocity in $\dfrac{\text{ft}}{\text{s}}$

$r =$ local radius in in.

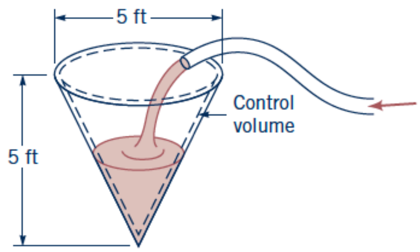
and $\int_0^R u r dr$ is evaluated numerically with the trapezoidal rule with unequal intervals.

The result is:

$$\dot{m} = 2\pi\left(0.00238\frac{\text{slug}}{\text{ft}^3}\right)\int_{r=0}^{r=\frac{3}{12}\text{ft}} u\frac{\text{ft}}{\text{s}}(r\,\text{ft})(dr\,\text{ft}) = \underline{\underline{0.0114\frac{\text{slugs}}{\text{s}}}}$$

Consider doing this problem in metric units. If you do the answer is: 0.0204 kg/s

Solution 5.27



From application of the conservation of mass principle to the control volume shown in the figure, we have

$$\frac{\partial}{\partial t}\int_{\text{CV}} \rho\, d\forall + \int_{\text{CS}} \rho \mathbf{V}\cdot\widehat{\mathbf{n}dA} = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q\int_0^t dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{12Q} = \frac{\pi(5\,\text{ft})^2(5\,\text{ft})\left(1728\frac{\text{in.}^3}{\text{ft}^3}\right)}{(12)\left(20\frac{\text{gal}}{\text{min}}\right)\left(231\frac{\text{in.}^3}{\text{gal}}\right)}$$

and

$$t = \underline{\underline{12.2\,\text{min}}}$$