Time: start Monday December 12, 2022 at 1 pm (24-hour take-home exam, the exam should take about 2-3 hours).

Allowed: Open-book, open-notes exam; specifically, you may use your textbook, a calculator, a computer, and the internet.

Not allowed: You are not allowed to communicate with any other human being about the exam except the instructor, Eric Pardyjak. No texting, no emailing, no talking, posting, etc.

Exam Process: Download the exam from Canvas at 1pm on Monday December Download the exam from Canvas at 1 pm on Monday December 12, 2022 you will have 24-hours to complete the exam; It must be uploaded onto Canvas by 1 pm Tuesday December 13, 2022. You don't have to physically download the exam; you may choose to use your own paper and then upload a pdf or photo of your work. Please write as neatly and as organized as possible. Much of the work we do in this class involves derivations, please show all your work and important steps. <u>You must sign the honor policy on the following page.</u>

<u>Note that there are some problems that are only for the ME 6700 class. If you are in ME 5700, do not do these problems.</u>

ME EN 5700/6700 – Intermediate Fluid Dynamics Honor Policy for Exams
Fall 2022

To protect the integrity of this course, I am requiring all students taking this exam to sign a pledge that you will not cheat on this exam (Final Exam ME EN 5700/6700). Please note that this exam is open book and open notes, however, all the work that you submit must be your own. You may consult material and problems in the book, in your notes, or online, but you are <u>not</u> allowed to discuss the material on this exam with your peers in the class or anyone else prior to 1pm on Tuesday December 13, 2022. Furthermore, you are not allowed to post questions to online forums. Anyone found to have cheated on this exam will automatically fail the exam.

I affirm that I, ______________________ (write your name) am submitting my own work for this exam. I have neither given nor received help on this exam. I have not consulted with anyone (other than the instructor) on questions nor looked at or copied another student's work. I understand the penalties for academic dishonesty on this exam is failure.

____
Signature

____
Date/Time

ME 5700/6700: Intermediate Fluid Dynamics
FINAL EXAM
December 12, 2022

NAME: ______________________

1. [8 points] Match the most appropriate single statement on the right with term/s on the left

___ Circulation about a curve	1. $\nabla \times \vec{V} = 0$
___ Irrotational	2. $\nabla \cdot \vec{\omega} = 0$
___ Isotropic	3. Independent of viscosity
___ displacement thickness	4. shear stress linearly related to the deformation-rate tensor
___ Solenoidal	5. Fluid velocity at boundary is the same as the boundary velocity
___ Ergotic	6. Ensemble, time & spatial averages are the same
___ momentum deficit thickness	7. The distance by which a wall would be moved in a frictionless flow to maintain the same mass flux in a real flow
	8. Invariant to coordinate rotation
___ vorticity	9. Vorticity flux through a surface
	10. Invariant to coordinate translation
	11. $\int_0^\infty \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$
	12. $\nabla \cdot \vec{V} = 0$
	13. $\nabla \times \vec{V}$

2. [3 points] In the thermal energy equation, please define the viscous dissipation rate. Use appropriate terms of the equations to help explain, if necessary.

3. [3 points] Please write down three characteristics of turbulence.

4. [3 points] Please provide the mathematical definition for separation of flow.

5. [3 points] Consider a boundary layer developing over a flat plat starting from the plate's leading edge. At what point does this boundary layer become fully developed?

6. [4 points] Explain why a vortex line cannot start or end in the fluid. Use appropriate equations to support your explanation.

7. [4 points] Describe the turbulent energy cascade.

8. [12 pts] Consider the velocity field given by the following components:

$$u = 16x^2 + y$$
$$v = 10$$
$$w = yz^2$$

a. [5 pts] Compute the circulation, Γ, for the flow field around the contour given below by integrating the velocity around the contour counterclockwise.

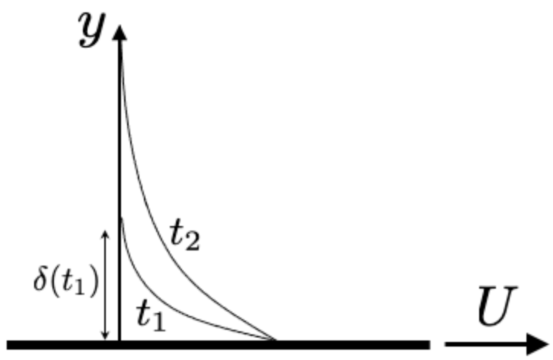
$0 \leq x \leq 10$	$y = 0$
$0 \leq y \leq 5$	$x = 10$
$0 \leq x \leq 10$	$y = 5$
$0 \leq y \leq 5$	$x = 0$

b. [5 pts] Now compute the following:

$$\int_A \vec{\omega} \cdot \hat{n} \; dA$$

c. [2 pts] How do your results in (a) and (b) compare? Explain.

9. [12 points] Consider an infinitely wide impulsively started plate as shown schematically below. The flow is fully incompressible. Velocity profiles are sketched in the flow for times t_1 and t_2.
   a. [5 pts] What is the vertical velocity in this flow? (Hint: use basic equations and show mathematically).
   b. [4 pts] What is the simplified horizontal momentum equation for this flow? State all assumptions.
   c. [3 pts] <u>Using scale analysis</u>, determine the time τ it will take for a fluid particle at height $\delta(\tau)$ above the plate to begin to move once the plate has been set in motion.



10. [24 points] For the following questions, assume that the velocity profile for an incompressible laminar boundary layer on a flat plate with no pressure gradient has the following form:

$$\frac{u}{U_\infty} = a_0 + a_1 \frac{y}{\delta} + a_2 \left(\frac{y}{\delta}\right)^2$$

a. [9 points] Determine a_0, a_1, a_2 and write out the equation for the <u>non-dimensional</u> velocity profile.

b. [6 points] Using the von Kàrmàn integral equation, determine δ/x. List any important assumptions needed to do this. **6700 Problem only**

c. [9 points] Determine δ^* , θ and τ_{wall} in terms of the Reynolds number based on x (the streamwise distance along the plate). **<u>6700 Problem only</u>**

11. [10 points] Consider the vorticity equation as written below.

$$\underbrace{\frac{\partial \vec{\omega}}{\partial t}}_{i} + \underbrace{\left(\vec{V} \cdot \nabla\right)\vec{\omega}}_{ii} = \underbrace{\left(\vec{\omega} \cdot \nabla\right)\vec{V}}_{iii} + \underbrace{\nu \nabla^2 \vec{\omega}}_{iv}$$

a. [5 points] Describe the physical meaning of each term.

i.

ii.

iii.

iv.

b. [5 points] Simplify this vorticity equation for steady, incompressible, 2D flow.

12. [16 points] Consider the steady state, inviscid vorticity equation for a baratropic fluid in a non-rotating coordinate system.
    a. [3 points] Write down the equation using index notation.
    b. [10 points] Use Reynolds decomposition to derive a time averaged vorticity equation. Show all work.
    c. [3 points] Explain the physical meaning of each term.

13. [16 points] Consider steady, fully developed, 2D, incompressible water flow through a channel of height h = 3 cm and length L = 3 m subject to a pressure drop of 200 kPa.

   a. [4 points] Simplify the Navier-Stokes equations for this problem (neglect entrance effects). Begin with the following continuity equation and momentum equations. Show your work and justification.

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

b. [8 points] Using scale analysis on the simplified set of equations (assuming the flow is laminar), estimate the velocity of the water in the channel. When finished, be sure to provide a number in meters per second.

c. [4 points] Is this value physically possible? Why or why not?