Quiz 3: Conservation of Mass and Other Prior Concepts,

September 18, 2025

Q1.- Write the expression that:

(a) Determines Conservation of Mass in a Lagrangian framework?

$$\implies \frac{dm}{dt} = \frac{DM}{Dt} = 0 \tag{1}$$

(b) Determines Conservation of Mass in an Eulerian framework?

$$\implies \frac{\partial \rho}{\partial t} + \frac{\partial(\rho\, u_i)}{\partial x_i} = 0 \tag{2}$$

Q2.- Write the equation of Conservation of Mass for an incompressible fluid.

$$\implies \frac{\partial u_i}{\partial x_i} = 0 \qquad \text{or} \qquad \frac{D\rho}{Dt} = 0 \tag{3}$$

Q3.- In an incompressible flow, can the density of the fluid change in space? (*Justify your response.*)

$\implies$ Yes.

Justification: By expanding the equation for Conservation of Mass under the condition of incompressibility ($\partial u_i/\partial x_i = 0$), this can be written as

$$\frac{D\rho}{Dt} = 0 = \frac{\partial \rho}{\partial t} + u_i \frac{\partial(\rho)}{\partial x_i} \tag{4}$$

It is clear that $u_i \frac{\partial \rho}{\partial x_i} \neq 0$ while at the same time, $\frac{D\rho}{Dt} = 0$. In this case, $\frac{\partial \rho}{\partial t}$ would also have to be different than zero.

Q4.- What is the relation between the streamfunction and the streamlines?

$\implies$ $d\psi = 0$ along the streamlines.

Q5.- Choose whether the following statements are True or False.

- **(True/False)** In a stagnation flow, the flow is highly rotational. $\Longrightarrow$ False

- **(True/False)** In solid body rotation, a fluid element besides experiencing rotation, it also experiences important shear deformation. $\Longrightarrow$ False

- **(True/False)** Vorticity is a vector tangent to the direction of the velocity field variation. $\Longrightarrow$ False