ME EN 7210 Optimal Controls
Homework #2 **Due Tuesday, Jan. 27 at the beginning of class**

This assignment MUST be typeset in either LaTex or MS Word. Any hand-written work will not be accepted and if submitted, the entire assignment will receive a zero grade!

Show all your work – do not solve using Matlab, but work out the problem by hand for these problems!

Problem 1

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find $\|A\|_p$, the p-norm of matrix A.

Problem 2

Let A and B be two real matrices of size *n* x *n*. Prove that if A and B are both positive definite, then so is the sum A+B. Note: this is a proof, not showing several examples with numbers and concluding that it's true for all matrices A and B of size *n* x *n*.

Problem 3

(a) Determine the definiteness of the following matrix:

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

(b) For what numbers b is the following matrix positive semi-definite?

$$\begin{pmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{pmatrix}$$

Problem 4

For an inverted pendulum system, the objective is to maintain an upright position of the pendulum on a cart. The linearized state equations are:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -x_3(t) + 0.2u(t)$$
$$\dot{x}_3(t) = x_4(t)$$
$$\dot{x}_4(t) = 10x_3(t) - 0.2u(t)$$

where, $x_1(t) =$ is horizontal linear displacement of the cart, $x_2(t) =$ is linear velocity of the cart, $x_3(t) =$ is angular position of the pendulum from vertical line, $x_4(t) =$ is angular velocity, and $u(t) =$ is the horizontal force applied to the cart. Formulate a performance index to keep the pendulum in the vertical position with as little energy as possible.

Problem 5

(a) Consider the following function of two variables:

$$f(x, y) = x^2 + y^2$$

Determine the stationary points and classify them into maxima, minima and saddles.

(b) Consider the following function of two variables:

$$f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$$

Determine the stationary points and classify them into maxima, minima and saddles.

Problem 6

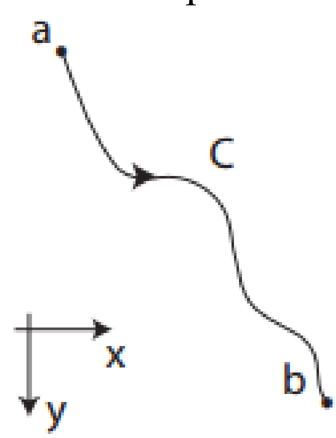
In this problem, you will show that the shortest distance between two points is a straight line connecting the two points. Let $P_1=(x_1, y_1)$ and $P_2=(x_2, y_2)$ be two points on a 2D plane. Find the third point $P_3=(x_3, y_3)$, such that $d_1=d_2$, where d_1 is the distance from P_3 to P_1 and d_2 is the distance from P_3 to P_2.

Problem 7

You work in a box-making factory and you are the lead box designer. Your boss asks you to design a rectangular box with an open top that uses the least amount of metal and the box must have a capacity of 10 in^3. Assume that the length, width, and height of the box are: x, y, and z, respectively. Determine the dimensions that meet these specs!

Problem 8

Given two points a and b in a vertical plane as shown below:



Assume that a metal wire connects points a and b with some shape. Next, let a bead start at point a. What shape should the wire take so that the bead traverses from a to b under gravity in the shortest time? Assume there's no friction.