Aerospace Propulsion

Lecture 19

Rocket Propulsion II

IMAGINE

Rocket Propulsion: Part II

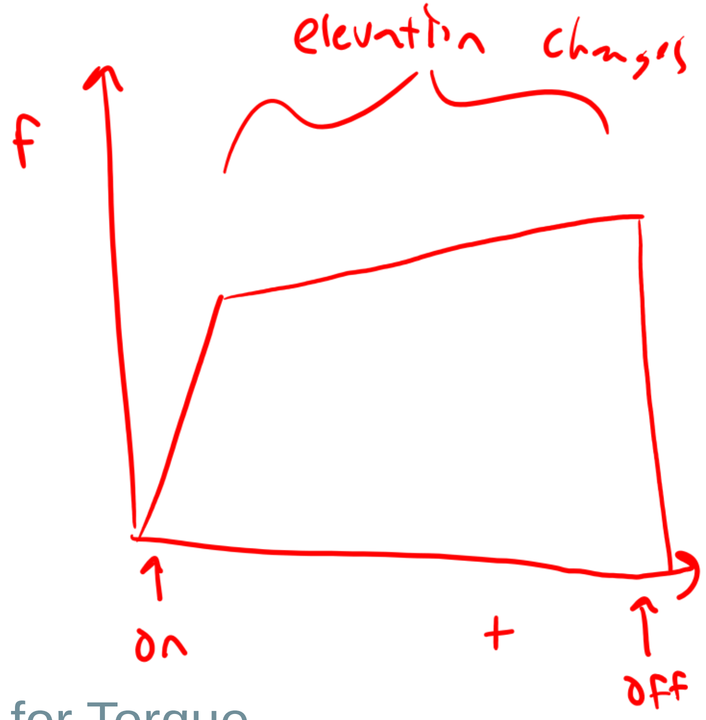
- Definitions
- Rocket Thrust
- Basic Rocket Analysis

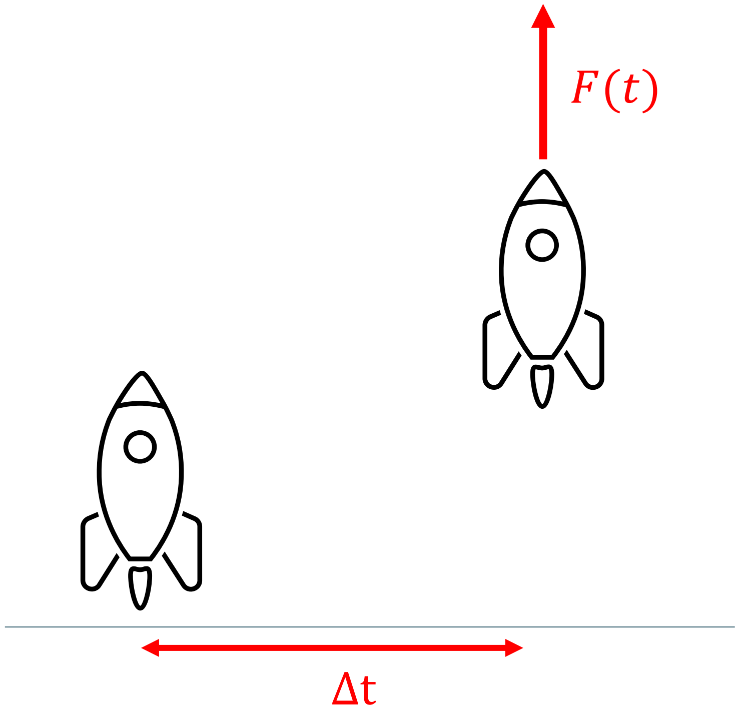
Definitions

- Major differences between air-breathing and rocket engines
  - Rockets keep all propellants on board
  - Rockets directly overcome gravity with thrust (low/no lift forces)
    - Every little bit of propellant is important

- Rocket capabilities are often measured in terms of how much total energy can be extracted from propellants

Definitions





- Total impulse
  - $I_t = \int_0^t F dt$
  - Thrust force F
    - May vary in time
    - T typically reserved for Torque
  - Time t
- For constant thrust with instant on/off
  - $I_t = F\Delta t$
- Proportional to total energy released by the propellant

Definitions

$I_s = \frac{\int_0^t F\,dt}{g_0 \int_0^t \dot{m}\,dt}$

Assume constant thrust

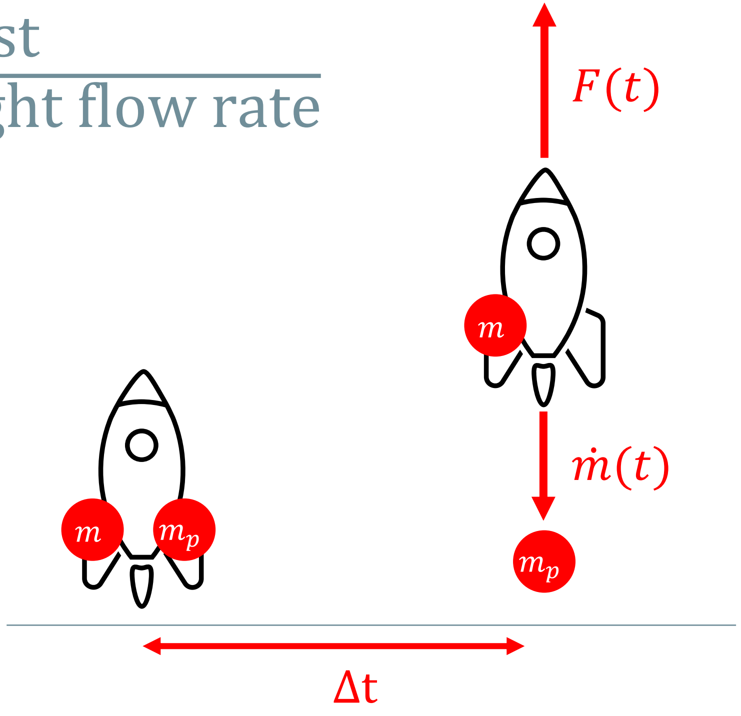
Define $\int_0^t \dot{m}\,dt = m_p$

$\rightarrow I_s = \frac{F\Delta t}{g_0 m_p} = \frac{I_t}{g_0 m_p}$

- Specific impulse
  - $I_s = \frac{\int_0^t F dt}{g_0 \int_0^t \dot{m} dt} = \frac{\text{Total impulse}}{\text{Propellant weight}} = \frac{\text{Thrust}}{\text{Propellant weight flow rate}}$

I_{sp}

  - Nominal earth gravity g_0
    - $g_0 = 9.8066$ m/s^2
  - Propellant mass flow rate $\dot{m}$
- Average value (useful over small Δt)
  - $I_s = \frac{I_t}{m_p g_0}$
    - Mass exhausted m_p
- Important/common way to describe rockets



Definitions

Low TSFC = good
High I_s = good

- Quick note – have we seen I_s before?
  - $I_s = \frac{\int_0^t F dt}{g_0 \int_0^t \dot{m} dt} \approx \frac{F}{\dot{m} g_0} \propto \frac{F}{\dot{m}}$
  - $TSFC = \frac{\dot{m}}{F}$ (air breathing propulsion)
  - $I_s \propto \frac{1}{TSFC}$
  - Different fields use different quantities
    - TSFC is historically <u>not</u> used in the field of rockets

$I_s = \frac{\int_0^t F dt}{g_0 \int_0^t \dot{m}\, dt}$

Assume, F, $\dot{m}$

$I_s = \frac{F \Delta t}{g_0 \dot{m} \Delta t}$

$= \frac{F}{\dot{m} \underbrace{g_0}_{\text{constant}}}$

Definitions

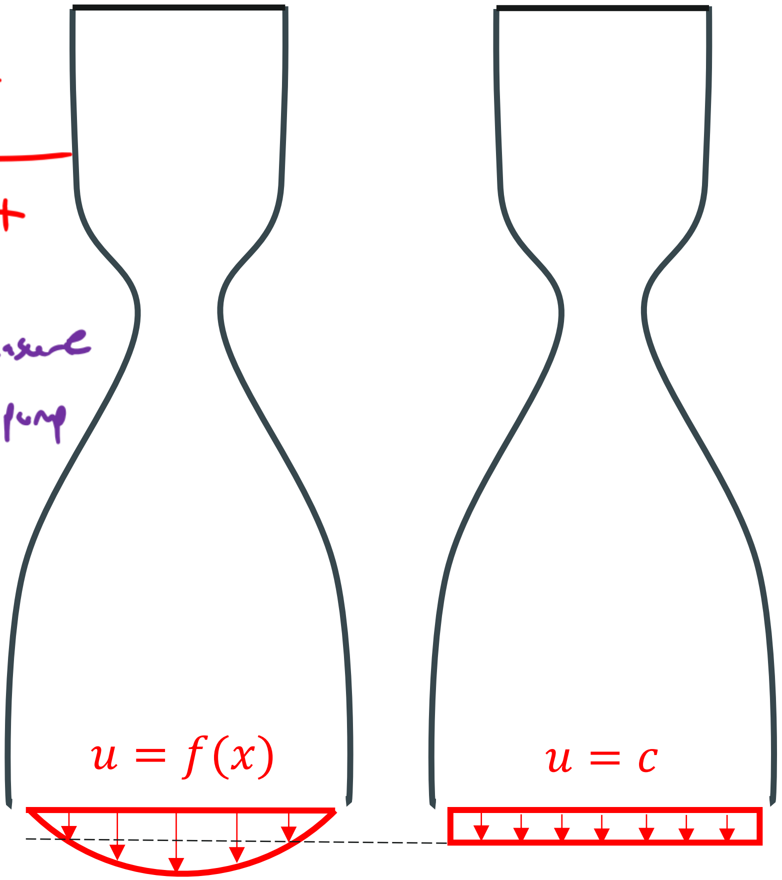
force sensor

Solid: $I_s = \dfrac{F \Delta t}{g_0 m_p}$ clock, scale

Liquid: $I_s = \dfrac{\int_0^t F\,dt}{g_0 \int_0^t \dot{m}\,dt}$

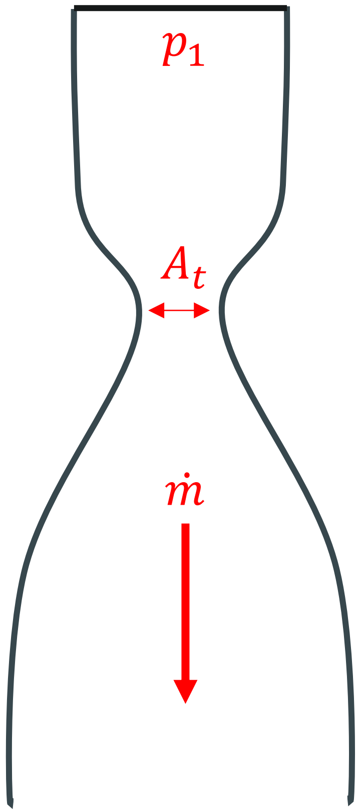
Can measure through pump

- Effective Exhaust Velocity
  - $c = I_s g_0 = F/\dot{m} (= V_e)$
  - Mean velocity leaving rocket nozzle
  - Note that this is basically a constant multiplied by specific impulse
    - Varies between source, I_s more common
- Measuring I_s and c for ground tests
  - Solid motor
    - Tough to measure flow rate, use m_p
  - Liquid Engine
    - Can measure flow rate (pumps), use $\dot{m}$

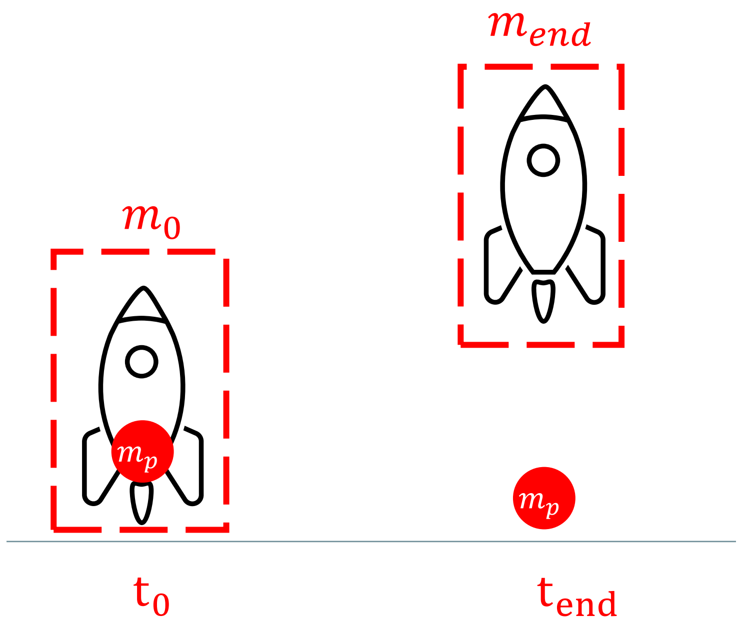


Definitions

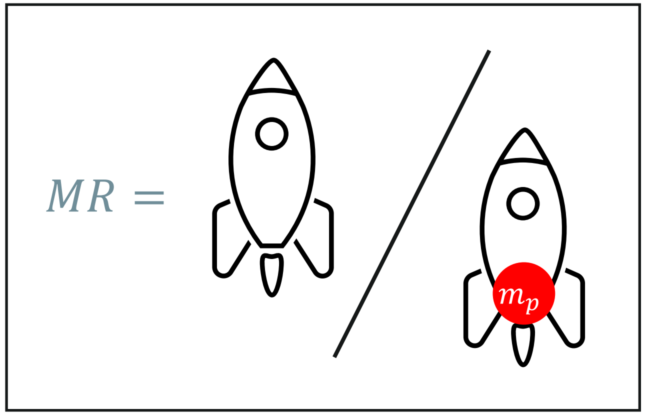
- Characteristic Velocity
  - $c^* = \frac{p_1 A_t}{\dot{m}}$
  - Both I_s and c are measured based on properties that come *after* the nozzle
    - Both quantities are measures of overall rocket efficiency
  - The characteristic velocity is a measure of the rocket's efficiency <u>excluding the nozzle</u>
    - Related to efficiency of combustion process
  - Note that the characteristic velocity is <u>not</u> related to any physical velocity, but has dimensions of velocity



Definitions



- Mass Ratio
  - $MR = \frac{m_{end}}{m_0}$
  - Final vehicle mass m_{end}
    - Often m_f is used, this is <u>not</u> fuel mass
  - Initial vehicle mass m_0
  - Represents how much of the rocket is <u>not</u> propellant
  - $m_0 = m_{end} + m_p$
    - Assuming all mass lost was propellant



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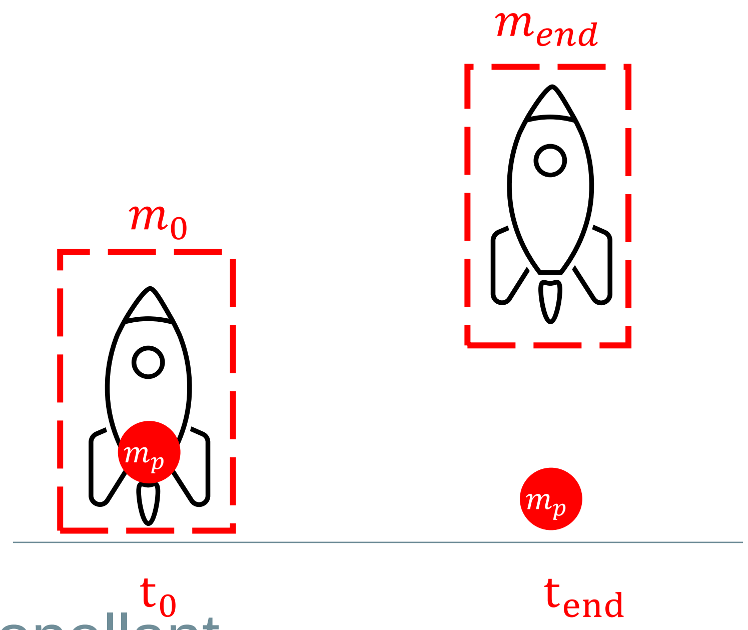
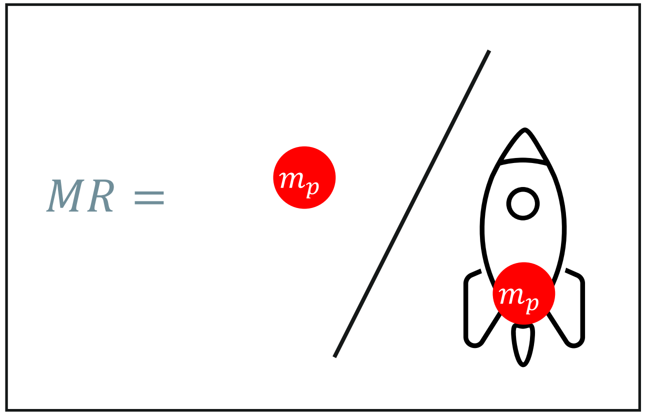


- Propellant Mass Fraction
  - $\zeta = \frac{m_p}{m_0} = \frac{m_0 - m_{end}}{m_0}$
  - Represents how much of the rocket is propellant

$$\zeta = \frac{M_0 - M_{end}}{M_0} = 1 - \frac{M_{end}}{M_0} = 1 - MR$$



Definitions

- Impulse to Weight Ratio $\frac{I_t}{w_0}$
  - Total impulse divided by total sea-level weight w_0
  - Assuming constant thrust and instant on/off
    - $\frac{I_t}{w_0} = \frac{I_t}{(m_{end}+m_p)g_0} = \frac{I_s}{m_{end}/m_p+1}$

$$\frac{I_t}{W_0} = \frac{I_t}{M_{total}\, g_0} = \frac{I_t}{(M_{end}+M_p)\, g_0} = \frac{I_s M_p g_0}{(M_{end}+M_p)\, g_0} = \frac{I_s}{\left(\frac{M_{end}}{M_p}\right)+1}$$

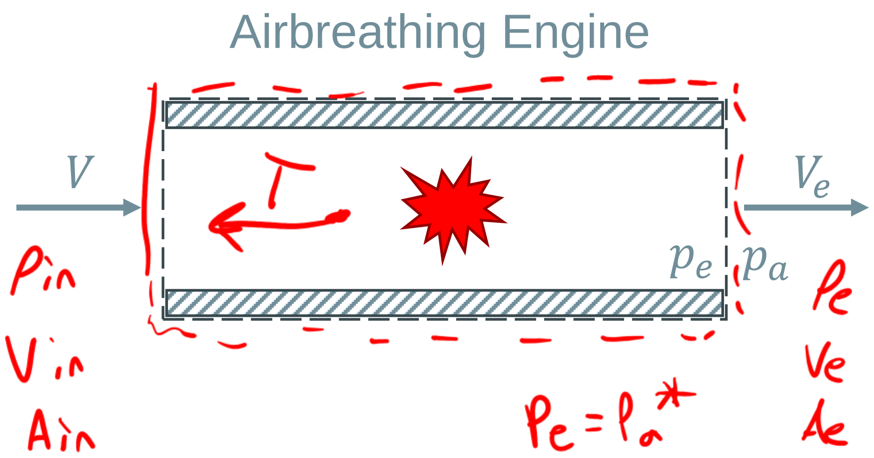
- Thrust to Weight Ratio $\frac{F}{w_0}$
  - Acceleration that an engine can apply to itself (in g_0's)

Rocket Thrust

- For airbreathing propulsion, we derived
  - $T = \dot{m}(V_e - V)$

- Airbreathing propulsion assumed:
  - Flow entering and exiting
  - Outlet pressure = ambient pressure
    - Not necessarily true for supersonic exhaust
    - Neglected over/under-expanded outlets

- These assumptions do not hold for rockets!



$$\iint \rho \vec{V}(\vec{V}\cdot\hat{n})\,dA = \sum F$$

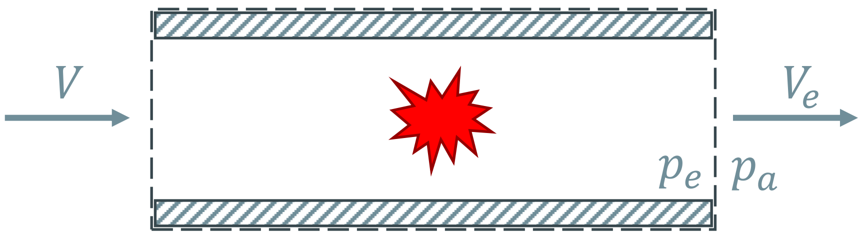
$$\rho_e V_e^2 A_e - \rho_{in} V_{in}^2 A_{in} = T$$

$$T = \dot{m}(V_e - V_{in})$$

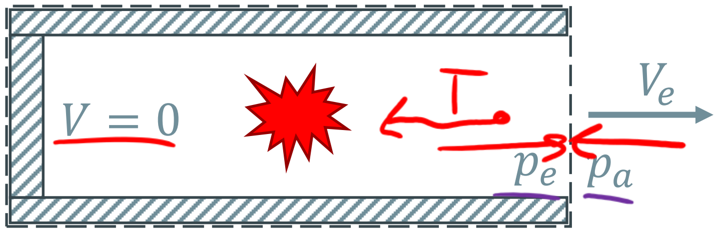
velocity of aircraft

Rocket Thrust

Airbreathing Engine



Rocket Engine



- For rocket propulsion:
  - Inlet velocity $V = 0$
  - $p_e \neq p_a$
    - Extreme case of vacuum ($p_a = 0$)
    - *Actually, they are equal at one specific elevation
    - Variable area rocket nozzles are rare
      - Most rockets are expendable
      - Multiple stages each designed for different p_a

$$\iint \rho \vec{V}(\vec{V}\cdot\hat{n})\,dA = \Sigma F$$

$$\rho_e V_e^2 A_e = T + p_a A_e - p_e A_e$$

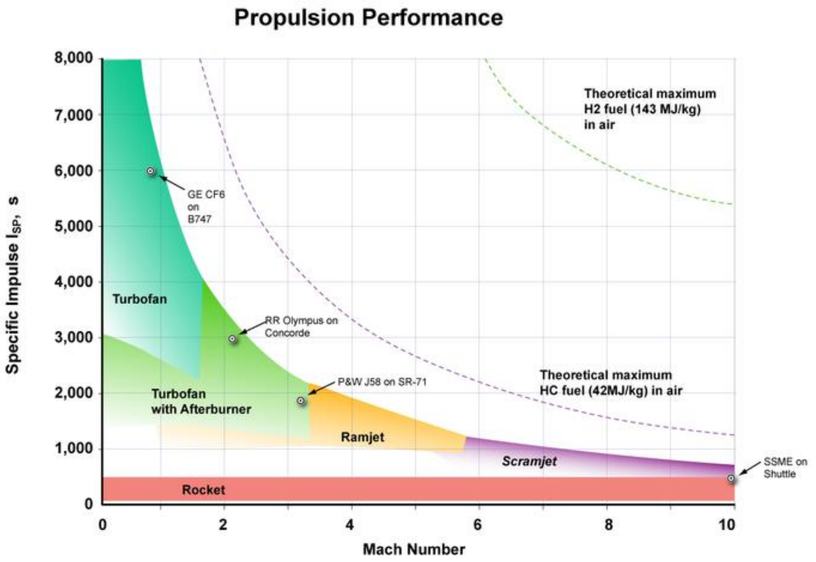
$$T = \dot{m} V_e + (p_e - p_a) A_e$$

- $F = \dot{m} V_e + (p_e - p_a) A_e$

Rocket Thrust

- $F = \dot{m}V_e + (p_e - p_a)A_e$
- Rocket thrust consists of two terms
  - First term – Momentum thrust
    - Differs from airbreathing propulsion because $V = 0$
  - Second term – Pressure thrust
    - Can give negative thrust
    - Negative thrust avoided by making sure $p_e > p_a$
  - Generally, momentum thrust is larger than pressure thrust
- Note that in a vacuum, $p_a = 0$
  - $F = \dot{m}V_e + p_e A_e$ (max thrust)

Rocket Thrust



- $F = \dot{m}V_e + (p_e - p_a)A_e$
- A few notes about rocket thrust
  - Unlike airbreathing engines, rocket engine thrust is <u>independent</u> of the flight speed of the rocket
  - Both airbreathing and rocket engine thrust is affected by elevation
    - Pressure (and temperature) decreases with elevation
    - Airbreathing engines encounter inlet air with varying properties
      - Thrust <u>decreases</u> with elevation
    - Rocket engines have a varying pressure thrust
      - Thrust <u>increases</u> with elevation
      - Note that specific impulse also increases with elevation

Rocket Thrust

- Power
  - Exhaust Power
    - $P_e = \frac{1}{2}\dot{m}V_e^2 = \frac{1}{2}\dot{w}g_0 I_s^2 = \frac{1}{2}FV_e$
    - Power from exhaust gases
  - Chemical Power
    - $P_c = \dot{m}(LHV)$
    - Same as airbreathing propulsion
  - Vehicle Power
    - $P_v = FV$
    - Power transmitted to vehicle flying at speed V

Assume $P_e = P_a$

$P_e = \frac{1}{2}\dot{m}V_e^2$

$= \frac{1}{2}\dot{m}V_eV_e$

$= \frac{1}{2}FV_e$

$\downarrow$

$\frac{1}{2}(\dot{m}g_0 I_s)V_e$

$\downarrow$

$\frac{1}{2}(\dot{m}g_0 I_s)I_s g_0$

$P_e = \frac{1}{2}\dot{m}g_0 I_s^2 g_0$

$= \frac{1}{2}\dot{w}g_0 I_s^2$

$I_s = \frac{F}{\dot{m}g_0}$

$\Rightarrow F = \dot{m}g_0 I_s$

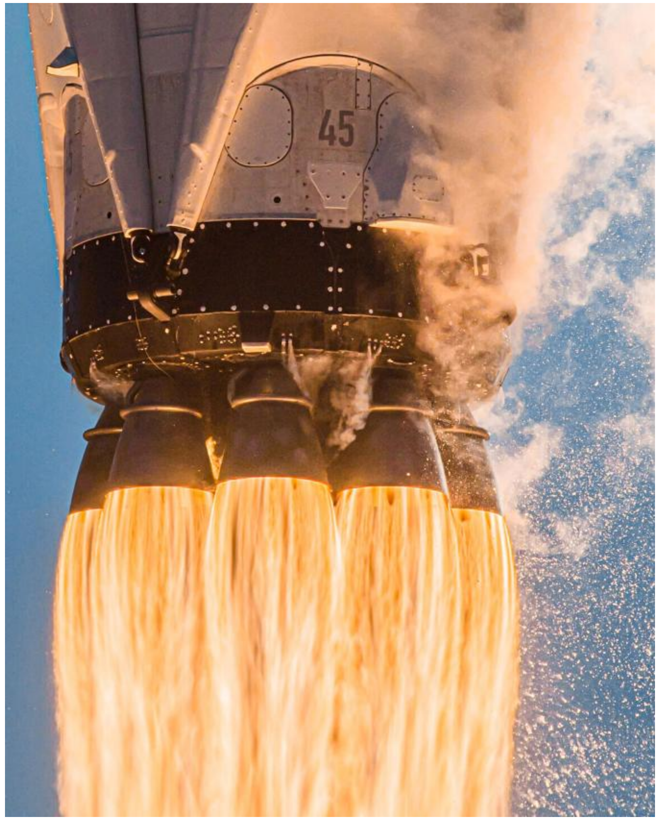
when $P_e = P_a$

$I_s = \frac{V_e}{g_0}$

$\Downarrow$

$V_e = I_s g_0$

Rocket Thrust

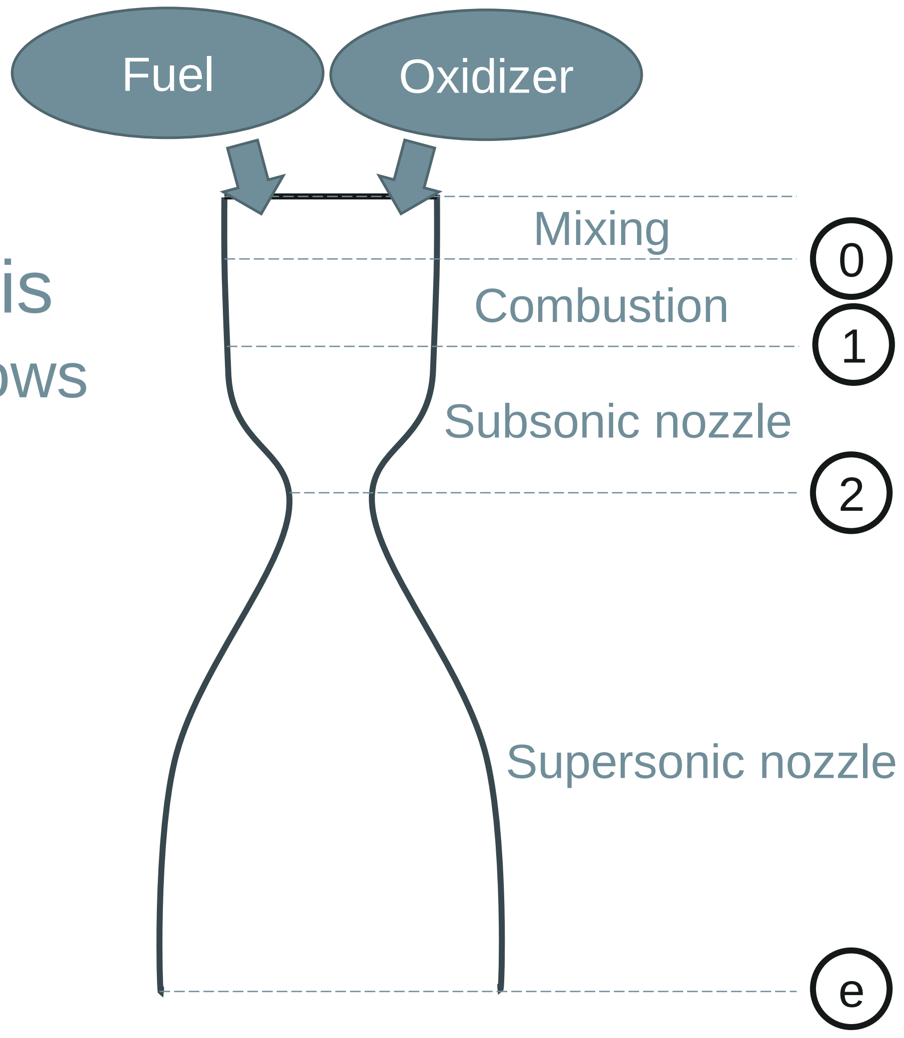


- Multiple propulsion systems
  - Consider multiple simultaneous sources of thrust
    - E.g., Falcon 9 simultaneously uses 9 first stage engines
    - We are not considering staging here
  - $F_{tot} = \sum F_i$
    - Total thrust equal to sum of individual thrusts
  - $\dot{m}_{tot} = \sum \dot{m}_i$
    - Total mass flow rate equal to sum of individual flow rates
  - $(I_s)_{tot} = \dfrac{F_{tot}}{g_0 \dot{m}_{tot}}$
    - Total specific impulse calculated with above two parameters

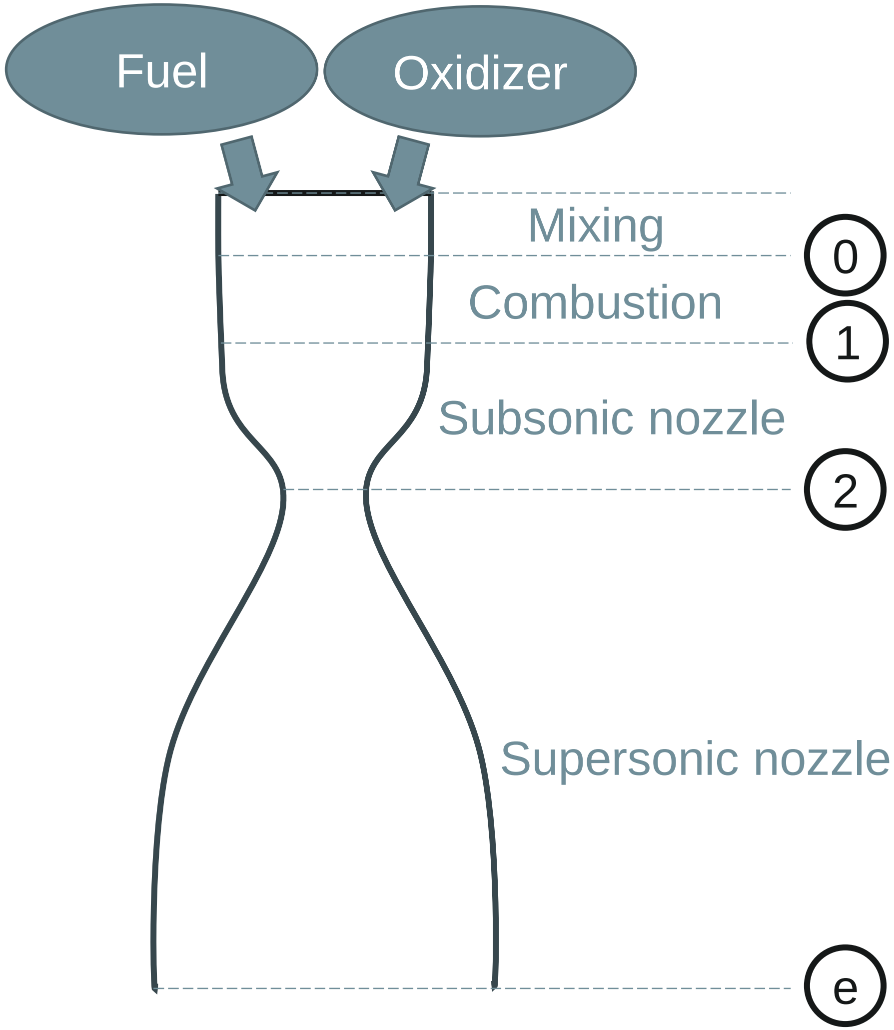
Basic Rocket Analysis

- Major assumptions of the basic analysis
  - Quasi-1D approach from compressible flows
  - Flow throughout the engine is isentropic
  - Propellants are perfectly mixed

- This is even simpler than a ramjet
  - No inlet, propellants already mixed
  - Combustor
  - Isentropic Nozzle



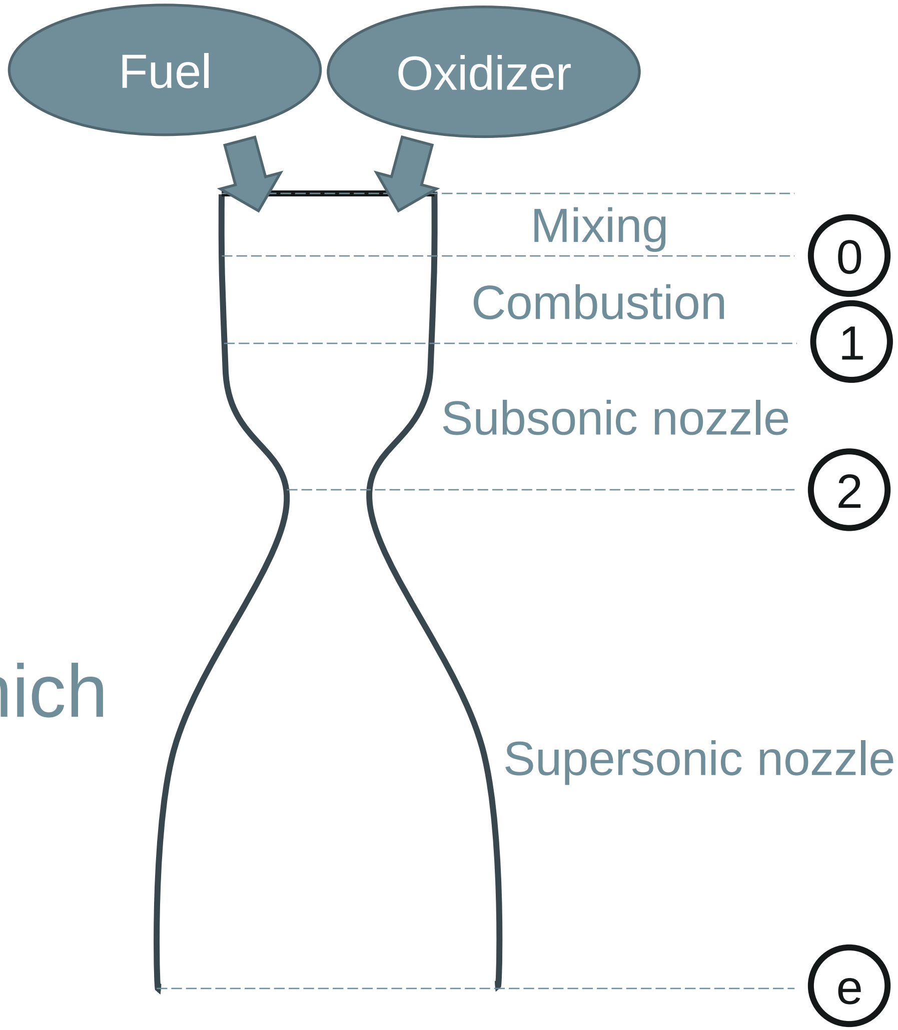
Basic Rocket Analysis



- Start at state 0
  - Assume small initial velocity
  - $T_{t0} = T_0$
  - $p_{t0} = p_0$

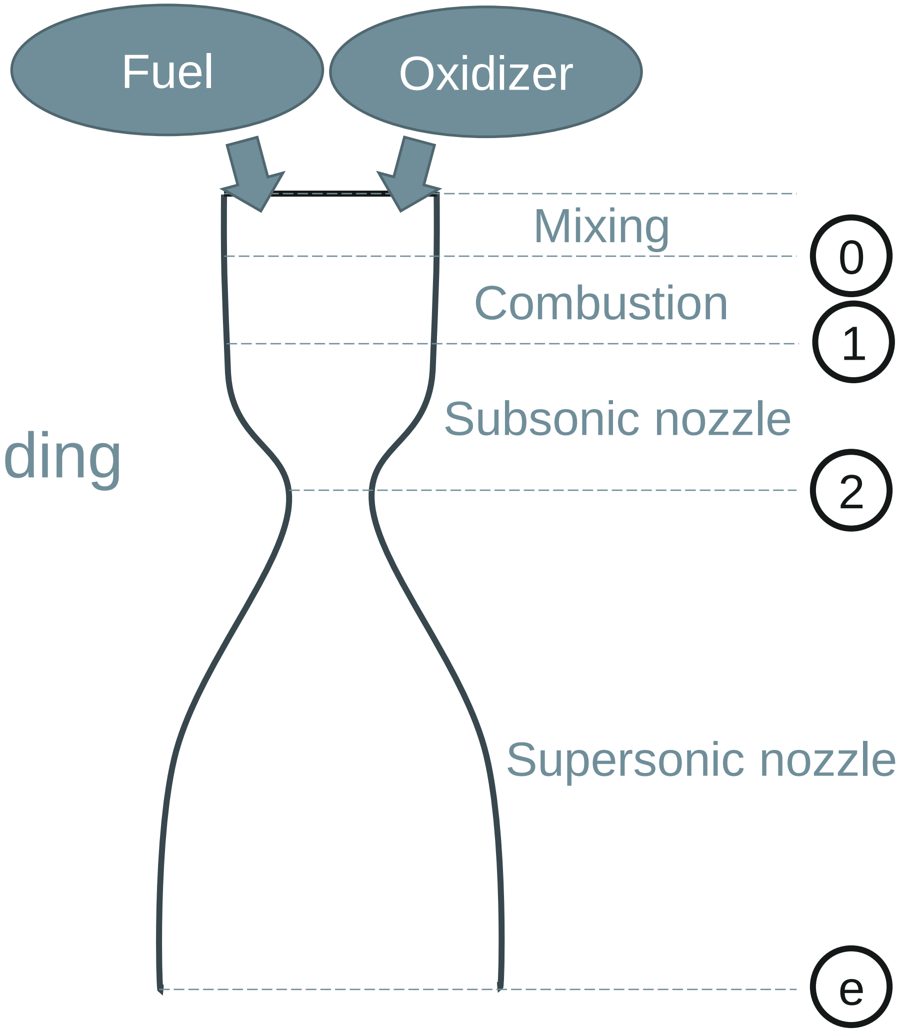
- State 0 to state 1 (combustion)
  - $T_{t1} = T_{t0} + \frac{\phi\left(\frac{F}{A}\right)_{st} LHV}{c_p} = T_{t0} + \frac{\left(\frac{F}{A}\right) LHV}{c_p}$
  - $p_{t1} = p_{t0}$

Basic Rocket Analysis



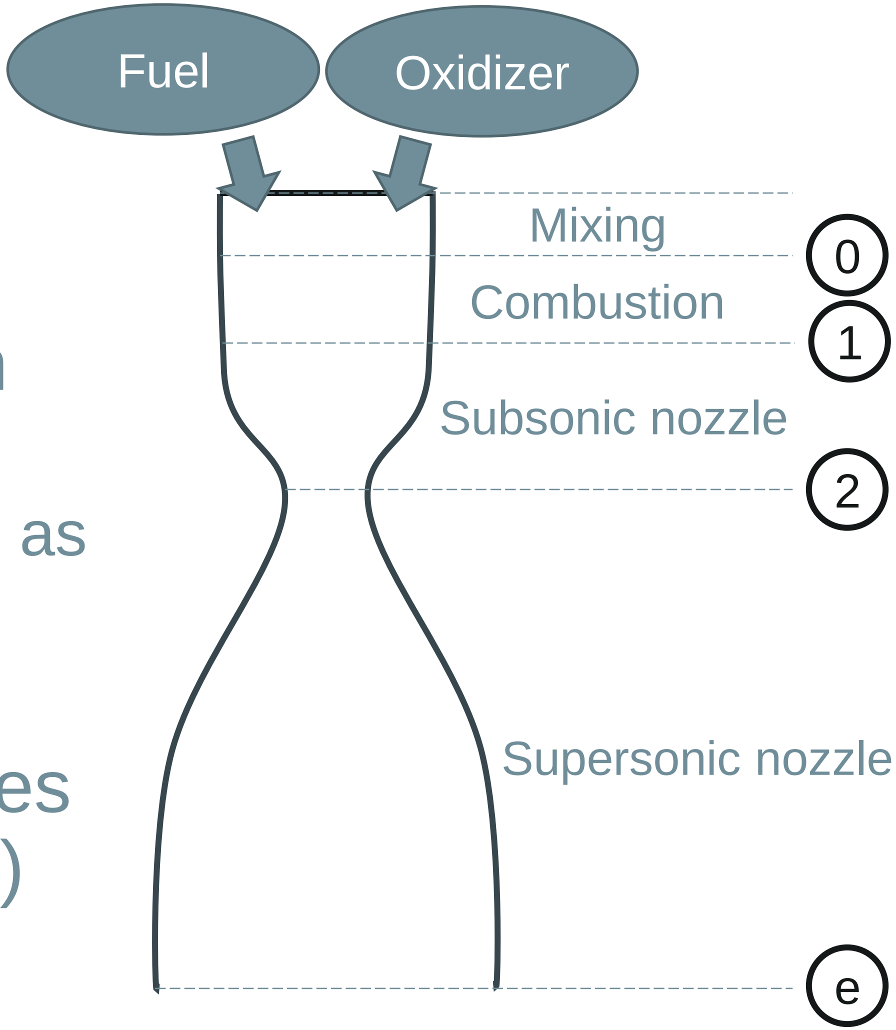
- State 1 to exhaust (isentropic nozzle)
  - $V_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{t1}\left[1-\left(\frac{p_a}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$
- This is valid for isentropic flow only, which assumes that $p_e = p_a$
- Thrust (assuming $p_e = p_a$)
  - $F = \dot{m}V_e$

Basic Rocket Analysis



- What are we expelling?
  - It's no longer mostly air
  - Rocket exhaust varies significantly depending on which propellants are used
- Lower molecular mass increases V_e
  - $V_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{t1}\left[1-\left(\frac{p_a}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$
  - $V_e = \sqrt{2\frac{\gamma}{\gamma-1}\left(\frac{\bar{R}}{\bar{m}}\right)T_{t1}\left[1-\left(\frac{p_a}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

Basic Rocket Analysis



- Note that since this is an isentropic nozzle, all our previous equations from compressible flows are also valid
  - Account for combustion before the nozzle as an increased temperature
- Our equations for non-isentropic nozzles are also valid (considering combustion)
  - Normal shock within nozzle
  - Over/under-expanded nozzles

Basic Rocket Analysis

- Relevant equations refresher

Isentropic Flows (Static)

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}$$

$$\frac{p_2}{p_1} = \left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right)^{\frac{1}{\gamma - 1}}$$

$$\frac{A_2}{A_1} = \frac{M_1}{M_2} \left(\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

Isentropic Flows (Stagnation)

$$T_{t1} = T_{t2}$$
$$p_{t1} = p_{t2}$$

Normal Shocks (Stagnation)

$$T_{t1} = T_{t2}$$

$$\frac{p_{t2}}{p_{t1}} = \left[\frac{\frac{\gamma + 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{1}{\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}} \right]^{\frac{1}{\gamma - 1}}$$

Normal Shocks (Static)

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)\left(\frac{2\gamma}{\gamma - 1} M_1^2 - 1\right)}{\left[\frac{(\gamma + 1)^2}{2(\gamma - 1)}\right] M_1^2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2}$$