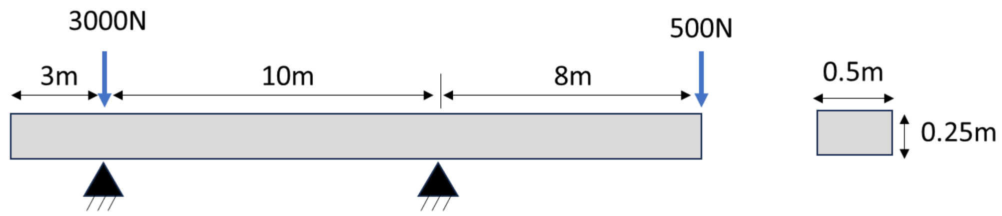
Homework # 5 – Design for Static Strength

Instructions:
Upload your homework as a PDF to Gradescope. Due dates are posted in Gradescope. Please make sure that your uploaded PDF file is readable. Show your work, no credit will be given for the answer only. Include your Matlab code of applicable (with comments).

Problem 1 (10 points)

Find the locations and magnitudes of the maximum tensile bending stress (σ_{max}) due to moment (M) and the maximum shear stress (τ_{max}) due to shear force (V).



Hand in

- Location and magnitudes of σ_{max} due to moment (M) and τ_{max} due to transverse shear

Problem 2 (20 points, 5 points each part)

A ductile material has a yield strength of $S_y = 200\ MPa$. Calculate the implied factor of safety against yielding for each of the failure theories for ductile materials (maximum shear stress, distortion energy), for each of the following cases of plane stress:

a) $\sigma_1 = 120\ MPa, \sigma_2 = 120\ MPa, \sigma_3 = 0$
b) $\sigma_1 = 0, \sigma_2 = -40\ MPa, \sigma_3 = -60\ MPa$
c) $\sigma_1 = 100\ MPa, \sigma_2 = 0, \sigma_3 = 0$
d) $\sigma_1 = 120\ MPa, \sigma_2 = 20\ MPa, \sigma_3 = 0$

(note: values given are all principal stresses)

Hand in

- Calculation of implied safety factors. Show work.

Problem 3 (25 points, 5 points for each of a-c, 10 points for d)

A brittle material has the following properties: $S_{ut} = 300\ MPa$, and $S_{uc} = 600\ MPa$. Calculate the implied factor of safety for the Brittle Coulomb-Mohr failure theory for the following cases of plane stress:

a) $\sigma_x = 150\ MPa, \sigma_y = 150\ MPa$
b) $\sigma_x = 80\ MPa, \tau_{xy} = 40\ MPa$
c) $\sigma_x = 150\ MPa, \sigma_y = -50\ MPa, \tau_{xy} = 50\ MPa$

(note: the values given are not principal stresses)

d) Use some computing platform (i.e., Matlab, Excel, Python, etc.) to plot the contour for the Brittle Mohr-Coulomb theory in the σ_B vs. σ_A plane. Make sure to scale your plot the same in the x- and y-direction and indicate and identify cases a) through c) with black square markers on this plot. Add a grid and axes labels to the plot.

Hand in

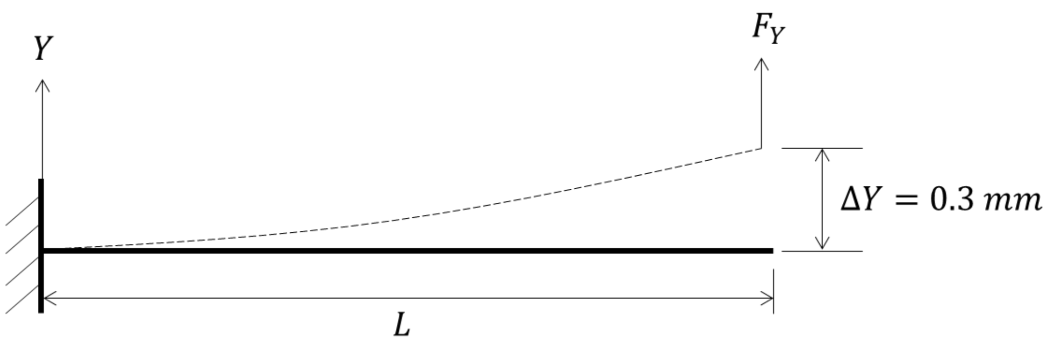
- Calculation of implied safety.
- Graphical representation of both failure criteria in the σ_B vs. σ_A plane. Indicate and identify the cases a) through c) on the graph.
- Your code (i.e., Matlab, python, etc.). If using a spreadsheet, turn in a screen shot of the spreadsheet.

Problem 4 (20 points)

The structure you are designing contains a cantilever beam of rectangular cross-section. The beam will be deflected $\Delta Y = 0.3\ mm$ until it hits a limit stop as shown in the figure below. The length of the beam, L, is $2.5\ mm$. The width of the beam is b and the thickness of the beam is h. b and h are free design variables, but must be kept within the following ranges: $b = [0.1\ mm, 0.3\ mm]$ $h = [0.01\ mm, 0.1\ mm]$. Select the values of b and h that maximize the factor of safety of the beam for a deflection of $\Delta Y = 0.3\ mm$. What is the maximum factor of safety?

The beam is made of polysilicon (a brittle material) and has the following properties: ($E = 160\ GPa, S_{ut} = 1.4\ GPa, S_{uc} = 20\ GPa$). Use the Brittle Coulomb-Mohr failure theory.

(Hint: F_Y could be any value. It just must deflect the beam $\Delta Y = 0.3\ mm$. The stiffness of a cantilever beam is defined by the ratio of tip displacement to end load (i.e. $k = \frac{F_Y}{\Delta Y}$) For a cantilever beam, is $k = 3EI/L^3$ where E is Young's modulus, I is the second moment of area, and L is the length of the beam.)



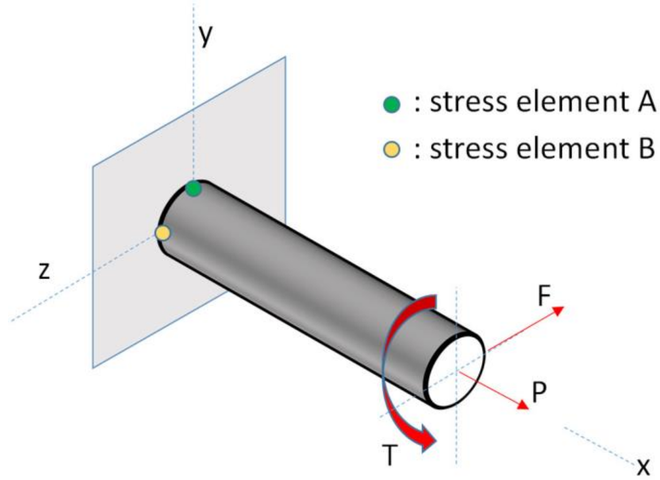
Hand in

- All calculations to find the optimal b, and h and the factor of safety.
- If you use Matlab, Excel or some other tool (you don't have to), turn in your code or a screen shot of the spreadsheet.

Problem 5 (20 points, 10 each part)

Consider a beam made of AISI 1006 Cold-drawn steel as shown in the figure ($S_y = 280\ MPa$). The relevant geometry and loading are:

- Length of beam (l): 100 mm
- Diameter of beam (d): 15 mm
- $F = 0.55kN$
- $P = 4\ kN$
- $T = 25\ Nm$



a) Determine the factor of safety based on the Distortion-energy theory for stress elements A and B.
b) Assume that the length is exact, but the diameter is $d = 15 \pm 1.2\ mm$ where the tolerance range is $\pm 3\sigma$. Considering only the stresses at points A and B, what proportion of parts will have factor of safety less than 1.3?

Hand in

- Calculation of implied safety factors.
- Matlab code if you use Matlab.

Problem 6 (15 points)

The figure below shows a cantilever beam with a circular cross-section of diameter d, made from 1035 hot rolled steel ($S_y = 270\ MPa$). The cantilever beam is end loaded with a force $F = 2000\ N$ resulting in a bending moment, an axial force $P = 15{,}000\ N$, and a torsional moment $M_t = 150\ Nm$ which acts in the yz-plane. The length of the beam is $L = 0.1\ m$. Using the Distortion-energy failure criterion, plot the factor of safety as a function of d in the range $25\ mm \leq d \leq 45\ mm$.

(Hint: Normal stress due to bending will contribute more than transverse shear stress.)

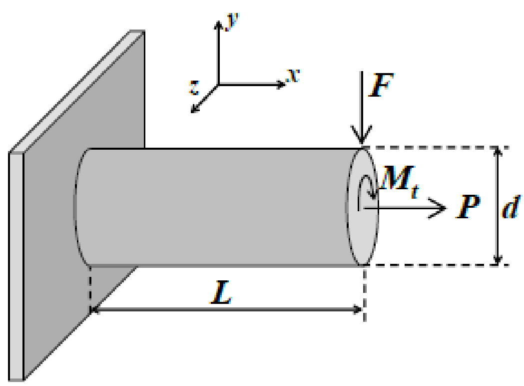


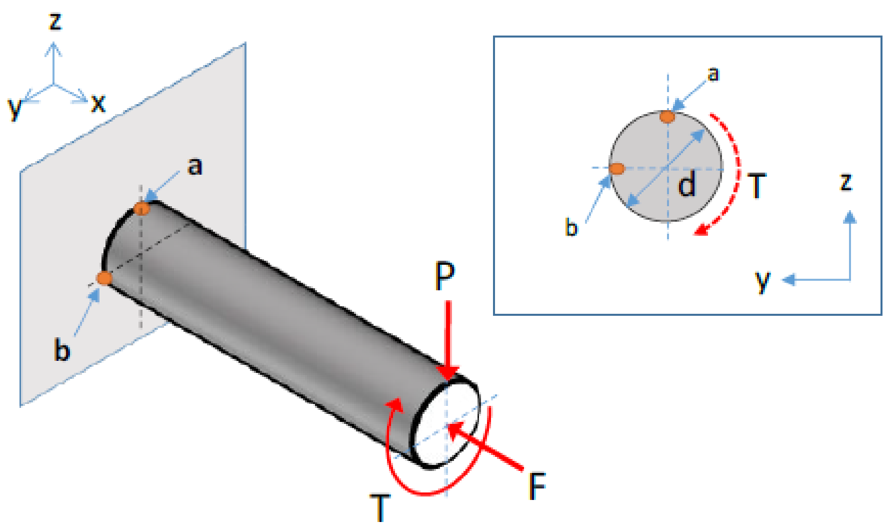
Figure : Cantilever beam

Hand in

- All calculations / equations used to find the relationship between d and the factor of safety.
- A graph showing the factor of safety as a function of d. (Label axes.)
- Any Matlab or python code you used, or a screen shot of your spreadsheet.

<u>Problem 7</u> (20 points, 10 each part)

Consider a solid round bar made of brittle material that has $S_{ut} = 30\ MPa, S_{uc} = 100\ MPa$. The bar is firmly fixed to the wall and is subject to: 1) an axial load of $F = 100\ kN$, 2) a bending load of $P = 5\ kN$, and 3) a torque of $T = 3\ kNm$. The length of the bar is $L = 500\ mm$ and the diameter is $d = 100\ mm$.



a) Determine the factor of safety at point "a" using Brittle Coulomb-Mohr theory.
b) Determine the factor of safety at point "b" using the Modified Mohr theory.

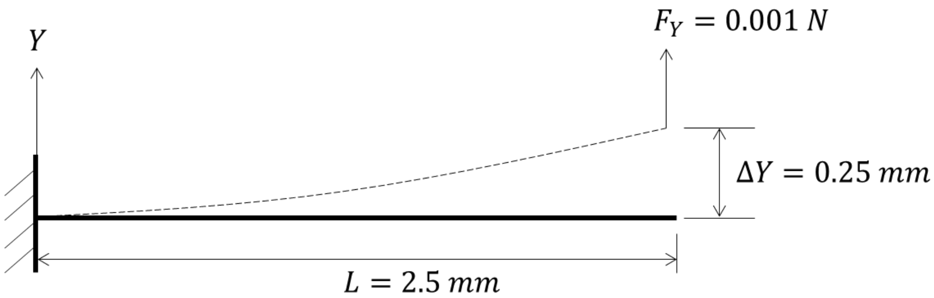
<u>Hand in</u>

- All factor of safety calculations.

Extra Credit Problem (15 points)

The structure you are designing contains a cantilever beam of rectangular cross-section similar to Problem 4, but there is no limit stop. Under the load $F = 0.001\ N$, the free end needs to deflect $0.25\ mm$. b and h are free design variables, but must be kept within the following ranges: $b = [0.1\ mm, 0.3\ mm]$ $h = [0.01\ mm, 0.1\ mm]$. Select the values of b and h that maximize the safety of factor of the beam.

The beam is made of polysilicon (a brittle material) and has the following properties: ($E = 160\ GPa, S_{ut} = 1.4\ GPa, S_{uc} = 20\ GPa$). Use the Brittle Coulomb-Mohr failure theory.



Hand in

- All calculations to find the optimal b, and h and the factor of safety.
- If you use Matlab, Excel or some other tool, turn in your code or a screen shot of the spreadsheet.