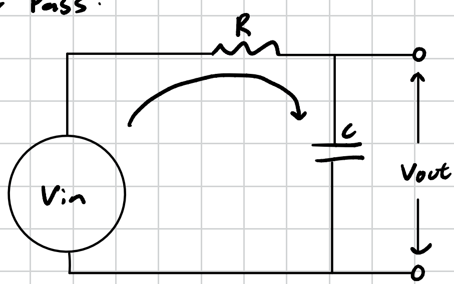
3. Pre-Lab Exercises

1. In lecture, we showed how to derive the transfer function of an RC circuit using impedances. Use this method to derive equations (5) and (6).

Low Pass:
$$\frac{V_{out}}{V_{in}} = \frac{1}{1+j(\omega/\omega_c)} = \frac{1}{1+j(f/f_c)} \tag{5}$$

High Pass:
$$\frac{V_{out}}{V_{in}} = \frac{j(\omega/\omega_c)}{1+j(\omega/\omega_c)} = \frac{j(f/f_c)}{1+j(f/f_c)} \tag{6}$$

Low Pass:



Total impedance: $R + \frac{1}{jwc}$

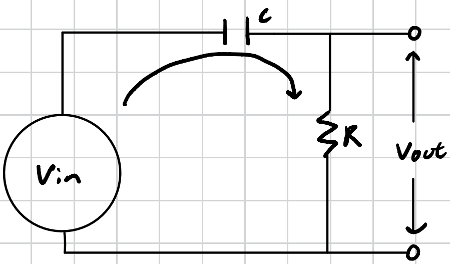
$i = \frac{V_{in}}{Z} = \frac{V_{in}}{R+\frac{1}{jwc}}$

$V_{out} = i Z_c = \frac{V_{in}}{R+\frac{1}{jwc}}\left(\frac{1}{jwc}\right)$

$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{jwc}}{R+\frac{1}{jwc}} = \frac{1}{1+jwcR}$

$\frac{1}{RC} = w_c \therefore \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1+j\left(\frac{w}{w_c}\right)} = \frac{1}{1+j\left(\frac{f}{f_c}\right)}}$

High Pass



Total impedance: $R + \frac{1}{jwc}$

$i = \frac{V_{in}}{Z} = \frac{V_{in}}{R+\frac{1}{jwc}}$

$V_{out} = i R_c = \frac{V_{in}}{R+\frac{1}{jwc}}(R)$

$\frac{V_{out}}{V_{in}} = \frac{R}{R+\frac{1}{jwc}} = \frac{Rjwc}{Rjwc+1}$

$\frac{1}{RC} = w_c \therefore \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{j\left(\frac{w}{w_c}\right)}{j\left(\frac{w}{w_c}\right)+1} = \frac{j\left(\frac{f}{f_c}\right)}{1+j\left(\frac{f}{f_c}\right)}}$

2. Show how equations (8) and (9) can be derived from equations (5) and (6). Hint: a complex number $a + bj$ has magnitude $\sqrt{a^2+b^2}$ and phase angle of $\tan^{-1}\left(\frac{b}{a}\right)$.

Low Pass: $$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(f/f_c)^2}}, \qquad \phi = -\tan^{-1}\left(\frac{f}{f_c}\right) \tag{8}$$

High Pass: $$\left|\frac{V_{out}}{V_{in}}\right| = \frac{f/f_c}{\sqrt{1+(f/f_c)^2}}, \qquad \phi = \tan^{-1}\left(\frac{f_c}{f}\right) \tag{9}$$

Low Pass:

(5) $\frac{V_{out}}{V_{in}} = \frac{1}{1+j\left(\frac{f}{f_c}\right)}$

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| = \frac{\sqrt{1^2}}{\sqrt{1^2+\left(j\left(\frac{f}{f_c}\right)\right)^2}} = \frac{1}{\sqrt{1+(f/f_c)^2}}}$$

$\phi = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$ numerator $- \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$ denominator

$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{(f/f_c)}{1}\right)$

$\phi = 0 - \tan^{-1}\left(\frac{f}{f_c}\right)$

$$\boxed{\phi = -\tan^{-1}\left(\frac{f}{f_c}\right)}$$

High Pass:

(6) $\frac{V_{out}}{V_{in}} = \frac{j(f/f_c)}{1+j(f/f_c)}$

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| = \frac{\sqrt{(j(f/f_c))^2}}{\sqrt{1^2+(j(f/f_c))^2}} = \frac{f/f_c}{\sqrt{1+(f/f_c)^2}}}$$

$\frac{j\left(\frac{f}{f_c}\right)}{1+j\left(\frac{f}{f_c}\right)}\left(\frac{1-j\left(\frac{f}{f_c}\right)}{1-j\left(\frac{f}{f_c}\right)}\right) = \frac{j\left(\frac{f}{f_c}\right)+\left(\frac{f}{f_c}\right)^2}{1+\left(\frac{f}{f_c}\right)^2}$

$\phi = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$ numerator $- \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$ denominator

$\phi = \tan^{-1}\left(f/f_c \Big/ \left(\frac{f}{f_c}\right)^2\right) - \tan^{-1}\left(\frac{0}{1}\right)$

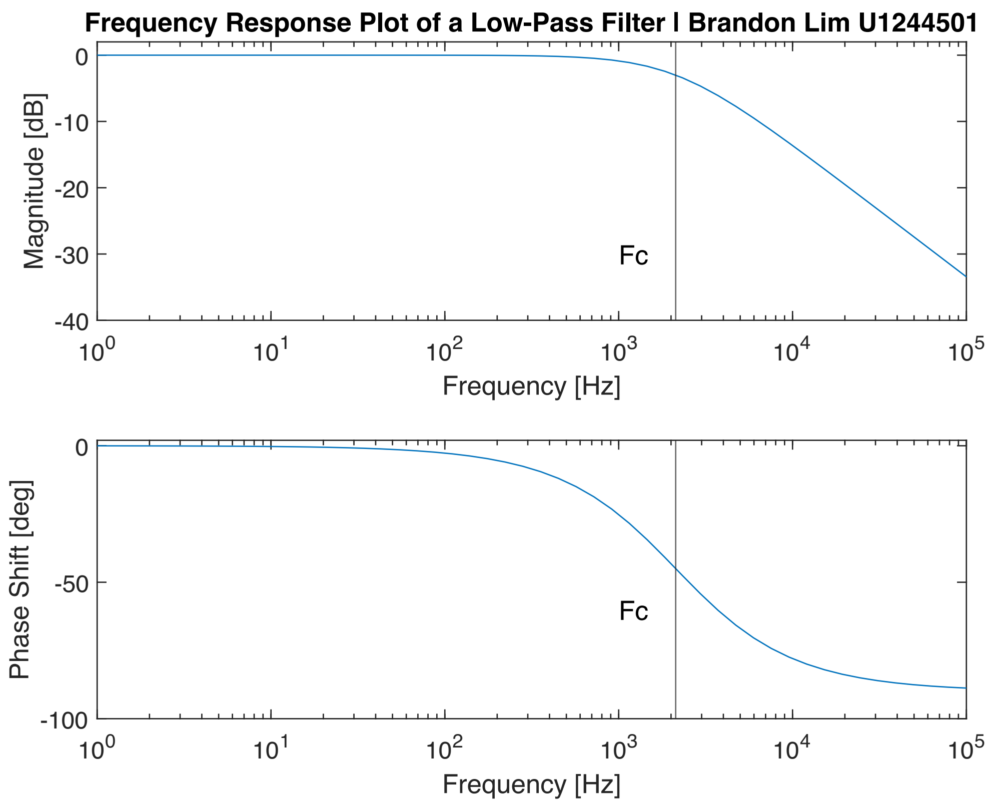
$\phi = \tan^{-1}\left(\frac{f}{f_c}\cdot\frac{f_c^2}{f^2}\right) = \tan^{-1}\left(\frac{f_c}{f}\right)$

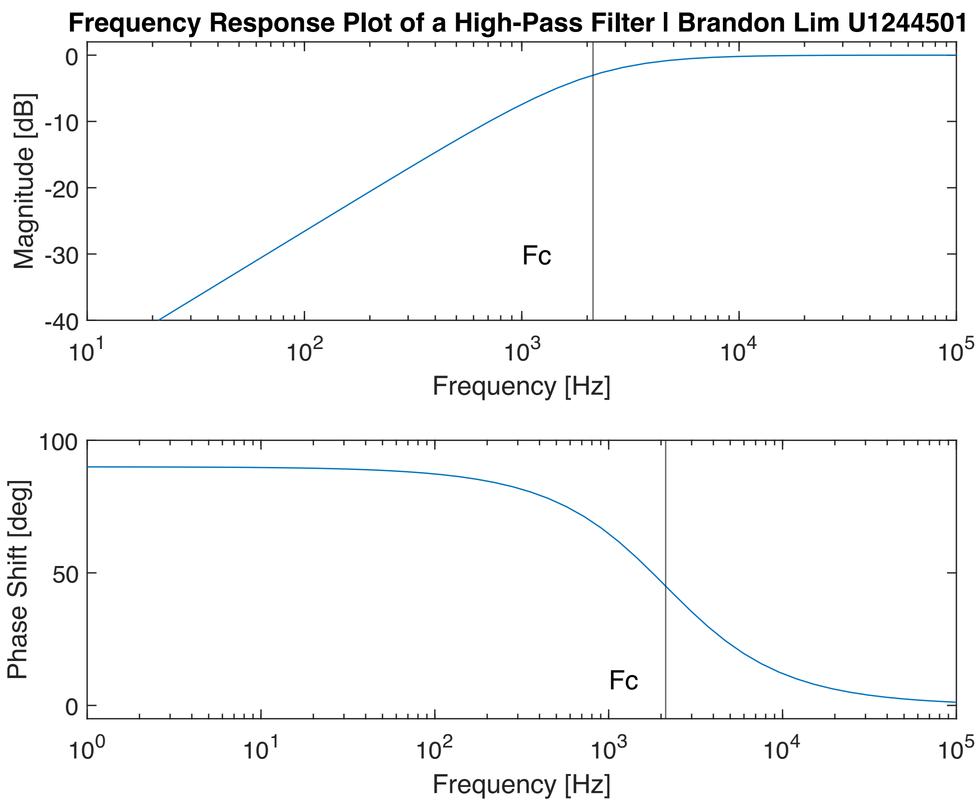
$$\boxed{\phi = \tan^{-1}\left(\frac{f_c}{f}\right)}$$

3. Using equations (8), (9) and (10), plot the frequency response of a low and high pass filter using the parameters indicated below. Calculate the Magnitude (dB) and phase (deg) for a range of logarithmically-spaced frequencies spanning 2 orders of magnitude above and below cut-off frequency (hint: use *logspace* function in MATLAB). Be sure to plot the frequencies on a log scale (i.e. *semilogx* in MATLAB) and properly label your axes. Also label the cut-off frequency on your plot. Include screenshots of your code and plots in line with this question with your name indicated. Also submit your .m files to canvas.

   a. Low-Pass filter with R = 4.7 kΩ and C = 0.1 μF

   b. High-Pass filter with R = 4.7 kΩ and C = 0.1 μF


Frequency Response Plot of a Low-Pass Filter | Brandon Lim U1244501
Magnitude [dB]
0
-10
-20
-30
-40
Fc
10^0
10^1
10^2
10^3
10^4
10^5
Frequency [Hz]
Phase Shift [deg]
0
-50
-100
Fc
10^0
10^1
10^2
10^3
10^4
10^5
Frequency [Hz]



Frequency Response Plot of a High-Pass Filter I Brandon Lim U1244501
Magnitude [dB]
0
-10
-20
-30
-40
Fc
10^1
10^2
10^3
10^4
10^5
Frequency [Hz]
Phase Shift [deg]
100
50
0
Fc
10^0
10^1
10^2
10^3
10^4
10^5
Frequency [Hz]

```
% Brandon Lim
clear, clc, close all
%Low Pass Filter
w = logspace(0,5);
R = 4.7*1000; %ohms
C = 0.1 * 10^-6; %farads

wc = 1/(R*C); %Cutoff Frequency
magnitudeNorm = 1./sqrt(1+((w.^2)/(wc.^2)));
magnitudeDB = 20*log10(magnitudeNorm);

phaseShift = -atan(w./wc) .* 180./pi;

figure
subplot(2,1,1)
semilogx(w,magnitudeDB)
ylim([-40,2])
title("Frequency Response Plot of a Low-Pass Filter | Brandon Lim U1244501")
xlabel("Frequency [Hz]")
ylabel("Magnitude [dB]")
hold on
subplot(2,1,1)
xline(wc)
text(10^3,-30,0,"Fc")

subplot(2,1,2)
semilogx(w,phaseShift)
ylim([-100,2])
xlabel("Frequency [Hz]")
ylabel("Phase Shift [deg]")
hold on
subplot(2,1,2)
xline(wc)
text(10^3,-60,0,"Fc")
%% High Pass
clear, clc, close all
w = logspace(0,5);
R = 4.7*1000; %ohms
C = 0.1 * 10^-6; %farads

wc = 1/(R*C); %Cutoff Frequency
magnitudeNorm = (w./wc)./(sqrt(1+((w.^2)/(wc^2))));
magnitudeDB = 20*log10(magnitudeNorm);

phaseShift = atan(wc./w) .* 180./pi;

figure
subplot(2,1,1)
semilogx(w,magnitudeDB)
```

```
ylim([-40,2])
title("Frequency Response Plot of a High-Pass Filter | Brandon Lim U1244501 ")
xlabel("Frequency [Hz]")
ylabel("Magnitude [dB]")
hold on
subplot(2,1,1)
xline(wc)
text(10^3,-30,0,"Fc")

subplot(2,1,2)
semilogx(w,phaseShift)
ylim([-5,100])
xlabel("Frequency [Hz]")
ylabel("Phase Shift [deg]")
hold on
subplot(2,1,2)
xline(wc)
text(10^3,10,"Fc")
```