Aerodynamics Test 1

Fall 2024

- *This test has 3 questions. Answer all of them. Each question has equal value. All parts of a given question have equal value.*
- *Solutions to this test must be prepared individually. Implicit or explicit collaboration with others (whether current or former students in the course or not) is strictly forbidden in accordance with academic code of conduct, which is in effect. This includes any communication of contents of the test before midnight on Friday 9/27/24.*
- *You may use the course text, any course material, and any of your notes in preparing your solution.*
- *You are asked submit a single PDF file containing your answers to all the questions. You are being given an extra 5 minutes on this test (85m minutes instead of 80). Use this time to make sure that your submission is clear and easily legible (including particularly the quality of any scanning). Unclear answers/papers will score zero.*

1. A flow has a velocity field $\boldsymbol{V} = \frac{A}{r^2}\boldsymbol{e_r} + \frac{B}{r}\boldsymbol{e_\theta} + \mathrm{C}\sin(\omega t)\,\boldsymbol{e_z}$ expressed using cylindrical coordinates (as defined in our Vector Algebra class slides). The symbols *A, B,* C and ω denote dimensional constants and *t* is time. Find, as explicit functions of position*,* expressions for
   (a) the time rate of change of velocity seen at a fixed point in space,
   (b) the acceleration of the fluid material,
   (c) the time rate of change of volume of a fluid particle as a fraction of its volume,
   (d) the average angular velocity of the fluid particles.

2. Answer the following:
   (a) The flow inside a pipe, spinning along its axis, takes place entirely in cylindrical shells so that there is no radial velocity component. Write down an explicit expression (in terms of cylindrical coordinates r, θ, z) for a streamfunction that describes one set of streamsurfaces of this flow.
   (b) For the flow in part (a), derive an expression for the non-zero cylindrical velocity components in terms of the derivative of the second streamfunction. You may assume that the flow is compressible and steady.
   (c) In general (and altogether separate from the above problem) which of the assumptions…
      i. constant density flow
      ii. perfect gas
      iii. steady flow
      iv. conservation of mass
      v. conservation of momentum
      vi. Stokes' hypothesis

   …are strictly necessary to infer; (i) the Navier Stokes Equations, (ii) that streamlines are identical to particle paths, (iii) Stokes' theorem. If none write "none".

3. Measurements of a two-dimensional supersonic flow are made using a technique where an intense laser beam is used to pick out fluid lines in a flow and cause them to fluoresce, so that their movement can be visually tracked. The flow is invariant in the z direction. The laser is used to pick out two fluid lines in the air flow AB and CD, at location 1 shown in Fig. 1, parallel to the x and y axes and each 10mm long. Instantaneous tracking of the lines at location 1 shows AB to be initially rotating clockwise at the rate of 4500 rad/s, and CD not to be rotating at all. After the marked fluid lines are allowed to convect a short distance downstream to location 2 they have the configuration shown in Fig. 2 and their lengths have changed as shown. Independent measurements show the density of the air at locations 1 and 2 to be 0.17 kg/m^3 and 0.32 kg/m^2 respectively.

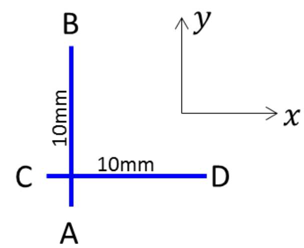

Fig. 1

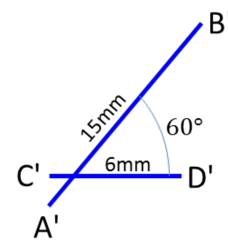

Fig. 2

(a) Is this flow irrotational? If so, justify your answer. If not, estimate the direction and magnitude of the vorticity at location 1.

(b) Subsequent time-resolved measurements reveal that this otherwise steady flow is contaminated by low-level turbulence. Consider turbulent eddies of unit vorticity initially oriented in the x and y directions at location 1. How will these eddies be oriented once they reach location 2, and what will their vorticity be as a percentage of their vorticity at location 1.

(c) State the assumptions you needed to make to answer question (b)