Block diagrams is a graphical representation for signals and transfer functions math!

Block Diagrams

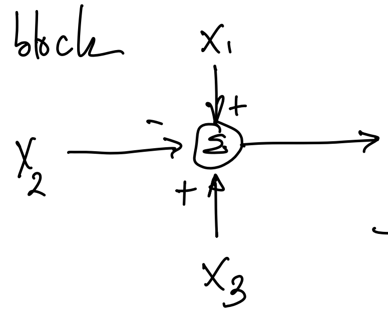
$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

Block diagram Rules

1) Basic block

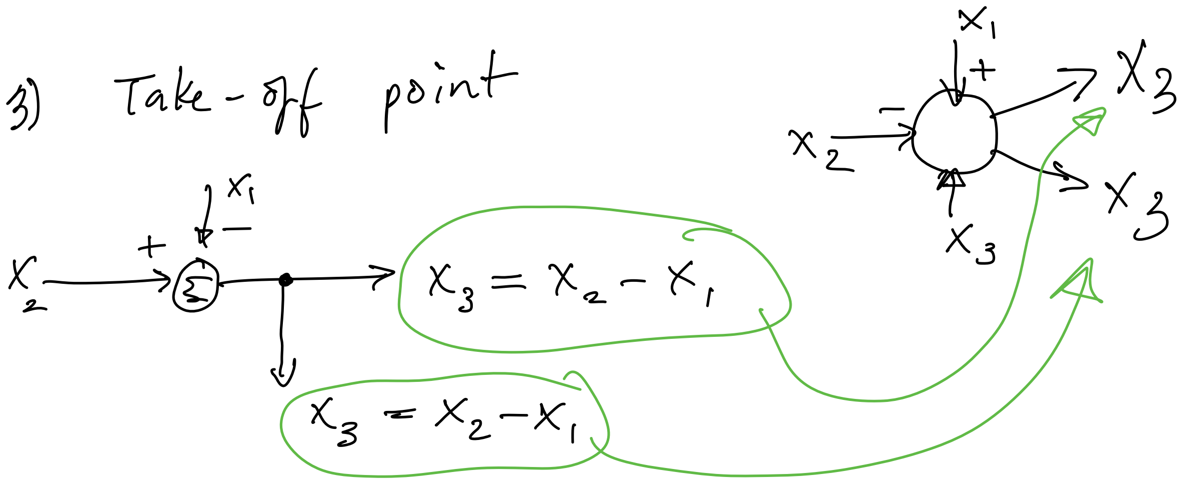
$$\underset{\text{Input}}{U(s)} \longrightarrow \boxed{T(s)} \longrightarrow \underset{\text{output}}{Y(s)} \Rightarrow \frac{Y(s)}{U(s)} = T(s)$$

2) Summing block



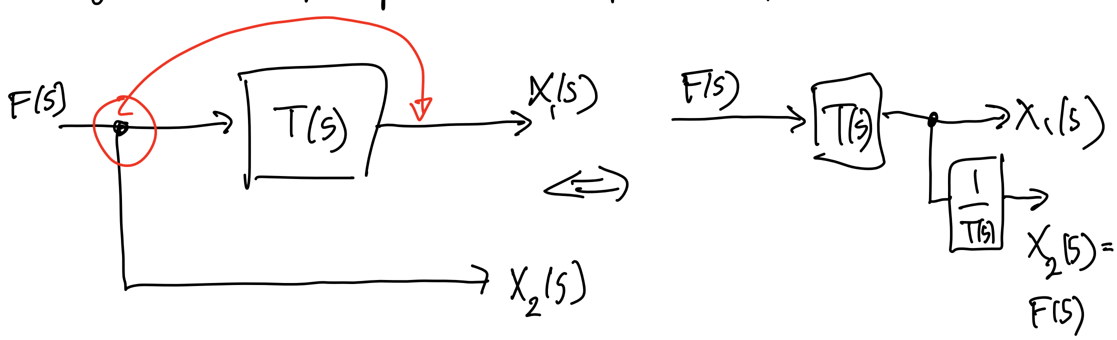
$$\underline{\underline{X_T = X_1 - X_2 + X_3}}$$

3) Take-off point

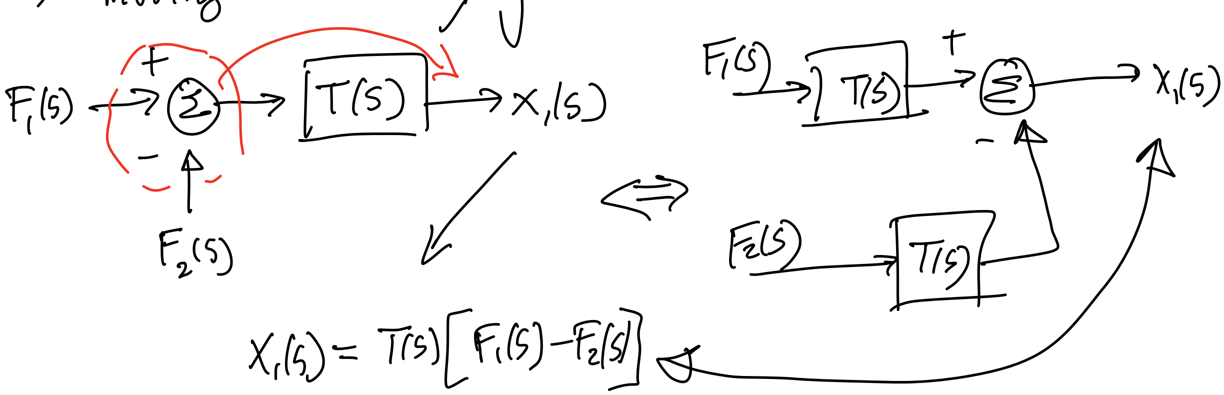


Block diagram Algebra

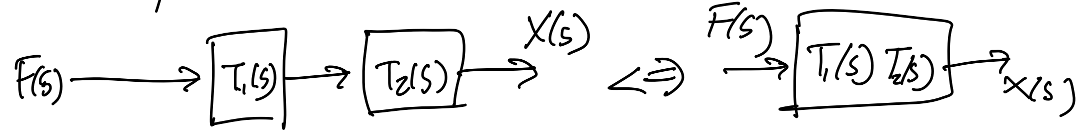
1) moving take-off point in front of a block:



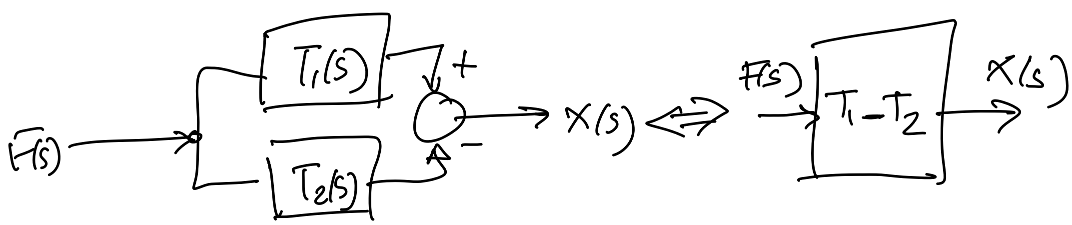
2) Moving summing junction ahead of a block:



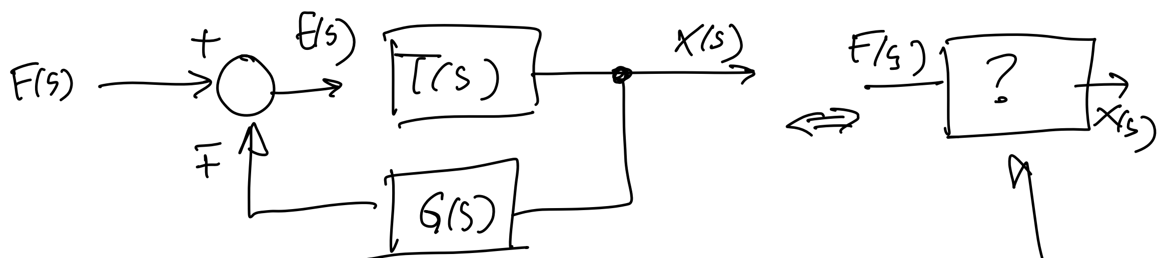
3) Cascading blocks



4) Summing of two blocks:



5) Simple feedback loop



Error E(s)

$E(s) = F(s) \mp G(s)\,X(s)$ (#1)

$X(s) = E(s)\,T(s)$ (#2)

Sub (#1) into (#2)

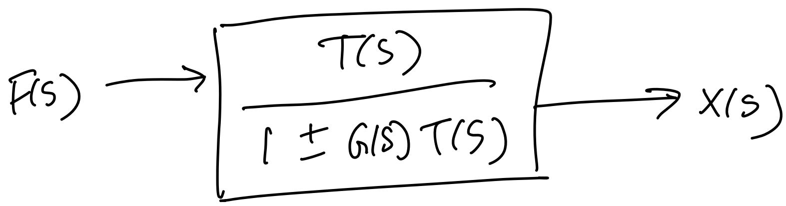
$\Rightarrow X(s) = [F(s) \mp G(s)X(s)]\,T(s)$ $\Rightarrow$ find $\dfrac{X(s)}{F(s)}$

$\Rightarrow X(s) = T(s)F(s) \mp \underbrace{G(s)T(s)X(s)}$

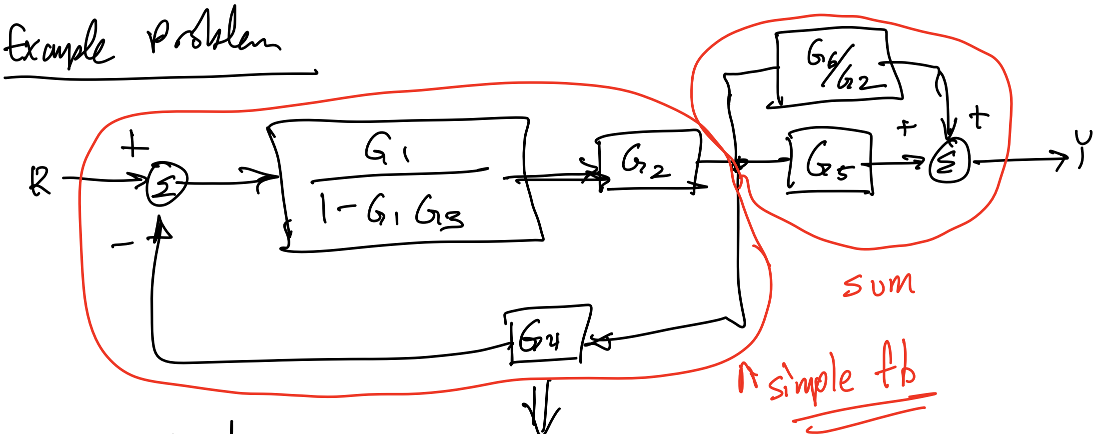
$\Rightarrow X(s) \pm G(s)T(s)X(s) = T(s)\,F(s)$

$X(s)\left[1 \pm G(s)T(s)\right] = T(s)\,F(s)$

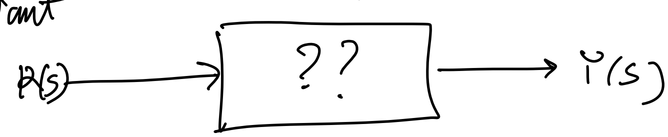
$$\Rightarrow \boxed{\frac{X(s)}{F(s)} = \frac{T(s)}{1 \pm G(s)\,T(s)}}$$

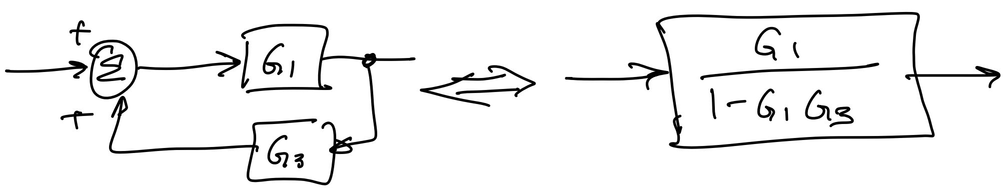


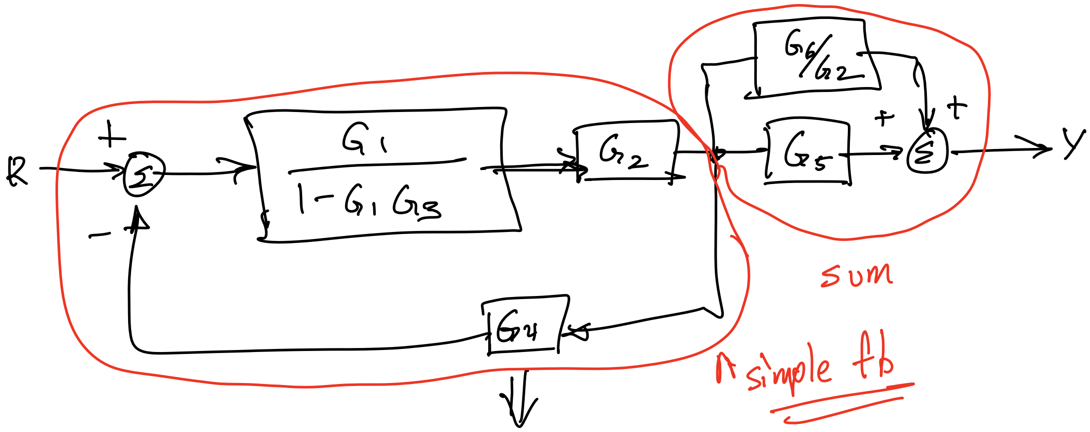
Example Problem



we want







$$R \rightarrow \boxed{\dfrac{\dfrac{G_1G_2}{1-G_1G_3}}{1+\dfrac{G_1G_2G_4}{1-G_1G_3}}} \rightarrow \boxed{\dfrac{G_6}{G_2}+G_5} \rightarrow$$

$$R \rightarrow \boxed{\dfrac{G_1G_2}{1-G_1G_3}\cdot\dfrac{1-G_1G_3}{(1-G_1G_3)+G_1G_2G_4}} \rightarrow \boxed{\dfrac{G_6+G_5G_2}{G_2}} \rightarrow Y$$

$$R \rightarrow \boxed{\dfrac{(G_1G_2-G_1^2G_3G_2)(G_6+G_5G_2)}{(1-G_1G_3)(1-G_1G_3)+(1-G_1G_3)(G_1G_2G_4)\,G_2}} \rightarrow Y$$

$$\boxed{\frac{Y(s)}{R(s)}=\frac{G_1G_2G_6+G_1G_2^2G_5-G_1^2G_2G_3G_6-G_1^2G_2^2G_3G_5}{1-2G_1G_3+G_1^2G_3^2+G_1G_2^2G_4-G_1^2G_2^2G_3G_4}}$$

Signal flow graphs + Mason's Rule

Mathematical tool to simplify block diagrams without doing block diagram algebra!

Signal flow graph

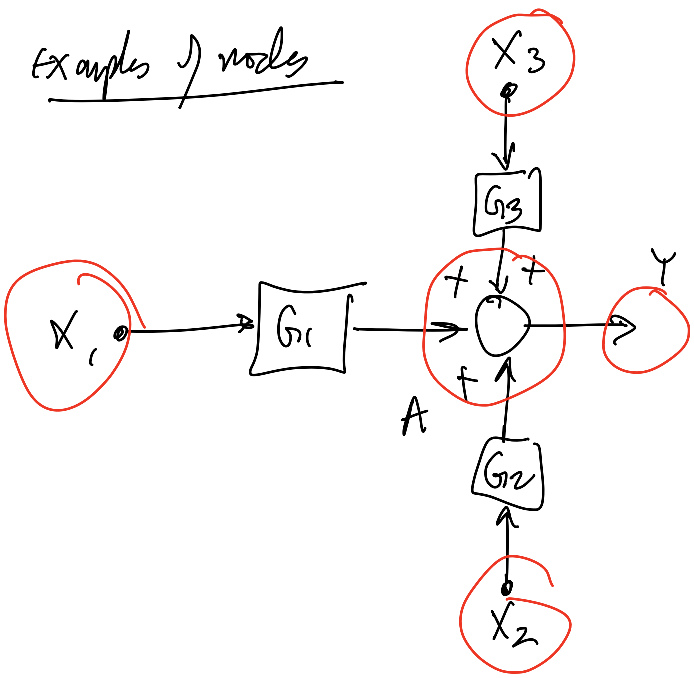
Characterize a system by a network of directed branches + associated gains (transfer function) connected at nodes.

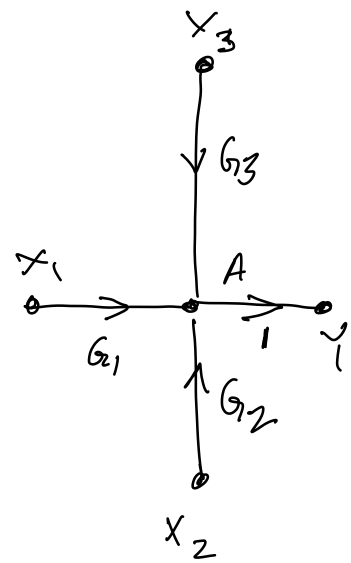
1) Node : Internal signals in the diagram such as the common input to several blocks or output of a summing junction. System inputs + outputs are also nodes.

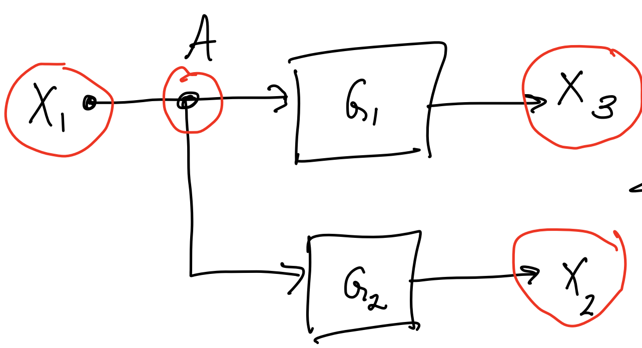
   * the input node has outgoing branches

   * the output node has incoming branches only.

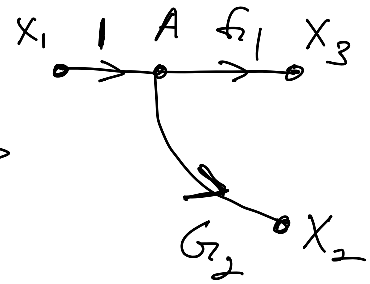
Examples of nodes



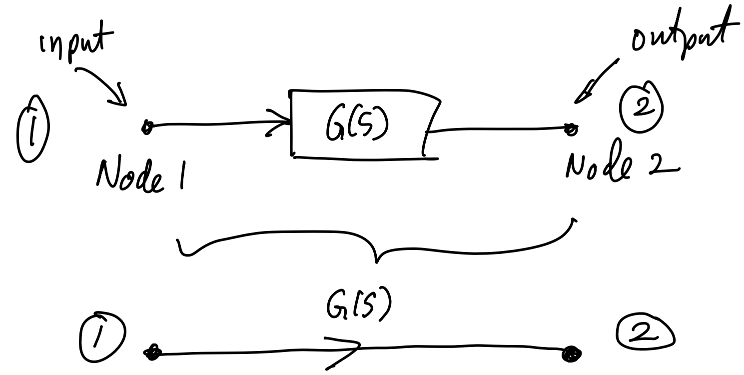




$\Longleftrightarrow$

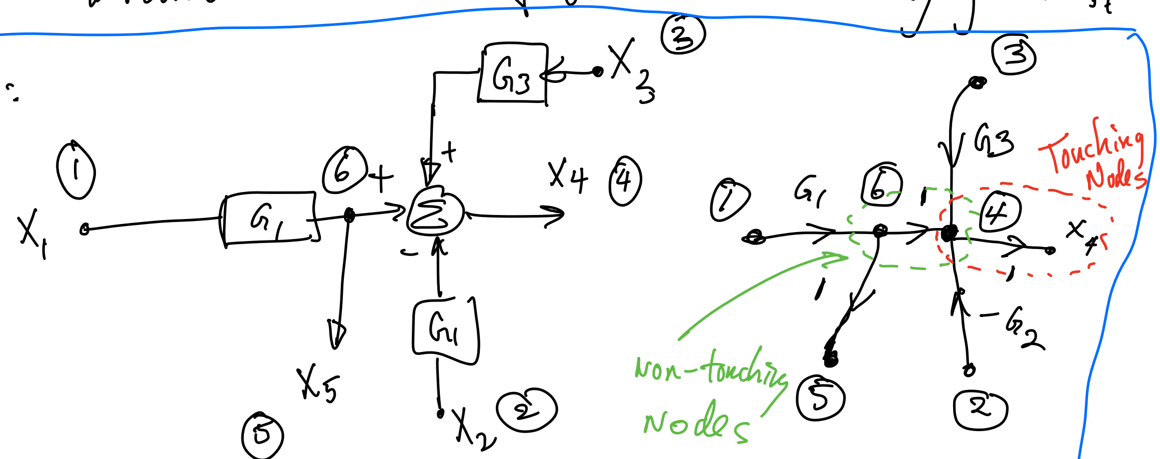


2. Branches : relate two nodes



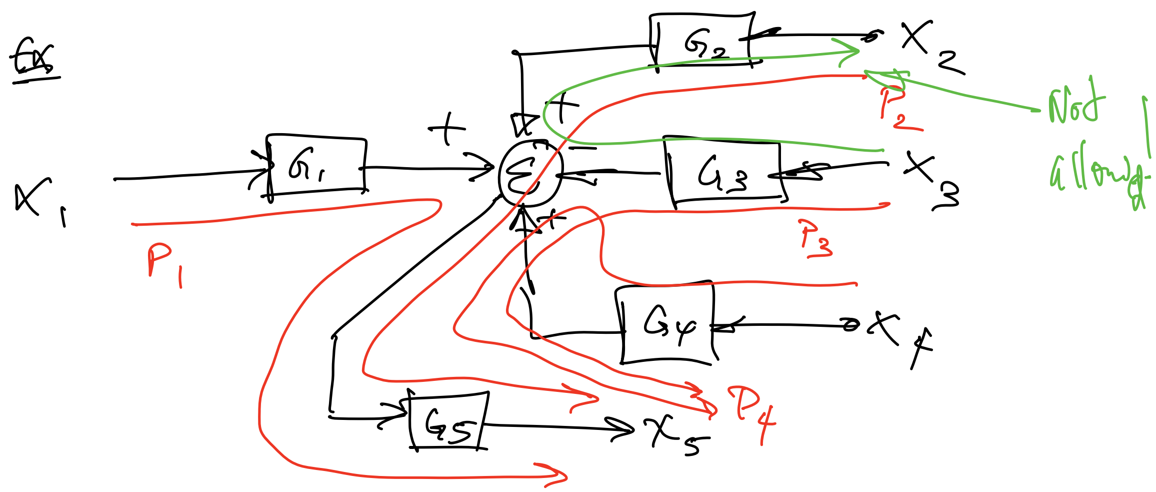
* when a node is a summing junction, the sign of the transfer function in the branch comes from the summing junction.

Ex:



3. Path: Sequence of connected blocks (branches); a route passing from one variable (signals) to another in the direction of signal flow. A path only include a variable <u>once</u>.

<u>Ex</u>

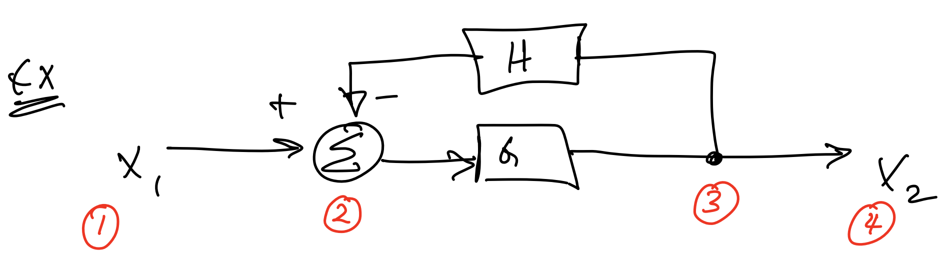


4. <u>Forward path:</u>

Path from desired input variable to a desired output variable, where you only include variable just once.

5. Loop: Any closed path that returns to its starting point. (Node)

<u>Ex</u>



6. Loop path: path that leads from a given variable back to same variable.

7. Loop and path gains:
Product of component gains (transfer functions) making up the path/loop.

8. Touching loops or paths: two loops/paths that share/have a common component (branch or Node)