Bode plotting example

$$G(s) = \frac{1000\,(s+3)}{s(s+12)(s+50)}$$ put into form as follows:

$$G(\omega) = \frac{5\,(j\omega(1/3)+1)}{j\omega\,(j\omega(1/12)+1)\,(j\omega(1/50)+1)} \Rightarrow \begin{array}{l} \tau_1 = 1/3 \\ \tau_a = 1/12 \quad \tau_b = 1/50 \end{array}$$

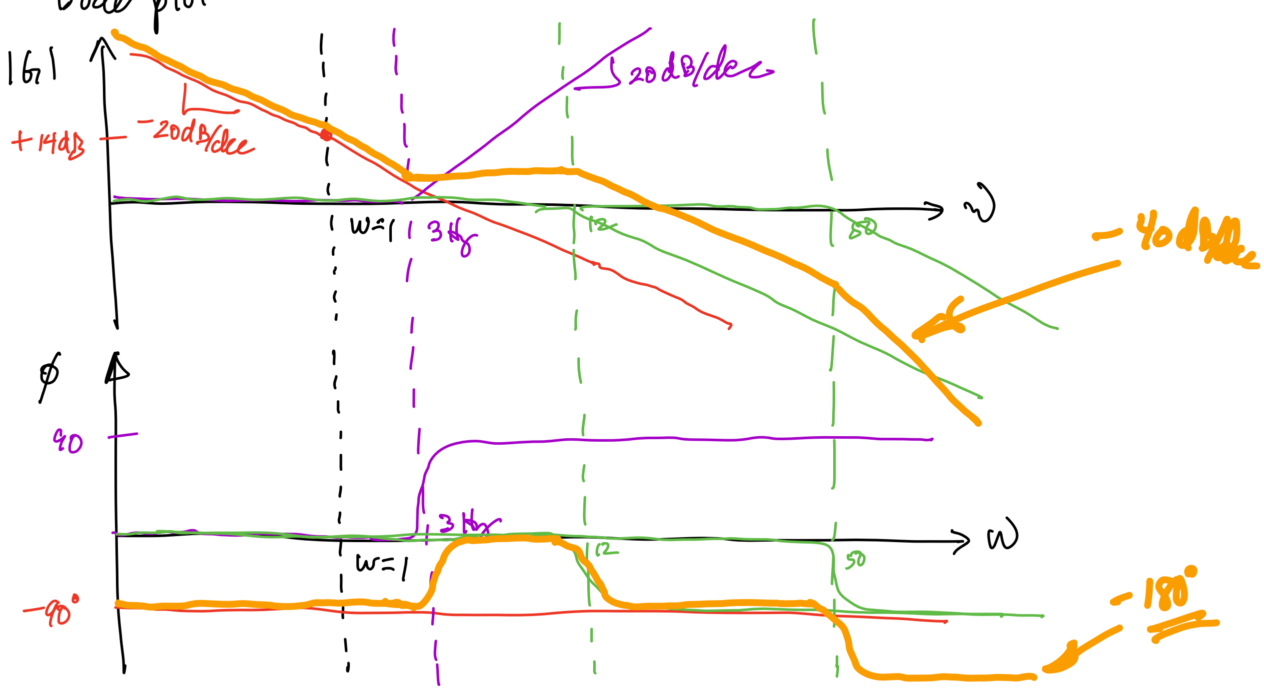
Step 1 : $\frac{5}{j\omega}$ term ; $K_0\,(j\omega)^n$

Intercept: $20\log 5 \approx 14.0\ dB$ (decibel)

$\log 5 = 0.70$

$n = -1 \Rightarrow -20dB/dec$

Bode plot



Step 2 $\quad (j\omega(1/3)+1)^n \qquad \tau_1 = 1/3$

$$\omega_{BP} = \frac{1}{\tau_1} = 1/\tfrac{1}{3} = 3 \text{ Hz}$$

$n = +1 \Rightarrow +20$ dB/dec slope from ω_{BP}

Step 3 $\quad (j\omega(1/12)+1)^{-1}$ and $(j\omega(1/50)+1)^{-1}$

$n = -1 \Rightarrow -20$ dB/dec from ω_{BP}

$$\omega_{BP_1} = \frac{1}{1/\tau_a} = \frac{1}{1/12} = 12 \text{ Hz}$$

$$\omega_{BP_2} = 1/\tfrac{1}{\tau_b} = 1/\tfrac{1}{50} = 50 \text{ Hz}$$

System Type Identification from Bode Plot

Ex: Type 0 system:

$$G(s) = \frac{3}{s+2} \Rightarrow G(s) = \frac{3}{2\left(j\omega(1/2)+1\right)} = \frac{3/2}{\underbrace{(j\omega(1/2)+1)}_{n=-1}}$$

Slope 0 dB on the Bode plot.

Ex Type 1 system

$$G(s) = \frac{3}{s(s+2)}$$

Intial slope of magnitude plot determines the system type:

Type 0 $\rightarrow$ zero slope

Type 1 $\rightarrow$ -20 dB/dec slope

Type 2 $\rightarrow$ -40 dB/dec slope

Type 3 $\rightarrow$ -60 dB/dec slope.

Steady-state error

Type 0 system $\rightarrow$ position constant K_p

slope is zero on Bode plot

Initial value of $G(j\omega)$ when $\omega = 0$

$$|G(j\omega=0)| = |K_0|$$

$$\Rightarrow K_p = K_0$$

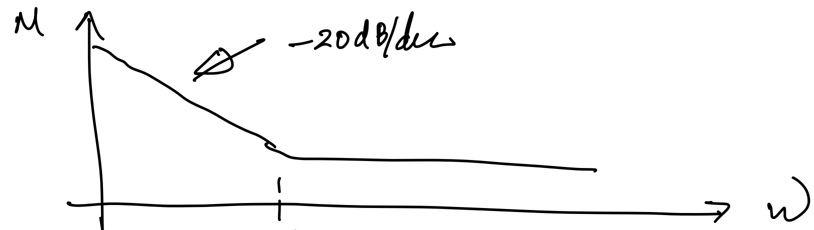
Then for unity feedback:

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K_0}$$

$$K_p = \lim_{s \to 0} s\,G(s)\,R(s)$$

Type 1

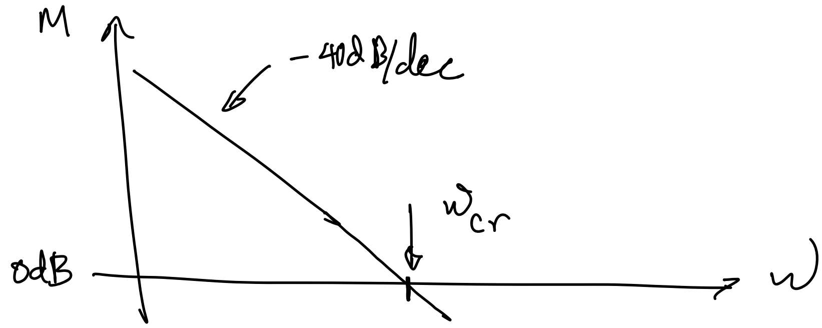
Initial slope is -20 dB/dec



$\omega_{BP} \to \omega_c$ (cut off frequency)

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\omega_{BP}} = \frac{1}{\omega_c}$$

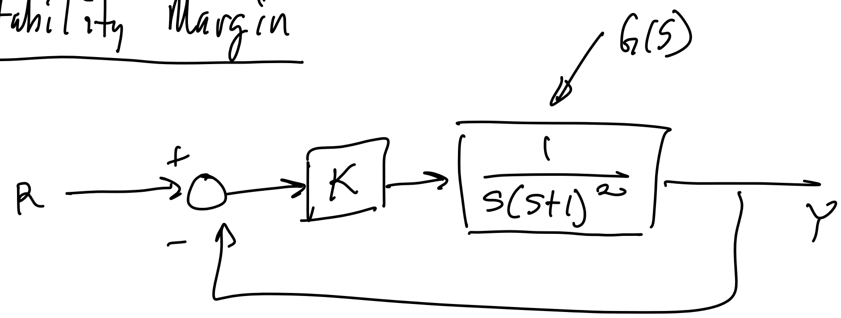
Type 2

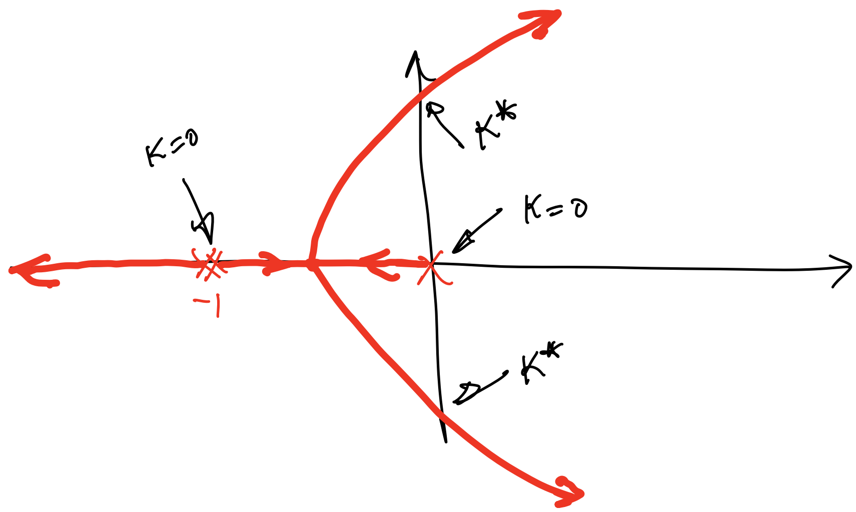


$$e_{ss} = \frac{1}{K_a}$$

$$\omega_{cr} = \sqrt{K_a} \Rightarrow K_a = \omega_{cr}^2$$

Stability Margin





From Root-locus method:

$$1 + K\hat{G}(s) = 0$$

where:

$$|K\hat{G}(s)| = 1$$

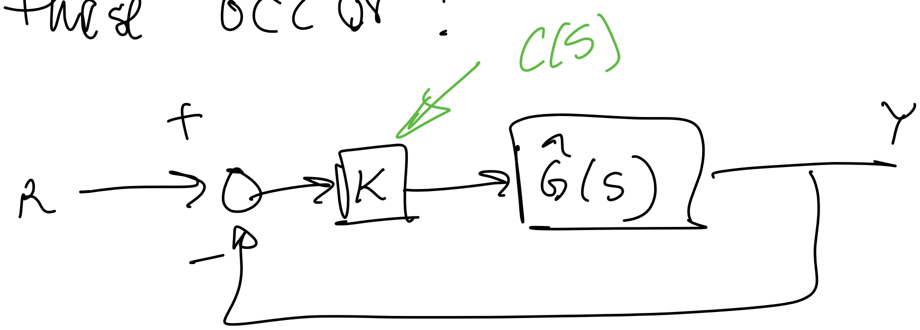
$$\angle K\hat{G}(s) = 180^\circ$$

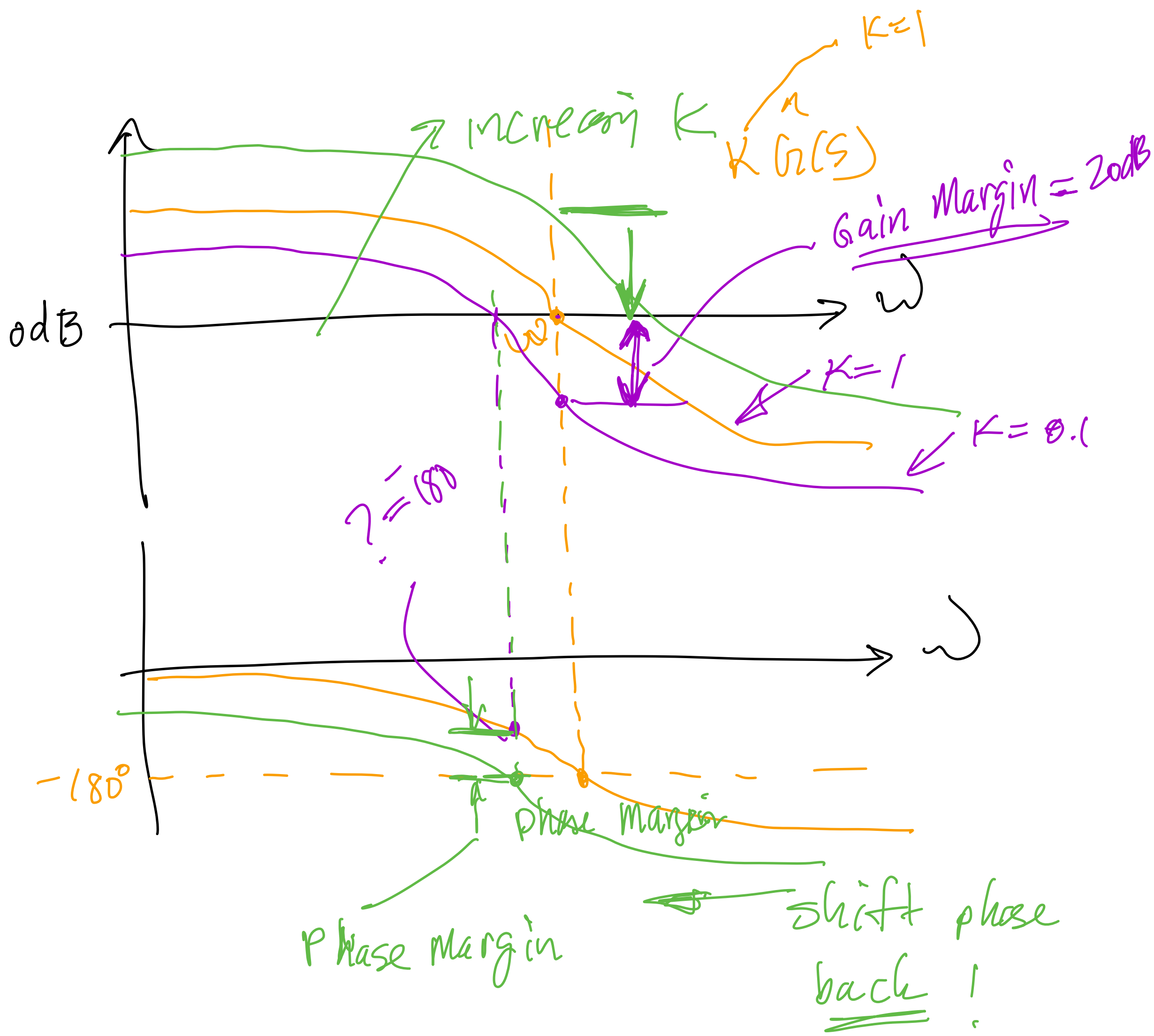
At the point of marginal instability $(s=j\omega)$ the following applies:

$$|K\hat{G}(j\omega)| = 1 \quad (0\,dB)$$

$$\angle K\hat{G}(j\omega) = 180^\circ = -180^\circ$$

Look at Bode plot to see if/when these occur!





increasing K
K=1
K G(S)
Gain Margin = 20dB
0dB
ω
K=1
K=0.1
ω
? = 180
ω
-180°
phase margin
Phase margin
shift phase back !