Homework #9
ME EN 5210/6210 & CH EN 5203/6203 & ECE 5652/6652
Linear Systems & State-Space Control

Use this page as the cover page on your assignment, submitted as a single pdf.

Problem 1

Is the following state equation controllable? Is it observable? Use the "Jordan form" method of Section 6.5, and show your work.

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \\ 3 & 2 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 2 & 2 & -1 & 3 & -1 & -1 & 1 \\ 1 & 3 & -1 & 2 & 0 & 0 & 0 \\ 0 & -4 & -1 & 1 & 1 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t)$$

Problem 2

Is it possible to find a set of b_{ij} and a set of c_{ij} such that the state equation below is controllable? Observable? Use the "Jordan form" method of Section 6.5.

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} b_{11} & b_{11} \\ b_{21} & b_{12} \\ b_{31} & b_{13} \\ b_{41} & b_{14} \\ b_{51} & b_{15} \end{bmatrix} \boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \end{bmatrix} \boldsymbol{x}(t)$$

This problem will give insight into how, during the design process, you can determine how many actuators and sensors that you need, and where to place them, in a given system.

Problem 3

Consider a continuous-time system of the form

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -15 & -79 & -145 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t)$$

(a) Verify that this system is controllable. You must show every step of your work, but you may use MATLAB to assist with your numerical calculations.

(b) We want to implement a sampled-date system using a microcontroller with a sampling period of $T = 1$ second. Use the MATLAB function *c2d* to do this for you. Verify that the system is still controllable after sampling. You may again use MATLAB to assist with your numerical calculations. Note: use a dot notation to pull a matrix out of a state-space-system *sys* in MATLAB (e.g., *sys.A*).

(c) Using Equation 6.67 in the textbook, write your own MATLAB script to calculate discrete equivalent A and B matrices, and verify that they match the matrices from Part (b). If you've done it correctly, your matrices should be very close, with only negligible numerical errors.

(d) A sampling period of 1 second is pretty long, so we may want to sample faster. Using Theorem 6.9 in the textbook, determine the fastest sampling period (i.e., the smallest *T*) for which we should expect to lose controllability due to sampling.

(e) Repeat Part (b) using your *T* from Part (d), and verify that the system is not controllable after sampling.

Problem 4

The following system is not controllable. Use the Kalman decomposition to reduce the state-space equation to a controllable one. Is this system stabilizable, even though it's not controllable? Is the reduced equation observable?

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \boldsymbol{x}(t)$$