ME EN 5200/6200 ECE 5615/6615 Classical Control

Exam 02 Details and Practice Problems

What's covered on Exam 2:

- Homeworks 6-9
- Lectures notes from Week 7 through Week 12, topics between steady-state error to PID design using root locus.
- Covers reading from Chapters 7 through 9 of the text.
- Exam 1 will be 50-minutes long, approximately 4-5 problems. Exam 1 will be closed notes and book, no electronic devices of any kind. Exam will be **in-person** on Thursday, November 26, starting at 3:40 -4:40 pm (50-minute exam). Please come prepared with something to write with.
- Laplace table below will be provided.

Topics Covered:

- Steady-state error and system type
- Disturbance rejection and sensor noise
- Root locus
- Design via root locus
- Lead/lag control
- PID control

Practice Problems – Note, some problems below have solutions while other do not, so it is encouraged that you do the problems and study with classmates and compare solutions.

Problem 1 (10 pts)

Answer the following questions:

(a) [2 pts] What information does the Routh-Hurwitz table give you?

Answer: Number of closed-loop poles in the RHP.

(b) [1 pts] What system type do you need to have a zero steady-state error for a parabolic reference input?

Answer: Type 3, because $1/s^3$ will then cancel $R(s) = 1/s^3$, leaving one "s" term when applying the final value theorem.

(c) [2 pts] What is the root locus?

Answer: Plot of the closed-loop pole locations for varying gain K.

(d) [2 pts] If an open-loop system is unstable, is it possible to stabilize it using feedback control?

Answer: Yes, in most cases, because you can affect the closed-loop poles by adding feedback to move poles from open right-half plan to the left-half plane.

(e) [3 pts] What is the transfer function for a PID controller and label terms?

Answer: $C(s) = u(s)/e(s) = kp + ki/s + kds$, where kp is proportional term, ki/s is the integral term, and kds is the derivative term.

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Problem 1 [10 pts]

(a) [5 pts] Describe the effects of a **lead controller** in a feedback system. Explain in terms of pole/zero location in the s-plane and also the effects on the root locus. Show sketches if needed to explain your reasoning.

(b) [5 pts] Describe the effects of a **lag controller** in a feedback system. Explain in terms of pole/zero location in the s-plane and also the effects on the root locus. Show sketches if needed to explain your reasoning.

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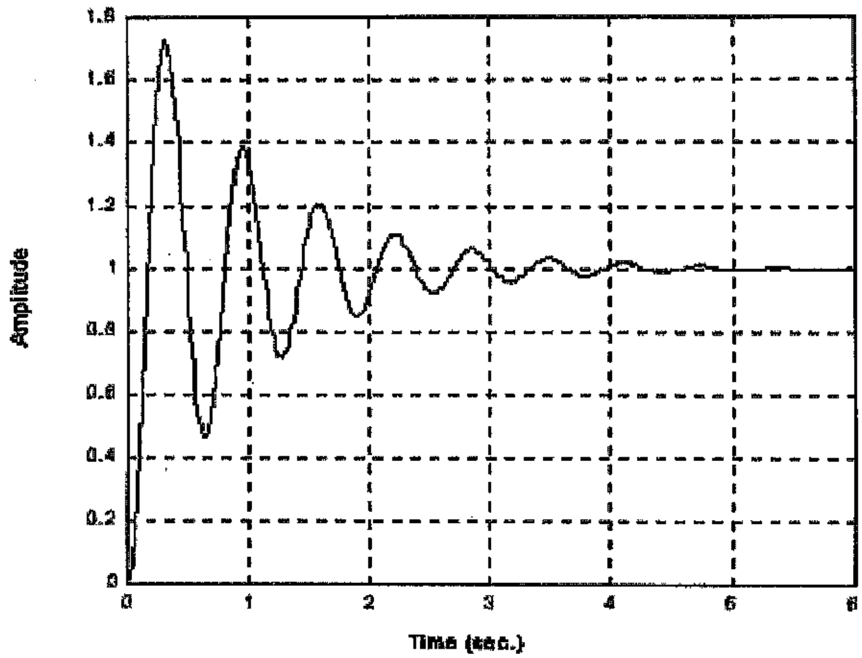
Problem 2 [5 pts]

The step response for a closed-loop system is shown below. The reference was a step of magnitude 4.5.



(a) [3 pts] What's the steady-state error?

Answer: We know R=4.5 step input and Y= 1.0 at steady state, so

ess = 4.5-1 = 3.5. Steady error is 3.5.

(b) [2 pts] What do you think the system type is? Explain you answer.

Answer: Because the steady-state error is a constant due to a step, then the system must be type 0.

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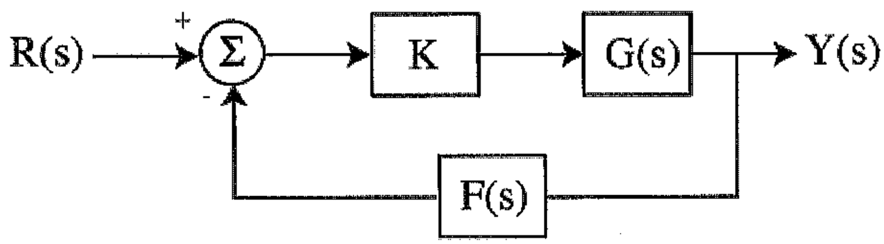
Problem 2 [5 pts]

Consider the system shown below:



$$G(s) = \frac{1}{(s+3)}; \quad F(s) = \frac{2}{s+B}$$

Put this system into unity feedback configuration. Show your work!

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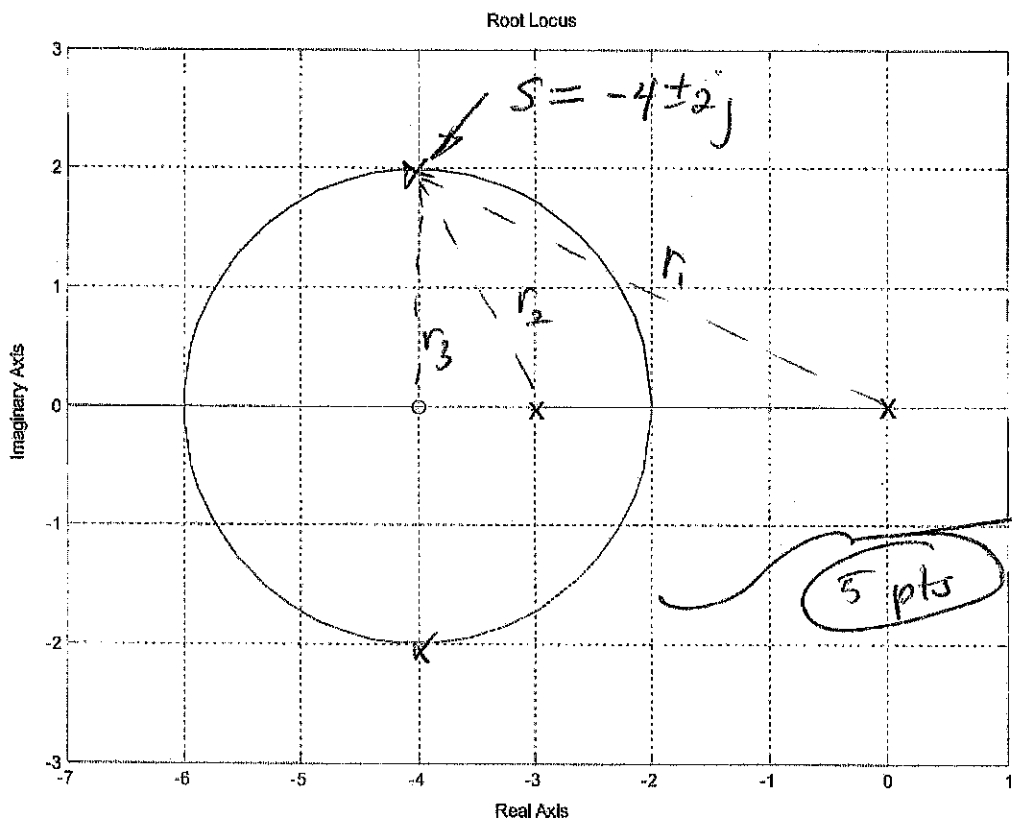
Problem 3 [10 pts]

Consider the root locus below. (a) What gain K would put the closed-loop poles at s = -4 ± 2j? (b) What's the closed-loop damping ratio for the K value from (a)?



$r_1 = \sqrt{20}$

$r_2 = \sqrt{5}$

$r_3 = 2$

$$\boxed{\zeta = \frac{4}{\sqrt{20}} = \frac{2\sqrt{5}}{5}}$$

From characteristic equation:

4 pts

$$1 + KG(s) = 0$$

$$\Rightarrow K = \left|\frac{-1}{G(s)}\right|_{s=-4\pm 2j} = \left|\frac{1}{\frac{\text{zeros}}{\text{poles}}}\right| = \frac{|\text{poles}|}{|\text{zeros}|}$$

$$\Rightarrow G(s) = \frac{s+4}{s(s+3)}$$

$$\Rightarrow K = \left|\frac{s(s+3)}{s+4}\right|_{s=-4\pm 2j} \Rightarrow K = \frac{r_1 \cdot r_2}{r_3} \Rightarrow K = \frac{\sqrt{100}}{2} = \frac{10}{2}$$

$$\boxed{K = 5}$$ 1pt

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Problem 4 [15 pts] (1/2)

A closed-loop negative unity feedback system has an open-loop transfer function given by (with K included):

$$G(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

Sketch the root locus by showing the details of at least three of the steps described in class. Show all your work for full credit.

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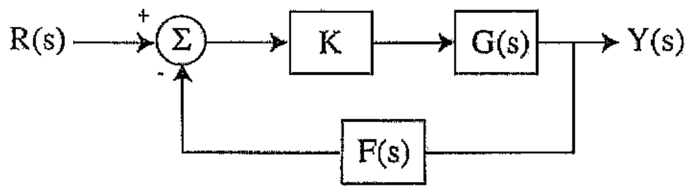
Problem 5 [10 pts]

Consider the system shown on the right, where

$$G(s)=\frac{1}{s^2+5s+6};\quad F(s)=\frac{s^2+3s+2}{H}$$



Let $K = 1$. Draw a root locus for the parameter $H \geq 0$. Note, you are drawing a root locus for the parameter H, not K! Show all your work and steps. Neatly draw the locus and label axes.

Characteristic Eq :

$$1 + K\,G(s)\,F(s) = 0 \qquad K=1$$

$$1+\left(\frac{1}{s^2+5s+6}\right)\left(\frac{s^2+3s+2}{H}\right) = 0$$

$$H(s^2+5s+6) + s^2+3s+2 = 0$$

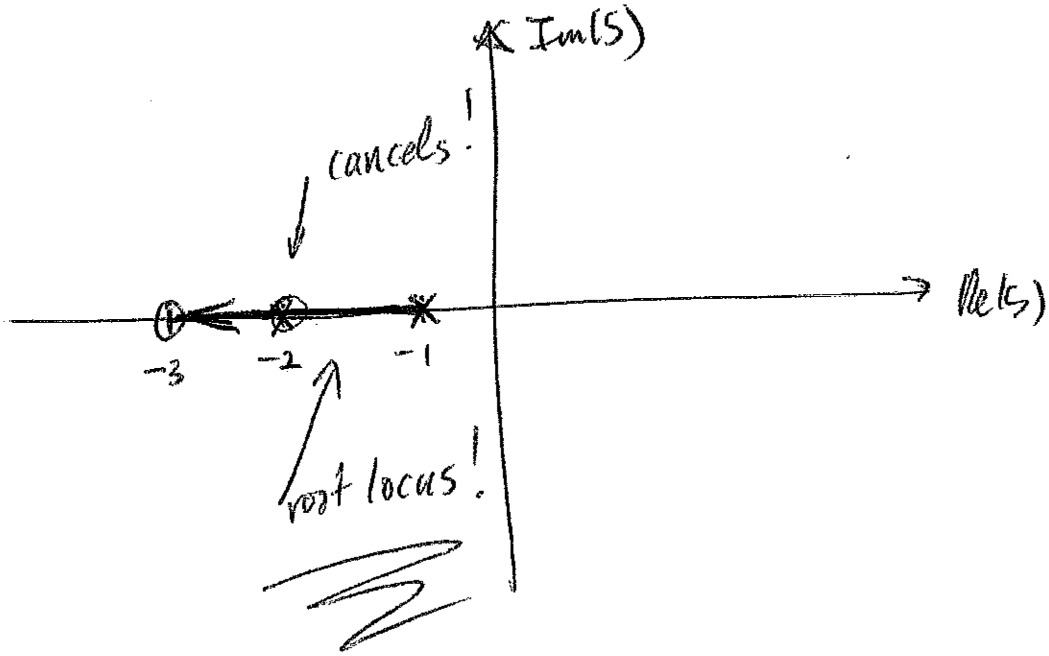
$$\Rightarrow\ 1 + H\underbrace{\frac{s^2+5s+6}{s^2+3s+2}}_{\text{open-loop!}} = 0$$

$$\frac{s^2+5s+6}{s^2+3s+2} = \frac{(s+3)\cancel{(s+2)}}{\cancel{(s+2)}(s+1)}$$

OL zeros

OL poles

pole cancels zero!

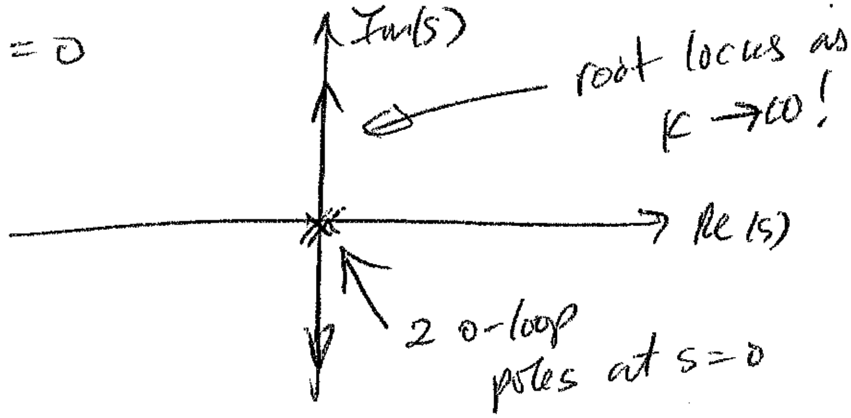


Problem 6 [18 pts]

Dr. Leang's tractor has an open-loop transfer function of $1/s^2$. Yes, what a hunk of junk! He needs to close the loop to get better performance. Help him by doing the following:

(a) [3 pts] If Dr. Leang closed the loop around his tractor with a proportional controller with gain K, what would the root locus for the closed-loop poles look like? Please draw the root locus. Label axes, identify open-loop poles, etc.

$1 + K\left(\frac{1}{s^2}\right) = 0$



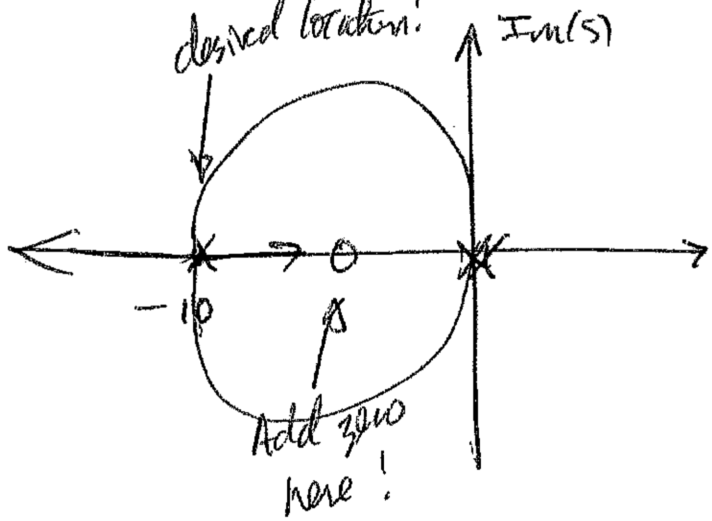
(b) [5 pts] Explain the behavior of the closed-loop system from Part (a) when the gain K increases. What will happen to the behavior of his tractor as K increases? Will this kind of controller be useful? Why and why not?

When K increases, system poles on $j\omega$-axis!

System just oscillates with frequency increasing as $K \to \infty$!

System is marginally stable!

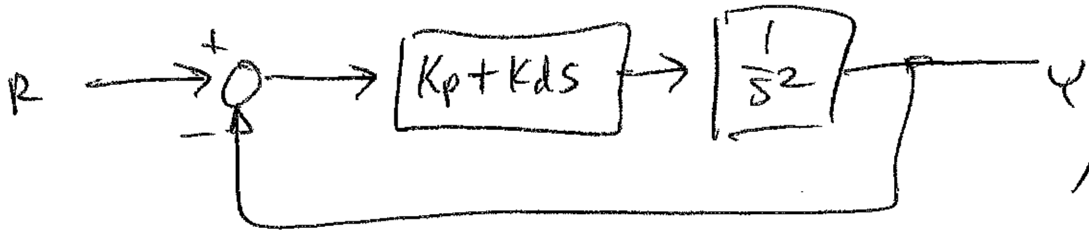
(c) [10 pts] He would like to design a feedback controller such that the closed-loop system has a damping ratio of 1 and a natural frequency of 10 rad/s. Help him design such a controller. Show closed-loop block diagram, give the transfer function for your controller, explain reason for controller, and provide appropriate values for the controller gains or poles/zeros for your controller. Is this a better controller compared to Part (b)?



3 pts

Controller: PD

2 pts



3 pts

Closed-loop characteristic equation

$$\frac{Y}{R} = \frac{(K_p + K_d s)\left(\frac{1}{s^2}\right)}{1 + [K_p + K_d s]\left[\frac{1}{s^2}\right]} \Rightarrow s^2 + K_p + K_d s = 0$$

Want poles at: $(s-10)^2 = (s+10)(s+10) = s^2 + 20s + 100$

Thus: $K_d = 20$ and $K_p = 100$, so controller is

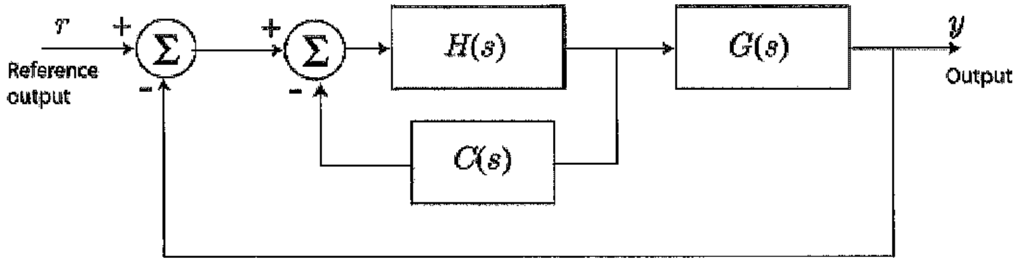
2 pts

$$C(s) = K_p + K_d s = 100 + 20s \Rightarrow \boxed{20(s+5)}$$

This controller is damped and meets specs. Better compared to (a)

Problem 7 (20 pts)

Consider the following system:



$$H(s) = \frac{1}{s^2(s+1)}; \quad G(s) = \frac{1}{s^2(s+3)} \quad C(s) = \frac{1}{s}$$

(a) [4 pts] Find the closed-loop transfer function

(b) [4 pts] Find the system type

(c) [4 pts] Find the steady-state error for a step with magnitude 5

(d) [4 pts] Find the steady-state error for a ramp input with slope 5

(e) [4 pts] Suppose the closed-loop poles are -3.0190, -1.3166, 0.3426 ± j0.7762, -0.3495, then how do you interpret the results in (c) and (d)?

a. For the inner loop:

$$G_1(s) = \frac{\frac{1}{s^2(s+1)}}{1+\frac{1}{s^3(s+1)}} = \frac{s}{s^4+s^3+1}$$

$$G_e(s) = \frac{1}{s^2(s+3)}\ G_1(s) = \frac{1}{s(s^5+4s^4+3s^3+s+3)}$$

$$T(s) = \frac{G_e(s)}{1+G_e(s)} = \frac{1}{s^6+4s^5+3s^4+s^2+3s+1}$$

b. From $G_e(s)$, system is Type 1.

c. Since system is Type 1, $e_{ss} = 0$

d. ; From $G_e(s)$, $K_v = \lim_{s\to 0} sG_e(s) = \frac{1}{3}$. Therefore, $e_{ss} = \frac{5}{K_v} = 15$.

e. Poles of T(s) = -3.0190, -1.3166, 0.3426 ± j0.7762, -0.3495. Therefore, system is unstable and results of (c) and (d) are meaningless