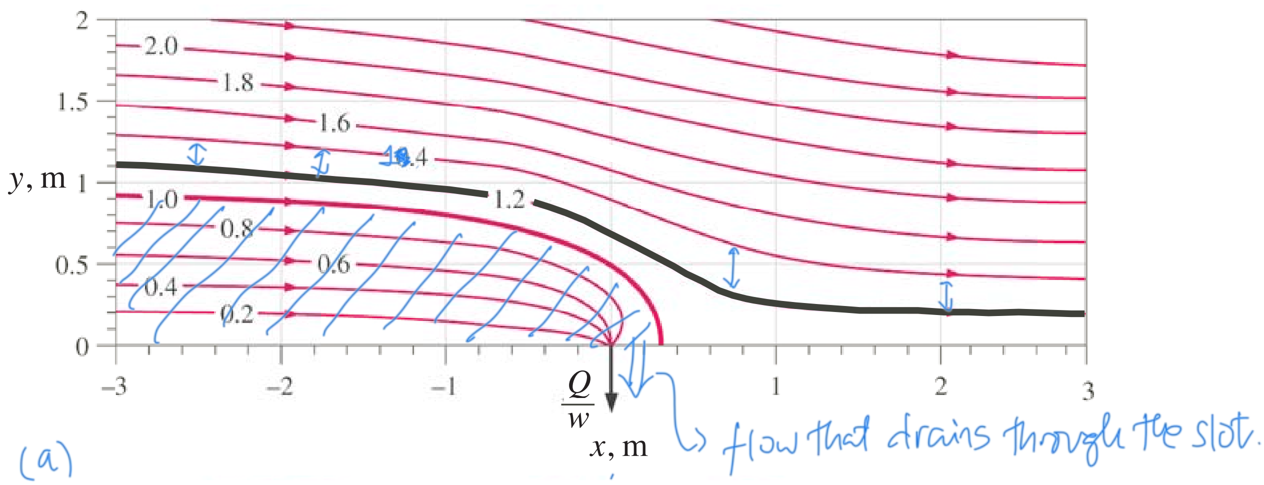
In Class Practice Problem – Boundary Layer Theory (Fall 2025)

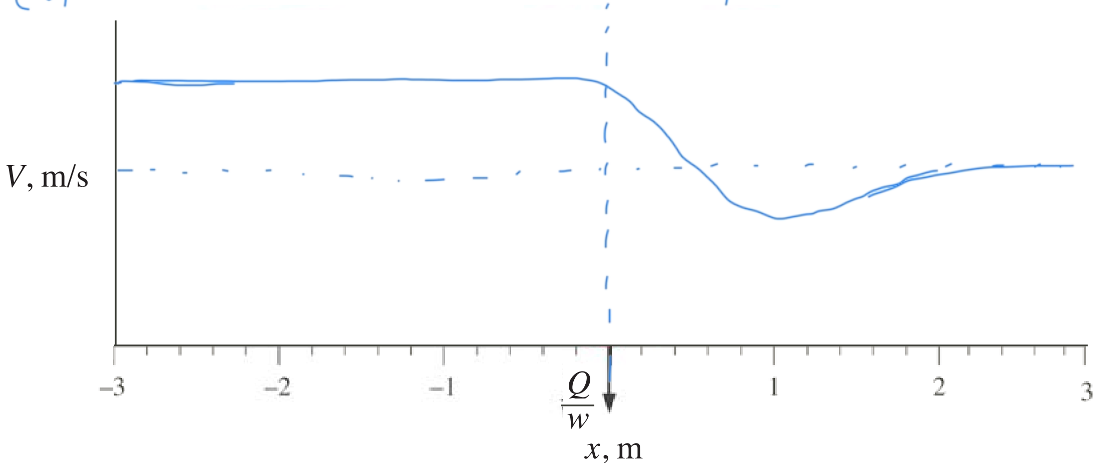
P1. Water drains through a narrow slot, located at $x = 0$, $y = 0$ on the bottom wall of a channel having a width of $w = 2$ m. The streamline pattern and corresponding values of the streamfunction ψ are shown below.

a In the figure provided, plot the magnitude of the velocity V as a function of x along the streamline drawn in black. [Be sure to include the correct values of V on the y-axis]

b What is the volume flow rate Q through the slot? [Hint: assume the streamline along the bottom wall between $-3 \le x \le 0.3$ has a value $\psi = 0$.]



(a)



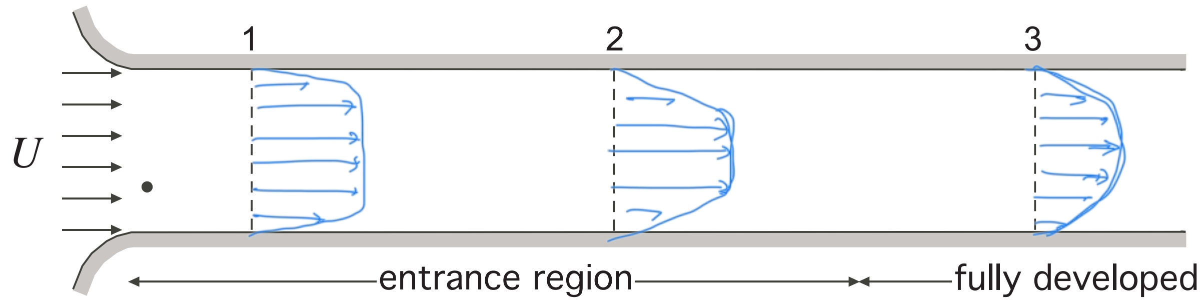
To estimate the velocity we use the fact that the volume flow rate between two streamlines is constant and equal to the difference in ψ values:

$$@\ x=-3: \quad u = \frac{\partial \psi}{\partial y} \simeq \frac{\Delta \psi}{\Delta y} = \frac{1.4-1.2}{0.15} = 1.33 \text{ m/s}$$

$$@\ x=3: \quad u = \frac{\partial \psi}{\partial y} \simeq \frac{\Delta \psi}{\Delta y} = \frac{1.4-1.2}{0.4-0.2} = 1 \text{ m/s}$$

(b) The volume flow rate $Q = \Delta \psi = 1 - 0 = 1 \ m^3/s$.

P2. Steady flow enters a two-dimensional channel with a uniform velocity U. Assume the pressure gradient $\partial p/\partial x$ remains constant throughout the entire length of the channel.



a. Draw (to-scale) the velocity profiles at locations 1, 2, and 3 on the figure provided.

b. Write the simplified x- AND y-momentum equations in the *entrance region*.

x-mom: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2}$

y-mom: $\frac{\partial p}{\partial y} = 0$

c. Write the simplified x- AND y-momentum equations in the *fully developed* region.

x-mom: $\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}$

y-mom: $\frac{\partial p}{\partial y} = 0$

d. Comment on the magnitude of the *vertical* velocity in the entrance region compared to the fully developed region.

The vertical velocity is zero everywhere in the fully developed region. Alternatively in the entrance zone a small (but non-negligible) vertical velocity exists due to the growth of the BL.

e. Write down the mathematical expression for the acceleration of the fluid particle marked by • in the figure.

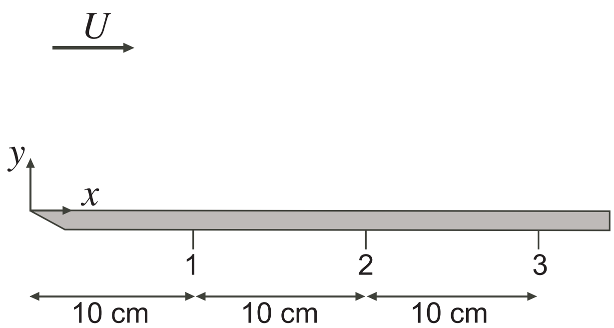
$$\frac{Du}{Dt} = \cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

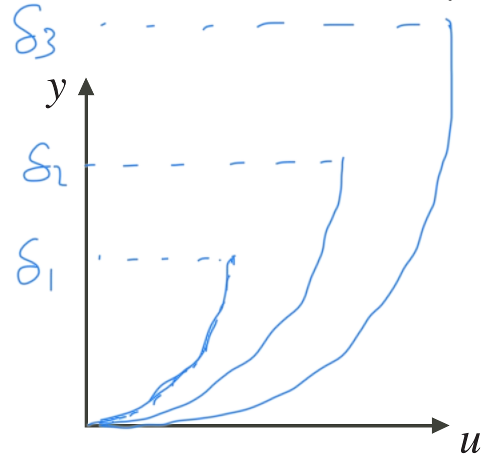
$$\frac{Dv}{Dt} = \cancel{\frac{\partial v}{\partial t}} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

f. Describe what happens to the momentum of the fluid particle marked by • as it moves down the channel. [Be specific and use appropriate fluids terminology]

As the particle enters the channel, it will eventually enter the BL wherein its x-momentum will decrease and its y-momentum will increase. As it continues downstream, x-momentum will increase due to the pressure gradient. In the fully developed region, momentum remains constant.

P3. Consider the steady flow over a flat plate as shown below. The freestream velocity is $U = 5$ m/s.





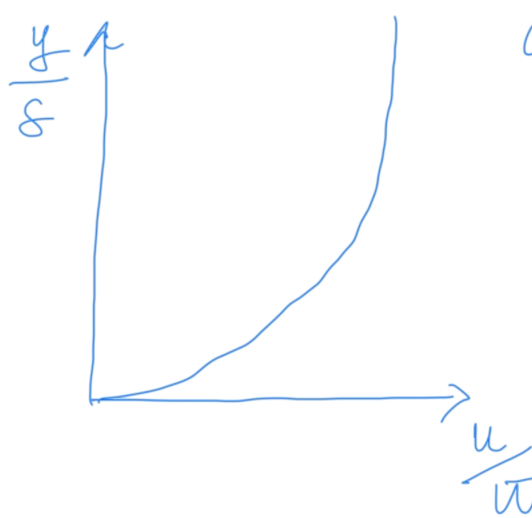
a. Is the flow laminar at location 3 for the case of <u>air</u>? What about for the case of <u>water</u>? [Use calculations to support your answer. Note, the viscosities of air and water are 1.5×10^{-5} m^2/s and 1×10^{-6} m^2/s, respectively.]

air: $Re = \dfrac{Ux}{\nu_a} = 1\times10^5 < Re_c \Rightarrow$ Laminar

water: $Re = \dfrac{Ux}{\nu_w} = 1.5\times10^6 > Re_c \Rightarrow$ Turbulent.

b. In the plot given, draw the *dimensional* velocity profiles at locations 1, 2, and 3 for the case of <u>air</u>.

c. How would the figure change if you plotted u/U versus y/δ instead, where δ denotes the local boundary layer thickness?



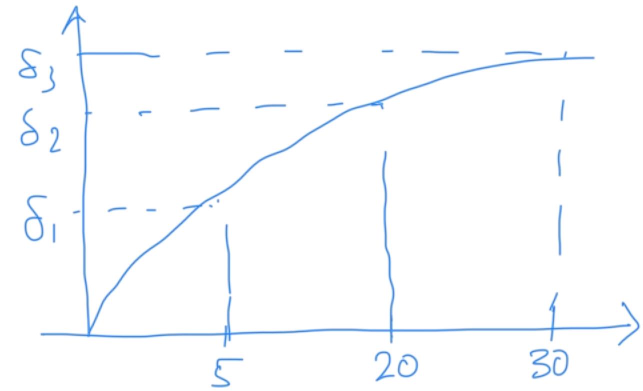
all profiles collapse.

d. For the case of <u>air</u>, plot the boundary layer thickness δ as a function of x over the range $0 \le x \le 30$ cm.

$\delta_1 = 5/\sqrt{Re_x} = 2.7\text{mm}$

$\delta_2 = 3.9\text{mm}$

$\delta_3 = 4.7\text{mm}.$



e. Consider flow through an octagonal pipe as shown in the figure. What is the increase in centerline velocity do to boundary layer growth over the pipe length L? Assume a negligible pressure gradient, that the flow is uniform over the inflow cross section with a flow rate Q_0 and an inflow velocity U_0, and that the wall boundary layers merge after some distance leading to a cylindrical inviscid core region. Using your answer and your own physical intuition, for $L/D \approx 1$ is the impact of the wall boundary layers important in this flow?