ME EN 5830/6830: Aerospace Propulsion Problem Set #10: Rocket Propulsion III and IV

Due date: 04/15/2024 by 11:59pm

## **Submission**

Assignments can only be submitted on Gradescope, which can be accessed through Canvas. If you have any questions about submission, please email the class TA, John Gardner at <a href="mailto:john.w.gardner@utah.edu">john.w.gardner@utah.edu</a>. Submissions will be automatically locked at the due date given above.

## Introduction

This problem set primarily covers the material from Lectures 20 and 21. The goal is for students to be able to understand how thrust produced by rockets affects the final speed and elevation of the rocket. After completion of the assignment, students should be able to:

- Compute maximum velocities after take-off with and without gravity.
- Compute maximum elevations reached by rockets.
- Compute maximum velocities for multi-stage rockets.

## Assignment

**Problem #1:** Consider the rocket analyzed in Problem Set #9. That rocket had an exhaust velocity of  $V_e = 2312$  m/s. In addition, with a functioning combustor, the rocket had a mass ratio of MR = 0.4161.

- a) Compute the maximum change in velocity achievable by this rocket assuming it is in deep space (no drag or gravity).
- b) In Problem Set #9, we found that with a functional combustor, the rocket burn time was  $t_{end} = 186$  s. Compute the maximum change in velocity achievable by this rocket now assuming it is taking off from earth (but continue to neglect drag). You may assume that the rocket is firing straight upwards and that gravity does not change significantly with altitude ( $g = g_0$ ).
- c) If this rocket were shot straight upwards from earth, what is the maximum height it would achieve (neglecting drag)? How does this compare to Low-Earth Orbit, which is defined approximately as 2,000 km from the surface of the earth? *Note: Recall that our rocket was based on a single stage of the Falcon 9 rocket, which in actuality has multiple stages.*
- d) For our computed mass ratio (MR=0.4161), what exhaust velocity would be required to reach a height of h=2000 km?

**Problem #2:** Consider the three stage Saturn V rocket (stage information given below, see figure):

| Stage (i) | $I_s$ (s) | Burn Time (s) | Total mass of $\underline{only}$ this stage including propellant $m_i$ (kg) | Propellant mass of $rac{ m only}{ m this}$ stage $m_{p,i}$ (kg) |
|-----------|-----------|---------------|-----------------------------------------------------------------------------|------------------------------------------------------------------|
| 1         | 304       | 161           | 2,286,217                                                                   | 2,150,999                                                        |
| 2         | 421       | 390           | 490,778                                                                     | 451,730                                                          |
| 3         | 421       | 475           | 119,900                                                                     | 106,600                                                          |
| Payload   | -         | -             | 100,000                                                                     | -                                                                |

a) Assuming no gravity or drag forces, and that separation of stages and start of subsequent engines is instantaneous, compute the final velocity for this rocket after take-off. Note: Even though we are not considering gravity as a force, you will still use  $g_0$  to convert from  $I_s$  to  $V_e$ .

