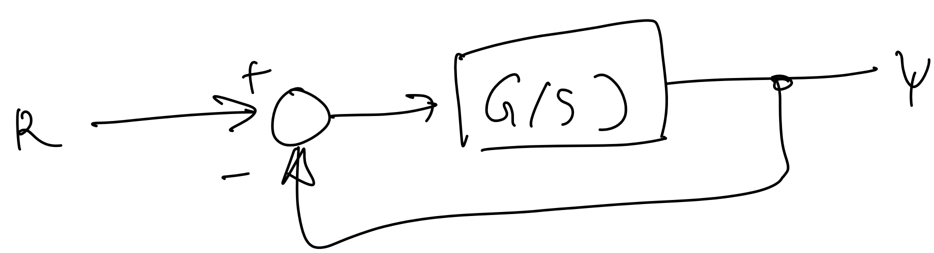
System Type

Recall unity feedback control system



where

$$G(s) = \frac{K(s+z_1)(s+z_2)\ldots}{s^n(s+p_1)(s+p_2)\ldots}$$

System type is defined by the number of pure integrators in the forward path between R and Y. In other words, the system type is the value of n.

Ex

$$G(s) = \frac{3}{s^2+2s+4} \Rightarrow n = 0$$

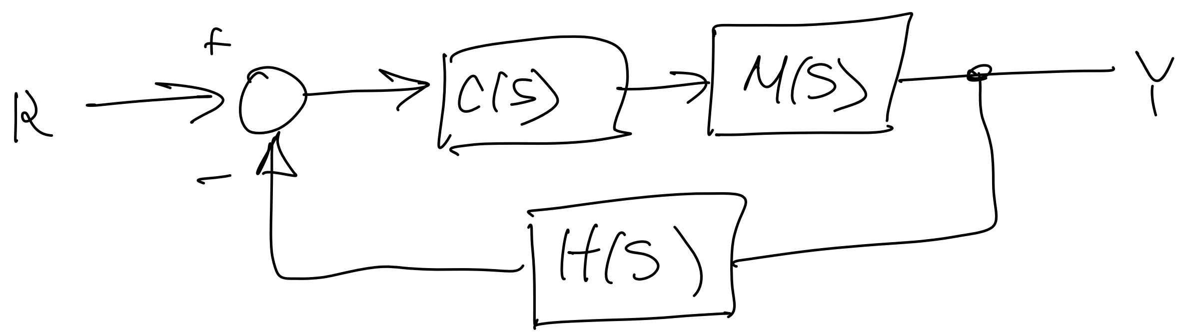
Type 0

$$G(s) = \frac{3s}{s^3+2s^2} = \frac{3}{s(s+2)}$$

$n = 1$ type 1

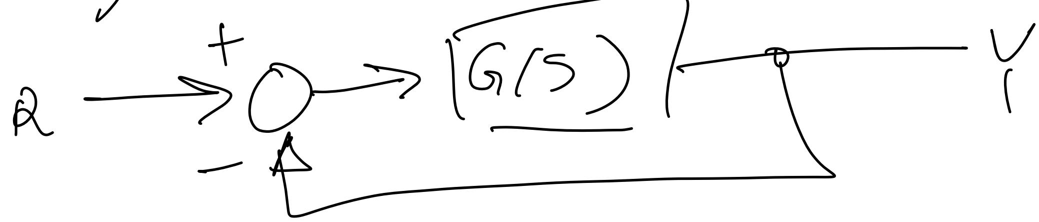
System type defines the steady state error for unity feed-back system.

what about non-unity feedback system? How do we find the system type?



what do we look at to get system type?

we need to put into this configuration:



$$E(s) = R(s) - Y(s)$$

For unity feedback:

$$E(s) = \left[\frac{1}{1+G(s)}\right] R(s)$$

For non-unity feedback:

$$E(s) = [1 - T(s)]\, R(s)$$

$$\Rightarrow T(s) = \frac{C(s)M(s)}{1+C(s)M(s)H(s)}$$

$$\Rightarrow E(s) = \left[1 - \frac{C(s)M(s)}{1+C(s)M(s)H(s)}\right] R(s)$$

$$= \left[\frac{1+C(s)M(s)H(s) - C(s)M(s)}{1+C(s)M(s)H(s)}\right] R(s)$$

Compare:

$$\frac{1 + C(s)M(s)H(s) - C(s)M(s)}{1 + C(s)M(s)H(s)} = \frac{1}{1+G(s)}$$

Solve for $G(s)$:

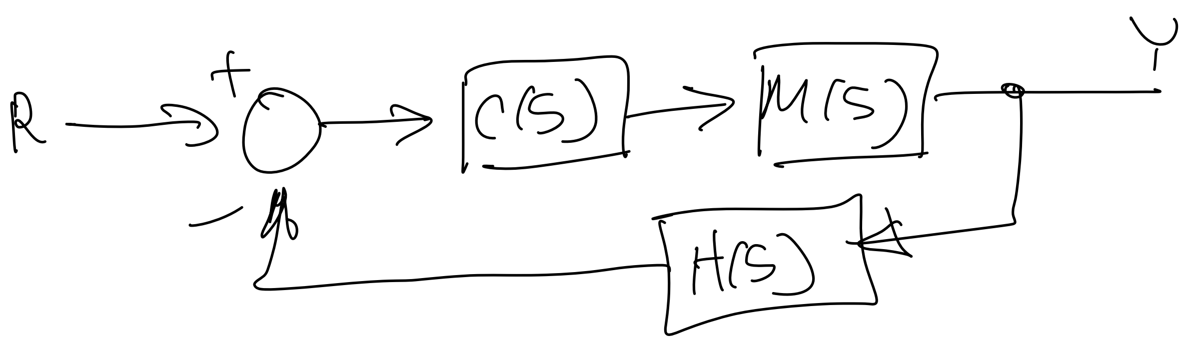
$$[1+G(s)][1 + C(s)M(s)H(s) - C(s)M(s)] = 1 + C(s)M(s)H(s)$$

$$\Rightarrow \cancel{1} + G(s) + \cancel{C(s)M(s)H(s)} + C(s)M(s)H(s)G(s) - C(s)M(s) - G(s)M(s)C(s) = \cancel{1} + \cancel{C(s)M(s)H(s)}$$

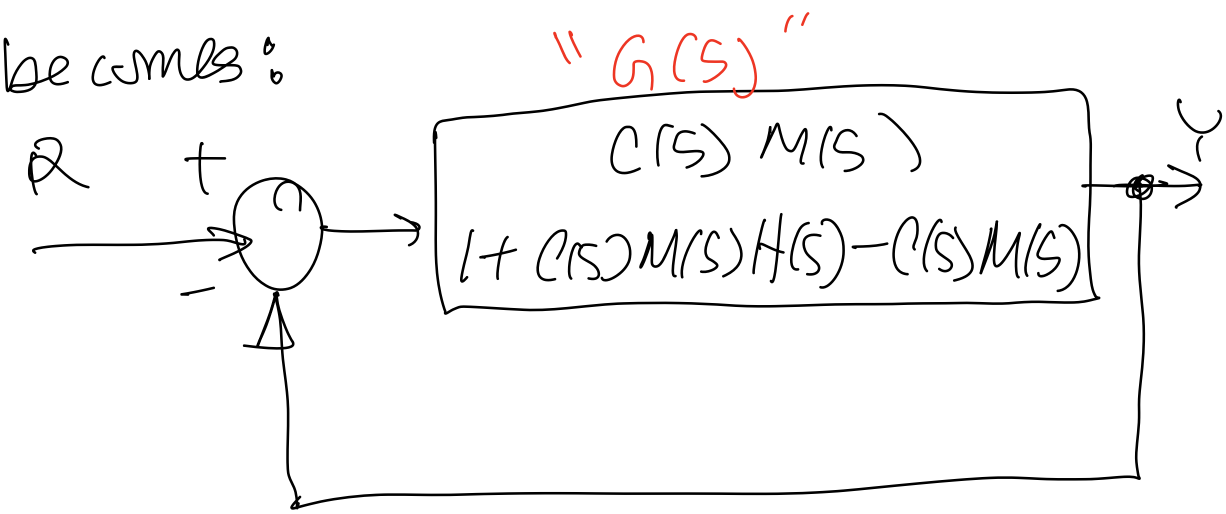
$$\Rightarrow [1 + C(s)M(s)H(s) - C(s)M(s)]G(s) = C(s)M(s)$$

$$\Rightarrow G(s) = \frac{C(s)M(s)}{1 + C(s)M(s)H(s) - C(s)M(s)}$$

Thus:



becomes:



So to determine system type, we look at new "G(s)"!!

Sensitivity

Def: The degree to which changes in system parameters affect system transfer function.

$$S_P^F = \lim_{\Delta P \to 0} \frac{\text{Fractional change in function } F}{\text{Fractional change in parameter } P}$$

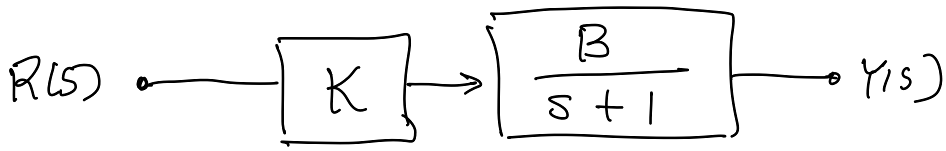
$S = 0 \Rightarrow$ Not sensitive

$S = 1 \Rightarrow$ 100% sensitive

$$S_P^F = \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P} = \frac{P}{F} \frac{\Delta F}{\Delta P}$$

$$\Rightarrow \boxed{S_P^F = \frac{P}{F} \frac{\partial F}{\partial P}}$$

Open-loop example



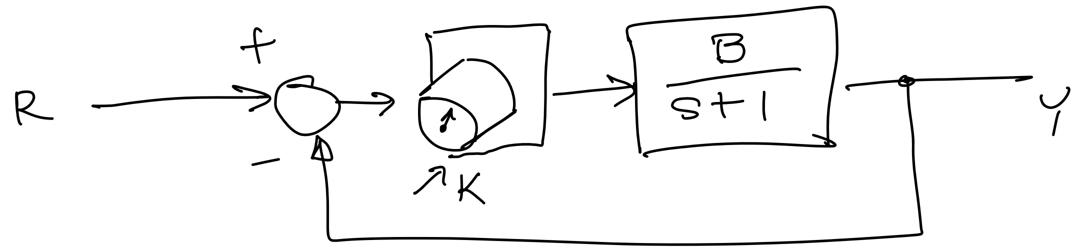
$$\frac{Y(s)}{R(s)} = H(s) = \frac{BK}{s+1}$$

What is $S_B^{H(s)} = \dfrac{B}{H(s)} \cdot \dfrac{\partial H(s)}{\partial B}$

$$S_B^{H(s)} = \frac{\cancel{B}}{\dfrac{\cancel{BK}}{\cancel{s+1}}} \cdot \left(\frac{\cancel{K}}{\cancel{s+1}} \right)$$

$$\Rightarrow \boxed{S_B^{H(s)} = 1 \quad \text{100\% sensitive}}$$

Closed-loop



$$H(s) = \frac{Y(s)}{R(s)} = \frac{KB}{s+1+KB}$$

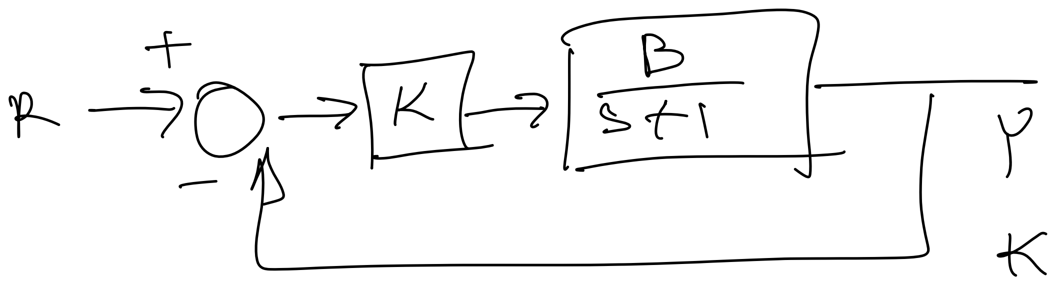
$$S_B^{H(s)} = \frac{B}{H(s)} \frac{\partial H(s)}{\partial B} = \frac{\cancel{B(s+1+BK)}}{\cancel{BK}} \cdot \frac{\cancel{K}(s+1+BK) - BK(K)}{(s+1+BK)^2}$$

$$S_B^{H(s)} = \frac{s+1+BK-BK}{s+1+BK}$$

$$\boxed{S_B^{H(s)} = 1 - \frac{BK}{s+1+BK}}$$

Look at s.s. $\Rightarrow$ $\underline{\underline{s=0}}$

$$\Rightarrow \boxed{S_B^{H(s=0)} = 1 - \frac{BK}{1+BK}}$$



$\underline{\underline{K \geq 0}}$

$$S_B^{H(s=0)} \in [0, 1)$$