

**ME 5700/6700: Intermediate Fluid Dynamics**  
**EXAM - I**  
**October 18, 2023**

NAME: \_\_\_\_\_

**The exam has two portions:**

- 1- **In-class (50 min).** The first part (questions 1-7) should be completed during the in-class exam time.
- 2- **Take-home (*till 10am tomorrow Oct 19th*).** The second part (questions 8-10) should be completed at home and submitted on canvas. You can also re-work just 1 in-class problem for increased credit. You are NOT allowed to talk with anyone about the take-home exam and must sign the honor policy below.

**In-class portion (65/100). You can use your 1-page equation sheet:**

1. **[4 points]** Explain the difference between Lagrangian and Eulerian approaches in fluid flow analysis.
2. **[4 points]** Strain rate interpretation: Describe what the trace of the strain rate tensor physically represents.
3. **[4 points]** Provide an example where the flow is incompressible but density is not constant. Mathematically state the conservation of mass equation in terms of density for this case.
4. **[3 points]** What is the definition of wall shear stress (WSS) in terms of traction?

5. [10 points] In magnetohydrodynamics, we have a PDE that relates the magnetic field  $\mathbf{B}$  (a vector) to the velocity vector  $\mathbf{v}$ . The equation in vectorial notation is ( $\nabla^2$  is Laplacian and  $\times$  is cross-product):

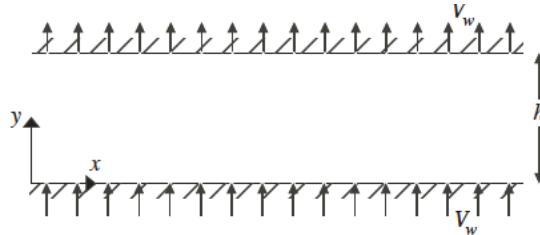
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla^2 \mathbf{B}$$

Write the equation in index notation (make sure the indices are consistent and follow the index laws).

6. [20 points] Consider a 2D velocity vector field given by  $u=x^2$  and  $v = -2xy-1$

- a. Compute the components of the strain rate tensor.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
- b. By just looking at the strain rate tensor (without any other calculation) is the flow incompressible? Why?
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
- c. Compute the vorticity vector. Is the flow irrotational?

7. [20 points] Consider flow in a channel (similar to Couette flow) but with porous walls. A constant vertical velocity  $V_w$  exists in the  $y$  direction at the top/bottom porous walls (instead of no-penetration). The flow is driven by a given pressure gradient  $\frac{\partial P}{\partial x}$ . The flow is assumed fully developed.



*x-component of the incompressible Navier–Stokes equation:*

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial x} + \rho g_x + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

*y-component of the incompressible Navier–Stokes equation:*

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial P}{\partial y} + \rho g_y + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

*z-component of the incompressible Navier–Stokes equation:*

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial P}{\partial z} + \rho g_z + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

- Use the continuity equation to find the  $y$  component of velocity  $v$  everywhere. (state assumptions)
- Start with the  $x$ -component of momentum equation and simplify it (state all assumptions). Write the boundary conditions needed to solve the equation.
- [Extra credit: +6]** Solve the equation to find the  $x$ -component of velocity  $u$ . To make the algebra easy assume  $\rho = \mu = V_w = \frac{\partial P}{\partial x} = 1$ .

**Take-home portion (35/100):**

In order to protect the integrity of this course, I am requiring all students taking this exam to sign a pledge that you will not cheat on this exam (Exam 1 ME EN 5700/6700). Please note that the take-home part is open book and open notes, however, all of the work that you submit must be your own. You may consult material and problems in the book, in your notes, but you are not allowed to discuss the material on this exam with your peers in the class or anyone else prior to the deadline (10 am Oct 19<sup>th</sup>). Furthermore, you are not allowed to post questions to online forums. **Anyone found to have cheated on this exam will automatically fail the exam.**

I affirm that I, \_\_\_\_\_ (write your name) am submitting my own work for this exam. I have neither given nor received help on this exam. I have not consulted with anyone (other than the instructor) on questions nor looked at or copied another student's work. I understand the penalties for academic dishonesty on this exam is failure.

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Signature

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Date/Time

Submit on a separate paper (upload to Canvas by 10am Thurs Oct 19<sup>th</sup>)

You can also submit one of the in-class questions that you thought you performed not so great and get increased partial credit based on your new submission.

8. [10 points] Pressure Poisson equation (PPE)

Consider the pressure Poisson equation

$$\frac{\partial}{\partial x_i} \left( \frac{\partial P}{\partial x_i} \right) = -\rho \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j}$$

Write down this equation for a 3D flow in terms of P, and 3D velocity components u,v,w (i.e., convert from index notation to a typical equation).

9. [10 points] Rotation tensor:

In the kinematics lectures, we learned that a line element evolves according to the velocity gradient as

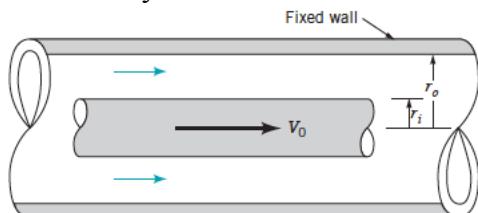
$$\frac{d}{dt} (\delta x_i) = \delta x_j \frac{\partial V_i}{\partial x_j}$$

Let's look into the role of rotation tensor part of the velocity gradient. Plug in the rotation tensor  $R_{ij}$  in the velocity gradient in this equation and demonstrate that the action of rotation tensor does not change the length of the line element. That is,

$$\frac{d}{dt} (\delta x_i^2) = 0$$

10. [15 points] Consider the Poiseuille flow we derived in class with the same assumptions (steady, fully developed, driven by a given pressure gradient, etc.). We plug in another cylinder inside the tube and the inner cylinder is moving with a velocity  $V_0$  as shown. The inner moving cylinder has radius  $r_i$  and the outer fixed cylinder has radius  $r_o$ . Use cylindrical equation form of Navier-Stokes (just like how we derived Poiseuille flow) and find the velocity profile. Note the fluid domain here is  $r_i < r < r_o$

**Hint:** A key difference with Poiseuille flow here is the boundary conditions.



11. [OPTIONAL: Bonus] Redo one of the in-class questions that you think you did not perform that well (you will not get penalized if this is worse solution!).