
Fluid Mechanics (ME EN 5700/6700)

Exam 1, Fall 2012

(Open Book, Open Notes, Closed neighbor)

0. [0 pt] What is your name?

1. [8 pt] Continuum Hypothesis

(a) Explain why the continuum hypothesis is important in the study of fluid dynamics

(b) Give an example of a flow in which the continuum hypothesis would be invalid. Why?

2. [8 pt] Write the following vector quantities in index notation.

(a) $\vec{u} \cdot (\vec{\nabla} \times \vec{u})$

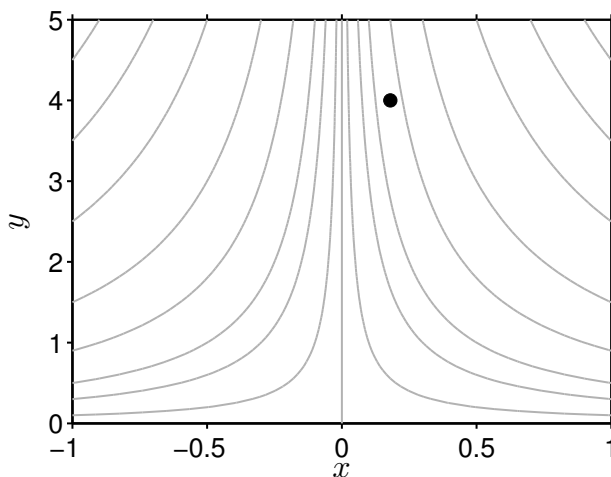
(b) $(\vec{\nabla} \times \vec{a}) \cdot (\vec{c} \times \vec{b})$

3. [8 pt] Conservation of Mass

- (a) Write the conservation of mass equation for a fluid in the Lagrangian view.
- (b) Write the conservation of mass equation for a fluid in the Eulerian view.
- (c) Can a *steady* flow with a non-uniform density field of the form $\rho = \rho(x, y)$ be incompressible? Why or Why not? [Show your work.]

4. [8 pt] List and explain the key assumptions required in the derivation of a constitutive equation (i.e., stress tensor closure) for a Newtonian fluid.

5. [10 pt] The image below illustrates the streamline pattern in a flow at a given time t . A fluid particle is indicated by the black circle.

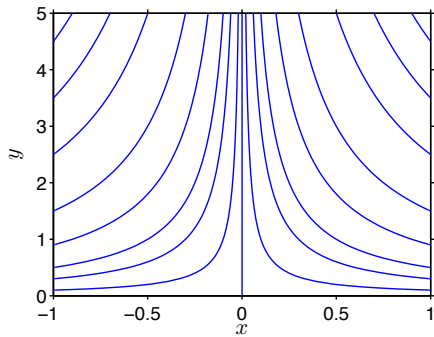


- Define the concept of *streamline* (in words).
- Provide the mathematical definition of the *streamfunction*, ψ .
- How are the streamlines related to the streamfunction?
- From this image alone, can you predict the pathline traced out by the fluid particle (black circle) that originated at $(x, y) = (-0.5, 2)$ at time $t = 0$? Why or why not?

6. [20 pt] Consider unsteady, stagnation point flow in a plane, where the velocity components are

$$u = (1 + 2t)x \quad \text{and} \quad v = y. \quad [\text{note it should be } v = -y]$$

The streamline pattern is shown below. Note that although the magnitude of the horizontal velocity is changing in time, the streamline pattern remains the same.

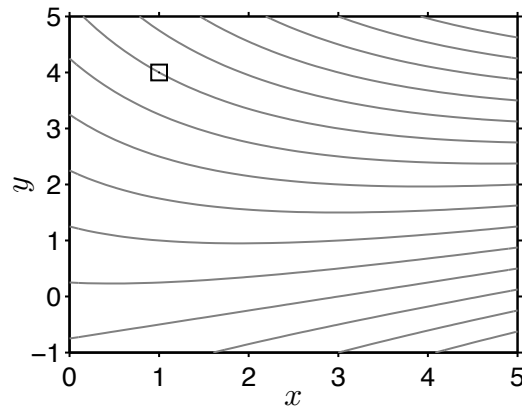


- (a) Is the flow incompressible? Show your work.

- (b) Assume that the density is uniform in space. Derive an expression for the density as a function of time, $\rho(t)$.

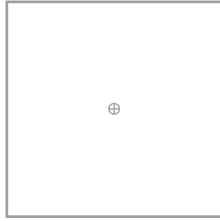
- (c) What is the local time rate of change of momentum? [Remember that momentum is a vector quantity.]

7. [26 pt] Streamlines of the flow in a converging duct is shown below. The horizontal and vertical velocity components associated with this flow are, respectively: $u = 0.5x + 1.5$ and $v = -0.5y + 0.25x$.



- (a) Determine the components in the strain-rate tensor, e_{ij} .
- (b) Determine the components in the vorticity vector, ω_i .
- (c) On the image provided, draw the pathline of the fluid particle that originates at point $(x = 1, y = 4)$ at time $t = 0$.

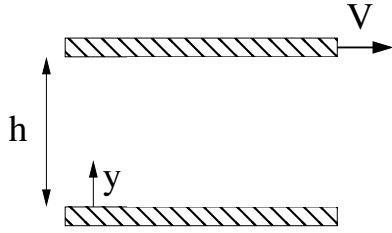
- (d) Consider an initially square particle with centroid located at $(x = 1, y = 4)$, as shown in the streamline plot above. How will the fluid particle look after a infinitesimal time Δt later? [Support your reasoning with calculations]



- (e) Derive expressions for the components of the acceleration of a generic fluid particle.

- (f) Explain how the momentum of the fluid particle at $(x = 1, y = 4)$ is changing with respect to time. [Support your answer with calculations]

8. [12 pt] Consider steady flow between two flat plates (separated by a distance h) resulting from the constant motion of the top plate a velocity U_0 .



- (a) Using the continuity equation derive an equation for the vertical velocity v . [state assumptions and show your work.]
- (b) Simplify the Navier-Stokes equations (start with the full equation, x-component **only**) for this flow and provide appropriate boundary and initial conditions. [State assumptions and show your work.]
- (c) Solve for the horizontal velocity u . [State assumptions and show your work.]
- (d) Explain what the role of viscous diffusion is in this flow at steady state. What is happening to a fluid particle in this flow field at steady-state? [Discuss momentum transport and kinematics.]