Quiz 4: Conservation of Mass and Momentum Equation,

September 25, 2025

Q1.- Write Cauchy's equation of motion (expanding the material derivative term).
hint: Before jumping to your notes, try to write it down by discussing with your working partner/s and thinking a bit.

Response:

Cauchy's equation of motion is:

$$\rho \frac{\partial u_i}{\partial t} + \rho\, u_j \frac{\partial u_i}{\partial x_j} = \rho\, g_i + \frac{\partial \tau_{ij}}{\partial x_j}. \tag{1}$$

Q2.- What is the physical meaning of the term $\rho u_j \frac{\partial u_i}{\partial x_j}$ mean?

Response:This term is the advection term, and accounts for the fact that the momentum of a fluid particle might be changing as it passes through a fixed point in space (apparent acceleration).

Q3.- How does one generically write the non-penetration boundary condition for fluid flows?

Response:

$$\vec{u} \cdot \hat{n} = 0. \tag{2}$$

Q4.- What about the non-slip condition?

Response:The no-slip boundary condition states that the velocity component of the fluid tangential to a stationary solid is zero,

$$\vec{u_1} \cdot \hat{t} = 0 \qquad \text{(along the surface)}. \tag{3}$$

Q5.- Write down Cauchy's Equation of motion, and simplify it down for a steady flow, contained within two infinitely long plates, where the top one moves at a certain velocity.

Response:
In the streamwise-direction:

$$\cancel{\rho \frac{\partial u}{\partial t}} + \rho \left[\cancel{u \frac{\partial u}{\partial x}} + \cancel{v \frac{\partial u}{\partial y}} \right] = \cancel{\rho g_x} + \left[\cancel{\frac{\partial \tau_{xx}}{\partial x}} + \frac{\partial \tau_{xy}}{\partial y} \right]. \tag{4}$$

In the spanwise-direction:

$$\cancel{\rho \frac{\partial v}{\partial t}} + \rho \left[\cancel{u \frac{\partial v}{\partial x}} + \cancel{v \frac{\partial v}{\partial y}} \right] = \rho g_y + \left[\cancel{\frac{\partial \tau_{yx}}{\partial x}} + \frac{\partial \tau_{yy}}{\partial y} \right]. \tag{5}$$