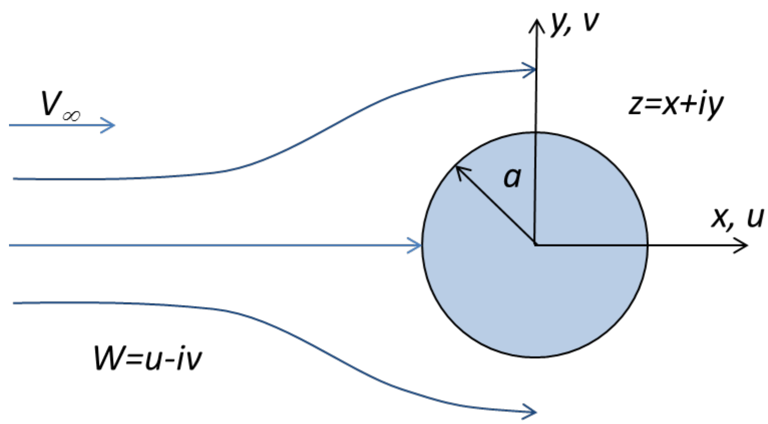
Homework 4

Vorticity and Ideal Flow

Your answers to these questions, and what you learn from them, will be greatly enhanced through collaboration and discussion amongst your discussion group and in the recitation. This is actively encouraged. However, once you have decided how to answer these, the final solutions must be prepared individually.

1. The flow past a circular cylinder of radius a centered at the origin, produced by a free stream of velocity V_∞ in the positive x direction, is described by the 'complex' velocity field $W = V_\infty - V_\infty a^2/z^2$. The real and imaginary parts of W are the x component and the *negative* of the y component of velocity, respectively. The real and imaginary parts of z are the x and y coordinates (no negative here). This ideal flow solution is pretty realistic over the upstream half of the cylinder. Consider now a real situation where the free stream flow includes some low level turbulence. An eddy of initial vorticity Ω_∞ is convected towards the cylinder along its stagnation streamline. Estimate (in terms of the parameters given) the average vorticity in the eddy when it passes through the point $z = -2a$ point if the eddy is initially aligned with the (a) x axis, and (b) the y axis. (c) Another eddy, initially aligned with the y axis is convected along a path above the stagnation streamline and through the point $z = -2a + ia$ (where i is the square root of minus 1). Estimate the angle of the eddy, relative to the y axis (positive counter-clockwise) at this point.



2. (a) Show using the complex potential or velocity, that a doublet can be generated by a pair of equal-strength opposite-signed point vortices, brought infinitely close together, while holding the product of their strength and the distance between them constant. Give the doublet strength of this product, and determine the doublet direction as compared to the initial position of the vortices. (b) Explain the multivaluedness of the function $\log_e(z)$. What about $\log_e(z - A)$, where A is a complex constant?

3. Investigate the flow $W(z) = Az^2$ where A is a complex constant. Determine expressions for the polar velocity components, the streamfunction and velocity potential. Plot the streamlines and determine the influence of the argument of A.