Example: Residual stresses in an assembly (Ex. 12.3, U&F)

Given / find: see slide (I solved with tensile loading & release).

Sol'n: Structure doesn't yield until all materials do.

Loading (elastic + plastic)

- Yield loads
  - $P_a = \sigma_{a,yp} A_a = (320\,MPa)(500\,mm^2) = 160\,kN$
  - $P_s = \sigma_{s,yp} A_s = (240\,MPa)(750\,mm^2) = 180\,kN$
  - $\Rightarrow P_u = 2(160\,kN) + 180\,kN = 500\,kN$

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Load release (or re-load) (all elastic)

- Geometry: $\delta_a = \delta_s \Rightarrow \frac{P'_a L_a}{A_a E_a} = \frac{P'_s L_s}{A_s E_s}$

$$\Rightarrow \frac{P'_a (360\,mm)}{(500\,mm^2)(70\,GPa)} = \frac{P'_s (375\,mm)}{(750\,mm^2)(210\,GPa)} \Rightarrow P'_s = 4.32\,P'_a \quad (1)$$

- From equilibrium, $2P'_a + P'_s = 500\,kN \quad (2)$

$(1) \rightarrow (2)$: $2P'_a + 4.32P'_a = 500\,kN \Rightarrow P'_a = \frac{500\,kN}{6.32} = 79.1\,kN$

$\Rightarrow P'_s = 341.7\,kN$

- Superposition
  - $P_a - P'_a = 160\,kN - 79.1\,kN = 80.9\,kN$
  - $P_s - P'_s = 180\,kN - 341.7\,kN = -161.7\,kN$

  * Note that $2(80.9\,kN) - 161.7\,kN = 0$

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- Stress:
  - $\sigma_a = \frac{80.9\,kN}{500\,mm^2} = 162\,MPa$
  - $\sigma_s = \frac{-161.7\,kN}{750\,mm^2} = -216\,MPa$

* Note: assembly remains fully elastic thereafter for $P_u \leq 500\,kN$

CHECK

* ΔL @ largest deformation

$$\delta = \frac{P_a L_a}{A_a E_a} = \frac{(160\,kN)(360\,mm)}{(500\,mm^2)(70\,GPa)} = 0.00160\,m = 1.6\,mm$$

(compared to 360 mm)