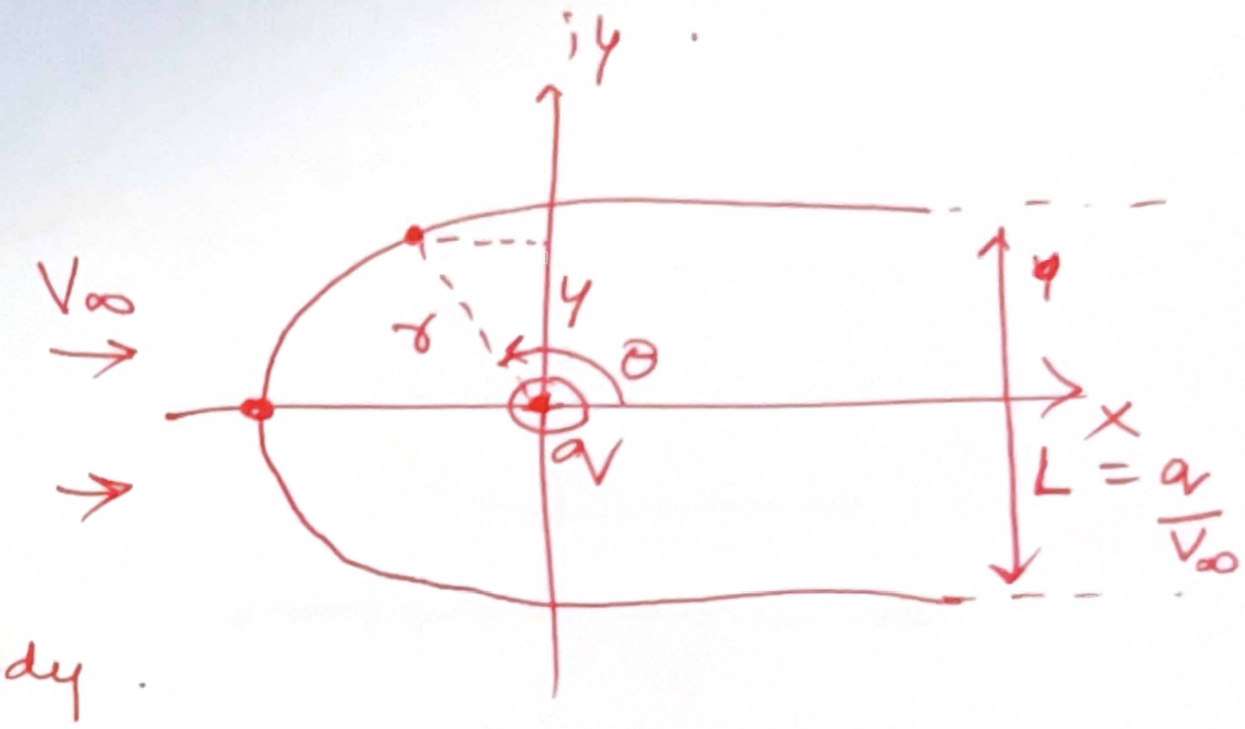
i) $\alpha = 0$, q at $z_1 = 0$.

$$W(z) = V_\infty + \frac{q}{2\pi z}$$

$$F(z) = V_\infty z + \frac{q}{2\pi}\log_e(z)$$



We know

$$\psi = \frac{q}{2V_\infty}\left(1 - \frac{\theta}{\pi}\right) \text{ on body.}$$

$$\therefore\ r = \frac{y}{\sin\theta}\ ;\quad \text{so } \underline{z \text{ on body}} = re^{i\theta} = \frac{y}{\sin\theta}e^{i\theta}$$

$$W(z) = V_\infty + \frac{q\sin\theta}{2\pi y e^{i\theta}} = V_\infty + \frac{V_\infty \sin\theta}{\pi\left(1 - \frac{\theta}{\pi}\right)}e^{-i\theta}$$

$$= V_\infty + \frac{V_\infty \sin\theta}{\pi - \theta}e^{-i\theta}$$

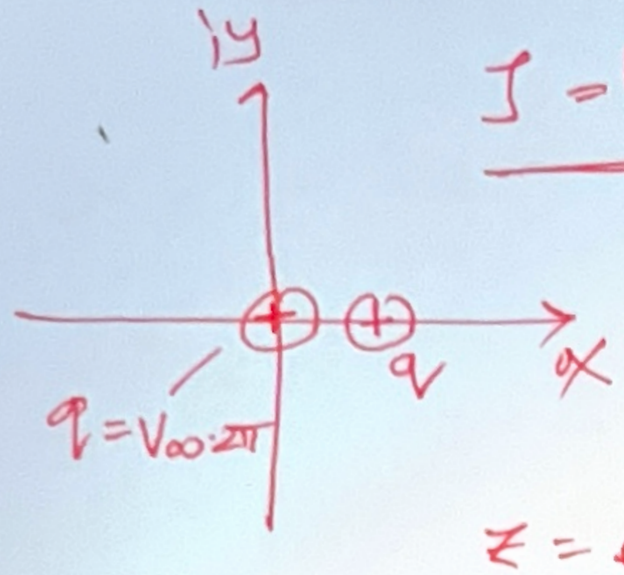
$$C_p = 1 - \frac{|W(z)|^2}{V_\infty^2} = 1 - \left|1 + \frac{\sin\theta}{\pi - \theta}e^{-i\theta}\right|^2.$$

$$= 1 - \left|\frac{\sin\theta\cos\theta}{(\pi - \theta)} + 1 - i\frac{\sin^2\theta}{(\pi - \theta)}\right|^2$$

$$= 1 - \left(1 + \frac{\sin^2\theta\cos^2\theta}{(\pi - \theta)^2} + \frac{2\sin\theta\cos\theta}{(\pi - \theta)} + \frac{\sin^4\theta}{(\pi - \theta)^2}\right)$$

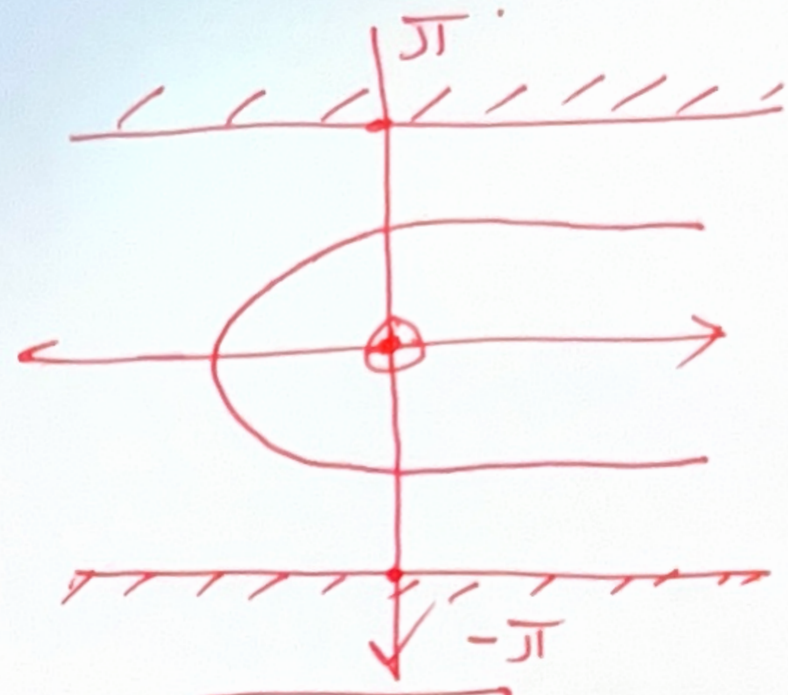
$$C_p = -\left(\frac{\sin^2 2\theta}{4(\pi - \theta)^2} + \frac{\sin 2\theta}{(\pi - \theta)} + \frac{\sin^4(\theta)}{(\pi - \theta)^2}\right)$$

b) Using log-mapping of a flow containing two sources as shown below.



$\zeta = \log_e z$

$z = e^{\zeta}$



z-plane

ζ-plane

$$F(z) = \frac{q}{2\pi}\log_e(z-1) + \frac{V_\infty \cdot 2\pi}{2\pi}\log_e z$$

$$\tilde{F}(\zeta) = F(z(\zeta)) = \frac{q}{2\pi}\log_e(e^{\zeta}-1) + V_\infty \zeta.$$

$$\therefore \tilde{F}(\zeta) = V_\infty \zeta + \frac{q}{2\pi}\log_e(e^{\zeta}-1)$$

$$\tilde{W}(\zeta) = W(z)\cdot\frac{dz}{d\zeta} = \left(\frac{q}{2\pi(e^{\zeta}-1)} + \frac{V_\infty}{e^{\zeta}}\right)e^{\zeta}$$

$$\tilde{W}(\zeta) = V_\infty + \frac{q}{2\pi\left(1-\frac{1}{e^{\zeta}}\right)}$$

Now. stagnation point $\Rightarrow \tilde{W}(\zeta) = 0 \Rightarrow V_\infty + \frac{q}{2\pi\left(1-\frac{1}{e^{\zeta}}\right)}$

$$\therefore \zeta_{stag.} = -\log_e\left(1 + \frac{q}{2\pi V_\infty}\right)$$

i) $z_{stag} = -0.3 = -\frac{q}{2\pi V_\infty} \Rightarrow q = 0.3(2\pi \cdot 10) = 18.84$

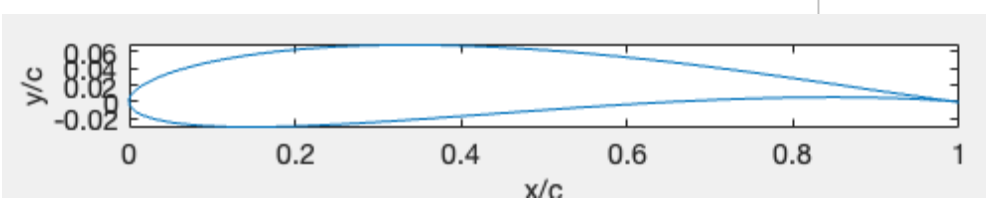
$$J_{stag} = -0.3 = -\log\left(1 + \frac{q}{2\pi V_\infty}\right)$$

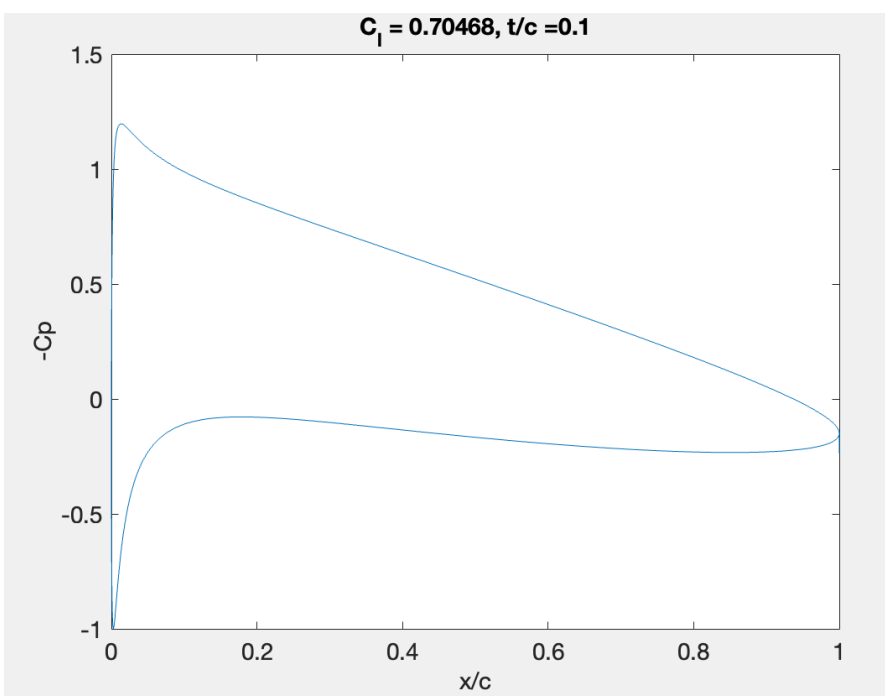
$$\Rightarrow e^{0.3} = 1 + \frac{q}{2\pi(10)}$$

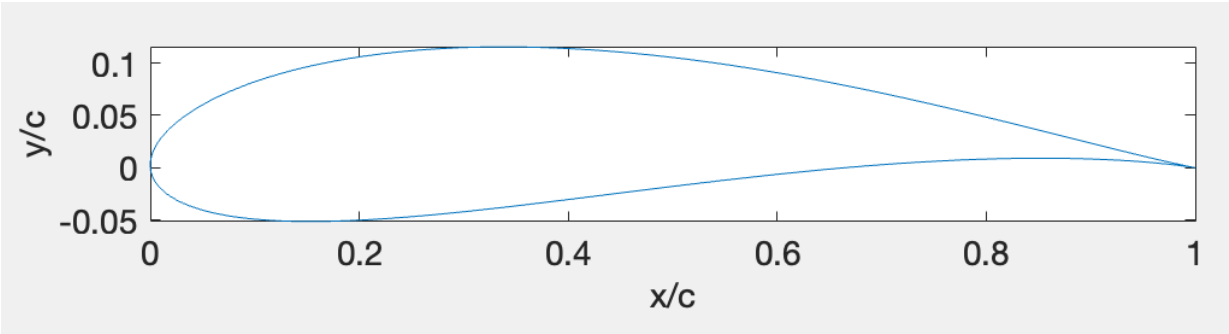
$$\Rightarrow \left(e^{0.3} - 1\right) 2\pi \cdot 10 = q = 0.349(2\pi \cdot 10) = 21.92$$

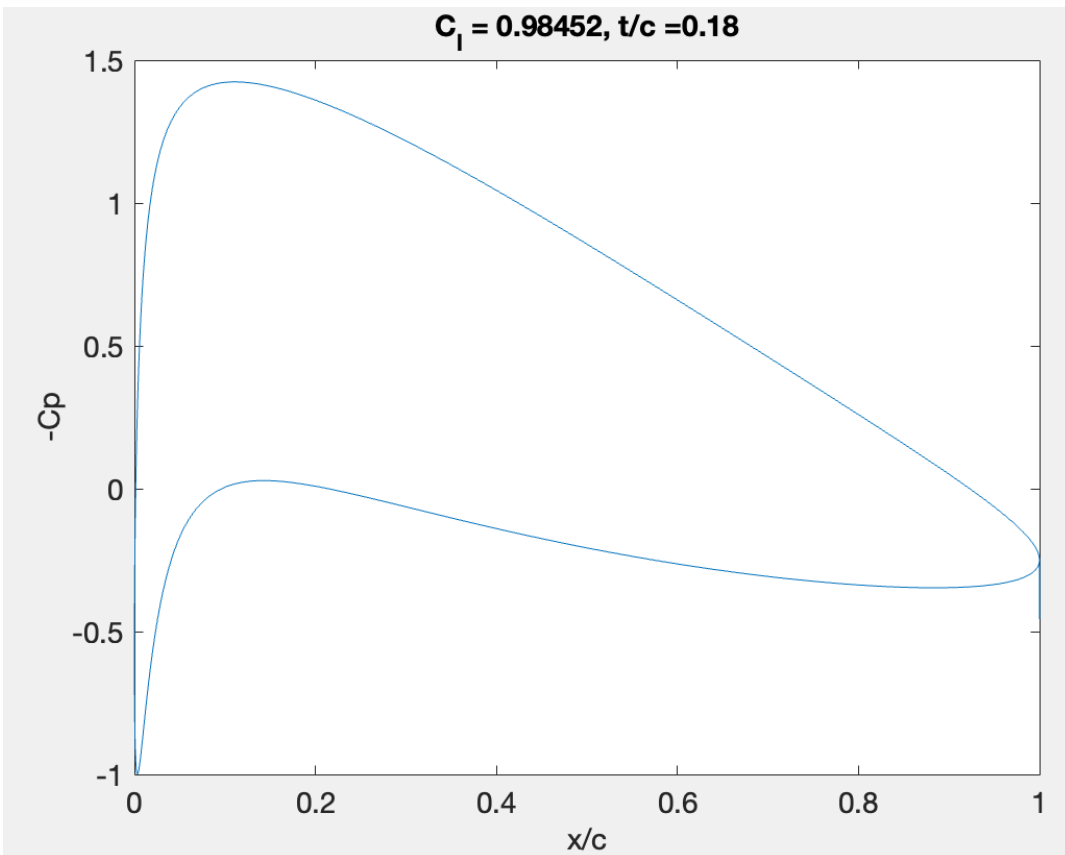
$\Rightarrow q_{channel} = 21.92$; $q_{free\text{-}flight} = 18.84$

```
C=1; Vinf = 1;
tc = 0.1 %input('Airfoil thickness to chord ratio :');
beta = 3*pi/180 %-input('zero lift angle of attack (deg) : ')*pi/180;
alpha = 3*pi/180;%input('Angle of attach (deg) :')*pi/180;

% Center of circle in unmapped zeta-plane
zeta1 = -0.7698*tc + i*( C + 0.7698*tc)*tan(beta);

%radius of circle
a = abs(C-zeta1);

% Define  points on circle
sigma = [0:360]/360*2*pi;
zeta = zeta1 + a*exp(i*sigma);

% Transform circle using Jukowski mapping
z = zeta + C.*C./zeta;
chord = max(real(z)) - min(real(z));
figure
plot((real(z)-min(real(z)))/chord, imag(z)/chord);
xlabel('x/c'); ylabel('y/c');
axis image

% Cp distribution
gamma = -4*pi*a*Vinf*sin(alpha + beta);
Wzeta = Vinf*exp(-i*alpha) - Vinf*a^2*exp(i*alpha)./(zeta - zeta1).^2 - i*gamma/2/pi./(zeta - zeta1);
dzetadz = 1./(1- C.*C./zeta.^2);

Wz = Wzeta.*dzetadz;

Cp = 1 - abs(Wz).^2/Vinf^2;

figure
plot((real(z)-min(real(z)))/chord,-Cp);
fontsize = 11;
```






0.1
0.05
0
-0.05
y/c
0
0.2
0.4
0.6
0.8
1
x/c


C_l = 0.98452, t/c =0.18
1.5
1
0.5
0
-0.5
-1
-Cp
0
0.2
0.4
0.6
0.8
1
x/c