ME EN 7210 Optimal Controls **Spring 2026**
Assignment #4 **Due Thursday, Feb. 19 at the beginning of class**

Problem 1

Read the two papers by Arechavaleta et al., on human walking and Mombaur et al., paper on optimal control for humanoids. Briefly answer the following questions about the two papers:

a) What is the main contribution of the paper?
b) How did they chose the objective (cost) function for their work? Please be very specific about this and describe in your own words the motivation and reasoning behind their choice. Also, you want to clearly articulate the variables or parameters they chose for their cost function. Basic hand-waving discussions or arguments will not be accepted.
c) How did they implement and validate their work?
d) What kind of results did they get and how well did it work?

Problem 2 Consider the extremization of a functional which is dependent on derivatives higher than the first derivative $\dot{x}(t)$ such as

$$J(x(t), t) = \int_{t_0}^{t_f} V(x(t), \dot{x}(t), \ddot{x}(t), t)dt.$$

with fixed-end point conditions. Show that the corresponding Euler-Lagrange equation is given by

$$\frac{\partial V}{\partial x} - \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial V}{\partial \ddot{x}}\right) = 0.$$

Hint: Use and follow the notes from class to show this – in other words, follow the formulation that was presented and show the same formulation but for the V(.) function given above.

Problem 3 For a second order system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -2x_1(t) + 3u(t)$$

with performance index

$$J = 0.5x_1^2(\pi/2) + \int_0^{\pi/2} 0.5u^2(t)dt$$

and boundary conditions $\mathbf{x}(0) = [0 \;\; 1]'$ and $\mathbf{x}(t_f)$ is free, find the optimal control.

Hint: There may not be an analytical solution, so you may have to solve for this using numerical techniques.