

Conversion from MPa to ksi: 1 ksi = 6.89 MPa

Conversion from mm to inch: 1 mm = 0.04 inch

Maximum stress from different loading conditions

Axial: $\sigma = F/A$

Bending:

- Transverse shear
 - o circular cross-section $\tau_{max} = 4V/3A$
 - o rectangular cross-section: $\tau_{max} = 3V/2A$
- Normal bending stress: $\sigma_{max} = My/I$ (where I is the area moment of inertia. See “Properties of Sections” below.

Torque: $\tau_{max} = Tr/I_p$ (where I_p (or J_p) is the polar moment of inertia. See “Properties of Sections” below.)

Design for static strength

Principal stresses (plane stress): $\sigma_A, \sigma_B = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Maximum shear stress (plane stress): $\tau_A, \tau_B = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Von Mises (or equivalent) stress (tri-axial): $\sigma_{eq} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$

Von Mises (or equivalent) stress (plane stress): $\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$

Von Mises (or equivalent) stress (plane stress – principal stresses): $\sigma_{eq} = \sqrt{\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2}$

Static failure criteria for ductile materials (assumes $\sigma_A > \sigma_B$)

	Maximum Shear Stress	Distortion Energy (Von Mises)
$\sigma_A \geq \sigma_B \geq 0$	$n = \frac{S_y}{\sigma_A}$	$n = \frac{S_y}{\sigma_{eq}}$
$\sigma_A \geq 0 \geq \sigma_B$	$n = \frac{S_y}{\sigma_A - \sigma_B}$	$n = \frac{S_y}{\sigma_{eq}}$
$0 \geq \sigma_A \geq \sigma_B$	$n = -\frac{S_y}{\sigma_B}$	$n = \frac{S_y}{\sigma_{eq}}$

($\sigma_{A,B}$: principal stresses, σ_{eq} : Von Mises stress, S_y : yield strength)

Static failure criteria for brittle materials (assumes $\sigma_A > \sigma_B$)

	Failure Theory		
	Maximum Normal Stress (MNS)	Brittle Coulomb-Mohr (BCM)	Modified-Mohr (MM)
$\sigma_A \geq \sigma_B \geq 0$	$n = \frac{S_{ut}}{\sigma_A}$	$n = \frac{S_{ut}}{\sigma_A}$	$n = \frac{S_{ut}}{\sigma_A}$
$\sigma_A \geq 0 \geq \sigma_B$	$n = \min \left[\frac{S_{ut}}{\sigma_A}, -\frac{S_{uc}}{\sigma_B} \right]$	$\frac{1}{n} = \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}}$	If $ \sigma_B < \sigma_A $, $n = \frac{S_{ut}}{\sigma_A}$
			If $ \sigma_B > \sigma_A $ $\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}}$
$0 \geq \sigma_A \geq \sigma_B$	$n = -\frac{S_{uc}}{\sigma_B}$	$n = -\frac{S_{uc}}{\sigma_B}$	$n = -\frac{S_{uc}}{\sigma_B}$

($\sigma_{A,B}$: principal stresses, S_{ut} : ultimate tensile strength, S_{uc} : ultimate compressive strength)

Design for fatigue strength

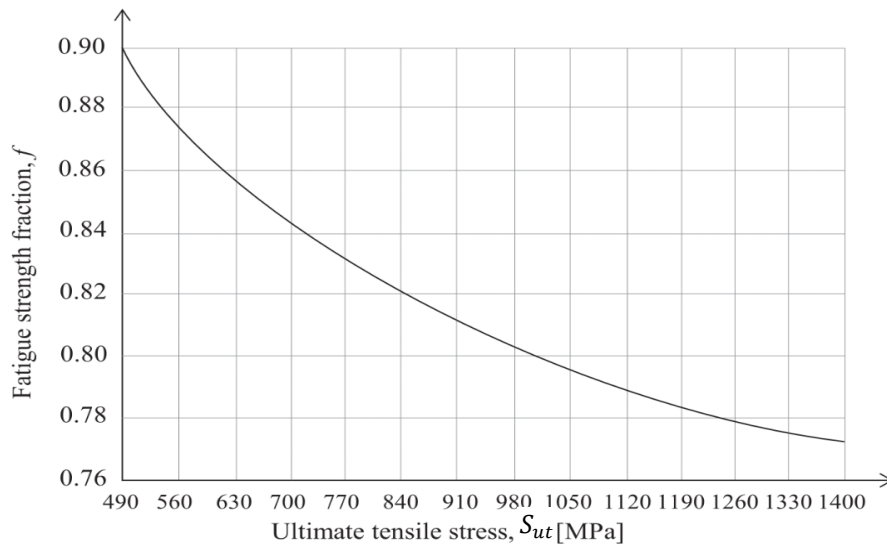
Fatigue life for $10^3 < N < 10^6$

$$S_f(N) = aN^b$$

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$b = -\frac{1}{3} \log \left(\frac{fS_{ut}}{S_e} \right)$$

Fatigue strength fraction, f



Endurance limit: $S'_e = 0.5S_{ut}$ when $S_{ut} < 1400 \text{ MPa}$
 $S'_e = 700 \text{ MPa}$ when $S_{ut} > 1400 \text{ MPa}$

Endurance limit modifying factors (Marin factors)

$$S_e = k_a k_b k_c k_d k_e k_m S'_e$$

$$- k_a = a S_{ut}^b$$

Surface Finish	Factor a		Exponent b
	S_{ut} , kpsi	S_{ut} , MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

$$- k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ [inch]} \\ 0.91d^{-0.157} & 2 \leq d \leq 10 \text{ [inch]} \\ 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ [mm]} \\ 1.51d^{-0.157} & 51 \leq d \leq 254 \text{ [mm]} \end{cases} \quad \text{for bending and torsion loading}$$

$$k_b = 1 \quad \text{for axial loading}$$

$$d_{eq} = 0.808\sqrt{bh} \quad \text{for rectangular cross-section with } b = \text{width and } h = \text{height}$$

$$d_{eq} = 0.37d \quad \text{for non-rotating shaft with circular cross-section of diameter } d$$

$$- k_c = \begin{cases} 1 & \text{bending or combined loading} \\ 0.85 & \text{axial loading} \\ 0.59 & \text{torsional loading} \end{cases}$$

$$- k_d = \frac{S_{ut,T}}{S_{ut,RT}}$$

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

$$- k_e = 1 - 0.08z \quad \text{where } z \text{ is from the standard normal distribution}$$

Fatigue failure criteria for fluctuating stress

- Soderberg: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$
- Mod-Goodman: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$
- Gerber: $\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$
- ASME-elliptic: $\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$

Calculating fatigue strength ($S_f(N)$) or number of cycles (N) for fluctuating stresses for $10^3 < N < 10^6$

Use equivalent fully reversed stress (σ_{rev}) for fluctuating stress

- Mod-Goodman: $\sigma_{rev} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}}$
- Gerber: $\sigma_{rev} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{S_{ut}}\right)^2}$

Von Mises stresses (alternating and midrange) for combined loading in fluctuating stress case

$$\sigma'_a = \left\{ \left[K_{f \text{ bending}} \sigma_{a \text{ bending}} + K_{f \text{ axial}} \frac{\sigma_{a \text{ axial}}}{0.85} \right]^2 + 3 \left[K_{fs \text{ torsion}} \tau_{a \text{ torsion}} \right]^2 \right\}^{\frac{1}{2}}$$

$$\sigma'_m = \left\{ \left[K_{f \text{ bending}} \sigma_{m \text{ bending}} + K_{f \text{ axial}} \sigma_{m \text{ axial}} \right]^2 + 3 \left[K_{fs \text{ torsion}} \tau_{m \text{ torsion}} \right]^2 \right\}^{\frac{1}{2}}$$

Fatigue stress concentration factor (K_f)

$$K_f = 1 + q(K_t - 1)$$

$$K_{fs} = 1 + q(K_{ts} - 1)$$

K_t : stress concentration factor

q : notch radius

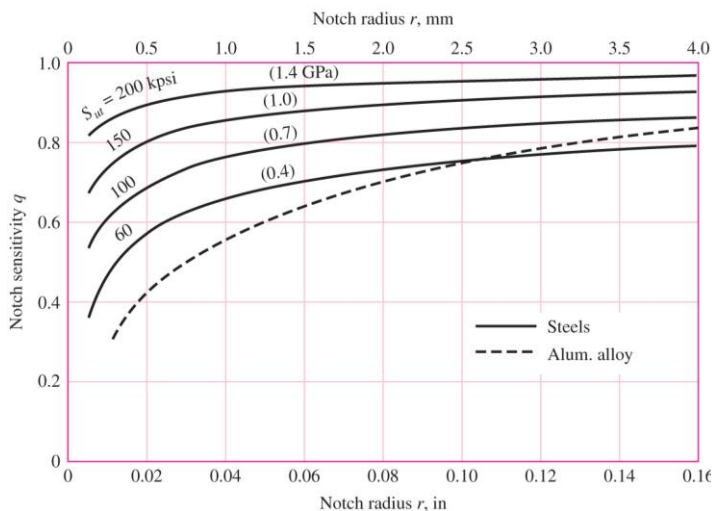
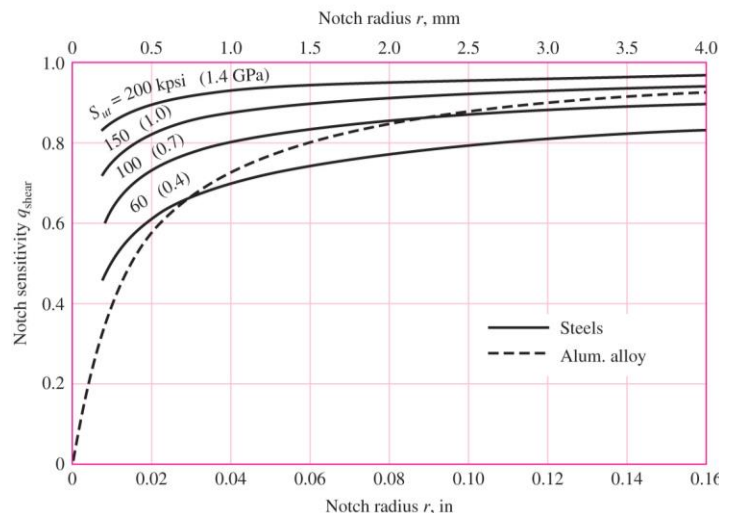
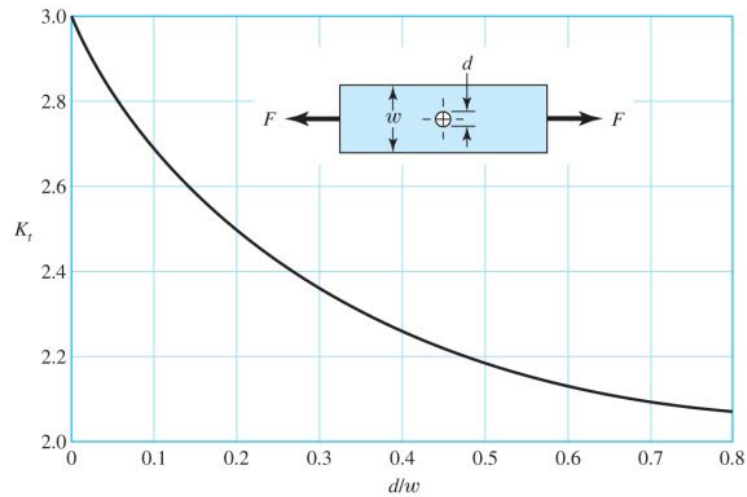
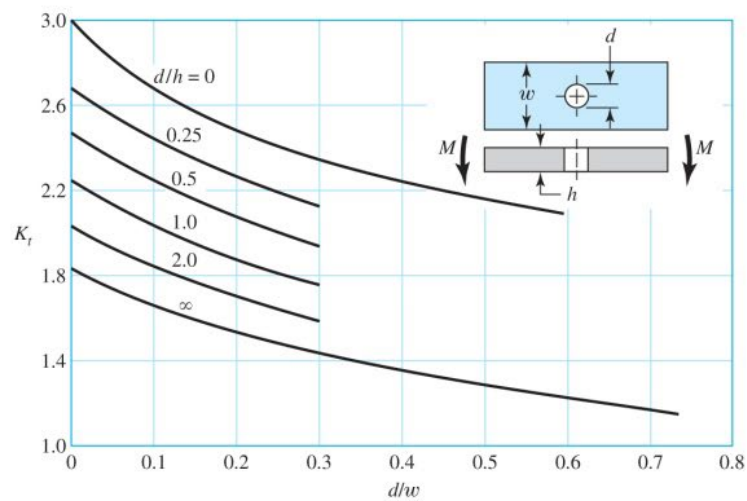
Notch sensitivity for bending or axial load**Notch sensitivity for torsional load**

Table A-15Charts of Theoretical Stress-Concentration Factors K_t^* **Figure A-15-1**

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

**Figure A-15-2**

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

**Figure A-15-3**

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

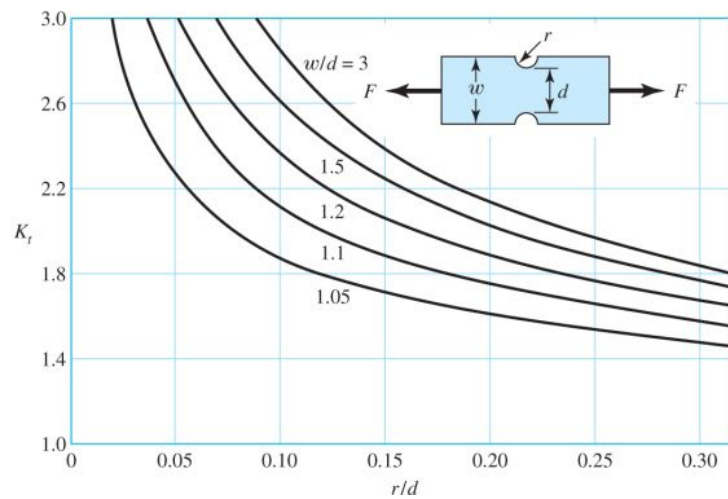
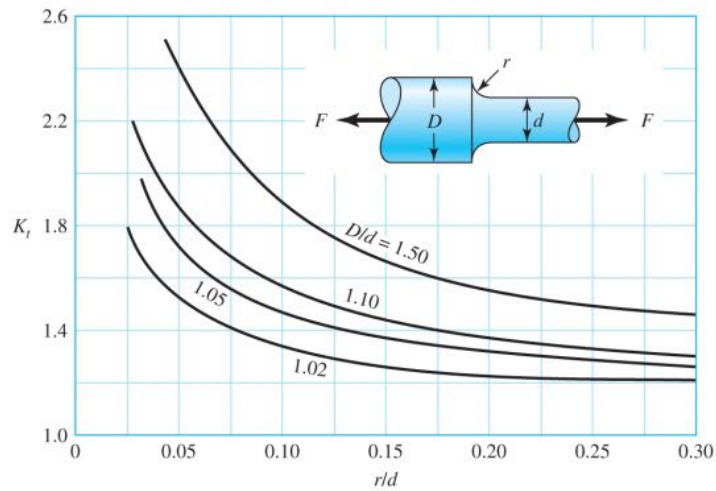
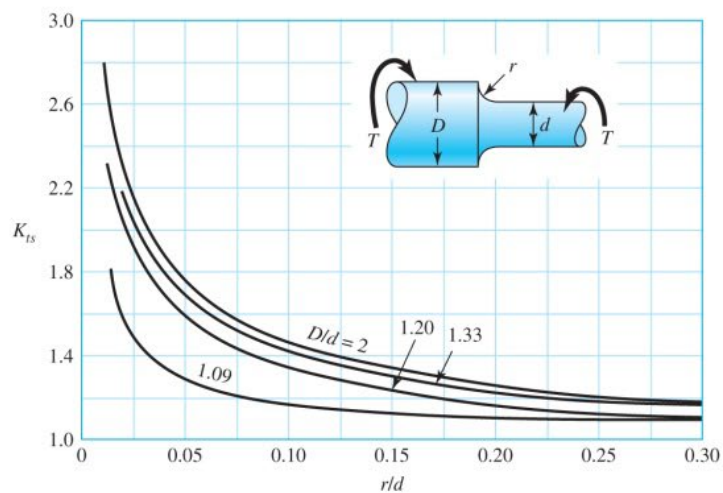


Table A-15Charts of Theoretical Stress-Concentration Factors K_t^* (Continued)**Figure A-15-7**

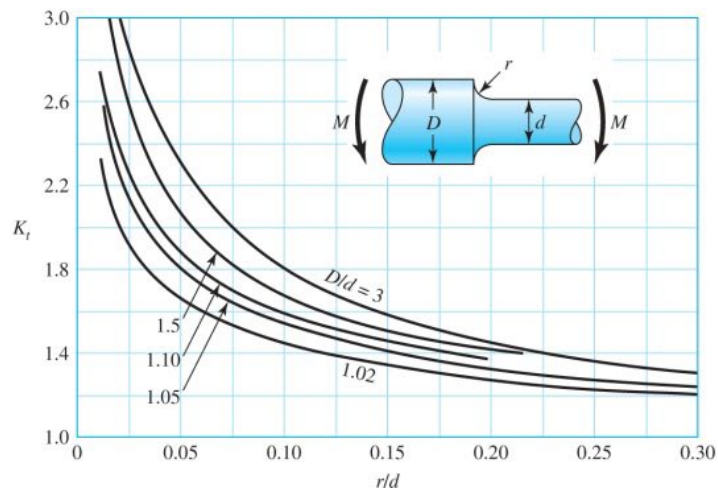
Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

**Figure A-15-8**

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

**Figure A-15-9**

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Part 1 Properties of Sections

A = area

G = location of centroid

$I_x = \int y^2 dA$ = second moment of area about x axis

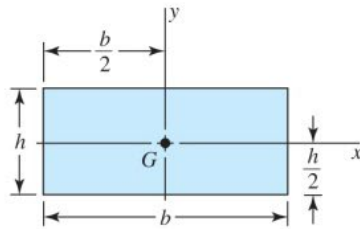
$I_y = \int x^2 dA$ = second moment of area about y axis

$I_{xy} = \int xy dA$ = mixed moment of area about x and y axes

$J_G = \int r^2 dA = \int (x^2 + y^2) dA = I_x + I_y$
= second polar moment of area about axis through G

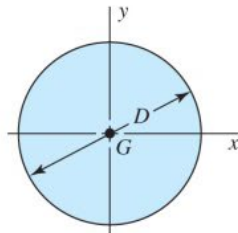
$k_x^2 = I_x/A$ = squared radius of gyration about x axis

Rectangle



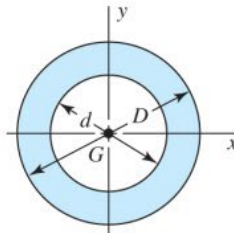
$$A = bh \quad I_x = \frac{bh^3}{12} \quad I_y = \frac{b^3h}{12} \quad I_{xy} = 0$$

Circle



$$A = \frac{\pi D^2}{4} \quad I_x = I_y = \frac{\pi D^4}{64} \quad I_{xy} = 0 \quad J_G = \frac{\pi D^4}{32}$$

Hollow circle



$$A = \frac{\pi}{4}(D^2 - d^2) \quad I_x = I_y = \frac{\pi}{64}(D^4 - d^4) \quad I_{xy} = 0 \quad J_G = \frac{\pi}{32}(D^4 - d^4)$$