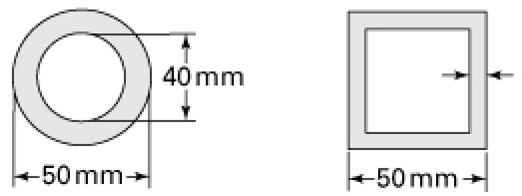
Homework 13 Solutions

1) The figure shows the cross sections of two aluminum alloy 2114-T6 bars that are used as compression members, each with effective length of L_e. Find (a) the wall thickness of the hollow square bar so that the bars have the same cross-sectional area and (b) the critical load of each bar. Given: $L_e = 3$ m and $E = 72\ GPa$ (from Table D.1).



(a) Same area:

$$\frac{\pi}{4}(d_o^2 - d_i^2) = b_o^2 - b_i^2$$

$$b_i^2 = b_o^2 - \frac{\pi}{4}(d_o^2 - d_i^2) = 50^2 - \frac{\pi}{4}(50^2 - 40^2)$$

or

$$b_i = 42.35\ mm \qquad t = \frac{1}{2}(b_o - b_i) = 3.83\ mm \qquad \blacktriangleleft$$

(b) Circular bar

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(50^4 - 40^4) = 181\times10^{-9}\ in.^4$$

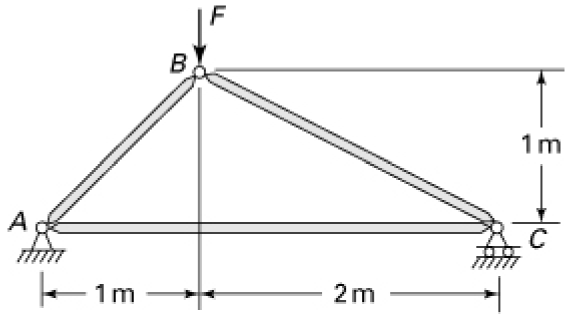
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(72\times10^9)(181\times10^{-9})}{(3)^2} = 14.29\ kN \qquad \blacktriangleleft$$

Square bar

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}(50^4 - 42.35^4) = 252.8\times10^{-9}\ m^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(72\times10^6)(252.8\times10^{-9})}{(3)^2} = 19.96\ kN \qquad \blacktriangleleft$$

2) Based on a factor of safety of $n = 1.8$, determine the maximum load F that can be applied to the truss shown. Given: Each column is of 50 mm-diameter aluminum bar having $E = 70\ GPa$.



$$L_{BC} = \sqrt{2^2 + 1^2} = 2.236\ m \qquad r = \frac{d}{4} = \frac{50}{4} = 12.5\ mm$$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (\cancel{40})^4 = \cancel{125.7(10^{-9})\ m^4} \ 3.07x10^{-7}, \qquad L_{AB} = \sqrt{1^2 + 1^2} = 1.414\ m$$

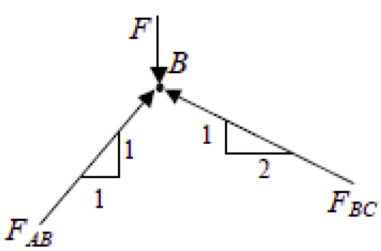
Bar BC $(L/r = 2{,}236/12.5) = 178$. Thus

$$(F_{BC})_{all} = \frac{P_{cr}}{n} = \frac{\pi^2(70)(1\cancel{25}.7)}{1.8(2.236)^2} = \cancel{9.65}\ kN \quad 23.55$$

Bar AB

$$(F_{AB})_{all} = \frac{P_{cr}}{n} = \frac{\pi^2(70)(1\cancel{25}.7)}{1.8(1.414)^2} = \cancel{24.1}\ kN \quad 58.89$$

Joint B



$$\sum F_x = 0: \frac{1}{\sqrt{2}} F_{AB} - \frac{2}{\sqrt{5}} F_{BC} = 0, \ F_{BC} = 0.791 F_{AB}$$

$$\sum F_y = 0: \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{5}} F_{BC} - F = 0, \ F = 1.061 F_{AB}$$

Solving

$$F = 1.061 F_{AB} \qquad F = 1.341 F_{BC}$$

The allowable value for F:

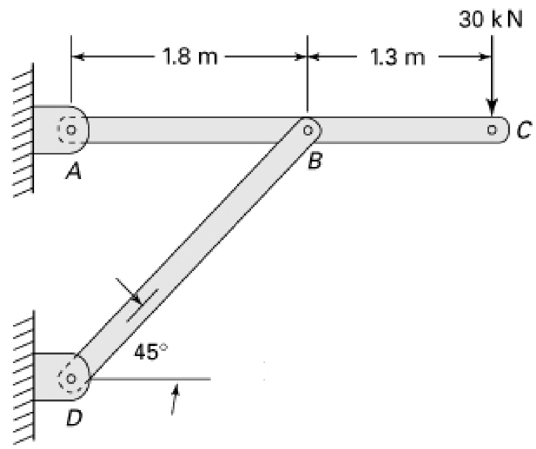
$$F < 1.341(\cancel{9.65}) = \cancel{12.93}\ kN \quad 31.6$$

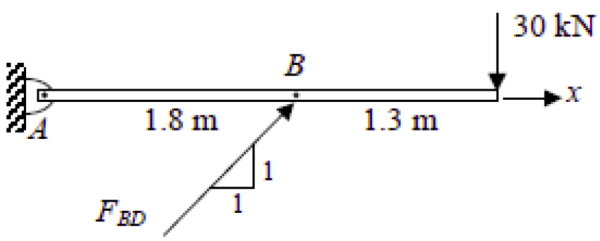
$$F < 1.061(\cancel{24.1}) = \cancel{25.6}\ kN \quad 62.4$$

Thus

$$F_{all} = \cancel{12.93\ kN} \qquad 31.6\ kN$$

3) Brace BD of the structure shown is made of a steel rod ($E = 210\ GPa$ and $\sigma_{yp} = 250\ MPa$) with a square cross section ($50\ mm$ on a side). Calculate the factor of safety n against failure by buckling.





$$\sum M_A = 0: \quad -30(3.1) + \frac{1}{\sqrt{2}} F_{BD}(1.8) = 0, \qquad F_{BD} = 73.1\ kN$$

$$I = b^4/12 = 50^4/12 = 520.8 \times 10^3\ mm^4, \qquad A = 50 \times 50 = 2.5 \times 10^3\ mm^2$$

$$r = \sqrt{\frac{I}{A}} = 14.4\ mm \qquad \frac{L}{r} = \frac{\cancel{1800}}{14.4} = 125$$

So,

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (210 \times 10^9)}{(125)^2} = 132.6\ MPa$$

We have

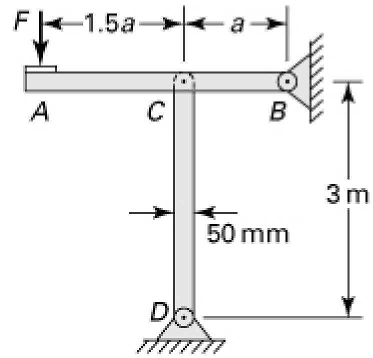
$$\sigma_{cr} < \sigma_{yp} \qquad \text{(solution is valid)}$$

$$(F_{BD})_{cr} = 132.6 \times 10^6 (2.5 \times 10^{-3}) = 331.5\ kN$$

and

$$n = \frac{(F_{BD})_{cr}}{F_{BD}} = \frac{331.5}{73.1} = \cancel{4.5} \quad 2.28$$

4) A horizontal rigid bar AB is supported by a pin-ended column CD and carries a load F. The column is made of steel bar having 50 by 50 mm square cross section, 3 m length, and $E = 200\ GPa$. What is the allowable value of F based a factor of safety of $n = 2.2$ with respect to buckling of the column?



We have

$$I = \frac{1(50)^4}{12} = 0.521 \times 10^6\ mm^4.$$

Hence

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200 \times 10^3)(0.521)}{(3)^2} = 114.3\ kN$$

$$P_{all} = \frac{P_{cr}}{n} = \frac{114.3}{2.2} = 52\ kN$$

Thus

$$F = \frac{P_{all}}{2.5} = \frac{52}{2.5} = 20.8\ kN \quad ◀$$