Aerospace Propulsion

Lecture 12

Airbreathing Propulsion II

IMAGINE U

Airbreathing Propulsion: Part II

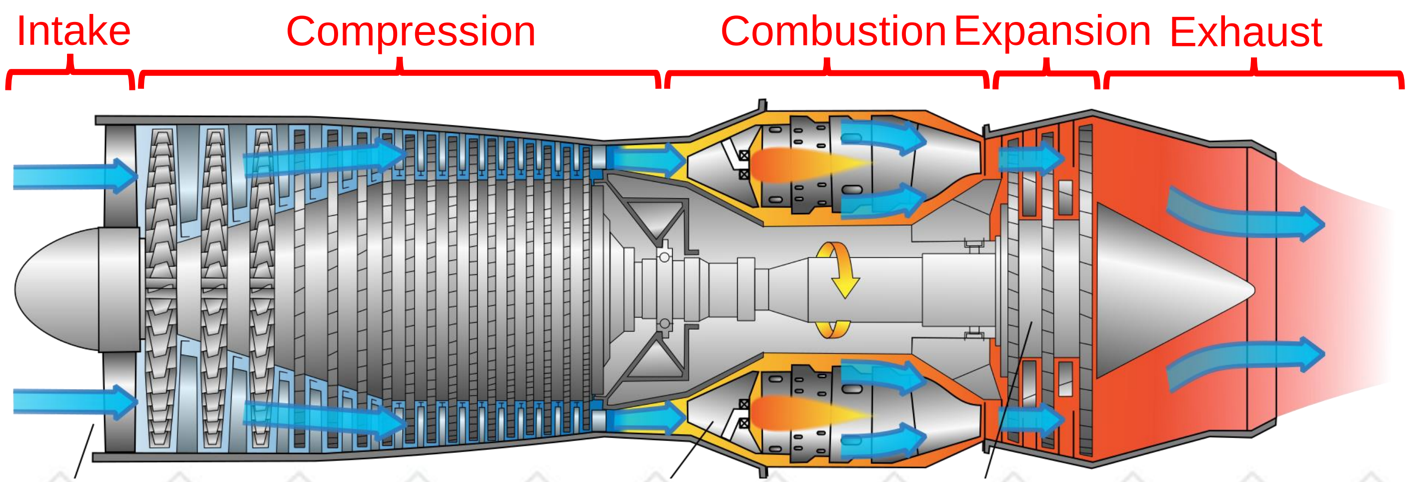
- First law for open cycles
- Ideal Brayton Cycle
- Turbojet Analysis

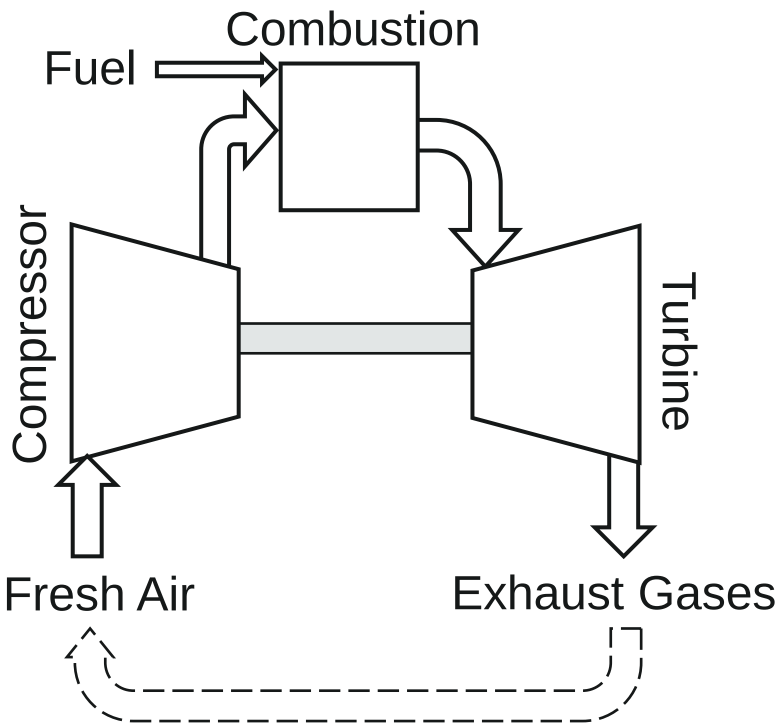
First law for open cycles

- So far, we generally considered closed systems
  - First Law: $Q_{in} - W_{out} = m\Delta h_t$
- Now, we consider open systems
  - First Law: $\dot{Q}_{in} - \dot{W}_{out} = \dot{m}\Delta h_t$
- Keep in mind, gas turbines are constantly flowing (at steady state)
  - We care about power, heat release rate and mass flow rate

Ideal Brayton Cycle

- Basic Turbojet represented by ideal Brayton cycle
  - 1-2: Isentropic compression
  - 2-3: Constant pressure heat addition
  - 3-4: Isentropic expansion
  - 4-1: Constant pressure heat rejection





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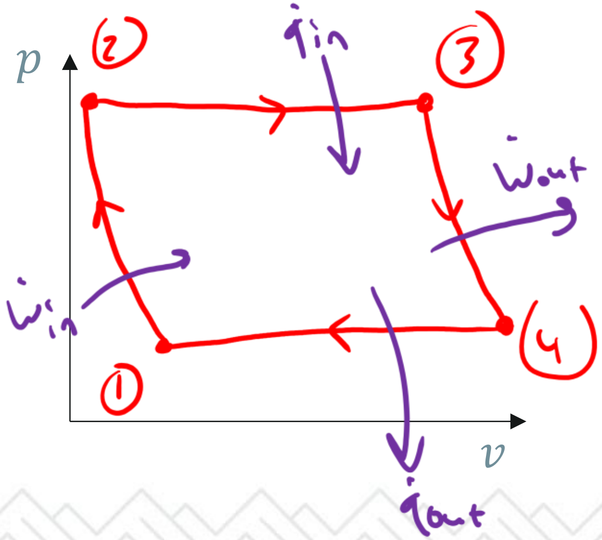

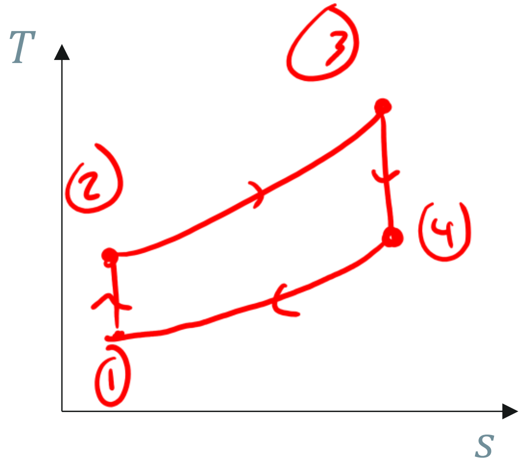

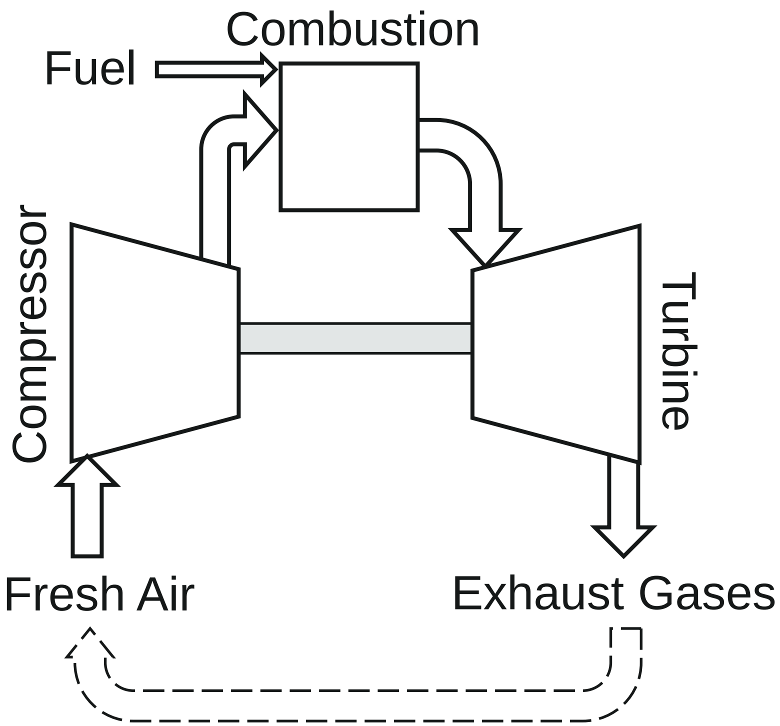

Assume ideal gas

Assume constant c_p

Ideal Brayton Cycle

- Basic Turbojet represented by ideal Brayton cycle
  - 1-2: Isentropic compression
  - 2-3: Constant pressure heat addition
  - 3-4: Isentropic expansion
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- Introduce new quantities
  - $r = \rho_2/\rho_1 = v_1/v_2$
  - $r_p = p_2/p_1$

Measure of compression in compressor

1→2: isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$$

2→3: isobaric heat addition

$$Q_{in} = m\Delta h$$

$$m_f(LHV) = m c_p (T_3 - T_2)$$

3→4: isentropic expansion

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$$

4→1: isobaric heat release

$$Q_{out} = m c_p (T_4 - T_1)$$

Ideal Brayton Cycle

- Ideal Brayton cycle efficiency
  - $\eta = \frac{W_{out} - W_{in}}{Q_{in}}$

$= 1 - \frac{Q_{out}}{Q_{in}}$

$= 1 - \frac{\dot{m} c_p (T_4 - T_1)}{\dot{m} c_p (T_3 - T_2)}$

$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$

$\eta = 1 - \frac{T_1 (T_4/T_1 - 1)}{T_2 (T_3/T_2 - 1)}$

$\eta = 1 - \frac{T_1}{T_2}$

$\eta = 1 - r_p^{\frac{1-\gamma}{\gamma}}$

$P_1 = P_4$
$P_2 = P_3$ $\Rightarrow \frac{P_4}{P_3} = \frac{P_1}{P_2}$

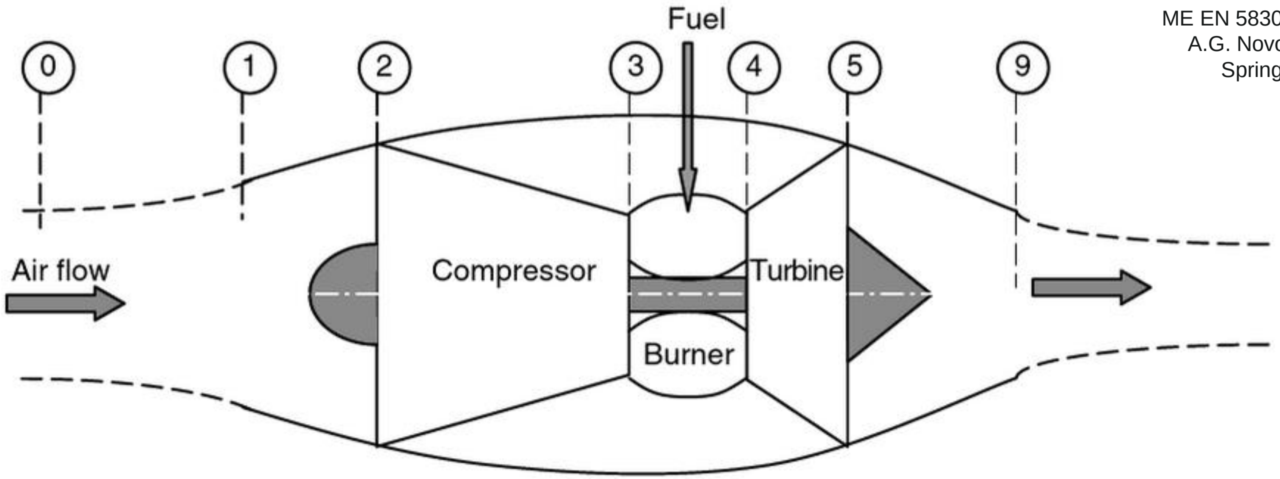
$\left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$

$\frac{T_4}{T_3} = \frac{T_1}{T_2}$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$

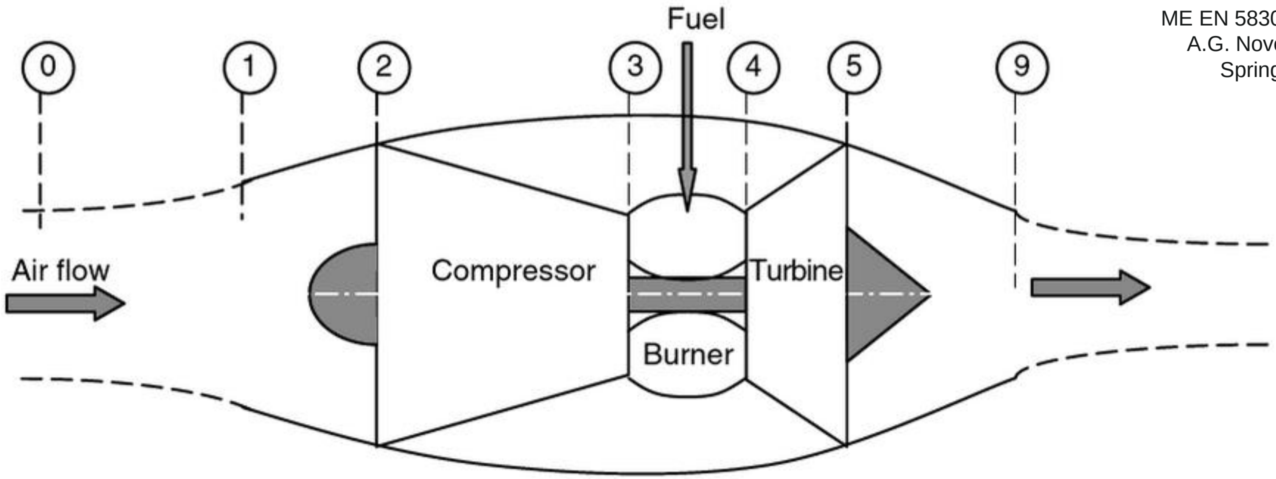
  - $\eta = 1 - r_p^{\frac{1-\gamma}{\gamma}} = \underbrace{1 - r^{1-\gamma}}_{\text{can show } r_p = r^{\gamma}}$

Turbojet Analysis



- Station Numbering
  - 0: Unperturbed flow
  - 1: Diffuser inlet
  - 2: Diffuser outlet/Compressor inlet
  - 3: Compressor outlet/Combustor inlet
  - 4: Combustor outlet/Turbine inlet
  - 5: Turbine outlet/Nozzle inlet
  - 6-8: Space for additional processes (later slides)
  - 9: Nozzle outlet

Turbojet Analysis

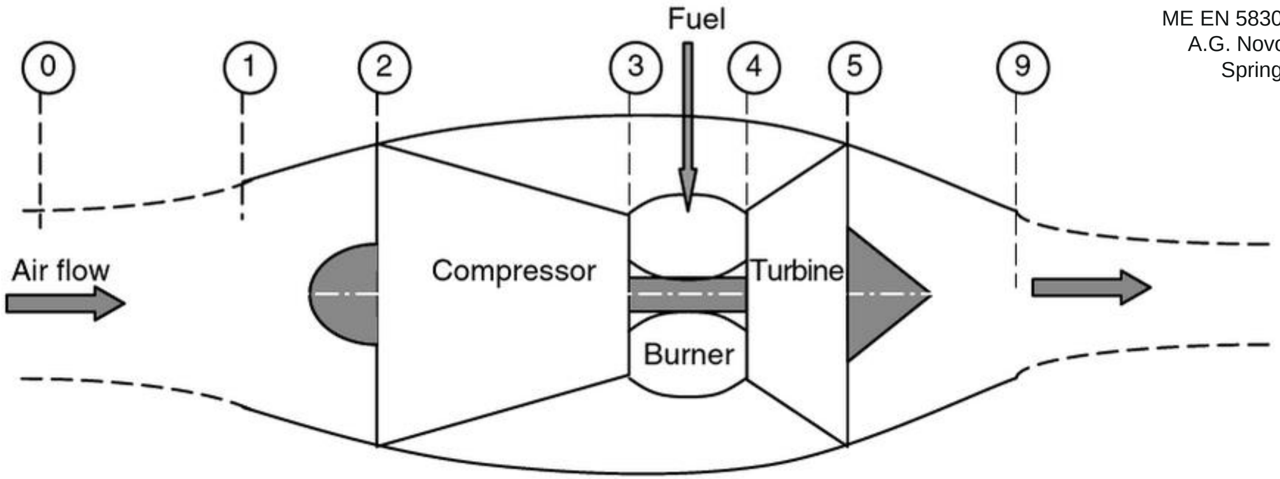


- Inlet (Diffuser) (0-2)
  - Delivers air to the compressor at the necessary Mach number
  - Commercial/Military aircraft fly at Mach numbers >0.7 (M_0)
  - Compressors are usually designed for inlet Mach numbers <0.6 (M_2)
    - Higher Mach numbers increase the risk of shocks

  - Decelerates flow
  - Increases pressure

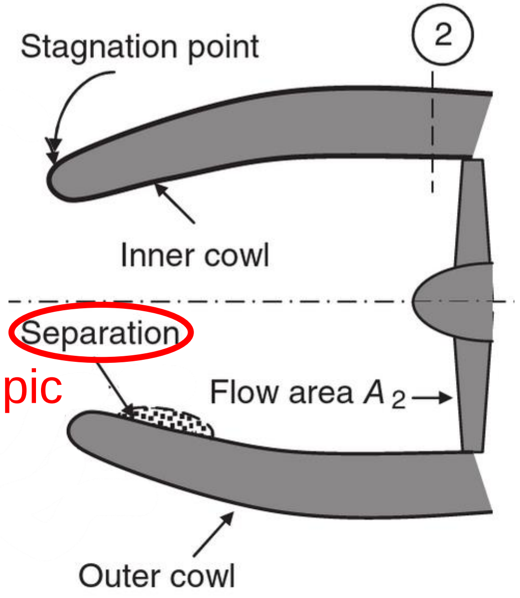
  - Try to avoid flow separation in the boundary layer

Turbojet Analysis

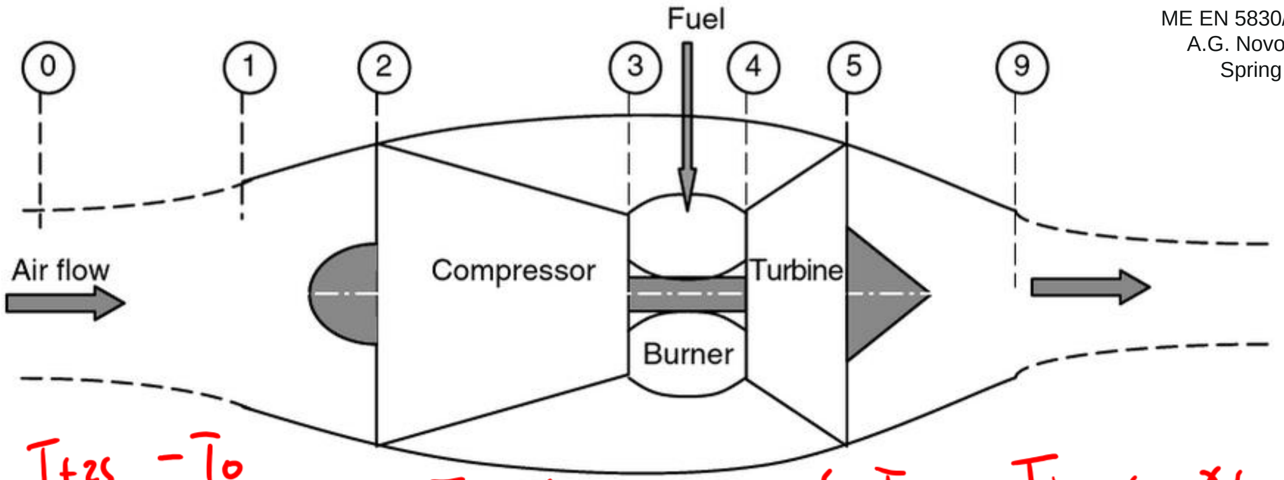


- Inlet (Diffuser) (0-2)
  - Ambient conditions (0) are not the same as inlet conditions (1)
  - We will lump all outside/inside acceleration/deceleration together
    - Consider 0-2 as a single process
  - No work input/output
  - Diffuser is adiabatic
  - $\Delta h_t = 0$
    - $T_{t2} = T_{t0} = T_0\left(1 + \frac{\gamma-1}{2} M_0^2\right)$
  - Diffuser is not necessarily isentropic
    - $\eta_d = \frac{h_{t2s}-h_0}{h_{t2}-h_0} = \frac{T_{t2s}-T_0}{T_{t2}-T_0}$

Non-isentropic



Turbojet Analysis



- Inlet (Diffuser) (0-2)
  - Diffuser outlet pressure

$$\eta_d = \frac{T_{t2s} - T_0}{T_{t2} - T_0} = \frac{T_{t2s}/T_0 - 1}{T_{t2}/T_0 - 1}$$

$$\boxed{\frac{T_{t2}}{T_0} = \frac{T_{t0}}{T_0} = \left(1+\frac{\gamma-1}{2}M_0^2\right)}$$

isentropic flow eqn.

$$\frac{T_{t2s}}{T_0} = 1 + \eta_d\left(\frac{T_{t2}}{T_0} - 1\right)$$

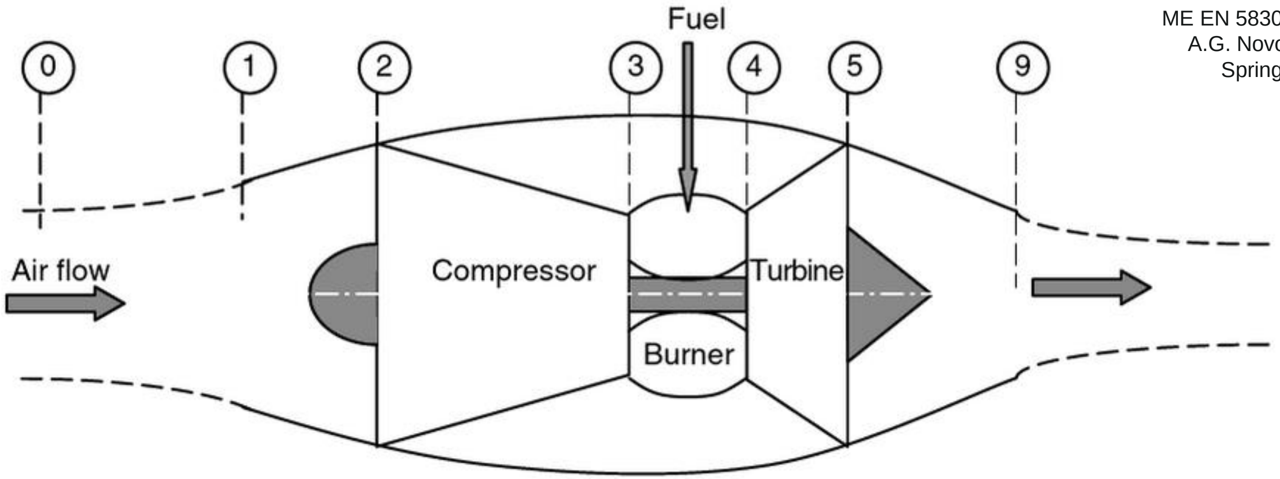
$$\left(\frac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \eta_d\left(\frac{T_{t2}}{T_0} - 1\right)$$

$$\left(\frac{p_{t2}}{p_0}\right) = \left[1 + \eta_d\left(\frac{\gamma-1}{2}M_0^2\right)\right]^{\frac{\gamma}{\gamma-1}}$$

- $\frac{p_{t2}}{p_0} = \left(1 + \eta_d \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}}$
- $\frac{p_{t2}}{p_{t0}} = \left(\frac{p_{t2}}{p_0}\right)\left(\frac{p_0}{p_{t0}}\right) = \frac{\left(1+\eta_d\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}$

stagnation pressure definition

Turbojet Analysis



- Compressor (2-3)
  - Increases the pressure and energy in the gas before the combustor
    - Power comes from turbine
  - Work input
  - Compressor is adiabatic
  - $\dot{W}_{in} = \dot{m}\Delta h_t$
  - Compressor is not necessarily isentropic
    - $\eta_c = \frac{h_{t3s}-h_{t2}}{h_{t3}-h_{t2}} = \frac{T_{t3s}-T_{t2}}{T_{t3}-T_{t2}}$
    - $\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$

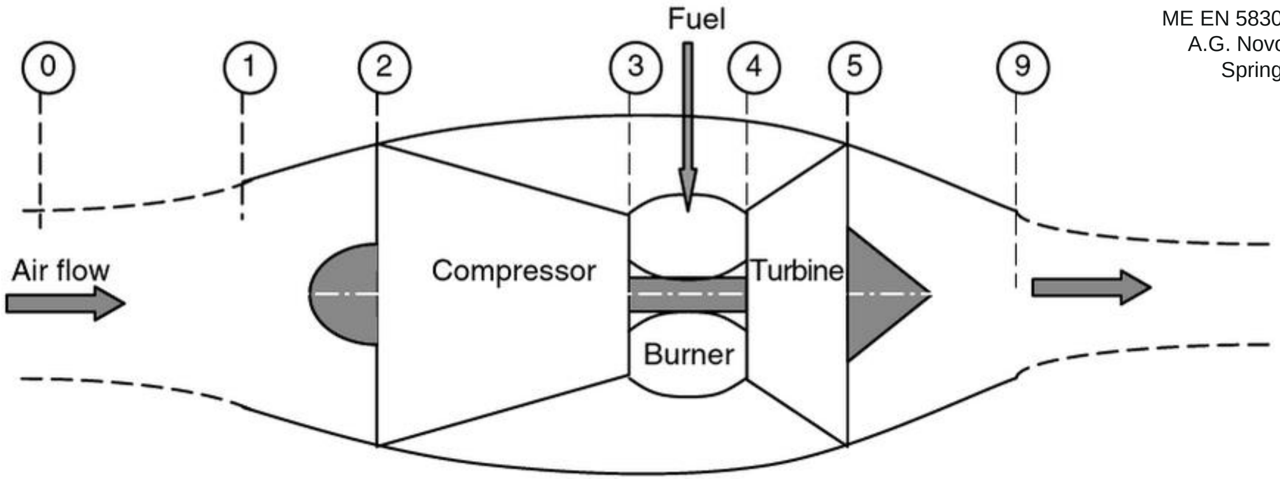
Pressure change across compressor
$r_p = \frac{p_{t3}}{p_{t2}}$

$$\eta_c = \frac{T_{t3s}-T_{t2}}{T_{t3}-T_{t2}} = \frac{T_{t3s}/T_{t2} - 1}{T_{t3}/T_{t2} - 1}$$

$$\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left(\frac{T_{t3s}}{T_{t2}} - 1\right)$$

$$\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Turbojet Analysis



- Burner (3-4)
  - Adds tremendous energy to the flow through combustion
  - Assume isentropic (in practice ~3% friction losses are typical)
  - Assume pressure is approximately constant between 3 and 4
    - $p_{t4} \approx p_{t3}$
  - No work input/output
  - Heat input from combustion
  - $\dot{Q}_{in} = \dot{m}\Delta h_t$
    - $T_{t4} = T_{t3} + \frac{\phi\left(\frac{F}{A}\right)_{st} LHV}{c_p} = T_{t3} + \frac{\left(\frac{F}{A}\right) LHV}{c_p}$

Combustion:

$\dot{Q}_{in} = \dot{m}_f (LHV)$

$= \dot{m}_a \left(\frac{F}{A}\right)(LHV)$

$= \dot{m}_a \left(\frac{F}{A}\right)_{st} \phi (LHV)$

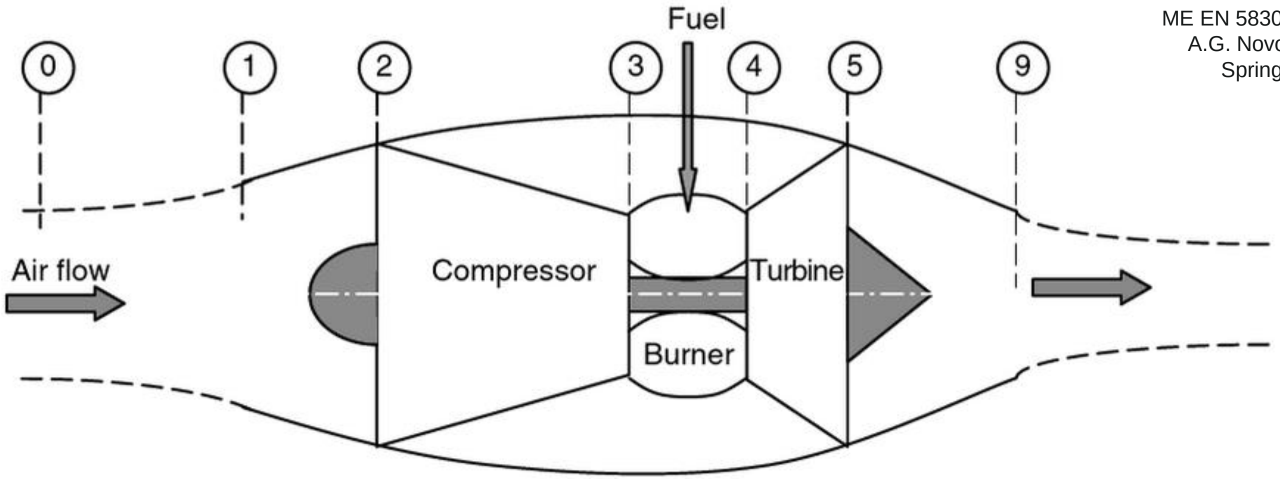
$\approx \dot{m} (F/A)_{st} \phi (LHV)$

First Law:

$\dot{Q}_{in} = \dot{m} c_p (T_{t4} - T_{t3})$

$\cancel{\dot{m}}(F/A)_{st}\phi (LHV) = \cancel{\dot{m}} c_p (T_{t4} - T_{t3})$

Turbojet Analysis



- Turbine (4-5)
  - Lowers pressure and extracts energy from flow to power compressor
  - Work output
  - "Ideal" turbine is adiabatic
  - $-\dot{W}_{out} = \dot{m}\Delta h_t$
  - Turbine is not necessarily isentropic
    - $\eta_t = \frac{h_{t4}-h_{t5}}{h_{t4}-h_{t5s}} = \frac{T_{t4}-T_{t5}}{T_{t4}-T_{t5s}}$
    - ①+② → $T_{t5} = T_{t4} - \frac{T_{t2}}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$
    - ①+②+③ → $p_{t5} = p_{t4}\left[1 - \frac{1}{\eta_c \eta_t}\frac{T_{t2}}{T_{t4}}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$

$\dot{W}_{in,C} = \dot{W}_{out,T} = \dot{m} C_p (T_{t3} - T_{t2})$

eqn. on slide 12

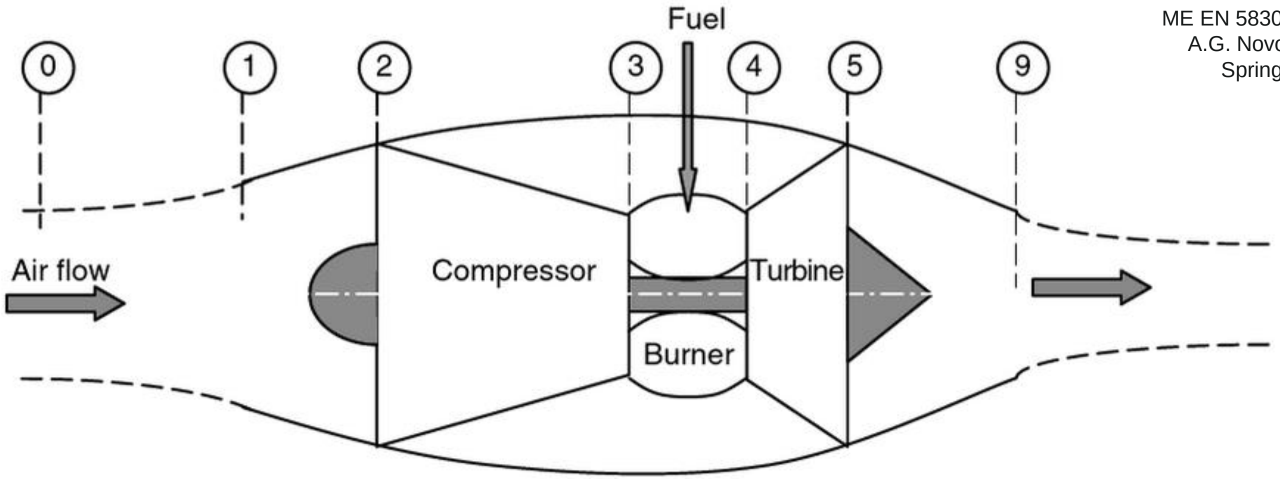
① $\dot{W}_{in,C} = \dot{W}_{out,T} = \frac{\dot{m} c_p T_{t2}}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$

② $\dot{W}_{out,T} = \dot{m} C_p (T_{t4} - T_{t5})$

$\eta_t = \frac{T_{t4}-T_{t5}}{T_{t4}-T_{t5s}}$ .... $\frac{T_{t5}}{T_{t4}} = 1 - \eta_T\left(1 - \left(\frac{T_{t5s}}{T_{t4}}\right)\right)$

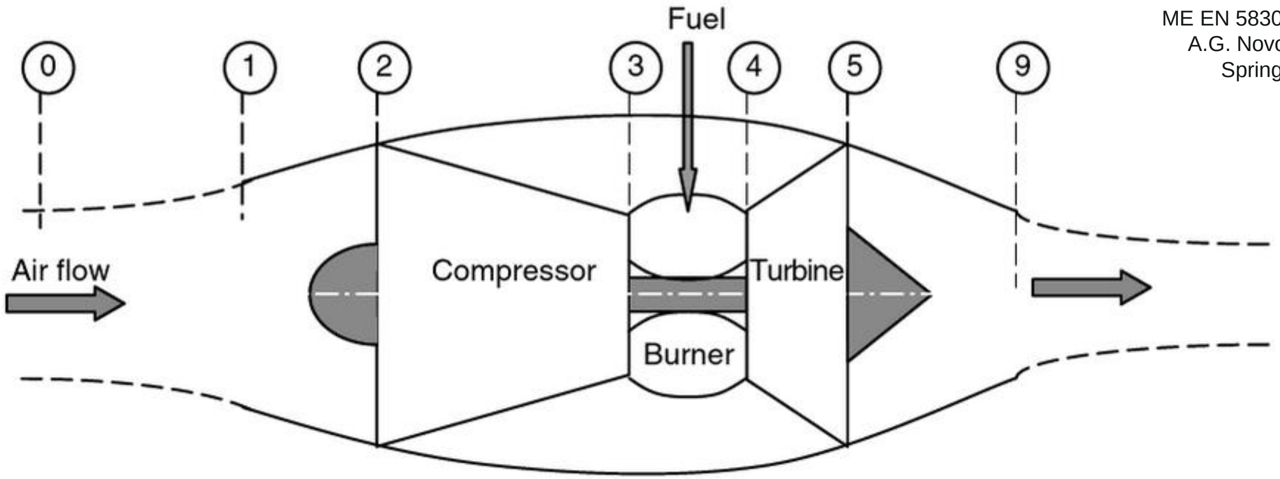
③ $\frac{T_{t5}}{T_{t4}} = 1 - \eta_T\left[1 - \left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right]$

Turbojet Analysis



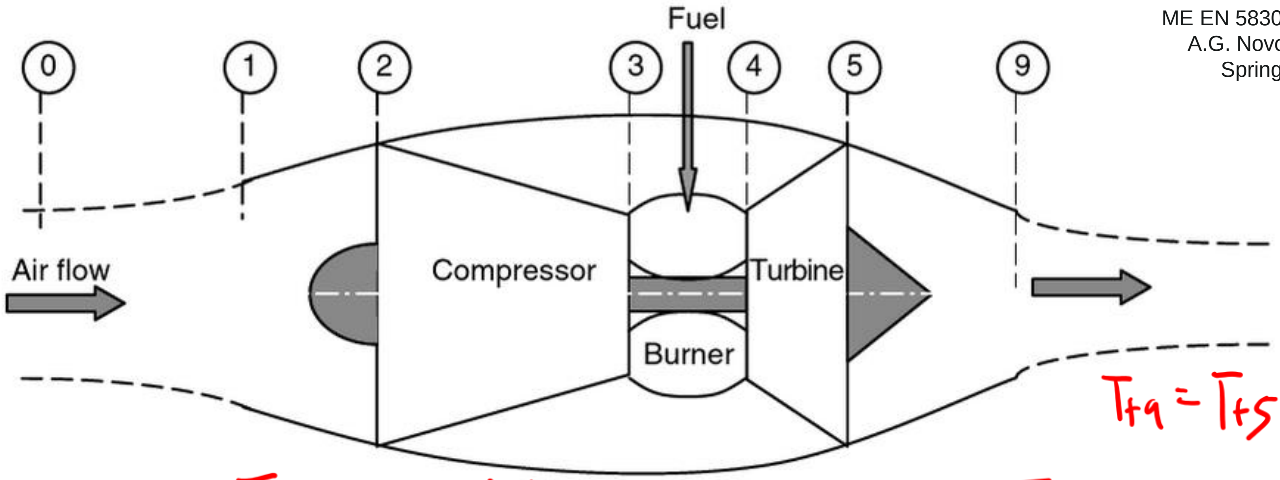
- Exhaust (Nozzle) (5-9)
  - Efficiently accelerates exhaust gas to increase thrust
  - Pressure at exit (9) is not necessarily the same as ambient
    - Recall what we learned about supersonic nozzles as an example
  - We will not consider nozzle shocks for ideal turbojets
  - We will lump acceleration in both ambient and nozzle together (i.e., assume $p_9 = p_a$)

Turbojet Analysis



- Exhaust (Nozzle) (5-9)
  - No work input/output
  - Nozzle is adiabatic
  - $\Delta h_t = 0$
    - $T_{t9} = T_{t5}$
  - Compute exhaust velocity from stagnation temperature ($T_t = T + \frac{1}{2c_p} V^2$):
    - $V_e = \sqrt{2c_p(T_{t9} - T_9)}$
  - Nozzle is not necessarily isentropic
    - $\eta_n = \frac{h_{t5} - h_9}{h_{t5} - h_{9s}} = \frac{T_{t5} - T_9}{T_{t5} - T_{9s}}$

Turbojet Analysis



$T_{t9} = T_{t5}$

- Exhaust (Nozzle) (5-9)
  - Exhaust Velocity

$$\eta_n = \frac{T_{t5} - T_a}{T_{t5} - T_{as}} = \frac{1 - T_a/T_{t5}}{1 - T_{as}/T_{t5}}$$

$$\Rightarrow \frac{T_a}{T_{t5}} = 1 - \eta_n \left(1 - \frac{T_{as}}{T_{t5}}\right)$$

$$\frac{T_a}{T_{t5}} = 1 - \eta_n \left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$T_9 = T_{t5} - T_{t5}\eta_n \left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$V_e = \sqrt{2c_p(T_{t9} - T_9)}$$

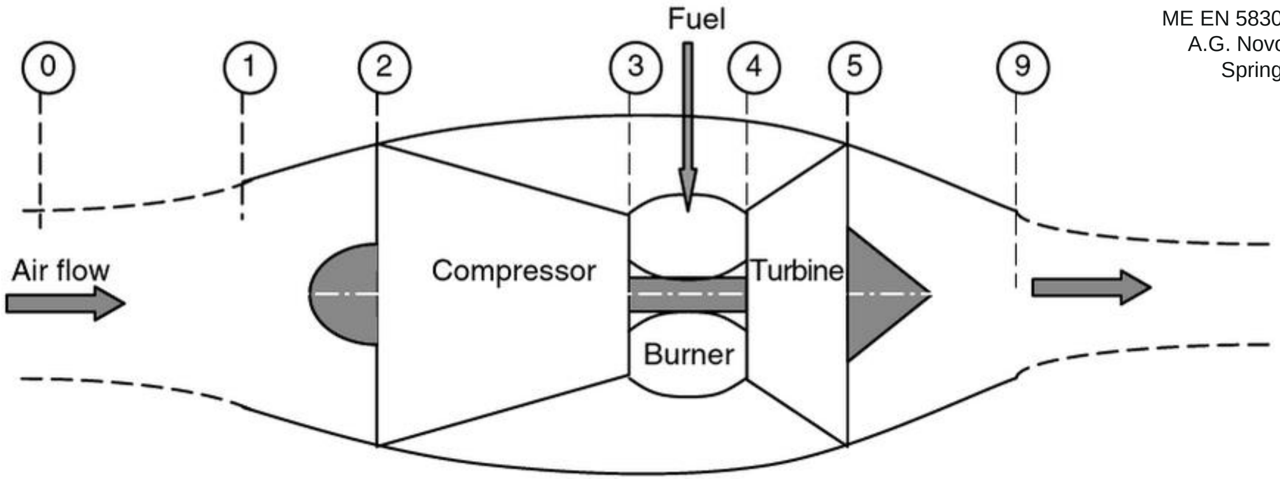
$$= \sqrt{2c_p(T_{t5} - T_a)}$$

$$= \sqrt{2c_p T_{t5} \eta_n \left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$c_p = \frac{\gamma}{\gamma - 1} R$$

- $V_e = \sqrt{2 \frac{\gamma}{\gamma-1} \eta_n R T_{t5} \left[1 - \left(\frac{p_a}{p_{t5}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

Turbojet Analysis



- Exhaust (Nozzle) (5-9)
  - $T = \dot{m}(V_e - V) = \dot{m}\left(V_e - M\sqrt{\gamma R T_a}\right)$

M_0 T_0

- Propulsive Efficiency
  - $\eta_p = \dfrac{TV}{\dot{m}\left[\dfrac{V_e^2}{2} - \dfrac{V^2}{2}\right]} = \dfrac{2V}{V_e + V}$