Fluid Mechanics (ME EN 5700/6700)
Exam 1, Fall 2013

(Open Book, Open Notes, Closed neighbor)

0. [0 pt] What is your name?

1. [8 pt] Continuum Hypothesis

(a) Explain why the continuum hypothesis is important in the study of fluid dynamics

(b) For a gas, when is the continuum hypothesis valid [Hint: think length scales]?

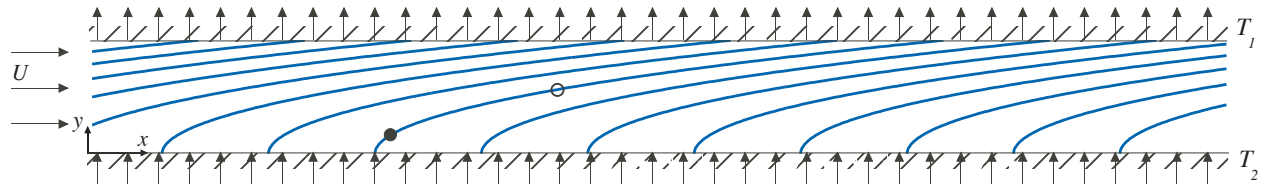**2.** [9 pt] Write the following vector quantities in index notation.

(a) $\vec{u} \times \left(\vec{\nabla} \times \vec{u} \right)$

(b) $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{u} \right)$

(c) $\vec{\nabla} \times \vec{\nabla} \phi$

3. [12 pt] Streamlines in a porous channel are shown. The top and bottom walls are heated to a temperature T_1 and T_2, respectively, where $T_2 > T_1$. The flow is steady. Consider the following equation:

$$\underbrace{\frac{DT}{Dt}}_{\text{I}} = \underbrace{\frac{\partial T}{\partial t}}_{\text{II}} + \underbrace{u_j \frac{\partial T}{\partial x_j}}_{\text{III}}$$

a. How is the equation useful in the given problem?

b. What is term I called?

c. Describe in words what happens to the time rate of change of the temperature of the fluid particle indicated by the black circle. Which term in the equation represents this?

d. Describe in words the time rate of change of the temperature at the fixed point indicated by the open circle. Which term in the equation represents this?

e. What is term III called? What is the sign of term III at the fixed point indicated by the open circle? [Support your answer]

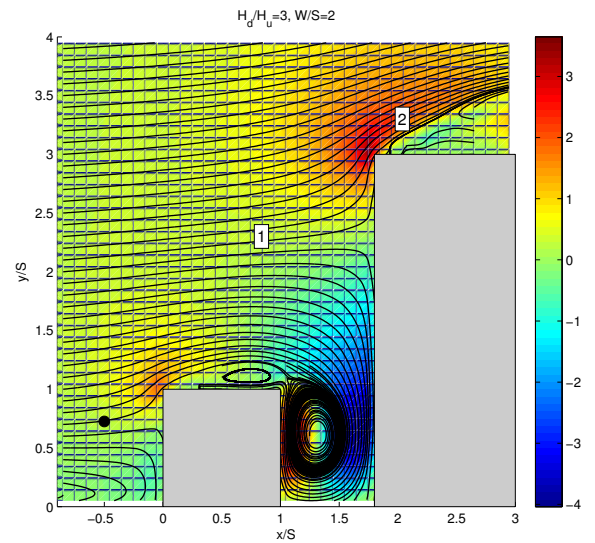**4.** [10 pt] A student claims that a two-dimensional flow with a non-constant density field in the form

$$\rho = a_0 + a_1\, x + a_2\, x^2\,,$$

where a_0, a_1, and a_2 are constants, is incompressible.

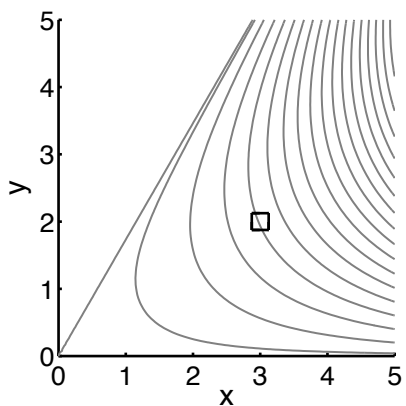a. Write the mathematical definition of incompressible flow.

b. Based on your knowledge of fluid dynamics, is the student's claim above correct or incorrect? [Show the work to support your answer]

5. [10 pt] The image below illustrates the streamline pattern in a flow at a given time t. A fluid particle is indicated by the black circle.

a. How are the streamlines related to the stream function?

b. From this image alone, can you predict the pathline of the particle indicated by the black circle?

c. What can you say about the magnitude of the velocity at point 1 compared to that at point 2?

6. [28 pt] Streamlines of the flow in a corner is shown below. The horizontal and vertical velocity components associated with this flow are, respectively: $u = x^2 - y^2$ and $v = -2xy$.

(a) Determine the components in the strain-rate tensor, e_{ij}.

(b) Determine the components in the vorticity vector, ω_i.

(c) On the image provided, draw the pathline of the fluid particle that originates at point $(x = 3, y = 2)$ at time $t = 0$.
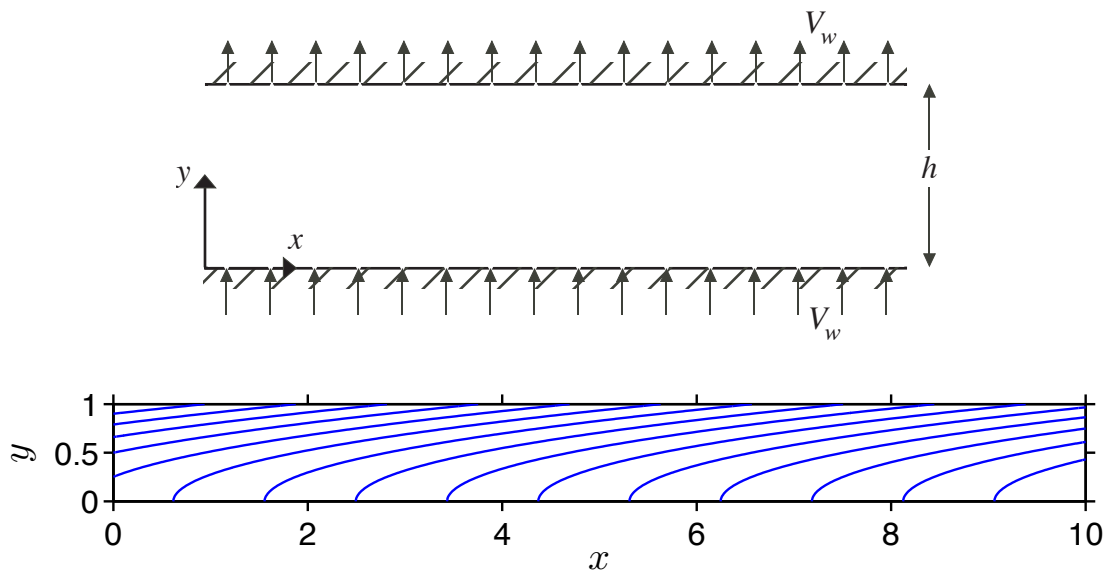
(d) Is the flow incompressible [show work]?

(e) Consider an initially square particle with centroid located at $(x = 3, y = 2)$, as shown in the streamline plot above. How will the fluid particle look after a infinitesmial time Δt later? [Support your reasoning with calculations]

(f) Derive expressions for the components of the acceleration of a generic fluid particle.

(g) Explain how the momentum of the fluid particle at $(x = 3, y = 2)$ is changing with respect to time. [Support your answer with calculations]

7. [23 pt] Consider planar flow in a channel with porous walls. A constant vertical velocity V_w exists at the top and bottom walls as shown. Flow is driven through the channel by a pressure gradient $\partial P / \partial x = -K$.

a. Use the continuity equation to find the vertical velocity v. [State assumptions]

b. Simplify the Navier-Stokes equation (x-component ONLY) for this flow and provide the appropriate boundary conditions. [State assumptions]

c. For the case where $V_w = 0$, explain the role of viscous diffusion in the fluid dynamics.

d. How would you expect the streamline pattern to change if the viscosity of the fluid was increased? [Suggestion: write the acceleration term in (b) from the Lagrangian viewpoint, then consider the motion of a fluid particle.]

e. [**Extra Credit**] Solve for the horizontal velocity component.