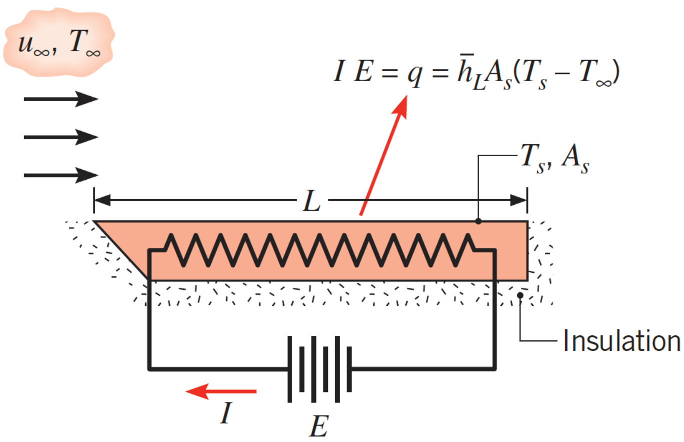
Laminar Flow Flat Plate Convection

Thermal Fluids and Energy Systems Lab

(ME EN 4650)



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Department of Mechanical Engineering
University of Utah

Based on Prof. M's slides

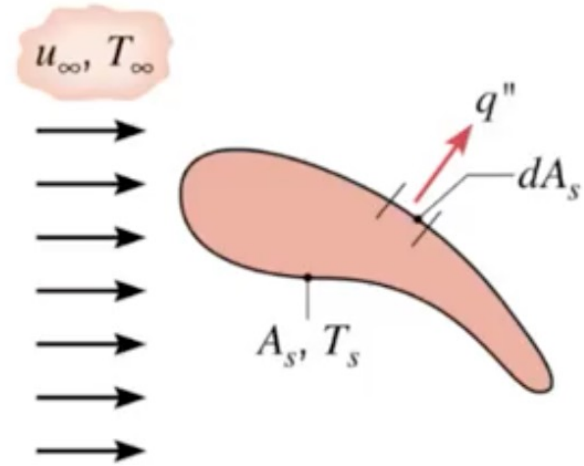
Objectives

- Measure the temperature as a distance of length along a heated flat plate using embedded thermocouples
- Calculate the local and average heat transfer coefficients and Nusselt numbers & compare with the theoretical predictions,
- Calculate the net heat flux from the surface

Convective Heat Transfer

What we want to know:

1. $Nu = f(x^*, Re, Pr)$
2. q'' (heat fluxes)
3. T_S (surface temperatures)



What we can measure:

1. P_{dyn} : Pitot-static probe (Bernoulli)
2. $T_s(x)$: using thermocouples
3. q'' : heat flux from heating element (V and R from a multimeter)
4. T_∞ : thermocouples

$$Nu = \frac{h\,L}{k}$$

$$Re = \frac{U_\infty L}{\nu}$$

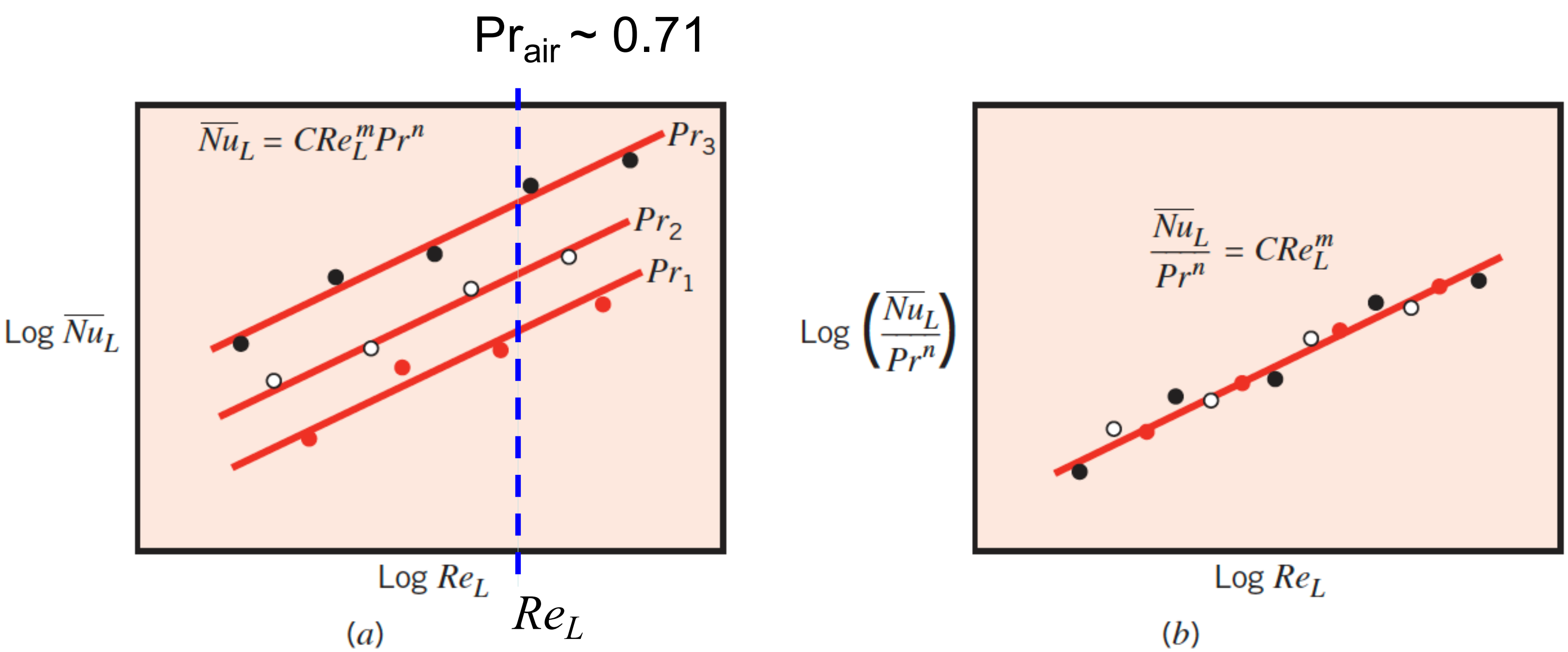
$$Pr = \frac{\nu}{\alpha}$$

Nusselt Number Relationships

$$\overline{Nu} = C Re_L^m Pr^n$$

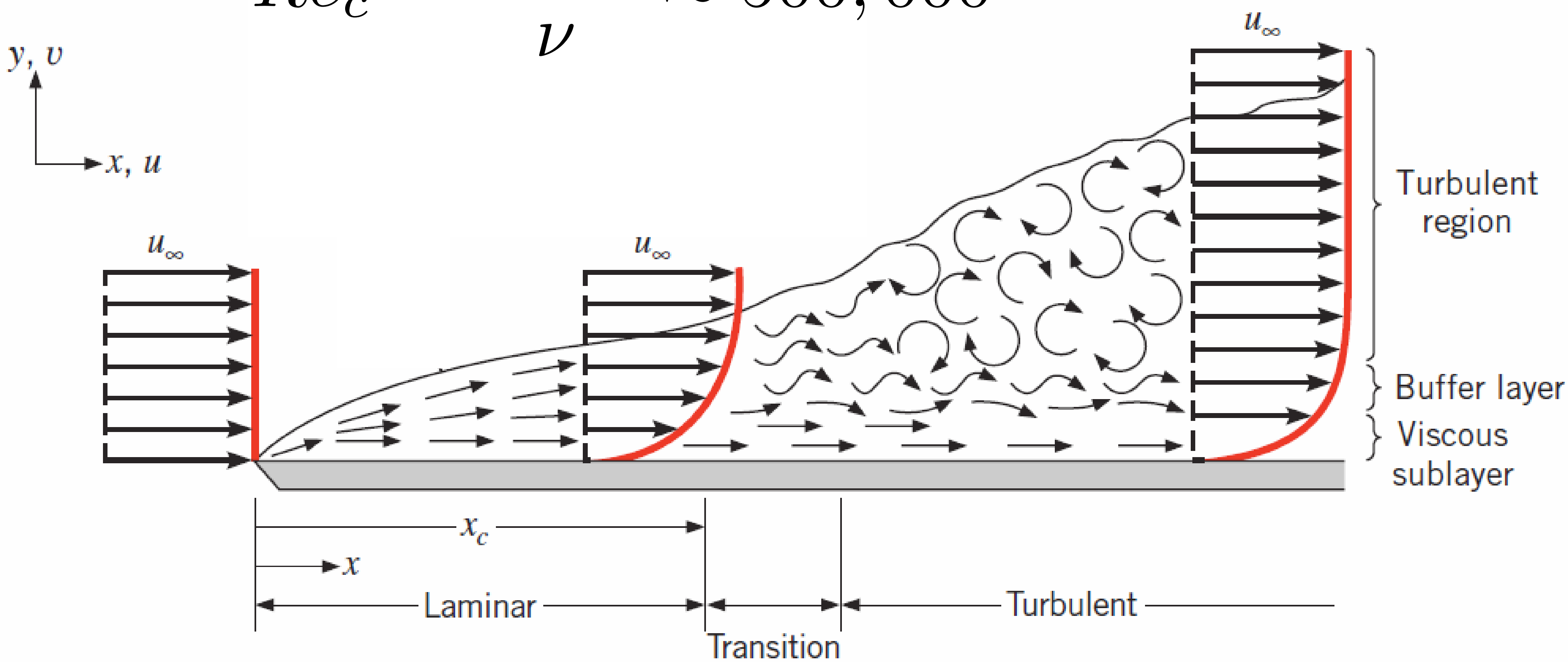
Nusselt Interpretations:

1. A non-dimensional temperature gradient at the wall
2. Ratio of convective heat transfer to pure conduction (i.e., if the fluid was motionless)

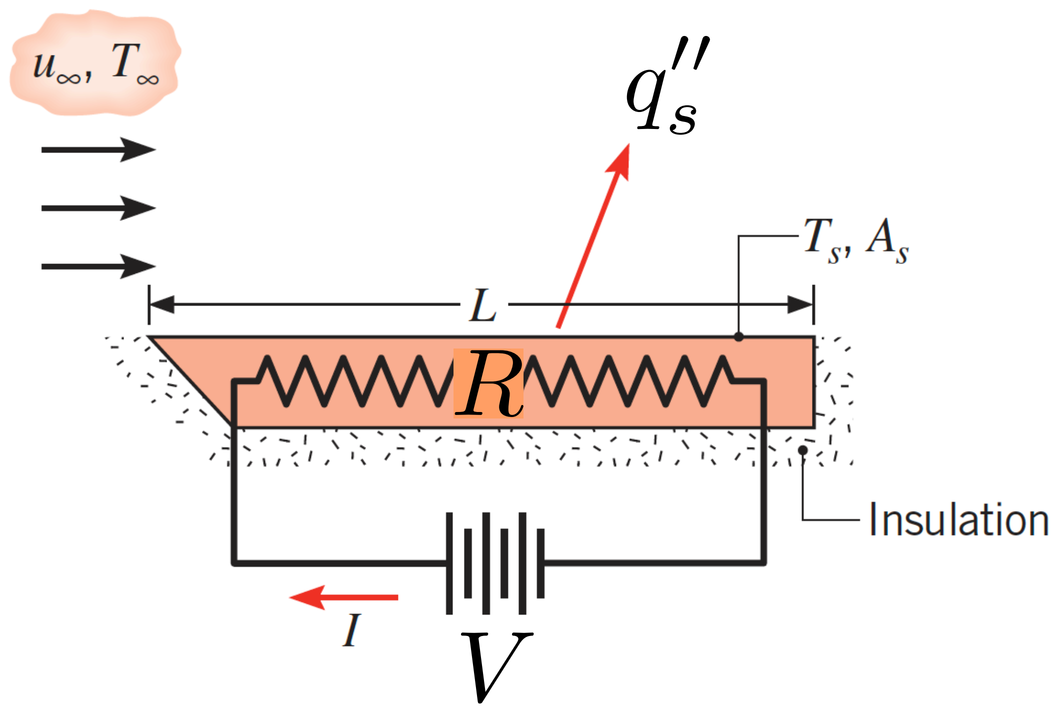


Flow over a Flat Plate

$$Re_c = \frac{U_\infty x}{\nu} \approx 500,000$$



Heat Transfer from a Heated Plate



$$P_e = \frac{V^2}{R}$$

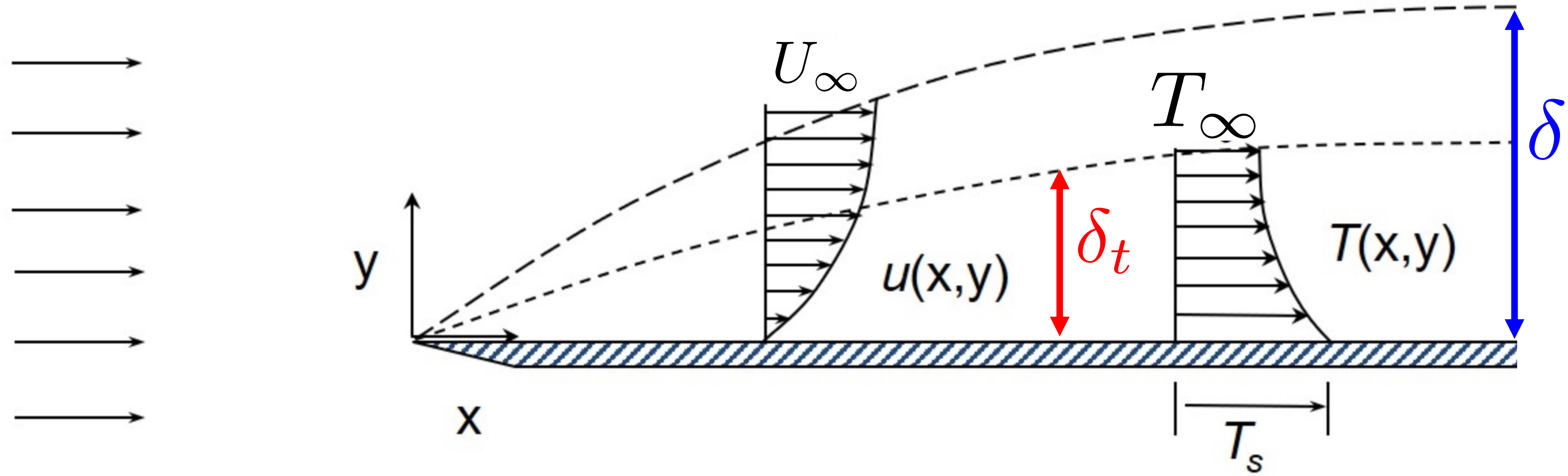
We'll assume a uniform heat-flux boundary condition

Heat flux (top surface):

$$q_s'' = \frac{P_e}{A_s}$$

Prandtl Number Heated Plate

U_∞, T_∞



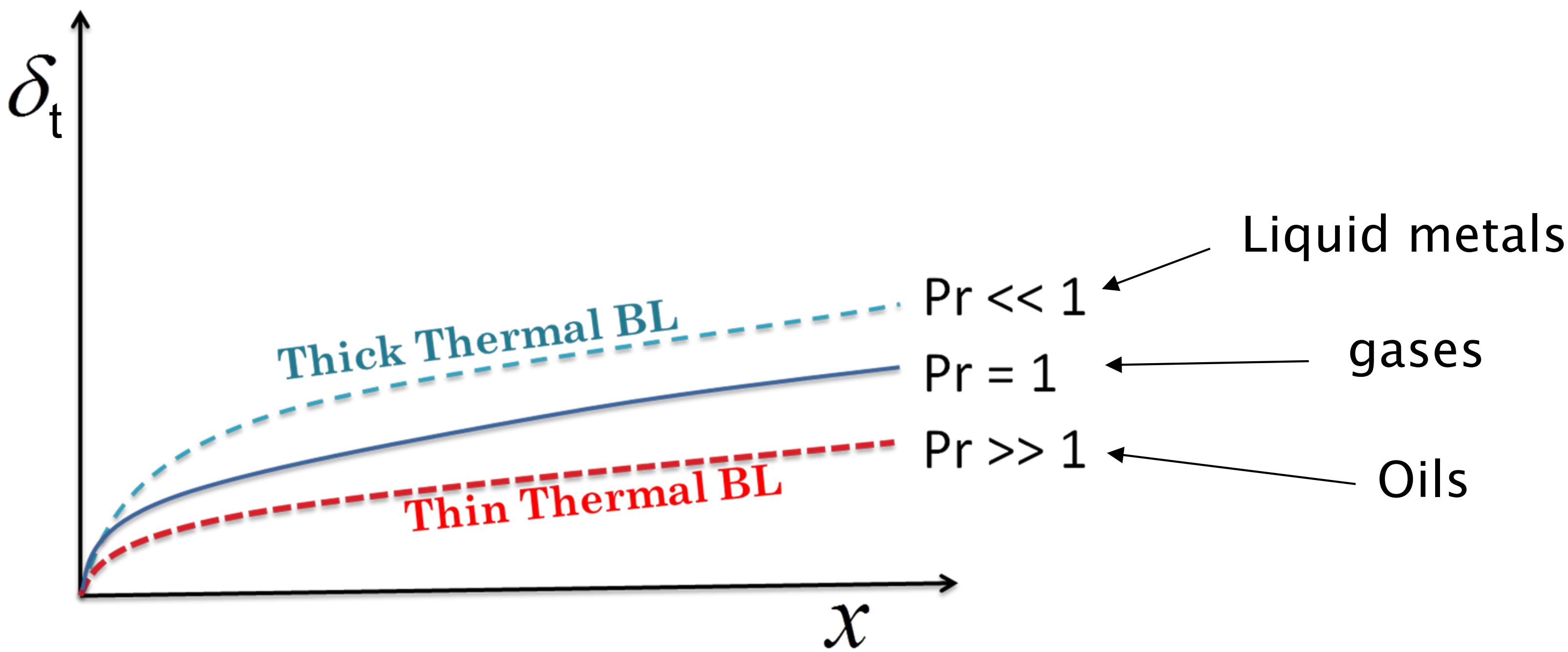
$$Pr = \frac{\nu}{\alpha}$$

Here, $Pr = \dfrac{\nu}{\alpha} > 1$ Since $\dfrac{\delta}{\delta_t} > 1$

This is the case for oils

Pr ~ 1 for gases ($\delta \sim \delta_t$)
Pr < 1 for liquid metals ($\delta_t > \delta$)

Effect of Prandtl Number on Boundary Layer Growth



Heat Transfer from a Heated Plate

Heat flux (top surface):

$$q''_s = \frac{P_e}{A_s}$$

Newton's Law of Cooling:

$$q''_s = h(x)[T_s - T_\infty]$$

At the wall:

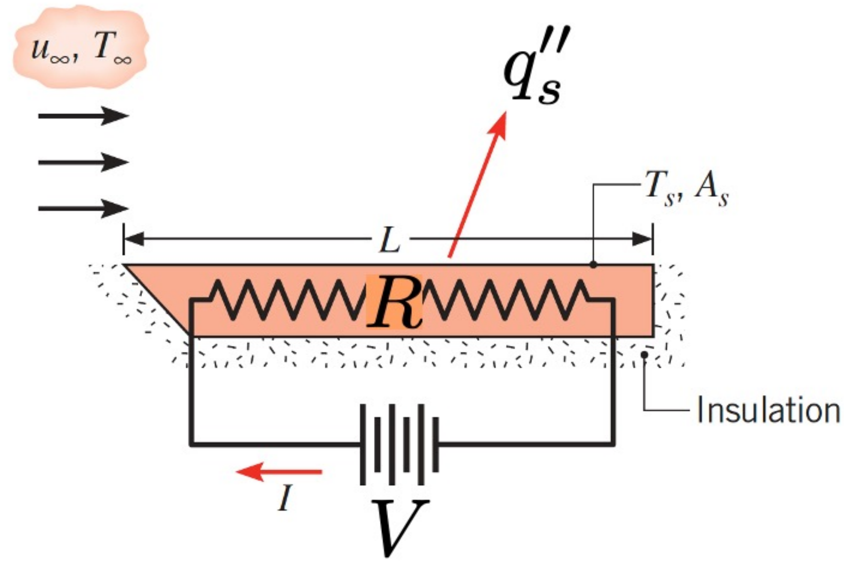
$$q''_s = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}$$

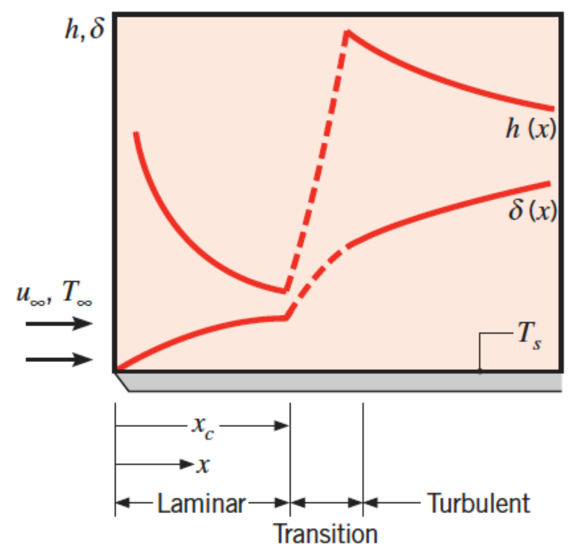
Local heat transfer coefficient:

$$h(x) = \frac{q''_s}{[T_s - T_\infty]}$$

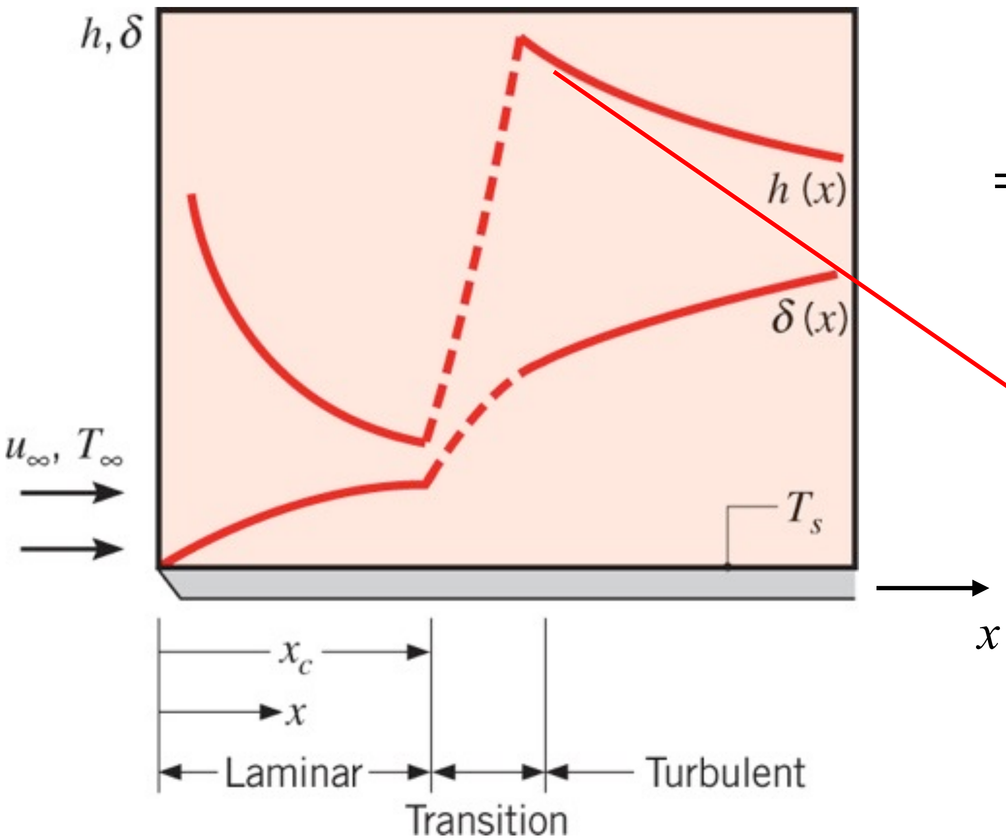
Local Nusselt number:

$$Nu_x = \frac{h(x)x}{k_f}$$





Heat Transfer from a Heated Plate



$$\Rightarrow x \uparrow \Rightarrow \delta_t \uparrow \Rightarrow \left.\frac{\partial T}{\partial y}\right|_{y=0} \downarrow \Rightarrow h \downarrow$$

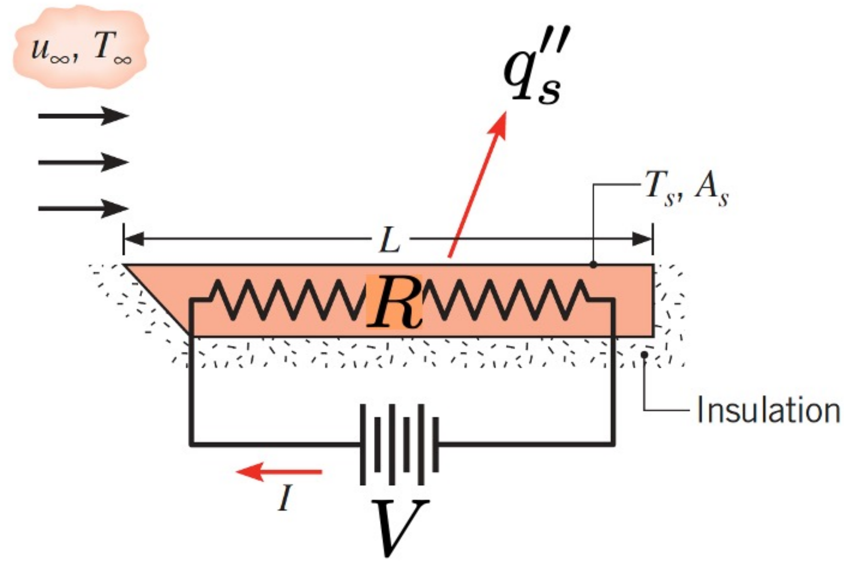
Turbulent mixing induces large temperature gradient at the wall

$$h = \frac{k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$

Heat Transfer from a Heated Plate

Heat flux (top surface):

$$q_s'' = \frac{P_e}{A_s}$$



Newton's Law of Cooling:

$$q_s'' = h(x)[T_s - T_\infty]$$

At the wall:

$$q_s'' = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}$$

Local heat transfer coefficient:

$$h(x) = \frac{q_s''}{[T_s - T_\infty]}$$

Local Nusselt number:

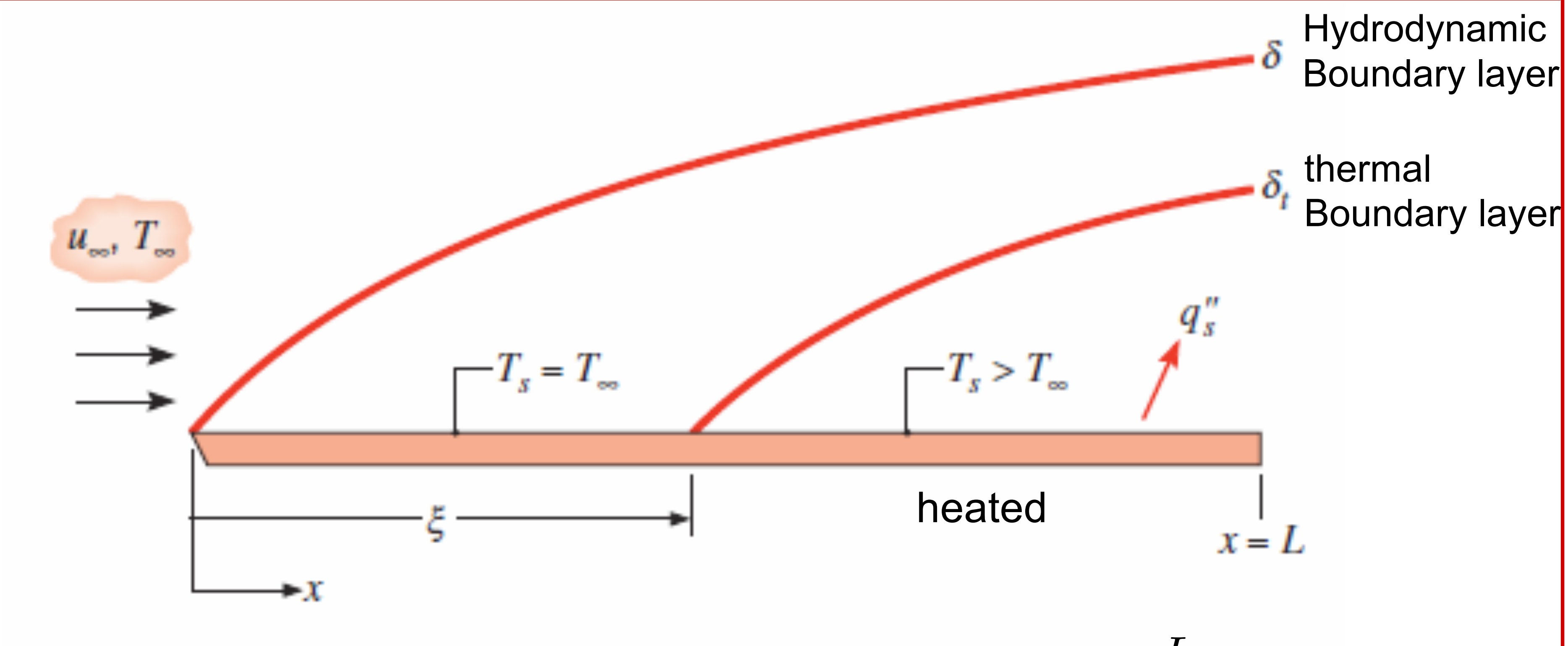
$$Nu_x = \frac{h(x)x}{k_f}$$

Evaluate k_f at film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

Look up or use Matlab script provided on CANVAS

Flat Plate with Unheated Starting Length

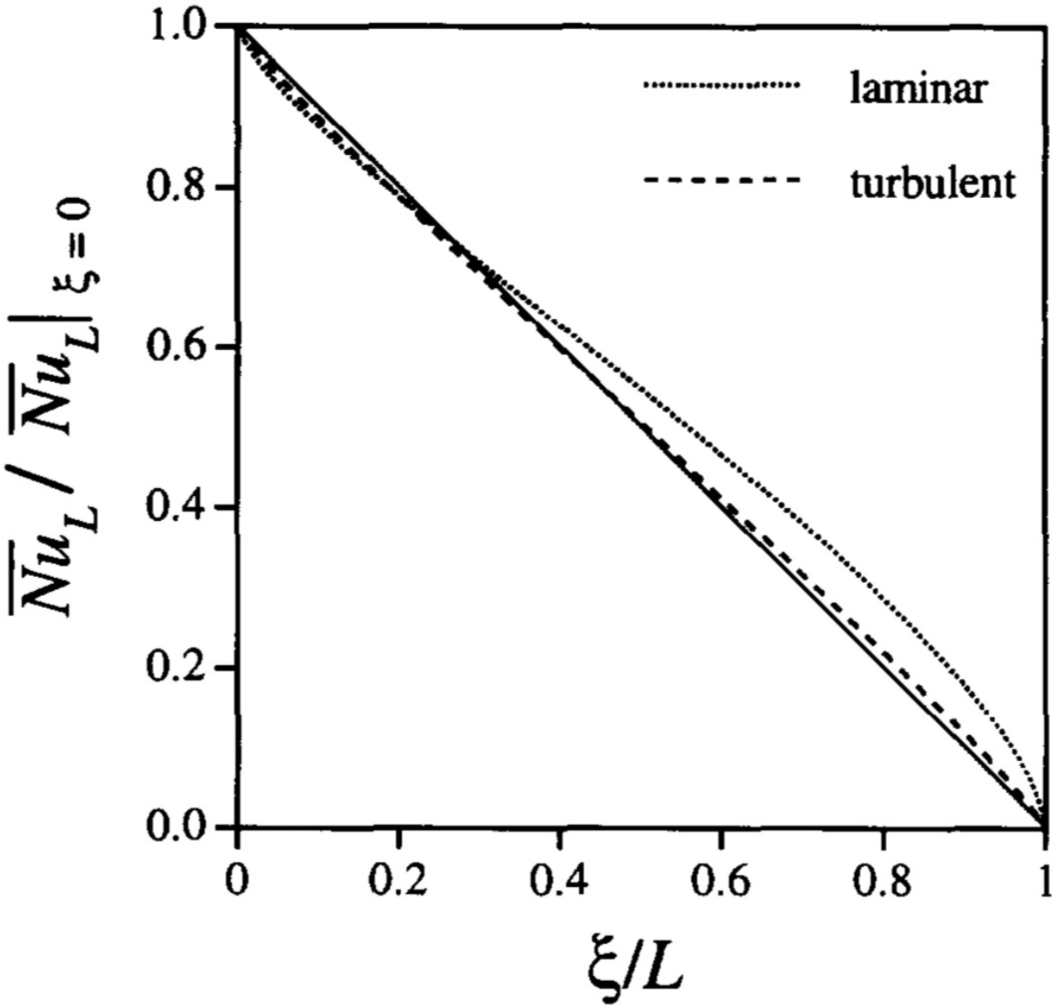


Average heat transfer coefficient: $$\overline{h}_L = \frac{1}{L-\xi}\int_{x=\xi}^{x=L} h(x)dx$$

Average Nusselt number:

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k_f}$$

Effect of Unheated Starting Length on Average Nusselt Number

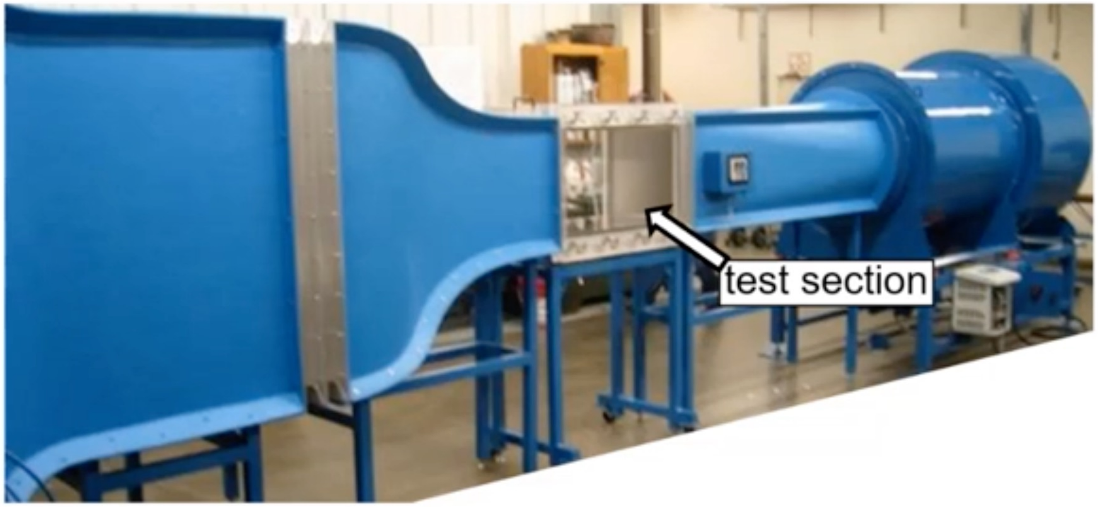


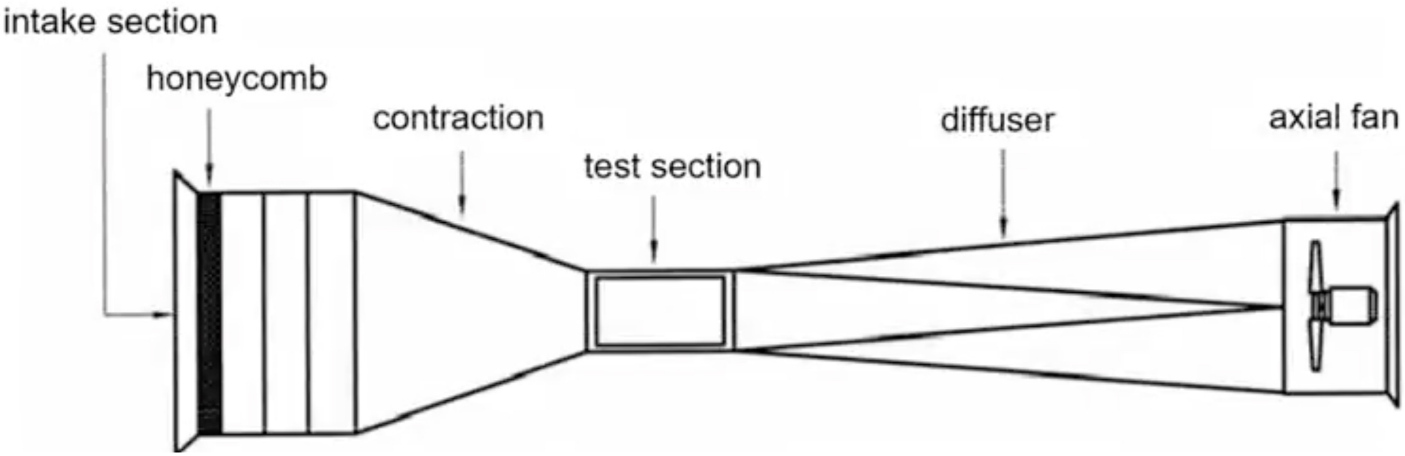
Measurements

Quantity	Symbol	Units	Instrument
Freestream dynamic pressure	P_{dyn}	mmHg	Pitot-static probe
Plate surface temperature	$T_s(x)$	°C	thermocouple
Freestream temperature	T_∞	°C	thermocouple
Heater voltage	V	VAC	multimeter
Heater resistance	Ω	Ohm	multimeter

Wind Tunnel

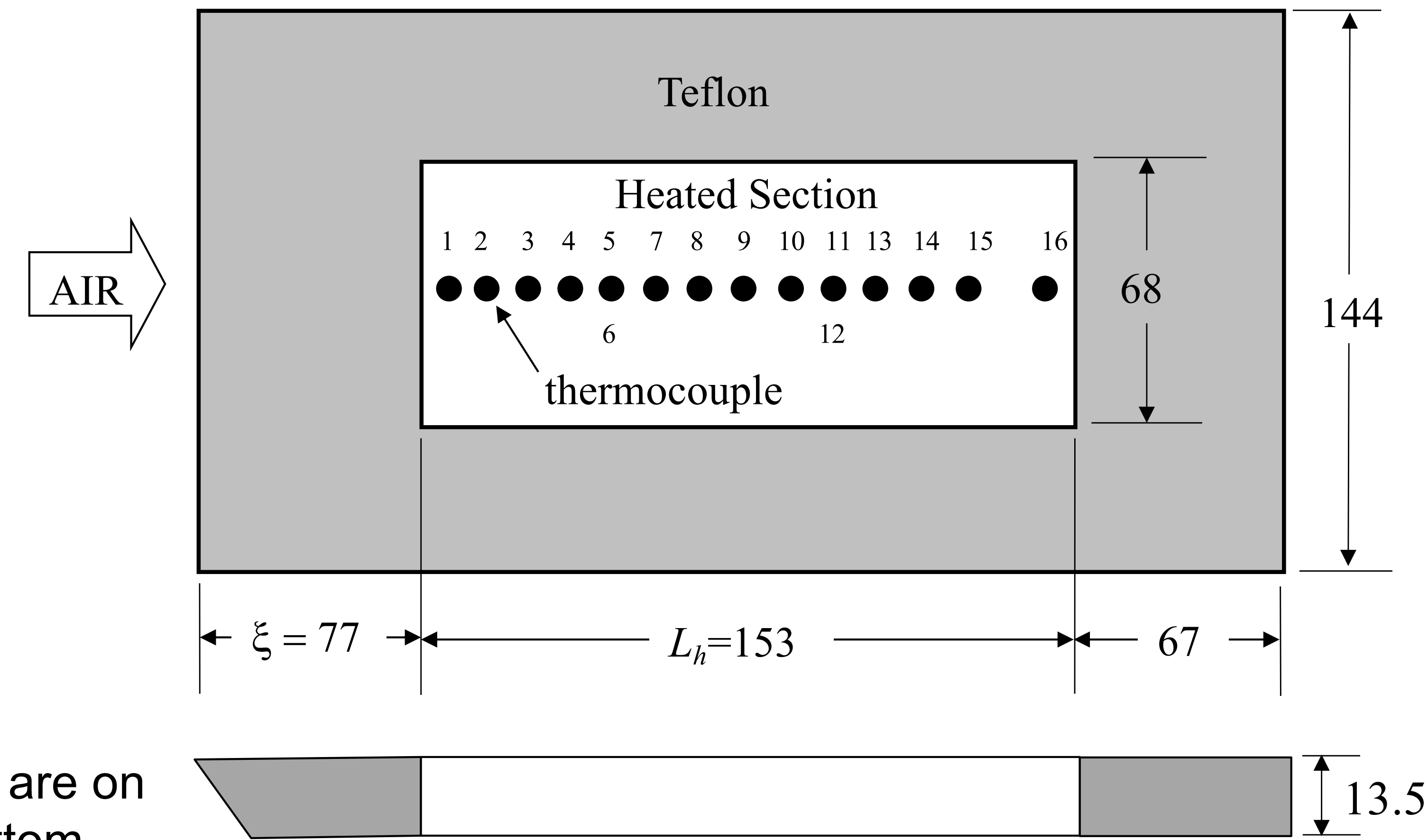
24’’ x 12’’ x 12” cross section





Experimental Apparatus

Heated Flat Plate with an Unheated Starting Length

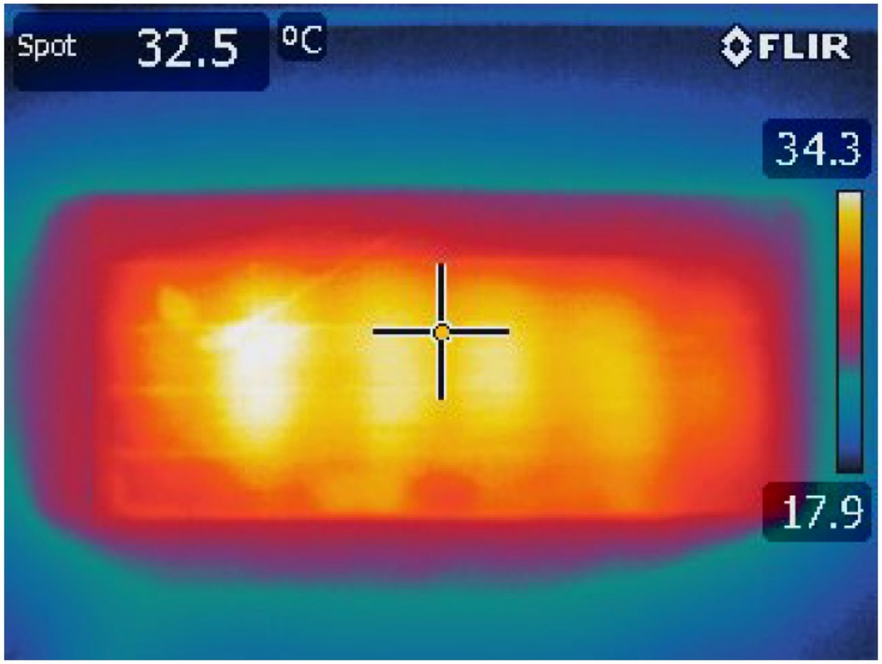


6 & 12 are on the bottom

For laminar flow: $Re_L < 5 \times 10^5$ which is ~ 40 m/s

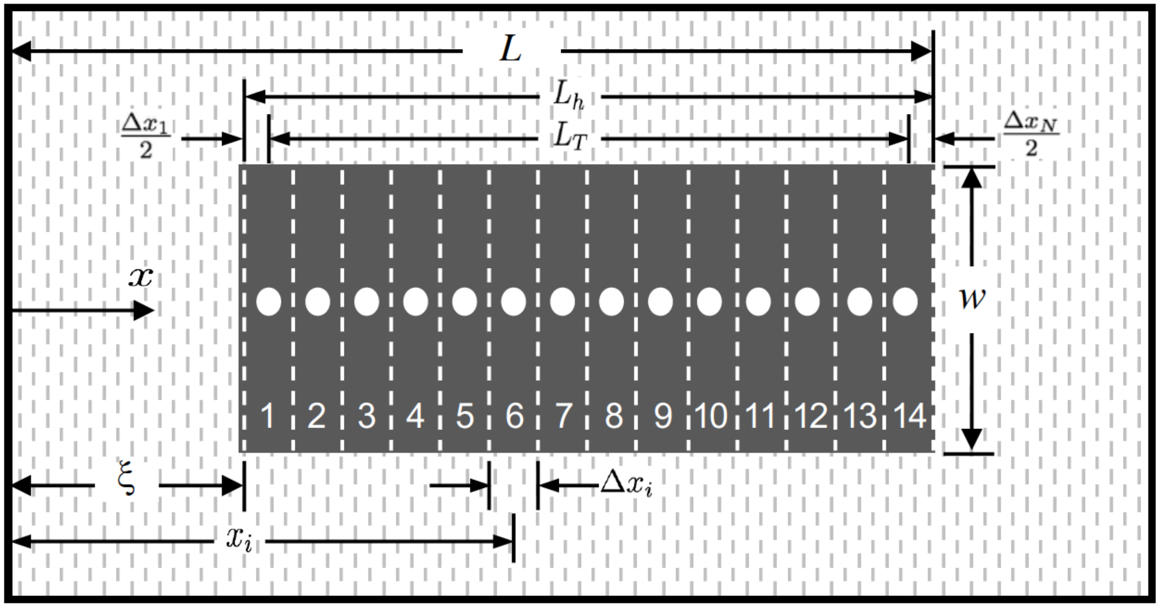
Units in mm

Photographs of Heated Plate



Assumption of constant heat flux?

Data Analysis: Measurements



Heat flux from the top surface: $q_s'' = \frac{P_e}{2A_p} = \frac{V^2}{R2L_h w}$

P_e – total power supplied to heater

Local heat transfer coefficient: $h(x_i) = \frac{q_s''}{T_s(x_i) - T_\infty}$

Average heat transfer coefficient: $\overline{h}_L = \frac{1}{L_T}\int_{x_1}^{x_{14}} h(x)dx$

Use Trapezoidal rule

Data Analysis: Measurements

Local Nusselt number: $Nu_x = \dfrac{h(x)x}{k_f}$

Use AirProperties.m script

Average Nusselt number: $$\overline{Nu}_L = \frac{\overline{h}_L L}{k_f}$$

Data Analysis: Theoretical Formulas

(Kays and Crawford, 1993; Ameel, 1997)

For Laminar Flow

Local Nusselt number:

$$Nu_{x,\mathrm{th}}(x) = \frac{0.453\, Re_x^{1/2}\, Pr^{1/3}}{\left[1 - (\xi/x)^{3/4}\right]^{1/3}}$$

Local heat transfer coefficient: $h_{x,\mathrm{th}}(x) = \left(\dfrac{\overline{k}_f}{x}\right) Nu_{x,\mathrm{th}}(x)$

Average heat transfer coefficient:

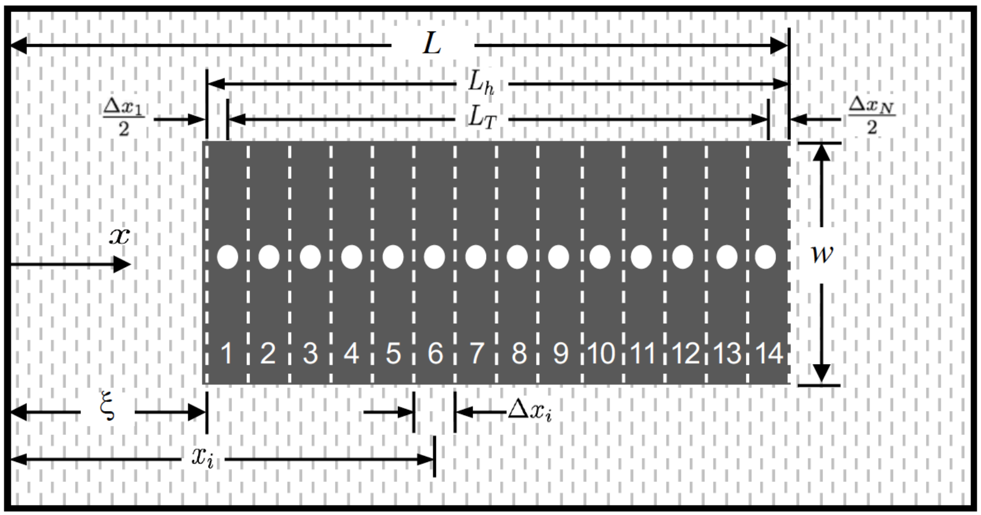
$$\overline{h}_{L,\mathrm{th}} = 2\left(\frac{\overline{k}_f}{L-\xi}\right)\left(0.453\, Re_L^{1/2}\, Pr^{1/3}\right)\left[1 - \left(\frac{\xi}{L}\right)^{3/4}\right]^{2/3}$$

Average Nusselt Number:

$$\overline{Nu}_{L,\mathrm{th}} = \frac{\overline{h}_{L,\mathrm{th}}\, L}{\overline{k}_f}$$

Assume all properties are constant and evaluated at the average film temperature

Data Analysis: Predictions from Theory



Newtons Law of Cooling

Estimate Heat Flux
(given measurements of surface temperature)

$$q''_{s,\mathrm{th}}(x_i) = h_{x,\mathrm{th}}(x_i)\boxed{[T_s(x_i) - T_\infty]}$$

Meas

$$q_{s,\mathrm{th}} = \frac{w\,L_h}{L_T}\int_{x=x_1}^{x=x_N} q''_{s,\mathrm{th}}(x)\,dx$$

Trapezoidal Rule

Estimate Surface Temperature
(given measurements of heat flux)

Meas

$$T_{s,\mathrm{th}}(x) = \boxed{T_\infty} + \frac{\boxed{q''_s}}{h_{x,\mathrm{th}}(x)}$$

Effect of Thermal Radiation

Local radiation heat flux (top surface):

$$q''_{\mathrm{rad}}(x_i) = \epsilon\,\sigma\left(T_s^4(x_i) - T_\infty^4\right)$$

Temperature need to be in Kelvin (absolute!)

$$\epsilon = 0.7$$

$$\sigma = 5.6703 \times 10^{-8}(\mathrm{W\,m^{-2}K^{-1}})$$

Now we can plot corrected surface temp

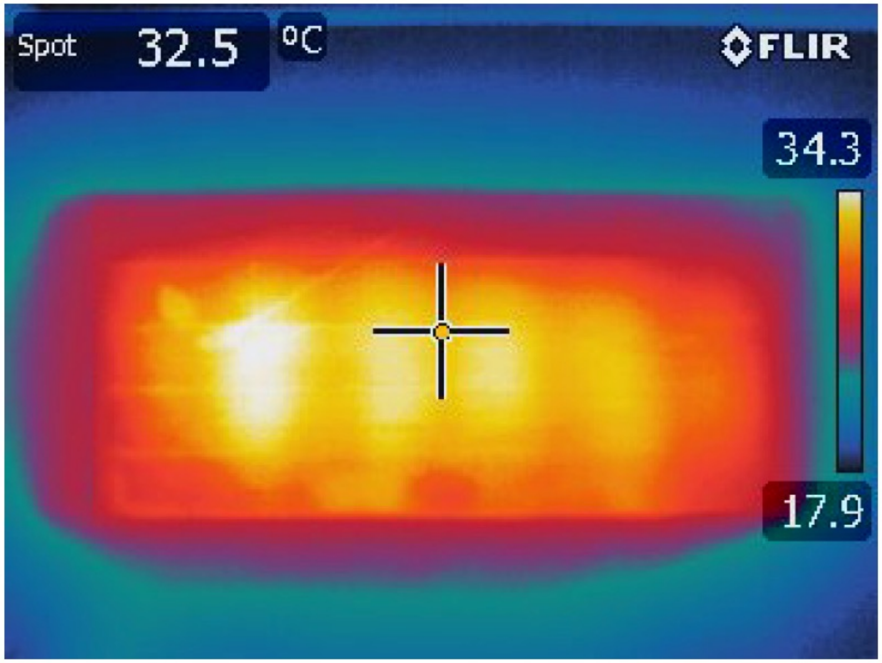
$$T_{s,\mathrm{th}}(x) = T_\infty + \frac{q''_s - q''_{\mathrm{rad}}}{h_{x,\mathrm{th}}(x)}$$

Average Heat Transfer Rate

$$q''_{\mathrm{rad},L} = \frac{1}{L_T}\int_{x=x_1}^{x=x_N} q''_{\mathrm{rad}}(x)\,dx$$

Total Heat Transfer Rate:

$$q_{\mathrm{rad,L}} = A_{hp}q''_{\mathrm{rad,L}}$$



Percent radiation lost to the surroundings

$$\frac{q''_{\mathrm{rad},L}}{q''_s} \cdot 100$$ ~15%

Data

- No physical lab – data are provided for you on CANVAS under RESOURCES – Experimental Tools: ConvectionData.dat

Data

%Thermocouple Number Thermocouple Temp. (C)
% Pbaro = 776-120.6 mmHg
% Tamb = 22.2 oC
% dynamic pressure = 0.1 in H2O
% voltage = 40.03 VAC
% resistance = 157.1 Ohms

1.0	30.8
2.0	33.6
3.0	34.4
4.0	35.8
5.0	38.2
6.0	37.6
7.0	37.9
8.0	39.6
9.0	39.3
10.0	39.7
11.0	40.0
12.0	39.8
13.0	40.9
14.0	40.0
15.0	40.6
16.0	41.0

Bernoulli

$$U_\infty = \sqrt{2\,P_{\mathrm{dyn}}/\rho}$$

Thermocouple	Location (mm)	Thermocouple	Location (mm)
1	85	9	153
2	92	10	162
3	102	11	173
4	112	12	173
5	123	13	186
6	123	14	196
7	134	15	209
8	143	16	219

Data

- AirProperties.m script provided for computed (Temps in Kelvin, Pressure in Pa, see Appendix)

```
[rho,mu,k,Cp] = AirProperties(T,P)
```

```
function [rho,mu,k,Cp] = AirProperties(T,P)
%-------------------------------------------------------------------
% function [rho,mu,k,Cp] = AirProperties(T,P)
%
% Properties of DRY air based on the ideal gas law. Note, for ideal
% gases, Cp, k, and mu are independent of pressure.
%
% INPUTS:
%    T      Temperature in Kelvin
%    P      Atmospheric pressure in Pascal
%
% OUTPUTS:
%    rho    density in kg/m^3
%    mu     absolute viscosity in kg/(m*s)
%    k      thermal conductivity in W/(m*K)
%    Cp     specific heat in J/(kg*K)
%
% M Metzger
%-------------------------------------------------------------------
```