Aerospace Propulsion

Lecture 8

Compressible Flows: Part II

Compressible Flows: Part II

- Isentropic Nozzles
- Area-Mach Number Relationships
- Converging-Diverging Nozzles
- Friction Losses in Nozzles

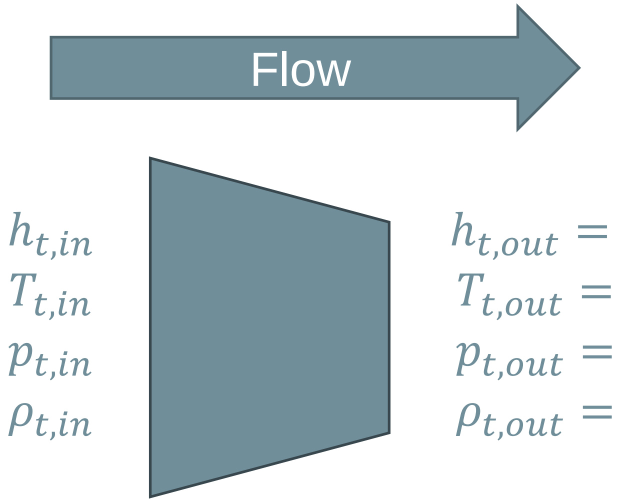
Isentropic Nozzles

- Recall our stagnation properties
  - $h_t = h + \frac{1}{2}V^2$
  - $T_t = T\left(1 + \frac{\gamma-1}{2}M^2\right)$
  - $p_t = p\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$
  - $\rho_t = \rho\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$

  - These are **constant** for an isentropic process

Isentropic Nozzles

- Consider an isentropic nozzle
  - How do stagnation properties vary?
- Enthalpy
  - $h_{t,in} = h_{t,out}$
  - $h_{in} + \frac{1}{2}V_{in}^2 = h_{out} + \frac{1}{2}V_{out}^2$
- Other variables
  - $T_{t,in}/T_{t,out} = 1$
  - $p_{t,in}/p_{t,out} = 1$
  - $\rho_{t,in}/\rho_{t,out} = 1$



$T_t = T\left(1 + \frac{\gamma-1}{2}M^2\right)$

$T_{t1} = T_{t2}$

Isentropic Nozzles

- State changes

* $\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$
- $\frac{p_2}{p_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$
- $\frac{\rho_2}{\rho_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{1}{\gamma-1}}$

- Area change

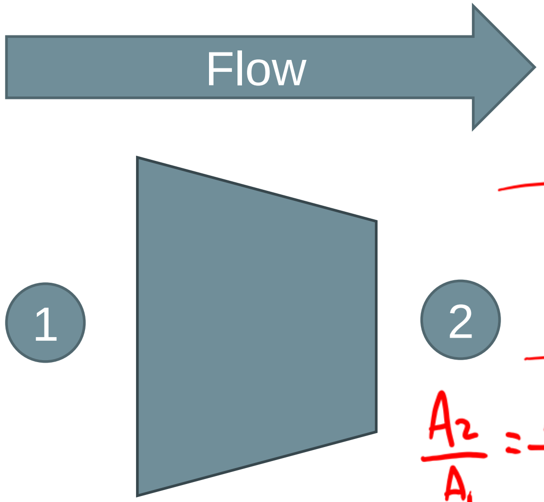
- $\frac{A_2}{A_1} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

In the isentropic case, all changes are related to the change in Mach number

Isentropic Nozzles

- Example
  - Consider an isentropic converging nozzle with air
  - $M_1 = 0.4$
  - $T_1 = 300\ \text{K}$
  - $p_1 = 101325\ \text{Pa}$
  - $\gamma = 1.4$
  - $R = 287 \dfrac{\text{J}}{\text{kg-K}}$
  - $M_2 = 0.8$

  - Compute $T_2, p_2, \rho_2, V_2, A_2/A_1$



$$T_2 = T_1 \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)} = (300\text{K})\frac{\left(1+\frac{1.4-1}{2}(0.4)^2\right)}{\left(1+\frac{1.4-1}{2}(0.8)^2\right)}$$

$$\boxed{T_2 = 274.47\text{K}}$$

$$P_2 = P_1 \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = (101325\text{Pa})\left(\frac{1+\frac{1.4-1}{2}(0.4)^2}{1+\frac{1.4-1}{2}(0.8)^2}\right)^{\frac{1.4}{1.4-1}}$$

$$\boxed{P_2 = 74{,}218\text{Pa}}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{(74{,}218\text{Pa})}{\left(287\frac{\text{J}}{\text{kg-K}}\right)(274.47\text{K})} = \boxed{0.942\frac{\text{kg}}{\text{m}^3}}$$

$$M = \frac{V}{a} \Rightarrow V = aM, \quad V_2 = M_2\sqrt{\gamma R T_2}$$

$$V_2 = (0.8)\sqrt{(1.4)\left(287\frac{\text{J}}{\text{kg-K}}\right)(274.47\text{K})}$$

$$\boxed{V_2 = 265.67\text{m/s}}$$

$$\frac{A_2}{A_1} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= \left(\frac{0.4}{0.8}\right)\left(\frac{1+\frac{1.4-1}{2}(0.8)^2}{1+\frac{1.4-1}{2}(0.4)^2}\right)^{\frac{1.4+1}{2(1.4-1)}} = \boxed{0.383 = \frac{A_2}{A_1}}$$

Area-Mach Number Relationships

- Recall our "one-dimensional" flows

Momentum:

$$dP + \rho V dV = 0$$

Mass:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

$$dV = -\left(\frac{dA}{A} + \frac{d\rho}{\rho}\right)V$$

Combine Mass/momentum

$$dP = \rho V^2\left(\frac{dA}{A} + \frac{d\rho}{\rho}\right)$$

$$dP = \rho V^2\left(\frac{dA}{A} + \frac{1}{a^2}\frac{1}{\rho}dP\right)$$

$$dP = \rho V^2\frac{dA}{A} + \frac{V^2}{a^2}dP$$

$$dP - M^2 dP = \rho V^2\frac{dA}{A}$$

$$dP(1-M^2) = \rho V^2\frac{dA}{A}$$

$$dP(1-M^2) = P\gamma\frac{V^2}{a^2}\frac{dA}{A}$$

$$dP(1-M^2) = P\gamma M^2\frac{dA}{A}$$

For isentropic System

$$d\rho = \frac{1}{a^2}dP$$

For ideal gas

$$a^2 = \gamma RT$$ and

$$P = \rho RT$$

Combine

$$\rho = \frac{P\gamma}{a^2}$$

- $\frac{dp}{dA} = \left(\frac{\gamma M^2}{1-M^2}\right)\frac{p}{A}$

Area-Mach Number Relationships

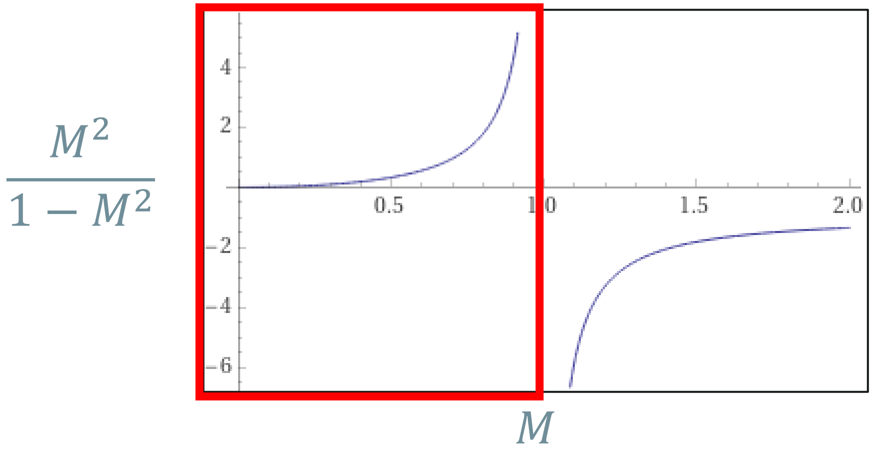
$$\frac{dp}{dA} = \gamma \underbrace{\left(\frac{M^2}{1-M^2}\right)}_{\text{plotted}} \frac{p}{A}$$

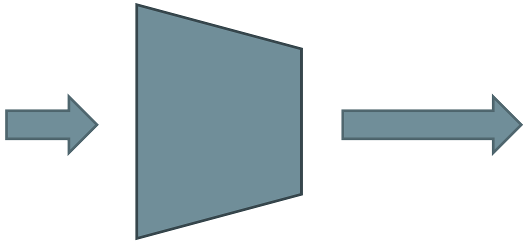
- For a subsonic flow $(M < 1)$
  - $\frac{dp}{dA} > 0$

$$\frac{dp}{dA} = \frac{dp}{dx}\frac{dx}{dA} > 0$$

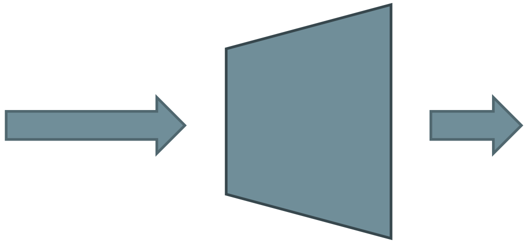
for subsonic nozzle, $\frac{dA}{dx} < 0$

then, $\frac{dp}{dx} > 0 \longrightarrow p_1 > p_2$ accelerates flow





Subsonic Nozzle



Subsonic Diffuser

Area-Mach Number Relationships

$$\frac{dp}{dA} = \gamma\left(\frac{M^2}{1-M^2}\right)\frac{p}{A}$$

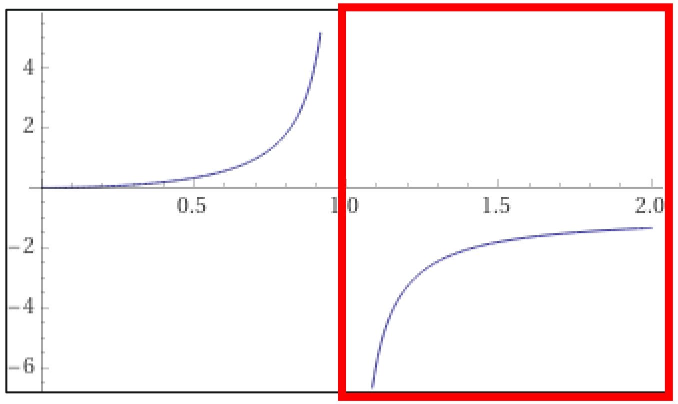
- For a supersonic flow $(M > 1)$
  - $\frac{dp}{dA} < 0$

$$\frac{dp}{dx}\frac{dx}{dA} < 0$$

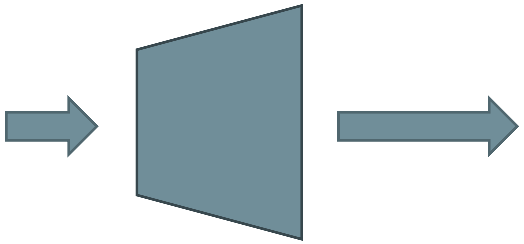
If $\frac{dA}{dx} < 0$

then $\frac{dp}{dx} > 0$ → $p_2 > p_1$ decelerates flow

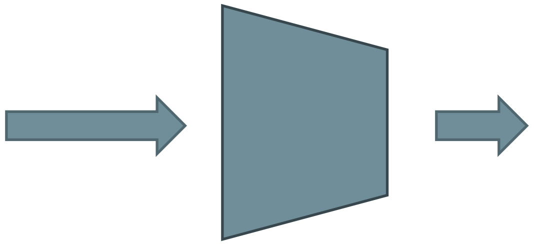
$\frac{M^2}{1-M^2}$



M



Supersonic Nozzle



Supersonic Diffuser

Area-Mach Number Relationships

$$\frac{dp}{dA} = \left(\frac{\gamma M^2}{1 - M^2}\right)\frac{p}{A}$$

$$\frac{d\rho}{dA} = \left(\frac{M^2}{1 - M^2}\right)\frac{\rho}{A}$$

$$\frac{dV}{dA} = -\left(\frac{1}{1 - M^2}\right)\frac{V}{A}$$

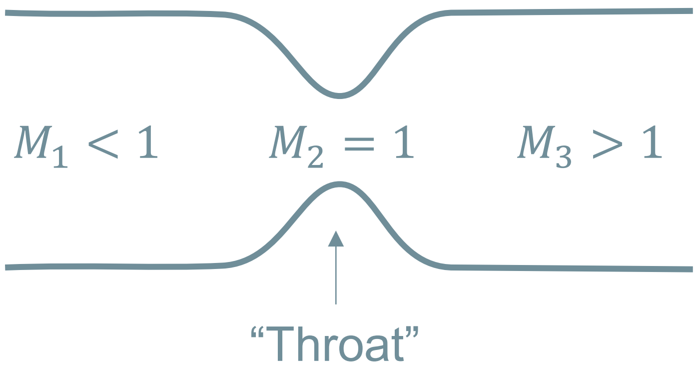
As the area decreases:

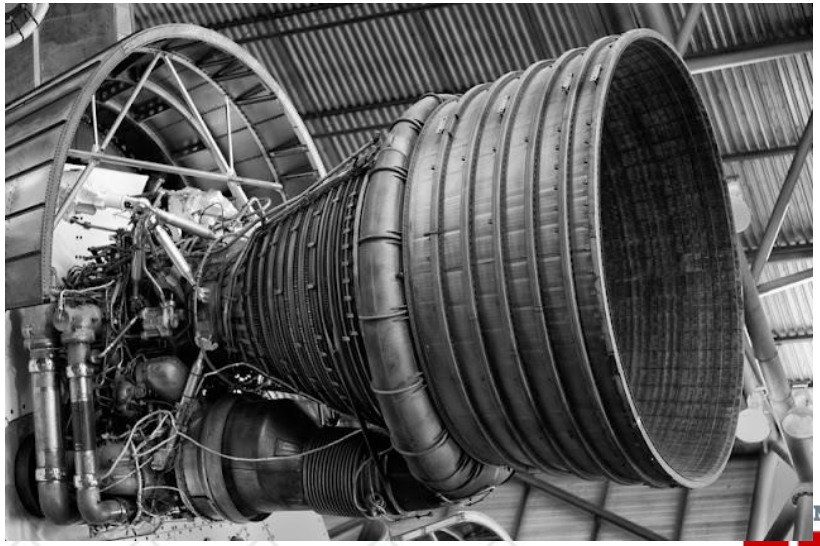
	Subsonic (M<1)	**Supersonic (M>1)**
Pressure (p)	Decreases	Increases
Density (ρ)	Decreases	Increases
Velocity (V)	Increases	Decreases

Can we ever accelerate a subsonic flow to $M > 1$?

Converging-Diverging Nozzles

- DeLaval nozzle
  - Converging nozzle until $M = 1$
  - Diverging nozzle afterwards
  - **Must reach $M = 1$ in throat**
    - Otherwise, $M_3 < 1$
- This requires specific design
  - Compute nozzle in two parts
    - $1 \rightarrow 2$ is the first (subsonic) process
    - $2 \rightarrow 3$ is the second (supersonic) process
  - We looked at a converging subsonic nozzle
  - Follow same principles, but set $M_2 = 1$





Converging-Diverging Nozzles

- DeLaval nozzle
  - Example: Assume Air, $M_1 = 0.5, \mathrm{A}_1 = 1\ \mathrm{m}^2, p_1 = 1\ \mathrm{MPa}, M_3 = 1.5$

$$A_2 = A_1 \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= (1 m^2)\left(\frac{0.5}{1}\right)\left(\frac{1+\frac{1.4-1}{2}(1)^2}{1+\frac{1.4-1}{2}(0.5)^2}\right)^{\frac{1.4+1}{2(1.4-1)}} = 0.746 m^2$$

$$A_3 = A_2 \frac{M_2}{M_3}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

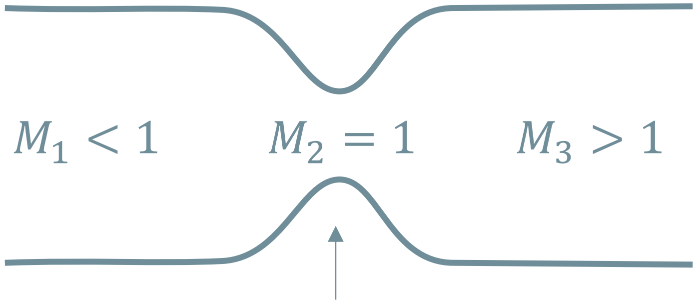
$$= (0.746 m^2)\left(\frac{1}{1.5}\right)\left(\frac{1+\frac{1.4-1}{2}(1.5)^2}{1+\frac{1.4-1}{2}(1)^2}\right)^{\frac{1.4+1}{2(1.4-1)}} = 0.874 m^2$$

$$p_2 = p_1\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{p_2 = 0.954\ MPa}$$

$$p_3 = p_2\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_3^2}\right)^{\frac{\gamma}{\gamma-1}}$$

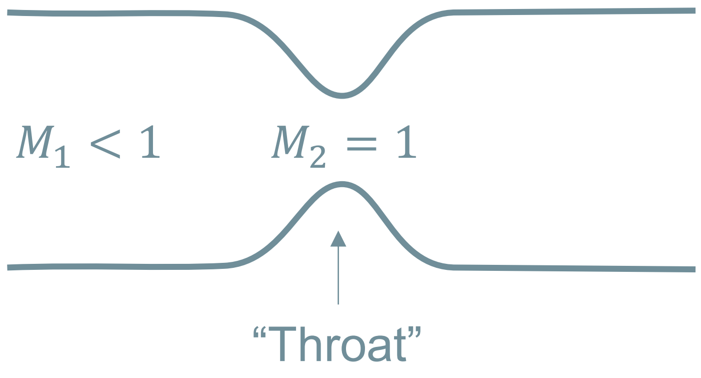
$$\boxed{p_3 = 0.492\ MPa}$$



  - Pressure plays a critical role in controlling this flow

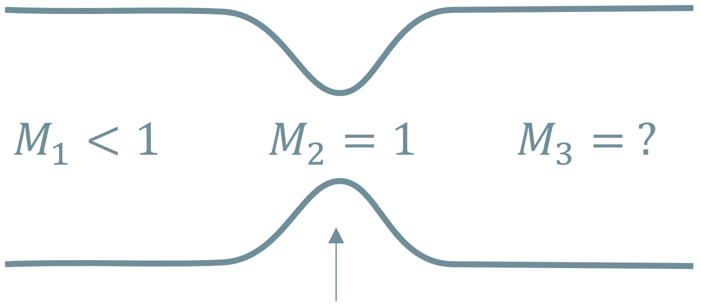
Converging-Diverging Nozzles

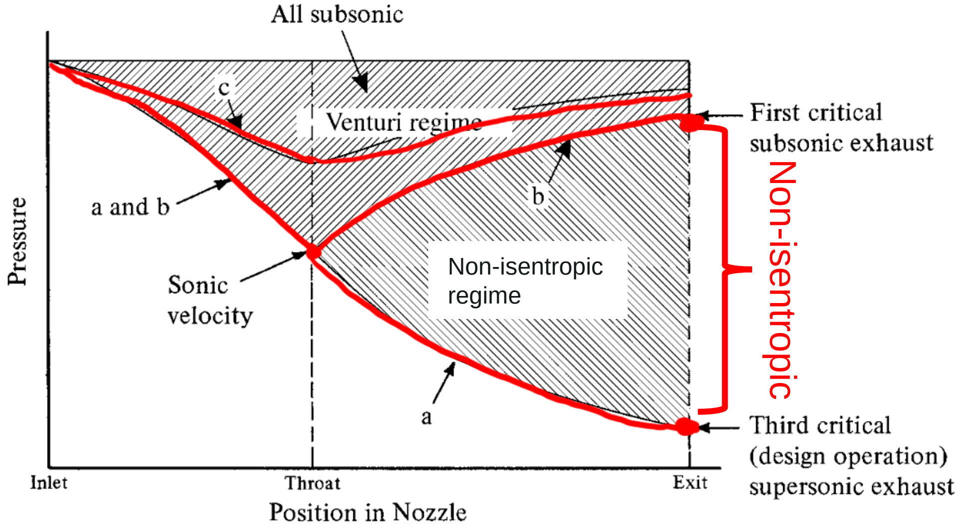
- Choked Flow
  - Cannot have $M_2 > 1$ since area decreases from 1 to 2
  - Flow must be at $M_2 = 1$ in the throat (otherwise $M_3 < 1$)
  - Max nozzle flow rate $\dot{m}_{\text{max}} = \rho_2 U_2 A_2$
  - This condition is called "choked flow"



Converging-Diverging Nozzles

- If the throat is subsonic, the diverging section will be as well
  - Venturi regime (e.g., c)
- If the throat is at $M_2 = 1$, the diverging section can be subsonic or supersonic depending on exit pressure
  - First critical - subsonic exhaust (b)
  - Third critical - supersonic exhaust (a)
- Discuss everything between later





Friction Losses in Nozzles

- Only isentropic nozzles and diffusers so far – how do we account for losses?

> Note: There are many ways to define these efficiencies based on different quantities. These are the ones we'll use in this course

- Isentropic efficiency of a nozzle:
  - $\eta_\eta = \frac{\Delta h_{actual}}{\Delta h_{ideal}} = \frac{h_{t1} - h_2}{h_{t1} - h_{2s}} = \frac{T_{t1} - T_2}{T_{t1} - T_{2s}}$

V_2^2/V_1^2

- Isentropic efficiency of a diffuser:
  - $\eta_d = \frac{\Delta h_{actual}}{\Delta h_{ideal}} = \frac{h_{t2} - h_1}{h_{t2s} - h_1} = \frac{T_{t2} - T_1}{T_{t2s} - T_1}$

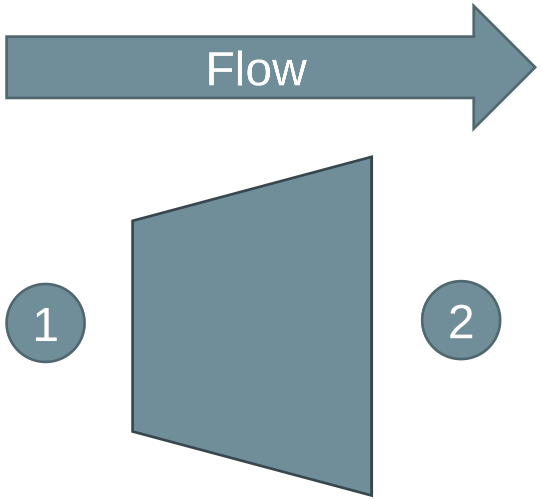
This form is valid for ideal gases with constant specific heat, which is a decent approximation when working with relatively small temperature increases (i.e., not combustion)

Friction Losses in Nozzles

$$T_{t1} = T_1\,(X)$$

$$T_1 = \frac{T_{t1}}{X}$$

- Example: Consider a non-isentropic nozzle with constant c_p
  - $M_1 = 3$
  - $T_1 = 600\ \mathrm{K}$
  - $\gamma = 1.4$
  - $M_2 = 6$
  - $\eta_n = 0.85$



$$T_{t1} = T_1\left(1 + \frac{\gamma-1}{2} M_1^2\right)$$

$$= (600\mathrm{K})\left(1 + \frac{1.4-1}{2}(3)^2\right) = \boxed{1680\mathrm{K}}$$

for isentropic case, $T_{t2} = T_{t1}$

$$T_{2s} = T_{t2s}\left(1 + \frac{\gamma-1}{2} M_2^2\right)^{-1}$$

$$T_{2s} = (1680\mathrm{K})\left(1 + \frac{1.4-1}{2}(6)^2\right)$$

$$\boxed{T_{2s} = 204.88\mathrm{K}}$$

- Find the **static** outlet temperature T_2

$$\eta_n = \frac{T_{t1} - T_2}{T_{t1} - T_{2s}} = 0.85 = \frac{(1680\mathrm{K}) - T_2}{(1680\mathrm{K}) - (204.88\mathrm{K})}$$

$$\rightarrow \boxed{T_2 = 426.15\mathrm{K}}$$