HW 4: Dimensional Analysis
Due: 11/11/2025

1. [**6700 only**] The slope of the free surface of a steady wave in one-dimensional flow in a shallow liquid layer is described by the equation

$$\frac{\partial h}{\partial x} = -\frac{u}{g}\frac{\partial u}{\partial x},$$

where h is the height of the free surface relative to some reference level and u is the velocity of the liquid.

(a) Use a length scale L and a velocity scale V_o to nondimensionalize this equation.

(b) What is the nondimensional parameter that characterizes this flow?

2. Consider the simplified momentum equation shown below for a steady, incompressible flow with constant viscosity and negligible body forces. The problem has a characteristic length scale of L and a velocity scale of U.

$$\underbrace{\rho\, u\,\frac{\partial u}{\partial x} + \rho\, v\,\frac{\partial u}{\partial y}}_{\text{I}} = \underbrace{-\frac{\partial p}{\partial x}}_{\text{II}} + \underbrace{\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{\text{III}}$$

(a) Determine the proper nondimensionalization of the *pressure* term if Term I is zero.

(b) Determine the proper nondimensionalization of the *pressure* term if Term III is zero.

(c) Provide real world examples when the two cases (a) and (b) would be valid.

3. A ship is cruising at 30 knots (15.43 m/s) and the passengers experience smoke pollution on the deck from the stack exhaust. A 20:1 scale model is to be studied to provide a hopeful solution.

(a) Should a water channel or a wind tunnel be used? State the relevant nondimensional parameter in this problem.

(b) What velocity should be used in the model study? Clearly show your work.

4. A 25:1 scale model of a submarine is being tested in a wind tunnel in which $p = 200$ kPa and $T = 300$ K. The expected speed of the prototype is 30 km/hr. Assume that the submarine will NOT operate near the free surface of the ocean.

(a) What should be the speed of the wind tunnel?

(b) What is the ratio of the model drag force over the prototype drag force, $(F_D)_m/(F_D)_p$?

5. A windmill is designed to operate at 20 rpm in a 15-mph wind and produce 300 kW of power. The blades are 175 ft in diameter. A 10:1 model is to be tested at 90-mph wind velocity.

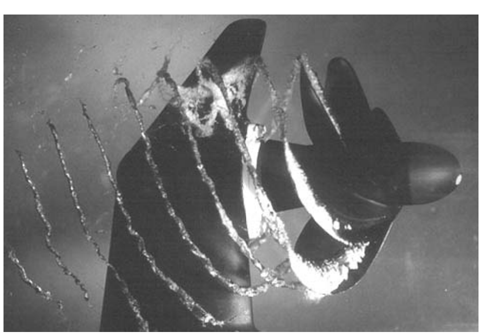
(a) What rotor speed should be used in the model experiment?

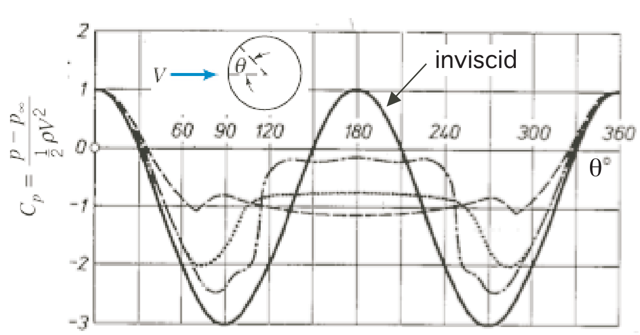
(b) What power should be expected in the model experiment?

6. Consider a circular cylinder of diameter D and length ℓ that is being towed in a water tank at a velocity U.

(a) Express the drag force in dimensionless form as a function of all of the relevant nondimensional parameters. Use Buckingham's Pi theorem.

(b) [**6700 only**] Estimate the maximum speed $U_{\max}$ at which the cylinder can be towed in the water tank without causing cavitation. The phenomenon of cavitation, i.e., the formation gas bubbles in a liquid flow, occurs when the local pressure drops below the vapor pressure of the liquid. The photograph below illustrates cavitation on a propeller. In this problem, assume that the onset of cavitation occurs at a cavitation number $Ca = 0.5$, where $Ca = \frac{p-p_v}{1/2\,\rho U^2}$ and p_v denotes the vapor pressure of the liquid. Note, the smaller Ca, the more likely the occurrence of cavitation, which is typically an undesirable phenomenon. In order to answer this question, we need to know something about the pressure distribution around a circular cylinder. The plot below shows the variation of the dimensionless pressure coefficient, $C_p = \frac{p-p_\infty}{1/2\,\rho U^2}$, for several different Reynolds numbers. For this problem, assume that the pressure reaches a minimum at $\theta \approx 90°$, as shown, where the pressure coefficient has a minimum value of $C_p = -2.4$. [HINT: to determine $U_{\max}$ you need to simultaneously match C_p and C_a.]





Fluids Terminology

Please define the following terminology. Type your responses. For each item, include a reference in parenthesis at the end of the definition as to where you obtained your information.

1. nondimensional parameter
2. dimensional analysis
3. geometric similarity
4. dynamic similarity
5. fundamental dimensions
6. Buckingham's Pi theorem
7. Reynolds number
8. Froude number
9. Mach number
10. Pressure coefficient
11. Drag coefficient