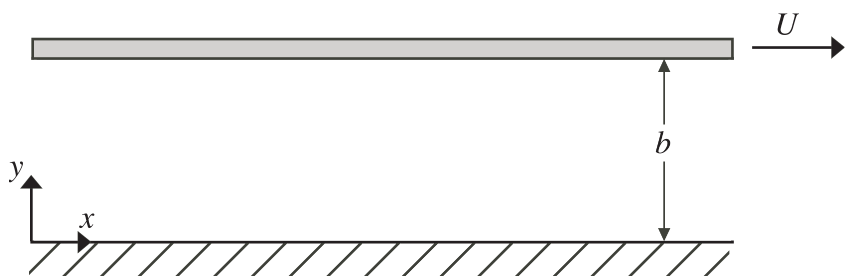
HW 3: Momentum and Energy Equation

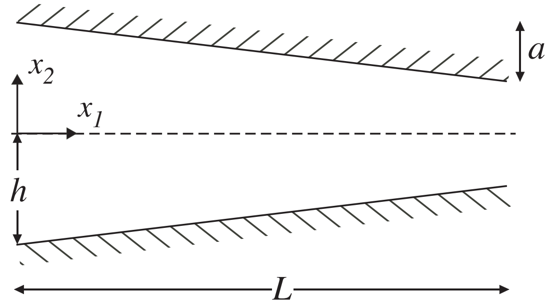
1. Consider incompressible, planar flow of a Newtonian fluid (with dynamic viscosity μ and density ρ) between two infinitely long parallel plates separated by a distance h, as shown. The upper plate moves at a constant speed of U, while the lower plate remains stationary. In addition, there is an applied pressure gradient along the x-direction ($dP/dx = -K$ where $K > 0$).



- **(a)** Simplify the x- and y-momentum equations for this flow.
- **(b)** Solve for the velocity field.
- **(c)** Plot u/U versus y/b for $\tilde{P} = 15, 10, 5, 0, -5, -10, -15$ where $\tilde{P} = b^2K/(\mu U)$. [Put all lines on a single figure]
- **(d)** Calculate the stress tensor, σ_{ij}.
- **(e)** Derive expressions for the shear stress acting on the upper and lower walls.

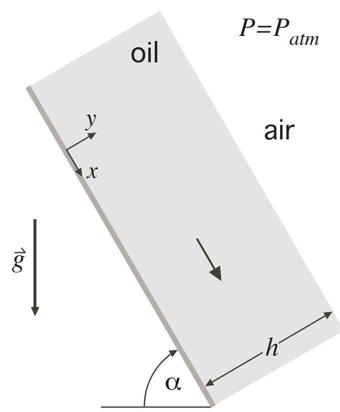
2. Consider steady, incompressible flow in a converging channel as shown below. Flow enters the channel uniformly with an inlet velocity of V_o. The horizontal velocity inside the channel is modeled as one-dimensional with the expression

$$u_1 = V_o \left(1 - \frac{a}{hL}\, x_1\right)^{-1} .$$



(a) Determine the expression for the vertical velocity u_2 as a function of both x_1 and x_2. (Hint: The appropriate boundary condition at the wall is $\vec{u} \cdot \hat{n} = 0$.)

3. Consider the flow of an oil film falling down an inclined wall as shown below.



(a) Simplify the x- and y-momentum equations for this flow. Include the effect of the hydrostatic pressure. [Clearly state your assumptions]

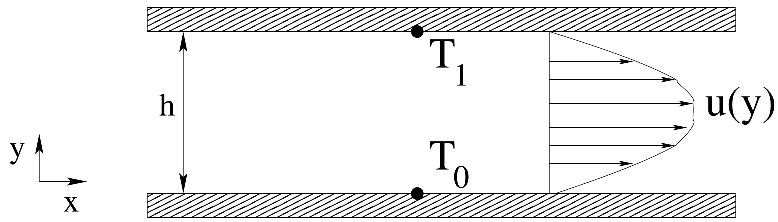
(b) Solve for the pressure inside the oil film as a function of y. [Hint: Use the y-momentum equation and the boundary condition $P(y = h) = P_{atm}$]

(c) Solve for the velocity of the oil, $u(y)$, using the x-momentum equation.

(d) Plot the nondimensional velocity $\tilde{u}$ versus y/h for the case $\alpha = 60°$, where $\tilde{u} = u\mu/(\rho g h^2)$.

(e) Derive an expression for the shear stress acting on the stationary wall.

4. The stream wise velocity profile is fully-developed, steady-state, incompressible flow between two infinite parallel plates driven by a constant stream wise pressure gradient dP/dx. Consider the case where the top and bottom plates are heated to constant temperatures T_0 and T_1 as shown in the figure.



(a) Solve for the velocity field.

(b) Starting with the energy equation, solve for the temperature distribution.

(c) Non dimensionalize your answer to part [b.] using the temperature difference $\Delta T = T_1 - T_0$ and then plot what the non dimensional profile should look like and how it will change with $\chi = \frac{h^4}{144k\Delta T}\left(\frac{dP}{dx}\right)^2$ for $T_1 > T_0$.