

Aerospace Propulsion

Lecture 14

Airbreathing Propulsion VI

Airbreathing Propulsion: Part VI

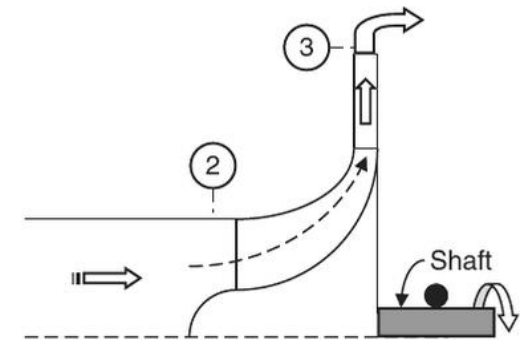
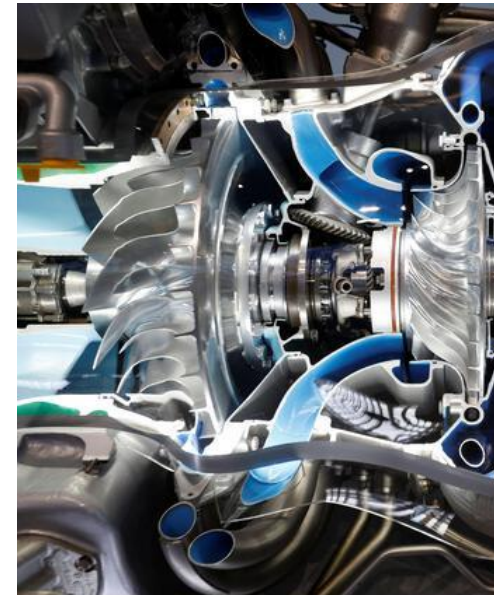
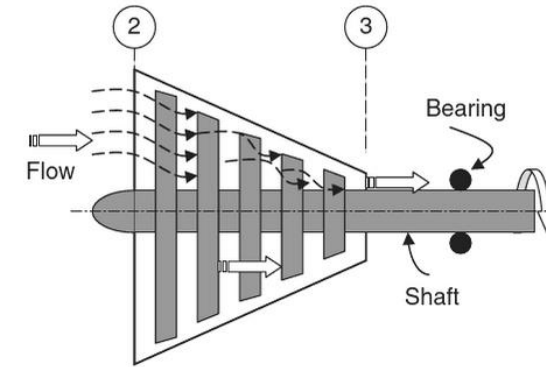
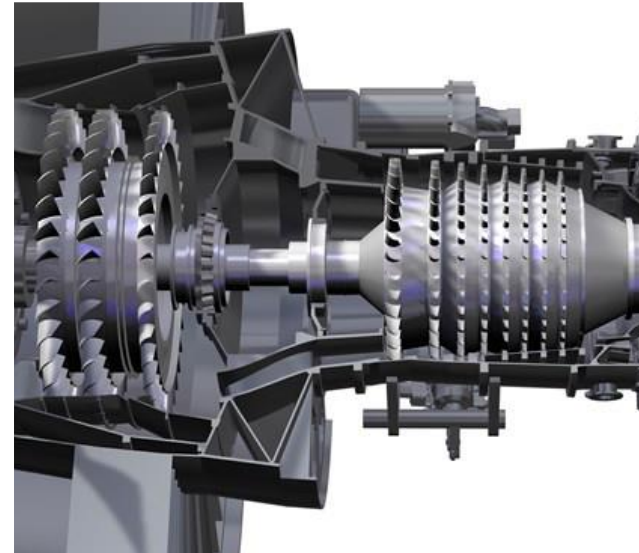
- Turbomachinery
- Compressor
- Turbine
- Practical Considerations
- Summary

Turbomachinery

- “Machines that exchange energy with a fluid through shaft rotation”
 - Compressors and Turbines
- Axial: Principal flow direction is parallel to turbomachine axis
- Radial: Principal flow direction is perpendicular to turbomachine axis

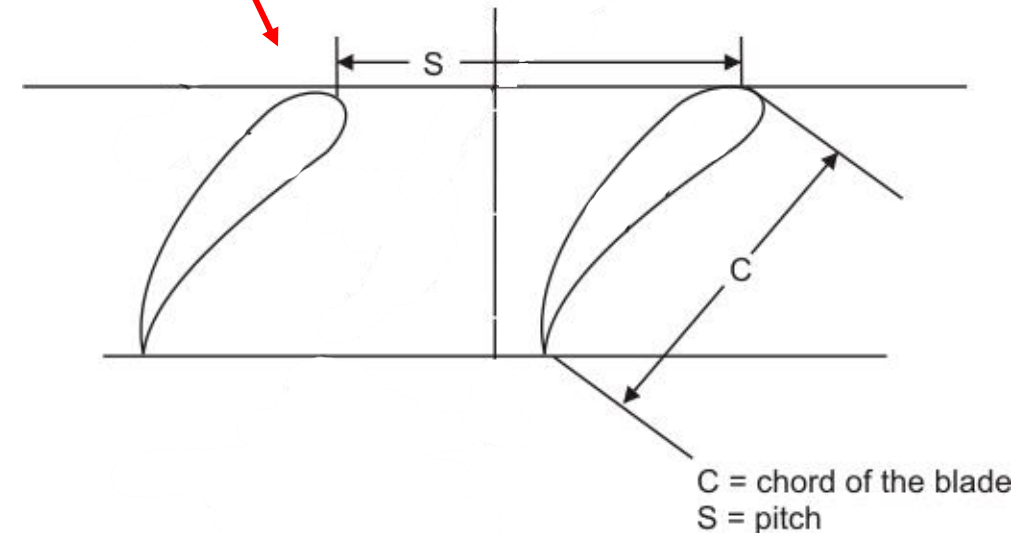
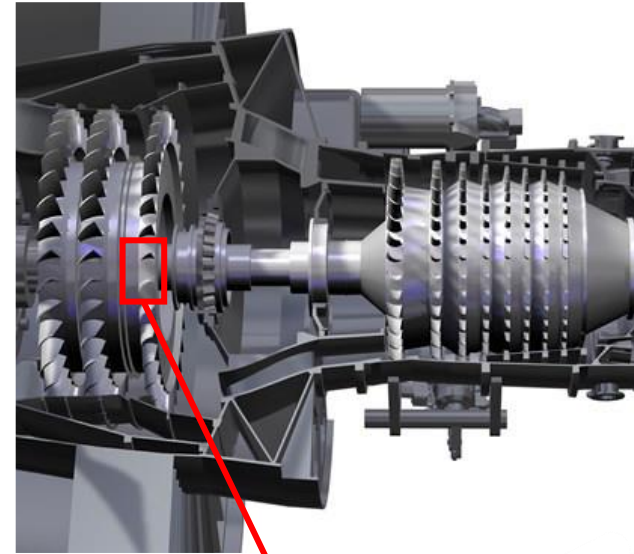
Turbomachinery

- Axial Turbomachinery
 - Primary form of turbomachinery in most aircraft applications
 - Relatively small entry area
 - Remains efficient even at high velocities
- Radial Turbomachinery
 - Not as common in aircraft engines, but does appear in slower aircraft
 - Often used in smaller turboprops
 - Flow turning requires larger entry area
 - Increased drag on engine



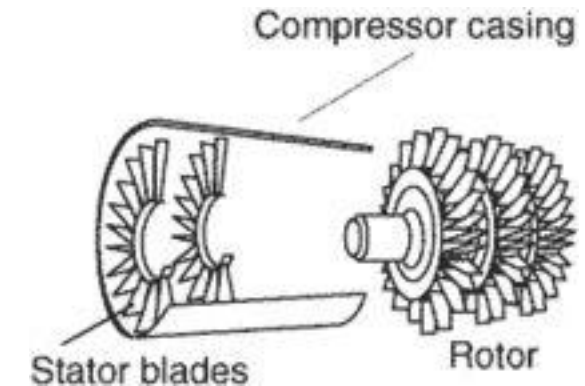
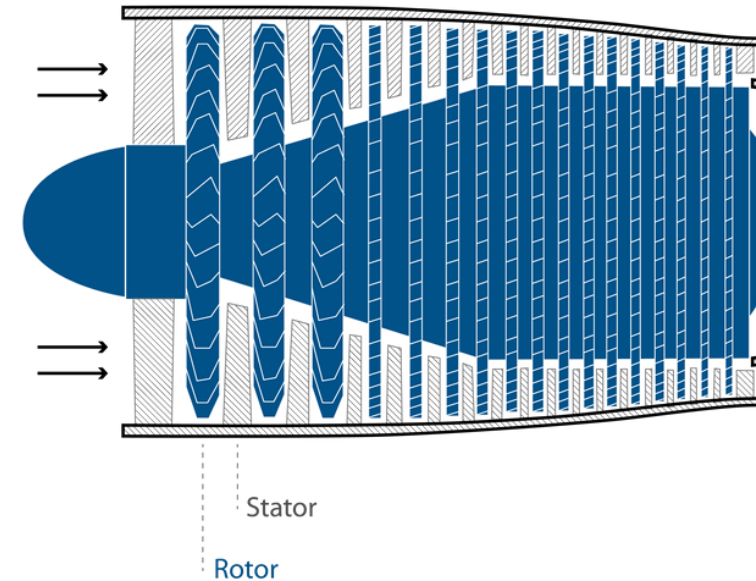
Turbomachinery

- Axial turbomachinery relies on a series of airfoil shaped “blades”
- Chord c : Longest linear distance of a blade
- Pitch s : Distance between consecutive blades
- Chord and pitch vary between stages



Turbomachinery

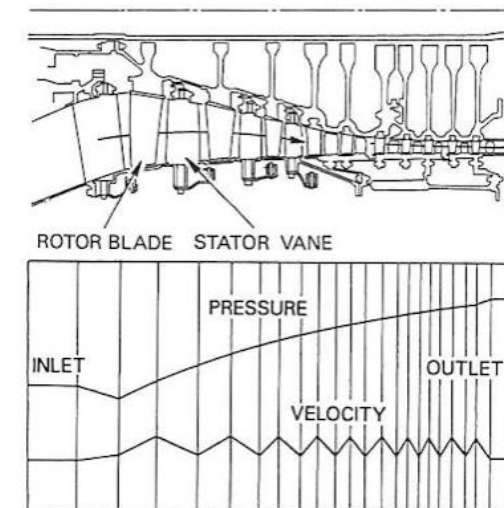
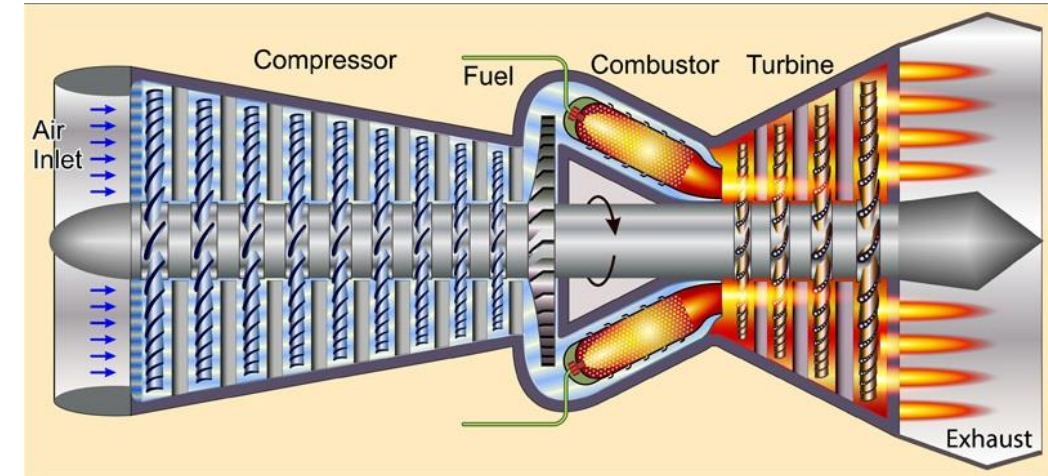
- Turbomachinery relies on a combination of stators and rotors
- Rotors
 - Moving blade that exchanges energy with fluid
- Stator
 - Stationary blade that turns flow
- A compressor/turbine “stage” is a single rotor/stator pair



Turbomachinery



- What happens to flow as it travels through turbomachinery?
 - Compressor
 - Density increasing
 - Flow area decreasing
 - Turbine
 - Density decreasing
 - Flow area increasing
- Conservation of mass
 - Essentially constant axial velocity through compressor/turbine

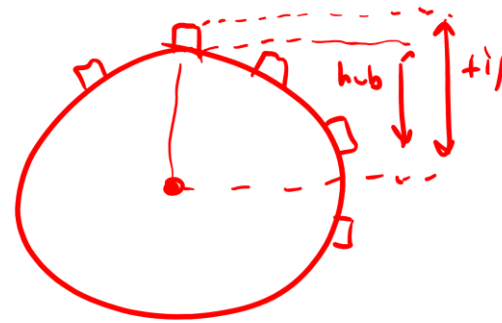


$$\dot{m} = \rho U A$$

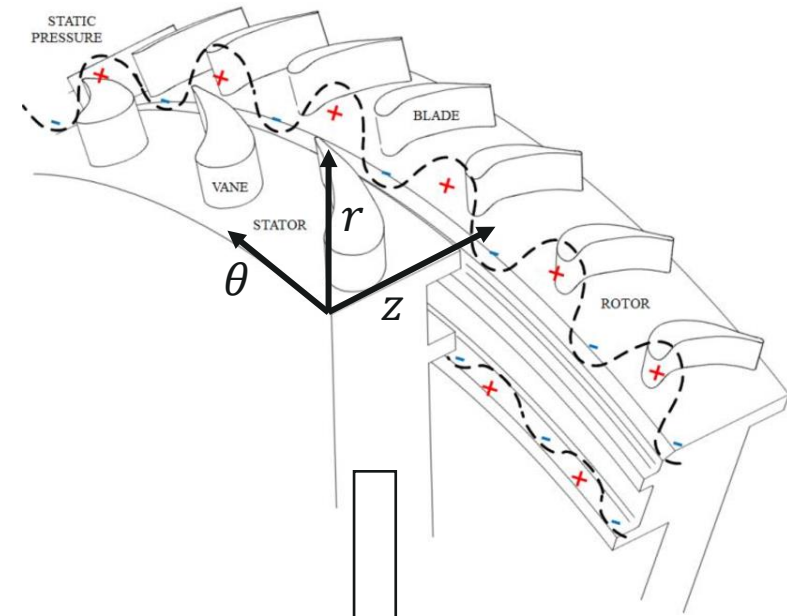
↑ ↓

$$U \approx \text{constant}$$

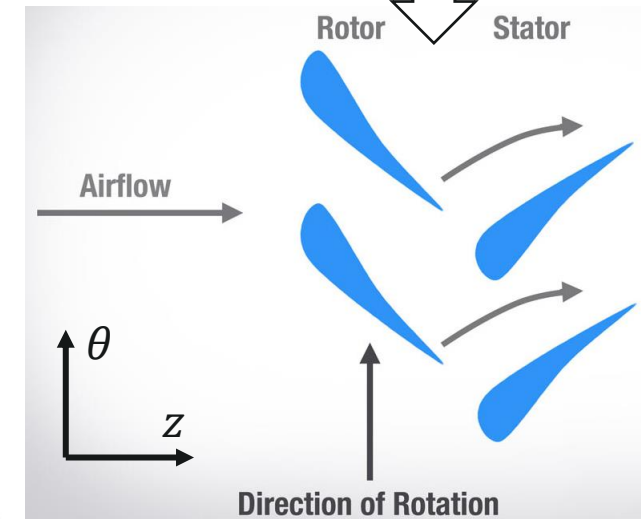
Turbomachinery



- Analyze turbomachinery by “unwrapping” it
 - Hub to tip ratio can be as small as 0.5 at lowest pressure stages but 0.9 or greater at high pressure stages
 - Neglect curvature effects
 - Neglect change in radius through a stage



- Rotational velocity of blade
 - $U = \omega r \hat{e}_\theta$
 - ω : rotational velocity
 - r : distance from centerline (radius)



Turbomachinery

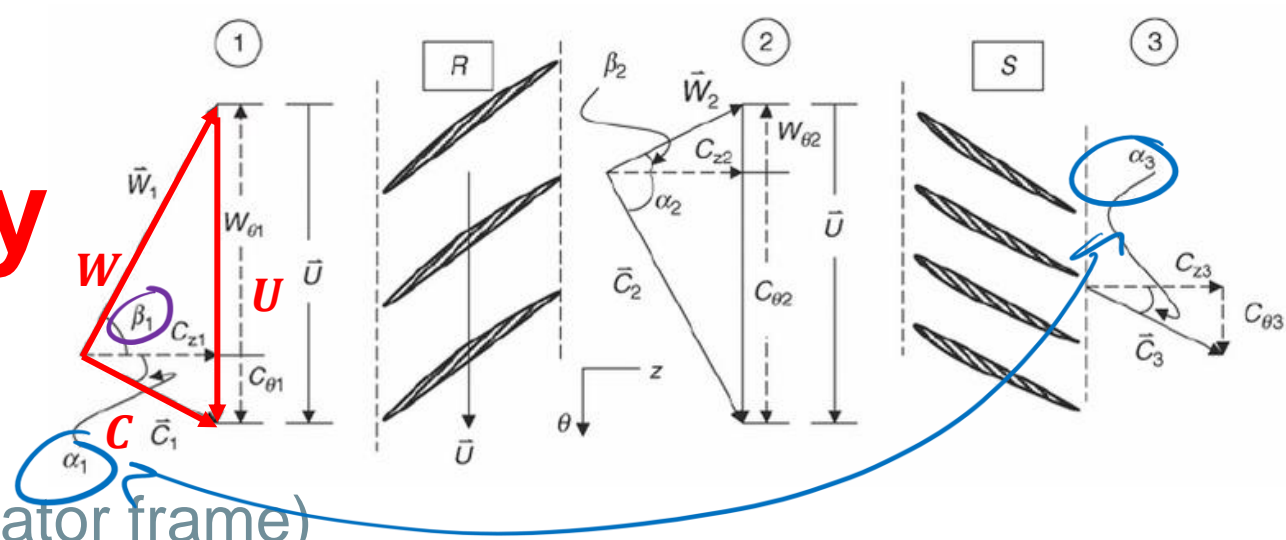
- Two reference frames used when analyzing turbomachinery
 - Absolute reference frame (stator frame)
 - Observer in a lab watching the machine run
 - Rotor spins
 - Stator does not move
 - Relative reference frame (rotor frame)
 - Observer attached to spinning rotor
 - Rotor does not move
 - Stator spins

Turbomachinery

• Velocity triangles

- U : Velocity of the rotor (stator frame)
- C : Absolute velocity of flow (stator frame)
- W : Relative velocity of flow (rotor frame)
- $C = U + W$

- α : Angle of absolute velocity relative to axial direction
 - At design point, this is same at entrance/exit angle of stage
- β : Angle of relative velocity relative to axial direction
 - At design point, this is same at entrance/exit angle of stage



Compressor

Axial velocity component

• Compressor is constant

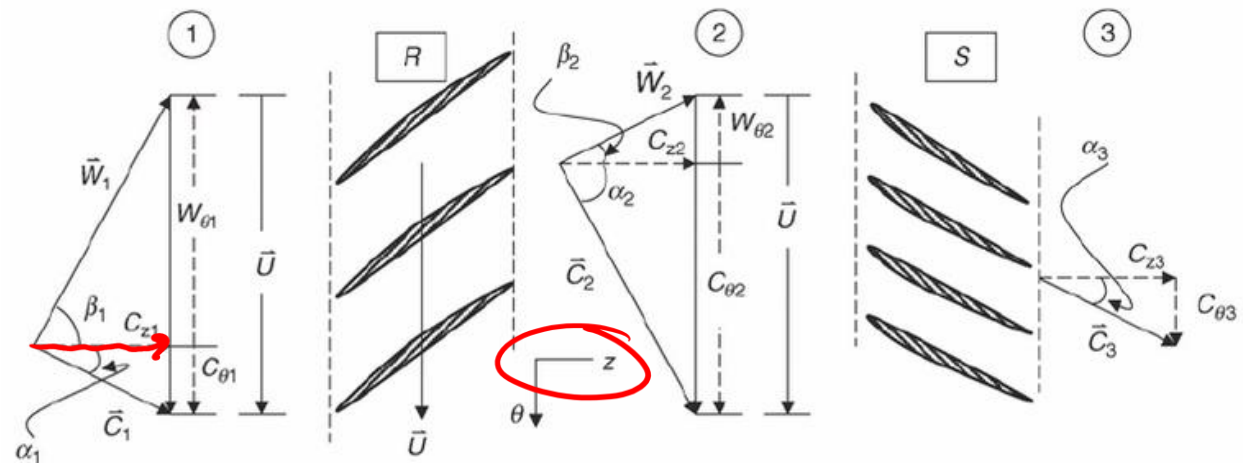
- Mechanically compresses air in gas turbine
- Rotor imparts angular momentum and power to fluid
- Stator removes angular momentum from fluid

• Velocity triangles

- $C_{zi} = C_i \cos \alpha_i$
- $W_{zi} = W_i \cos \beta_i$

~~✗~~ • Axial Velocity: $C_{z1} = C_{z2} = C_{z3} = W_{z1} = W_{z2} = W_{z3}$

• $\frac{C_1}{C_2} = \frac{\cos \alpha_2}{\cos \alpha_1}$ and $\frac{W_1}{W_2} = \frac{\cos \beta_2}{\cos \beta_1}$



$\dot{W} = FV = \tau \omega$

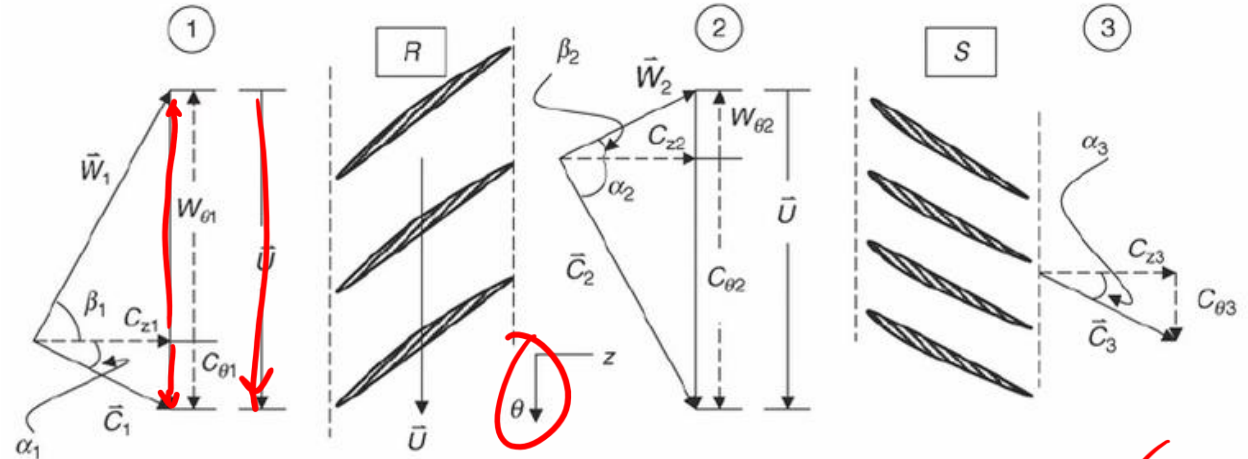
equal $\rightarrow C_{z1} = C_1 \cos \alpha_1 \Rightarrow C_1 \cos \alpha_1 = C_2 \cos \alpha_2$
 $\rightarrow C_{z2} = C_2 \cos \alpha_2$
 $\frac{C_1}{C_2} = \frac{\cos \alpha_2}{\cos \alpha_1}$

equal $\rightarrow W_{z1} = W_1 \cos \beta_1 \Rightarrow \frac{W_1}{W_2} = \frac{\cos \beta_2}{\cos \beta_1}$
 $\rightarrow W_{z2} = W_2 \cos \beta_2$

Compressor

• Velocity triangles

- $C_{\theta i} = C_i \sin \alpha_i = C_{zi} \tan \alpha_i$
- $W_{\theta i} = W_i \sin \beta_i = W_{zi} \tan \beta_i$
- $C_{\theta i} = W_{\theta i} + U$



$C_{\theta i} = C_i \sin \alpha_i$ from last slide, $C_i = \frac{C_{zi}}{\cos \alpha_i}$

$C_{\theta i} = C_{zi} \tan \alpha_i$

$C_{\theta 1} = C_1 \sin \alpha_1 = C_{z1} \tan \alpha_1$
 $C_{\theta 2} = C_2 \sin \alpha_2 = C_{z2} \tan \alpha_2 \Rightarrow \frac{C_{\theta 1}}{C_{\theta 2}} = \frac{C_{z1} \tan \alpha_1}{C_{z2} \tan \alpha_2} = 1$

$W_{\theta 1} = W_1 \sin \beta_1 = W_{z1} \tan \beta_1$
 $W_{\theta 2} = W_2 \sin \beta_2 = W_{z2} \tan \beta_2 \Rightarrow \frac{W_{\theta 1}}{W_{\theta 2}} = \frac{W_{z1} \tan \beta_1}{W_{z2} \tan \beta_2} = 1$

• $\frac{C_{\theta 1}}{C_{\theta 2}} = \frac{\tan \alpha_1}{\tan \alpha_2}$ and $\frac{W_{\theta 1}}{W_{\theta 2}} = \frac{\tan \beta_1}{\tan \beta_2}$

Assume constant radius for each stage

Compressor

• Torque and Power

• Torque ($\tau = rF = \Delta \text{Angular momentum}$)

- Both rotor and stator apply a torque to the flow (angular momentum)

$$\tau_{in} = \dot{m}(r_2 C_{\theta 2} - r_1 C_{\theta 1}) = \dot{m}r(C_{\theta 2} - C_{\theta 1}) = \dot{m}rC_z(\tan \beta_2 - \tan \beta_1)$$

$$\tau_{in} = \dot{m}r(C_{\theta 2} - C_{\theta 1})$$

$$\tau_{in} = \dot{m}rC_z(\tan \alpha_2 - \tan \alpha_1)$$

$$\tau_{in} = \dot{m}rC_z(\tan \beta_2 - \tan \beta_1)$$

• Power ($\dot{W} = FV = \tau\omega$)

- Only the rotor applies power to the flow

$$\dot{W}_{in} = \tau_{in}\omega = \dot{m}r\omega(C_{\theta 2} - C_{\theta 1}) = \dot{m}U(C_{\theta 2} - C_{\theta 1})$$

• Torque and Power

• Torque

- Both rotor and stator apply a torque to the flow (angular momentum)

$$\tau_{in} = \dot{m}(r_2 C_{\theta 2} - r_1 C_{\theta 1}) = \dot{m}r(C_{\theta 2} - C_{\theta 1}) = \dot{m}rC_z(\tan \beta_2 - \tan \beta_1)$$

• Power

- Only the rotor applies power to the flow

$$\dot{W}_{in} = \tau_{in}\omega = \dot{m}r\omega(C_{\theta 2} - C_{\theta 1}) = \dot{m}U(C_{\theta 2} - C_{\theta 1})$$

$$\text{Angular momentum} = \dot{m}rU$$

$$C_{\theta i} = W_{\theta i} + U$$

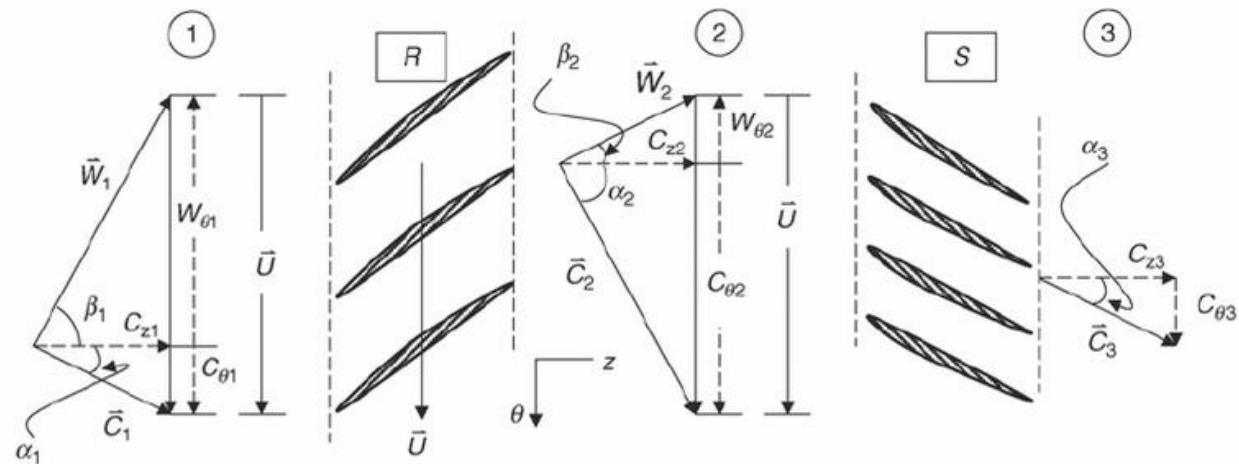
$$C_z \tan \alpha_i = W_z \tan \beta_i + U$$

\Downarrow

$$C_z \tan \alpha_2 = W_z \tan \beta_2 + U \quad (W_z = C_z)$$

$$\tan \alpha_2 = \tan \beta_2 + \frac{U}{C_z}$$

Compressor



• Stagnation Pressure

- Applying first law and combining with our power equation

- $\dot{W}_{12} = \dot{m}(h_{t2} - h_{t1})$

① → ② Adiabatic, work: $\dot{W}_{12} = \dot{m}(h_{t2} - h_{t1})$

② → ③ Adiabatic, no work $h_{t3} = h_{t2}$

Combine:

$$\dot{W}_{12} = \dot{m}(h_{t3} - h_{t1}) \quad \text{prev. slide}$$

$$= \dot{m} c_p (T_{t3} - T_{t1}) = \dot{m} U (C_{\theta 2} - C_{\theta 1})$$

$$T_{t3} = T_{t1} + \frac{U}{c_p} (C_{\theta 2} - C_{\theta 1})$$

①②③ • $\frac{p_{t3}}{p_{t1}} = \left[1 + \eta_{cs} \frac{U}{c_p T_{t1}} (C_{\theta 2} - C_{\theta 1}) \right]^{\frac{\gamma}{\gamma-1}}$

$$\frac{T_{t3}}{T_{t1}} = 1 + \frac{U}{c_p T_{t1}} (C_{\theta 2} - C_{\theta 1}) \quad (1)$$

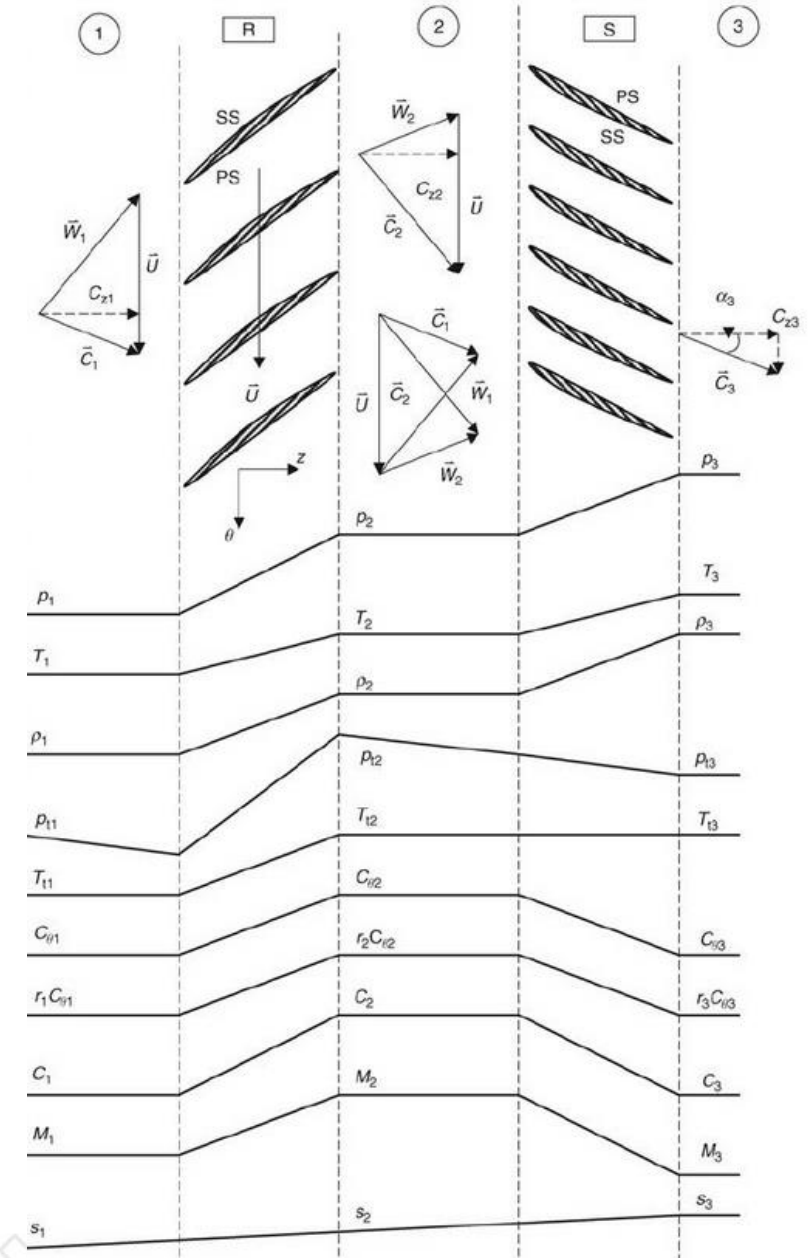
$$\eta_{cs} = \frac{h_{t3s} - h_{t1}}{h_{t3} - h_{t1}} = \frac{T_{t3s}/T_{t1} - 1}{T_{t3}/T_{t1} - 1}$$

$$\frac{T_{t3s}}{T_{t1}} = 1 + \eta_{cs} \left(\frac{T_{t3}}{T_{t1}} - 1 \right) \quad (2)$$

$$\frac{T_{t3s}}{T_{t1}} = \left(\frac{p_{t3}}{p_{t1}} \right)^{\frac{\gamma-1}{\gamma}} \quad (3)$$

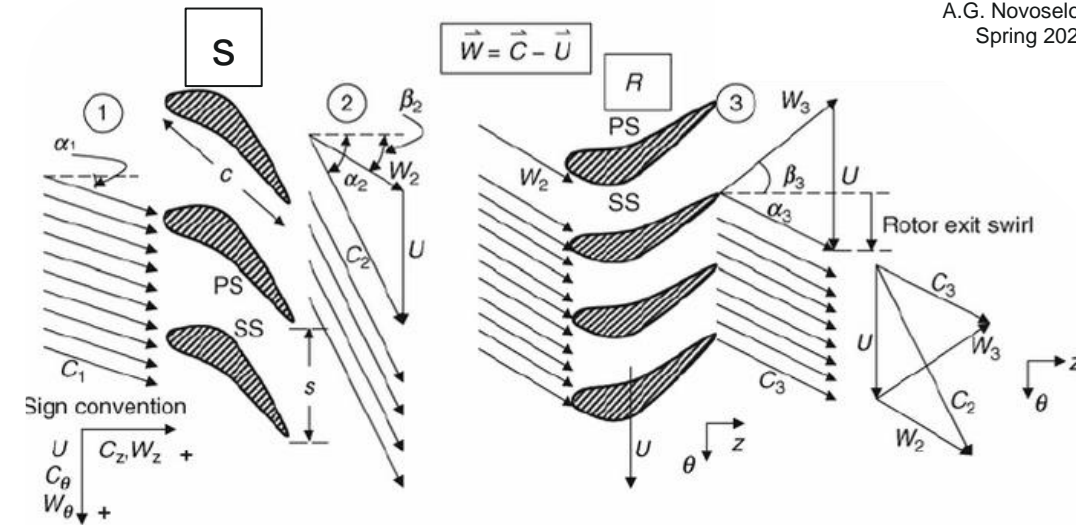
Compressor

- Stagnation Temperature
 - Previously derived stagnation temperature change across an arbitrary compressor
 - $\frac{T_{t3}}{T_{t1}} = 1 + \frac{1}{\eta_{cs}} \left(\frac{p_{t3}^{\frac{\gamma-1}{\gamma}}}{p_{t1}^{\frac{\gamma-1}{\gamma}}} - 1 \right)$ *← can apply to each stage*
- Each compressor stage
 - Increases stagnation pressure
 - Increases stagnation temperature
 - Does not change velocity (approximately)
 - Requires work input



Turbine

- Turbine extracts energy from flow
 - Stator adds angular momentum to fluid
 - Rotor removes angular momentum and power from fluid
- Velocity triangles
 - Axial velocity: $C_{z1} = C_{z2} = C_{z3} = W_{z1} = W_{z2} = W_{z3}$
 - $C_{zi} = C_i \cos \alpha_i$
 - $W_{zi} = W_i \cos \beta_i$
 - Tangential velocity
 - $C_{\theta i} = C_i \sin \alpha_i = C_{zi} \tan \alpha_i$
 - $W_{\theta i} = W_i \sin \beta_i = W_{zi} \tan \beta_i$
 - $C_{\theta i} = W_{\theta i} + U$



Stator first

Stator accelerates flow (in θ)

So rotor can extract
the energy

Turbine

- Torque and Power

- Torque

- $\tau_{out} = \dot{m}r(C_{\theta 2} - C_{\theta 3})$

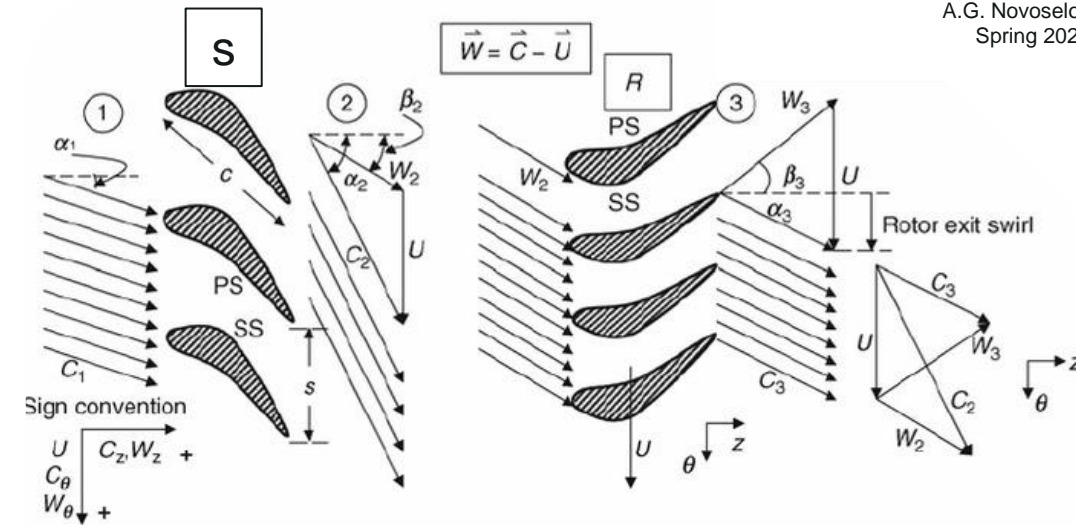
- Power

- $\dot{W}_{out} = \tau_{out}\omega = \dot{m}U(C_{\theta 2} - C_{\theta 3})$

- Stagnation Pressure

- First law + power

- $$\frac{p_{t3}}{p_{t1}} = \left[1 - \frac{1}{\eta_{ts}} \frac{U}{c_p T_{t1}} (C_{\theta 2} - C_{\theta 3}) \right]^{\frac{\gamma}{\gamma-1}}$$



$$h_{t1} = h_{t2} \quad (\text{no work / heat})$$

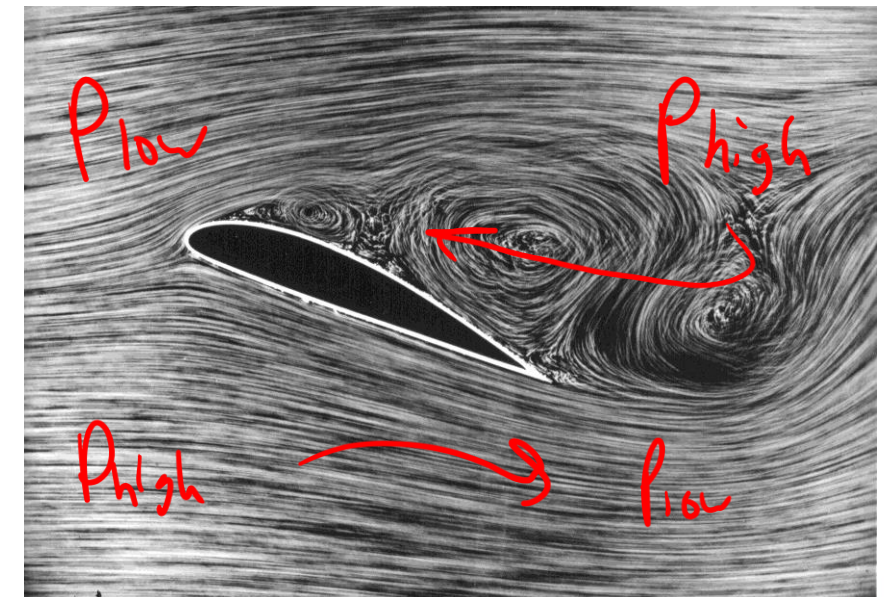
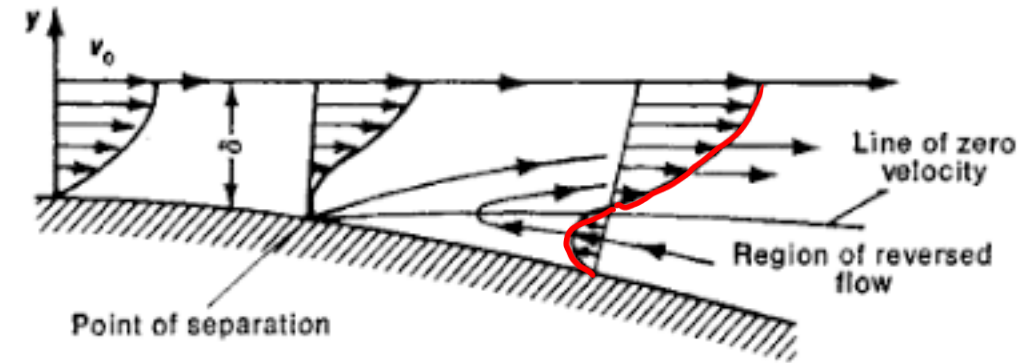
$$\dot{m} c_p (T_{t1} - T_{t3}) = \dot{m} U (C_{\theta 2} - C_{\theta 3})$$

$$T_{t3} = T_{t1} - \frac{U}{c_p} (C_{\theta 2} - C_{\theta 3})$$

$$\eta_{ts} = \frac{h_{t1} - h_{t3}}{h_{t1} - h_{t3s}}$$

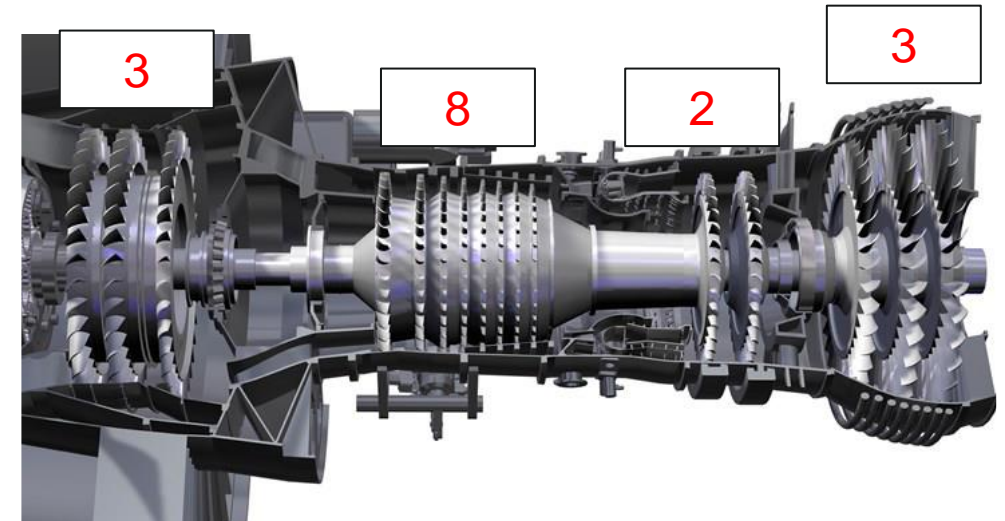
Practical Considerations

- Boundary layer separation
 - Detachment of a boundary layer from surface leading to a wake
 - Highly non-isentropic and inefficient
 - Leads to “stall”
 - Promoted by large turning angles
 - Promoted by increasing pressure and inhibited by decreasing pressure
 - Compressor: Pressure increases each stage
 - Turbine: Pressure decreases each stage



Practical Considerations

- Compressor stall
 - Can only handle small pressure jump and turning angle each stage
 - Many compressor stages
- Turbine stall
 - Can handle large pressure jumps and turning angles each stage
 - Few turbine stages
- Turbine blade cooling
 - To be discussed after combustors



Summary

- In the stator reference frame:
 - Stator and rotor turn flow (apply a torque)
 - Only rotor exchanges power
 - Stagnation temperature and pressure constant across (lossless) stator
 - Stagnation temperature and pressure change across rotor
- Compressor vs. Turbine
 - Stagnation pressure/temperature increase in compressor stage
 - Stagnation pressure/temperature decrease in turbine stage
 - Compressor order is rotor-stator
 - Turbine order is stator-rotor