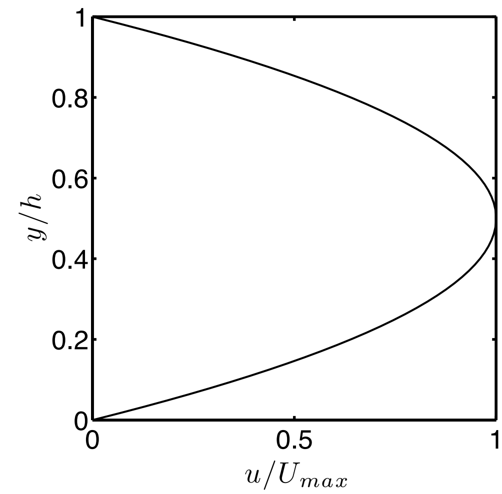
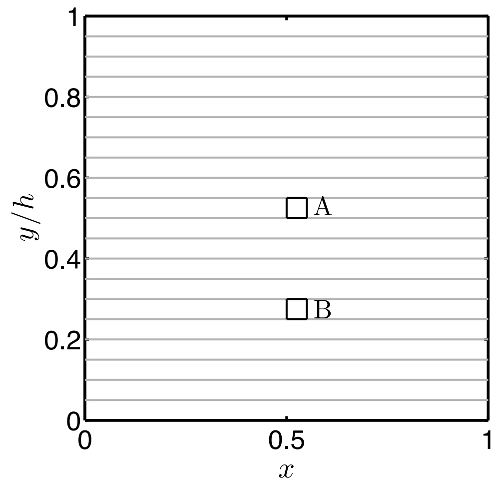
Collection of in Class Practice Problems (Material included in Lectures 1 to 9)

P1. Consider flow through a channel with infinite parallel plates. The velocity profile is shown in the left image, and the streamline pattern is shown in the right image. Two different *infinitesimal* fluid particles are marked in black and labeled “A” and “B”. The corresponding velocity field for this flow is

$$u = -4\,U_{max}[(y/h)^2 - y/h] \qquad \text{and} \qquad v = 0.$$





(a) Determine the components for the strain-rate tensor, e_{ij}.

(b) Determine the components for the vorticity vector, ω_i.

(c) Is the flow incompressible? [Show why or why not]

(d) Based on your results from parts a and b, describe how you expect the initially square fluid particle labeled “A” with centroid located at $y/h = 0.5$ to deform in this flow. [Draw a simple picture of this deformation to support your written interpretation]

(e) Based on your results from parts a and b, describe how you expect the initially square fluid particle labeled “B” with centroid located at $y/h = 0.25$ to deform in this flow. [Draw a simple picture of this deformation to support your written interpretation]

(f) [**6700 only**] Plot the vorticity profile, i.e., ω_3 versus y/h, for this flow. [Be sure to properly label your axes]

P2. A flow field in the xy plane has velocity components:

$$u = 3\,x + y \qquad \text{and} \qquad v = 2\,x - 3\,y.$$

The circulation is defined as the area integral of vorticity, i.e.,

$$\Gamma = \int_A \vec{\omega} \cdot d\vec{A}\,.$$

Show that the circulation around the circle $(x-1)^2 + (y-6)^2 = 4$ is equal to $4\,\pi$.

P3. Start with Leibnitz's theorem in the form:

$$\frac{D}{Dt}\int_{\mathcal{V}(t)} F\, d\mathcal{V} = \int_{\mathcal{V}} \left[\frac{\partial F}{\partial t} + \frac{\partial (F\, u_j)}{\partial x_j}\right] d\mathcal{V},$$

where it has been assumed that the surface bounding the volume $\mathcal{V}(t)$ moves with the velocity u_j. For any continuous fluid property $f = F/\rho$ (where ρ is the fluid density), show that the above equation simplifies to

$$\frac{D}{Dt}\int_{\mathcal{V}(t)} \rho f\, dV = \int_{\mathcal{V}} \rho \frac{Df}{Dt}\, dV.$$

P4. Consider a one-dimensional velocity field: $u = u(x,t)$, $v = 0$, and $w = 0$, where the density varies as

$$\rho = \rho_0 \, (2 - \cos \omega \, t).$$

Find an expression for $u(x,t)$ that satisfies mass conservation if $u(0,t) = U$.