21) Non-Dimensional Navier Stokes

$$x_i^* = \frac{x_i}{l}, \qquad t^* = \Omega t, \qquad u_j^* = \frac{u_j}{U},$$
$$p^* = \frac{p - p_\infty}{\rho U^2}, \qquad g_j^* = \frac{g_j}{g}$$
$$\left[\frac{\Omega l}{U}\right]\frac{\partial u^*}{\partial t^*} + (u^* \cdot \nabla^*)u^*$$
$$= -\nabla^* p^* + \left[\frac{gl}{U^*}\right] g^* + \left[\frac{\mu}{\rho U l}\right]\nabla^{*2} u^*$$

26) Blasius Solution

Similarity

$$\eta = \frac{y}{\delta(x)} = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}$$

Solution:

$$f(\eta) = \frac{\psi(x,y)}{\sqrt{\nu x U_\infty}} \to f''' + \frac{1}{2} f f'' = 0$$

30) Vorticity Introduction

Vorticity Definition

$$\omega = \nabla \times u, \qquad \omega_k = \epsilon_{ijk}\frac{\partial u_j}{\partial x_i}$$
$$\omega_1 = \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}, \omega_2 = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}, \omega_3 = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}$$

Vortex Lines

$$\frac{dx}{\omega_x} = \frac{dy}{\omega_y} = \frac{dz}{\omega_z}$$

Vorticity Continuity

$$\int \boldsymbol{\omega} \cdot \boldsymbol{n}\, dA = 0 \to \omega_1 A_1 = \omega_2 A_2$$

Vorticity at the Wall

$$0 = -\nabla P - \mu \nabla \times \boldsymbol{\omega}$$

34) Turbulence Intro

Kolmogorov Scale

$$\eta = \left(\frac{\nu^3}{\epsilon = 2\nu S_{ij} S_{ij}}\right)^{\frac{1}{4}}, \tau = \left(\frac{\nu}{\epsilon}\right)^{1/2}, V = (\nu\epsilon)^{\frac{1}{4}}, \frac{\eta}{L} = Re^{-\frac{3}{4}}$$

Statistics in Turbulence

$$u = \bar{u} + u', \qquad \bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u\, dt$$

Properties

$$\bar{f'} = 0, \qquad \bar{\bar{f}} = \bar{f}, \qquad \overline{\bar{f}g} = \bar{f}\bar{g}, \qquad \overline{f+g} = \bar{f} + \bar{g},$$
$$\overline{fg} = \bar{f}\bar{g} + \overline{f'g'}, \qquad \overline{f'^2} \neq 0$$

27) Von-Karman BL MM Integral Eqn.

$$\frac{1}{\rho}\tau_w = \frac{d}{dx}[U_e^2\theta] + U_e\delta^*\frac{dU_e}{dx} \to \frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2}$$

28) Falkner-Skan BL Solution

External Flow Conditions

$$U_e(x) = bx^m, \qquad Re_x = \frac{bx^{m+1}}{\nu}$$

Solution

$$f''' + \alpha f f'' + \beta(1 - f'^2) = 0$$
$$\alpha = \frac{\delta}{\nu}\frac{d}{dx}(U_e\delta), \qquad \beta = \frac{\delta^2}{\nu}\frac{dU_e}{dx}$$

29) Pressure Gradients – NS @ Wall

$$\mu\frac{\partial^2 u}{\partial y^2}\bigg|_{y=0} = \frac{dp}{dx} \to \text{Sep.} \to \tau_w\frac{du}{dy}\bigg|_{y=0} = 0$$

32) Vector Identities

$$\nabla \times (\nabla\phi) = 0$$
$$\nabla \cdot (\nabla \times \mathrm{A}) = 0$$
$$\nabla \times (\nabla \times \mathrm{A}) = \nabla(\nabla \cdot \mathrm{A}) - \nabla^2 A$$
$$(A \cdot \nabla)A = \frac{1}{2}\nabla \mathrm{A}^2 - A \times (\nabla \times \mathrm{A})$$
$$A \times B = -B \times A$$
$$\nabla \times (\mathrm{A} \times \mathrm{B}) = \mathrm{A}(\nabla \cdot \mathrm{B}) - \mathrm{B}(\nabla \cdot \mathrm{A}) + (\mathrm{B} \cdot \nabla)\mathrm{A} - (\mathrm{A} \cdot \nabla)\mathrm{B}$$

35) Reynolds Averaged Navier Stokes

$$u = \bar{u} + u', \qquad P = \bar{P} + P'$$

Continuity

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \qquad \frac{\partial u_i'}{\partial x_i} = 0$$

Momentum

$$\bar{u}_j\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial}{\partial x_j}\left(\overline{u_i' u_j'}\right)$$

Closure

Reynolds Stress Tensor: $\tau_t = \rho\left(\overline{u_i' u_j'}\right)$, Eddy viscosity μ_t, Prandtl mixing length l

$$\tau_t 2\mu_t\overline{S_{ij}}, \qquad \mu_t = \rho l^2\sqrt{2\overline{S_{ij}} \cdot\cdot \overline{S_{ij}}}$$

Energy equation

$$\rho_0 c_p\left(\frac{\partial \bar{T}}{\partial t} + \bar{u}_j\frac{\partial \bar{T}}{\partial x_j}\right) = -\frac{\partial Q_j}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(-k\frac{\partial \bar{T}}{\partial x_j} + \rho_0 c_p\overline{\bar{u}_j T'}\right)$$

24/25) Boundary Layer Equations

Navier Stokes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2},$$

Displacement & Momentum Thickness

$$\delta^* = \int_0^\delta\left(1 - \frac{u}{U_e}\right)dy, \qquad \theta = \int_0^\infty\frac{u}{U_e}\left(1 - \frac{u}{U_e}\right)dy$$

Wall Shear Stress

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}, \qquad u_n = -\frac{1}{2\mu}\nabla \cdot \vec{\tau}\delta_n^2, \qquad \tau_w = \mu\frac{du}{dy}\bigg|_0$$

Boundary Conditions

$$u_0 = 0, \qquad u_\delta = U, \qquad \frac{\partial u}{\partial y}\bigg|_\delta = 0$$

Scale Analysis

$$x^* = \frac{x}{L}, \qquad y^* = \frac{y}{\delta}, \qquad u^* = \frac{u}{U_e}, \qquad v^* = \frac{v}{V_s}, \qquad P^* = \frac{P - P_\infty}{\rho U_e^2}$$
$$\frac{\delta}{L} \approx \frac{1}{\sqrt{Re}}, \qquad \frac{\delta^2}{\nu} \approx \frac{L}{U_e}$$

32/33) Vorticity Transport Equation

General

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u} + \nu\nabla^2\boldsymbol{\omega}$$
$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u} + \nu\nabla^2\boldsymbol{\omega}$$
$$\frac{\partial \omega_i}{\partial t} + u_j\frac{\partial \omega_i}{\partial x_j} = \omega_j\frac{\partial u_i}{\partial x_j} + \nu\frac{\partial^2 \omega_i}{\partial x_j^2}$$

Rotating reference frame

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} + 2\Omega) \cdot \boldsymbol{\nabla u} + \frac{1}{\rho^2}\nabla\rho \times \nabla p + \nu\nabla^2\boldsymbol{\omega}$$
$$\frac{D\omega_n}{Dt} = (\omega_j + 2\Omega_j)\frac{\partial u_n}{\partial x_j} + \frac{\epsilon_{nqi}}{\rho^2}\frac{\partial\rho}{\partial x_q}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 \omega_n}{\partial x_j^2}$$

2D Vorticity

$$\omega_x = \omega_y = 0, \qquad \frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\omega} = \nu\nabla^2\boldsymbol{\omega}$$

37) Turbulent Boundary Layer

Inner Layer (viscous sublayer)

$$U^+ = \frac{U}{u_\star} = f(y^+) = f\left(\frac{y u_\star}{\nu}\right), \qquad u_\star^2 \equiv \frac{\tau_w}{\rho}$$

Outer Layer

$$\frac{U_\infty - U}{u_\star} = F\left(\frac{y}{\delta}\right) = f(\zeta)$$

Overlap Layer

$$U^+ \equiv \frac{U}{u_\star} = f(y^+) = \frac{1}{\kappa}\ln(y^+) + B, F(\zeta) = -\frac{1}{\kappa}\ln(\zeta) + A$$

Intermediate Fluid Dynamics Final Reference Sheet

Zane Frey

21) Non-Dimensional Energy

$$\epsilon \equiv \frac{1}{\rho}\tau_{ij}S_{ij}, \qquad \rho\frac{Dh}{Dt} = \frac{Dp}{Dt} + \rho\epsilon + \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right)$$
$$\epsilon^* = \frac{\rho_0 l^2\epsilon}{\mu_0 U^2}, \qquad \mu^* = \frac{\mu}{\mu_0}, \qquad k^* = \frac{k}{k_0}, \qquad T^* = \frac{T - T_0}{T_w - T_0}$$
$$\rho^*\frac{DT^*}{Dt^*} = \left[\frac{U^2}{c_p(T_w - T_0)}\right]\frac{Dp^*}{Dt^*} + \left[\frac{U^2}{c_p(T_w - T_0)}\frac{\mu_0}{\rho_0 U l}\right]\rho^*\epsilon^* + \left[\frac{k_0}{c_p\mu_0}\frac{\mu_0}{\rho_0 U l}\right]\nabla^*(k^*\nabla^* T^*)$$

31) Kelvin's Circulation Theo.

$$\Gamma \equiv \oint_C \boldsymbol{u} \cdot ds = \int_A \boldsymbol{\omega} \cdot \boldsymbol{n}\, dA$$
$$\frac{D\Gamma}{Dt} = \int_c\left(\frac{1}{\rho}\frac{\partial\tau_{ij}}{\partial x_j}\right)dx_i = 0$$

Vortex Pressure

$$\rho\boldsymbol{a} = -\nabla P$$

36) Turbulence Energy Budget

$$\bar{E} = \frac{1}{2}U_i^2, \qquad \overline{S_{ij}} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right), \qquad \bar{e} = \frac{1}{2}u_i^2, \qquad \overline{S_{ij}'} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Mean Kinetic Energy (MKE)

$$\frac{\partial \bar{E}}{\partial t} + U_j\frac{\partial \bar{E}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(-\frac{U_j P}{\rho_0} + 2\nu U_i\overline{S_{ij}} - \overline{u_i u_j}U_i\right) - 2\nu\bar{S}_{ij}\bar{S}_{ij} + \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \frac{g}{\rho_0}\bar{\rho}U_3$$

Turbulent Kinetic Energy (TKE)

$$\frac{\partial \bar{e}}{\partial t} + U_j\frac{\partial \bar{e}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(-\frac{\overline{p u_j}}{\rho_0} + 2\nu\overline{u_i S_{ij}'} - \frac{1}{2}\overline{u_i^2 u_j}\right) - 2\nu\overline{S_{ij}' S_{ij}'} - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$$

Navier-Stokes Equations

General momentum for incompressible

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-\nabla p+\rho g+\mu\nabla^2\boldsymbol{u},\qquad \rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\rho g_i+\mu\frac{\partial^2 u_i}{\partial x_j\partial x_j}$$

Newtonian Rectangular:

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial x}+v_z\frac{\partial v_x}{\partial x}\right)=\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)-\frac{\partial p}{\partial z}+\rho g_z$$

Newtonian Cylindrical:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$$
$$=\mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right]+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)-\frac{\partial p}{\partial r}+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$=\mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right]+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right)-\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)-\frac{\partial p}{\partial z}+\rho g_z$$

Stress Tensor (Newtonian)

Rectangular

$$\tau_{ii}=\mu\left(2\frac{\partial v_i}{\partial i}-\frac{2}{3}(\nabla\cdot\mathbf{v})\right),$$

$$\tau_{ij}=\tau_{ji}=\mu\left(\frac{\partial v_i}{\partial j}+\frac{\partial v_j}{\partial i}\right)$$

Cylindrical

$$\tau_{rr}=\mu\left(2\frac{\partial v_r}{\partial r}-\frac{2}{3}(\nabla\cdot\mathbf{v})\right)$$

$$\tau_{\theta\theta}=\mu\left(2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right)-\frac{2}{3}(\nabla\cdot\mathbf{v})\right)$$

$$\tau_{zz}=\mu\left(2\frac{\partial v_z}{\partial z}-\frac{2}{3}(\nabla\cdot\mathbf{v})\right)$$

$$\tau_{r\theta}=\tau_{\theta r}=\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$$

$$\tau_{\theta z}=\tau_{z\theta}=\mu\left(\frac{\partial v_\theta}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)$$

$$\tau_{zr}=\tau_{rz}=\mu\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)$$

$$\nabla\cdot\mathbf{v}=\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}$$

Rate of Strain Tensor

Rectangular

$$S_{xx}=\frac{\partial v_x}{\partial x}$$

$$S_{yx}=S_{xy}=\frac{1}{2}\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)$$

Cylindrical

$$S_{rr}=\frac{\partial v_r}{\partial r}$$

$$S_{\theta r}=S_{r\theta}=\frac{1}{2}\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$$

$$S_{\theta\theta}=\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}$$

$$S_{\theta z}=S_{z\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta}+\frac{\partial v_\theta}{\partial z}\right)$$

$$S_{zz}=\frac{\partial v_z}{\partial z}$$

$$S_{rz}=S_{zr}=\frac{1}{2}\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right)$$

Energy Equation

$$\rho c_p\frac{DT}{Dt}-\beta T\frac{dp}{dt}=2\mu S_{ij}^2+\lambda(\nabla\cdot\boldsymbol{u})^2+\nabla\cdot(k\nabla T)$$

Continuity

Rectangular (x,y,z)

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cylindrical (r,θ,z)

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cauchy's Equation of Motion

$$\rho\frac{Du_i}{Dt}=\rho g_i+\frac{\partial\sigma_{ij}}{\partial x_j},\qquad \rho\frac{D\boldsymbol{u}}{Dt}=\rho\boldsymbol{g}+\nabla\cdot\boldsymbol{\sigma}$$

Constitutive eq:

$$\sigma_{ij}=-p\delta_{ij}+\tau_{ij}=-p\delta_{ij}+2\mu S_{ij}+\lambda(\nabla\cdot\vec{u})\delta_{ij}$$

Reynolds Transport Theorem

$$\left.\frac{dN}{dt}\right|_{sys}=\frac{\partial}{\partial t}\int_{CV}\eta\rho dV+\int_{CS}\eta\rho\vec{V}\cdot d\vec{A}$$

Momentum Equation

$$\frac{d}{dt}\int_{V(t)}\rho\boldsymbol{u}(x,t)dV=\int_{V(t)}\rho\boldsymbol{g}dV+\int_{A(t)}\mathbf{f}(\mathbf{n},\mathbf{x},t)dA$$

Material Derivative

$$\rho\frac{D\boldsymbol{u}}{Dt}=\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=\rho\left(\frac{\partial u_i}{\partial t}+u_j\partial_j u_i\right)$$

Calculus Review

Gradient of Scalar

$$\nabla\phi=\frac{\partial\phi}{\partial x}\hat{\imath}+\frac{\partial\phi}{\partial y}\hat{\jmath}+\frac{\partial\phi}{\partial z}\hat{k}$$

Divergence

$$\nabla\cdot\vec{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}=\frac{\partial A_i}{\partial x_i}\qquad \nabla\cdot T=\frac{\partial T_{ij}}{\partial x_i}$$

Curl

$$\nabla\times\vec{A}=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\hat{\imath}+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\hat{\jmath}+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\hat{k}$$

Laplacian

$$\nabla^2\phi=\nabla\cdot\nabla\phi=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}$$

Product Rule

$$\nabla\cdot(\nabla^2\boldsymbol{u})=\nabla^2(\nabla\cdot u)$$

General Dot Product

$$\boldsymbol{u}\cdot\boldsymbol{v}=u_i v_i=u_1v_1+u_2v_2+u_3v_3$$

Double Dot Product

$$A_{ij}B_{ji}=\mathbf{A}:\mathbf{B}=\mathrm{tr}(\mathbf{AB}^T)$$

Index Notation

Einstein Notation

$$A_{ij}x_j=c_i$$

Kronecker delta

$$\delta_{ij}=\begin{cases}1 \text{ if } i=j\\0 \text{ if } i\neq j\end{cases}$$

$$\delta_{ii}=3,\qquad \delta_{ij}\delta_{jk}=\delta_{ik},\qquad \delta_{ij}A_{jk}=A_{ik}$$

Alternating Tensor

$$\epsilon_{ijk}=\begin{cases}1 \text{ if } ijk=123{,}231{,}312\\ 0 \text{ if indicies equal}\\ -1 \text{ if } ijk=321{,}213{,}132\end{cases}$$

Epsilon-delta

$$\epsilon_{ijk}\epsilon_{klm}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$$

Cross Product

$$u\times v=w,$$
$$w_i=\epsilon_{ijk}u_jv_k\qquad \epsilon_{ijk}\partial_i u_j$$

Intermediate Fluid Dynamics Midterm Reference Sheet

Zane Frey

Stream Func.

2D Incompressible

$$u=\frac{\partial\psi}{\partial y}$$
$$v=-\frac{\partial\psi}{\partial x}$$

Potential Function

Irrotational flow $\qquad \boldsymbol{u}=\nabla\phi$

Kinematics

$$\frac{\partial u_i}{\partial x_j}=S_{ij}+R_{ij},\qquad S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\qquad R_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right)$$

$$\nabla\cdot\vec{v}=\mathrm{tr}(S)=S_{ii}\qquad \boldsymbol{R}_{ij}=-\frac{1}{2}\epsilon_{ijk}\omega_k$$

Traction

$$f_i=\sum_{j=1}^{3}\sigma_{ij}n_j\,,$$
$$\mathbf{f}=\mathbf{n}\cdot\boldsymbol{\sigma}$$

Comma Notation

$$\phi_{,i}=\partial\phi/\partial x_i$$

Gradient, divergence, curl of a vector

$$A_{i,j}=\frac{\partial A_i}{\partial x_j},\quad \nabla\cdot\mathbf{A}=A_{i,i}=\frac{\partial A_i}{\partial x_i}$$

$$\nabla\times\mathbf{A}=\epsilon_{ijk}A_{k,j}=\epsilon_{ijk}\frac{\partial A_k}{\partial x_j}$$

Visualizations

Pathline

$$\frac{d\vec{x}_p}{dt}=\vec{u}_p$$

Streakline

$$\frac{d\vec{x}_{str}}{dt}=\vec{u}_p$$

Streamline

$$\frac{dy}{dx}=\frac{v}{u}$$

Stokes Hypothesis

Bulk viscosity

$$\zeta=\lambda+\frac{2}{3}\mu=0\quad \lambda=-\frac{2}{3}\mu$$

Stokes'/Gauss' Theorem

$$\int\int_A(\nabla\times\mathbf{u})\cdot\mathbf{n}\,dA=\int_C\mathbf{u}\cdot\mathbf{t}\,ds$$

$$\int\int\int_V\nabla\cdot\mathbf{u}\,dV=\int\int_A\mathbf{n}\cdot\mathbf{u}\,dA$$