Advanced Mechanics of Materials Vocabulary

Stress and Strain

- Vector Transformation: $\boldsymbol{a}' = \boldsymbol{M}\boldsymbol{a}$, where $\boldsymbol{M}$ is the transformation, or rotation or direction cosine matrix
  - $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{x}'\cdot\boldsymbol{x} & \boldsymbol{x}'\cdot\boldsymbol{y} & \boldsymbol{x}'\cdot\boldsymbol{z} \\ \boldsymbol{y}'\cdot\boldsymbol{x} & \boldsymbol{y}'\cdot\boldsymbol{y} & \boldsymbol{y}'\cdot\boldsymbol{z} \\ \boldsymbol{z}'\cdot\boldsymbol{x} & \boldsymbol{z}'\cdot\boldsymbol{y} & \boldsymbol{z}'\cdot\boldsymbol{z} \end{bmatrix}$
- Tensor (2nd order) Transformation: $\boldsymbol{T}' = \boldsymbol{M}\boldsymbol{T}\boldsymbol{M}^{\boldsymbol{T}}$
- Stress vector on arbitrary surface with normal unit vector $\boldsymbol{n}$: $\boldsymbol{t} = \boldsymbol{T}\boldsymbol{n} = \boldsymbol{\tau} + \boldsymbol{\sigma}$

Material Response

- Hooke's Law (Voigt Notation)

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

Failure Theory

- Max distortion energy (Von Mises): $(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) = 2\sigma_{yp}^2$
- Coulomb-Mohr: $\frac{\sigma_1}{\sigma_u} - \frac{\sigma_3}{\sigma_u'} = 1$

Fracture Mechanics

- Stress intensity factor: $K = \lambda\sigma\sqrt{\pi a}$

Fatigue

- $N_{cr} = N_f \left(\frac{\sigma_{cf}}{\sigma_f}\right)^{1/b}$, where $b = \frac{ln(\sigma_f/\sigma_e)}{ln(N_f/N_e)}$
- Modified Goodman: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_u} = 1$
- Soderberg: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_{yp}} = 1$
- Gerber: $\frac{\sigma_a}{\sigma_{cr}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$
- SAE: $\frac{\sigma_a}{\sigma_{cr}} + \frac{\sigma_m}{\sigma_f} = 1$

Dynamic Loading

- Impact factor (vertical drop): $K = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$

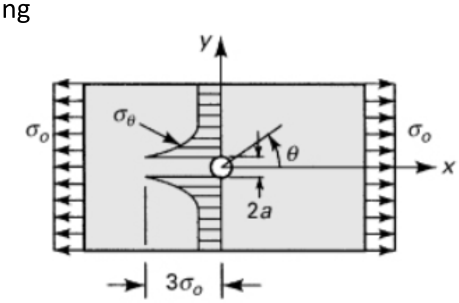
Stress Concentrations

- Stress distribution around a hole in a flat plate under uniaxial loading

$$\sigma_r = \tfrac{1}{2}\sigma_o\left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

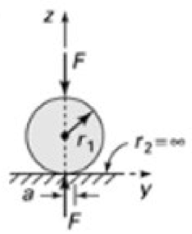
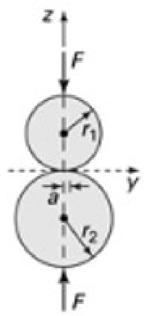
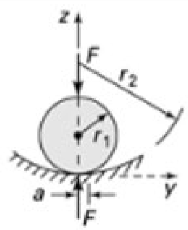
$$\sigma_\theta = \tfrac{1}{2}\sigma_o\left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = -\tfrac{1}{2}\sigma_o\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$



Contact

Table 3.2. Maximum Pressure P_o and Deflection δ of Two Bodies in Contact

Configuration	Spheres: $p_o = 1.5\frac{F}{\pi a^2}$	Cylinders: $p_o = \frac{2}{\pi}\frac{F}{aL}$
A 	Sphere on a Flat Surface $a = 0.880\sqrt[3]{Fr_1\Delta}$ $\delta = 0.775\sqrt[3]{F^2\frac{\Delta^2}{r_1}}$	Cylinder on a Flat Surface $a = 1.076\sqrt{\frac{F}{L}r_l\Delta}$ For $E_1 = E_2 = E$: $\delta = \frac{0.579F}{EL}\left(\frac{1}{3} + \ln\frac{2r_1}{a}\right)$
B 	Two Spherical Balls $a = 0.880\sqrt[3]{F\frac{\Delta}{m}}$ $\delta = 0.775\sqrt[3]{F^2\Delta^2\cancel{n}}$ ← m	Two Cylindrical Rollers $a = 1.076\sqrt{\frac{F\Delta}{Lm}}$
C 	Sphere on a Spherical Seat $a = 0.880\sqrt[3]{F\frac{\Delta}{n}}$ $\delta = 0.775\sqrt[3]{F^2\Delta^2 n}$	Cylinder on a Cylindrical Seat $a = 1.076\sqrt{\frac{F\Delta}{Ln}}$

Note: $\Delta = \frac{1}{E_1} + \frac{1}{E_2}$, $m = \frac{1}{r_1} + \frac{1}{r_2}$, $n = \frac{1}{r_1} - \frac{1}{r_2}$ * all formulae in this table assume $\nu = 0.3$

Stress distributions: Stresses below the surface along the load axis (for $\nu = 0.3$): (left) two spheres; (right) two parallel cylinders. Note: All normal stresses are compressive.