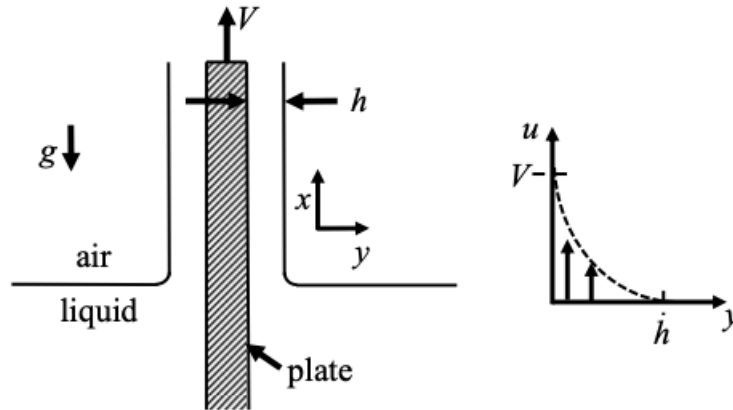


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**Collection of in Class Practice Problems**  
**(Material included in 13-16)**


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**P1.** A continuous plate is pulled vertically up out of a liquid at a constant velocity  $V$  (see figure). As a result of this motion, a thin layer of liquid of thickness  $h$  forms on the plate surface with the liquid velocity at the outer edge of the layer ( $u(h)$ ) equal to zero (see profile sketch on the right hand side of the figure). Assume the flow is steady state, fully developed in the  $x$  direction (vertical as indicated), and uniform into the page.



- (a) Solve for the plate normal ( $y$ -direction)  $v$  velocity in the liquid using conservation of mass (state all assumptions).

For a 2D incompressible flow, conservation of mass is,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

which for a fully developed flow in the  $x$ -direction, means that any derivative in the  $x$ -direction  $\rightarrow 0$ ,

$$\frac{\partial v}{\partial y} = 0 \quad (2)$$

Upon integration of this equation,  $v = C$ .

Using the Boundary Conditions,

$$v(y = h) = 0 \rightarrow C = 0 \quad (3)$$

Hence,  $v = 0$  everywhere, similar to previous problems we had seen.

- (b) Simplify the  $x$ -direction momentum equation (Navier-Stokes) for this problem (state all assumptions).

For an incompressible flow the N-S equation in the  $x$ -direction is:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\rho g - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} \quad (4)$$

If the flow is steady,  $\partial_t \rightarrow 0$ , and fully developed,  $\partial_x \rightarrow 0$

Then the previous equation becomes,

$$\rho v \frac{\partial u}{\partial y} = -\rho g + \mu \frac{\partial^2 u}{\partial y^2} \quad (5)$$

and from part (a),  $v = 0$ , hence  $\Rightarrow \boxed{0 = -\rho g + \mu \frac{\partial^2 u}{\partial y^2}}$

- (c) Obtain an expression for the liquid  $x$ -direction velocity profile.

From part (b) we have that,

$$0 = -\rho g + \mu \frac{\partial^2 u}{\partial y^2} \Rightarrow \mu \frac{\partial^2 u}{\partial y^2} = \rho g \quad (6)$$

Integrating once, it gives:

$$\frac{du}{dy} = \frac{\rho g}{\mu} y + C_1 \quad (7)$$

One can find  $C_1$  by noting that the shear stress,

$$\tau_{xy} = \mu \frac{du}{dy} = 0 \text{ at } y = h, \quad (8)$$

hence  $C_1 = -\frac{\rho g h}{\mu}$

Using  $C_1$  and integrating a second time, one obtains,

$$u = \frac{\rho g}{2\mu} y^2 + -\frac{\rho g h}{\mu} y + C_2 \quad (9)$$

In this case,  $C_2$  can be found using the non-slip condition,  $u(y = 0) = V \rightarrow C_2 = V$ .

Therefore, the final profile equation is:

$$u = \frac{\rho g}{2\mu} y^2 + -\frac{\rho g h}{\mu} y + V \quad (10)$$

- (d) Using your velocity profile from the previous part, find an expression for the layer thickness  $h$ .

The layer thickness can be deduced by assuming that  $u(y = h) = 0$ , so that

$$0 = \frac{\rho g}{2\mu} h^2 + -\frac{\rho g h}{\mu} h + V \quad (11)$$

Which upon rearranging and solving for  $h$ ,

$$h = \sqrt{\frac{V\mu}{\rho, g}} \quad (12)$$