# Chapter 10

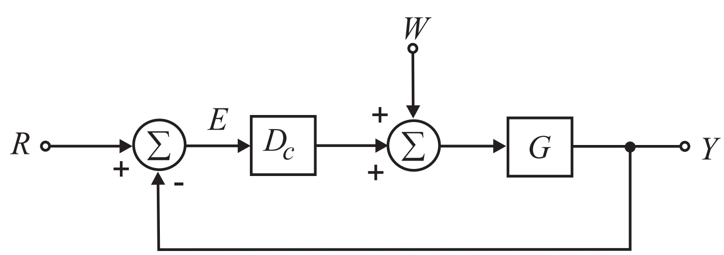
# Control-System Design: Principles and Case Studies

## Problems and Solutions for Chapter 10

1. Of the three types of PID control (proportional, integral, or derivative), which one is the most effective in reducing the error resulting from a constant disturbance? Explain.

   **Solution:**

   Integral control is the most effective in reducing the error due to constant disturbances.




Problem 10.1: Block diagram for showing integral control is the most effective means of reducing steady-state errors.

Using the above block diagram,

$$
\begin{aligned}
Y &= G(W + ED_c), \\
E &= R - Y = R - G(W + ED_c), \\
E &= \frac{1}{1 + D_c G} R - \frac{G}{1 + D_c G} W, \\
e_\infty &= \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \left( \frac{1}{1 + D_c G} R - \frac{G}{1 + D_c G} W \right).
\end{aligned}
$$

Writing $G(s) = \frac{n_G(s)}{d_G(s)}$, and using a step input $R(s) = \frac{k_r}{s}$, and a step disturbance $W(s) = \frac{k_w}{s}$, we can show that integral control leads to zero steady-state error, while proportional and derivative control, in general, do not.

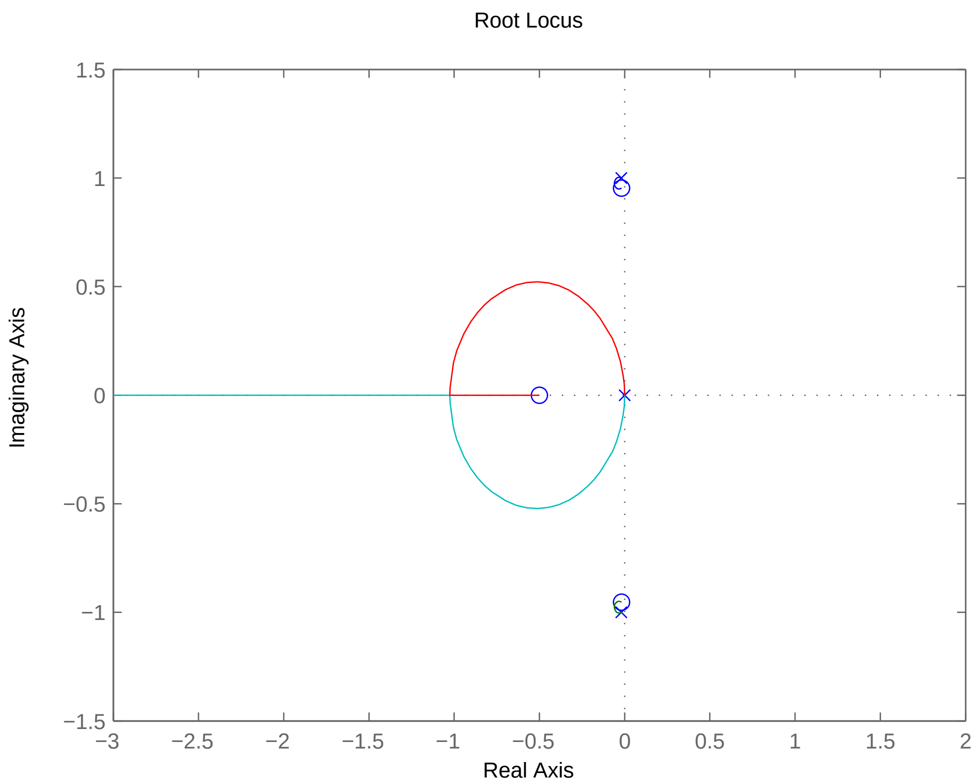
$$\begin{aligned}
\text{Integral control }, D_c(s) &= \frac{1}{s},\ e_\infty = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\left(\frac{d_G s k_r}{d_G s + n_G} + \frac{n_G s k_w}{d_G s + n_G}\right) = 0, \text{ if } n_G(0) \neq 0, \\
\text{Proportional control }, D_c(s) &= K_p,\ e_\infty \neq 0, \\
\text{Derivative control }, D_c(s) &= s,\ e_\infty = k_r - G(0)k_w \neq 0, \text{ if } d_G(0) \neq 0.
\end{aligned}$$

This analysis assumes that there are no pole-zero cancellations between the plant, $G$, and the compensator, $D_c$. In general, proportional or derivative control will not have zero steady-state error.

2. Is there a greater chance of instability when the sensor in a feedback control system for a mechanical structure is not collocated with the actuator? Explain.

**Solution:**

Yes. For comparison, see the following two root loci which were taken from the discussion in the text on satellite attitude control. In Fig. 10.26, the sensor and the actuator are collocated, resulting in a stable closed-loop system with PD control. In Fig. 10.5, the sensor and the actuator are not collocated creating an unstable system with the same PD control.




Problem 10.2: [Text Fig. 10.26] PD control of satellite: collocated.

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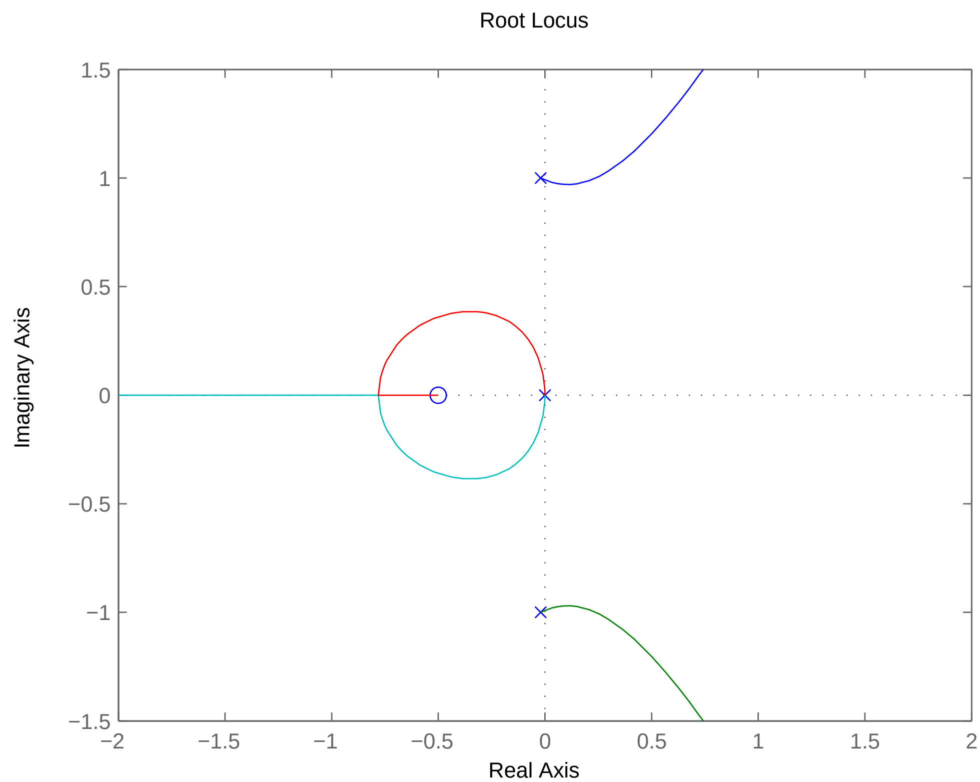



Problem 10.2: [Text Fig. 10.5] PD control of satellite: non-collocated

3. Consider the plant $G(s) = 1/s^3$. Determine whether it is possible to stabilize this plant by adding the lead compensator

$$D_c(s) = K\frac{s+a}{s+b}, \quad (a < b).$$

(a) What is the maximum phase margin of the resulting feedback system?

(b) Can a system with this plant, together with any number of lead compensators, be made unconditionally stable? Explain why or why not.

**Solution:**

(a) $G(s) = 1/s^3$ has phase angle of -270° for all frequencies. The maximum phase lead from a compensator $D_c(s) = K\frac{s+a}{s+b}$ is 90° with $\frac{b}{a} = \infty$. In practice a lead compensator with $\frac{b}{a} = 100$ contributes phase lead of approximately 80°. Hence the closed-loop system will be unstable with PM $= -10°$. To have PM $\approx 70°$ we need, for example, a double lead compensator $D_c(s) = \frac{(s+a)^2}{(s+b)^2}$ with $\frac{b}{a} = 100$.

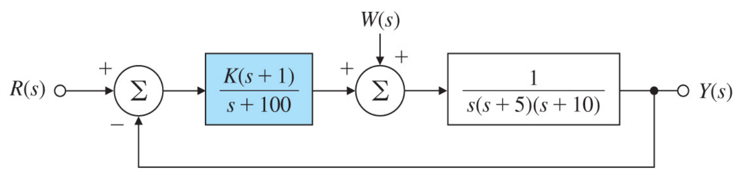



Figure 10.93: Control system for Problem 10.4

(b) No, this plant cannot be made "unconditionally stable" because the root locus departure angles from the three poles at the origin are $\pm$ 60°. For low enough gain, the poles are always in the right-half-plane. If we try positive feedback, one pole departs at 0° so again, one pole starts into the right-half-plane. For low-enough gain, the system will be unstable.

4. Consider the closed-loop system shown in Fig.10.93.

(a) What is the phase margin if $K = 70,000$?

(b) What is the gain margin if $K = 70,000$?

(c) What value of $K$ will yield a phase margin of $\sim 70°$?

(d) What value of $K$ will yield a phase margin of $\sim 0°$?

(e) Sketch the root locus with respect to $K$ for the system, and determine what value of $K$ causes the system to be on the verge of instability.

(f) If the disturbance $w$ is a constant and $K = 10,000$, what is the maximum allowable value for $w$ if $y(\infty)$ is to remain less than 0.1? (Assume $r = 0$.)

(g) Suppose the specifications require you to allow larger values of $w$ than the value you obtained in part(f) but with the same error constraint $[|y(\infty)| < 0.1]$. Discuss what steps you could take to alleviate the problem. Control system for Problem 10.4.

**Solution:**

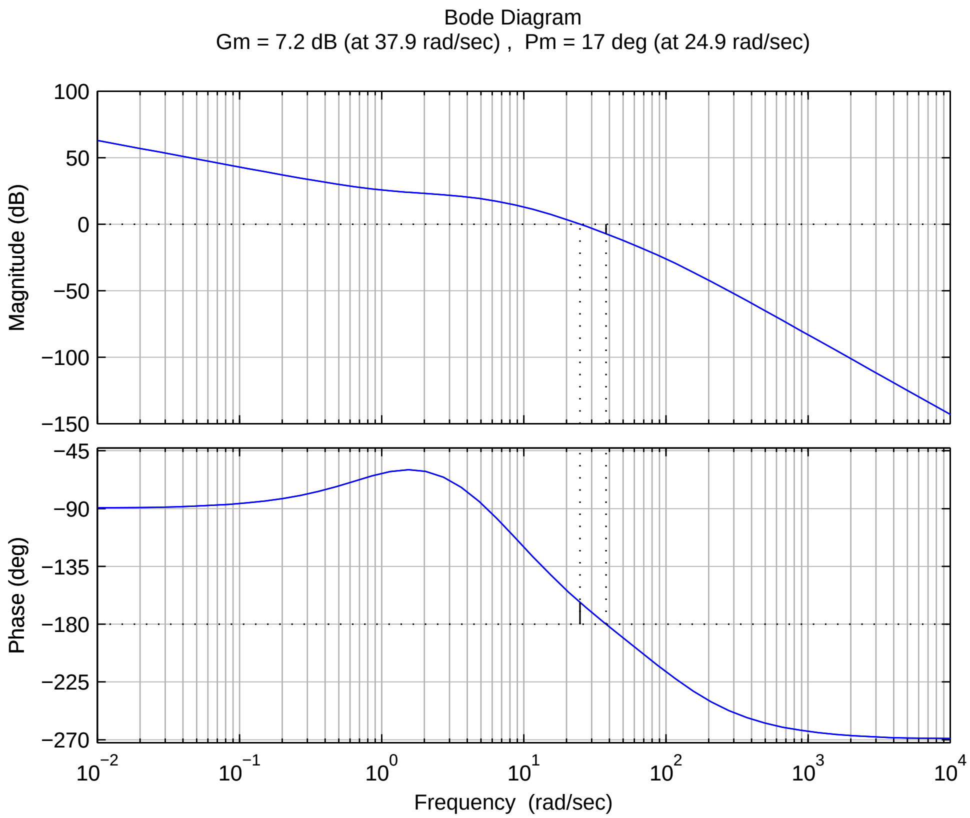
(a) To determine the phase and gain margin of the system given in Fig. 10.88, we produce the Bode plot of the loop gain shown on the next page (using MATLAB's `margin` command),

$$KD_c(s)G(s) = \frac{K(s+1)}{s(s+5)(s+10)(s+100)} = \frac{14(s+1)}{s(s/5+1)(s/10+1)(s/100+1)},$$

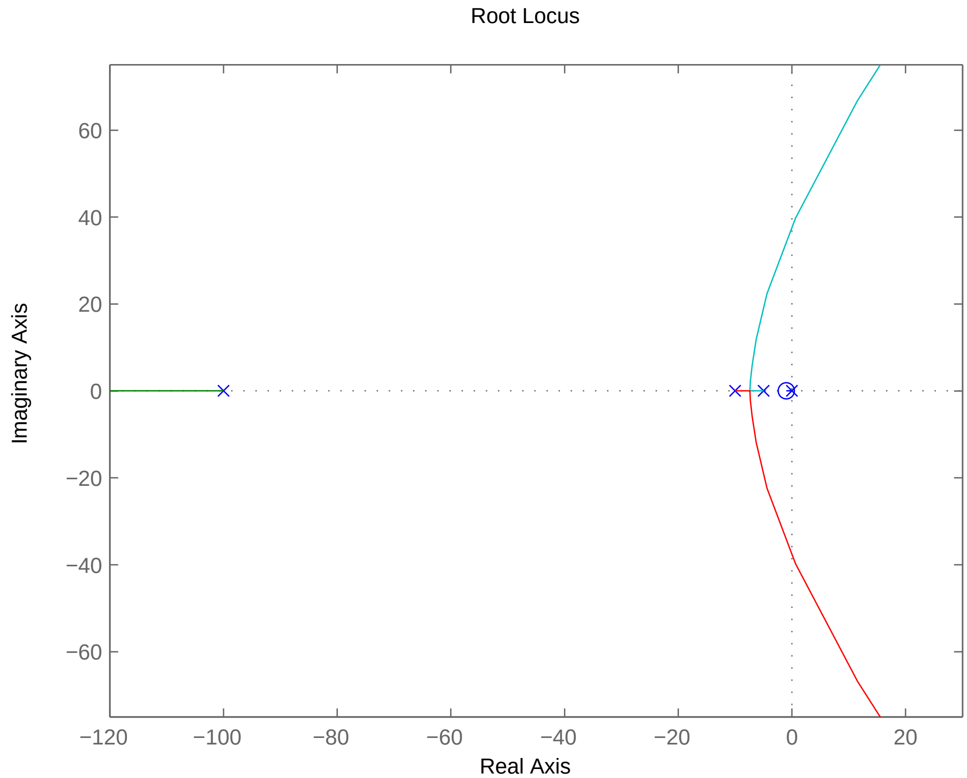
where $K = 70,000$. The Bode plot is shown on the next page along with the phase and gain margin. From the figure, the phase margin is about 17° near $\omega = 25.0$ rad/sec.

(b) The gain margin, from the figure, is approximately 7.2 db at $\omega = 38.0$ rad/sec. Therefore, the gain and phase margins are

$$\begin{aligned} \text{Gain Margin} &= 7.2 \text{ db}, \\ \text{Phase Margin} &= 17.01°. \end{aligned}$$




Bode plot for Problem 10.4.




Root Locus for Problem 10.4.

(c) A phase margin of 70° requires the magnitude to cross the 0 db line near a frequency of $\omega = 8.3$ rad/sec. Hence, the magnitude frequency response must be attenuated by 15 db, or the loop gain multiplied by 0.178. Therefore,

$$K_{70^\circ} = 0.178, \ K = 12,500.$$

(d) A phase margin of 0° results from amplifying the gain by exactly the gain margin value found in part (b). Hence, we amplify the loop gain by 7.2 db, or 2.293.

$$K_{0^\circ} = 2.293, \quad K = 160,500.$$

(e) The root locus of the system is given (using MATLAB's rlocus command). The value of $K$ that causes the system to be on the verge of stability is the gain where the root loci cross the $j\omega$ axis. This value of $K$ can be calculated algebraically or can be determined by the use of the MATLAB command rlocfind. In addition, the result from part (d) can be used since zero phase and gain margin translate to the system being on the verge of instability. Hence, the range of $K$ for stability is $0 < K < 160,500$.

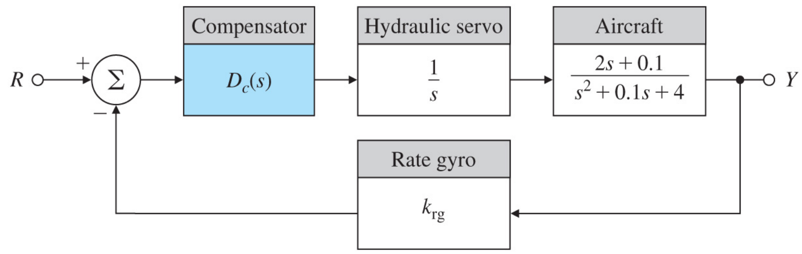



Figure 10.94: Block diagram for aircraft-attitude rate control

(f) With $R = 0$ and the disturbance labeled as $w$, we can write the transfer function from $W(s)$ to $Y(s)$ to determine the steady-state output value due to a constant disturbance input.

$$Y(s) = \frac{G}{1 + KD_cG} W(s),$$

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{G}{1 + KD_cG} W(s).$$

If $w(t)$ is constant, $w(t) = c$, then $W(s) = c/s$, so we have,

$$y_{ss} = \frac{100c}{K}.$$

(g) Therefore, with $K = 10000$ and $y < 0.1$, we have $c < 10$. Since $y_{ss} = 100c/K$, we can increase the gain $K$ to obtain the same error specification, $y_{ss}$, given larger values of $c$. However, this will sacrifice system stability and possibly transient performance. In this case, integral control can be added to reduce the steady-state output error to zero.

5. Consider the system shown in Fig.10.94, which represents the attitude rate control for a certain aircraft.

(a) Design a compensator so that the dominant poles are at $-2 \pm 2j$.

(b) Sketch the Bode plot for your design, and select the compensation so that the crossover frequency is at least $2\sqrt{2}$rad/sec and PM $\geq 50°$.

(c) Sketch the root locus for your design, and find the velocity constant when $\omega_n > 2\sqrt{2}$ and $\zeta \geq 0.5$.

**Solution:**

(a) With a constant gain compensator, $D_c(s) = K$, the root locus of,

$$D_c(s)G(s) = \frac{2K(s + 0.05)}{s(s^2 + 0.1s + 4)} = \frac{num}{den}.$$

does not pass through $-2\pm 2j$. Therefore we need compensation of at least a lead network. Let,

$$D_c(s) = K\frac{s+z}{s+p}.$$

Using the angle criterion, at the closed-loop pole location $s = -2 + 2j$, we can write an expression for the angle contribution from the lead network zero, $\phi_z$, and lead network pole, $\phi_p$.

$$\sum \phi_{z_i} - \sum \phi_{p_i} = -180^\circ \Longrightarrow \phi_z - \phi_p + 134^\circ - 180^\circ - 135^\circ - 116^\circ = -180^\circ.$$

So we have, $\phi = \phi_z - \phi_p = 117^\circ$. In MATLAB,

$$\mathsf{PHI} = \mathit{180}/\mathsf{pi} * [\,\mathsf{angle}(\mathsf{polyval}\ (\mathsf{n}um, \mathsf{s})\ /\mathsf{polyval}(\mathsf{d}en, \mathsf{s}))\ -\ \mathsf{pi}\,].$$

With selection of $z = 0.4$, we get $p = 11.7$. So that our lead design is,

$$D_c(s) = K\frac{s + 0.4}{s + 11.7}.$$

To find the compensator gain, $K$, we can utilize the magnitude criterion at the desired dominant closed-loop pole locations. We find that,

$$|D_c(s)G(s)|_{s=-2\pm j2} = 1 \Longrightarrow K = 17.0.$$

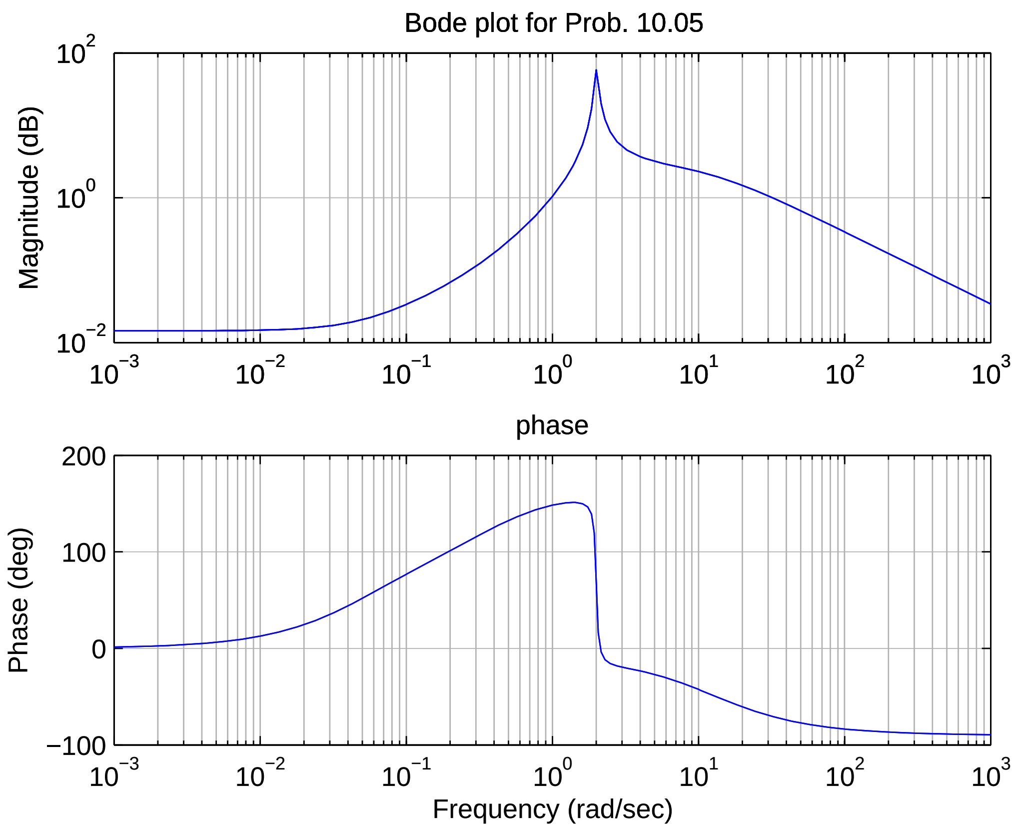
So the lead design is,

$$D_c(s) = 17\frac{s + 0.4}{s + 11.7}.$$

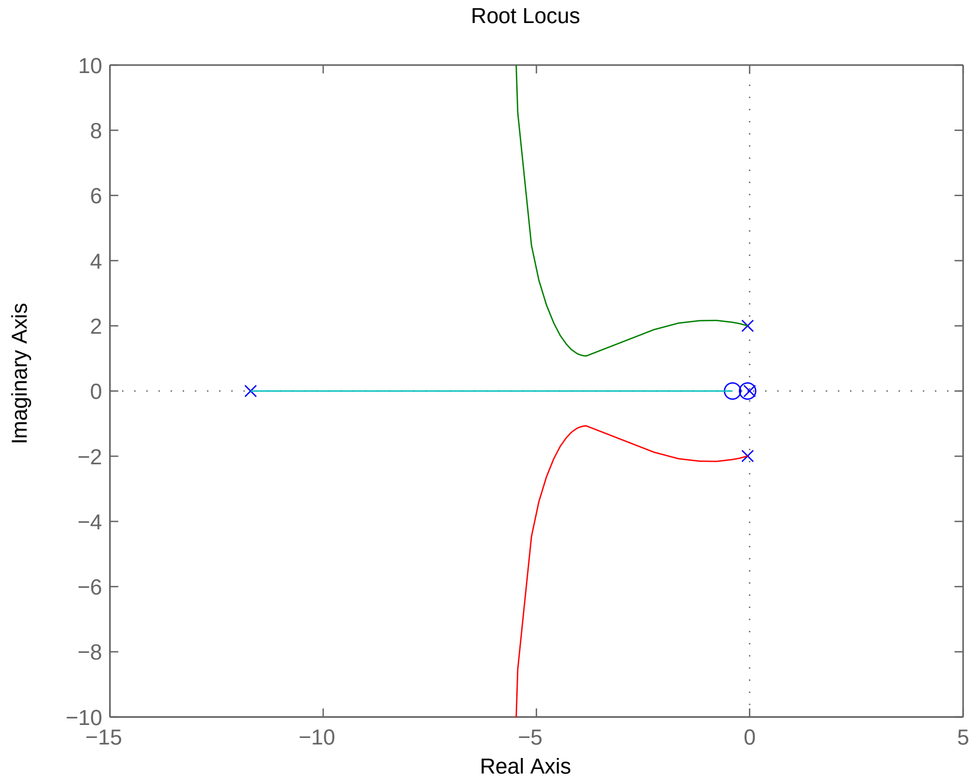
(b) The Bode plot of the system loop transfer function,

$$D_c(s)G(s) = 34\frac{(s + 0.4)(s + 0.05)}{s(s + 11.7)(s^2 + 0.1s + 4)},$$

is shown on the next page using MATLAB's Bode command. As the plot shows $\omega_c = 3$ and PM $= 67.3^\circ$. Therefore, both of the specifications are met by our design.




Problem 10.5 PD control of an aircraft: Bode plot.




Problem 10.5 PD control of an aircraft: root locus.

(c) The root locus plot is shown above using MATLAB's rlocus command. The velocity constant is most easily found from either the Bode plot or from,

$$K_v = \lim_{s\to 0} sD_c(s)G(s).$$

For our compensated system, $K_v = 0.0145$.

6. Consider the block diagram for the servomechanism drawn in Fig.10.95. Which of the following claims are true?

(a) The actuator dynamics (the pole at 1000 $rad/sec$) must be included in an analysis to evaluate a usable maximum gain for which the control system is stable.

(b) The gain $K$ must be negative for the system to be stable.

(c) There exists a value of $K$ for which the control system will oscillate at a frequency between 4 and 6rad/sec.

(d) The system is unstable if $|K| > 10$.

(e) If $K$ must be negative for stability, the control system cannot counteract a positive disturbance.

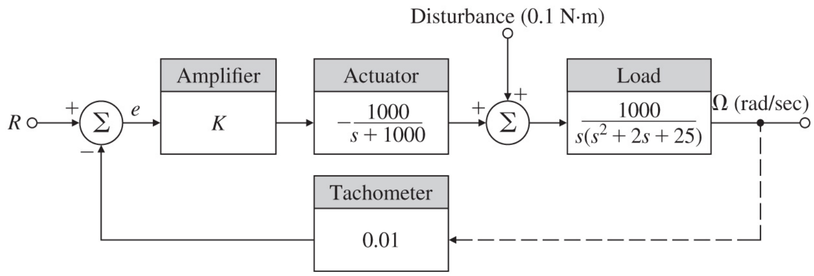



Figure 10.95: Servomechanism for Problem 10.6

(f) A positive constant disturbance will speed up the load, thereby making the final value of $e$ negative.

(g) With only a positive constant command input $r$, the error signal $e$ must have a final value greater than zero.

(h) For $K = -1$ the closed-loop system is stable, and the disturbance results in a speed error whose steady-state magnitude is less than 5rad/sec.

**Solution:**

(a) True. Even though it is tempting to approximate the actuator dynamics as infinitely fast, and hence, not important, the actuator pole dramatically alters the root-locus plot of the system to be controlled. The root locus shown on the next page is for the system without the actuator pole. The root locus for the entire system is also shown. Note that two very different root loci result.

(b) True. On a root locus plot, the pole at $s = 0$ will immediately move into the right-half plane unless the gain is negative. The root locus of the system for negative gain $K$ is shown on the next page.

(c) True. A gain of $K = -4.99$ produces imaginary poles at $s = \pm 5j$.

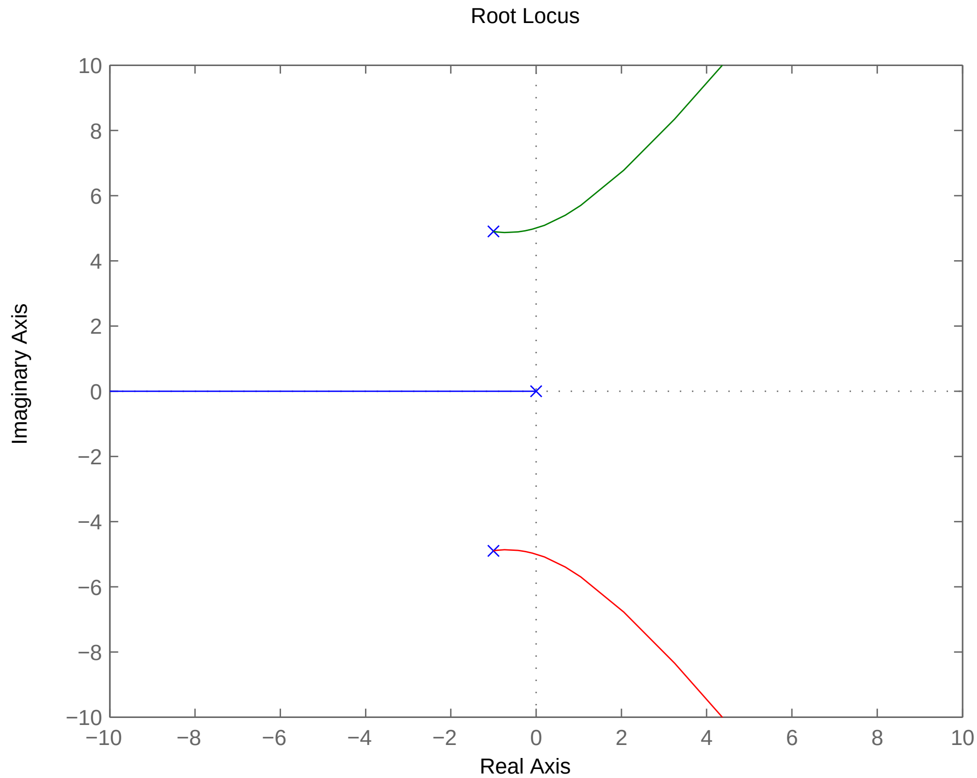
(d) True. The system is unstable for any gain $K > 0$, and is unstable for $K < -5$. Therefore, it is true that the system is unstable for $|K| > 10$.

(e) False. Since the actuator has a negative DC gain, a positive disturbance will cause a negative feedback signal to the load.

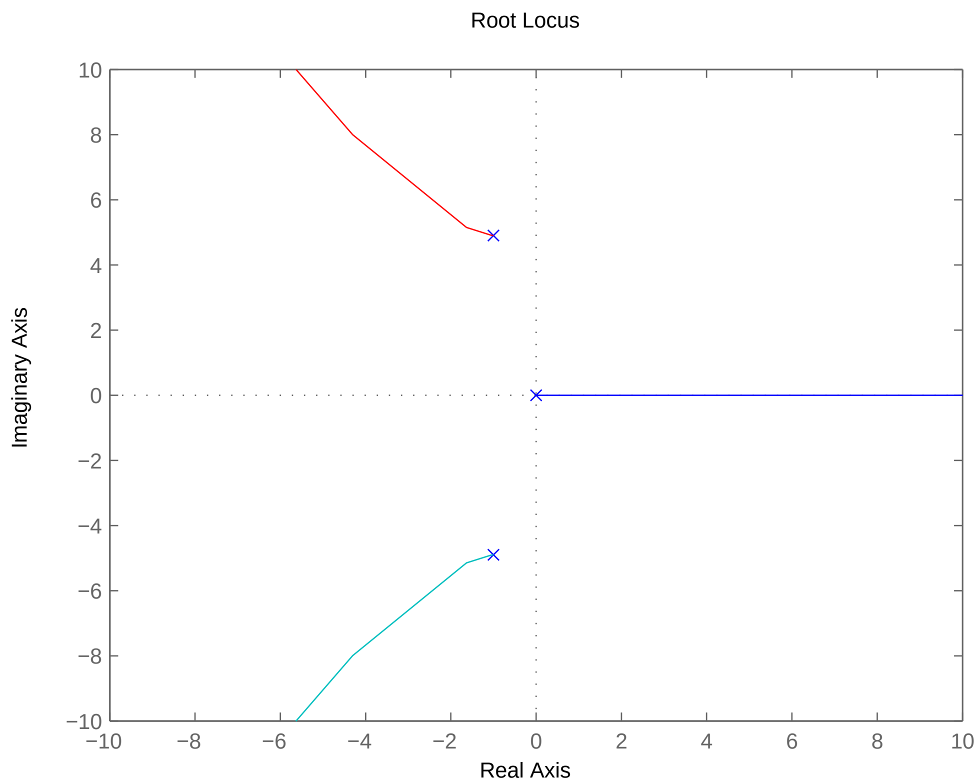
(f) True. The disturbance will speed up the load, resulting in a negative error. The closed-loop system has a DC gain from the disturbance, $d$, to the error signal, $e$, of $-1$. Therefore, the final value of the error due to a disturbance will be $-d$.

(g) False. The closed-loop system will result in an error signal equal to zero, if the disturbance is zero. The DC gain from the reference input to the error signal is zero. In addition, a position disturbance will cause a negative steady-state error.

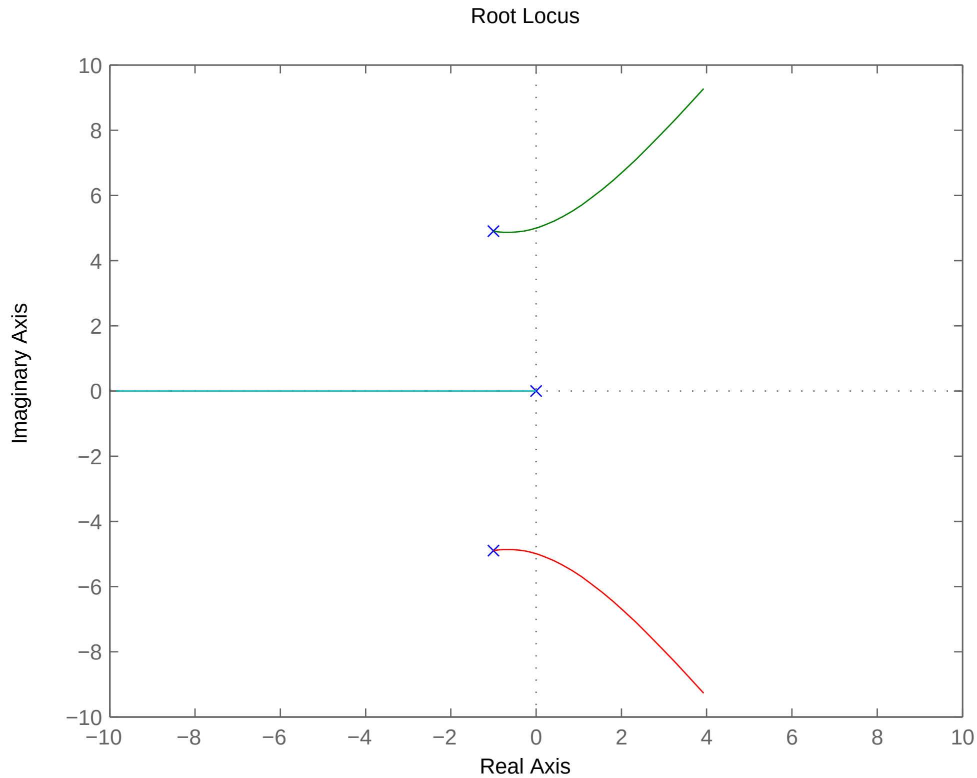
(h) False. The steady-state speed error due to the disturbance of .1, is 10 rad/sec, since the DC gain from $d$ to $y$ is 100. The error signal, $e$, is -0.1.




Problem 10.6 Servo mechanical root locus plot: without actuator.




Problem 10.6 Servo mechanical root locus plot: with actuator dynamics.




Problem 10.6 Servo mechanical root locus plot: for negative gain.

7. A stick balancer and its corresponding control block diagram are shown in Fig.10.96. The control is a torque applied about the pivot.

   (a) Using root-locus techniques, design a compensator $D_c(s)$ that will place the dominant roots at $s = -5 \pm 5j$ (corresponding to $\omega_n = 7$rad/sec, $\zeta = 0.707$).

   (b) Use Bode plotting techniques to design a compensator $D_c(s)$ to meet the following specifications:

   - steady-state $\theta$ displacement of less than 0.001 for a constant input torque $T_d = 1$,
   - Phase Margin $\geq 50°$,
   - Closed-loop bandwidth $\cong 7$rad/sec.

   **Solution:**

   (a) To have the compensated plant root locus go through the pole location $s = -5 \pm 5j$, we employ a lead compensator,

$$D_{c1}(s) = K\frac{s+z}{s+p}.$$

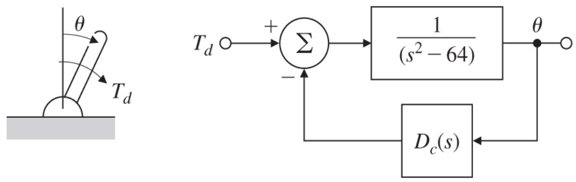



Figure 10.96: Stick balancer for Problem 10.7

Using the angle criterion,

$$\sum \phi_{z_i} - \sum \phi_{p_i} = -180^\circ,$$

at the closed-loop pole location $s = -5 + 5j$, we can write an expression for the angle contribution from the lead network zero, $\phi_z$, and lead network pole, $\phi_p$. We have,

$$\phi_z - \phi_p - 59^\circ - 159^\circ = -180^\circ,$$

or,

$$\phi = \phi_z - \phi_p = 38^\circ.$$

In MATLAB,

$$PHI \ = \ 180/pi \ * [\ angle(polyval \ \ (n, s) \ \ /polyval(d, s)) \ \ - \ \ pi].$$

So we have, $\phi = \phi_z - \phi_p = 38^\circ$. With selection of $z = 10$, we get $p = 45.7$. So that our lead design is,

$$D_{c1}(s) = K\frac{s+10}{s+45.7}.$$

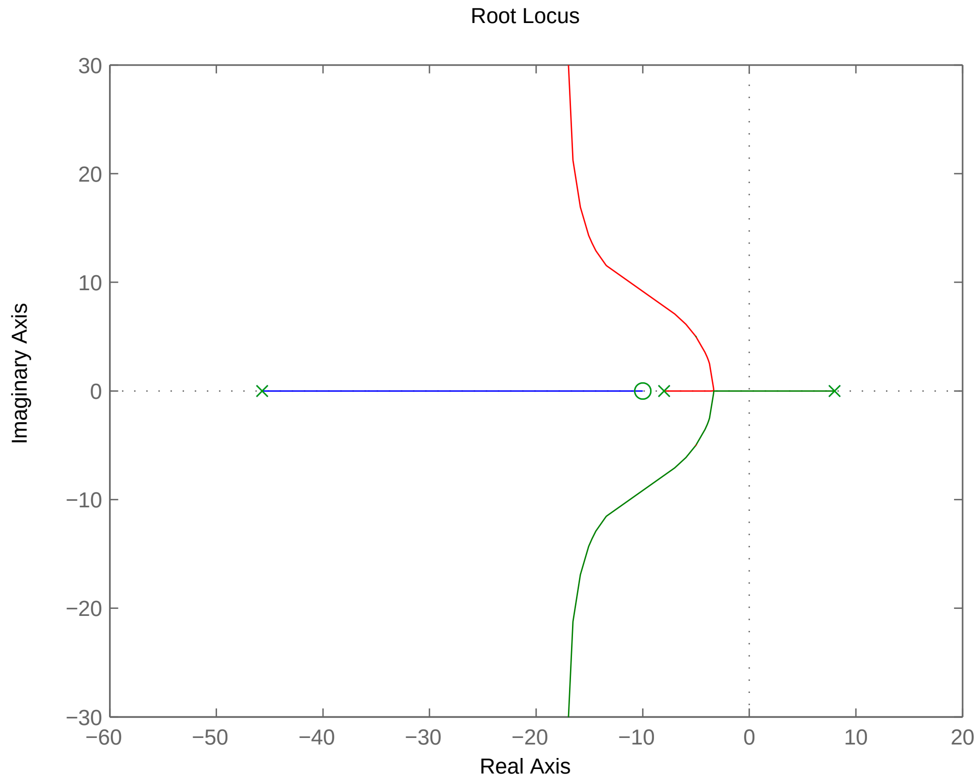
To find the compensator gain, $K$, we can utilize the magnitude criterion at the desired dominant closed-loop pole locations. We find that,

$$|D_{c1}(s)G(s)|_{s=-5\pm 5j} = 1 \Rightarrow K = 471.$$

Therefore, we have the compensator,

$$D_{c1}(s) = 471\frac{s+10}{s+45.7}.$$

The root locus plot of the compensated plant is shown using MATLAB's `rlocus` command.




Problem 10.7: Root locus of stick balancer compensated system.

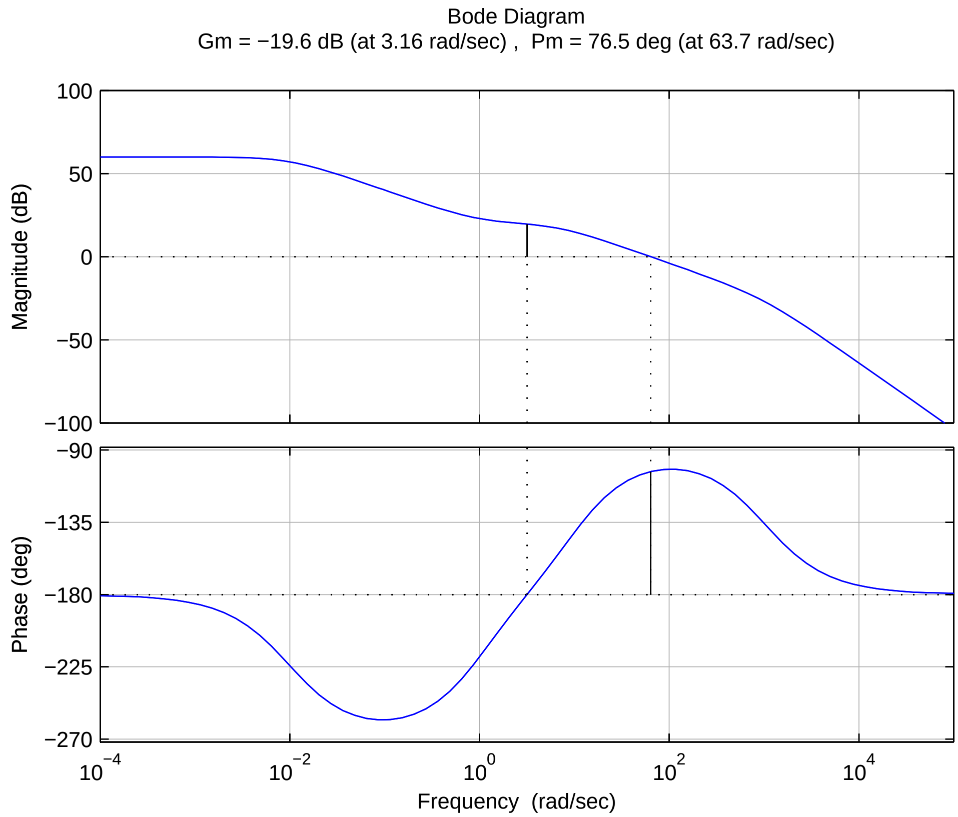
(b) We need a lag network in addition to a lead network to get the required $K_p$. Let,

$$D_{c2}(s) = 64000\frac{(s+1)(s/10+1)}{(s/0.01+1)(s/1000+1)}.$$

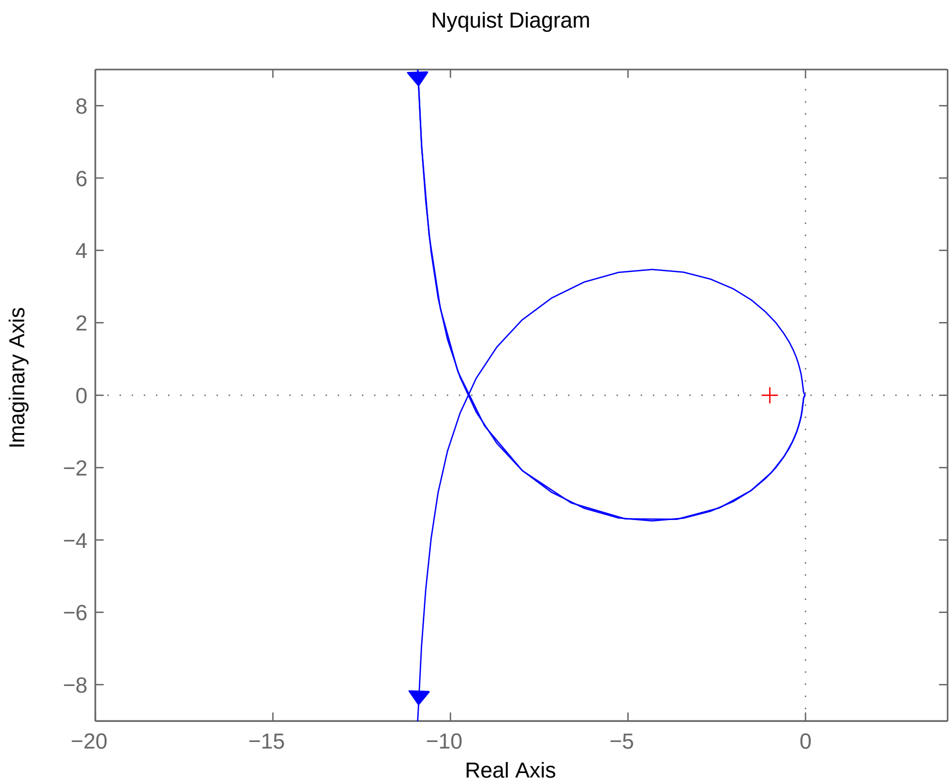
This compensator will meet our design specifications. The Bode plot of $D_{c2}(s)G(s)$ is shown on the next page using MATLAB's Bode command. Note that the phase margin is near 75 degrees. The 0 db cross-over frequency, $\omega_c$, is approximately 64 rad/sec. Hence, the bandwidth is near 64 rad/sec. The steady-state displacement to a unity constant input torque is,

$$\theta_{ss} = \lim_{s\to 0}\frac{G(s)}{1+G(s)D_c(s)} = 1.56\times 10^{-5} < 0.001.$$

Notice that this is an *unstable* open-loop system and the Bode plot must be interpreted carefully. A Nyquist plot is useful here. One is given for this compensator and plant using MATLAB's nyquist command as shown on the next page.




Problem 10.7 Frequency design method for stick balancer: Bode plot of compensated system.




Problem 10.7 Frequency design method for stick balancer: Nyquist plot of compensated system.

8. Consider the standard feedback system drawn in Fig.10.97.

(a) Suppose,

$$G(s) = \frac{2500\ K}{s(s+25)}.$$

Design a lead compensator so that the phase margin of the system is more than 45°; the steady-state error due to a ramp should be less than or equal to 0.01.

(b) Using the plant transfer function from part(a), design a lead compensator so that the overshoot is less than 25% and the 1% settling time is less than 0.1sec.

(c) Suppose

$$G(s) = \frac{K}{s(1+0.1s)(1+0.2s)},$$

and let the performance specifications now be $K_v = 100$ and PM $\geq 40°$. Is the lead compensation effective for this system? Find a lag compensator, and plot the root locus of the compensated system.

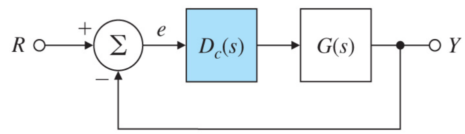



Figure 10.97: Block diagram of a standard feedback control system

(d) Using $G(s)$ from part(c), design a lag compensator such that the peak overshoot is less than 20% and $K_v = 100$.

(e) Repeat part(c) using a lead-lag compensator.

(f) Find the root locus of the compensated system in part(e), and compare your findings with those from part(c).

Block diagram of a standard feedback control system.

**Solution:**

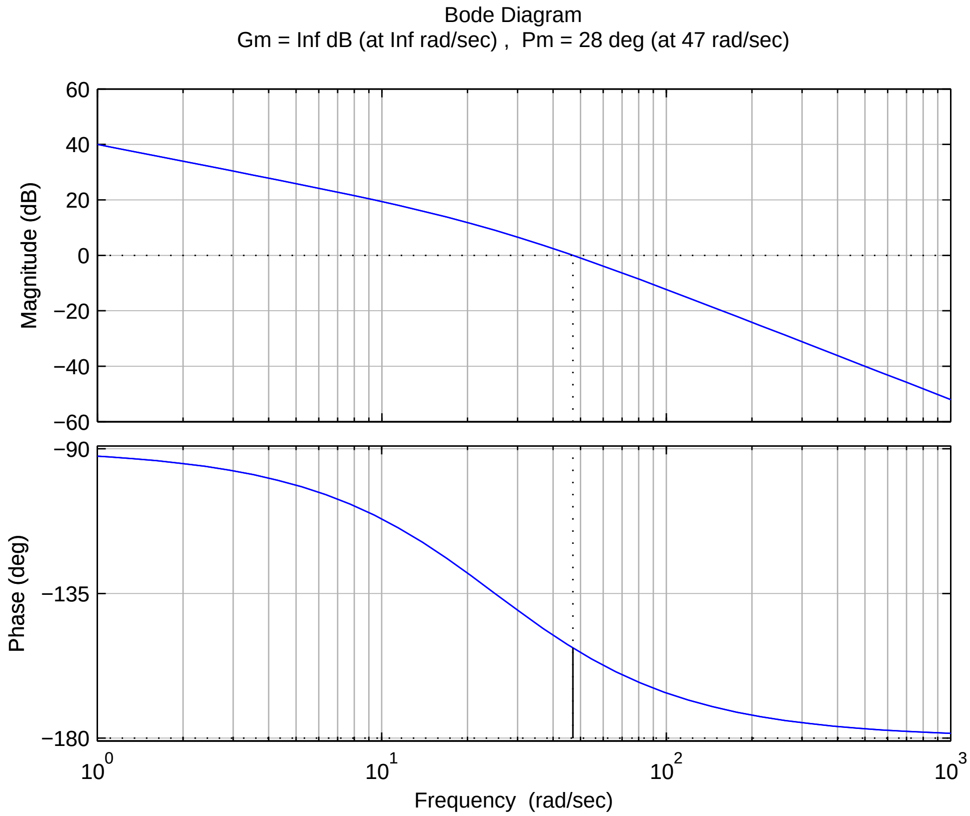
(a) The design specification of steady-state error provides information for the design of $K$.

$$\begin{aligned} e_\infty &= \frac{1}{K_v} = 0.01 \Longrightarrow K_v = 100. \\ K_v &= \lim_{s\to 0} sG(s) = 100K \Longrightarrow K = 1. \end{aligned}$$

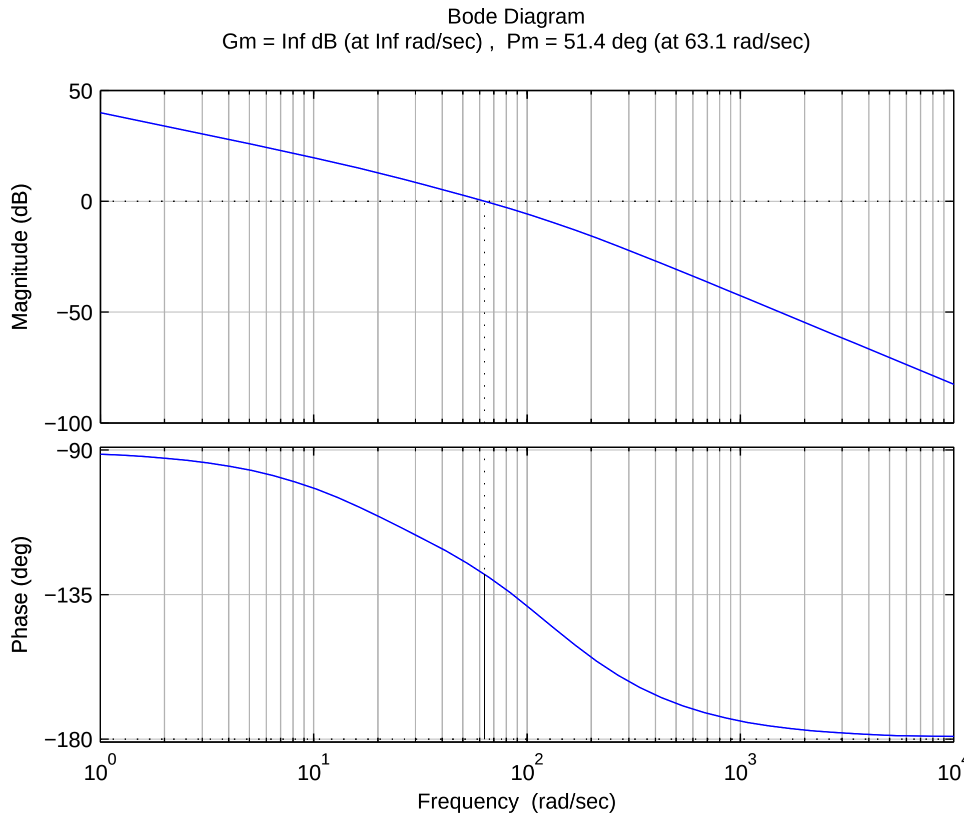
The Bode plot of,

$$G(s) = \frac{2500}{s(s+25)},$$

is given, using MATLAB's `margin` command and shows that the phase margin is approximately 30°. Therefore, we need 15° of phase lead. We select 30° of phase lead.




Problem 10.8: Frequency response of $G(s)$.




Problem 10.8: Frequency response of $D_c(s)G(s)$.

From text Fig. 6.53 of the text, we have $\frac{1}{\alpha} = 3$. Now, we need to find the frequency such that $|G(j\omega)| = \sqrt{\alpha} = 0.58$. From the Bode plot of $G(s)$, this results in $\omega = 63.5$ rad/sec. This frequency will be the crossover frequency of $D_c(s)G(s)$, i.e., $\omega_c = 63.5$ rad/sec. So the lead compensator is,

$$D_c(s) = \frac{\frac{s}{\omega} + 1}{\frac{s}{\frac{\omega}{\alpha}} + 1} = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1},$$

such that $\omega = \omega_c\sqrt{\alpha} = z \simeq 37$ and $\omega/\alpha = p \simeq 110$. Therefore, we have,

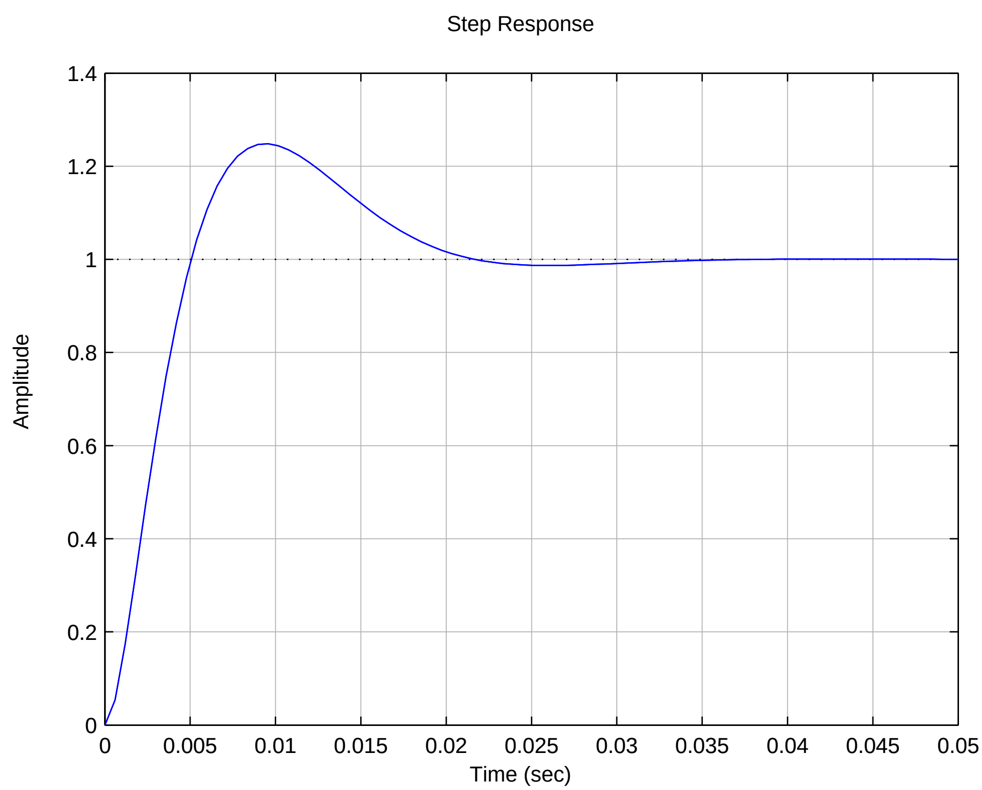
$$D_c(s) = \frac{s/37 + 1}{s/110 + 1}.$$

The Bode plot of the compensated system is shown on the previous page. The phase margin is $51°$ and $\omega_c = 63.2$ rad/sec.

(b) For $M_p = 25\%$, let $\zeta = 0.4$. For $t_s < 0.1$, let $\zeta\omega_n \approx 4.6/0.1 = 46$. Thus, $\omega_n = 115$ rad/sec, and $s = -46 \pm j105$. We set the lead zero at $s = 1.5 * \mathsf{abs}(s) = -172$ and compute the pole to be at $s = -1284$ using the angle criterion. The Bode plot and step response show the specifications are met with an additional gain of 20. Therefore, the compensator is,

$$D_c(s) = 74.65\frac{s+172}{s+1284}.$$

The closed-loop step response is shown below (using MATLAB's `step` command).




Step response of the closed-loop system for Problem 10.8 (b).

(c) The design specification of steady-state error provides information for the design of $K$.

$$K_v = \lim_{s\to 0} sG(s) = K \Longrightarrow K = 100.$$

The Bode plot of,

$$G(s) = \frac{100}{s(s/5+1)(s/10+1)},$$

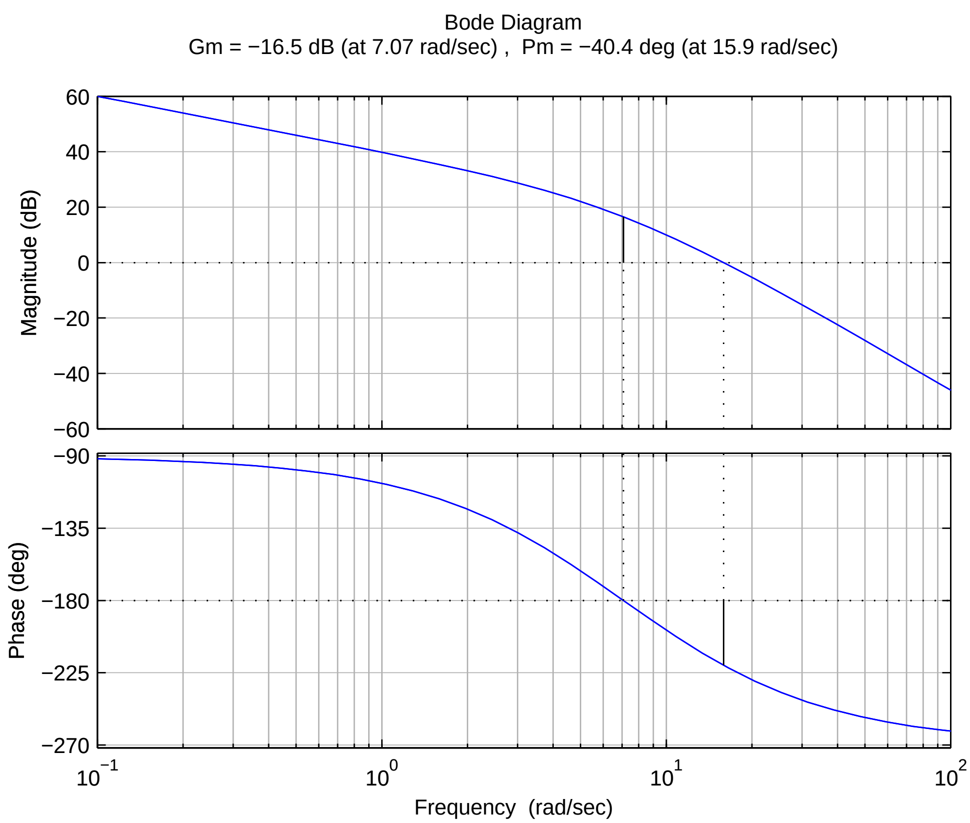
shows that the phase margin (using MATLAB's `margin`) is $-40°$. This is shown below. Therefore, we need a phase lead of greater than $80°$. A lead compensation $D_c(s) = \frac{(s+a)}{(s+b)}$ can not achieve this phase margin requirement. Hence, we try a lag network. We find the frequency such that phase margin of $G(j\omega)$ is our phase margin specification plus $10°$, or phase margin equals $50°$. At $\omega = 2.5$ rad/sec, the phase of $G(j\omega)$ is $-130°$ and $|G(j\omega)| = \alpha = 34.7$. This $\omega$ will be crossover frequency of $D_c(j\omega)G(j\omega)$, $\omega_c = 2.5$. Now, select the zero of $D_c(s)$ one decade below $\omega_c$, which implies $\omega = 0.25$ rad/sec. This results in $\frac{\omega}{\alpha} = 0.25/34.7 = 0.0072$. The lag network is thus,

$$D_c(s) = \frac{s/0.25 + 1}{s/0.0072 + 1},$$

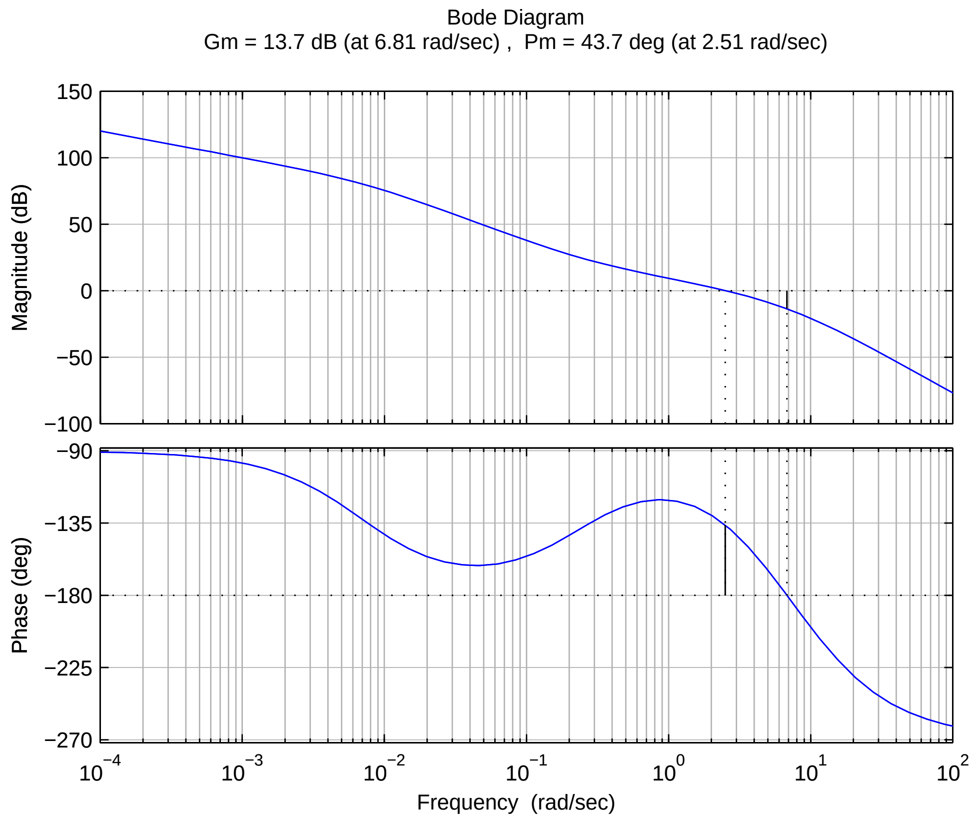
and the loop gain is,

$$D_c(s)G(s) = \frac{100(s/0.25 + 1)}{s(s/0.0072 + 1)(s/5 + 1)(s/10 + 1)}.$$

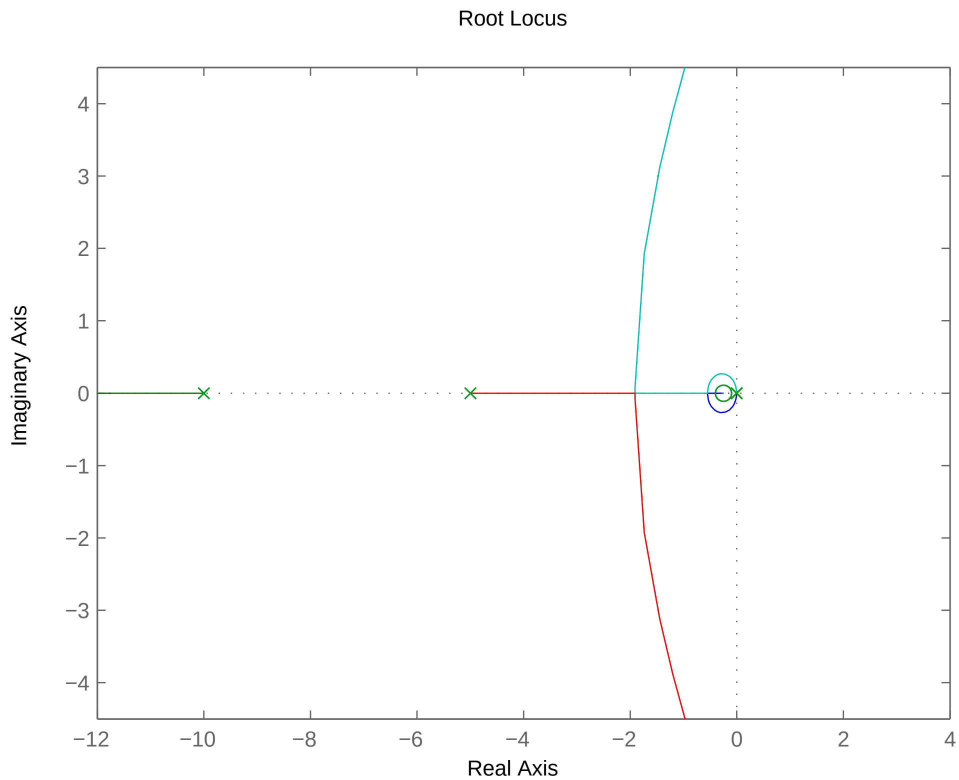
The Bode plot, root locus, and step responses are given on the next two pages (using MATLAB's `bode`, `rlocus`, `step` commands). Note that the design produces a phase margin of $43.7°$.



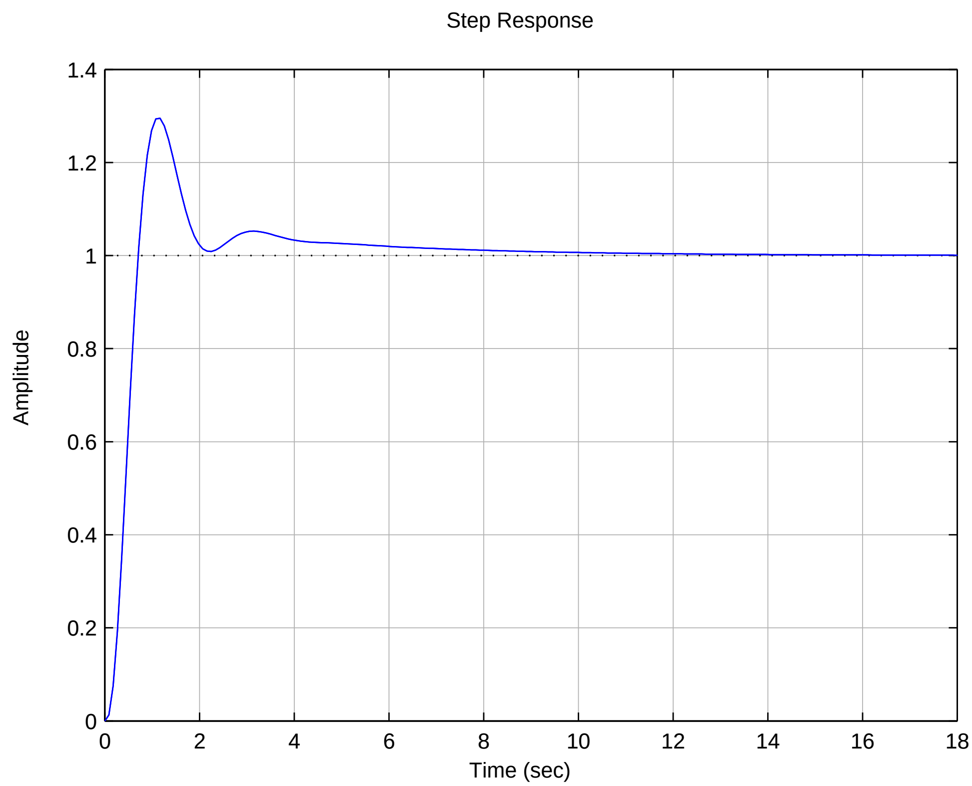

Bode plot of $G(s)$ for Problem 10.8 (c).




Bode plot of $D_c(s)G(s)$ for Problem 10.8 (c).




Root locus of $D_c(s)G(s)$ for Problem 10.8 (c).




Step response of closed-loop system for Problem 10.8 (c).

(d) We can design a lag compensator using root locus methods. The velocity constant requires the plant gain to be equal to 100, since,

$$K_v = \lim_{s\to 0} sG(s) = K \Longrightarrow K = 100.$$

Therefore,

$$G(s) = \frac{100}{s(1+0.1s)(1+0.2s)} = \frac{5000}{s(s+5)(s+10)}.$$

The root locus plot of $G(s)$ is shown below using MATLAB's rlocus command. For an overshoot specification of $M_p = 20\%$, we chose $\zeta = 0.46$. We can find the desired closed-loop pole locations by finding the intersection of the root locus shown with the constant damping line for $\zeta = 0.46$. This results in desired dominant poles at $s = -1.61 \pm 3.11j$. However, $K_v = K$ of $G(s)$ at these pole locations is 2.875, since,

$$\left|\frac{K}{s(s/5+1)(s/10+1)}\right|_{s=-1.61\pm j3.11} = 1 \Longrightarrow K = 2.875.$$

Therefore, we need to raise $K_v$ to 100. This implies using a lag compensator with $\alpha = \frac{100}{2.875} = 34.8$. If we select the compensator zero at $s = 0.1$, the pole location is $s = \frac{0.1}{\alpha} =$

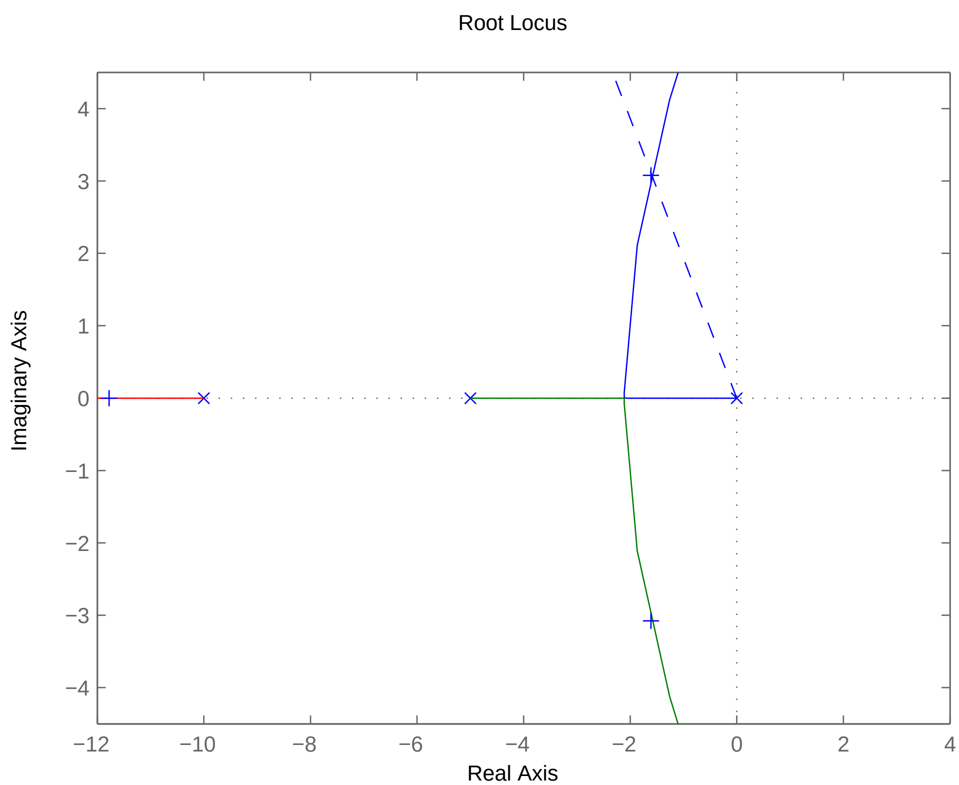
0.003. Hence,

$$D_c(s) = \frac{s/0.1 + 1}{s/0.003 + 1},$$

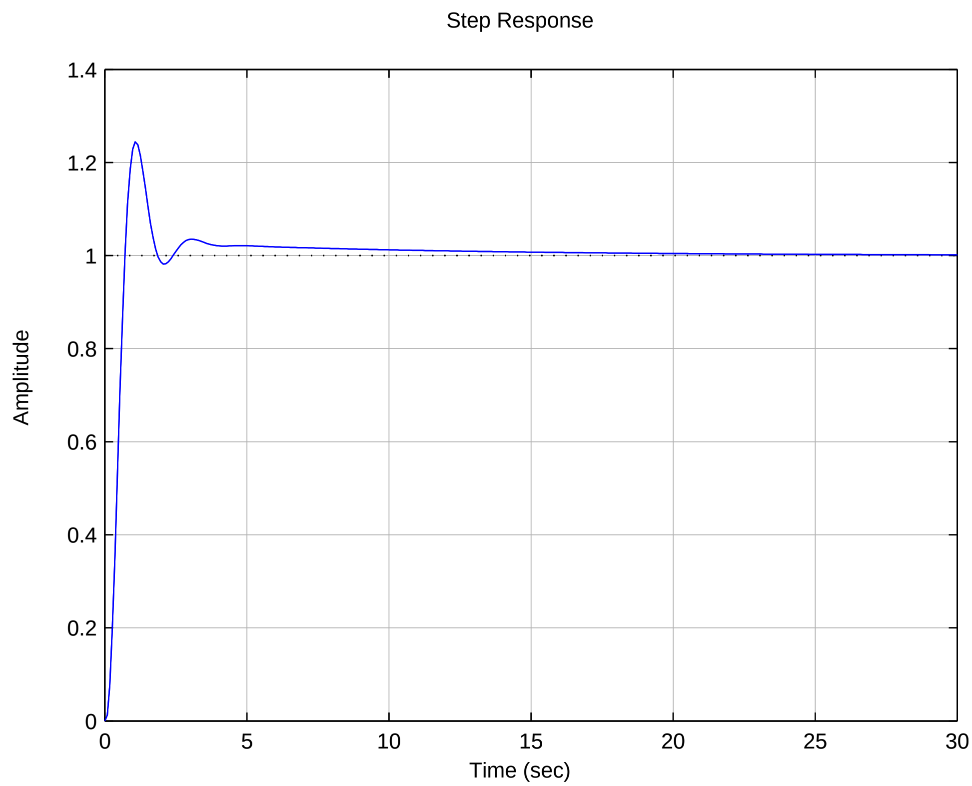
and the loop gain is,

$$D_c(s)G(s) = \frac{100(s/0.1 + 1)}{s(s/.003 + 1)(s/5 + 1)(s/10 + 1)}.$$

The step response of the closed-loop system is given on the next page (using MATLAB's `step` command). Note the small slow transient in the step response from the lag compensator.




Root locus for Problem 10.8 (d).




Step response of closed-loop system for Problem 10.8 (d).

(e) Again, the design specification of steady-state error provides information for the design of $K$.

$$K_v = 100 \Longrightarrow K = 100.$$

As mentioned in part (c), the phase margin for,

$$G(s) = \frac{100}{s(s/5+1)(s/10+1)},$$

is $-40^\circ$. First, we select the cross-over frequency, $\omega_c$. From the Bode plot of $G(s)$ given,

$$\angle G(j\omega) = -180^\circ \Longrightarrow \omega_c = 7.0.$$

With $\omega_c = 7.0$ we need $40^\circ$ more lead. From Fig. 6.52 in the text, an $\alpha = 0.1$ will provide $55^\circ$of lead. We select the lead such that zero location is $s = \omega = \omega_c\sqrt{\alpha} = 2.21$. The lead pole location is $s = \frac{\omega}{\alpha}$. So we have:

$$D_{lead}(s) = \frac{s/2.21+1}{s/22.1+1}.$$

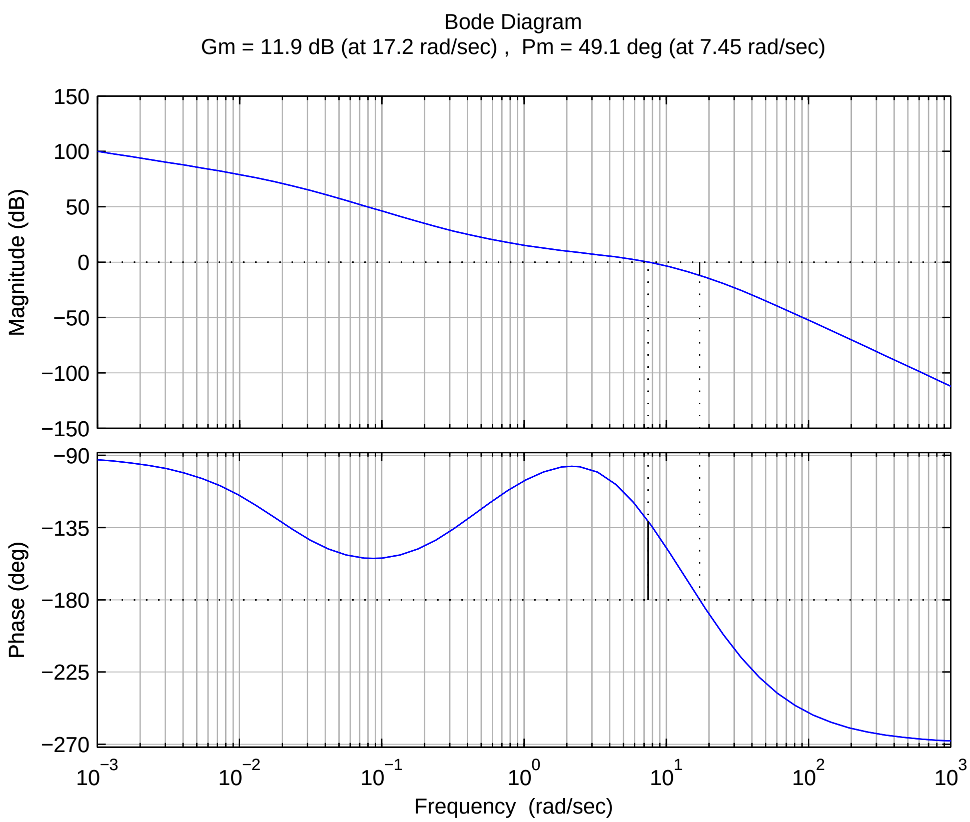
Now, we select the zero of the lag at least one decade lower than $\omega_c$. With $\alpha$ of the lag equal to 20, we have,

$$D_{lag}(s) = \frac{s/0.7+1}{s/0.035+1}.$$

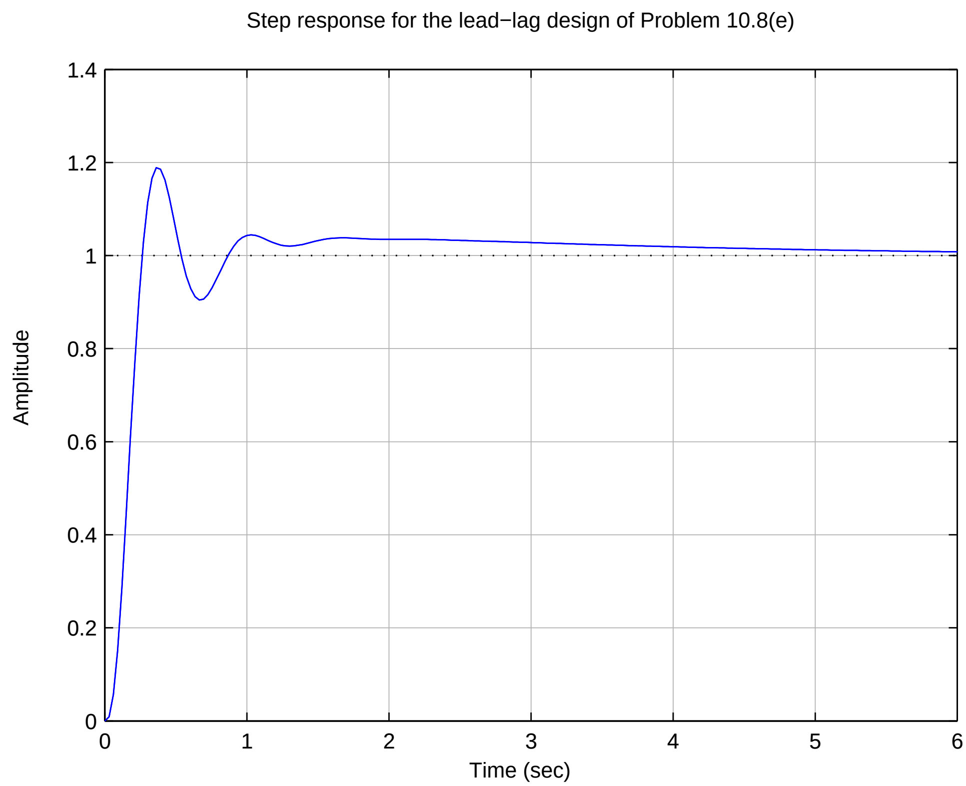
The lead-lag compensator is,

$$D_c(s) = \frac{(s/0.7+1)(s/2.21+1)}{(s/0.035+1)(s/22.1+1)}.$$

The system Bode plot and step response appear on the next page.

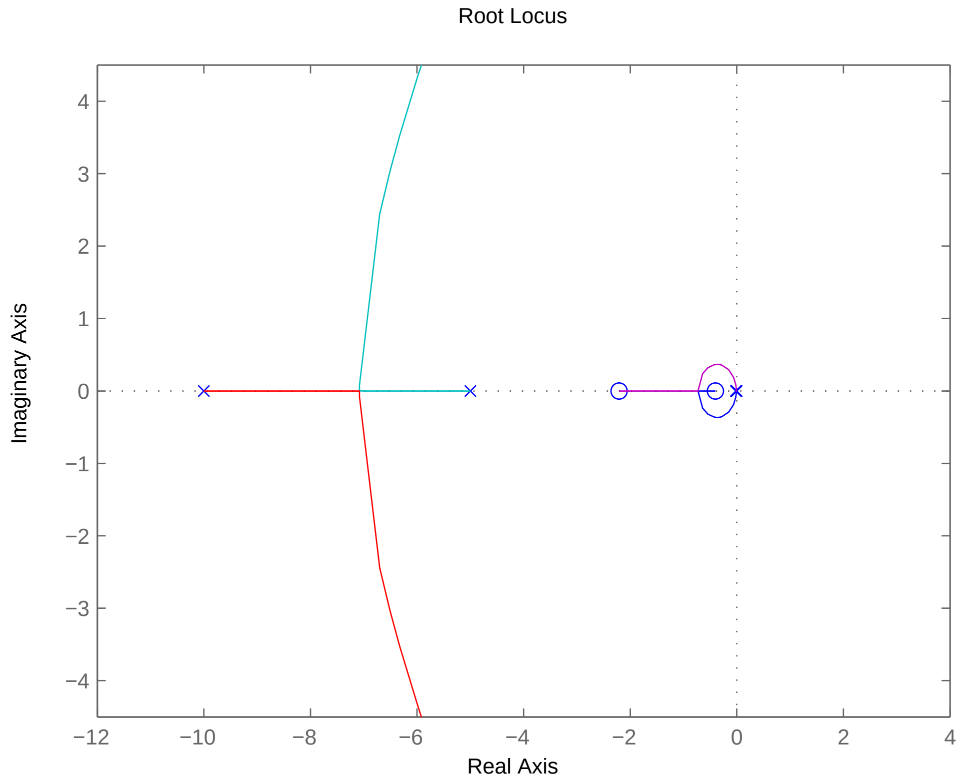



Bode plot of $G(s)D_c(s)$. for Problem 10.8 (e).




Step response for Problem 10.8 (e).

(f) The root locus plot of $D_c(s)G(s)$ from part (d) is shown on the next page.




Root locus for Problem 10.8 (e).

The main difference between the designs of part (c) and part (e) is that with lead-lag we have higher $\omega_c$, and hence higher bandwidth, and also lower rise time and lower overshoot.

9. Consider the system in Fig.10.97, where

$$G(s) = \frac{300}{s(s+0.225)(s+4)(s+180)}.$$

The compensator $D_c(s)$ is to be designed so that the closed-loop system satisfies the following specifications:

1.
   - zero steady-state error for step inputs,
   - PM $= 55°$, GM $\geq 6$ db,
   - gain crossover frequency is not smaller than that of the uncompensated plant.

   (a) What kind of compensation should be used and why?

   (b) Design a suitable compensator $D_c(s)$ to meet the specifications.

   **Solution:**

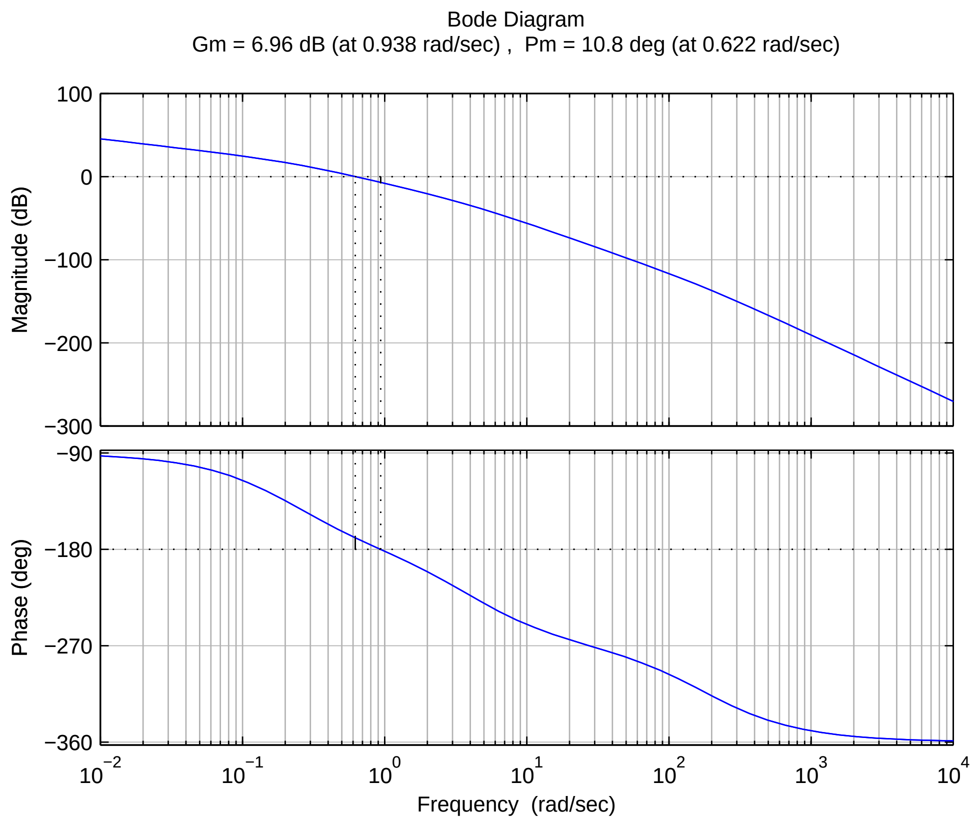
The Bode plot of $G(s)$ is shown on the next page. From the figure, the phase margin is $10.8°$ and $\omega_c = 0.623$.

1. (a) Since we need $55° - 10° = 45°$ of phase lead, a single lead network will do the job.

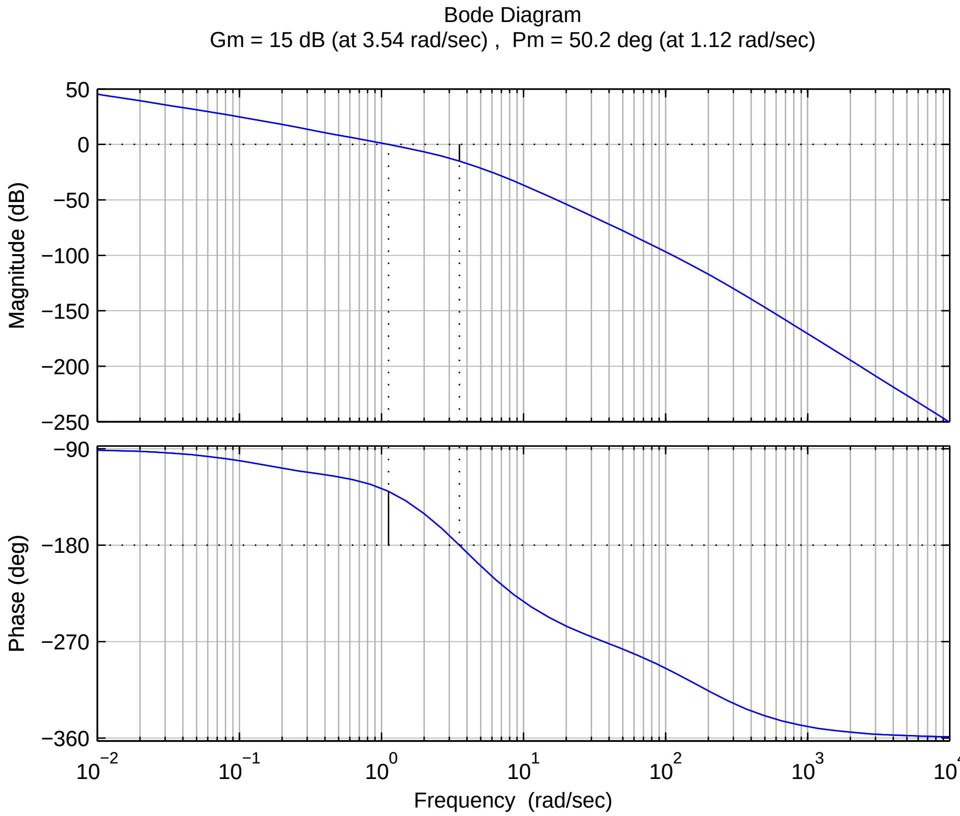
   (b) From a phase lead requirement of $45°$, we have $\frac{1}{\alpha} \approx 10$. Note that you can use either Fig. 6.52 of the text, or $\sin(\phi) = \frac{1-\alpha}{1+\alpha}$ where $\phi$ is the required phase lead in radians. Now we find the frequency, $\omega$, of $G(j\omega)$ such that $|G(j\omega)| = \sqrt{\alpha} = 0.32$. We find $\omega = 1.11$ which will be the $\omega_c$ of the compensated system. The zero of lead network is chosen as $s = \omega_c\sqrt{\alpha} = 0.35$. The pole location is located at $s = \frac{\omega_c}{\alpha} = 3.5$. Hence, the compensator and the loop gain are,

$$\begin{aligned} D_c(s) &= \frac{s/0.35+1}{s/3.5+1}, \\ D_c(s)G(s) &= \frac{1.8519(s/0.35+1)}{s(s/3.5+1)(s/0.225+1)(s/4+1)(s/180+1)}. \end{aligned}$$

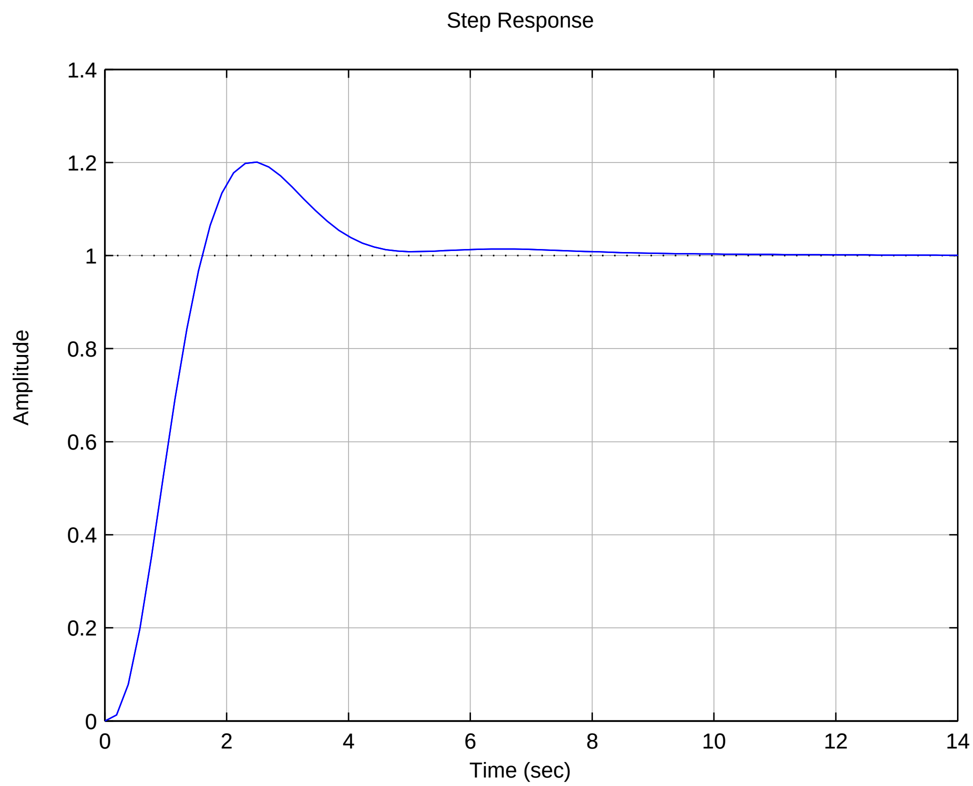
   The Bode plot of $D_c(s)G(s)$, the compensated system is shown on the next page using MATLAB's `margin` command. As the figure shows, $\omega_c = 1.1$, which is larger than the crossover frequency of the uncompensated plant, $G(s)$. The Bode plot shows a phase margin of $55°$ and a gain margin of 15 db. Both specifications meet the requirements. Finally, since $D_cG$ is a Type I system, the steady-state error, $e_\infty$, due to a step function is zero as shown on the next page.




Bode plot of $G(s)$ for Problem 10.9.




Lead design for Problem 10.9: Bode plot of the compensated system.




Step response of closed-loop system for Problem 10.9.

10. We have discussed three design methods: the root-locus method of Evans, the frequency-response method of Bode, and the state-variable pole-assignment method. Explain which of these methods is *best* described by the following statements (if you feel more than one method fits a given statement equally well, say so and explain why):

1. (a) This method is the one most commonly used when the plant description must be obtained from experimental data.
   (b) This method provides the most direct control over dynamic response characteristics such as rise time, percent overshoot, and settling time.
   (c) This method lends itself most easily to an automated (computer) implementation.
   (d) This method provides the most direct control over the steady-state error constants $K_p$ and $K_v$.
   (e) This method is most likely to lead to the *least complex* controller capable of meeting the dynamic and static accuracy specifications.
   (f) This method allows the designer to guarantee that the final design will be unconditionally stable.

(g) This method can be used without modification for plants that include transportation lag terms, for example,

$$G(s) = \frac{e^{-2s}}{(s+3)^2}.$$

This method is the one most commonly used when the plant description must be obtained from experimental data.

**Solution:**

(a) Frequency response method is the most convenient for experimental data because the sinusoidal steady-state records can be obtained directly in the laboratory. Either the root locus or state variable design generally requires a separate system identification effort between the experimental data and the construction of a model suitable for the design method.

(b) Either the root-locus or state variable pole assignment are the most direct for control over dynamic response. The pole-zero characteristics are the items of concentration in these two design methods.

(c) The state variable pole-assignment is most easily programmed because, once the specifications are given, the design is completely algorithmic. In the other methods, a trial and error cycle is required and while the analysis may be done by a computer the design is not easily implemented.

(d) The frequency response method of Bode shows the error constant (either $K_p$ or $K_v$) directly on the graph. State variable or root locus require a separate calculation for these numbers. (Using Truxal's formula, however, the state variable pole-assignment method can be used to give a specific control over $K_p$ or $K_v$ ).

(e) The root locus or Bode method will give the least complex controller. These techniques begin with gain alone and then add network compensation only as necessary to meet the specifications; whereas the state variable technique requires a controller of complexity comparable to that of the plant right from the start.

(f) Either the root locus, whereby the locus is required to be entirely in the left half plane up to the operating gain, or the Bode method whereby the phase margin is required to be positive for all frequencies below crossover to allow the designer to guarantee unconditionally stable behavior. The state variable design technique does not permit this guarantee.

(g) The frequency response technique can be used immediately for transportation lag, while the root locus requires a small modification and the state variable design method requires an approximation.

11. Lead and lag networks are typically employed in designs based on frequency response (Bode) methods. Assuming a type 1 system, indicate the effect of these compensation networks on each of the listed performance specifications. In each case, indicate the effect as "an increase," "substantially unchanged," or "a decrease." Use the second-order plant $G(s) = K/[s(s+1)]$ to illustrate your conclusions.

(a) $K_v$

(b) Phase margin

(c) Closed-loop bandwidth

(d) Percent overshoot


Figure 10.98: Spirit of Freedom balloon

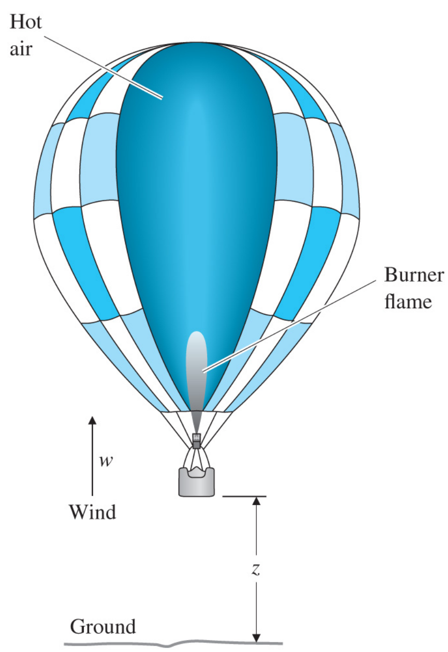



Figure 10.99: Hot-air balloon

(e) Settling time

**Solution:**

| | **Lead** | **Lag** |
|---|---|---|
| $K_v$ | Unchanged | Increased |
| Phase margin | Increased | Unchanged |
| Closed loop bandwidth | Increased | Unchanged |
| Percent overshoot | Decreased | Unchanged |
| Settling time | Decreased | Unchanged |

1. 12. *Altitude Control of a Hot-air Balloon*: American solo balloonist Steve Fossett landed in the Australian outback aboard *Spirit of Freedom* on July 3rd, 2002, becoming the first solo balloonist to circumnavigate the globe (see Fig. 10.98). The equations of vertical motion for a hot-air balloon (Fig.10.99), linearized about vertical equilibriumare

$$\delta\dot{T} + \frac{1}{\tau_1}\delta T = \delta q,$$
$$\tau_2\ddot{z} + \dot{z} = a\delta T + w,$$

where

$$\begin{aligned} \delta T &= \text{deviation of the hot} - \text{air temperature from the equilibrium} \\ \text{temperature where buoyant force } &= \text{weight}, \\ z &= \text{altitude of the balloon}, \\ \delta q &= \text{deviation in the burner heating rate from the equilibrium rate} \\ &\quad \text{(normalized by the thermal capacity of the hot air)}, \\ w &= \text{vertical component of wind} - \text{velocity}, \\ \tau_1, \tau_2, a &= \text{parameters of the equations.} \end{aligned}$$

An altitude-hold autopilot is to be designed for a balloon whose parametersare

$$\tau_1 = 250 \text{ sec}, \quad \tau_2 = 25 \text{ sec}, \quad a = 0.3 \text{ m/(sec} \cdot {}^\circ\text{C)}.$$

Only altitude is sensed, so a control law of theform

$$\delta q(s) = D_c(s)[z_d(s) - z(s)],$$

will be used, where $z_d$ is the desired (commanded) altitude.

(a) Sketch a root locus of the closed-loop eigenvalues with respect to the gain $K$ for a proportional feedback controller, $\delta q = -K(z - z_d)$. Use Routh's criterion (or let $s = j\omega$ and find the roots of the characteristic polynomial) to determine the value of the gain and the associated frequency at which the system is marginally stable.

(b) Our intuition and the results of part (a) indicate that a relatively large amount of lead compensation is required to produce a satisfactory autopilot. Because Steve Fossett was a millionaire, he could afford a more complex controller implementation. Sketch a root locus of the closed-loop eigenvalues with respect to the gain $K$ for a double-lead compensator, $\delta q = D_c(s)(z_d - z)$, where,

$$D_c(s) = K\left(\frac{s+0.03}{s+0.12}\right)^2.$$

(c) Select a gain $K$ for the lead-compensated system to give a crossover frequency of 0.06rad/sec.

(d) Sketch the magnitude portions of the Bode plots (straight-line asymptotes only) for the open-loop transfer functions of the proportional feedback and lead-compensated systems.

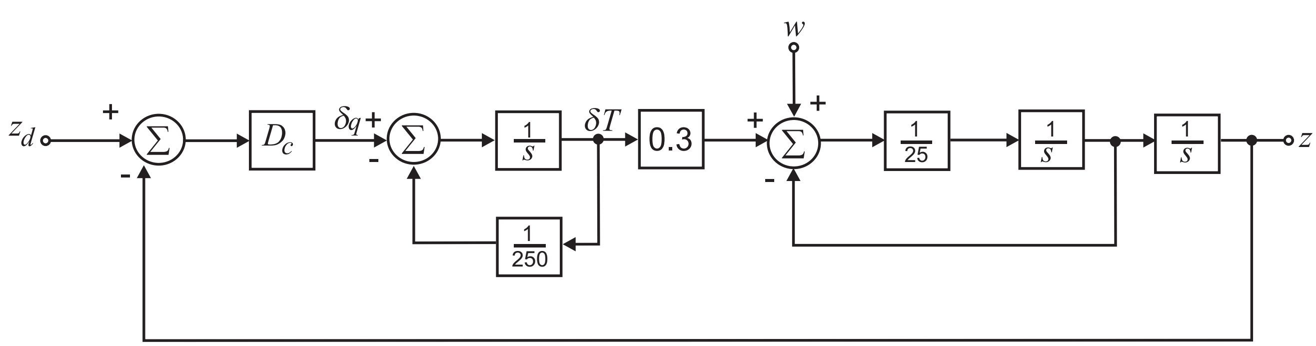
(e) With the gain selected in part(d), what is the steady-state error in altitude for a steady vertical wind of 1m/sec? (Be careful: First find the closed-loop transfer function from $w$ to the error.)

(f) If the error in part(e) is too large, how would you modify the compensation to give higher low-frequency gain? (Give a qualitative answer only.)

**Solution:**

$$\begin{aligned}
\delta\dot{T} + \frac{1}{\tau_1}\delta T &= \delta q \Longrightarrow \delta T = \frac{1}{s+\frac{1}{\tau_1}}\delta q = \frac{\tau_1}{\tau_1 s + 1}\delta q = G_1(s)\delta q, \\
\tau_2 \ddot{z} + \dot{z} &= a\delta T + w \Longrightarrow z = \frac{1}{s(\tau_2 s + 1)}(a\delta T + w) = G_2(s)\delta T + G_3(s) w.
\end{aligned}$$

The block diagram of the system is shown below.




Problem 10.12: Block diagram for balloon problem with only altitude measurement.

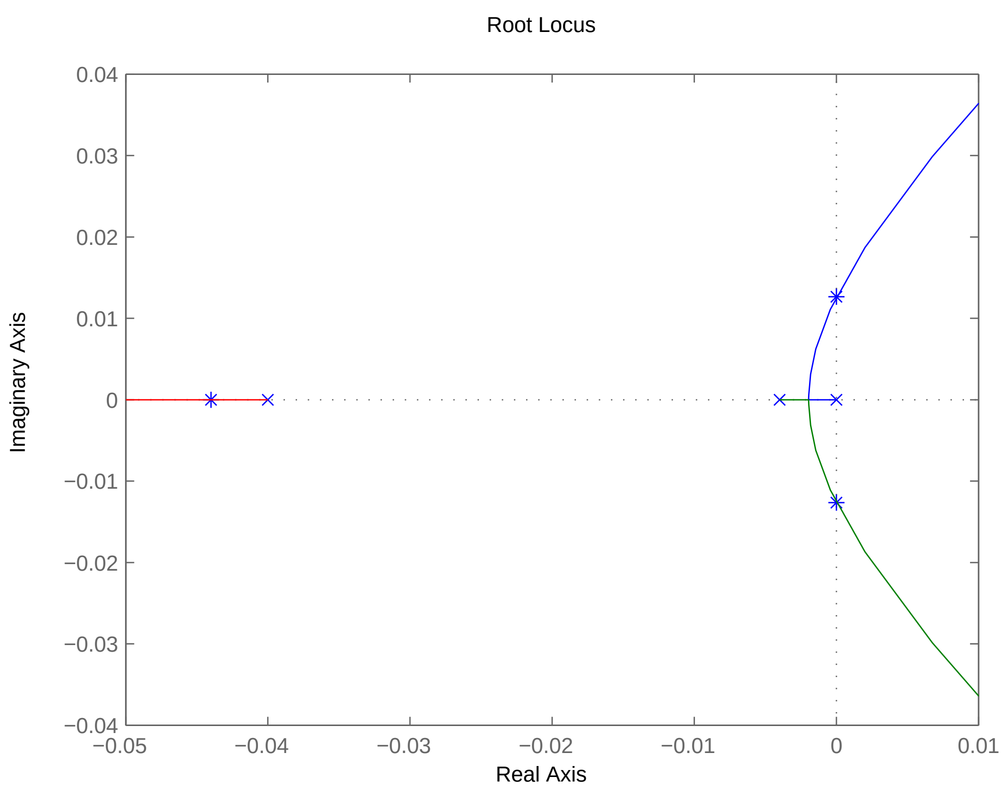
1. (a) With $D_c(s) = K$, the open-loop transfer function is,

$$D_c G_1 G_2 = K\left(\frac{\tau_1}{\tau_1 s + 1}\right)\left(\frac{a}{s(\tau_2 s + 1)}\right) = \frac{75K}{s(250s+1)(25s+1)}.$$

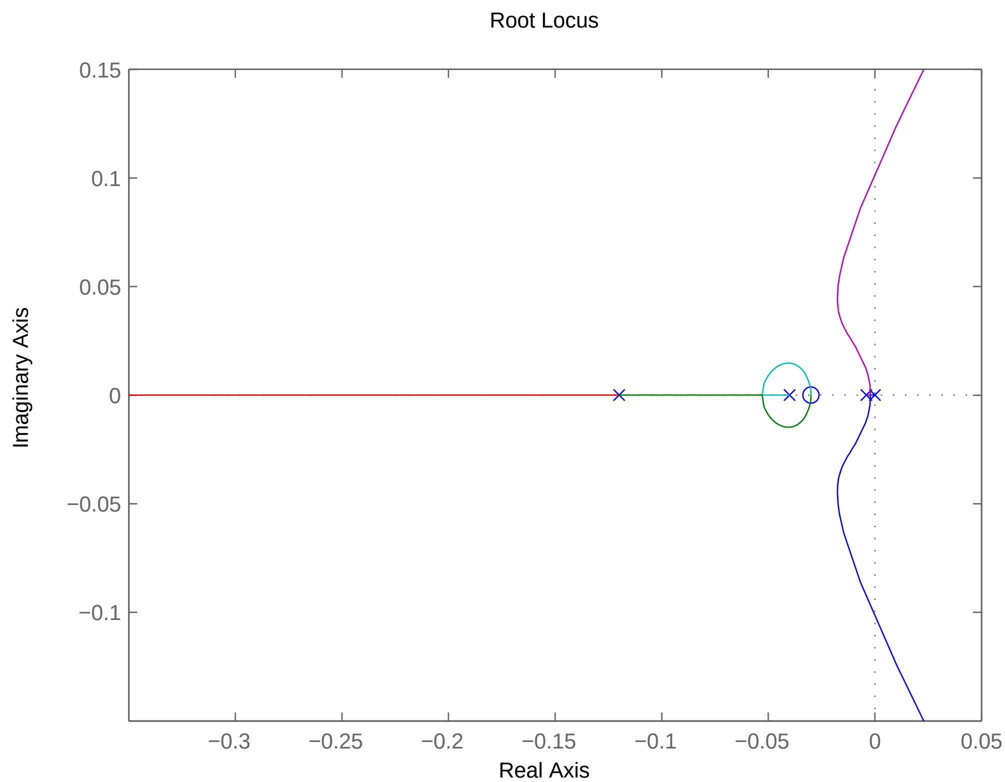
The closed-loop system roots are found from the numerator of the equation $1 + D_c G_1 G_2 = 0$. We can find the closed-loop roots which are on the imaginary axis by setting $s = j\omega$ (i.e., constrain the solution to lie on the $j\omega$ axis) and then equating the real and imaginary parts to zero. We find,

$$\begin{aligned} \tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + s + Ka\tau_1 &= 0, \\ &\Longrightarrow Ka\tau_1 - (\tau_1+\tau_2)^2 = 0, \\ \omega - \tau_1\tau_2\omega^3 &= 0. \end{aligned}$$

The result is $K = 5.87 \times 10^{-4}$ and $\omega = 0.01265$. Note that the system is unstable for $K > 5.87 \times 10^{-4}$. The next figure shows the root locus plot of $D_c G_1 G_2$.




Problem 10.12: Root locus for balloon altitude control system with $D_c(s) = K$.




Problem 10.12: Root locus for balloon altitude control system with double-lead compensation.

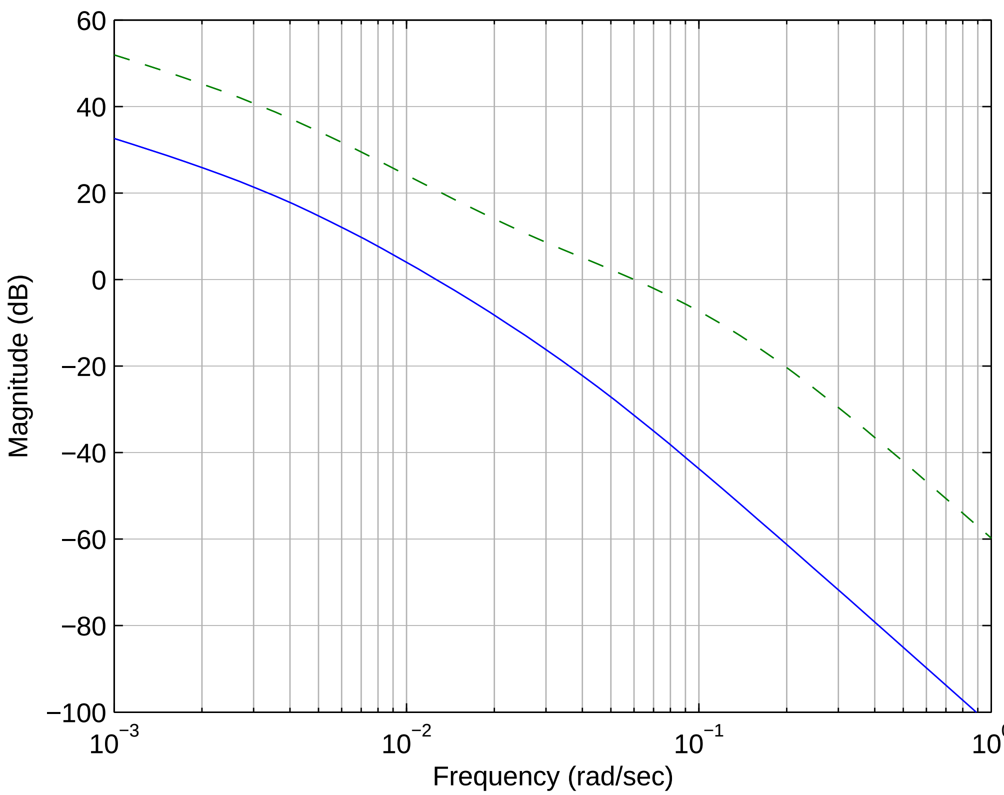
(b) The root locus using a double lead compensator is shown above. The open-loop transfer function used is,

$$D_cG_1G_2 = K\left(\frac{s+0.03}{s+0.12}\right)^2\left(\frac{\tau_1}{\tau_1 s+1}\right)\left(\frac{a}{s(\tau_2 s+1)}\right).$$

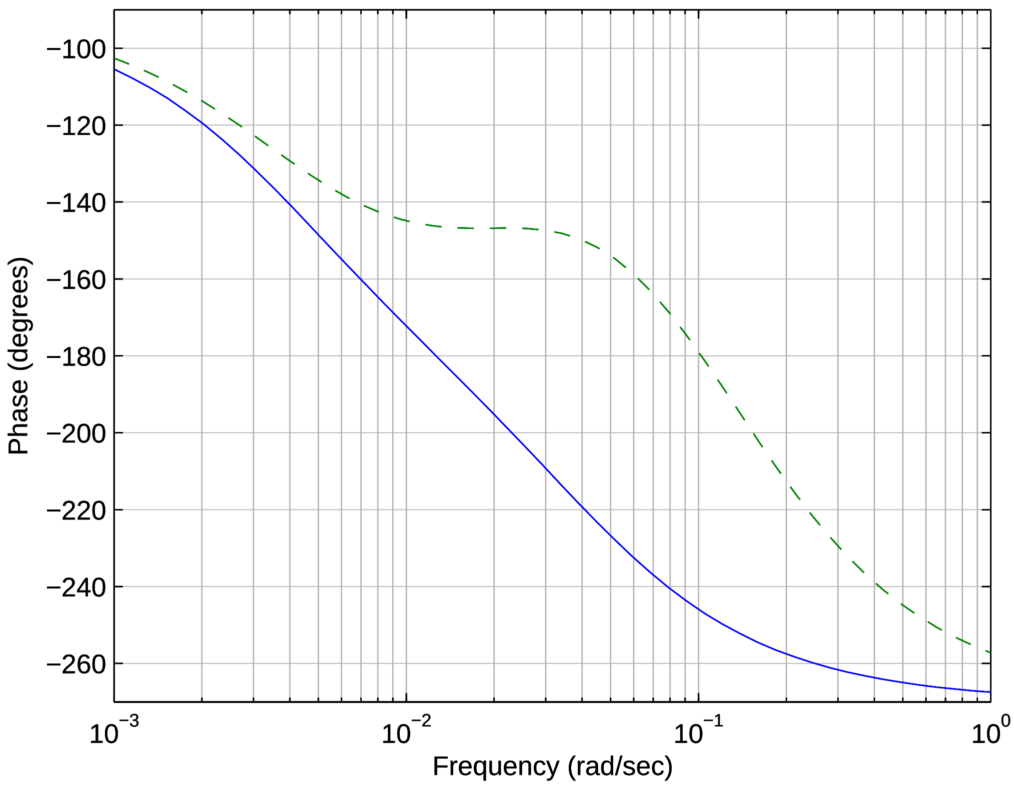
(c) To find $K$ such that $\omega_c = 0.06$,

$$|D_cG_1G_2|_{\omega=0.06} = 1 \Longrightarrow K = 0.0867.$$

(d) In order to plot the Bode plots, we need to specify which values for $K$ we are going to use. For the Bode plot of the proportional compensator, we use $K = 5.87\times10^{-4}$ from part (b) (the case where the closed-loop system is marginally stable). For the Bode plot of the double lead compensator, we use $K = 0.0867$ from part (d) (the gain when the crossover frequency is 0.06 rad/sec). The following figures show Bode magnitude and phase plots for the balloon control system for both cases. The solid line (blue) corresponds to the proportional compensator and the dashed line (green) corresponds to the double lead compensator.




Problem 10.12: Bode magnitude plots for proportional control (blue), and double lead compensation (green).




Problem 10.12: Bode phase plots for proportional control (blue), and double lead compensation (green).

Using the notation from part (a), we have (suppressing the Laplace variable $s$),

$$\begin{aligned} Z &= G_3W + D_cG_1G_2E, \\ E &= Z_d - Z = Z_d - G_3W - D_cG_1G_2E, \\ &\Longrightarrow E(1 + D_cG_1G_2) = Z_d - G_3W, \\ &\Longrightarrow E = (1 + D_cG_1G_2)^{-1}(Z_d - G_3W). \end{aligned}$$

Using a unit step on $w(t)$, i.e., $W(s) = 1/s$, and ignoring $z_d$ because it is not involved in the transfer function from $w$ to $e$, we have,

$$e_\infty = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{-sG_3}{1 + D_cG_1G_2} W = -2.46\ m.$$

(e) We can add a lag network at low frequency to boost the $K_v$ ($e_\infty = 1/K_v$). This will not affect the crossover frequency, $\omega_c = 0.06$ rad/sec. For example,

$$D_c(s) = \left(\frac{s + 0.02}{s + 0.002}\right)^2 \left(\frac{s + 0.03}{s + 0.12}\right)^2,$$

will increase $K_v$ by a factor of 100 or equivalently reduce the error by factor of 0.01, which implies $e_\infty = -0.0246$ m.

13. Satellite-attitude control systems often use a reaction wheel to provide angular motion. The equations of motion for such a system are

$$\begin{aligned} \text{Satellite}: &\quad I\ddot{\phi} = T_c + T_{ex}, \\ \text{Wheel}: &\quad J\dot{r} = -T_c, \\ \text{Measurement}: &\quad \dot{Z} = \dot{\phi} - aZ, \\ \text{Control}: &\quad T_c = -D(s)(Z - Z_d), \end{aligned}$$

where,

$$\begin{aligned} J &= \text{ moment of inertia of the wheel,} \\ r &= \text{ wheel speed,} \\ T_c &= \text{control torque,} \\ T_{ex} &= \text{disturbance torque,} \\ \phi &= \text{angle to be controlled,} \\ Z &= \text{measurement from the sensor,} \\ Z_d &= \text{reference angle,} \\ I &= \text{satellite inertia } (1000 \text{ kg/m}^2), \\ a &= \text{sensor constant } (1 \text{ rad/sec}), \\ D_c(s) &= \text{compensation.} \end{aligned}$$

(a) Suppose $D_c(s) = K_0$, a constant. Draw the root locus with respect to $K_0$ for the resulting closed-loop system.

(b) For what range of $K_0$ is the closed-loop system stable?

(c) Add a lead network with a pole at $s = -1$ so that the closed-loop systemhas a bandwidth $\omega_{BW} = 0.04$ rad/sec, a damping ratio $\zeta = 0.5$, and compensation givenby,

$$D_c(s) = K_1 \frac{s+z}{s+1}.$$

Where should the zero of the lead network be located? Draw the root locus of the compensated system, and give the value of $K_1$ that allows the specifications to bemet.

(d) For what range of $K_1$ is the system stable?

(e) What is the steady-state error (the difference between $Z$ and some reference input $Z_d$) to a constant disturbance torque $T_{ex}$ for the design of part(c)?

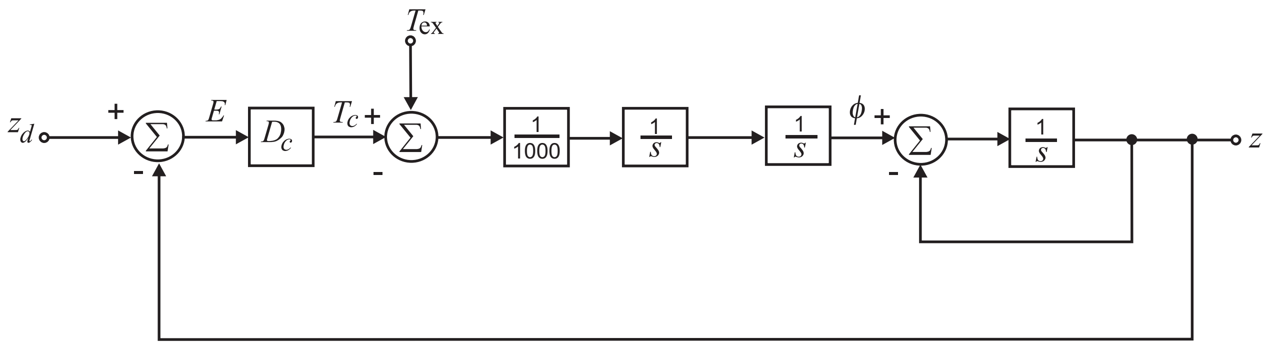
(f) What is the type of this system with respect to rejection of $T_{ex}$?

(g) Draw the Bode plot asymptotes of the *open-loop* system, with the gain adjusted for the value of $K_1$ computed in part(c). Add the compensation of part(c), and compute the phase margin of the closed-loop system.

(h) Write state equations for the open-loop system, using the state variables $\phi$, $\dot{\phi}$, and $Z$. Select the gains of a state-feedback controller $T_c = -K_\phi \phi - K_\phi \dot{\phi}$ to locate the closed-loop poles at $s = -0.02 \pm 0.02j\sqrt{3}$.

**Solution:**

The block diagram is shown below




Problem 10.13: Block diagram for satellite attitude control problem.

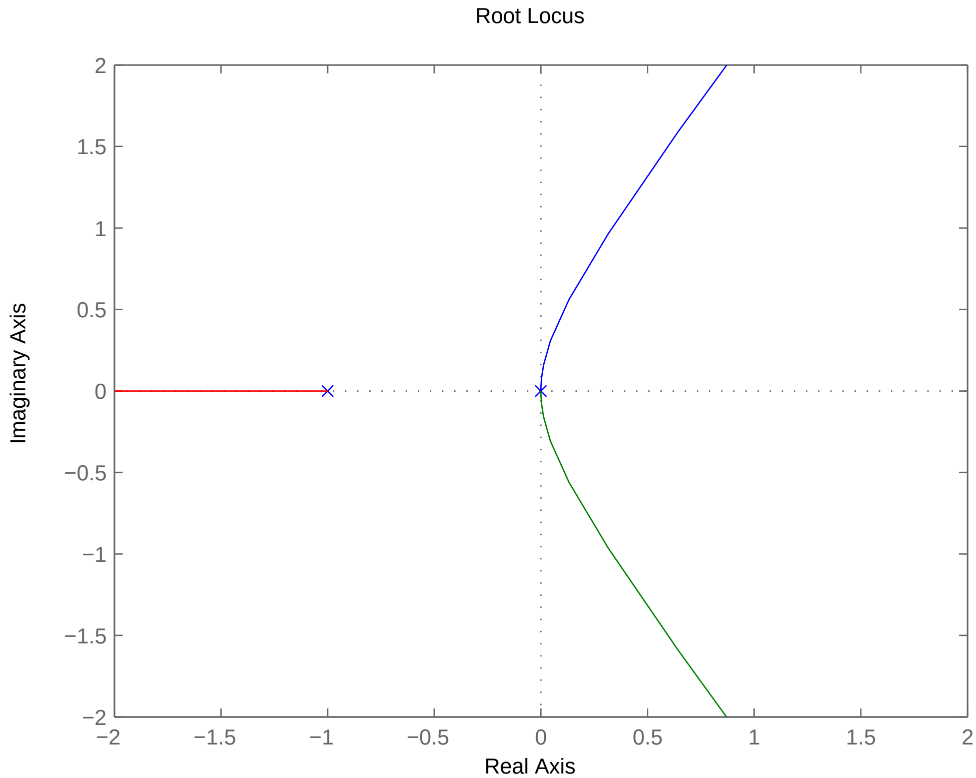
1. (a) With the transfer function from the measurement to the satellite's angle is,

$$\frac{\Phi}{Z} = \frac{D_c G}{1 + D_c GH}.$$

To form the root locus, we use,

$$D_c GH = \frac{K_0/I}{s^2(s+1)} = \frac{0.001 K_0}{s^2(s+1)}.$$

The root locus is shown on the next page.




Problem 10.13: Root locus for satellite problem.

(b) From the root locus, using MATLAB's rlocus command, the system is unstable for any value of $K_0$.

(c) With $\omega_n = 0.04$ and $\zeta = 0.5$, the closed-loop poles are at $s = -0.02 \pm 0.02\sqrt{3}j$. Using the phase angle criterion,

$$\begin{aligned}
\sum \phi_{z_i} - \sum \phi_{p_i} &= -180^\circ, \\
\phi_z - 120^\circ - 120^\circ - 2^\circ - 2^\circ &= -180^\circ, \\
&\Longrightarrow \phi_z = 64^\circ.
\end{aligned}$$

We can now calculate the location of the zero,

$$z = \frac{0.02\sqrt{3}}{\tan \phi_z} + 0.02 \Longrightarrow z = 0.0369.$$

So the compensator is,

$$D_c(s) = K_1 \frac{s + 0.0369}{s + 1}.$$

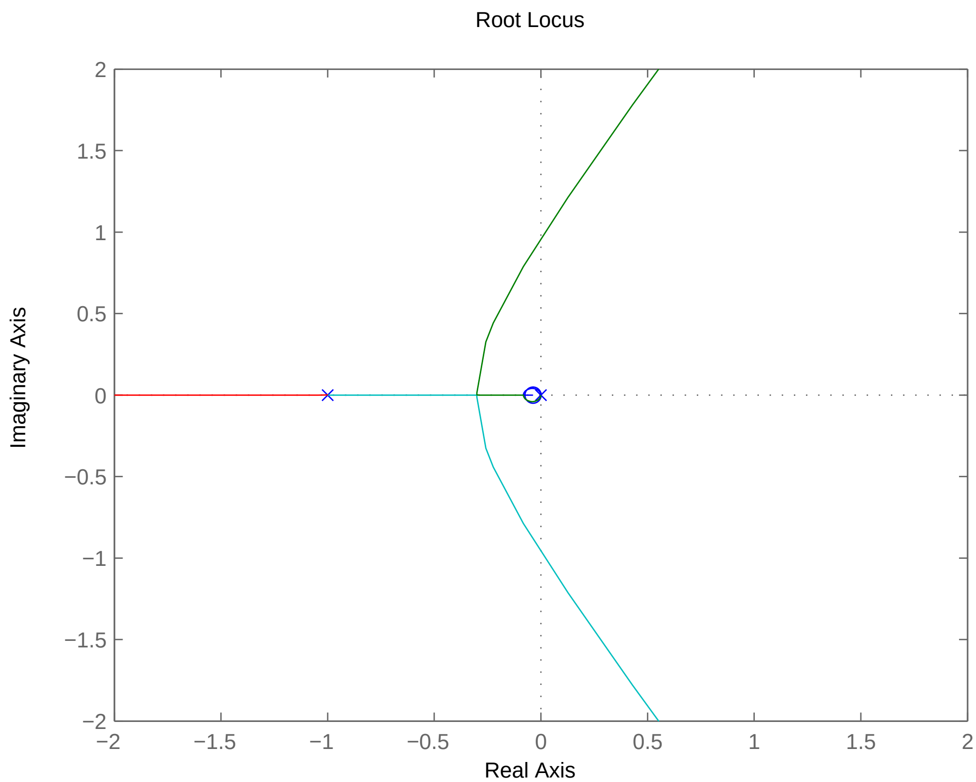
To find value of $K_1$ at $s = -0.02 \pm 0.02\sqrt{3}$, we set,

$$|D_c GH|_{s=-0.02+j0.02\sqrt{3}} = 1.$$

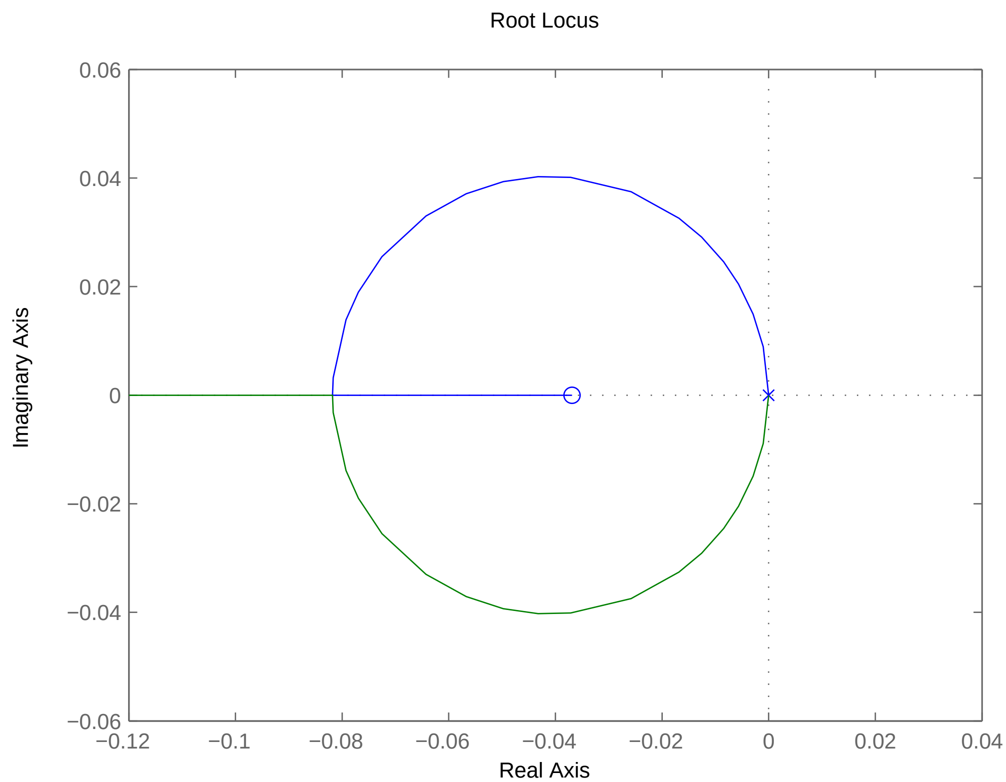
Solving for $K_1$ yields $K_1 = 39.92$. We plot the root locus of,

$$D_c GH = \frac{\frac{K_1}{I}(s + 0.0369)}{s^2(s+1)^2},$$

using MATLAB's rlocus command as shown on the next page.




Problem 10.13: Root locus for satellite problem with lead network.




Problem 10.13: Root locus for satellite problem with lead network: detailed view.

(d) To find the range of $K_1$ for which the system is stable, we use Routh's method on the numerator of $1 + D_cGH = 0$, i.e.,

$$s^4 + 2s^3 + s^2 + 0.001K_1 s + 3.69 \times 10^{-5} K_1 = 0.$$

This leads to the stable region $0 < K_1 < 1852.4$.

(e) We need to find the transfer function from $T_{ex}$ to $e$. In the Laplace domain (suppressing the $s$ for clarity),

$$\begin{aligned}
\Phi &= G(T_{ex} + D_c E), \\
E &= Z_d - Z = Z_d - H\Phi = Z_d - HGT_{ex} - HGD_cE, \\
&\Longrightarrow (1 + HGD_c)E = Z_d - HGT_{ex}, \\
&\Longrightarrow E = (1 + HGD_c)^{-1} Z_d - (1 + HGD_c)^{-1} HGT_{ex}.
\end{aligned}$$

Thus the steady-state error from a unit step input on $T_{ex}$ can be calculated using the Final

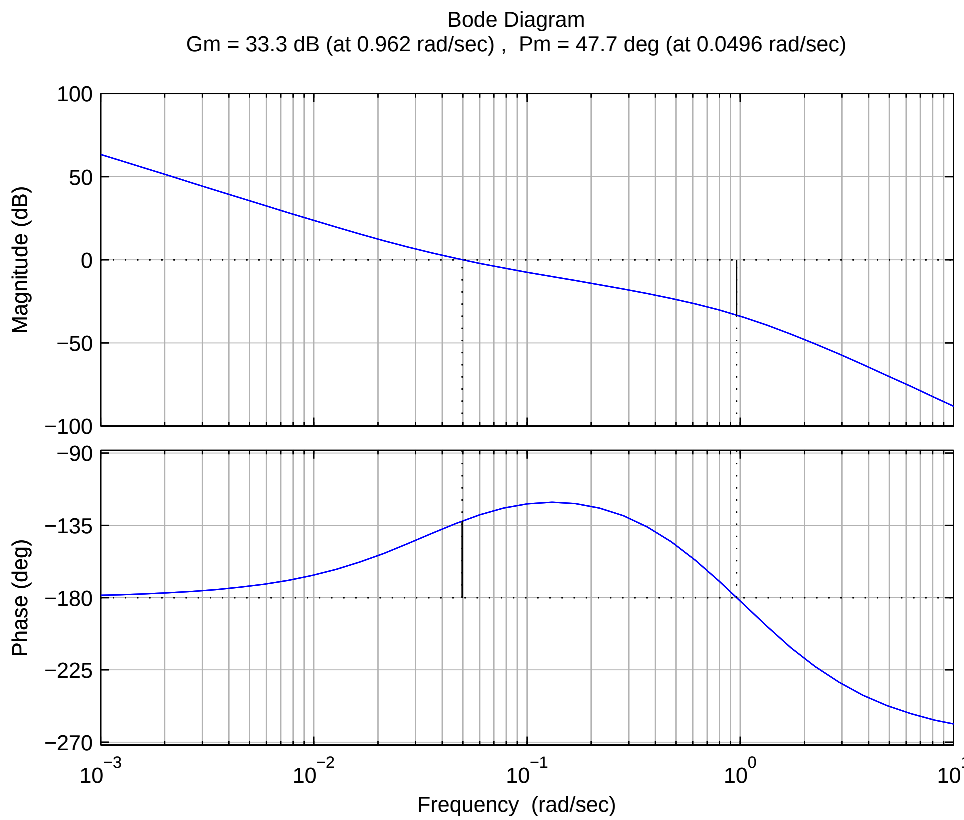
Value Theorem. With $Z_d$ and $T_{ex} = 1/s$, we find,

$$\begin{aligned} e_\infty &= \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} -\frac{sHGT_{ex}}{1+HGD_c} \\ &= \lim_{s\to 0} -\frac{HG}{1+HGD_c} = -\frac{1}{K_1 z} = -0.679. \end{aligned}$$

Because the system is linear, the steady-state error for any other size step input can be determined by simply scaling this result.

(f) Type 0.

(g) The Bode plot of $D_cGH$ is shown below using MATLAB's margin command. The phase margin is approximately 50° at $\omega_c = 0.05$ rad/sec and the gain margin is approximately 33 db at $\omega = 1$ rad/sec.




Problem 10.13: Bode plot for satellite problem.

(h) Taking $\mathbf{x} = [x_1\ x_2\ x_3]^T = [\phi\ \dot{\phi}\ z]^T$, $u = T_c$, and $w = T_{ex}$, we have,

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{B}_1 w, \\ y &= \mathbf{C}\mathbf{x}, \end{aligned}$$

where,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{1000} \\ 0 \end{bmatrix}, \ \mathbf{B}_1 = \begin{bmatrix} 0 \\ \frac{1}{1000} \\ 0 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Since state feedback will only use $\phi$ and $\dot{\phi}$, we have $K_z = 0$. Thus, we can only expect to place arbitrary at most two of the control poles,

$$\begin{aligned} \det(s\mathbf{I} - \mathbf{A} + \mathbf{BK}) &= \begin{bmatrix} s & -1 & 0 \\ K_\phi/1000 & s + K_{\dot{\phi}}/1000 & 0 \\ -1 & 0 & s+1 \end{bmatrix} \\ &= (s+1)(s^2 + K_{\dot{\phi}}/1000s + K_\phi/1000). \end{aligned}$$

So in order to get the desired closed-loop roots we need,

$$\alpha_c(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 0.04s + 0.0016.$$

Equating coefficients gives $K_\phi = 1.6$, and $K_{\dot{\phi}} = 40$. We can also use MATLAB's `place` command.

14. Three alternative designs are sketched in Fig.10.100 for the closed-loop control of a system with the plant transfer function $G(s) = 1/s(s+1)$. The signal $w$ is the plant noise and may be analyzed as if it were a step; the signal $v$ is the sensor noise and may be analyzed as if it contained power to very high frequencies.

1. (a) Compute values for the parameters $K_1$, $a$, $K_2$, $K_T$, $K_3$, $d$, and $K_D$ so that in each case (assuming $w = 0$ and$v = 0$),
$$\frac{Y}{R} = \frac{16}{s^2 + 4s + 16}.$$
Note that in system III, a pole is to be placed at$s = -4$.

(b) Complete the following table, expressing the last entries as $A/s^k$ to show how fast noise from $v$ is attenuated at high frequencies:

| System | $K_v$ | $\frac{y}{w}\big\vert_{s=0}$ | $\frac{y}{v}\big\vert_{s\to\infty}$ |
|---|---|---|---|
| I | | | |
| II | | | |
| III | | | |

(c) Rank the three designs according to the following characteristics (the best as "1," the poorest as"3"):

| | I | II | III |
|---|---|---|---|
| Tracking | | | |
| Plant-noise rejection | | | |
| Sensor-noise rejection | | | |

**Solution:**

(a)

$$\begin{aligned} I \quad &: \quad \frac{Y}{R} = \frac{K_1}{s^2 + as + K_1}, \\ &\Longrightarrow K_1 = 16, \ a = 4. \end{aligned}$$

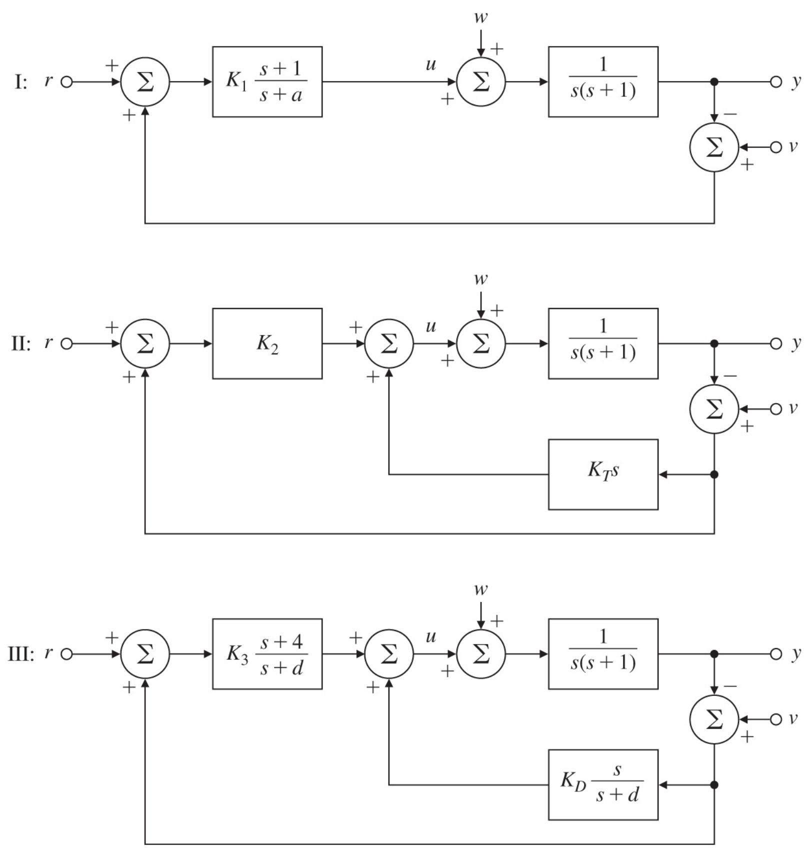



Figure 10.100: Alternative feedback structures for Problem 10.14

$$
\begin{aligned}
II \quad &: \quad \frac{Y}{R} = \frac{K_2}{s^2 + (1 + K_T)s + K_2}, \\
&\Longrightarrow \; K_2 = 16, \; K_T = 3.
\end{aligned}
$$

$$
\begin{aligned}
III \quad &: \quad \frac{Y}{R} = \frac{K_3(s+4)}{s^3 + (1+d)s^2 + (K_D + d + K_3)s + 4K_3}, \\
\frac{Y}{R} \quad &= \quad \frac{K_3(s+4)}{(s+4)(s^2 + \frac{d+K_D}{4}s + K_3)}, \; \text{ and } \; K_D = 3(d-4) \\
&\Longrightarrow \; K_3 = 16, d = 7, K_D = 9.
\end{aligned}
$$

(b) $K_v$:

$$
\begin{aligned}
E(s) \quad &= \quad R - Y = R - \frac{16}{s^2 + 4s + 16}R = \frac{s^2 + 4s}{s^2 + 4s + 16}R, \; R(s) = \frac{1}{s^2}, \\
e_\infty \quad &= \quad \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \frac{1}{4}, \\
K_v \quad &= \quad \frac{1}{e_\infty} \Longrightarrow K_v = 4 \; \text{ for all the designs.}
\end{aligned}
$$

$\frac{Y}{W}|_{s=0}$:

$$
\begin{aligned}
I \quad &: \quad \frac{Y}{W}|_{s=0} = \frac{a}{K_1} = \frac{1}{4}. \\
II \quad &: \quad \frac{Y}{W}|_{s=0} = \frac{1}{K_2} = \frac{1}{16}. \\
III \quad &: \quad \frac{Y}{W}|_{s=0} = \frac{d}{4K_3} = \frac{7}{64}.
\end{aligned}
$$

$\frac{Y}{V}|_{s\to\infty}$ :

$$
\begin{aligned}
I \quad &: \quad \frac{Y}{V}|_{s\to\infty} = \frac{K_1}{s(s+a) + K_1}|_{s\to\infty} \simeq \frac{K_1}{s^2} = \frac{16}{s^2}. \\
II \quad &: \quad \frac{Y}{V}|_{s\to\infty} = \frac{K_2 + K_T s}{s^2 + (1+K_T)s + K_2}|_{s\to\infty} \simeq \frac{K_T}{s} = \frac{3}{s}. \\
III \quad &: \quad \frac{Y}{V}|_{s\to\infty} = \frac{K_3(s+4) + K_D s}{s(s+d)(s+1) + K_3(s+4) + K_D s}|_{s\to\infty} = \frac{K_3 + K_D}{s^2} = \frac{25}{s^2}.
\end{aligned}
$$

Filling the table, and ranking the three designs:

| **System** | $K_v$ | $\frac{Y}{W}\|_{s=0}$ | $\frac{Y}{V}\|_{s\to\infty}$ | **tracking** | **Plant noise rejection** | **Sensor noise rejection** |
|---|---|---|---|---|---|---|
| I | 4 | 1/4 | $16/s^2$ | Same | 3 | 1 |
| II | 4 | 1/16 | $3/s$ | Same | 1 | 3 |
| III | 4 | 7/65 | $25/s^2$ | Same | 2 | 2 |

15. The equations of motion for a cart-stick balancer with state variables of stick angle, stick angular velocity, and cart velocityare

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 31.33 & 0 & 0.016 \\ -31.33 & 0 & -0.216 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ -0.649 \\ 8.649 \end{bmatrix} u,$$
$$y = [\ 10 \quad 0 \quad 0\ ]\mathbf{x},$$

where the output is stick angle, and the control input is voltage on the motor that drives the cart wheels.

1. (a) Compute the transfer function from $u$ to $y$, and determine the poles and zeros.
   (b) Determine the feedback gain $\mathbf{K}$ necessary to move the poles of the system to the locations $-2.832$ and $-0.521 \pm 1.068j$, with $\omega_n = 4$rad/sec.
   (c) Determine the estimator gain $\mathbf{L}$ needed to place the three estimator poles at $-10$.
   (d) Determine the transfer function of the estimated-state-feedback compensator defined by the gains computed in parts(b) and(c).
   (e) Suppose we use a reduced-order estimator with poles at $-10$, and $-10$. What is the required estimator gain?
   (f) Repeat part(d) using the reduced-order estimator.
   (g) Compute the frequency response of the two compensators.

   **Solution:**

   (a) The transfer function (using MATLAB's tf) is,

$$G(s) = \frac{-0.649(s+0.0028)}{(s-5.59)(s+5.606)(s+0.2)}.$$

   (b) With $\alpha_c = (s+2.832)(s+2.084 \pm 4.272j)$, the feedback gains are calculated using the Ackermann's formula or equating $\alpha_c$ with $\det(s\mathbf{I} - \mathbf{A} + \mathbf{BK})$. The result using MATLAB's place command is,

$$\mathbf{K} = [-101.2 \quad 14.18 \quad -0.2796].$$

   (c) The estimator gains with $\alpha_e(s) = (s+10)^3$ are calculated using the Ackermann's formula or equating $\alpha_e$ with $\det(s\mathbf{I} - \mathbf{A} + \mathbf{LC})$. The result using MATLAB's acker command is,

$$\mathbf{L} = [2.98 \quad 32.5 \quad 5850.6]^T.$$

   (d) The compensator transfer function can be obtained from (using MATLAB's ss2tf),

$$D_c(s) = -\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{BK} + \mathbf{LC})^{-1}\mathbf{L} = \frac{0.2398(s+5.60)(s-3.06)}{(s+23.4 \pm j22.1)(s-9.98)}.$$

   Notice that the compensator is *unstable*.

   (e) For reducing order estimator

$$\alpha_e(s) = (s+10)^2 \tag{1}$$

and matching coefficients of $det(s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{L}\mathbf{A}_{ab}) = 0$ where,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 31.33 & 0 & 0.016 \\ -31.33 & 0 & -0.216 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ -0.649 \\ 8.649 \end{bmatrix},$$

and coefficients of $\alpha_e(s) = (s+10)^2$ will yield (or using MATLAB's `acker` command),

$$\mathbf{L} = [19.8 \quad 5983]^T.$$

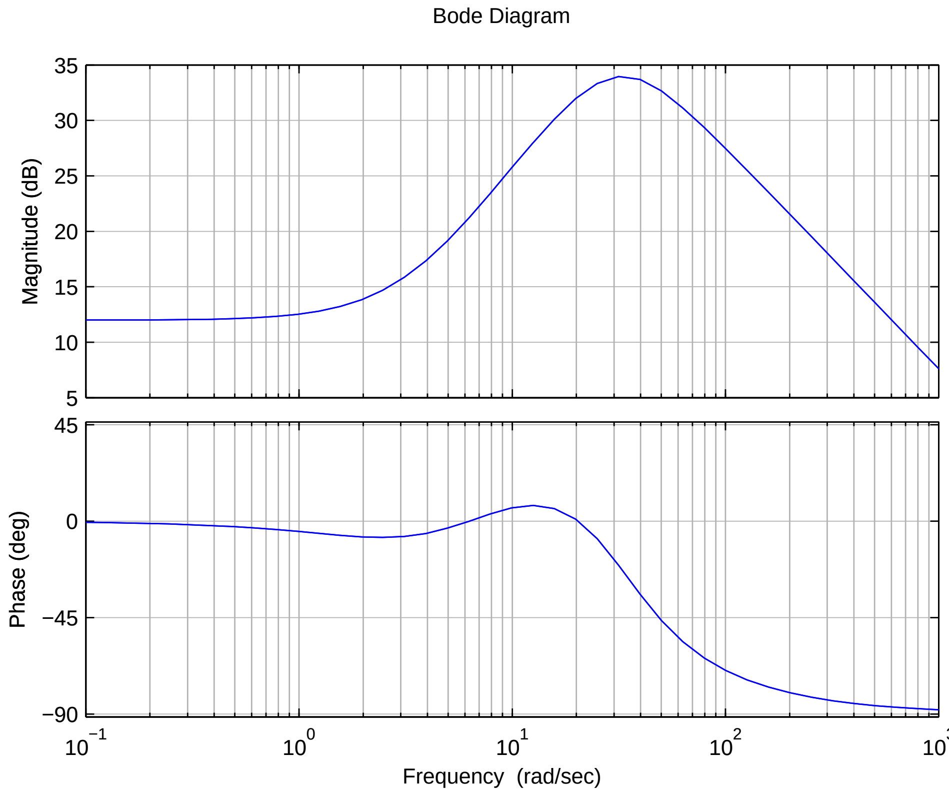
(f) The reduced order compensator is,

$$\begin{aligned}
\mathbf{A}_r &= \mathbf{A}_{bb} - \mathbf{L}\mathbf{A}_{ab} - (\mathbf{A}_b - \mathbf{L}\mathbf{B}_a)\mathbf{K}_b, \\
\mathbf{B}_r &= \mathbf{A}_r\mathbf{L} + \mathbf{A}_{ba} - \mathbf{L}\mathbf{A}_{aa} - (\mathbf{B}_b - \mathbf{L}\mathbf{B}_a)\mathbf{K}_a, \\
\mathbf{C}_r &= -\mathbf{K}_b, \\
D_r &= -\mathbf{K}_a - \mathbf{K}_b\mathbf{L}, \\
D_{cr}(s) &= \mathbf{C}_r(s\mathbf{I} - \mathbf{A}_r)^{-1}\mathbf{B}_r + D_r.
\end{aligned}$$

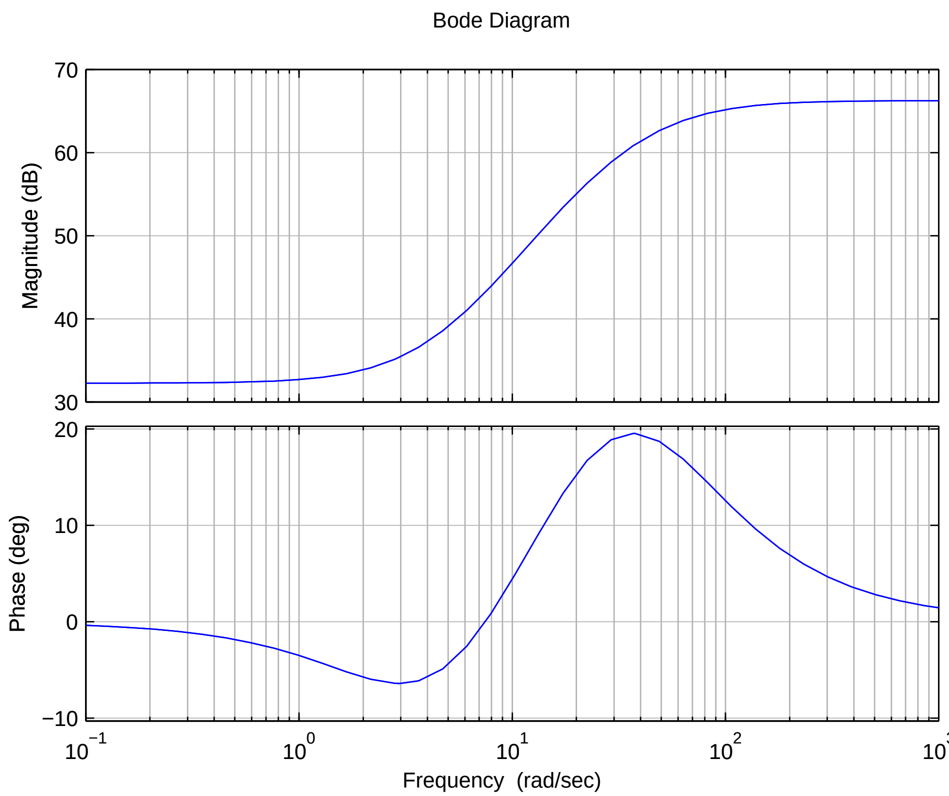
These calculations yield (using MATLAB's `ss2tf`),

$$D_{cr}(s) = \frac{2055(s+5.58)(s-3.69)}{(s+48.2)(s-21.4)}.$$

(g) The frequency responses of the two compensators follow. .




Frequency response of the compensator.




Frequency response of the reduced order compensator.

16. A 282-ton Boeing747 is on landing approach at sea level. If we use the state given in the case study (Section10.3) and assume a velocity of 221ft/sec (Mach 0.198), then the lateral-direction perturbation equations are,

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.0890 & -0.989 & 0.1478 & 0.1441 \\ 0.168 & -0.217 & -0.166 & 0 \\ -1.33 & 0.327 & -0.975 & 0 \\ 0 & 0.149 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0.0148 \\ -0.151 \\ 0.0636 \\ 0 \end{bmatrix} \delta r,$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix}.$$

The corresponding transfer function is (using MATLAB's ss2tf),

$$G(s) = \frac{r(s)}{\delta r(s)} = \frac{-0.151(s+1.05)(s+0.0328 \pm 0.414j)}{(s+1.109)(s+0.0425)(s+0.0646 \pm 0.731j)}.$$

(a) Draw the uncompensated root locus [for $1 + KG(s)$] and the frequency response of the system. What type of classical controller could be used for this system?

(b) Try a state-variable design approach by drawing a symmetric root locus for the system. Choose the closed-loop poles of the system on the SRL to be

$$\alpha_c(s) = (s + 1.12)(s + 0.165)(s + 0.162 \pm 0.681j),$$

and choose the estimator poles to be five times faster at

$$\alpha_e(s) = (s + 5.58)(s + 0.825)(s + 0.812 \pm 3.40j).$$

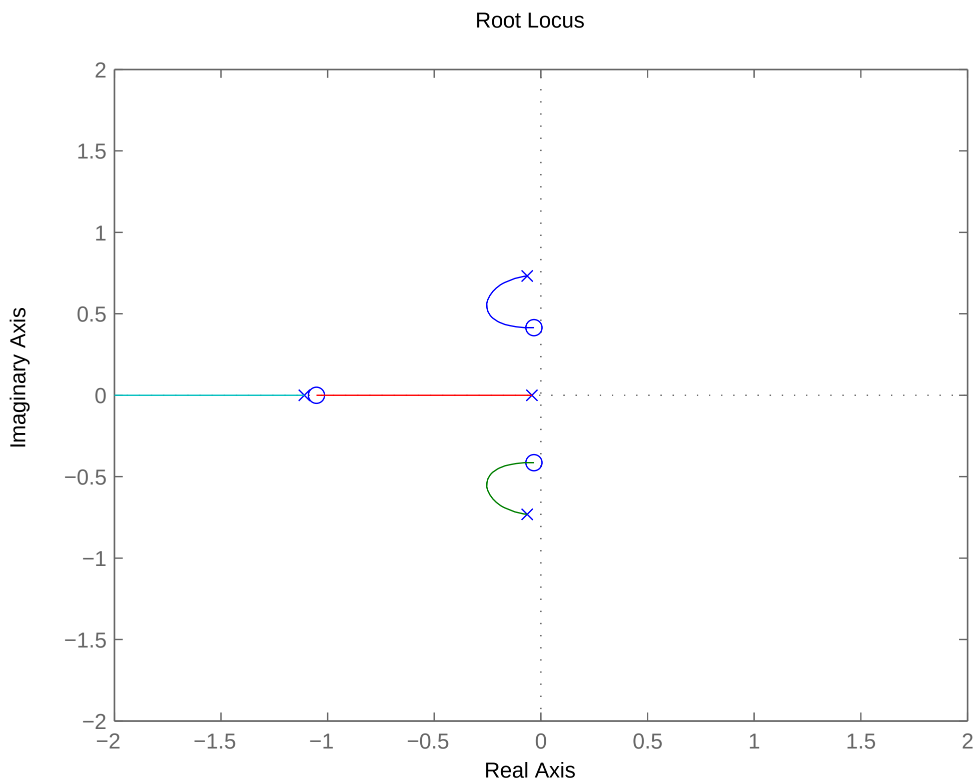
(c) Compute the transfer function of the SRL compensator.

(d) Discuss the robustness properties of the system with respect to parameter variations and unmodeled dynamics.

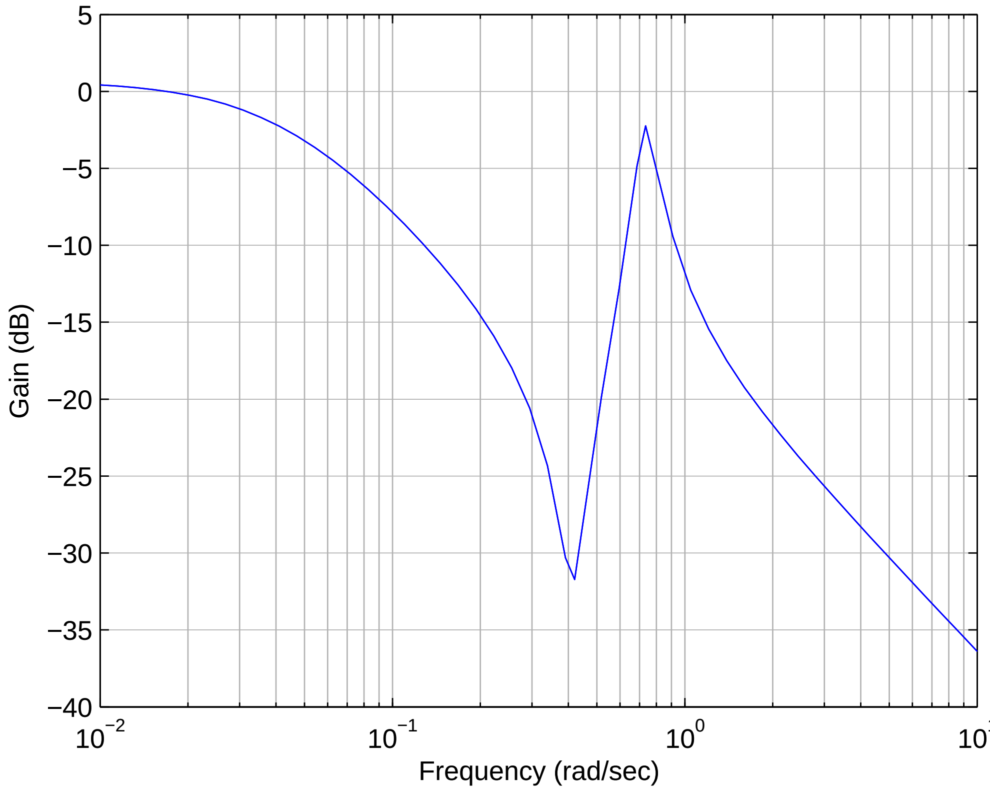
(e) Note the similarity of this design to the one developed for different flight conditions earlier in the chapter. What does this suggest about providing a continuous (nonlinear) control throughout the operating envelope?

**Solution:**

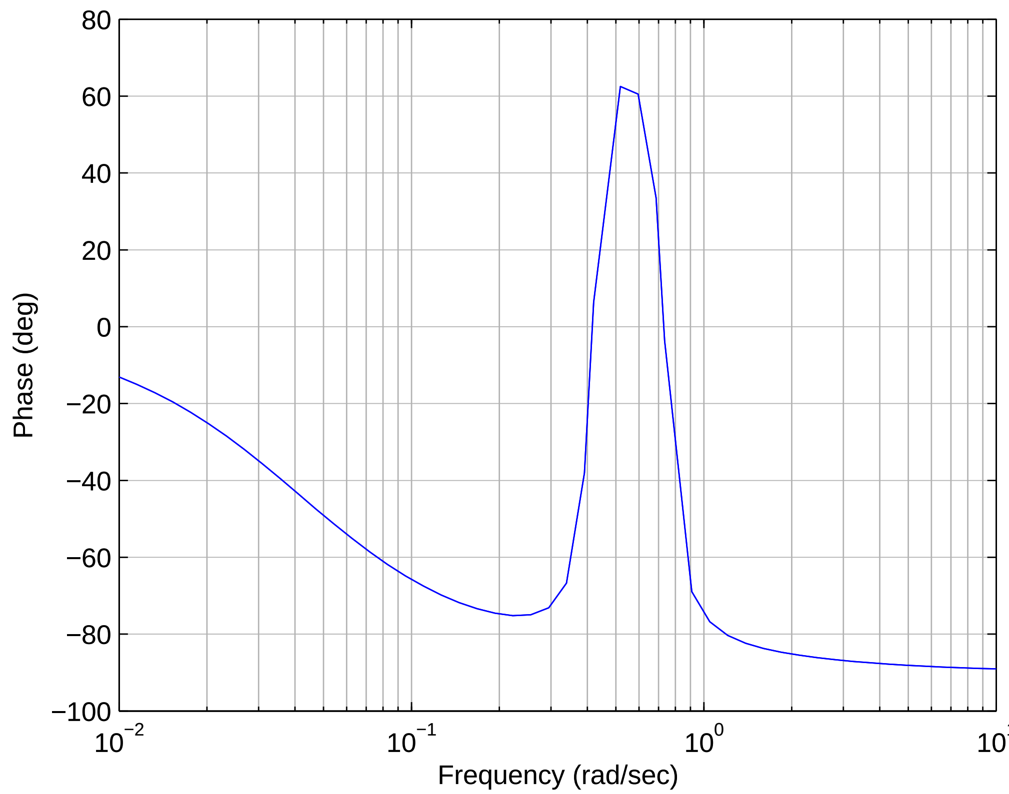
(a) The root locus (using MATLAB's rlocus command) and Bode plots (using MATLAB's bode command) are shown on the next two pages. From the figures, we see that a classical lag network could be used to lower the resonant gain.




Problem 10.16: Root locus for Boeing 747 problem.

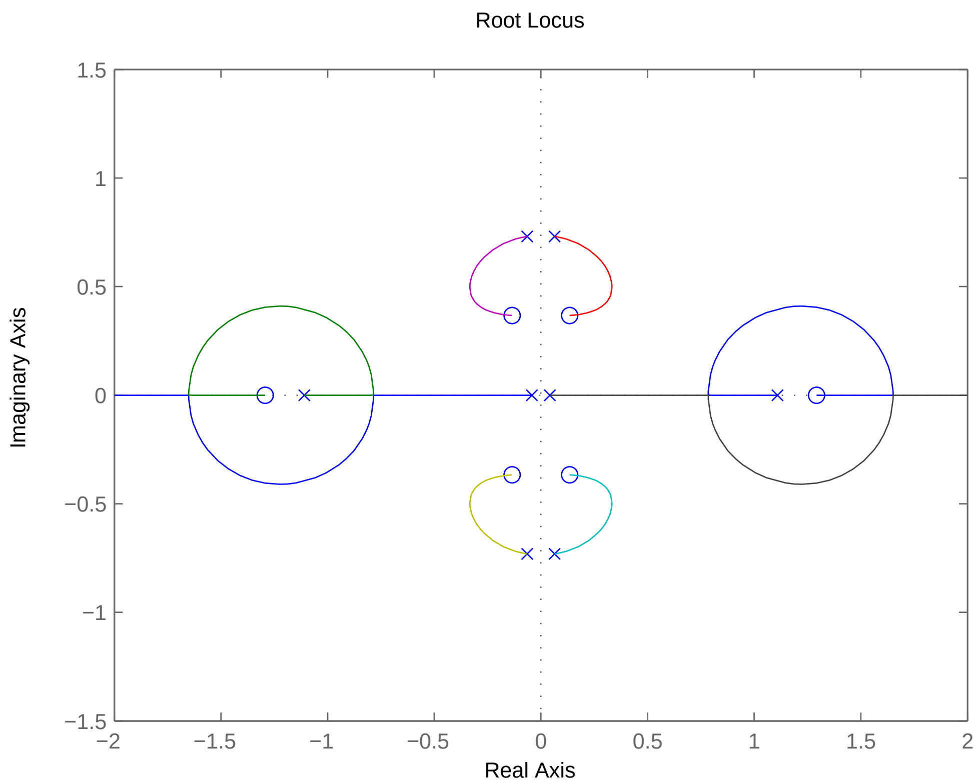



Problem 10.16: Bode magnitude plot for Boeing 747 problem.




Problem 10.16: Bode phase plot for Boeing 747 problem.

(b) The symmetric root locus $1 + kG(s)G(-s) = 0$ is shown below using MATLAB's rlocus command.




Problem 10.16: Symmetric root locus for Boeing 747 problem.

With the closed-loop poles of the system on the symmetric root locus at $\alpha_c(s) = (s + 1.12)(s + 0.165)(s + 0.162 \pm j0.681)$, the controller feedback gains are (using Ackermann's formula or matching the coefficients or using MATLAB's `place` command),

$$\mathbf{K} = [0.0308 \quad -2.122 \quad 0.112 \quad -0.034].$$

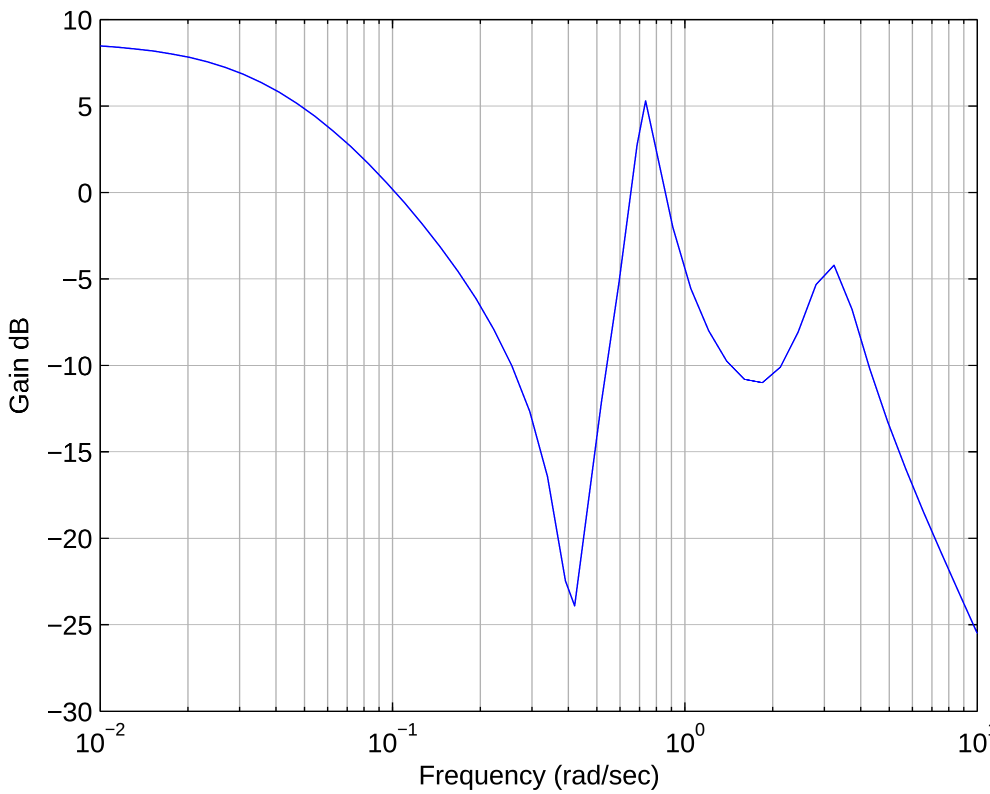
Similarly, the estimator gains with the estimator poles at $\alpha_e(s) = (s + 5.58)(s + 0.825)(s + 0.812 \pm j3.4)$ are found using Ackermann's formula or matching the coefficients or using MATLAB's `place` command. The estimator gains are,

$$\mathbf{L} = [154 \quad 6.75 \quad 39.53 \quad 973.98]^T.$$

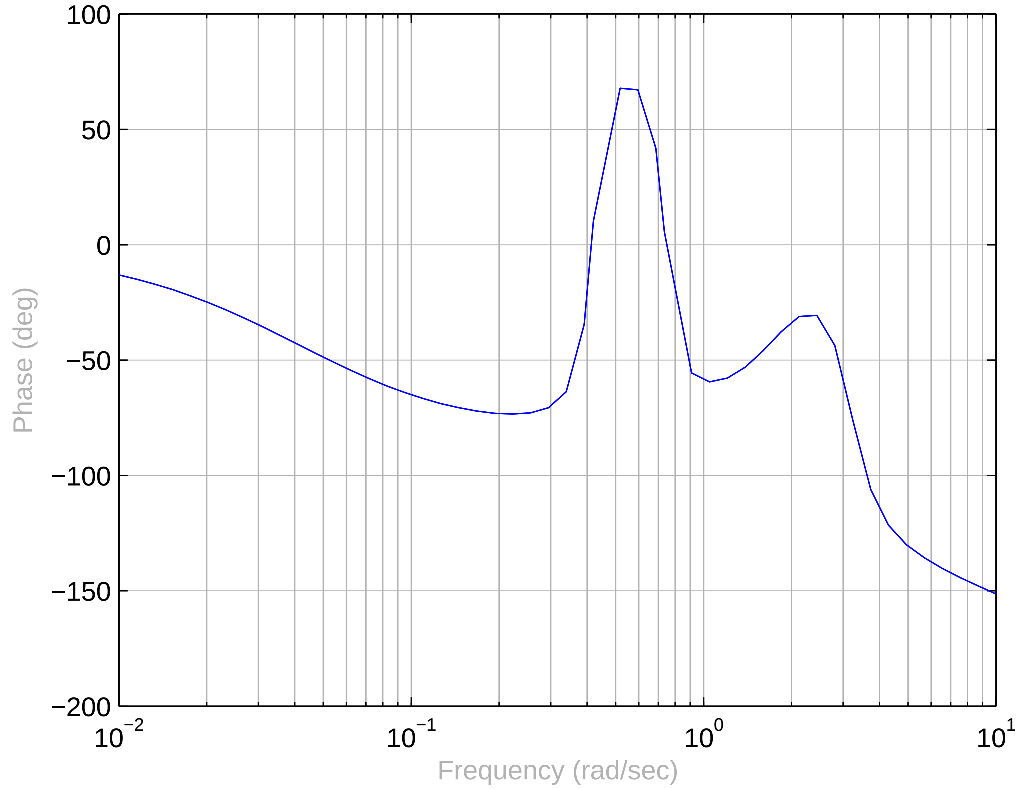
(c) The compensator transfer function is given by,

$$\begin{aligned} D_c(s) &= \frac{-38.25s^3 - 111.5s^2 - 215.1s - 136}{s^4 + 8.36s^3 + 24.02s^2 + 78.17s + 53.80} \\ &= \frac{-38.247(s + 0.94479)(s + 0.9851 \pm j1.6713)}{(s + 6.2987)(s + 0.85187)(s + 0.60319 \pm j3.1086)}. \end{aligned}$$

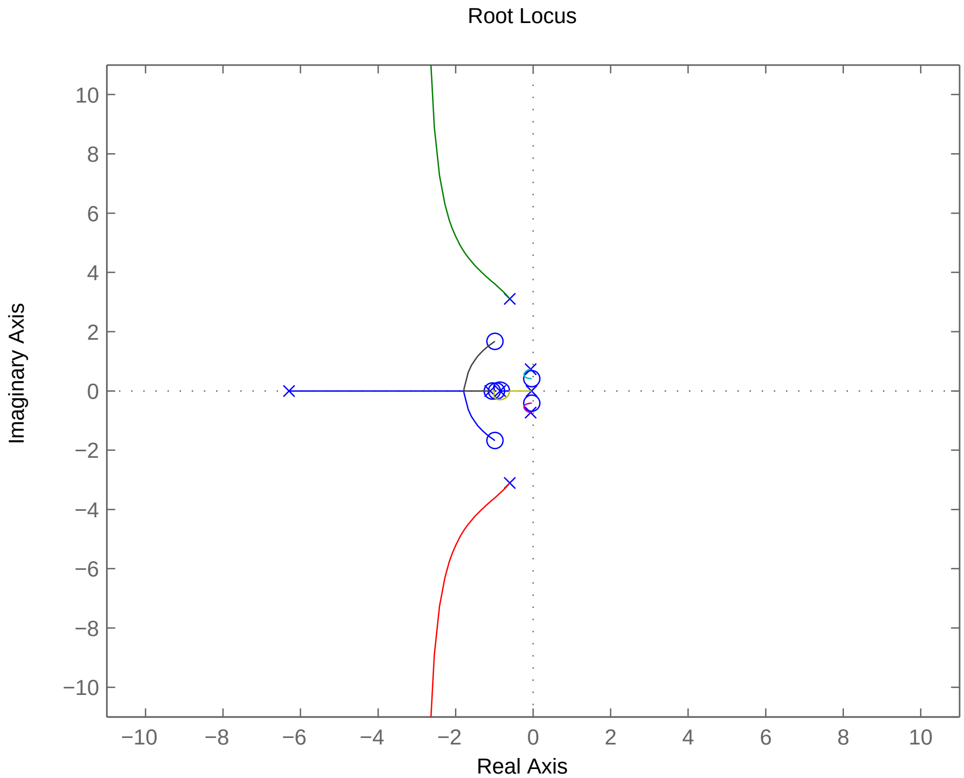
(d) The compensated Bode plot is shown below using MATLAB's `bode` command. Because the phase is always less than $-180°$, we would expect the system to be very robust with respect to gain changes. The phase margin also indicates good robustness.



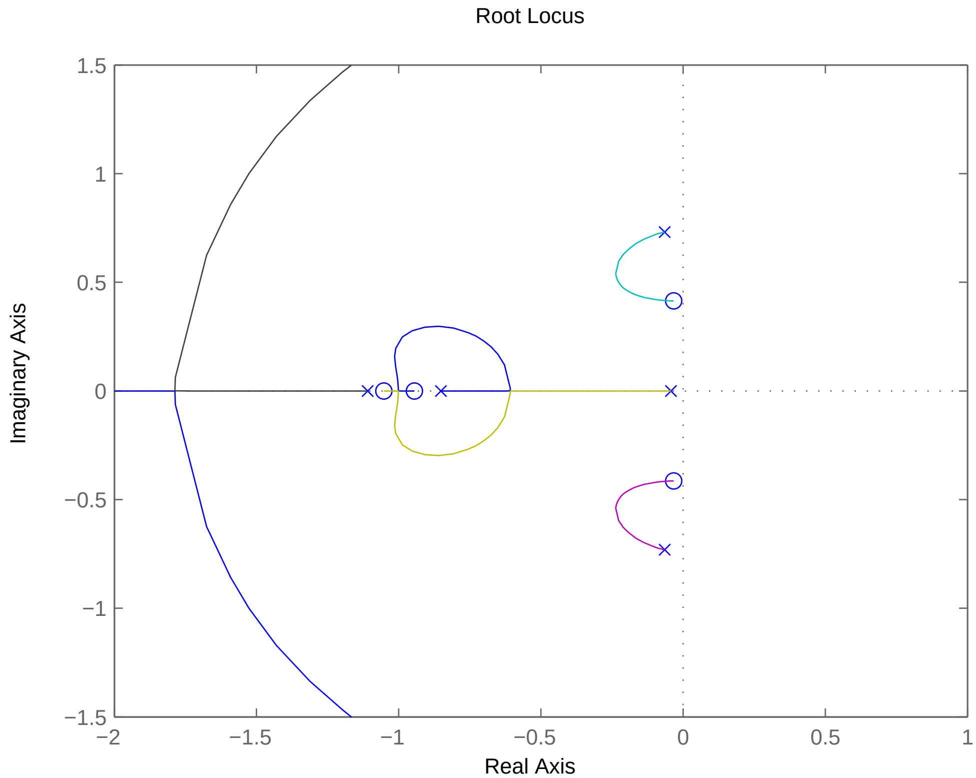

Problem 10.16: Bode magnitude for compensated system.




Problem 10.16: Bode phase plot for compensated system.




Problem 10.16: Root locus of compensated system for Boeing 747.




Problem 10.16 Root locus of compensated system for Boeing 747: Detailed view.

(e) Because a similar compensator stabilizes both cases, we would expect one compensator to be satisfactory over a wide range of flight conditions.

17. (Contributed by Prof. L.Swindlehurst) The feedback control system shown in Fig.10.101 is proposed as a position control system. A key component of this system is an armature-controlled DC motor. The input potentiometer produces a voltage $E_i$ that is proportional to the desired shaft position: $E_i = K_p\theta_i$. Similarly, the output potentiometer produces a voltage $E_0$ that is proportional to the actual shaft position: $E_0 = K_p\theta_0$. Note that we have assumed that both potentiometers have the same proportionality constant. The error signal $E_i - E_0$ drives a compensator, which in turn produces an armature voltage that drives the motor. The motor has an armature resistance $R_a$, an armature inductance $L_a$, a torque constant $K_t$, and a back-emf constant $K_e$. The moment of inertia of the motor shaft is $J_m$, and the rotational damping due to bearing friction is $B_m$. Finally, the gear ratio is $N : 1$, the moment of inertia of the load is $J_L$, and the load damping is $B_L$.

1. (a) Write the differential equations that describe the operation of this feedback system.

   (b) Find the transfer function relating $\theta_0(s)$ and $\theta_i(s)$ for a general compensator $D_c(s)$.

   (c) The open-loop frequency-response data shown in Table10.2 were taken using the armature voltage $v_a$ of the motor as an input and the output potentiometer voltage $E_0$ as the output.

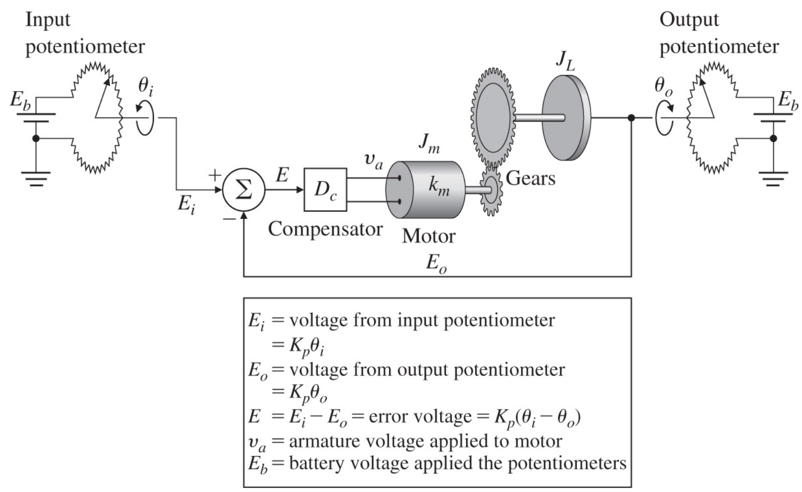



Figure 10.101: A servomechanism with gears on the motor shaft and potentiometer sensors

Assuming that the motor is linear and minimum-phase, make an estimate of the transfer function of the motor,

$$G(s) = \frac{\theta_m(s)}{V_a(s)},$$

where $\theta_m$ is the angular position of the motor shaft.

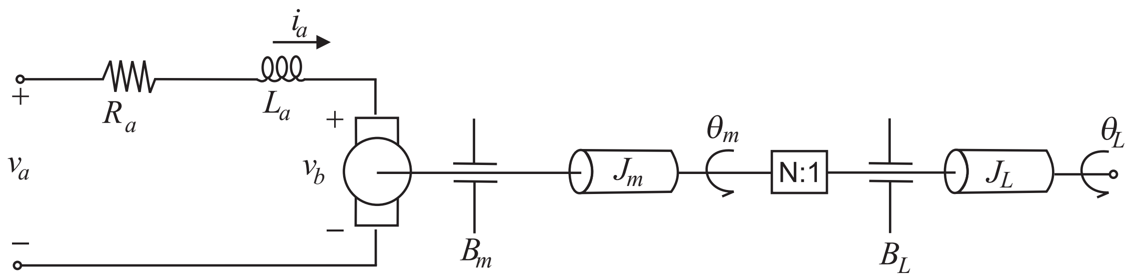
(d) Determine a set of performance specifications that are appropriate for a position control system and will yield good performance. Design $D_c(s)$ to meet these specifications.

(e) Verify your design through analysis and simulation using MATLAB.

**Solution:**

(a) First of all, we describe the motor dynamics in more detail. This is illustrated below.

| *Frequency (rad/sec)* | $\left\lvert\frac{E_0(s)}{V_a(s)}\right\rvert(db)$ | *Frequency (rad/sec)* | $\left\lvert\frac{E_0(s)}{V_a(s)}\right\rvert(db)$ |
|---|---|---|---|
| 0.1 | 60.0 | 10.0 | 14.0 |
| 0.2 | 54.0 | 20.0 | 2.0 |
| 0.3 | 50.0 | 40.0 | −10.0 |
| 0.5 | 46.0 | 60.0 | −20.0 |
| 0.8 | 42.0 | 65.0 | −21.0 |
| 1.0 | 40.0 | 80.0 | −24.0 |
| 2.0 | 34.0 | 100.0 | −30.0 |
| 3.0 | 30.5 | 200.0 | −48.0 |
| 4.0 | 27.0 | 300.0 | −59.0 |
| 5.0 | 23.0 | 500.0 | −72.0 |
| 7.0 | 19.5 | | |




Problem 10.17: DC motor.

The figure defines a few additional variables not mentioned in the problem statement: $i_a$ is the armature current, $v_b$ is the back emf, and $\theta_m$ is the angular position of the motor. Using Kirchhoff's voltage laws we can write,

$$v_a - v_b = R_a i_a + L_a \frac{di_a}{dt}. \qquad (1)$$

The torque of the motor, $T$ is proportional to the armature current. Thus,

$$T = K_t i_a. \qquad (2)$$

The back emf, $v_b$, is proportional to the angular speed. Hence,

$$v_b = K_e \frac{d\theta_m}{dt}. \qquad (3)$$

At the point of contact of the gears, we assign an equal and oppositely directed force $F$. (Since we do not know this force, we will eliminate it momentarily). Using Newton's law of motion, we have

$$\begin{aligned} J_m \ddot{\theta}_m + B_m \dot{\theta}_m &= T - F r_m, && (4) \\ J_L \ddot{\theta}_o + B_L \dot{\theta}_o &= -F r_o, && (5) \end{aligned}$$

where $r_m$ is the radius of the gear connected to the motor shaft and $r_o$ is the radius of the gear connected to the output shaft. The minus sign on both terms arises because of the defined directions for $\theta_m$ and $\theta_o$. And from the gear ratio information, we have

$$r_o = N r_m \Longrightarrow \theta_m = -N\theta_o \qquad (6)$$

(b) First, we will find the transfer function from $v_a$ to $\theta_o$ (the plant) and then we will find the closed loop transfer function. From the gear ratio information, we can combine Eq. (4) and (5) to eliminate $\theta_m$.

$$\begin{aligned} -NT &= -NK_t i_a = (N^2 J_m + J_L)\ddot{\theta}_o + (N^2 B_m + B_L)\dot{\theta}_o, && (7) \\ \ddot{\theta}_o &= -\underbrace{\frac{(N^2 B_m + B_L)}{(N^2 J_m + J_L)}}_{=\alpha}\dot{\theta}_o - \underbrace{\frac{N K_t}{(N^2 J_m + J_L)}}_{=\beta} i_a. && (8) \end{aligned}$$

Next, we combine Eq. (1) and (3) to get,

$$\frac{di_a}{dt} = -\frac{R_a}{L}i_a + \frac{NK_e}{L}\dot{\theta}_o + \frac{1}{L}v_a. \qquad (9)$$

Thus, if we define that state $\mathbf{x} = [\theta_o\ \dot{\theta}_o\ i_a]^T$, we have,

$$\begin{bmatrix} \dot{\theta}_o \\ \ddot{\theta}_o \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\alpha & -\beta \\ 0 & \frac{NK_e}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \theta_o \\ \dot{\theta}_o \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix} v_a.$$

The output equation is $y = [1\ 0\ 0]\mathbf{x}$. This state space realization can then be converted to transfer function form. We find,

$$G(s) = \frac{\Theta(s)}{V_a(s)} = \frac{-\beta/L_a}{s\left[s^2 + (\alpha + R_a/L_a)s + (\frac{R_a\alpha}{L_a} + \frac{\beta NK_e}{L_a})\right]}.$$

And so the closed-loop transfer function is,

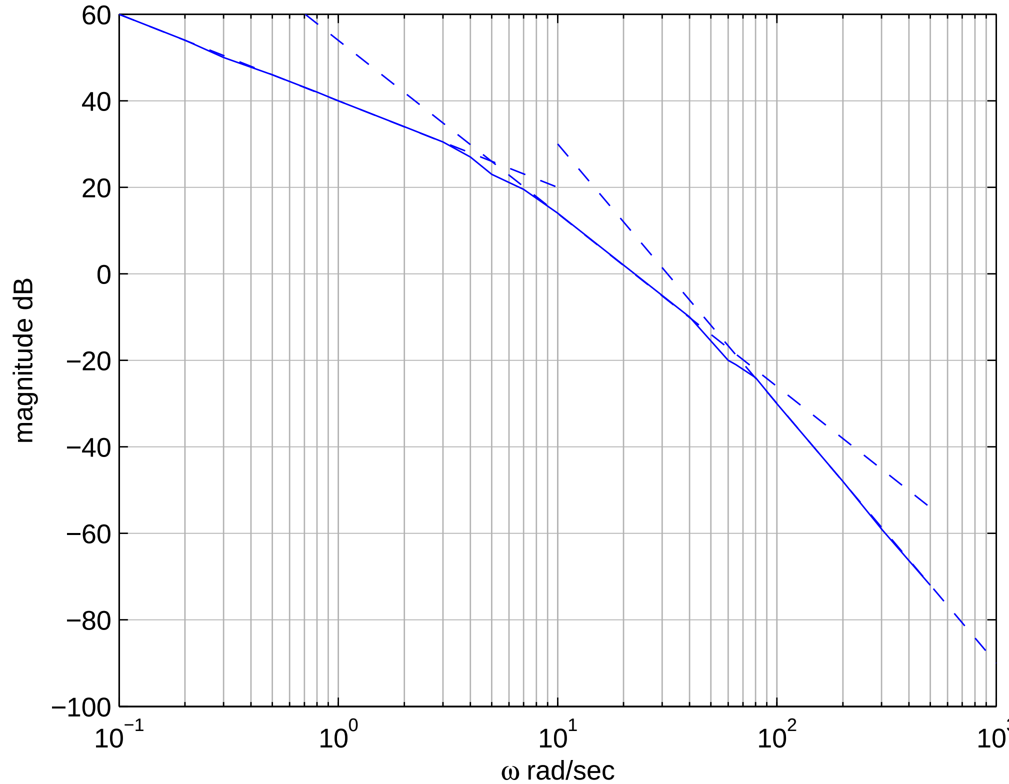
$$\frac{\Theta_o(s)}{\Theta_i(s)} = \frac{K_pG(s)D_c(s)}{1 + K_pG(s)D_c(s)}.$$

(c) The figure on the next page shows three straight lines fit through the frequency response data. From this information, we can estimate the transfer function of the plant, $G(s)$. From the figure, the poles appear to be located at $\omega = 5$ rad/sec and $\omega = 70$ rad/sec. Keeping the sign convention from part (b), we have,

$$G(s) = \frac{-\beta/L_a}{s(s+5)(s+70)}.$$

The gain, $\beta/L_a$, is determined by picking a particular value of $\omega$, say $\omega = 1$, comparing the calculated transfer function with the frequency response data. We find,

$$G(s) = \frac{-35700}{s(s+5)(s+70)}.$$



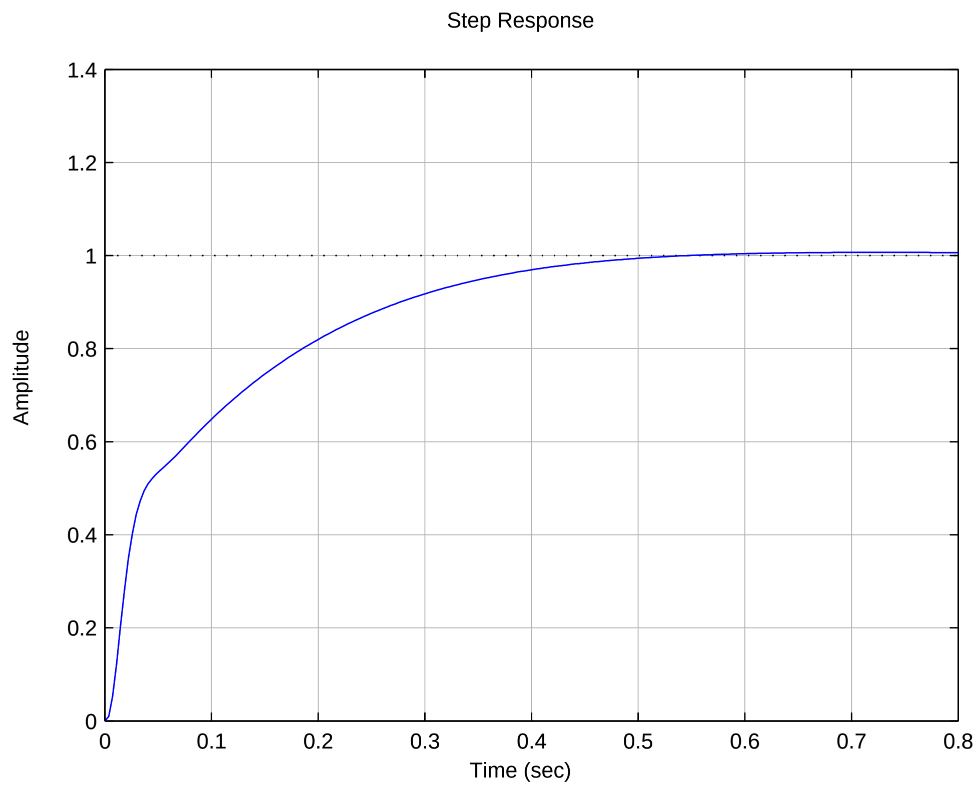

Problem 10.17: Straight line fits to frequency response.

(d) For a positioning system, we would like to keep the overshoot small, less than 1% (say). And we also like a reasonably fast rise time. For this plant, let's try to obtain $\omega_n = 6$ rad/sec (for the dominant roots). Which translates, using the rule of thumb for a dominant second order system, to $t_r = 1.8/6 = 0.3$ sec.

(e) Both of the time domain specifications are met using a double lead compensator,

$$G(s) = 50\frac{(s+9)^2}{(s+200)^2}.$$

The corresponding step response is shown on the next page using MATLAB's `step` command. It has an overshoot of 0.68% and a rise time of 0.2681 sec.




Problem 10.17: Step response of position control system.

18. Design and construct a device to keep a ball centered on a freely swinging beam. An example of such a device is shown in Fig.10.102. It uses coils surrounding permanent magnets as the actuator to move the beam, solar cells to sense the ball position, and a hall-effect device to sense the beam position. Research other possible actuators and sensors as part of your design effort. Compare the quality of the control achievable for ball-position-feedback only with that of multiple-loop feedback of both ball and beam position.

**Solution:**

See Text Figure 10.102.

19. Design and construct the magnetic levitation device shown in Figure 9.1. You may wish to use LEGO components in your design.

1. **Solution:** see K. A. Lilienkamp and K. Lundberg,"Low-cost magnetic levitation project kits for teaching feedback system design," *Proceeding of the American Control Conference*, pp. 1308-1313, 2004.

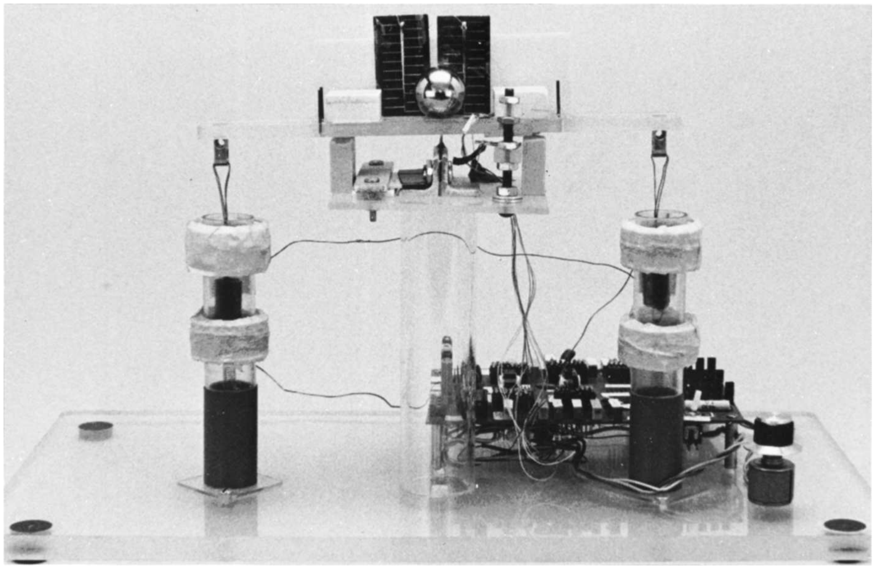
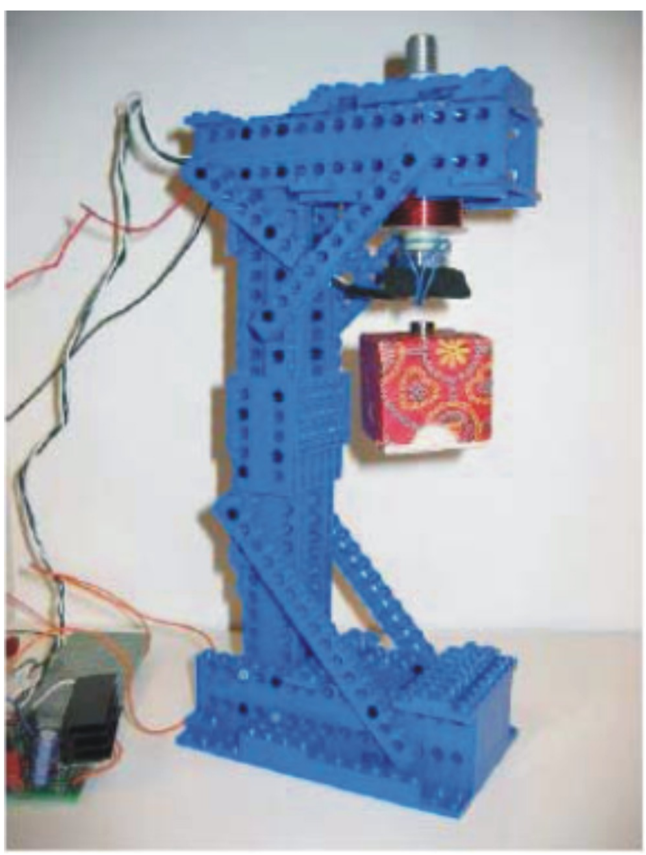


Figure 10.102: Ball-balancer design example



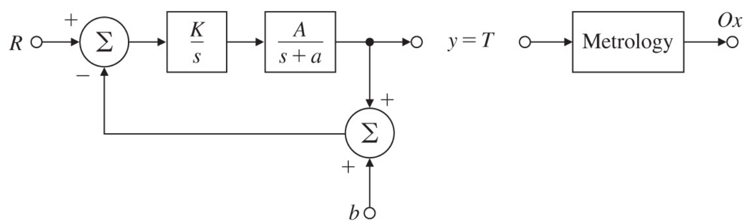



Figure 10.103: RTP system

Magnetic levitation system [see above Reference by K. A. Lilienkamp and K. Lundberg].

20. Design and build a Sun tracker using an Arduino board and related software.
**Solution:** See several prototypes on the Internet.

21. *Run-to-Run Control*: Consider the rapid thermal processing (RTP) system shown in Fig. 10.103. We wish to heat up a semiconductor wafer, and control the wafer surface temperature accurately using rings of tungsten halogen lamps. The output of the system is temperature $T$ as a function of time, $y = T(t)$. The system reference input $R$ is a desired step in temperature ($700^{\circ} C$) and the control input is lamp power. A pyrometer is used to measure the wafer center temperature. The model of the system is first order and an integral controller is used as shown in Figure 10.105. Normally, there is not a sensor bias ($b = 0$).

a. Suppose the system suddenly develops a sensor bias $b \neq 0$, where $b$ is known. What can be done to ensure zero steady-state tracking of temperature command $R$ despite the presence of the sensor bias?

1. (a) Now assume $b = 0$. In reality, we are trying to control the thickness of the oxide film grown ($Ox$) on the wafer and not the temperature. At present no sensor can measure $Ox$ in real time. The semiconductor process engineer must use an off-line equipment (called *metrology*) to measure the thickness of the oxide film grown on the wafer. The relationship between the system output temperature and $Ox$ is nonlinear and given by

$$\text{Oxide thickness } = \int_0^{t_f} pe^{-\frac{c}{T(t)}} \, dt,$$

where $t_f$ is the process duration, and $p$ and $c$ are known constants. Suggest a scheme in which the center wafer oxide thickness $Ox$ can be controlled to a desired value (say, $Ox = 5000$ Å) by employing the temperature controller and the output of the metrology.
**Solution:**

(a) We just increase $R$ by $+b$, i.e., replace $R$ by $(R + b)$ to cancel the sensor bias.

(b) Since there is a direct relationship between the temperature and oxide thickness, we could use the results of metrology to adjust the reference temperature until a desired thickness is obtained. We can do one "run" and measure the oxide thickness. Let us say, the metrology yields an oxide thickness of 5050 Angstrom (50 Å higher than desired). We would then lower the temperature, R, and try again. This is called "run-to-run" control and a linear static model can be used to provide the control adjustments. In effect, we will be "closing" the loop on metrology with a discrete integrator [1]. The recipe is adjusted from "run-to-run" using the following simple algorithm based on the attributes of the product produced in the previous run or runs [1]. Let $k = 1, 2, \ldots$, denote the run number, $r_k$ the recipe variable used during run $k$, $y_k$ the product quality attribute (oxide thickness) produced at the end of run $k$, and $e_k$ the normalized product quality error, defined as,

$$
\begin{aligned}
e_k &= y_k(i_{center}) - y_{des}(i_{center}), \\
e_k(i_{center)} &= Ox(i_{center}) - 5000. \qquad (1)
\end{aligned}
$$

where $y_{des}(i_{center})$ is the desired center oxide thickness at the center wafer node, $i_{center}$. The simplest choice for run-to-run control is to correct the previous recipe by an amount proportional to the current error. Thus, for run $k = 1, 2, \ldots$, adjust the recipe according to,

$$
\begin{aligned}
r_k &= r_{nom} + u_k \\
u_k &= u_{k-1} - \Gamma e_{k-1} \qquad u_0 = 0. \qquad (2)
\end{aligned}
$$

where $r_{nom}$ is the nominal recipe, $u_k$ is the correction to the nominal recipe for run $k$, and $\Gamma$ is the control design gain. $\Gamma$ is determined experimentally as follows. We would step up $R$ and measure the associated change in $Ox$, i.e., perturb $R$ by $\delta R$ and measure the associated output perturbation $\delta Ox$. Therefore,

$$\Gamma = \frac{\delta R}{\delta Ox}$$

is the control design gain. It is important to emphasize that (1)-(2) constitute the complete run-to-run algorithm. Also (2) has the same form as a gradient descent optimization algorithm. It is possible to choose the run-to-run control gain matrix $\Gamma$ and to analyze the algorithm under a variety of assumptions about how $u_k$ effects $e_k$ [1]. It can be shown that most of the widely used run-to-run algorithms are in the form of (2) for different choices of $\Gamma$. For more details see Reference [1].

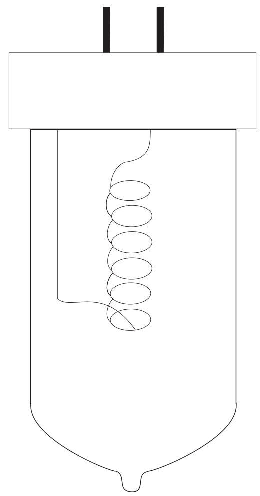
**Reference:**

[1] R. L. Kosut, D. de Roover, A. Emami-Naeini, J. L. Ebert, "Run-to-Run Control of Static Systems," in P*roc. 37th IEEE Conf. Decision Control*, pp. 695-700, December 1998.

22. Develop a nonlinear model for a tungsten halogen lamp and simulate it in Simulink.

**Solution:**

Discovered in 1959, a tungsten halogen bulb is similar to an ordinary incandescent bulb with the filament made from tungsten but the fill gas is a halogen compound, usually iodine or bromine. A schematic of the lamp is shown below. The idea behind the use of the halogen is to re-deposit the evaporated tungsten molecules back onto the filament. The Tungsten atoms evaporate off of the hot filament and condense onto the cooler inside wall of the bulb. However, halogen reacts with the tungsten and re-evaporates the deposited tungsten which reaches the hot filament again. This process is known as the "halogen cycle", and extends the lifetime of the bulb. In order for the halogen cycle to work, the bulb surface must be very hot, generally over 250°C. The bulb is made from quartz. Tungsten halogen lamps are now commonly used in rapid thermal processing (RTP) in semiconductor manufacturing [1].



Problem 10.21: Schematic of a tungsten halogen lamp.

Consider the following physical parameters for the lamp,

$$
\begin{aligned}
\text{total emissivity } \epsilon &\cong 0.4, \\
\text{density } \rho &= 19300\ [kg/m^3], \\
\text{specific heat } c &\cong 150\ [J/kgK]\ (\text{ at } T = 1000K), \\
\text{electrical resistivity } \rho_e &= 21.9 \times 10^{-8}\ [\Omega m]\ (\text{ at } T = 1000K), \\
\text{Stefan-Boltzman constant } \sigma &= 5.67 \times 10^{-8}\ [W/m^2K^4].
\end{aligned}
$$

Lamp design is based on a specification of maximum temperature, maximum applied voltage, and maximum delivered power. Using the following notation,

$$
\begin{aligned}
d &= \text{filament diameter }, [mm], \\
L &= \text{filament length }, [m], \\
T &= \text{filament temperature }, [K], \\
T_{\max} &= \text{max imum filament temperature }, [K], \\
T_{\infty} &= \text{ambient (room) temperature }, [K], \\
T_{e0} &= \text{temperature at which resistivity } \rho_e \text{ is specified }, [K], \\
V &= \text{normalized lamp voltage } (0 \le V \le 1), \\
V_{\max} &= \text{maximum applied voltage }, [V], \\
P_{\max} &= \text{maximum power }, [W], \\
I &= \text{current }, [A],
\end{aligned}
$$

the electrical resistivity is given by the relationship [3],

$$
\rho_e = \rho_{e0} \left( \frac{T}{T_{e0}} \right)^{1.2}. \tag{2}
$$

For example at $T_{e0} = 1000K$, we find $\rho_{e0} = 21.9\times10^{-8}\ \Omega m$. Employing the energy balance technique [1] leads to the equation,

$$\frac{1}{4}\pi d^2 L\rho c\,\dot{T} = -\,\epsilon\sigma\pi dL(T^4 - T_\infty^4) + \frac{\pi d^2 V_{\max}^2}{4\rho_{e0}L\big(\frac{T}{T_{e0}}\big)^{1.2}}V^2. \tag{3}$$

It is interesting to note that if we specify the maximum radiant power desired, $P_{\max}$, then the filament diameter, $d$, and length $L$, of the filament are specified for a given $T_{\max}$ and $V_{\max}$. From Eq. (3), in the steady-state, $\dot{T} = 0$, and with $V = 1$,

$$P_{\max} = \frac{\pi d^2 V_{\max}^2}{4\rho_{e0}L\big(\frac{T_{\max}}{T_{e0}}\big)^{1.2}} = \epsilon\sigma\pi dL(T_{\max}^4 - T_\infty^4), \tag{4}$$

and we can solve Eqs. (3) and (4), for d and L as follows. Equation (4) can be written as,

$$\alpha\frac{d^2}{L} = \beta dL, \tag{5}$$

where,

$$\alpha = \frac{\pi V_{\max}^2}{4\rho_{e0}\big(\frac{T_{\max}}{T_{e0}}\big)^{1.2}}, \tag{6}$$

and,

$$\beta = \epsilon\sigma\pi\left(T_{\max}^4 - T_\infty^4\right), \tag{7}$$

are prescribed. Then, the filament diameter, $d$, is simply related to filament length, $L$, by,

$$d = \frac{\beta}{\alpha}L^2. \tag{8}$$

Using Eq. (4), (5), and (8), and the maximum radiant power is related to the filament length by,

$$P_{\max} = \frac{\beta^2}{\alpha}L^3. \tag{9}$$

Using Eq. (9) together with Eqs. (6) and (7), we can then solve for the filament length , $L$, as,

$$L = \left(\frac{P_{\max}\alpha}{\beta^2}\right)^{\frac{1}{3}} = \left[\frac{P_{\max}\pi V_{\max}^2}{4\rho_{e0}\big(\frac{T_{\max}}{T_{e0}}\big)^{1.2}\pi^2\epsilon^2\sigma^2\left(T_{\max}^4 - T_\infty^4\right)^2}\right]^{\frac{1}{3}}. \tag{10}$$

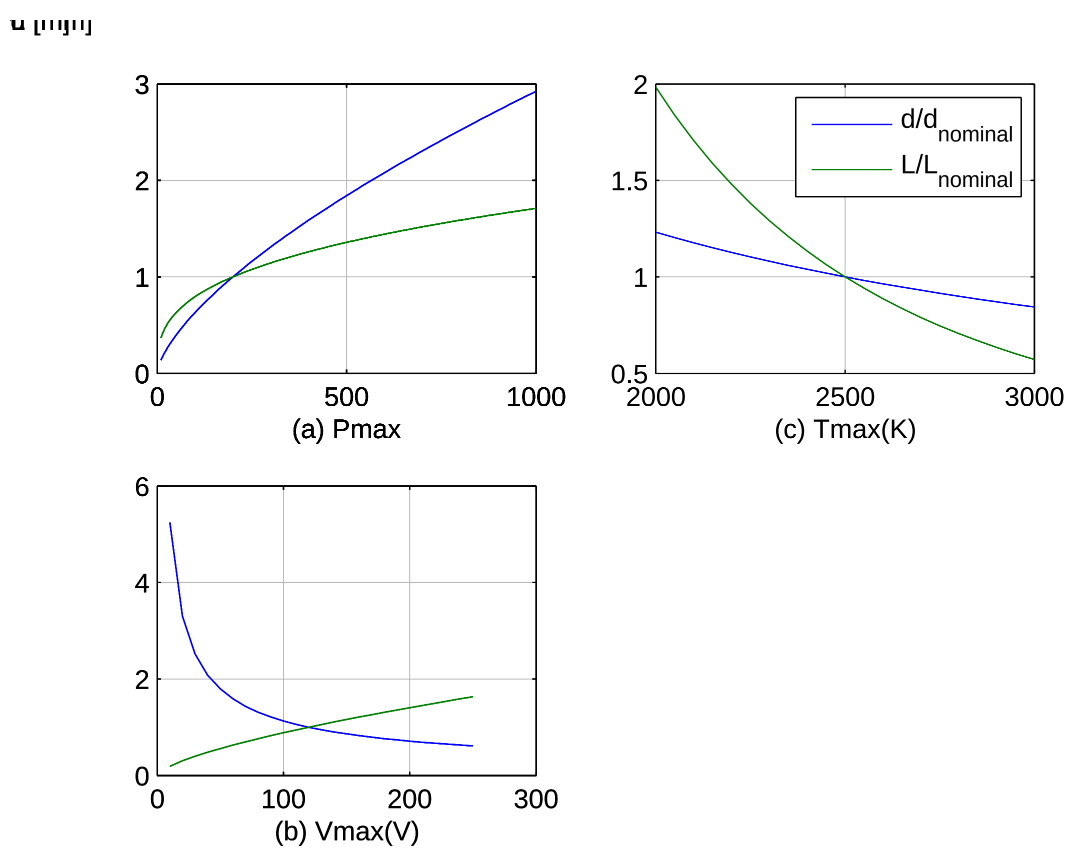
This can be simplified further by assuming that the maximum filament temperature is much higher than the ambient temperature, $T_{\max}^4 \gg T_\infty^4$, so that,

$$L \cong \left[\frac{P_{\max}\pi V_{\max}^2}{4\rho_{e0}\big(\frac{T_{\max}}{T_{e0}}\big)^{1.2}\pi^2\epsilon^2\sigma^2 T_{\max}^8}\right]^{\frac{1}{3}}. \tag{11}$$

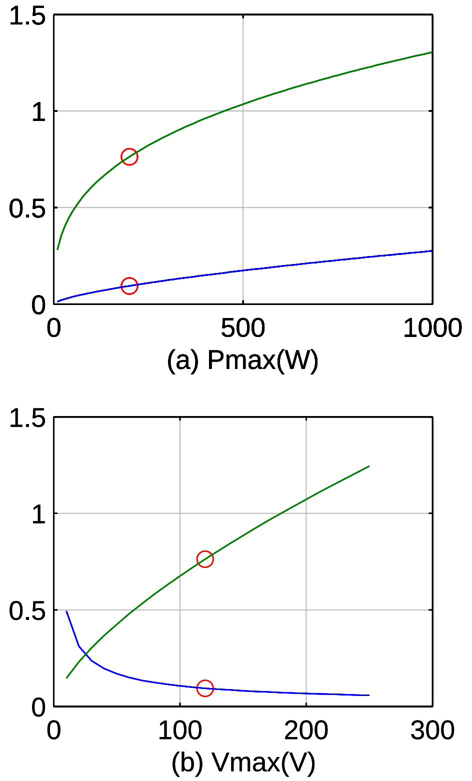
We can solve for the filament length and diameter in terms of the given quantities $P_{\max}$, $V_{\max}$, and $T_{\max}$ as well,

$$\begin{aligned} L &\propto P_{\max}^{\frac{1}{3}} V_{\max}^{\frac{2}{3}} T_{\max}^{-3.07}, \\ d &\propto P_{\max}^{\frac{2}{3}} V_{\max}^{-\frac{2}{3}} T_{\max}^{-0.93}. \end{aligned} \tag{12}$$

Figures (a), (b), and (c) on top of the next page show plots of the functional relationship between filament diameter and length as a function of the maximum power, $P_{\max}$, maximum temperature, $T_{\max}$, and maximum voltage, $V_{\max}$, respectively as well as the nominal operating point corresponding to $P_{\max} = 200W$, $T_{\max} = 3000K$, and , $V_{\max} = 120V$. Figure (a) shows that increasing $P_{\max}$ requires increases in both the filament diameter and length. As seen from Figure (b), increasing the maximum temperature, $T_{\max}$, requires decreasing the filament length but is relatively insensitive to the filament diameter. Figure (c) shows that increasing the maximum voltage, $V_{\max}$, requires increasing the filament length but decreasing the filament diameter. Figures (a), (b), and (c) on the bottom of the next page show the same relationships as in the top figure but in terms of the normalized diameter, $\frac{d}{d_{no\min al}}$, and normalized filament length, $\frac{L}{L_{no\min al}}$.




Problem 10.20: Lamp design parameters.




Problem 10.21: Lamp design parameters: normalized.

Assume the nominal operating temperature is denoted by $T_0$. Re-writing Equation (3) in terms of the normalized temperature, we have

$$\left(\frac{\dot{T}}{T_0}\right) = -\frac{4\epsilon\sigma T_0^3}{\rho c d}\left[\left(\frac{T}{T_0}\right)^4 - \left(\frac{T_\infty}{T_0}\right)^4\right] + \frac{V_{\max}^2}{4\rho_{e0}\rho c L^2 T_0}\frac{V^2}{\left(\frac{T}{T_{e0}}\right)^{1.2}}. \tag{13}$$

Let us define the normalized temperature, $x = \frac{T}{T_0}$,and re-write Eq. (13) as the nonlinear first-order system,

$$\dot{x} = -A(x^4 - x_\infty^4) + B\frac{V^2}{x^{1.2}}, \tag{14}$$

where,

$$\begin{aligned} A &= \frac{4\epsilon\sigma T_0^3}{\rho c d}, \\ B &= \frac{V_{\max}^2}{4\rho_{e0}\rho c L^2 T_0}. \end{aligned}$$

Linearizing Equation (14) about the nominal (normalized) temperature, we find the first-order linear dynamic model for the lamp as,

$$\dot{x} = -4Ax_0^3 x - B\frac{1.2}{x_0^{2.2}}V^2, \tag{15}$$

which means that the lamp time constant is,

$$\tau = \frac{1}{4Ax_0^3} \cong \frac{\rho c d}{16\epsilon\sigma T_0^3 x_0^3}. \tag{16}$$

In terms of lamp current, we have,

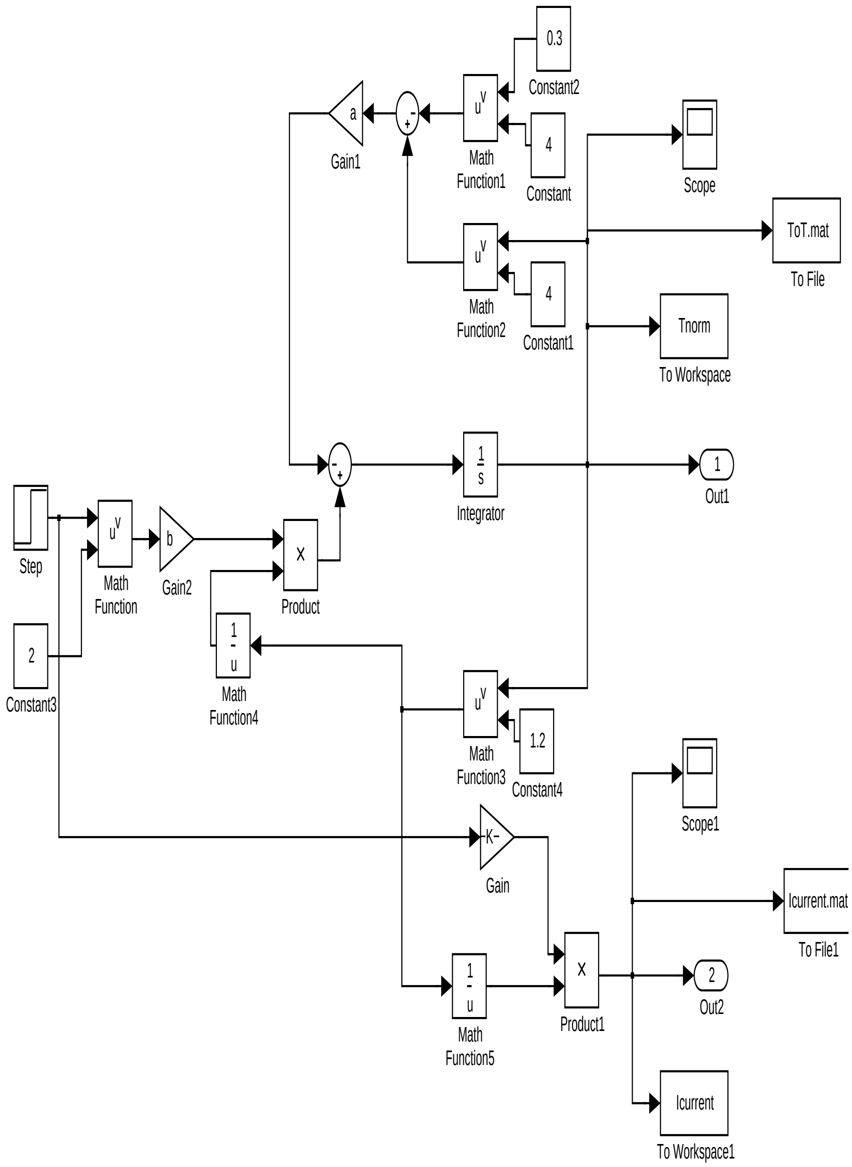
$$I = \frac{\pi d^2 V_{\max}}{4\rho_{e0} L\left(\frac{T}{T_{e0}}\right)^{1.2}} V. \tag{17}$$

Equation (16) implies that fast lamp response requires high filament temperature and low filament diameter. Typical values for the lamp filament time constant range from 0.5 to 2 seconds.

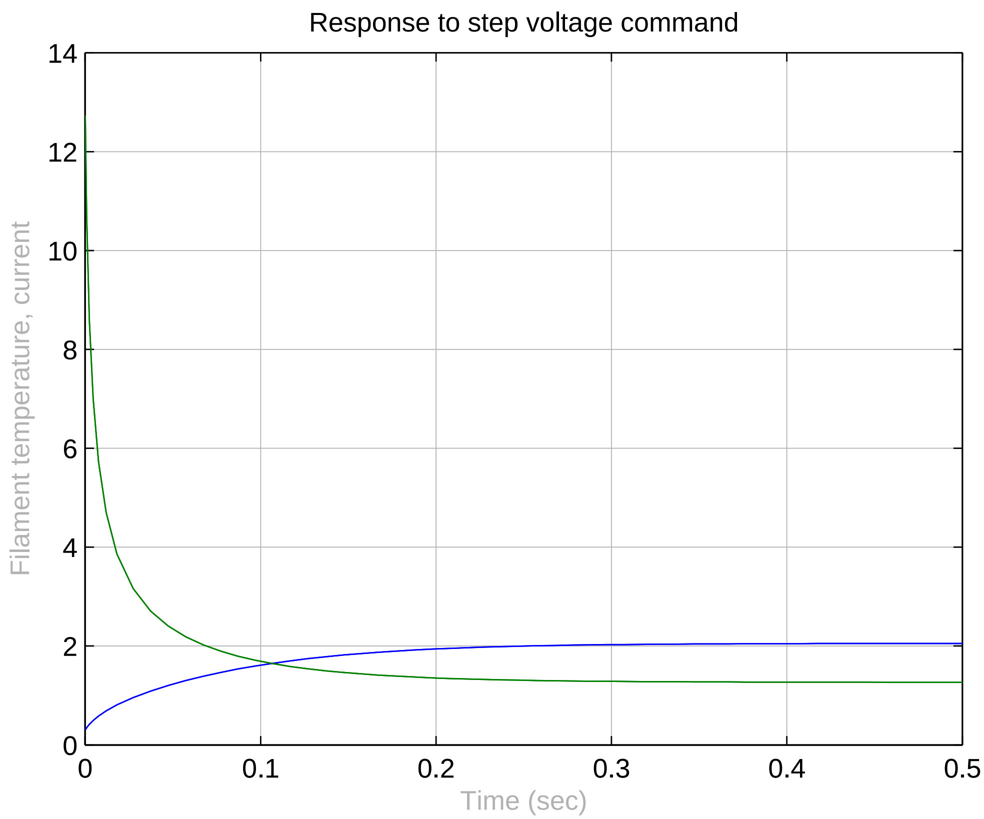
The output of the lamp may be considered to be current, normalized filament temperature, or radiative power,

$$y_{lamp} = \begin{bmatrix} I \\ x \\ P_{\max} \end{bmatrix}. \tag{18}$$

A nonlinear simulation for the lamp model may be implemented in Simulink as shown on the next page. The results of the simulation show the temperature and current response of the lamp to a step voltage command input as shown. The figure shows the fast lamp filament temperature response with a time constant of 0.07 seconds, and also shows that the current initially surges but quickly drops to a steady-state within approximately 0.3 seconds.




Problem 10.21: Simulink diagram for lamp model.




Problem 10.21: Lamp response to a step voltage command.

**References:**

[1] Reynolds, W. C., and H. C. Perkins, *Engineering Thermodynamics*, McGraw-Hill, 1977.

[2] A. Emami-Naeini, et al., “Modeling and Control of Distributed Thermal Systems,” *IEEE Tran. Contrl. Syst. Tech.*, pp. 668-683, September 2003.

[3] Lide, D. R., Ed., *Handbook of Chemistry and Physics*, CRC Press, 1993-1994.

[4] Modest, M. F., *Radiative Heat Transfer*, McGraw-Hill, 1993.

23. Develop a nonlinear model for a pyrometer. Show how temperature can be deduced from the model.

**Solution:**

Temperature measurement can be done by a variety of methods including thermocouples, resistive temperature detectors (RTDs), and pyrometers [1]. A pyrometer is a non-contact temperature sensor and measures the Infrared (IR) radiation which is directly a function of the temperature. It is known that objects emit radiant energy proportional to $T^4$ where $T$ is the temperature of the object. Among the advantages of pyrometers are that they have very fast response time, can be used to measure the temperature of moving objects (e.g., a rotating semiconductor wafer), and in vacuum for semiconductor manufacturing.

The single-wavelength pyrometer measures the total energy emitted from a surface at a given wavelength. To understand the operation of a pyrometer, we need to review some concepts from radiation heat transfer [2-3]. The emissivity of an object, $\epsilon$, is defined as the ratio of the energy flux emitted by a surface to that from a black body at the same temperature:

$$\epsilon = \frac{\int \epsilon_\lambda I_{b\lambda}(T) d\lambda}{\int I_{b\lambda}(T) d\lambda}, \tag{19}$$

where,

$$\begin{aligned}
\epsilon &= \text{total emissivity}, \\
\epsilon_\lambda &= \text{spectral emissivity}, \\
T &= \text{absolute temperature}, \;\; K, \\
\lambda &= \text{wavelength of radiation}, \;\; \mu m, \\
I_{b\lambda} &= \text{spectral black body intensity.}
\end{aligned}$$

The frequency, $\nu$, is given by,

$$\nu = \frac{c_0}{\lambda_0},$$

where,

$$\begin{aligned}
c_0 &= \text{speed of light in vacuum } = 2.998 \times 10^8 m/s, \\
\lambda_0 &= \text{wavelength of light in vacuum.}
\end{aligned}$$

The Plank's law of radiation states that the spectral radiance of a blackbody, or spectral intensity, $I_{b\nu}$, in a dielectric medium as a function of the wavelength and temperature is,

$$I_{b\nu}(T) = \frac{2h\nu^3 n^2}{c_o^2(e^{\frac{h\nu}{kT}} - 1)}, \tag{20}$$

where,

$$\begin{aligned}
h &= \text{Planck}\,'\,\text{s constant } = 6.626 \times 10^{-34} Js, \\
k &= \text{Boltzmann}\,'\,\text{s constant } = 1.3806 \times 10^{-23} J/K, \\
n &= \text{real refractive index ( } n = 1 \text{ for most gases)},
\end{aligned}$$

and the frequency and wavelength are related by,

$$\nu = \frac{c}{\lambda}, \tag{21}$$

where $c$ is the speed of light in the medium and is given by,

$$c = \frac{c_0}{n}.$$

After some manipulation, we can re-write Eq. (20) as [2],

$$I_{b\lambda}(T) = \frac{2hc_0^2}{n^2\lambda^5(e^{\frac{hc_0}{n\lambda kT}} - 1)}. \tag{22}$$

The total black-body radiation intensity, $I_b$, is obtained by integrating over all frequencies or wavelength [2]

$$\begin{aligned} I_b(T) &= \int I_{\nu b}(T) d\nu \\ &= \frac{n^2 \sigma T^4}{\pi}. \end{aligned} \tag{23}$$

The black body emissive flux is given by,

$$q_{\lambda b}(T) = \frac{C_1}{n^2 \lambda^5 (e^{\frac{C_2}{n\lambda T}} - 1)}, \tag{24}$$

where,

$$\begin{aligned} C_1 &= 2\pi h c_0^2 = 3.7419 \times 10^{-16},\ W/m^2, \\ C_2 &= \frac{h c_0}{k} = 14388\ \mu m K. \end{aligned} \tag{25}$$

Integrating over all wavelengths $\lambda$, we obtain the total black body emissive flux [2],

$$\begin{aligned} q_b(T) &= \int_0^\infty q_{\lambda b}(T) d\lambda \\ &= n^2 \sigma T^4, \end{aligned} \tag{26}$$

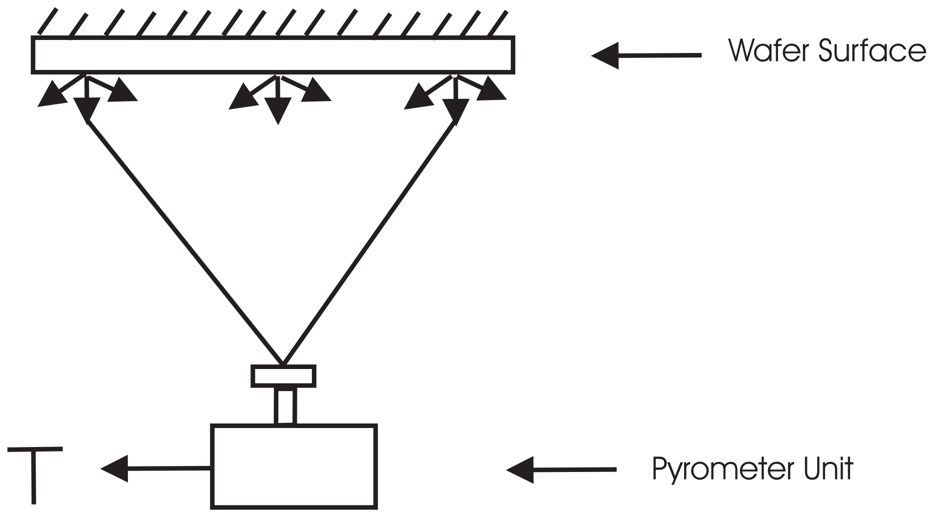
where $\sigma$ is the Stefan-Boltzman constant, $\sigma = 5.67 \times 10^{-8}\ W/(m^2 K^4)$.

The temperature may be determined from Eq. (20) as,

$$T = \frac{C_2}{\lambda} \frac{1}{\ln(1 + \frac{\epsilon_\lambda C}{I})}, \tag{27}$$

where,

$$C = \frac{C_1}{\lambda^5}. \tag{28}$$




Problem 10.22: Schematic of temperature measurement using pyrometry.

The figure on the bottom of the previous page shows the schematic of temperature measurement of a semiconductor wafer using pyrometry where, for this particular application, response time and view angle are very important. A two color pyrometer is also used for applications where absolute temperature measurement is important. The measurement can then be used for feedback control purposes, e.g., pyrometers are now routinely used in control of rapid thermal processing (RTP) systems in semiconductor manufacturing [4].

**References**:

[1] Fraden, J., *Handbook of Modern sensors: Physics, Designs, and Applications*, Springer, 1996.

[2] Ozisik, M. N., *Radiative Transfer and Interactions with Conduction and Convection*, Wiley-Interscience, 1973.

[3] Siegel, R. and J. R. Howell, *Thermal Radiation Heat Transfer*, Second Ed., Hemisphere Publishing Corp., 1981.

[4] A. Emami-Naeini, et al., "Modeling and Control of Distributed Thermal Systems," *IEEE Tran. Contrl. Syst. Tech.*, pp. 668-683, September 2003.

24. Repeat the RTP case study design by summing the three sensors to form a single signal to control the average temperature. Demonstrate the performance of the linear design, and validate the performance on the nonlinear Simulink simulation.

**Solution:**

A linear model for the system was derived in the text as,

$$\begin{aligned} \dot{\mathbf{T}} &= \mathbf{A}_3\,\mathbf{T} + \mathbf{B}_3\,\mathbf{u}, \\ \mathbf{y} &= \mathbf{C}_3\mathbf{T} + \mathbf{D}_3\,\mathbf{u}, \end{aligned} \tag{29}$$

where $\mathbf{y} = [T_{y1}\ T_{y2}\ T_{y3}]^T$ and,

$$\mathbf{A}_3 = \begin{bmatrix} -0.0682 & 0.0149 & 0.0000 \\ 0.0458 & -0.1181 & 0.0218 \\ 0.0000 & 0.04683 & -0.1008 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0.3787 & 0.1105 & 0.0229 \\ 0.0000 & 0.4490 & 0.0735 \\ 0.0000 & 0.0007 & 0.4177 \end{bmatrix},$$

$$\mathbf{C}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The three open-loop poles are computed from MATLAB and are located at $-0.0527, -0.0863$, and $-0.1482$. Since we tied the three lamps into one actuator and are only using the *average* temperature for feedback, the linear model is then:

$$\mathbf{A} = \begin{bmatrix} -0.0682 & 0.0149 & 0.0000 \\ 0.0458 & -0.1181 & 0.0218 \\ 0.0000 & 0.04683 & -0.1008 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.5122 \\ 0.5226 \\ 0.4185 \end{bmatrix},$$

$$\mathbf{C}_{avg} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}, \; D = [0],$$

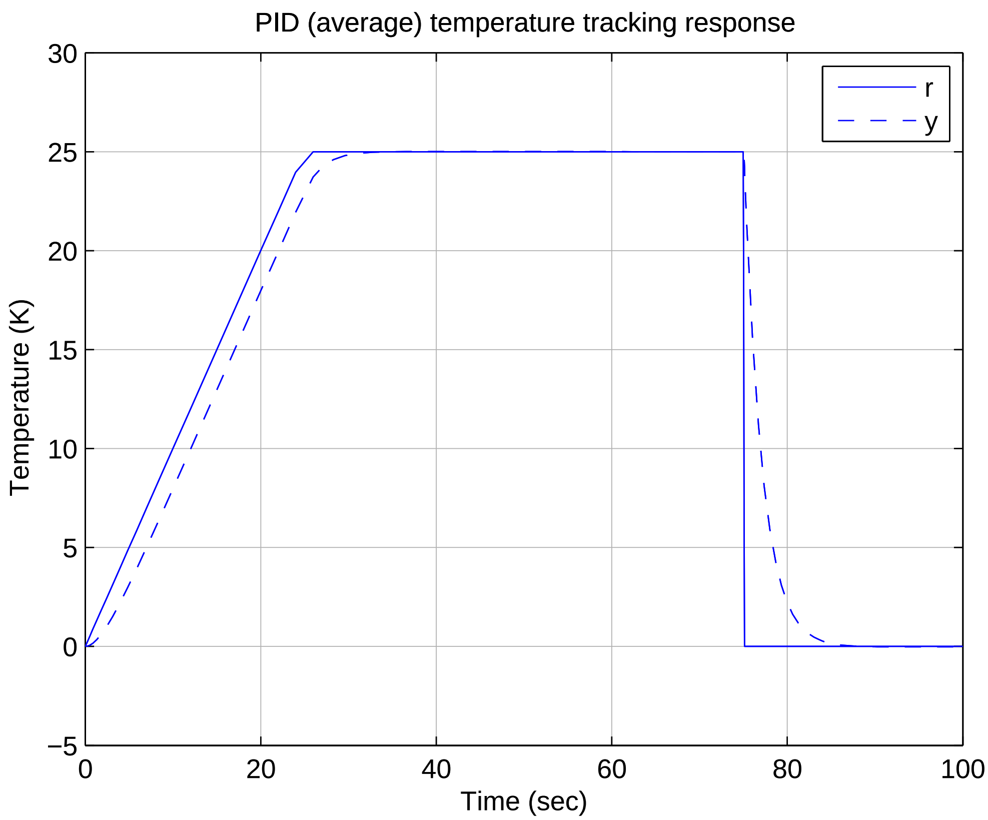
resulting in the transfer function,

$$G(s) = \frac{T_{yavg}(s)}{V_{cmd}(s)} = \frac{0.4844(s+0.0878)(s+0.1485)}{(s+0.1482)(s+0.0527)(s+0.0863)}.$$

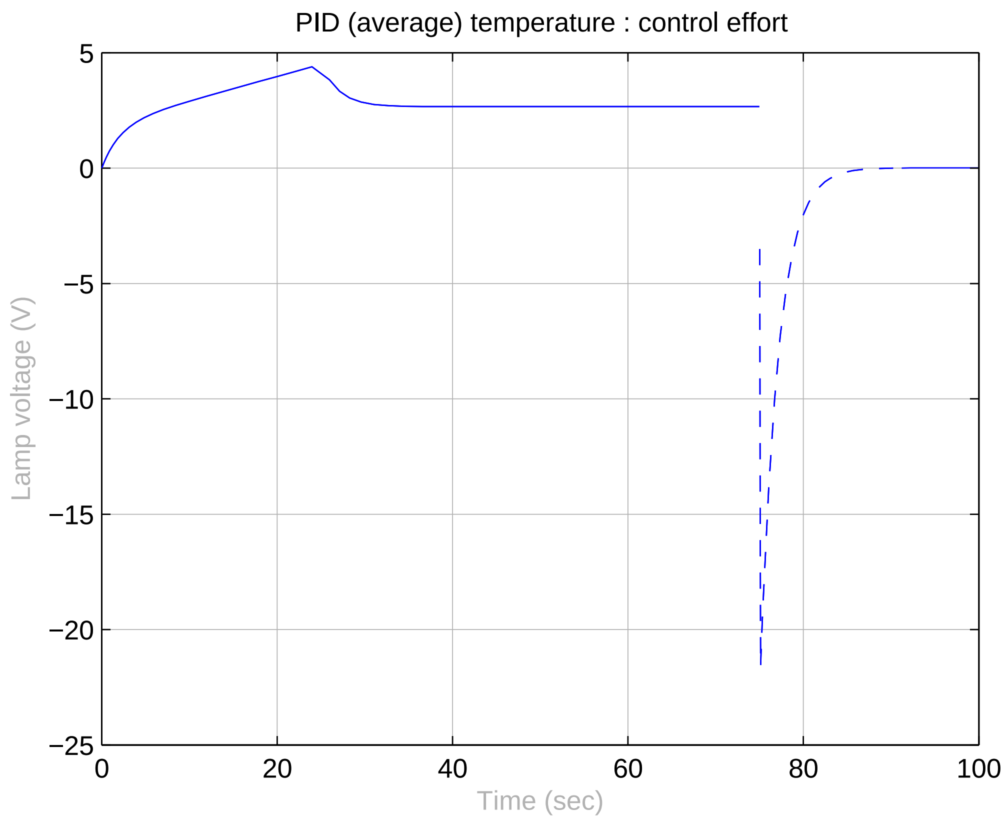
We may try a simple PI controller of the form,

$$D_c(s) = \frac{(s+0.0527)}{s},$$

so as to cancel the effect of the slower pole. The linear closed-loop response is shown as well as the associated control effort The system response follows the commanded trajectory with a time delay of approximately 2 sec and no overshoot. The lamp has its normal response until 75 sec and goes negative (shown in dashed) to try to follow the sharp drop in commanded temperature. As mentioned in the text, this behavior is not possible in the system as there is no means of active cooling and the lamps do saturate low. There is no explicit means of controlling the temperature nonuniformity using the PI controller.




Problem 10.23: Linear closed-loop RTP response for PI controller.




Problem 10.23: RTP Linear response for PI: control effort.

Next we design a state-space based controller. As in the text, we use the error space approach for inclusion of integral control and employ the linear quadratic gaussian technique of Chapter 7. The error system is,

$$\begin{bmatrix} \dot{e} \\ \dot{\boldsymbol{\xi}} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{C}_{avg} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} e \\ \boldsymbol{\xi} \end{bmatrix} + \begin{bmatrix} D \\ \mathbf{B} \end{bmatrix} \mu, \tag{30}$$

where,

$$\mathbf{A_s} = \begin{bmatrix} 0 & \mathbf{C}_{avg} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}, \mathbf{B_s} = \begin{bmatrix} D \\ \mathbf{B} \end{bmatrix}.$$

and $e = y - r$, $\boldsymbol{\xi} = \dot{\mathbf{T}}$ with $\mu = \dot{u}$. For state feedback design, the LQR formulation of Chapter 7 is used

$$\mathcal{J} = \int_0^\infty \{\mathbf{z}^T \mathbf{Q}\, \mathbf{z} + \rho\mu^2\} dt,$$

where $\mathbf{z} = [e\ \boldsymbol{\xi}^T]^T$. Note that $\mathcal{J}$ has been chosen in such a way as to penalize the tracking error, $e$, the control, $u$, as well as the differences in the three temperatures– therefore, the performance index should include a term of the form,

$$10\left\{(\dot{T}_1-\dot{T}_2)^2+(\dot{T}_1-\dot{T}_3)^2+(\dot{T}_2-\dot{T}_3)^2\right\},$$

and hence minimizes a measure of the *temperature non-uniformity.* As in the text, the factor of ten is used as the relative weighting between the error state and the plant state. The state and control weighting matrices, $\mathbf{Q}$ and $R$, are then,

$$\mathbf{Q}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 20 & -10 & -10\\ 0 & -10 & 20 & -10\\ 0 & -10 & -10 & 20 \end{bmatrix},\quad R=\rho=1.$$

The following MATLAB command is used to design the feedback gain,

```
[K]=lqr(A,B,Q,R).
```

The resulting feedback gain matrix computed from MATLAB is,

$$\mathbf{K}=[K_1 \;\; : \mathbf{K}_0],$$

where,

$$K_1 \;\; =1,\ \mathbf{K}_0 \;\; =\begin{bmatrix} 0.7344 & 0.9344 & 0.3921 \end{bmatrix},$$

which results in the internal model controller of the form,

$$\begin{aligned} \dot{x}_c &= B_c\, e, \\ u &= C_c\, x_c-\mathbf{K}_0\;\; \mathbf{T}, \end{aligned} \tag{31}$$

with $x_c$ denoting the controller state and,

$$B_c=-K_1=-1,\ C_c=1.$$

The resulting state-feedback closed-loop poles computed using MATLAB's `eig` command are at $-0.5395\pm 0.4373j, -0.1490$, and $-0.0879$. The full-order estimator was designed with the same process and sensor noise intensities used in the text as the estimator design knobs,

$$R_w=1,\ R_v=0.001.$$

The following MATLAB command is used to design the estimator,

```
[L]=lqe(A,B,C,Rw,Rv).
```

The resulting estimator gain matrix is,

$$\mathbf{L}=\begin{bmatrix} 16.142\\ 16.4667\\ 13.1975 \end{bmatrix},$$

with estimator error poles at $-15.3197, -0.1485$, and $-0.0878$. The estimator equation is,

$$\dot{\hat{\mathbf{T}}}=\mathbf{A}\hat{\mathbf{T}}+\mathbf{B}u+\mathbf{L}(y-\mathbf{C}\hat{\mathbf{T}}). \tag{32}$$

With the estimator, the internal model controller equation is modified as in the text

$$\begin{aligned} \dot{x}_c &= B_c\, e, \\ u &= C_c\, x_c - \mathbf{K}_0\ \hat{\mathbf{T}}. \end{aligned} \tag{33}$$

The closed-loop system equations are given in the text,

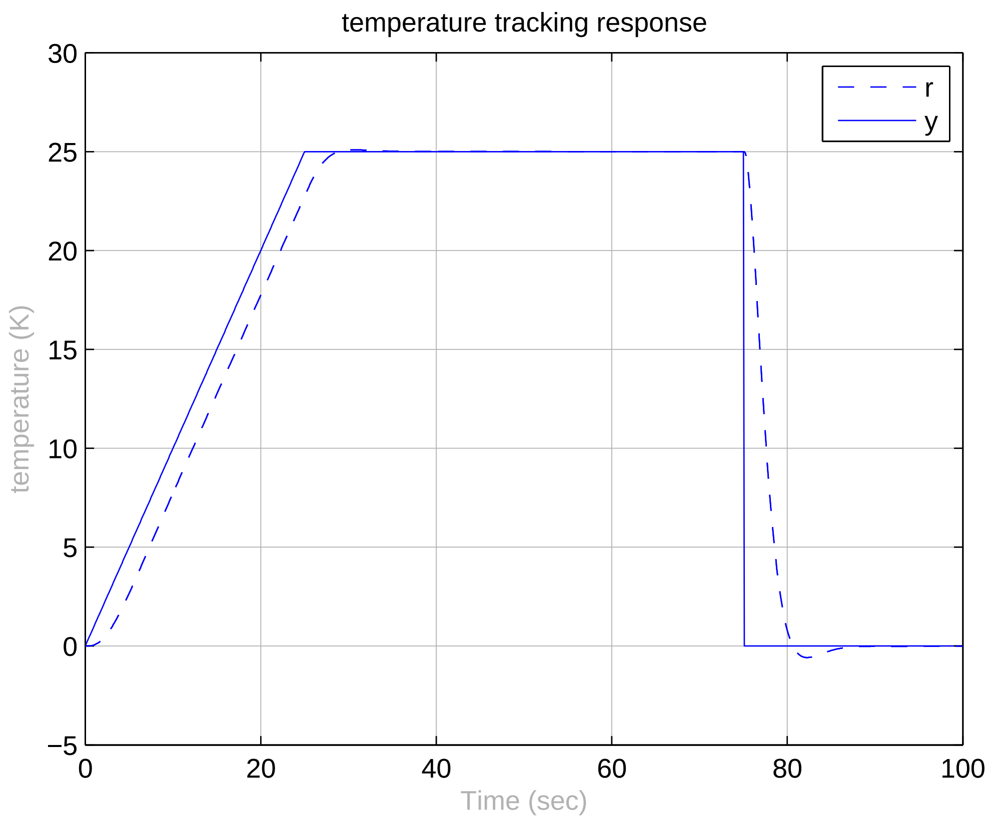
$$\begin{aligned} \dot{\mathbf{x}}_{cl} &= \mathbf{A}_{cl}\, \mathbf{x}_{cl} + \mathbf{B}_{cl}\, r, \\ y &= \mathbf{C}_{cl}\, \mathbf{x}_{cl} + \mathbf{D}_{cl}\, r, \end{aligned} \tag{34}$$

where $r$ is the reference input temperature trajectory, the closed-loop state vector is $\mathbf{x}_{cl} = [\mathbf{T}^T\ x_c^T\ \hat{\mathbf{T}}^T]^T$ and the system matrices are,

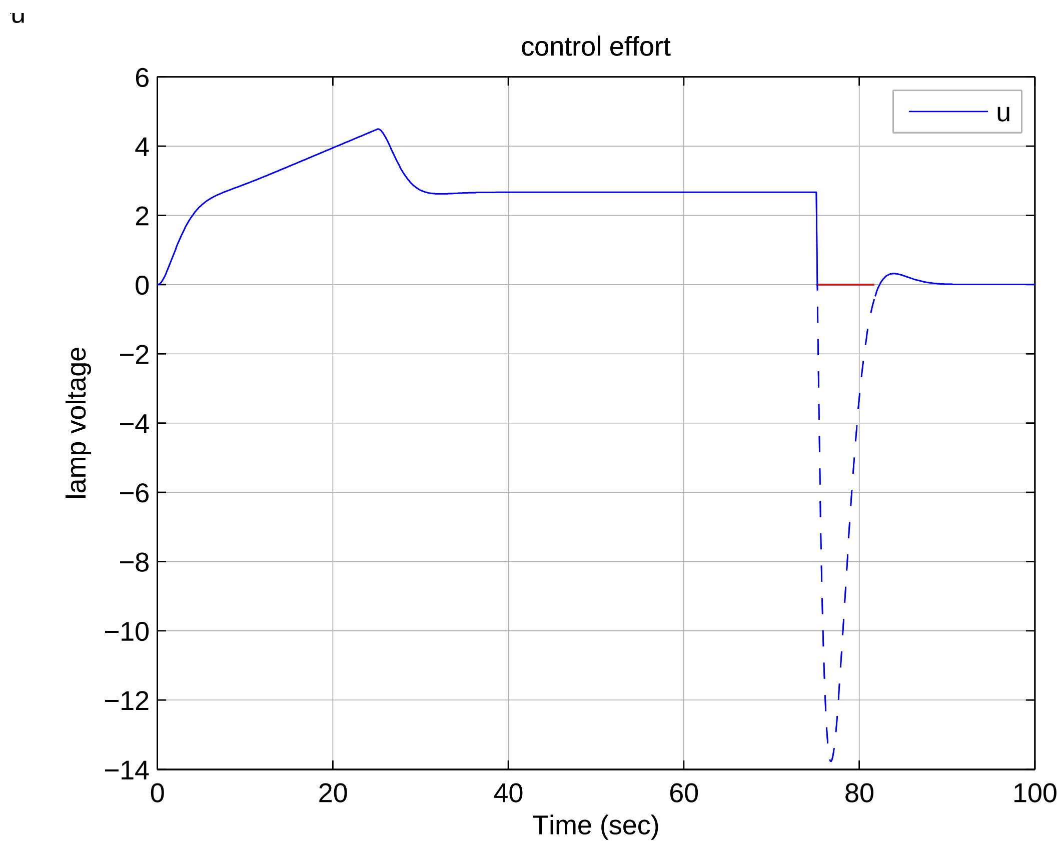
$$\begin{aligned} \mathbf{A}_{cl} &= \begin{bmatrix} \mathbf{A} & \mathbf{B}C_c & -\mathbf{B}\mathbf{K}_0 \\ B_c\mathbf{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}\mathbf{C} & \mathbf{B}C_c & \mathbf{A} - \mathbf{B}\mathbf{K}_0 - \mathbf{L}\mathbf{C} \end{bmatrix}, \quad \mathbf{B}_{cl} = \begin{bmatrix} \mathbf{0} \\ -B_c \\ \mathbf{0} \end{bmatrix}, \\ \mathbf{C}_{cl} &= \begin{bmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \mathbf{D}_{cl} = [0], \end{aligned}$$

with closed-loop poles (computed with MATLAB) located at $-0.5395 \pm 0.4373j, -0.1490,\ -0.0879,$ $-15.3197, -0.1485$ and $-0.0879$ as expected. The closed-loop control structure is as shown in text Figure 10.85.

The linear closed-loop response and the associated control effort are shown. The commanded temperature trajectory, $r$, is a ramp from 0°C to 25°C with a 1°C/sec slope followed by 50 sec soak time and drop back to 0°C. The system tracks the commanded temperature trajectory – albeit with a time delay of approximately 2 seconds for the ramp and a maximum of 0.0216°C overshoot. As expected the system tracks a constant input asymptotically with zero steady-state error. The lamp command increases as expected to allow for tracking the ramp input, reaches a maximum value at 25 sec and then drops to a steady-state value around 35 sec. The normal response of the lamp is seen from 0 to 75 sec followed by negative commanded voltage for a few seconds corresponding to fast cooling. Again, the negative control effort voltage (shown in dashed lines) is physically impossible as there is no active cooling in the system. Hence in the nonlinear simulations, commanded lamp power must be constrained to be strictly non-negative. Note that the response from $75 - 100$ sec is that of the (negative) step response of the system.

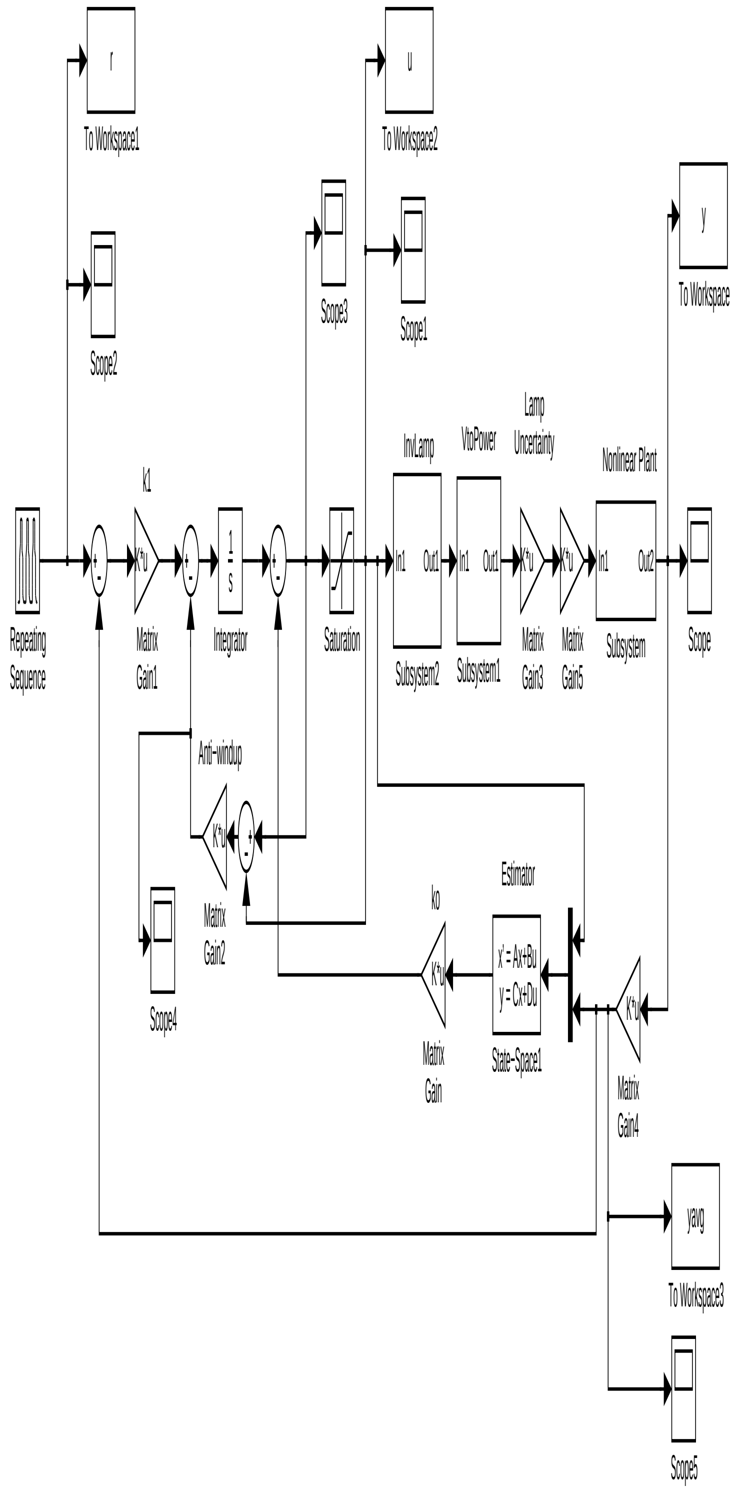



Problem 10.23: RTP linear (average) temperature tracking response.



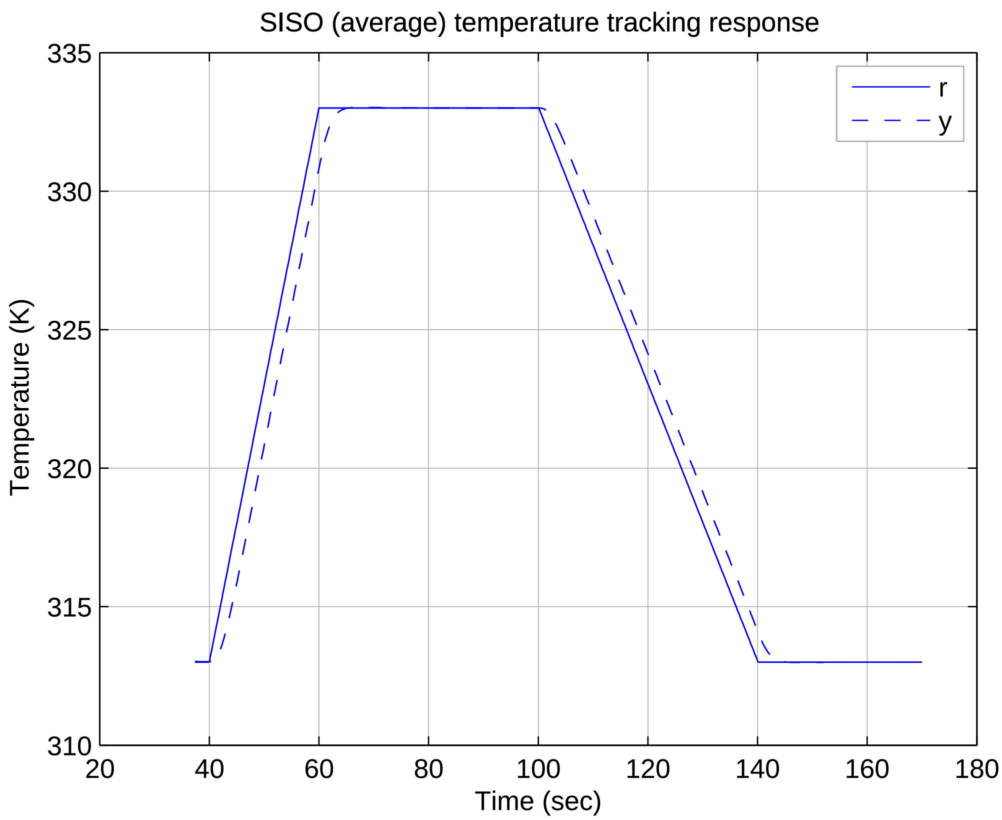

Problem 10.23 RTP linear (average) : control effort.

The nonlinear closed-loop system was simulated in Simulink as shown on the next page. In the diagram, Gain*4* $= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$. As in the text, the model was implemented in temperature units of degrees Kelvin and the ambient temperature is 301K. The nonlinear plant model is the implementation of text Eq. 10.48. The voltage range for system operation is between 1 to 4 volts as seen from the diagram. As in the text, a saturation nonlinearity is included for the lamp as well as integrator anti-windup logic to deal with lamp saturation. The nonlinear dynamic response and the control effort are shown. Note that the nonlinear response is in general agreement with the linear response.

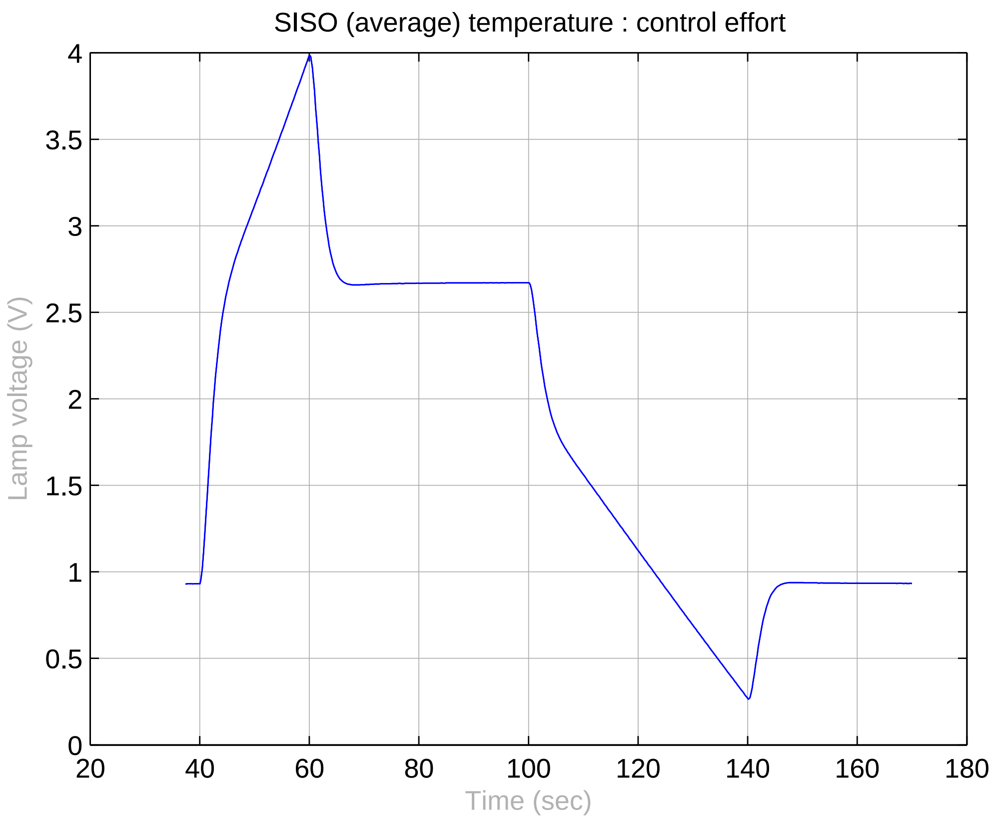

r
To Workspace1
Scope2
u
To Workspace2
Scope3
Scope1
y
To Workspace
K1
InvLamp
VtoPower
Lamp
Uncertainty
Nonlinear Plant
Repeating
Sequence
K*u
Matrix
Gain1
1
s
Integrator
Saturation
In1
Out1
Subsystem2
In1
Out1
Subsystem1
K*u
Matrix
Gain3
K*u
Matrix
Gain5
In1
Out2
Subsystem
Scope
Anti−windup
K*u
Matrix
Gain2
Scope4
ko
K*u
Matrix
Gain
Estimator
x' = Ax+Bu
y = Cx+Du
State−Space1
K*u
Matrix
Gain4
yavg
To Workspace3
Scope5

Problem 10.23: Simulink diagram for nonlinear closed-loop RTP system to control average temperature.

y




Problem 10.23: Nonlinear closed-loop response.




Problem 10.23: Nonlinear closed-loop response: control effort.

25. One of the steps in semiconductor wafer manufacturing during photolithography is performed by placement of the wafer on a heated plate for a certain period of time. Laboratory experiments have shown that the transfer function from the heater power, $u$, to the wafer temperature, $y$, is given by

$$\frac{y(s)}{u(s)} = G(s) = \frac{0.09}{(s+0.19)(s+0.78)(s+0.00018)}$$

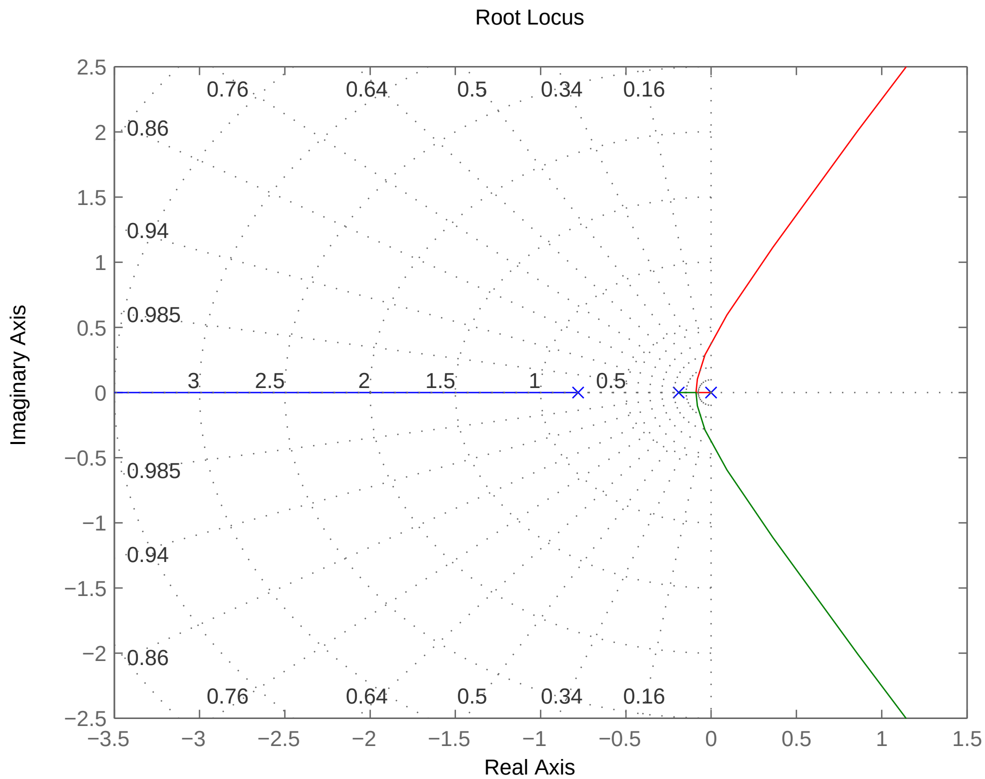
1. (a) Sketch the 180° root locus for the uncompensated system.

   (b) Using the root locus design techniques, design a *dynamic* compensator, $D(s)$, such that the system meets the following time-domain specifications

      i. $M_p \leq 5\%$
      ii. $t_r \leq 20\,\text{sec}$
      iii. $t_s \leq 60\,\text{sec}$
      iv. Steady-state error to a 1°C step input command $< 0.1$°C.
      Draw the 180° root locus for the compensated system.

**Solution:**

The uncompensated root locus is shown in the following figure.




Uncompensated root locus.

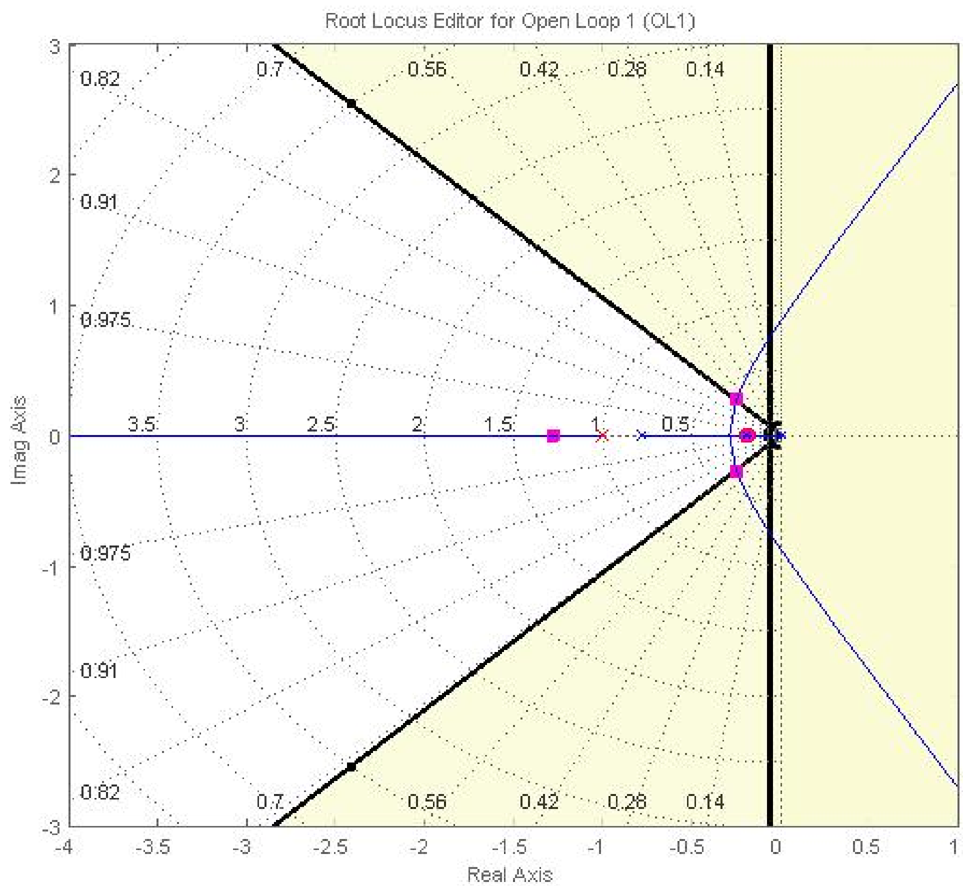
(a) First, convert the time domain specifications to $s$-plane specifications:

$$\begin{aligned} M_p &\leq 5\% \Rightarrow \varsigma \geqslant 0.707 \\ t_r &\leq 20\,\text{sec} \Rightarrow \omega_n \geqslant 0.09 \\ t_s &\leq 60\,\text{sec} \Rightarrow \sigma \geqslant 0.0767 \end{aligned}$$

(b) At this point there are several design methods one can use for this problem. **Method I**: We know that a pure integrator will improve steady-state behavior and we nearly have a pure integrator with the pole at -0.00018. Thus, we can cancel another (stable) pole with a lead network zero, and place a fast lead pole to appropriately shape the root locus. One possible lead compensator is

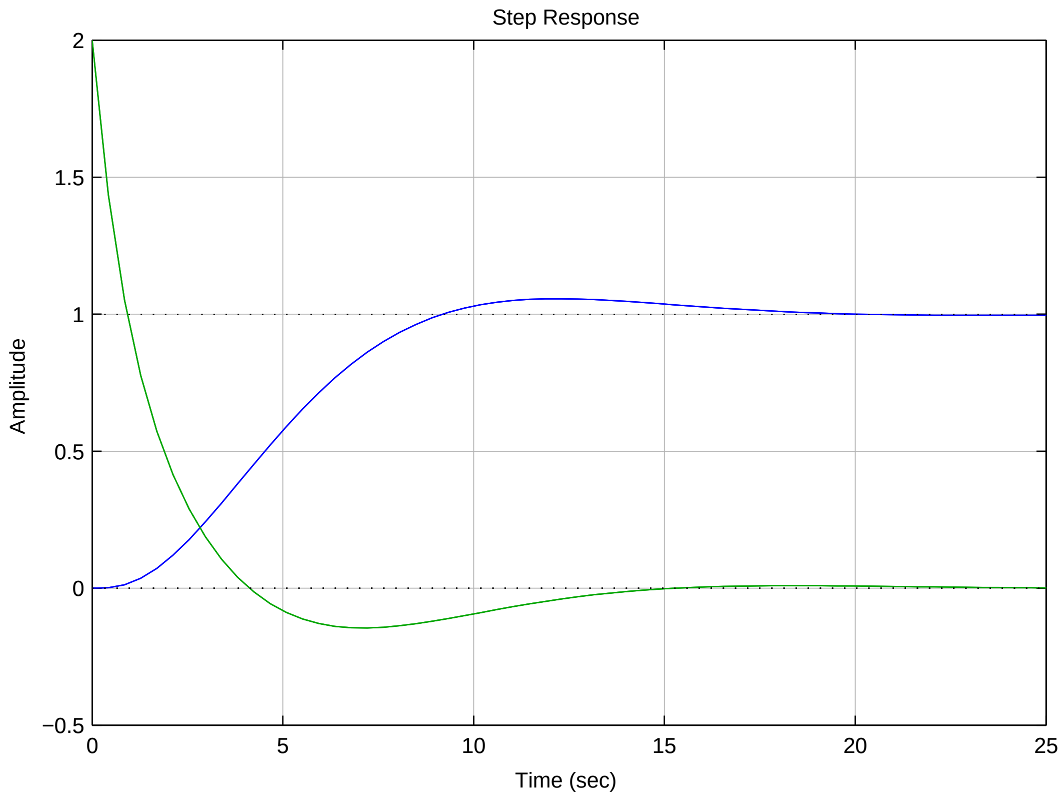
$$D_c(s) = K_c \frac{s + 0.19}{s + 1}$$

We can display the compensated root locus and s-plane regions in RLTool in the following figure.




RLTool: Compensator I.

Choosing $K_c = 2$ gives us the closed-loop poles within the specifications at $s = -0.24 \pm j0.28$. The results from RLTool follows.




RLTool: Step response ($y$) and control effort ($u$) for Compensator I.

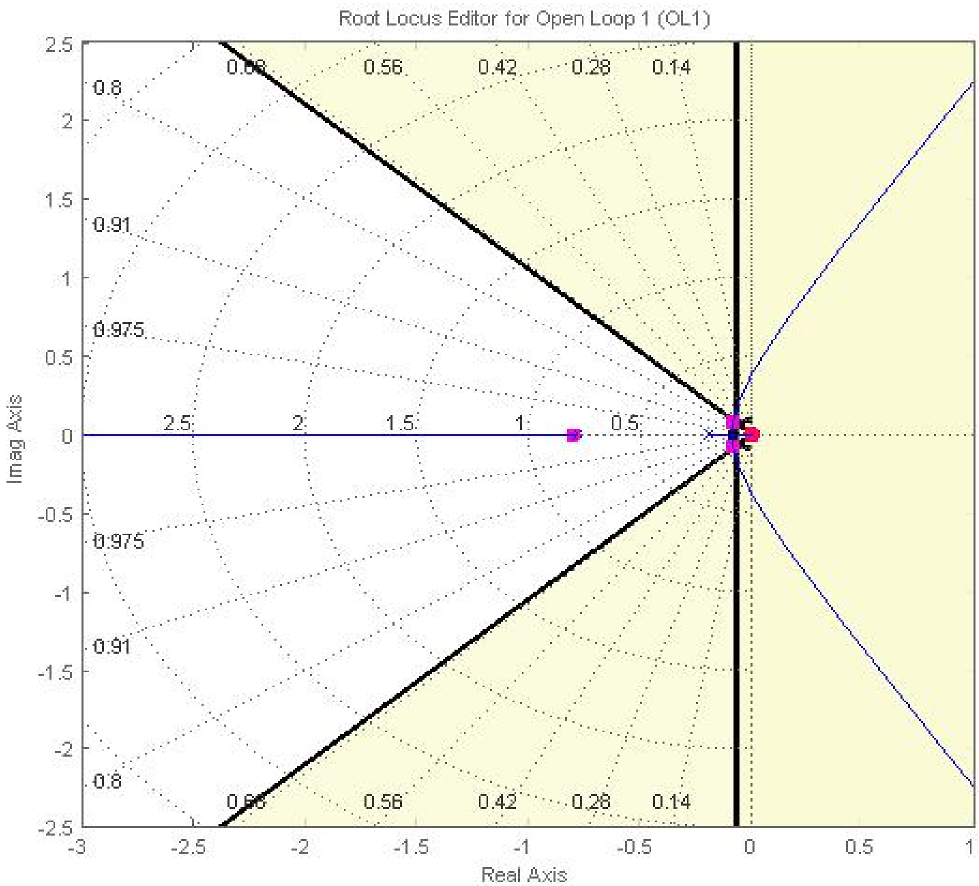
**Method II:** We can cancel out the nearly pure integrator with our compensator zero, and add a pure integrator. It is possible to achieve the time domain specifications with this compensator. The compensator becomes

$$D_c(s) = K\frac{s + 0.00018}{s}$$

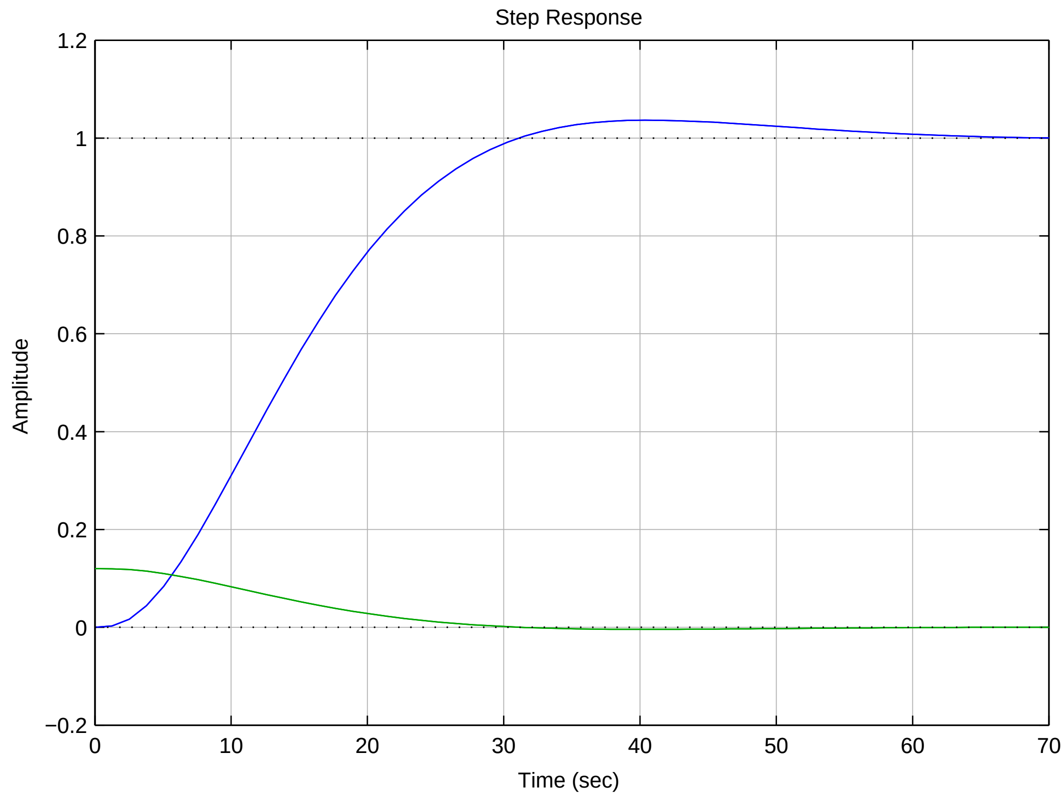
Applying the magnitude condition with the desired poles at $s = -008 \pm j0.08$, we get

$$K = 0.12$$

We can display the compensated root locus and $s$-plane regions in RLTool in the following figure.




RLTool: Compensator II.




RLTool: Step response ($y$) and control effort ($u$) for Compensator II.

26. *Excitation-Inhibition Model from Systems Biology* (Yang and Iglesias, 2005): In *Dictyostelium* cells, the activation of key signaling molecules involved in chemoattractant sensing can be modeled by the following third order linearized model. The external disturbance to the output transfer function is:

$$\frac{y(s)}{w(s)} = S(s) = \frac{(1-\alpha)s}{(s+\alpha)(s+1)(s+\gamma)}$$

where, $w$ is the external disturbance signal proportional to chemoattractant concentration, and $y$ is the output which is the fraction of active response regulators. Show that there is an alternate representation of the system with the "plant" transfer function

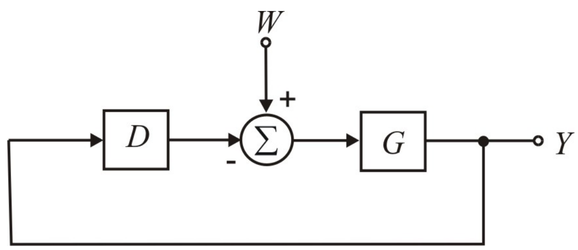
$$G(s) = \frac{(1-\alpha)}{s^2 + (1+\alpha+\gamma)s + (\alpha+\gamma+\alpha\gamma)}$$

and the "feedback regulator"

$$D_c(s) = \frac{\alpha\gamma}{(1-\alpha)s}$$

It is known that $\alpha \neq 1$ for this version of the model. Draw the feedback block diagram of the system showing the locations of the disturbance input and the output. What is the significance of this particular representation of the system? What hidden system property does it reveal? Is the disturbance rejection a robust property for this system? Plot the disturbance rejection response of the system for a unit step disturbance input. Assume the system parameter values are $\alpha = 0.5$ and $\gamma = 0.2$.

1. **Solution:** From the following figure we have:



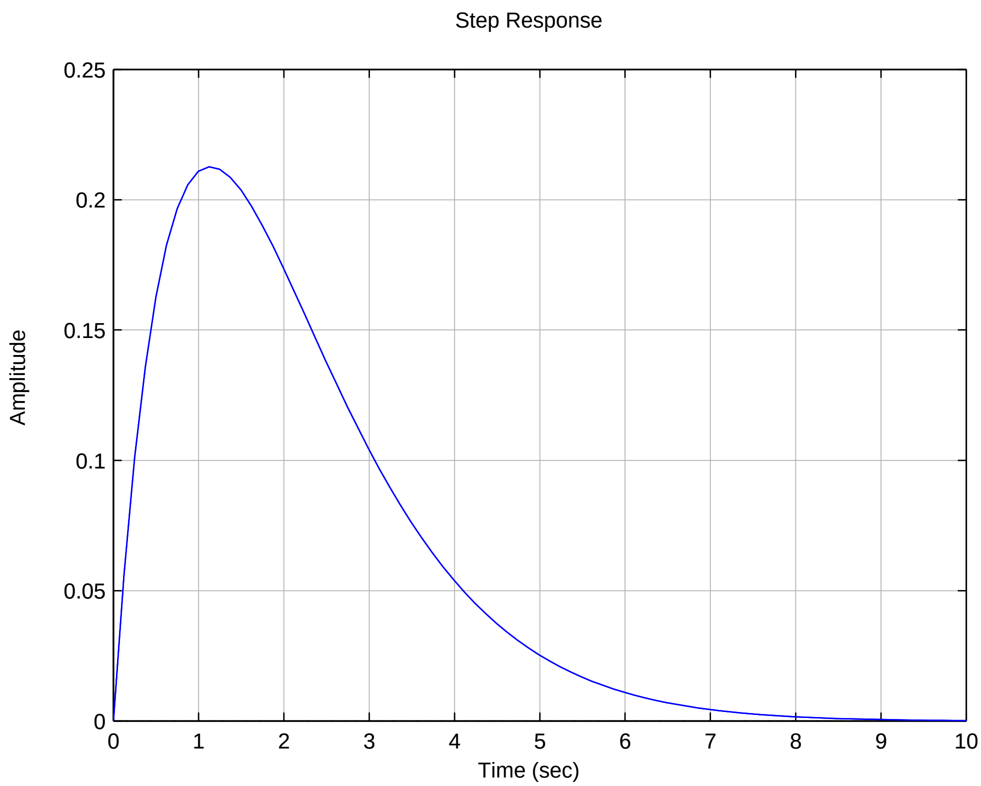

Problem 10.26: Feedback loop representation.

$$
\begin{aligned}
\frac{Y(s)}{W(s)} &= \frac{\frac{(1-\alpha)}{s^2+(1+\alpha+\gamma)s+(\alpha+\gamma+\alpha\gamma)}}{1+\frac{(1-\alpha)}{s^2+(1+\alpha+\gamma)s+(\alpha+\gamma+\alpha\gamma)}\frac{\alpha\gamma}{(1-\alpha)s}} \\
&= \frac{(1-\alpha)s}{(s+\alpha)(s+1)(s+\gamma)}
\end{aligned}
$$

The significance of this particular representation is that it reveals the internal model, namely the pure *integrator*. Hence the system is Type I with respect to disturbance rejection. It rejects constant disturbances in a robust fashion. For $\alpha = 0.5$ and $\gamma = 0.2$, we have

$$G(s) = \frac{0.5s}{s^2 + 1.7s + 0.8}$$

The disturbance response is shown in the following figure.




Problem 10.26: Disturbance rejection response for a unit step disturbance.

27. When a powerful industrial robot is used for delicate tasks such as polishing glass, it needs to be able to precisely control the force applied to the workpiece (see Figure 10.104). One way to approximately measure the applied force is to measure the deflection of a stiff spring on the robot's end-effector. The robot-workpiece interaction can be modeled as shown below, where it is assumed that the workpiece and the base are both fixed in place, and only the robot moves:

Assume that the robot's mass is $m = 8$kg, the damping friction in the robot is $b = 150$ Nsec/m and the spring constant for the force sensor is $k = 8000$N/m. The robot's motor dynamics (the transfer function from control command to applied force) can be modeled as a first-order system with a pole at -100. Hence the plant transfer function relating the applied force to the robot-workpiece interaction force is:

$$G(s) = \frac{k}{ms^2 + bs + k} \tag{35}$$

A closed-loop system is to be designed to control the robot-workpiece interaction force, as shown in Figure 10.105: (a) Sketch the open-loop step response for the combined actuator/robot system (without feedback control). (b) Let $N(s) = 0$. Sketch the root locus for the combined actuator/robot system assuming $D_c(s) = K$. Discuss the usefulness of a proportional control for decreasing the settling time. Discuss the effect of actuator dynamics on closed-loop stability with proportional

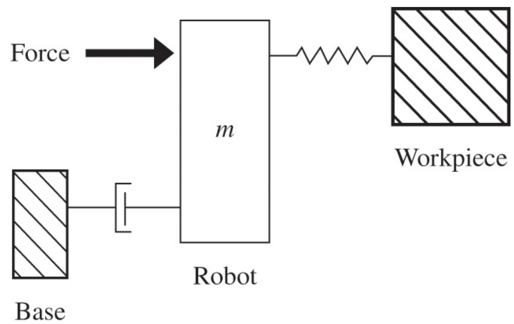



Figure 10.104: Robot system for Problem 10.27

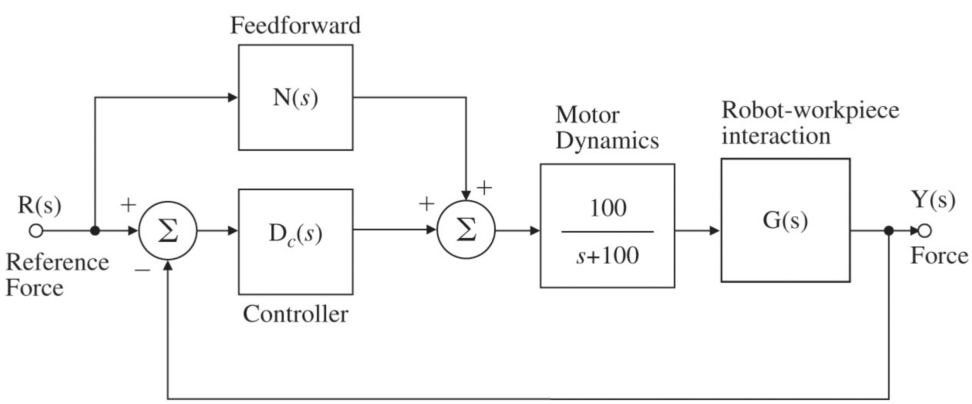
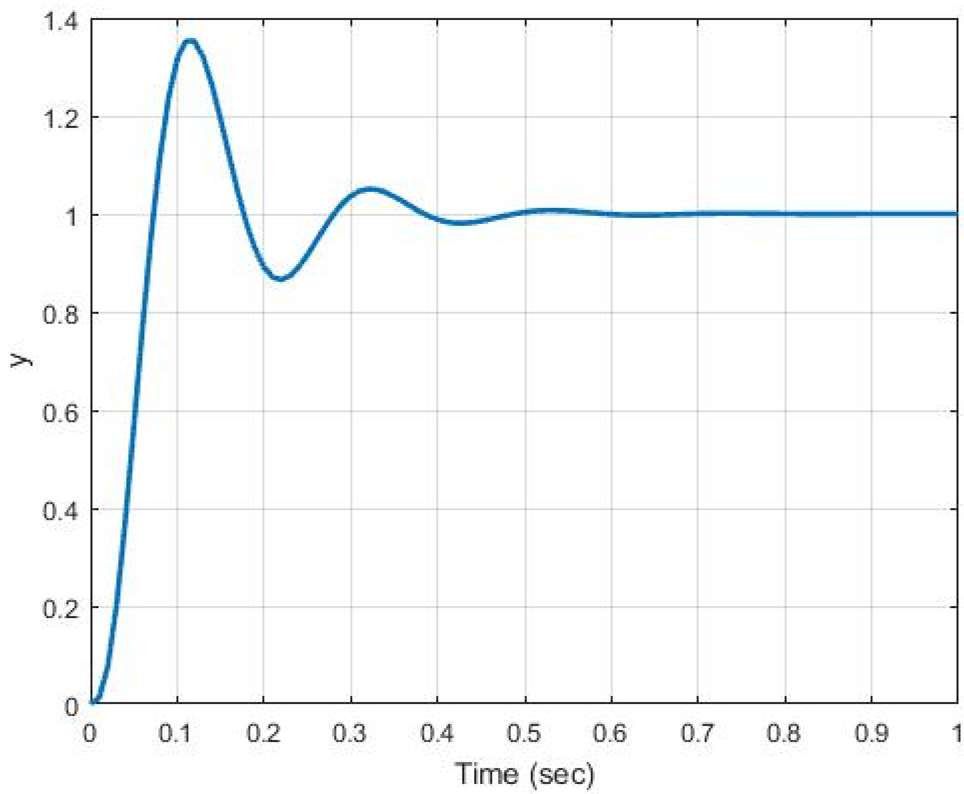



Figure 10.105: Feedback control system for Problem 10.27

control. (c) Now let $N(s) = N$ =constant. Derive the closed-loop transfer function for a generic controller $D_c(s)$. Using your derivation, explain why this feedback control structure employing the feedforward term $N(s)$, with the reference force entering the dynamics in two places, may be preferable to the standard feedback only structure for this problem. For which value of $N$ is the system able to track a step reference input with zero steady-state error?

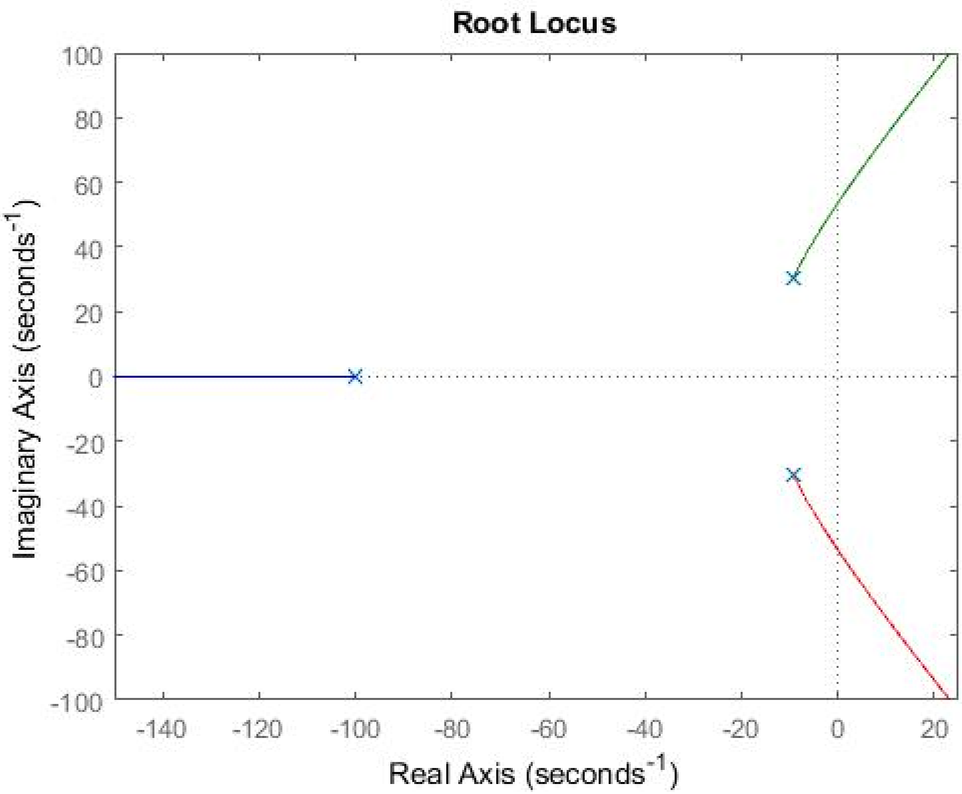
(a)

$$G_1(s) = \frac{100}{s+100}\frac{1000}{s^2+18.75s+1000} \tag{36}$$




Problem 10.27: step response.

(b)




Problem 10.27: root locus.

The presence of the actuator dynamics makes two branches of the root locus head toward the RHP.

(c)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1(s)(N(s) + D_c(s))}{1 + D_c(s)G_1(s)} \tag{37}$$

There is the possibility of adding zeros for greater control over the transient response.

$$T(\infty) = \frac{N + D_c(0)}{1 + D_c(0)} \tag{38}$$

So if $N = 1$ we will always get zero steady-state error to a step input.**10.28**. A vibration isolation strut uses a voice coil actuator. It is in parallel with a spring that has a natural frequency with the load of 10 Hz. A seismometer has been chosen for the feedback and one for feedforward. Each seismometer proof mass is supported on a spring which produces a natural frequency of the instrument of 4 Hz and a shunt resistor provides eddy current damping to give a damping factor $\varsigma = 1$. See Figures 100.106(a) and (b).

(a) Include this instrument dynamics in the sensing (feedback) loop and modify the compensation to provide a factor of 30 attenuation of ground motion at 10 Hz. (b) Assuming you can match scale factors and model parameters to 5%, how much improvement could you make to the feedback attenuation by using feedforward from a second seismometer mounted on the ground? (c) How would

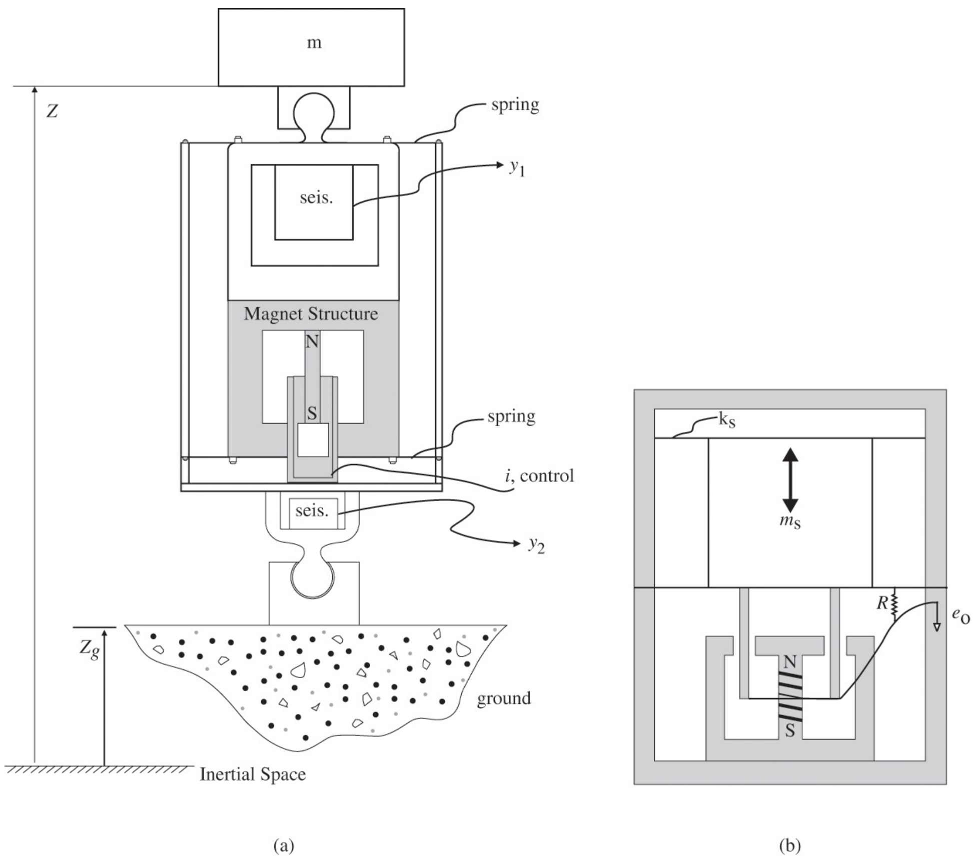



Figure 10.106: (a) Strut schematic to isolate mass m from ground motion zg, (b) Schematic of seismometer

you handle the instrument dynamics if you want to feedforward ground motion at low frequencies ($\omega$ <10 Hz)?

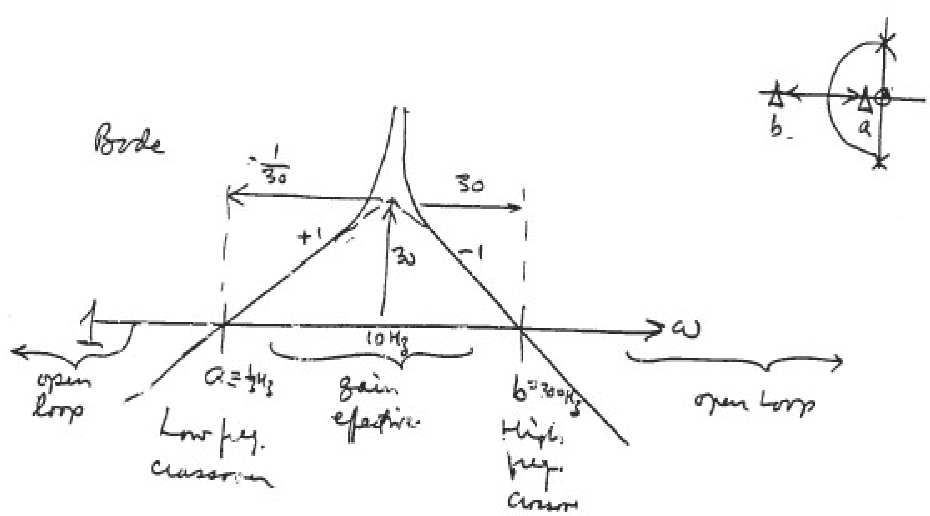
**Solution:**

(a)

$$m\ddot{z} = -k(z - z_g) + F \tag{39}$$
$$y_1 = e_{o1} = K(\dot{z}_{pm} - \dot{z}) \tag{40}$$
$$m_s\ddot{z}_{pm} = -k_s(z_{pm} - z) - b(\dot{z}_{pm} - \dot{z}) \tag{41}$$
$$y_{ff=}e_{o2} = K(\dot{z}_{pm} - \dot{z}_g) \tag{42}$$
$$m_s\ddot{z}_{pm} = -k_s(z_{pm} - z_g) - b_s(\dot{z}_{pm} - \dot{z}_g) \tag{43}$$
$$\frac{y}{z} = K\left(\frac{-m_s s^2}{m_s s^2 + b_s s + k_s}\right) \tag{44}$$
$$F = Bi \tag{45}$$
$$\tag{46}$$
$$\tag{47}$$

Control law: $i = K_P y$ for case without seismometer dynamics.

$$(ms^2 + k)z = +kz_g + BK_PK(-1)sz \tag{48}$$
$$(ms^2 + BK_PKs + k)z = kz_g \tag{49}$$
$$K_P\left(\frac{BK}{m}\right) = -\frac{s^2 + k/m}{s} \tag{50}$$




Problem 10.28(a): Bode frequency response.

If we include $H = \frac{ms^2}{ms^2+bs+k} = \frac{s^2}{(s+c)^2}$ with -1 included with $K$, we could get a +1 slope by adding a double lag compensation.

Let us say:

$$e = 3\omega_{cl} \tag{51}$$
$$d = \frac{\omega_{cl}}{3} \tag{52}$$

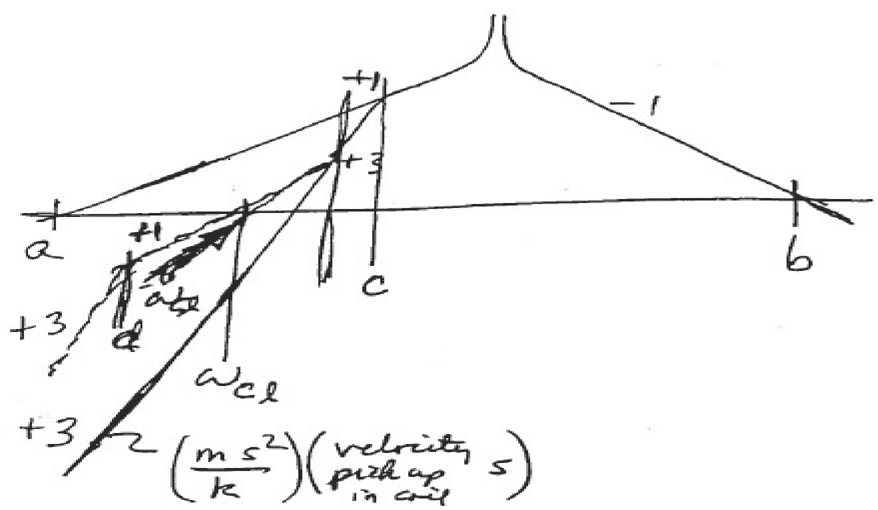
Here these frequencies would be

$$e \simeq 2.5Hz \tag{53}$$
$$d \simeq 0.25Hz \tag{54}$$

This is close enough to $e \to c$ $(4Hz)$ that one might as well make the compensation break at $c$

$$F = \left[-K\left(\frac{s^2}{s+c^2}\right)\right]\left[\frac{(s+c)^2}{(s+d)^2}\right]K_P \tag{55}$$

In the above Equation, the first term is the seismometer and the second term is the compensation. $c = 4Hz$ and crossover as before at $0.3Hz$.




Now it will look about the same– may need to reduce $d'$ to 0.05Hz or so if PM at $\omega_{cl}$ is too small.
(b)

$$i = K_P y_1 - kz_g \approx K_P y_1 - K_{FF}\int y_2 dt \tag{56}$$
$$(ms^2 + BK_P Ks + k)z = kz_g + K_{FF}\int(-\dot{z}_g)dt \tag{57}$$
$$= (k - K_{FF}K)z_g = \left(\frac{k - K_{FF}K}{k}\right)kz_g \tag{58}$$

If we can match $k$ and $K_{FF}K$ to 5% you should get a 20 : 1reduction in all frequencies for which the model works.

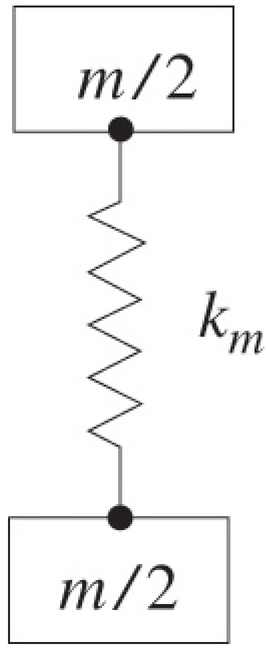



Figure 10.107: Schematic of elastic load for Problem 10.29

(c)

$$K_{FF}(s)\int y dt = \frac{K_{FF}}{s} y \equiv z_g \tag{59}$$

$$\frac{K_{FF}}{s}\left(\frac{\left(-ms^2\right)s}{ms^2+bs+k}\right)z_g = z_g \tag{60}$$

$$K_{FF}(s) = \frac{ms^2+bs+k}{m(s+f)^2} \tag{61}$$

$f$ should be below the lowest frequency for good fidelity.

**10.29**. Suppose that the load in Problem 10.28 is now elastic. Model the isolated mass as having two parts separated by a spring stiffness such that the free vibration frequency is 50 Hz as shown in Figure 10.107. (a) How would you alter your feedback control law from Problem 10.28? (b) What effect would this have on your feedforward implementation? (c) Would you need another sensor to control $z_{2\in}$, the position of the upper half of the isolated mass? What would you measure? How would you put it into the feedback control law?

**Solution:**

(a)

$$\frac{m}{2}\ddot{z}_2 = -k_m(z_2 - z_1)\sqrt{\frac{2mk_m}{2}} \tag{62}$$

$$\frac{m}{2}\ddot{z}_1 = -k_m(z_1 - z_2) + F - k(z_1 - z_g) \tag{63}$$

$$\begin{bmatrix} \frac{m}{2}s^2 + k_m & -k_m \\ -k_m & \frac{m}{2}s^2 + k_m + k \end{bmatrix} \begin{bmatrix} z_2 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ F + kz_g \end{bmatrix} \tag{64}$$

$$\frac{z_1}{F} = \frac{\frac{ms^2}{2} + k_m}{\left(\frac{m}{2}s^2 + k_m\right)\left(\frac{m}{2}s^2 + k_m + k\right) - k_m^2} \tag{65}$$

$$= \frac{\left(\frac{m}{2}s^2 + k_m\right)}{\left(\frac{m}{2}\right)\left(s^4 + [(2k_m + k)]\frac{2}{m}s^2 + \frac{4kk_m}{m^2}\right)} \tag{66}$$

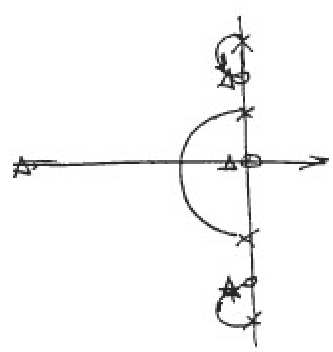
and feedback

$$y = sz_1 \tag{67}$$

two real roots and 2 roots near zeros

$$s = \pm j\sqrt{\frac{2k_m}{m}} \tag{68}$$

which is frequency of $m/2$ on $k_m$ viz. feedback locks $z_1$ to I.S. so you can get good control on $z_1$.



Root locus for Problem 10.29(a).

(b) No change.

(c) Yes. Measure $\dot{z}_2$ or $(z_2 - z_1)$ or some measure of energy in motion of $z_2$ to damp.

If you measure $\dot{z}_2$ you want force $F_2$ on $\frac{m}{2}$ at $z_2$. $F_2 = -b\dot{z}_2$. You do not have a force directly on $z_2$ . So you must form

$$F_2 = -k_m(z_2 - z_1) \tag{69}$$

by commanding $z_1$ and measuring $z_2$ (or $\dot{z}_2$)

$$-b\dot{z}_2 = -k_m z_2 + k_m z_{1c} \tag{70}$$

$$\dot{z_{1c}} = \dot{z}_2\left(1 - \frac{b}{k}s\right) \tag{71}$$

Note this suggests a RHP zero–something you probably would not have thought of just looking at a rootlocus for $\dot{z}_{2/F}$. Then

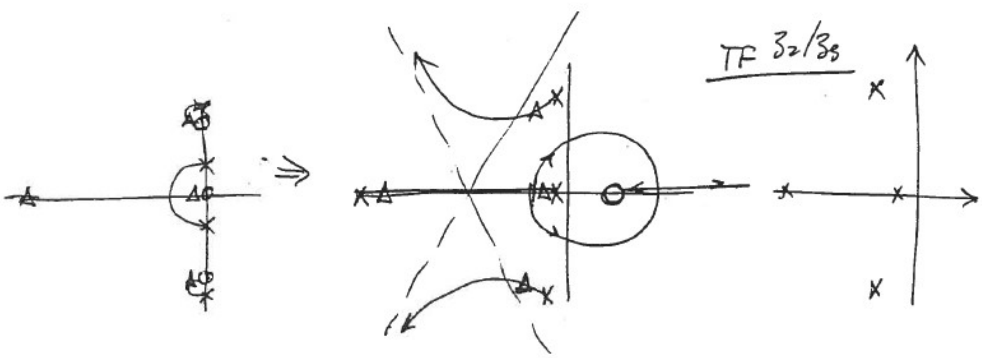
$$F = -K_P\dot{z}_1 + K_D(\dot{z_{1c}} - \dot{z}) \tag{72}$$

$$= -(K_P + K_D)\dot{z}_1 + K_D\dot{z}_2\left(1 - \frac{b}{k}s\right) \tag{73}$$

$$= -(K_P + K_D)y_1 + K_D y_2\left(1 - \frac{b}{k}s\right) \tag{74}$$

$$\begin{bmatrix} \frac{m}{2}s^2 + k_m & -k_m \\ -k_m - K_D\left(1 - \frac{b}{k}s\right) & \frac{m}{2}s^2 + K_P s + k_m + k \end{bmatrix}\begin{bmatrix} z_2 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ kz_g \end{bmatrix} \tag{75}$$

$$\frac{z_2}{z_g} = \frac{k_m k}{\Delta(s)} \tag{76}$$




Root loci for Problem 10.29(c).

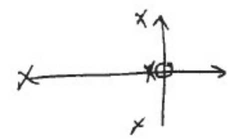
Other implementation: make control additive

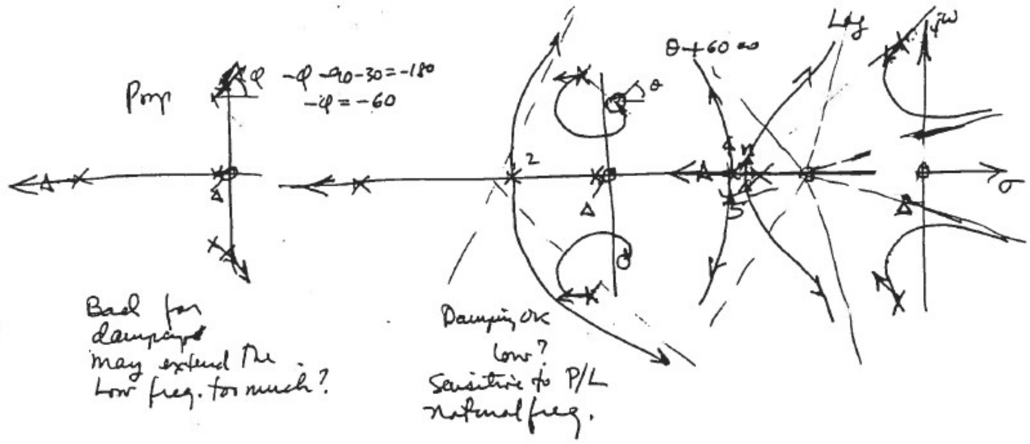
$$F = -K_P\dot{z}_1 + K_2(s)\dot{z}_2 \tag{77}$$

If we close the loop on the first term above, then

$$\frac{\dot{z}_2}{F} = -\frac{k_m}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)}s \tag{78}$$

Then use proportional control first and then insert notch filters.






Problem 10.29(c).

**10.30** The six transfer functions for the Mikrokopter quadrotor (Bergamasco and Lovera, 2014) identified from flight experiments are:

$$G_h = \frac{az}{f_h} = \frac{1.2498(s+0.3451)}{(s+16.49)(s+5.309)(s+1.933)} \tag{79}$$

$$G_\psi = \frac{r}{u_\psi} = \frac{0.077646(s+5.475)(s-0.2086)}{(s+11.03)(s^2+0.2838s+0.06947)} \tag{80}$$

$$G_{\theta 1} = \frac{q}{u_\theta} = \frac{0.22016(s+0.2579)(s-0.2596)}{(s+1.865)(s^2-1.285s.067)} \tag{81}$$

$$G_{\theta 2} = \frac{ax}{u_\theta} = \frac{-0.011659(s-3.271)(s+3.681)}{(s+1.865)(s^2-1.285s+8.067)} \tag{82}$$

$$G_{\varphi 1} = \frac{p}{u_\varphi} = \frac{-0.20194(s^2+0.09235s+0.2532)}{(s+1.82)(s^2-1.388s+10.02)} \tag{83}$$

$$G_{\varphi 2} = \frac{ay}{u_\varphi} = \frac{-0.00359(s-9.182)(s+4.164)}{(s+1.82)(s^2-1.388s+10.02)} \tag{84}$$

where $\varphi$ is roll angle, $\theta$ is pitch angle, $\psi$ is yaw angle and $p$= $\varphi$= roll rate, = = pitch rate, $r$ = $\psi$ = yaw rate as discussed in Chapter 2. $ax$, $ay$, and $az$ are the measurements of the components of the acceleration of the quadrotor along the three body axes. $f_h$ is the control input for the vertical (up and down) motion. The outputs provided by the inertial sensors are $ax$, $ay$, $az$, $p$, $q$, and $r$. (a) Find the order of the system and the number of transmission zeros at infinity for each of the six transfer functions. (b) Find the DC gain for each of the six transfer functions. (c) Use Matlab to plot the unit impulse and step responses for each of the six transfer functions. Is there anything peculiar about the yaw step response? (d) Classify each of the six transfer functions with regard to stability, minimum-phase or non-minimum phase system.

**Solution:**

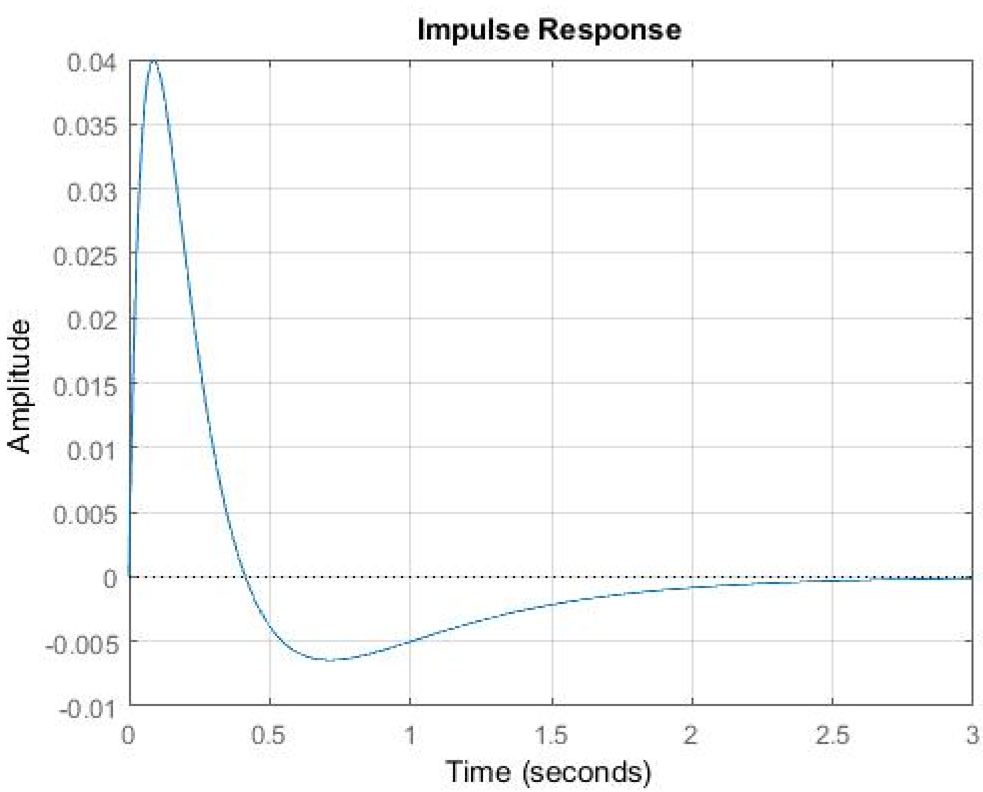
(a)

| order | transmission zeros at infinity |
|---|---|
| $n = 3$ | $n - m = 3 - 1 = 2$ |
| $n = 3$ | $n - m = 3 - 2 = 1$ |
| $n = 3$ | $n - m = 3 - 2 = 1$ |
| $n = 3$ | $n - m = 3 - 2 = 1$ |
| $n = 3$ | $n - m = 3 - 2 = 1$ |
| $n = 3$ | $n - m = 3 - 2 = 1$ |

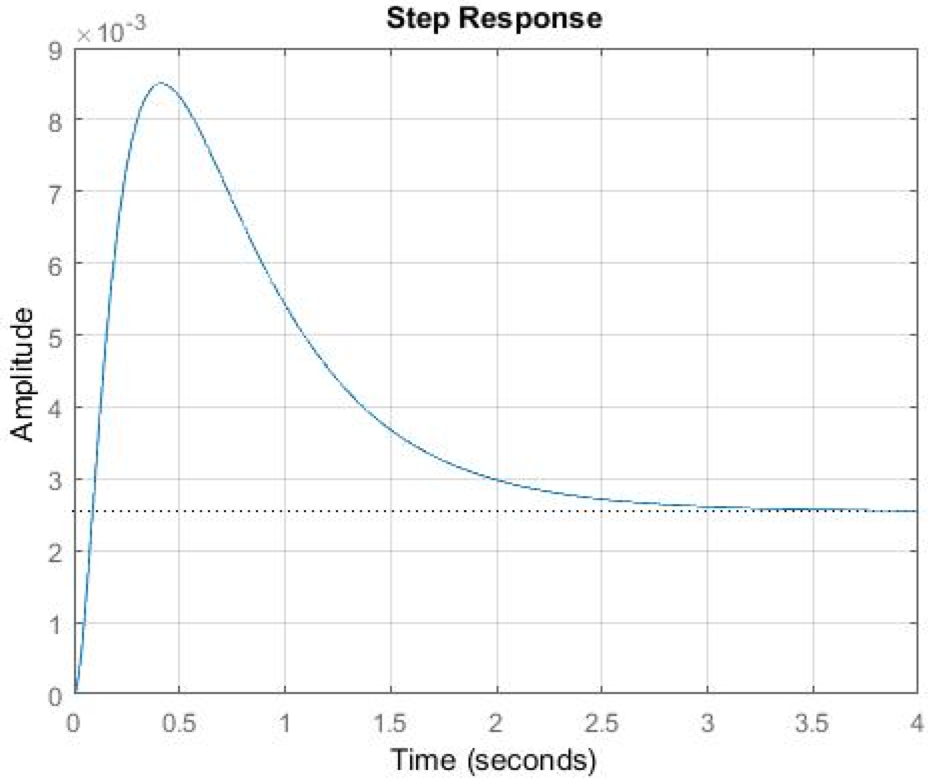
(b)

| DC Gain |
|---|
| 0.0025 |
| $-0.1157$ |
| Unstable |
| Unstable |
| Unstable |
| Unstable |

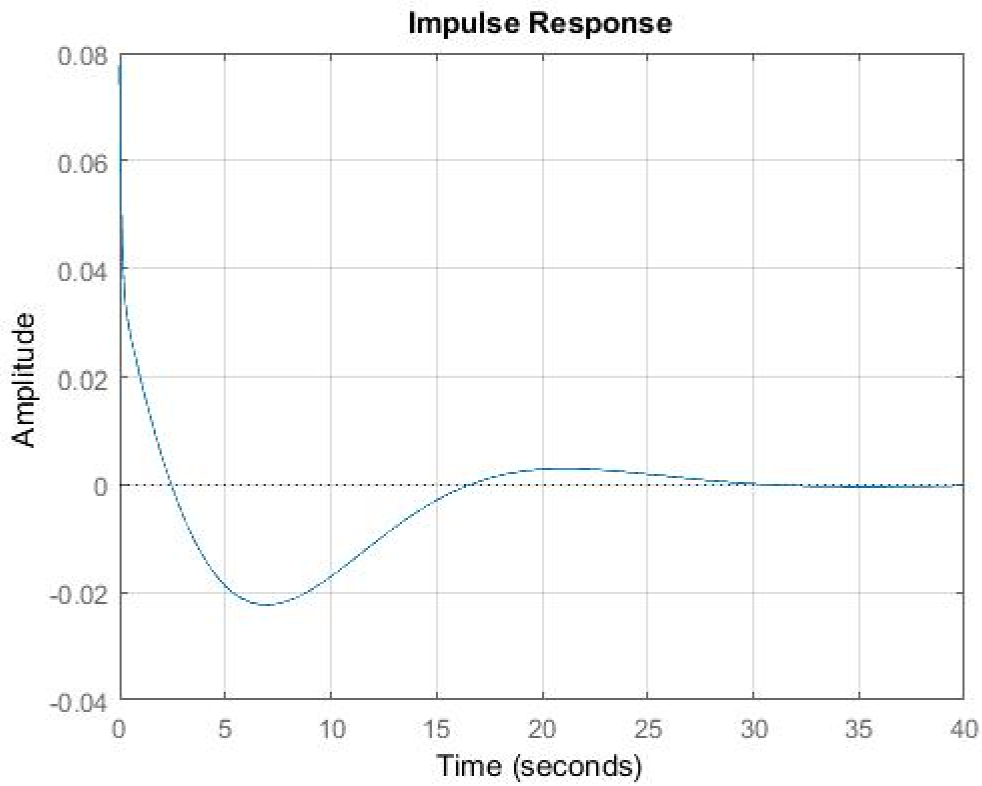
(c)



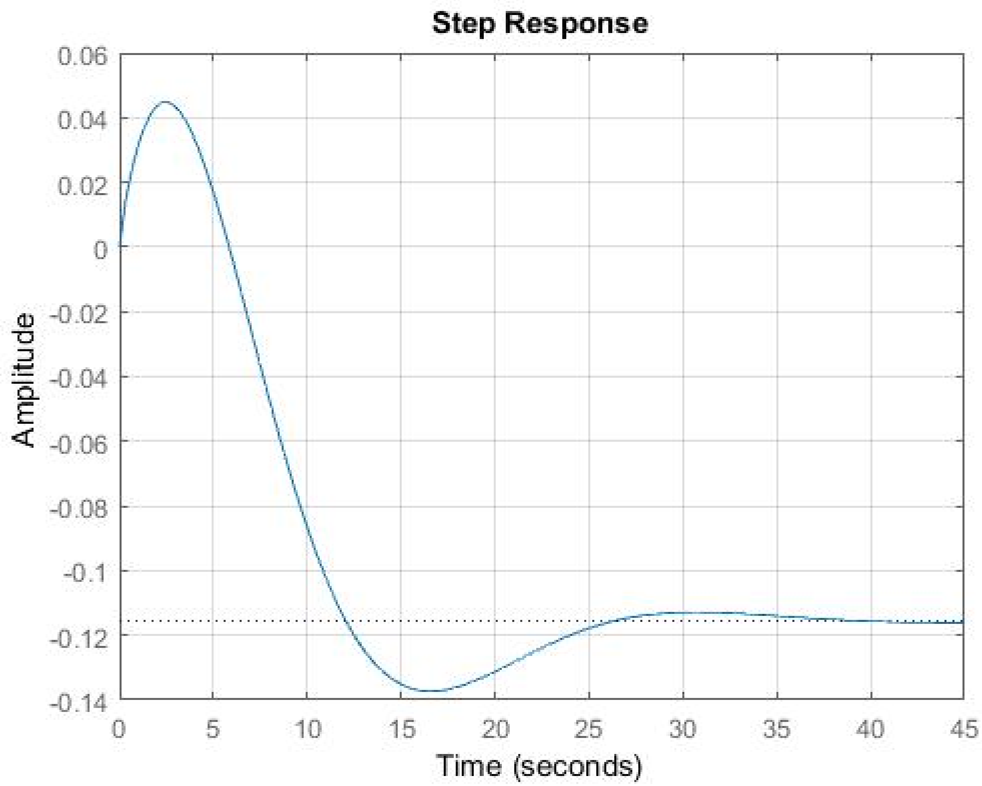

Problem 10.30(c): Impulse response for Gh.




Problem 10.30(c): step response for Gh.




Problem 10.30(c): Impulse response for Gpsi.




Problem 10.30(c): step response for Gpsi showing non-minimum phase behavior.

The others are unstable.

| | Stability | MP/NMP |
|---|---|---|
| | Stable | MP |
| | Stable | NMP |
| (d) | Unstable | NMP |
| | Unstable | NMP |
| | Unstable | MP |
| | Unstable | NMP |

**10.31** For the quadrotor Problem 10.30, (a) Find a second-order transfer function approximation for the third-order transfer function from, $f_h$ to $az$. (b) Find a second-order transfer function approximation for the third-order transfer function from $u_\psi$ to $r$. For each part, compare step responses (using Matlab) to demonstrate how good your approximation is as compared to the original transfer function.

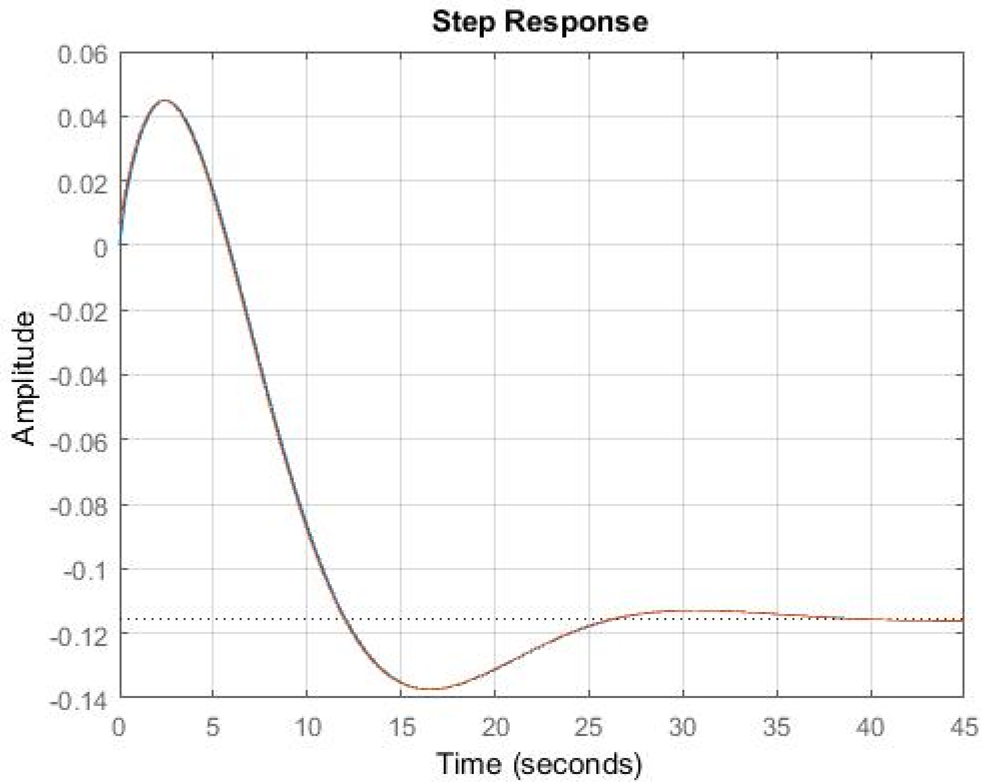
**Solution:**

(a)

$$G_{h1} = \frac{1.2498(s+0.3451)}{(16.49)(s+5.309)(s+1.933)} \tag{85}$$

(b)

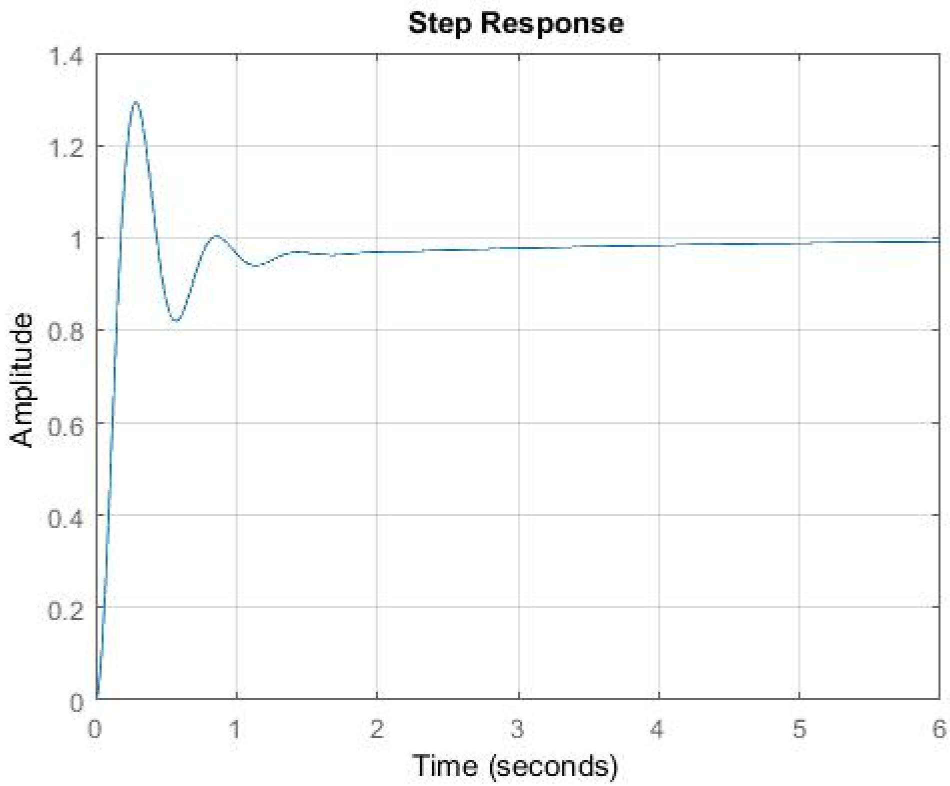
$$G_{\psi 1} = \frac{0.077646(s+5.475)(s-0.2086)}{(11.03)(s^2+0.2838s+0.06947)} \tag{86}$$




(b) **10.32** For the quadrotor Problem 10.30, design a dynamic controller (PI or PID) for the transfer function from $f_h$ to $az$ so that the rise time ($t_r$) is one sec or less and there is zero steady-state error to a step reference input. Use Matlab to show the resulting closed-loop step response for your design, and demonstrate that the design specifications have been met.

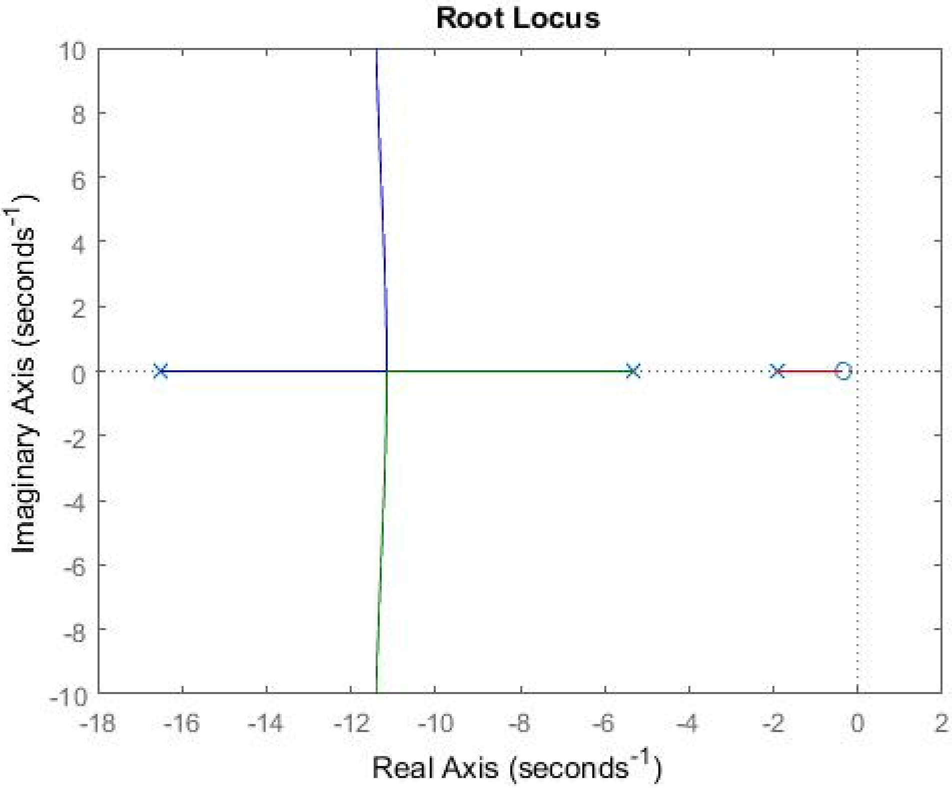
**Solution:**

$$D_c(s) = 100\frac{(s+16.49)}{s} \tag{87}$$



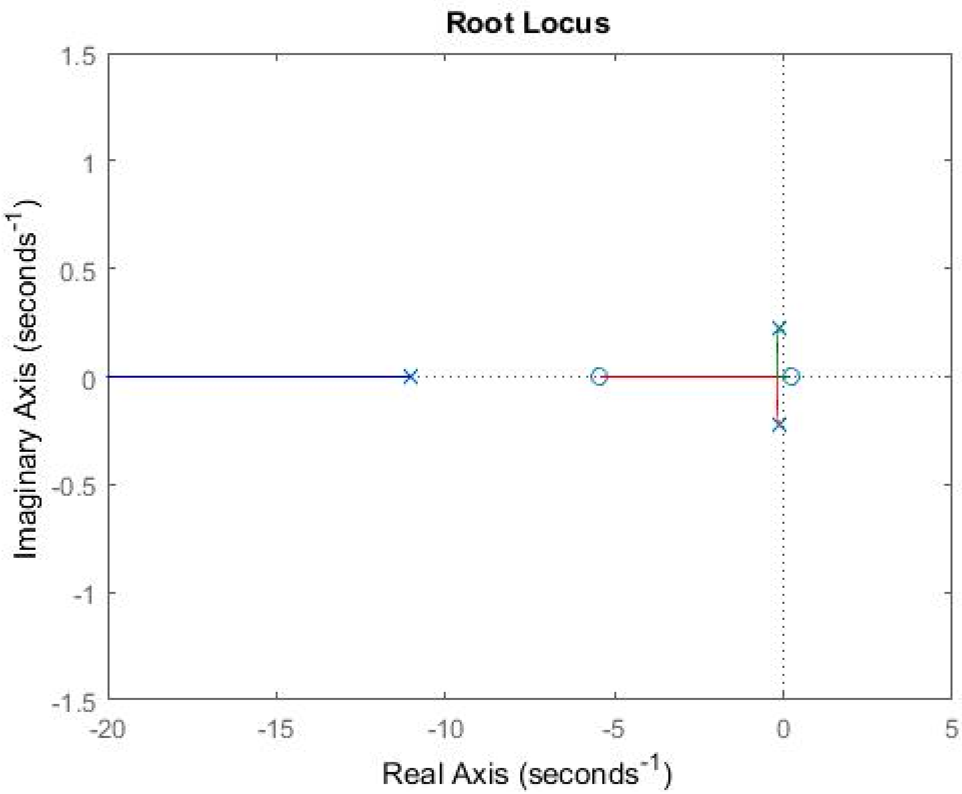

**10.33** For the quadrotor Problem 10.30, draw the 180° root locus for the six transfer functions for this system using Matlab. In each case, specify the range of the root locus gain, $K$, for which the closed-loop system is stable.

(a) $K > 0$.



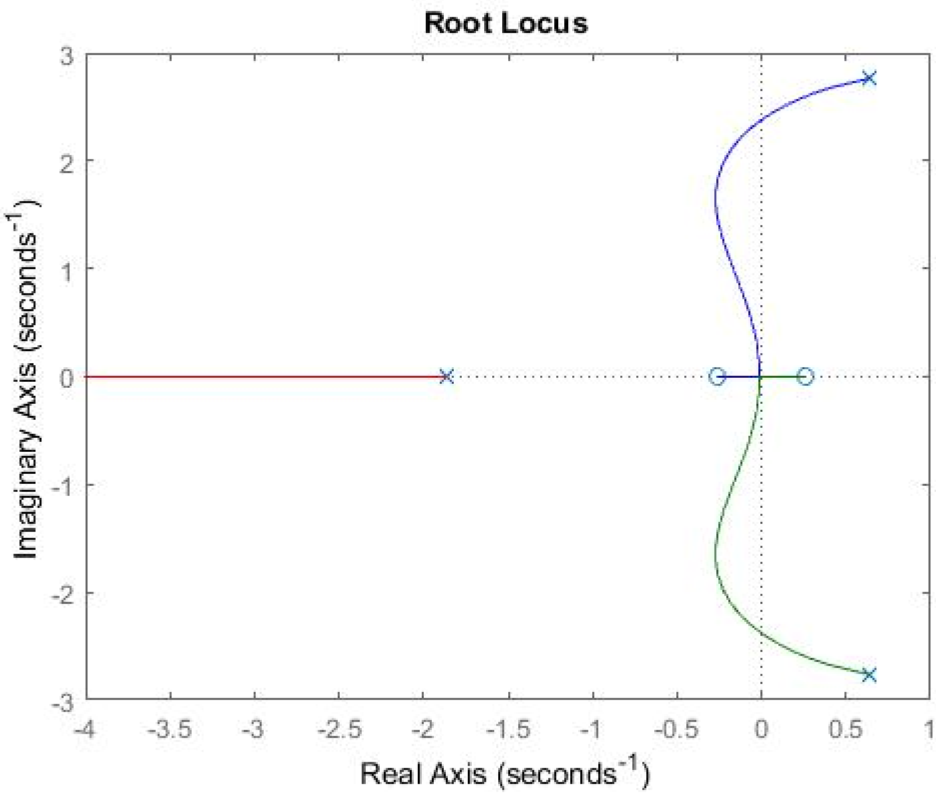

Problem 10.33(a): root locus.

$(b) K < 8.64.$

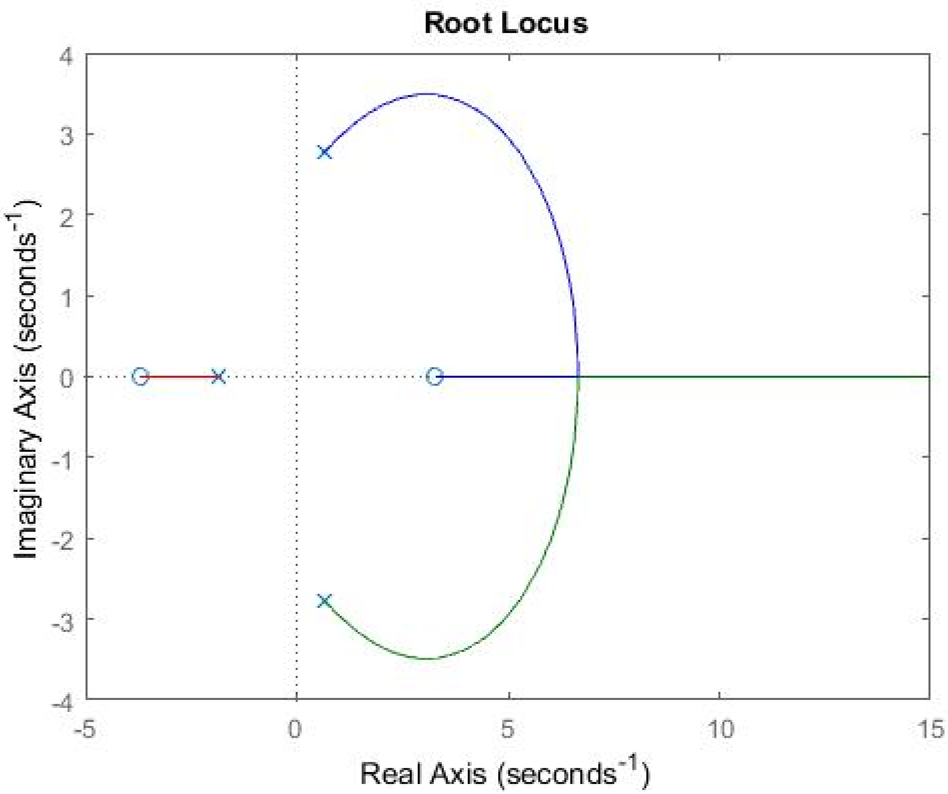



Problem 10.33(b): root locus.

(c) $9.36 < K < 1020.7$.

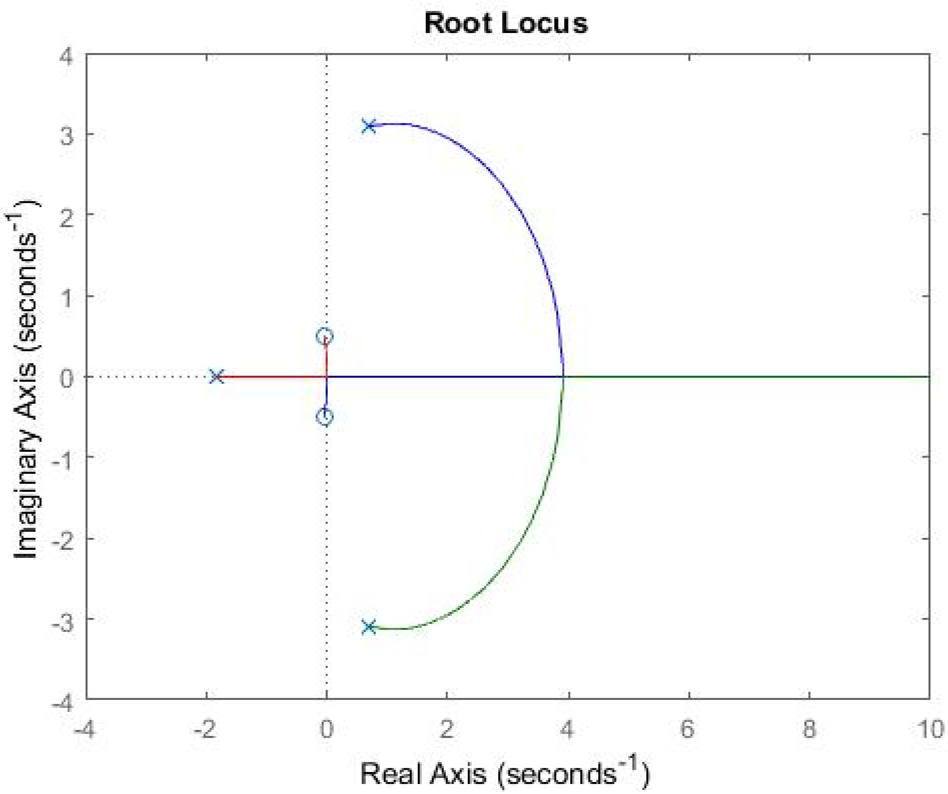



Problem 10.33(c): root locus.



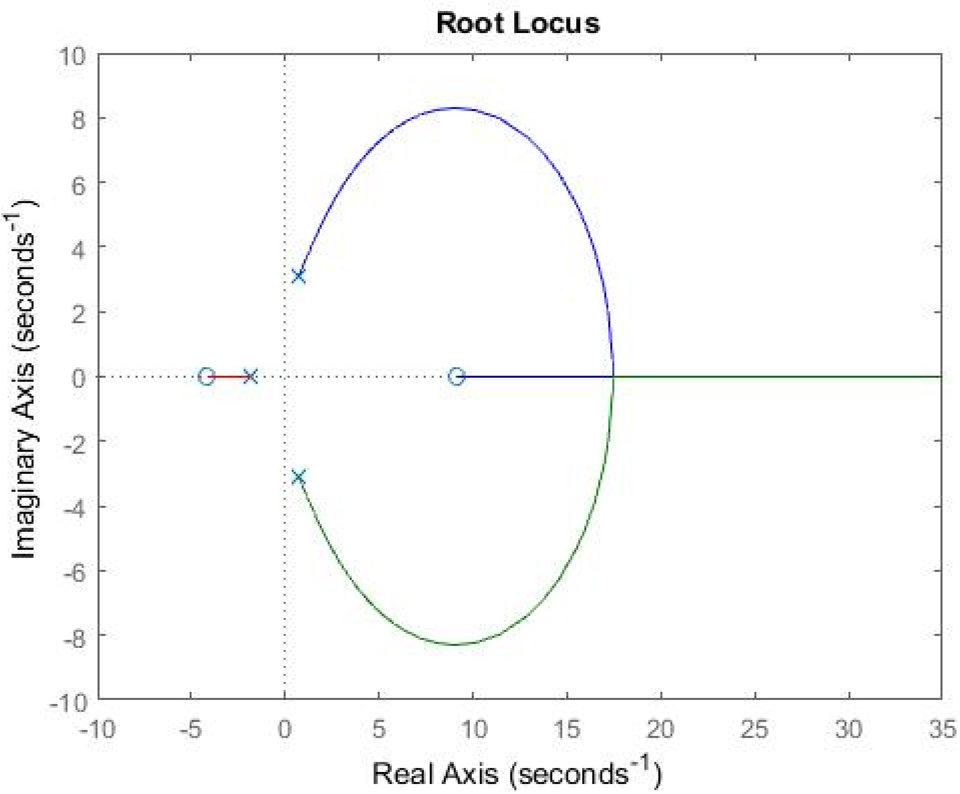

Problem 10.33(d): root locus.

(d) Unstable for $K > 0$.




Problem 10.33(e): root locus.

(e) Unstable for $K > 0$.



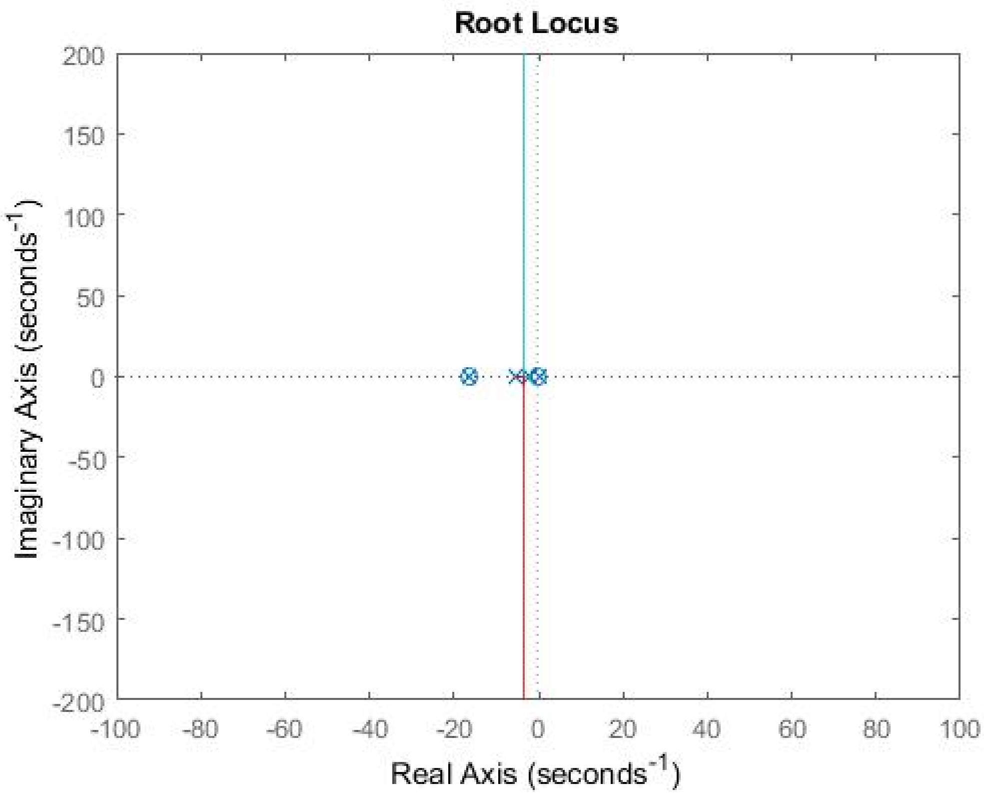

Problem 10.33(f): root locus.

(f) Unstable for $K > 0$.

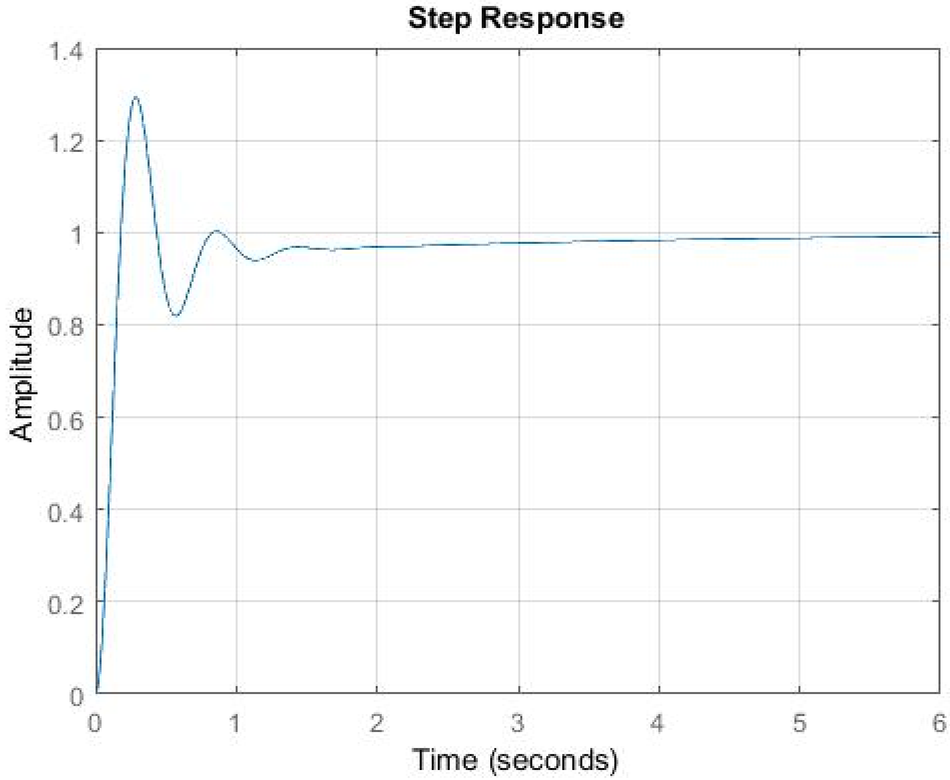
**10.34** For the quadrotor Problem 10.30, using root locus techniques, design a dynamic controller for the transfer function from $f_h$ to $az$ so that the rise time ($t_r$) is one sec or less and there is zero steady-state error to a step reference input. Use Matlab to show the resulting closed-loop step response for your design, and demonstrate that the design specifications have been met.

**Solution:** Let us use the same $D_c(s)$ as in Problem 10.32:

$$D_c(s) = 100\frac{(s + 16.49)}{s} \tag{88}$$

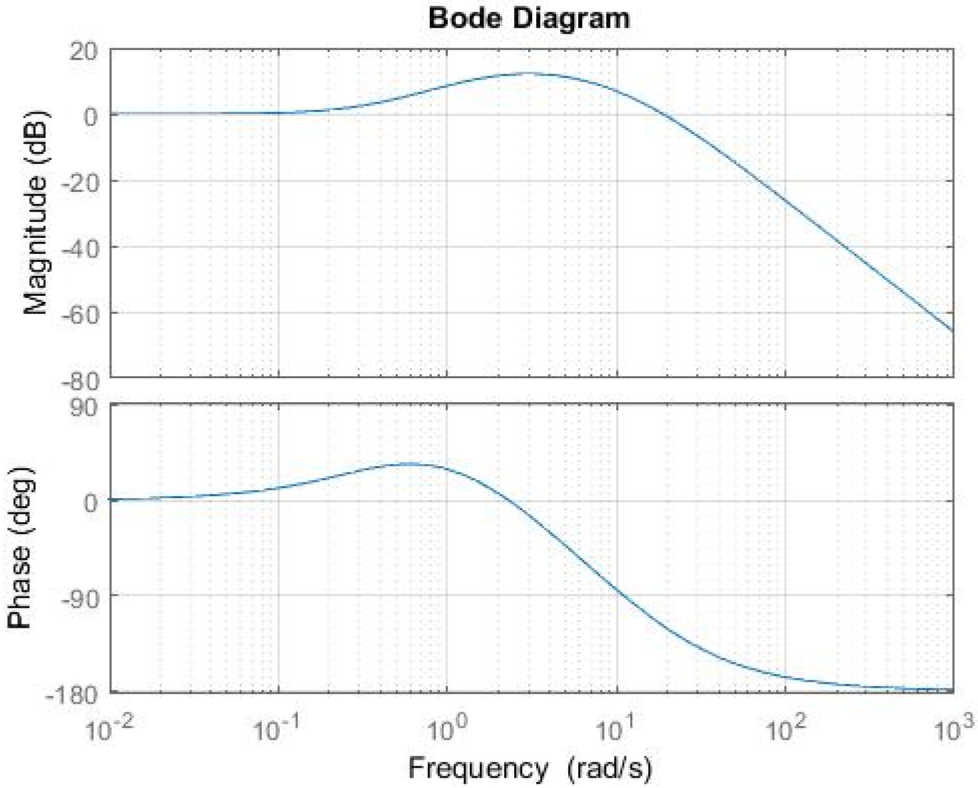



Problem 10.34: root locus.




Problem 10.34: step response.

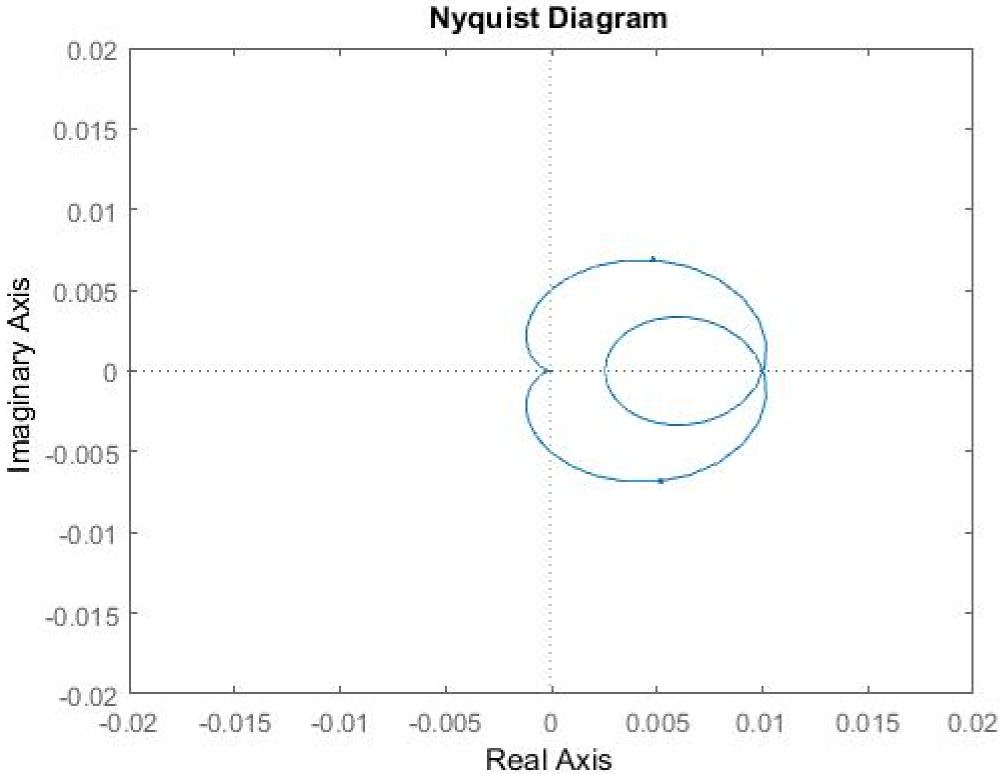
**10.35** For the quadrotor Problem 10.30, draw the Bode plot for the transfer function from $f_h$ to $az$. Adjust the transfer function gain so that the low-frequency gain is unity (0-db) prior to plotting the Bode frequency response.




Problem 10.35: Frequency response.

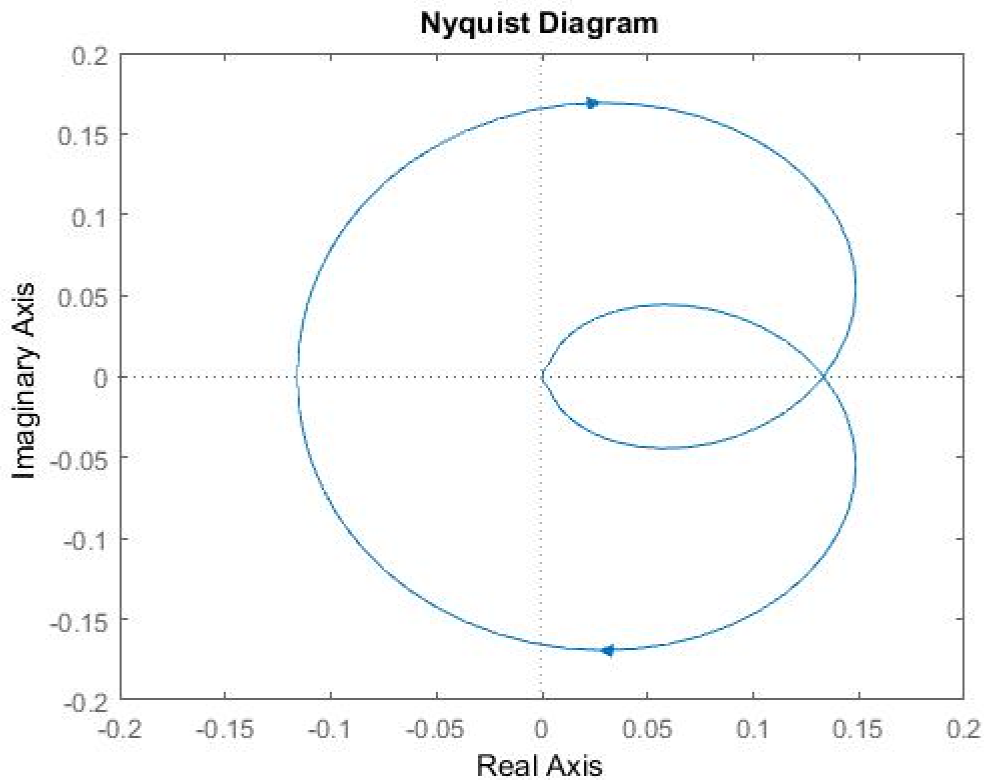
**10.36** For the quadrotor Problem 10.30, draw the Nyquist plots for the six transfer functions for this system using Matlab. In each case, specify the range of the gain, $K$ (both positive and negative), for which the closed-loop system is stable.

(a) $-\frac{1}{K} < 0$ or $-\frac{1}{K} > 0.01$. So $K > 0$.



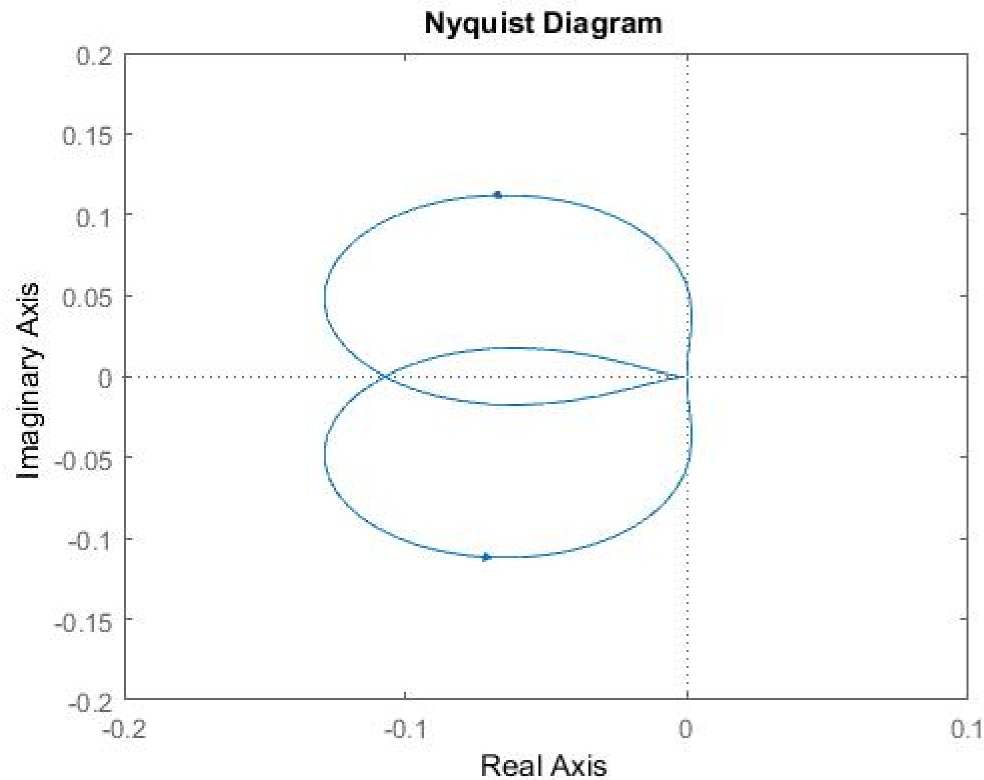

Problem 10.36(a) : Nyquist plot.

(b) $-\frac{1}{K} < -0.116$ or $-\frac{1}{K} > 0.113$. So $-7.5188 < K < 8.6207$.




Problem 10.36(b) : Nyquist plot.

(c) $-\frac{1}{K} < -0.00116$ or $-\frac{1}{K} > -0.107$. So $9.3458 < K < 862$.




Problem 10.36(c) : Nyquist plot.

(d) $-\frac{1}{K} < 0.101, -\frac{1}{K} < 0.00933$ or $-\frac{1}{K} > 0.018$. $-100 < K < -56$. (e) $-\frac{1}{K} < 0.101$ or $-\frac{1}{K} > 0$. So $K < -9.901$.

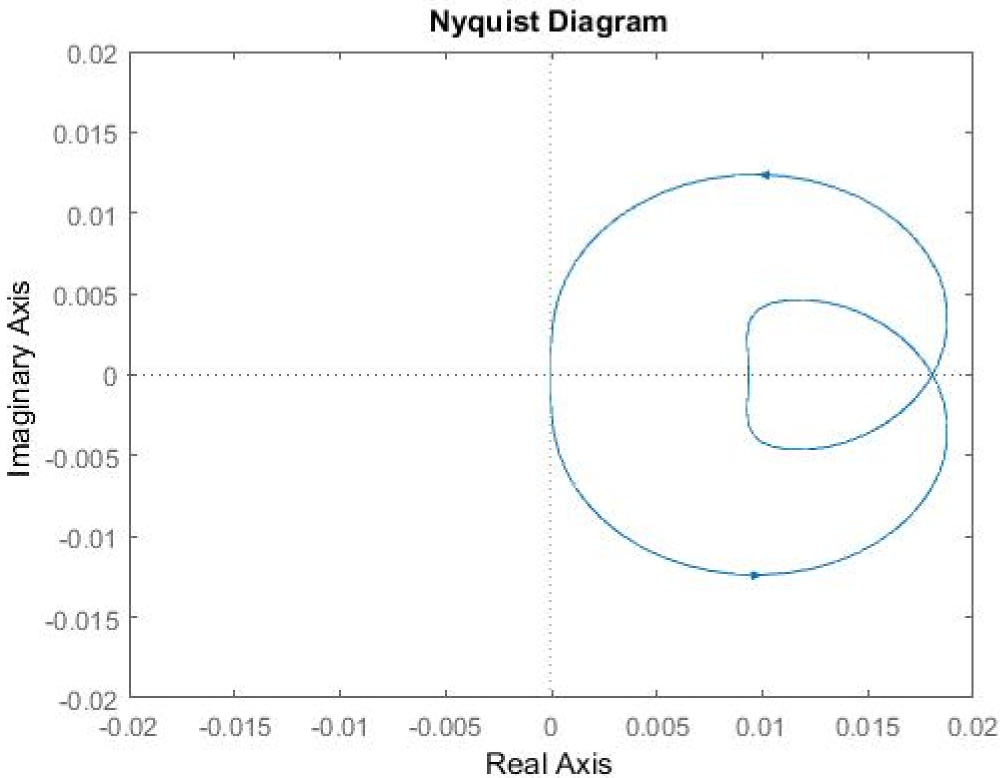
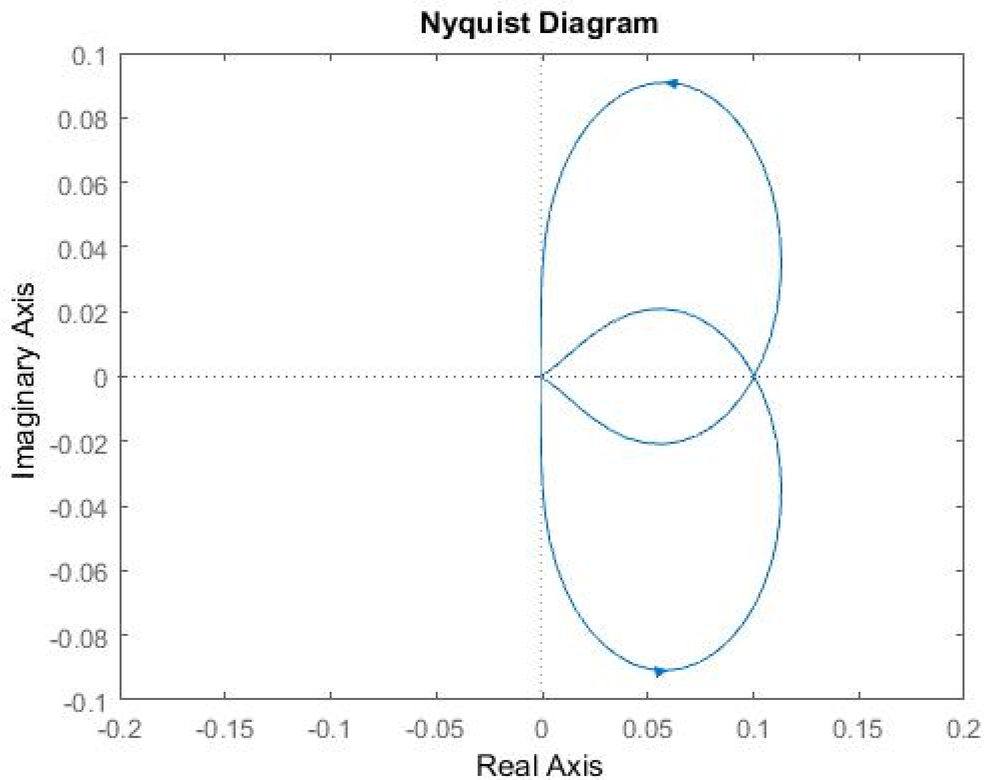
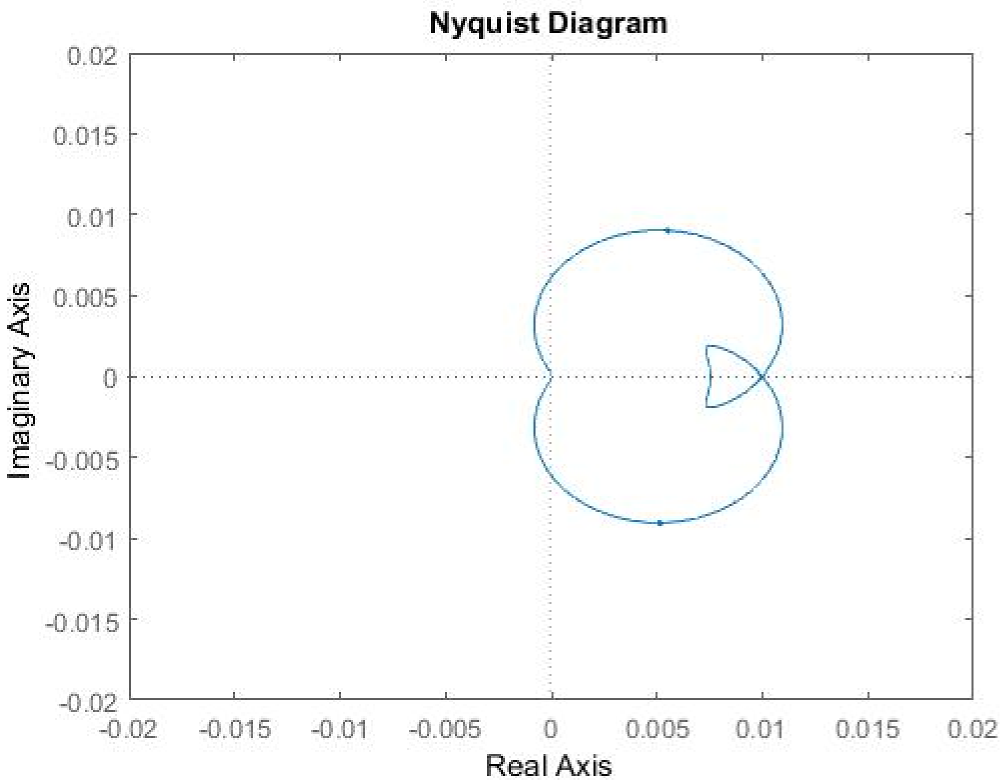



Figure 10.108: Problem 10.36(d) : Nyquist plot.




Problem 10.36(e) : Nyquist plot.

(f) $-\frac{1}{K} < 0.00997 or - \frac{1}{K} > 0.00752$so $-132.9787 < K < -100.3009$.

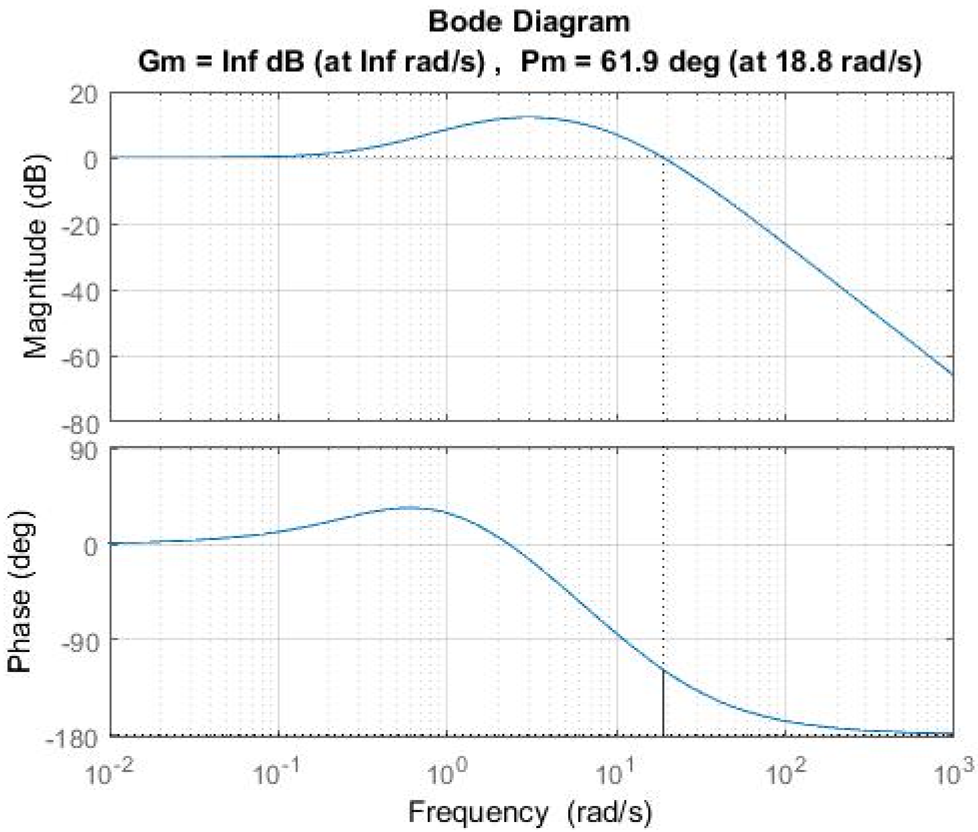



Problem 10.36(f) : Nyquist plot.

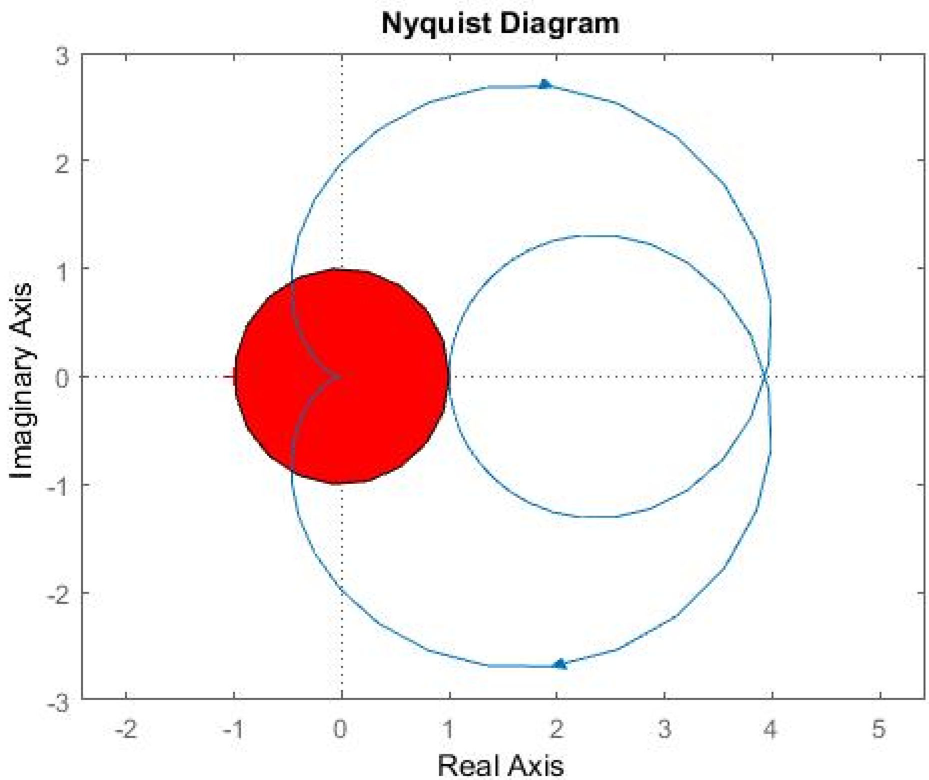
**10.37** For the quadrotor Problem 10.30, draw the Bode plot for the transfer function from $f_h$ to $az$. Adjust the transfer function gain so that the low-frequency gain is unity (0-db) prior to plotting the Bode frequency response. Compute the values of the PM and GM from the Bode plot, and the corresponding Nyquist plot. How do the values from Bode and Nyquist compare?

**Solution:**

From the Nyquist plot we see that GM $= \infty$ and PM $= 61.9°$ which is exactly the same obtained from Bode.




Problem 10.37: Bode frequency response.




Problem 10.37: Nyquist plot.