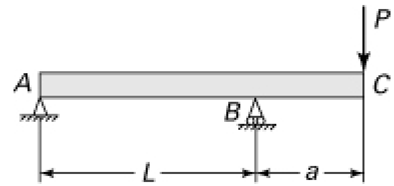
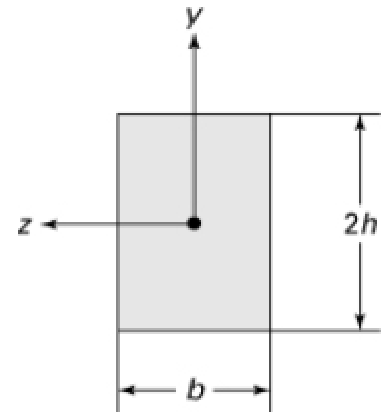
Final Exam Practice Problems

Problem 1
For the beam and loading shown, use Castigliano's theorem to determine the slope at point C. (Ans: $\frac{Pa}{6EI}(2L + 3a)$)



Problem 2
A steel rectangular beam with cross section shown is subjected to a moment about the z-axis 1.3 times greater than M_{yp}. Calculate (a) the half-thickness, e, of the elastic core; and (b) the values of the residual stress at both $y = e$ and $y = h$ following release of loading. Given: $\sigma_{yp} = 240\ MPa$, $b = 60\ mm$, and $h = 40\ mm$. (Ans: (a) $e = 25.3\ mm$; (b) $\sigma(e) = 43\ MPa$, $\sigma(h) = -72\ MPa$)



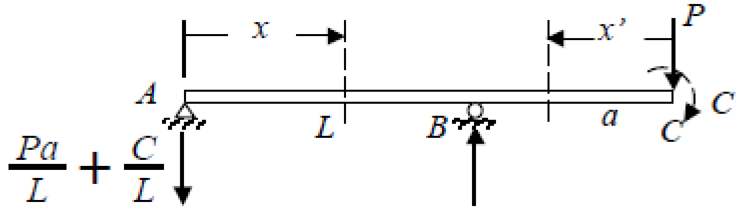
Problem 3
The nail shown has a length of $8\ cm$. Given $\sigma_{yp} = 345\ MPa$ and $E = 200\ GPa$, determine the radius of gyration r (recall $I = Ar^2$) required so that the risks of buckling and of yielding by compression due to a vertical load are equivalent. Make your determination based on the following assumptions:

- the force applied to the nail head has no transverse, or horizontal, component
- the nail is free to rotate at its base
- the nail cannot rotate at the head due to constraints imposed by the nail gun

(Ans: $r = 0.74\ mm$)

Solutions

Problem 1



<u>Segment *AB*</u>

$$M_1 = -P\frac{a}{L}x - \frac{C}{L}x$$

<u>Segment *BC*</u>

$$M_2 = -Px' - C$$

$$\frac{\partial M_1}{\partial C} = -\frac{x}{L}, \qquad \frac{\partial M_2}{\partial C} = -1.$$

For C=0, we have :

$$\theta_C = \frac{1}{EI}\int_0^L (-\frac{Pax}{L})(-\frac{x}{L})dx + \frac{1}{EI}\int_0^a -Px'(-x')dx'$$

$$= \frac{Pa}{EIL^2}\left|\frac{x^3}{3}\right|_0^L + \frac{P}{EI}\left|\frac{x'^2}{2}\right|_0^a$$

$$= \frac{Pa}{6EI}(2L+3a)$$

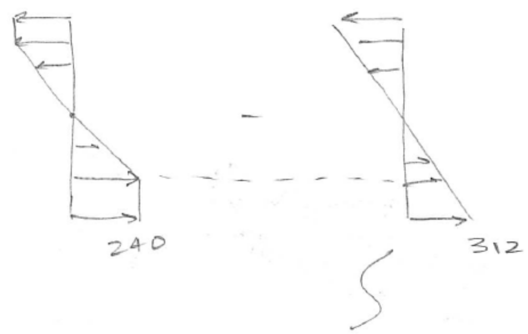
Problem 2

a) $M = 1.3\, M_{yp} = \frac{3}{2} M_{yp}\left[1 - \frac{1}{3}\left(\frac{e}{h}\right)^2\right]$

$$\Rightarrow 1.3 = \frac{3}{2} - \frac{1}{2}\left(\frac{e}{h}\right)^2 \Rightarrow \cancel{\tfrac{1}{2}}\left(\frac{e}{\cancel{h}}\right)^2 = \left(\frac{3}{2} - 1.3\right) 2h^2$$

$$\Rightarrow e = h\sqrt{2(1.5 - 1.3)} = 0.63\,h = 0.63(40) = \underline{25.3 \text{ mm}}$$

b)



$$\sigma_h = \frac{Mh}{I} = \frac{1.3\, M_{yp} h}{I}$$

$$\cdot\ \sigma_{yp} = \frac{M_{yp} h}{I} \Rightarrow M_{yp} = \sigma_{yp} \frac{I}{h}$$

$$\Rightarrow \sigma_h = \frac{1.3\cancel{h}}{\cancel{I}} \sigma_{yp} \frac{\cancel{I}}{\cancel{h}} = 1.3\, \sigma_{yp} = 1.3(240 \text{ MPa})$$

$$= 312 \text{ MPa}$$

$$\cdot\ \sigma_e = \frac{Me}{I} = \frac{1.3\, M_{yp} e}{I} = \frac{1.3\, e}{\cancel{I}} \sigma_{yp} \frac{\cancel{I}}{h}$$

$$= 1.3\,(240 \text{ MPa}) \frac{25.3}{40} = 197.3 \text{ MPa}$$

$$\approx 197 \text{ MPa}$$

$$\Rightarrow \sigma_e = 240 - 197 = \underline{43 \text{ MPa}}$$

$$\Rightarrow \sigma_h = 240 - 312 = \underline{-72 \text{ MPa}}$$

Problem 3

- Failure by buckling $P_{cr} = \frac{\pi^2 E A}{(L_e/r)^2} \quad \left(= \frac{\pi^2 E I}{L_e^2} \right)$

- Yielding: $P_y = \sigma_{yp} A$

- Let $P_{cr} = P_y \Rightarrow \frac{\pi^2 E \cancel{A}}{(L_e/r)^2} = \sigma_{yp} \cancel{A}$

$$\Rightarrow (L_e/r)^2 = \frac{\pi^2 E}{\sigma_{yp}} \quad \Rightarrow \quad \frac{L_e}{r} = \sqrt{\frac{\pi^2 E}{\sigma_{yp}}}$$

$$\Rightarrow \quad r = \frac{L_e}{\pi} \sqrt{\frac{\sigma_{yp}}{E}}$$

- For pinned-fixed case, $L_e = 0.7 L$

$$\Rightarrow \quad r = \frac{(0.7)(8\,cm)}{\pi} \sqrt{\frac{345 \times 10^6\,Pa}{200 \times 10^9\,Pa}} = \underline{0.074\,cm} = 0.74\,mm$$

$$= 0.00074\,m$$