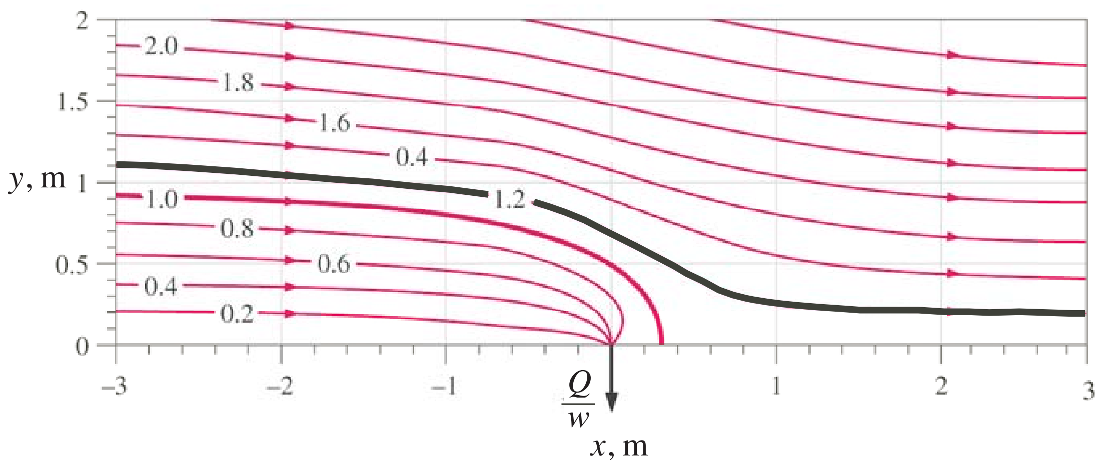
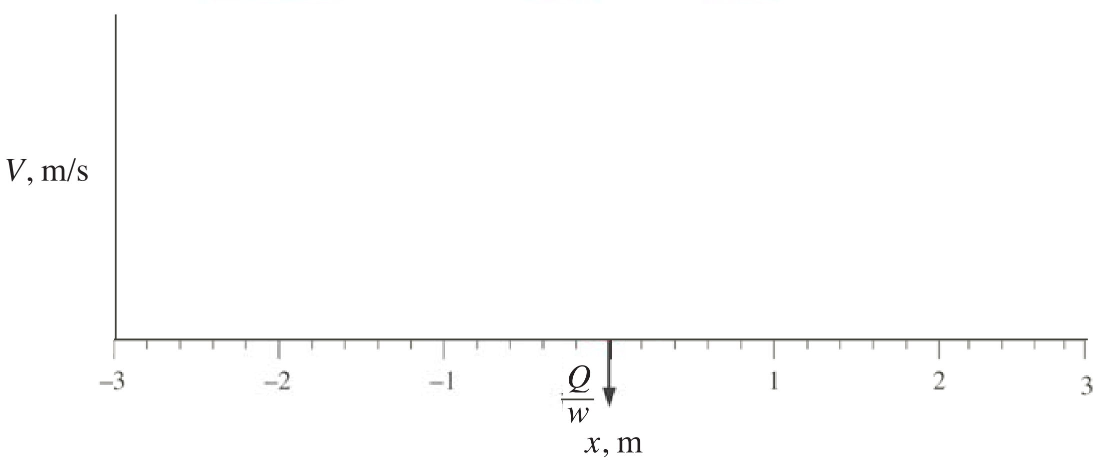
In Class Practice Problem – Boundary Layer Theory (Fall 2025)

P1. Water drains through a narrow slot, located at $x = 0$, $y = 0$ on the bottom wall of a channel having a width of $w = 2$ m. The streamline pattern and corresponding values of the streamfunction ψ are shown below.

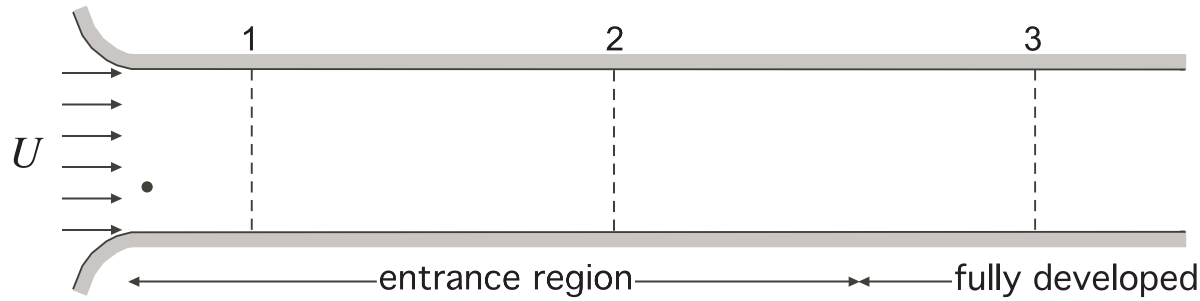
a In the figure provided, plot the magnitude of the velocity V as a function of x along the streamline drawn in black. [Be sure to include the correct values of V on the y-axis]

b What is the volume flow rate Q through the slot? [Hint: assume the streamline along the bottom wall between $-3 \leq x \leq 0.3$ has a value $\psi = 0$.]





P2. Steady flow enters a two-dimensional channel with a uniform velocity U. Assume the pressure gradient $\partial p/\partial x$ remains constant throughout the entire length of the channel.



a. Draw (to-scale) the velocity profiles at locations 1, 2, and 3 on the figure provided.

b. Write the simplified x- AND y-momentum equations in the *entrance region*.

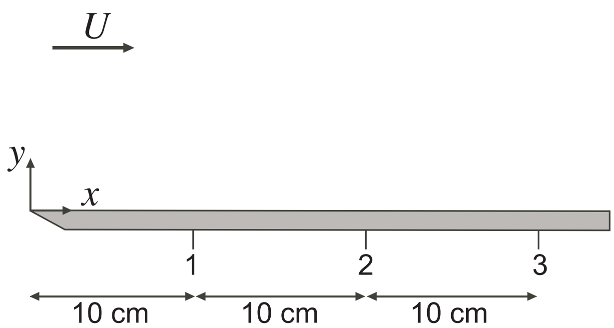
c. Write the simplified x- AND y-momentum equations in the *fully developed* region.

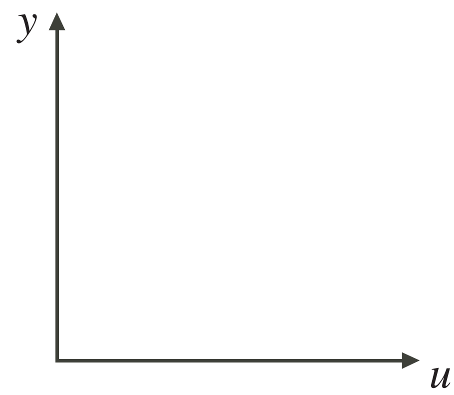
d. Comment on the magnitude of the *vertical* velocity in the entrance region compared to the fully developed region.

e. Write down the mathematical expression for the acceleration of the fluid particle marked by • in the figure.

f. Describe what happens to the momentum of the fluid particle marked by • as it moves down the channel. [Be specific and use appropriate fluids terminology]

P3. Consider the steady flow over a flat plate as shown below. The freestream velocity is $U = 5$ m/s.





a. Is the flow laminar at location 3 for the case of <u>air</u>? What about for the case of <u>water</u>? [Use calculations to support your answer. Note, the viscosities of air and water are 1.5×10^{-5} m^2/s and 1×10^{-6} m^2/s, respectively.]

b. In the plot given, draw the *dimensional* velocity profiles at locations 1, 2, and 3 for the case of <u>air</u>.

c. How would the figure change if you plotted u/U versus y/δ instead, where δ denotes the local boundary layer thickness?

d. For the case of <u>air</u>, plot the boundary layer thickness δ as a function of x over the range $0 \leq x \leq 30$ cm.

e. Consider flow through an octagonal pipe as shown in the figure. What is the increase in centerline velocity do to boundary layer growth over the pipe length L? Assume a negligible pressure gradient, that the flow is uniform over the inflow cross section with a flow rate Q_0 and an inflow velocity U_0, and that the wall boundary layers merge after some distance leading to a cylindrical inviscid core region. Using your answer and your own physical intuition, for $L/D \approx 1$ is the impact of the wall boundary layers important in this flow?