Fluid Mechanics (ME EN 5700/6700)
Final exam, Fall 2012

(Open Book, Open Notes, Closed neighbor)

0. [1 pt] What is your name?

1. [15 pt] Short answer questions.

a. Define the no-slip condition in words?

b. Mathematically define separation.

c. List five properties that are characteristic of all turbulent flows and give a brief description of each.

1. –

2. –

3. –

4. –

5. –

d. What is the 'closure problem' for a turbulent flow?

2. [8 pt] Consider the governing equation for the time rate of change of the kinetic energy ($E \equiv \frac{1}{2} u_i u_i$) of the fluid as shown below.

$$\underbrace{\rho \frac{\partial E}{\partial t}}_{I} + \underbrace{\rho\, u_i \frac{\partial E}{\partial x_i}}_{II} = \underbrace{\rho\, g_i\, u_i}_{III} + \underbrace{\frac{\partial u_i\, \tau_{ij}}{\partial x_j}}_{IV} + \underbrace{P \frac{\partial u_i}{\partial x_i}}_{V} - \underbrace{\phi}_{VI}$$

a. Describe the physical meaning of each term I–VI.

I –

II –

III –

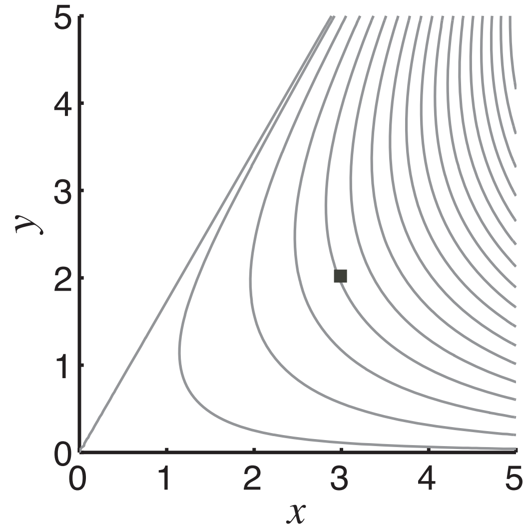
IV –

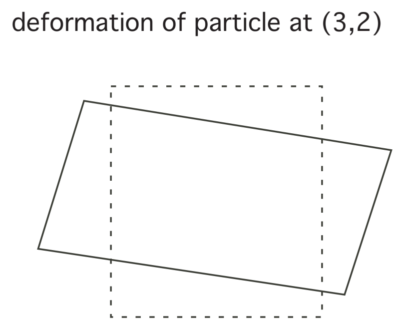
V –

VI –

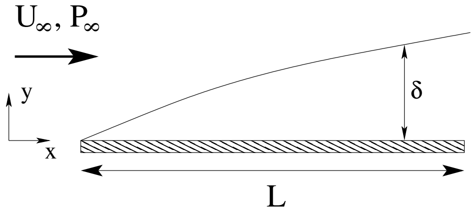
b. How is the *mechanical* energy equation above directly related to the *thermal* energy equation (be specific)?

3. [14 pt] Consider incompressible flow in a corner, the streamlines of which are drawn in the left figure below. The corresponding streamfunction is $\psi = x^2 y - y^3/3$. The fluid particle located at $x=3$, $y=2$ deforms as shown in the right figure. Based on this write the corresponding strain-rate tensor and vorticity at this instant in space-time.





4. [18 pt] Two dimensional (2D) incompressible boundary layer development over a flat plate



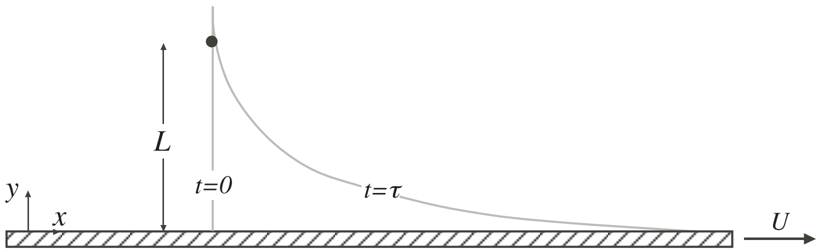
(a) For the flow given in the figure, using variables given in the figure, give the appropriate non dimensional forms for:

- the stream wise velocity u
- the stream wise distance x
- the vertical height y
- the differential pressure $P - P_\infty$

(b) Using your non dimensional variables from (a), derive a scaling law for the non dimensional vertical velocity $\tilde{v}$ in the boundary layer [hint start with the 2D incompressible conservation of mass].

(c) Starting with the steady state x direction conservation of momentum and using your answers to (a) and (b), derive the boundary-layer equations (thin flow assumption) for the flow shown in the figure [to receive full credit make sure to state all assumptions]. Make sure to cancel all terms that should be small (compared to the other terms in the equation) or zero.

5. [15 pt] Consider incompressible flow due to an impulsively started flat plate as shown. The plate is infinitely long in the x direction. Initially, the velocity u of the fluid above the plate is $u(t=0) = 0$. At $t = \delta t$, the plate is accelerated instantaneously to a constant velocity of U.



a. What is the vertical velocity in this flow? [Show your work]

b. Write down the simplified governing equation (momentum) for this flow. What do the terms in the equation represent? [Hint: There are only two terms.]

c. *Estimate* the time τ it will take for a fluid particle at a height L above the plate to begin to move once the plate has been set in motion. [Hint: Nondimensionalize your equation from a.]

6. [15 pt] We are interested in using polyethylene micro spheres of density ρ_p and diameter D_p as surrogates for fungal spores in a field experiment but we first need to determine how their terminal velocity V_t is related to real spores. We want to know this for for a wide range of diameters. We know there is some functional relationship of the form $V_t = f(D_p, \mu, \Delta\rho, g)$ where $\Delta\rho = \rho_p - \rho$, ρ is the density of air, μ is the dynamic viscosity of air, and g represents the gravitational constant.

a. Briefly state Buckingham's Pi theorem (in words).

b. Show how you can use dimensional analysis in order to write the *nondimensional* terminal velocity as a function of the other relevant *nondimensional* parameter(s) in the flow.

c. If we wanted to empirically determine the functional relationship between the nondimensional terminal velocity and the other nondimensional variables, how many experiments would we need to perform?

d. [*extra credit* 5 pt] On the back of the page, describe how you would perform the necessary experiment/experiments to determine V_t.

7. [14 pt] The impact of a pressure gradient on boundary layers.

a. The steady-state boundary layer x-momentum equation for flow driven by a spatially varying freestream velocity can be written as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

where $U_e = U_e(x)$ is the spatially varying freestream velocity. In this equation, the pressure gradient $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ from the original boundary layer equations (for constant freestream velocity) has been replaced by $U_e\frac{dU_e}{dx}$. What are the assumptions that we must make to justify this substitution?

b. Consider flow over a circular cylinder. Draw the boundary layer profiles at locations 1, 2, and 3 in the plot given below. Note, δ denotes the local boundary layer thickness and U represents the local freestream velocity. Assume the boundary layers remain laminar and that the flow separates at point 3. [The Blasius profile is provided for reference.]