ME EN 5830/6830: Aerospace Propulsion
Problem Set #5: Compressible Flows III and IV
Due date: 02/13/2024 by 11:59pm

Submission

Assignments can only be submitted on Gradescope, which can be accessed through Canvas. If you have any questions about submission, please email the class TA, John Gardner at john.w.gardner@utah.edu. Submissions will be automatically locked at the due date given above.

Introduction

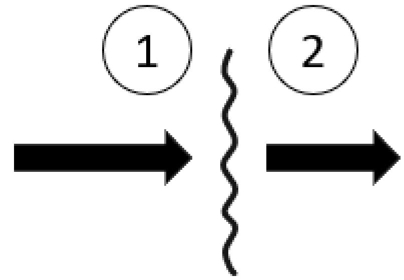
This problem set primarily covers the material from Lectures 9 and 10. The goal is for students to work with non-isentropic supersonic flows that lead to normal and oblique shocks, as well as look at how these shocks interact with nozzles. By completion, students will be able to

- Compute state changes across a normal shock
- Compute state changes across an oblique shock
- Compute state changes across multiple oblique shocks
- Calculate states within a nozzle operating at the second limit

Assignment

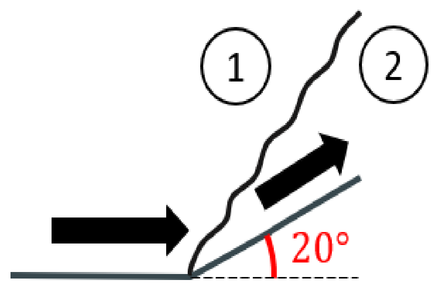
Problem #1: Consider air flowing at $T_1 = 200$ K, $p_1 = 10$ kPa, $M_1 = 2$. This supersonic flow encounters three different types of non-isentropic shocks.

a) Consider that this flow encounters a normal shock, that brings it from state 1 given above, to state 2.



1. Compute the Mach number after the shock M_2
2. Compute the temperature after the shock T_2
3. Compute the pressure after the shock p_2
4. Compute the ratio of stagnation pressure p_{t2}/p_{t1} across the shock. *Remember, we have lost a good deal of energy across this shock because it is non-isentropic*

b) Instead of encountering a normal shock, consider the above flow encountering a turning wall which leads to a single oblique shock, that brings it from state 1 given above, to state 2 (which is different from part a). Assume the wall turns by an angle $\theta = 20°$. At this inlet Mach number and deflection angle, the shock angle is $\beta = 53.42°$.



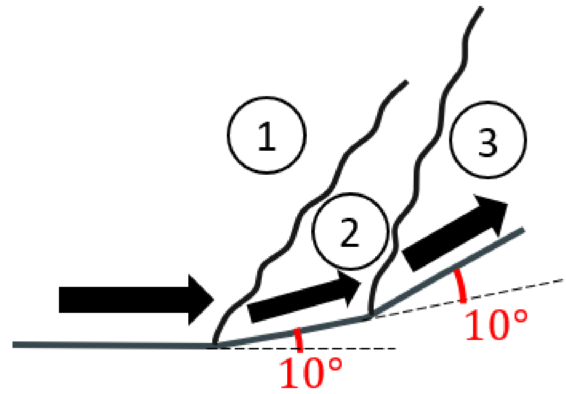
1. Compute the normal Mach number at state 1 M_{n1}
2. Compute the normal Mach number after the shock at state 2 M_{n2}
3. Compute the Mach number after the shock M_2
4. Is this a "strong" or "weak" oblique shock? Why?
5. Compute the temperature after the shock T_2
6. Compute the pressure after the shock p_2
7. Compute the ratio of stagnation pressure p_{t2}/p_{t1} across the shock. How does this value compare to that you found in part a)?

c) Instead of encountering a single oblique shock, consider the above flow encountering a wall with two turns which lead to two oblique shocks that bring the flow from state 1 given above, to a final state 3. Assume each turn in the wall has an angle $\theta = 10°$. At this inlet Mach number and deflection angle, the first shock angle is $\beta_{12} = 39.31°$.

1. Compute $M_2, T_2, p_2, p_{t2}/p_{t1}$

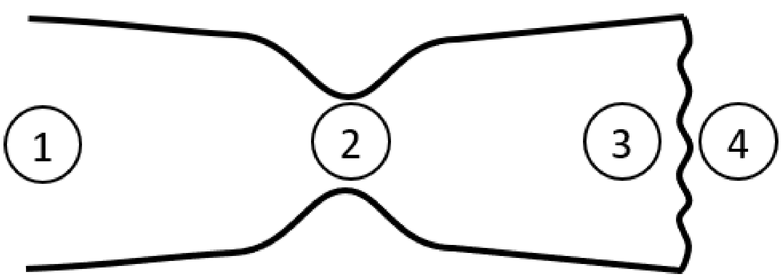
2. If you've correctly calculated M_2, the second shock angle would be $\beta_{23} =$ 49.38°. Assuming this angle, compute $M_3, T_3, p_3, p_{t3}/p_{t2}$

3. Compute the ratio of stagnation pressure p_{t3}/p_{t1}. How does this compare to the ratios of stagnation pressure computed in part a) and b)?



d) Rank the three shock systems (i.e., 1 normal, 1 oblique, or 2 obliques) in terms of efficiency (i.e., most efficient is closest to isentropic).

Problem 2: Consider air flowing through a converging-diverging nozzle operating at the second limit (i.e., with a normal shock occurring right at the outlet). The nozzle has an area ratio $A_4/A_2 = 3$.



a) Compute the Mach number at state 3 (just before the normal shock)

b) Compute the Mach number at state 4 (just after the normal shock)

c) Assume that the inlet Mach number is $M_1 = 0$ and that the outlet pressure $p_4 = 1$ bar. Compute the inlet pressure required for this nozzle to operate at the second limit.

d) <u>**Conceptual:**</u> If the outlet pressure were slightly increased, what would (qualitatively) happen to the shock? What would happen to the strength the shock?