

Differential Forms of the Divergence

$$\begin{aligned}
 \text{div} \vec{A} &= \nabla \cdot \vec{A} \\
 \text{Cartesian} \quad \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} &= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \vec{A} \\
 \text{Cylindrical} \quad \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} &= \left(\vec{e}_r \frac{\partial}{\partial r} + \frac{\vec{e}_\theta}{r} \frac{\partial}{\partial \theta} + \vec{e}_z \frac{\partial}{\partial z} \right) \cdot \vec{A} \\
 \text{Spherical} \quad \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} &= \left(\vec{e}_r \frac{\partial}{\partial r} + \frac{\vec{e}_\theta}{r} \frac{\partial}{\partial \theta} + \frac{\vec{e}_\phi}{r \sin \theta} \frac{\partial}{\partial \phi} \right) \cdot \vec{A}
 \end{aligned}$$