

1. Consider laminar, two-dimensional flow inside the boundary layer that develops over a flat plate.
- a. Write the governing equations, clearly stating all assumptions.

ANSWER

x-momentum: $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$

y-momentum: $\frac{\partial p}{\partial y} = 0$

cons. of mass: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

assumptions: steady flow, incompressible, Newtonian fluid,
 $Re_x \gg 1$

NOTE: In non-dimensional variables, the appropriate equations are

x-mom: $\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$

y-mom: $\frac{\partial \tilde{p}}{\partial \tilde{y}} = 0$

cons. mass: $\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$

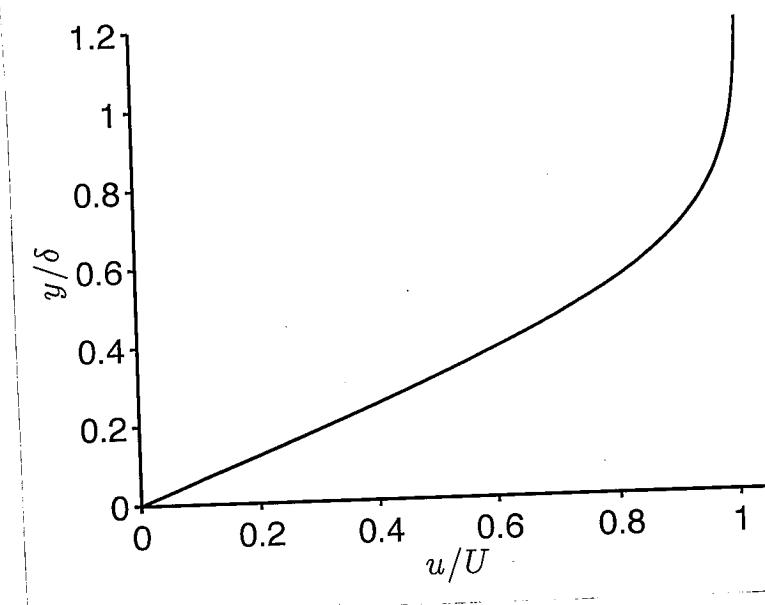
- b. Is the boundary layer fully developed? Why or why not?

ANSWER

The boundary layer is not fully developed because in a semi-infinite domain such as that over a flat plate, the boundary layer thickness continues to grow indefinitely in the x-direction.

I. C. Plot u/U versus y/δ from the Blasius solution.

ANSWER



d. Calculate δ^*/δ , θ/δ , C_f , Re_δ from the Blasius data.

ANSWER

The definition of the displacement thickness is

$$\delta^* = \int_0^\infty (1 - \frac{u}{U}) dy$$

Since the data are given in terms of y/δ we need to integrate with respect to y/δ NOT y . Therefore, divide both sides by δ :

$$\rightarrow \delta^*/\delta = \int_0^\infty (1 - \frac{u}{U}) d(y/\delta)$$

We evaluate this integral numerically using the trapezoidal rule.

The definition of the momentum deficit thickness is

$$\theta = \int_0^\infty \frac{u}{U} (1 - \frac{u}{U}) dy$$

Again, we divide by δ because the data are given in terms of y/δ !

1. d. (cont.)

$$\rightarrow \frac{\theta}{\delta} = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) \frac{dy}{y/\delta}$$

Again, we evaluate this integral numerically.

The definition of the skin friction coefficient is

$$C_f = \frac{T_w}{\frac{1}{2} \rho U^2} = \frac{u \frac{dy}{dy} \Big|_{y=0}}{\frac{1}{2} \rho U^2}$$

Because the data are given in terms of u/U and y/δ , we need to multiply the right hand side by $(\frac{u}{U})(\frac{\delta}{y})$, which gives

$$C_f = \frac{u \frac{dy}{dy} \Big|_{y/\delta=0}}{\frac{1}{2} \rho U^2} = \frac{\frac{dy}{dy} \Big|_{y/\delta=0}}{\frac{1}{2} \rho U^2 \underbrace{[\rho U \delta / u]}_{= Re_{\delta}}}$$

$$\rightarrow C_f Re_{\delta} = 2 \frac{\frac{dy}{dy} \Big|_{y/\delta=0}}{\frac{dy}{dy} \Big|_{y/\delta=0}}$$

The derivative is evaluated from the data using a second-order accurate forward finite difference, which calculates the slope at the wall based on the first three values of u/U .

The results are:

$S/\delta = 0.337$
$\theta/\delta = 0.125$
$C_f Re_{\delta} = 3.3$

1.e. Plot the nondimensional vertical velocity $v/U_{\infty}^{1/2}$ versus y/δ .

ANSWER

We are only given information about the horizontal velocity; however, we can derive the vertical velocity using the incompressible condition.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since we are given data in the form u/U we need to divide the equation by U .

$$\frac{\partial(u/U)}{\partial x} + \frac{\partial(v/U)}{\partial y} = 0.$$

Also, we know that $u/U = f(y/\delta)$, i.e., u/U is a function of y/δ and we also know that $\delta = \delta(x)$. Therefore, to determine the first partial derivative, we need to invoke the chain rule

$$\frac{\partial(u/U)}{\partial x} = \underbrace{\frac{\partial(u/U)}{\partial(y/\delta)} \frac{\partial(y/\delta)}{\partial x}}$$

determined numerically by taking finite difference of data

We know that in a laminar boundary layer, $\delta = 4.9 \sqrt{Vx/U}$. Thus,

$$\frac{\partial(y/\delta)}{\partial x} = y \frac{d\delta^{-1}}{dx} = y \frac{d[4.9 \sqrt{Vx/U}]}{dx} = \frac{y}{4.9} \sqrt{\frac{U}{V}} \frac{d\delta^{-1/2}}{dx} = \frac{y}{4.9} \sqrt{\frac{U}{V}} \left[-\frac{1}{2} \delta^{-3/2} \right]$$

$$\frac{\partial(y/\delta)}{\partial x} = -\frac{y}{9.8} \sqrt{\frac{U}{Vx^3}}$$

Plugging back into the incompressibility condition gives

$$\frac{\partial(u/U)}{\partial(y/\delta)} \left[\frac{y}{9.8} \sqrt{\frac{U}{Vx^3}} \right] = \frac{\partial(v/U)}{\partial y}$$

1.e. (cont.)

○ Multiply both sides by dy and integrate

$$\underbrace{9.8 \sqrt{\frac{U}{Vx^3}} \int_0^y \frac{d(\frac{U}{V})}{d(\frac{y}{s})} y dy}_{\text{Left Hand Side}} = \underbrace{\int_0^y \frac{d(\frac{U}{V})}{dy} dy}_{= \frac{U}{V}}$$

Since data are given in terms of y/s , we cannot integrate with respect to y . So, we must multiply and divide the left hand side by s^2 ,

$$\underbrace{9.8 \sqrt{\frac{U}{Vx^3}} \int_0^{y/s} s^2 \frac{d(\frac{U}{V})}{d(\frac{y}{s})} \frac{y}{s} d(\frac{y}{s})}_{\text{Left Hand Side}} = \frac{U}{V}$$

Now, we replace s^2 by its definition: $s = 4.9 \sqrt{\frac{Vx}{U}}$, thus $s^2 = 24.01 \frac{Vx}{U}$. Pulling s^2 outside the integral since it does not

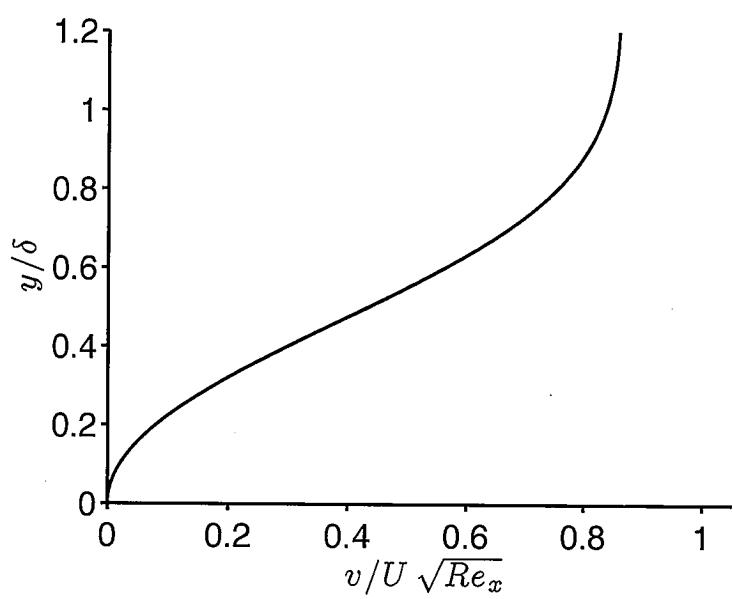
depend on y gives

$$\underbrace{9.8 \sqrt{\frac{U}{Vx^3}} (24.01) \frac{\sqrt{Vx}}{U}}_{= 2.45 \sqrt{\frac{Vx^2}{Ux^3}}} \int_0^{y/s} \frac{d(\frac{U}{V})}{d(\frac{y}{s})} \frac{y}{s} d(\frac{y}{s}) = \frac{U}{V}$$

this integral must be determined numerically from the data using the trapezoidal rule

Therefore, we have

$$\rightarrow \frac{U}{V} Re_x^{-1/2} = 2.45 \int_0^{y/s} \frac{d(\frac{U}{V})}{d(\frac{y}{s})} \frac{y}{s} d(\frac{y}{s})$$



1.F. Plot the nondimensional vorticity profile $-\Omega \theta / U$ versus y/θ .

ANSWER

In a boundary layer, we can approximate the vorticity as

$$\Omega \approx -\frac{\partial y}{\partial x}$$

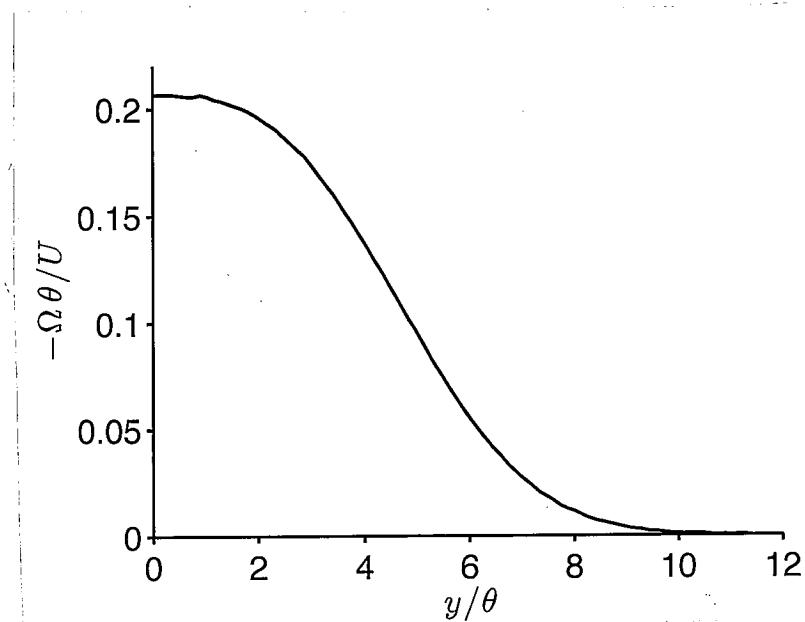
Since the data are provided in nondimensional form we need to multiply and divide by δ/U .

$$\Omega = -\frac{U/d(y/\delta)}{d(y/\delta)}$$

Then, the nondimensional vorticity is

$$-\frac{\Omega \theta}{U} = -\left(\frac{-U}{\delta} \frac{d(y/\delta)}{d(y/\theta)}\right) \frac{\theta}{U} = 0.125 \underbrace{\frac{d(y/\delta)}{d(y/\theta)}}_{\text{calculate numerically using finite difference of the data}}$$

Similarly, we can rewrite y/θ as $y/\theta = (y/\delta)/(\theta/\delta) = 8 y/\delta$



2. A flat plate (4m wide, 1m long) is immersed in Kerosene at 20°C ($\nu = 2.29 \times 10^{-6} \text{ m}^2/\text{s}$, $\rho = 880 \text{ kg/m}^3$). The freestream velocity is $U_\infty = 0.5 \text{ m/s}$

- a. What is the critical Reynolds number based on momentum thickness?

ANSWER

The critical Reynolds number based on x is given as $Re_x \approx 5 \times 10^5$. Therefore, we only need to convert Re_x to Re_θ .

$$Re_x = \frac{U_\infty x}{\nu} \quad \text{and} \quad Re_\theta = \frac{U_\infty \theta}{\nu}$$

For a flat plate,

$$\theta = 0.664 \sqrt{\frac{U_\infty x}{\nu}}.$$

Plugging this value of θ into the definition for Re_θ gives

$$Re_\theta = \frac{U_\infty}{\nu} 0.664 \sqrt{\frac{U_\infty x}{\nu}}$$

$$Re_\theta = 0.664 \sqrt{\frac{U_\infty x}{\nu}}$$

$$Re_\theta = 0.664 \sqrt{Re_x}$$

Plug in the critical value for Re_x

$$(Re_\theta)_{\text{critical}} = 0.664 \sqrt{5 \times 10^5}$$

$$\rightarrow (Re_\theta)_{\text{critical}} = 470$$

2. (cont.)

- b. Verify that Re_x is below the critical value everywhere along the plate.

ANSWER:

Since Re_x increases with distance along the plate, we only need to verify that the value of Re_x at the end of the plate is below the critical value for transition to turbulence.

$$\text{At } x=1\text{m: } Re_x = \frac{U_\infty(1\text{m})}{\nu} = \frac{(0.5 \text{ m/s})(1\text{m})}{2.29 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\rightarrow Re_x = 2.2 \times 10^5$$

The critical Reynolds number for a flat plate is generally accepted as $(Re_x)_{cr} \approx 5 \times 10^5$.

- c. Show that the boundary layer thickness and wall shear stress at the center of the plate is $\delta = 0.74 \text{ cm}$ & $T_w = 0.2 \text{ N/m}^2$.

ANSWER

For a flat plate, the Blasius solution tells us that

$$\delta = 4.9 \left(\frac{Vx}{U_\infty} \right)^{1/2} \quad \text{and} \quad T_w = 0.332 \rho U_\infty^2 \left(\frac{V}{U_\infty x} \right)^{1/2}$$

Plugging in the given values:

$$\rightarrow \delta = 4.9 \left(\frac{(2.29 \times 10^{-6} \text{ m/s})(0.5 \text{ m})}{0.5 \text{ m/s}} \right)^{1/2} = 0.10074 \text{ m}$$

$$\rightarrow T_w = 0.332 \left(800 \text{ kg/m}^3 \right) (0.5 \text{ m/s})^2 \left(\frac{2.29 \times 10^{-6} \text{ m/s}}{(0.5 \text{ m/s})(0.5 \text{ m})} \right)^{1/2} = 0.2 \text{ N/m}^2$$

Z. (cont.)

- d. Show that the boundary thickness and wall shear stress at the trailing edge is $\delta = 1.05 \text{ cm}$ and $\tau_w = 0.14 \text{ N/m}^2$

Answer

Plugging in the given values to the relations in part (c):

$$\rightarrow \delta = 4.9 \left(\frac{(2.29 \times 10^{-6} \text{ m}^2/\text{s})(1 \text{ m})}{0.5 \text{ m/s}} \right)^{1/2} = \underline{0.0105 \text{ m}}$$

$$\rightarrow \tau_w = 0.332 (800 \text{ kg/m}^3)(0.5 \text{ m/s})^2 \left(\frac{2.29 \times 10^{-6} \text{ m}^2/\text{s}}{(0.5 \text{ m/s})(1 \text{ m})} \right)^{1/2} = \underline{0.14 \text{ N/m}^2}$$

- e. Show that the total skin friction drag along one side of the plate is 1.14 N.

Answer

In order to calculate the total skin, we need to integrate the wall shear stress over the area of the plate.

$$F_D = \iint_{\substack{y=0 \\ x=0}}^{w=L} \tau_w \, dx \, dy \quad \text{where } L = 1 \text{ m}, \quad w = 4 \text{ m}$$

We can write the wall shear stress using the Blasius solution

$$\tau_w = 0.332 \rho U_\infty^2 \left(\frac{v}{U_\infty x} \right)^{1/2}$$

The wall shear stress is independent of y :

$$F_D = 0.332 \rho U_\infty^2 w \left(\frac{v}{U_\infty} \right)^{1/2} \underbrace{\int_0^L x^{-1/2} \, dx}_{2L^{1/2}}$$

$$F_D = 0.332 (800 \text{ kg/m}^3)(0.5 \text{ m/s})^2 (4 \text{ m}) \left(\frac{2.29 \times 10^{-6} \text{ m}^2/\text{s}}{0.5 \text{ m/s}} \right)^{1/2} 2 (1 \text{ m})^{1/2}$$

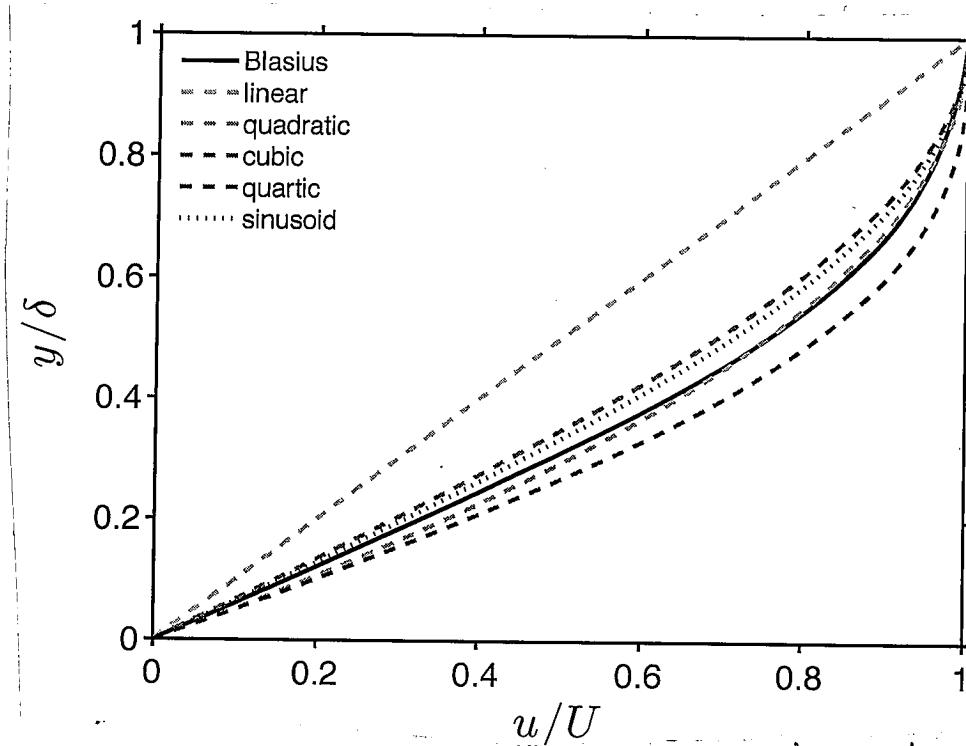
$$\rightarrow F_D = 1.14 \text{ N}$$

3. Compare the Blasius solution to several different polynomial approximations and a sinusoidal function.

- a. Plot u/U versus y/δ for all of the given profiles

ANSWER

Note, it only makes sense to plot the approximate profiles between $0 \leq y/\delta \leq 1$.



- b. Calculate δ^*/δ , θ^*/δ , $C_{f, \text{Reg}}$ for each profile along with the percent error relative to the Blasius solution.

ANSWER

Because we are given functional forms for the approximate profile we can perform the necessary integrations and differentiation analytically.

Linear:
$$\delta^*/\delta = \int_0^1 (1 - y/\delta) d(y/\delta) = \int_0^1 (1 - y/\delta) d(y/\delta) = y/\delta - \frac{1}{2} (y/\delta)^2 \Big|_0^1$$

$$= 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

3. b. (cont.)

$$\text{Linear} : \frac{y}{s} = \int_0^1 \frac{u}{v} \left(1 - \frac{u}{v}\right) d\left(\frac{y}{s}\right) = \int_0^1 \frac{y}{s} \left(1 - \frac{y}{s}\right) d\left(\frac{y}{s}\right) = \frac{1}{2} \left(\frac{y}{s}\right)^2 - \frac{1}{3} \left(\frac{y}{s}\right)^3 \Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} = 0.1667$$

$$C_F \cdot R_{FS} = 2 \left. \frac{d(y/v)}{d(y/s)} \right|_{y/s=0} = 2 \left. \frac{d}{d(y/s)} \left[\frac{y}{s} \right] \right|_{y/s=0} = 2(1) \Big|_{y/s=0} = 2$$

$$\text{Quadratic} : \frac{s^*}{s} = \int_0^1 \left(1 - \frac{y}{v}\right) d\left(\frac{y}{s}\right) = \int_0^1 1 - 2 \frac{y}{s} + \left(\frac{y}{s}\right)^2 d\left(\frac{y}{s}\right) = \frac{y}{s} - \left(\frac{y}{s}\right)^2 + \frac{1}{3} \left(\frac{y}{s}\right)^3 \Big|_0^1$$

$$= 1 - 1 + \frac{1}{3} = \frac{1}{3} = 0.333$$

$$\frac{y}{s} = \int_0^1 \frac{u}{v} \left(1 - \frac{u}{v}\right) d\left(\frac{y}{s}\right) = \int_0^1 2 \frac{y}{s} - \left(\frac{y}{s}\right)^2 - \underbrace{\left[2 \frac{y}{s} - \left(\frac{y}{s}\right)^2 \right]^2}_{4 \left(\frac{y}{s}\right)^2 - 4 \left(\frac{y}{s}\right)^3 + \left(\frac{y}{s}\right)^4} d\left(\frac{y}{s}\right)$$

$$= \left(\frac{y}{s}\right)^2 - \frac{5}{3} \left(\frac{y}{s}\right)^3 + \left(\frac{y}{s}\right)^4 - \frac{1}{5} \left(\frac{y}{s}\right)^5 \Big|_0^1 = 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15} = 0.133$$

$$C_F \cdot R_{FS} = 2 \left. \frac{d(y/v)}{d(y/s)} \right|_{y/s=0} = 2 \left. \frac{d}{d(y/s)} \left[2 \frac{y}{s} - \left(\frac{y}{s}\right)^2 \right] \right|_{y/s=0} = 2 \left[2 - 2 \left(\frac{y}{s}\right) \right] \Big|_{y/s=0} = 4$$

$$\text{Cubic} : \frac{s^*}{s} = \int_0^1 \left(1 - \frac{y}{v}\right) d\left(\frac{y}{s}\right) = \int_0^1 1 - \frac{3}{2} \frac{y}{s} + \frac{1}{2} \left(\frac{y}{s}\right)^3 d\left(\frac{y}{s}\right)$$

$$= \frac{y}{s} - \frac{3}{4} \left(\frac{y}{s}\right)^2 + \frac{1}{8} \left(\frac{y}{s}\right)^4 \Big|_0^1 = 1 - \frac{3}{4} + \frac{1}{8} = \frac{3}{8} = 0.375$$

$$\frac{\Theta}{s} = \int_0^1 \frac{u}{v} \left(1 - \frac{u}{v}\right) d\left(\frac{y}{s}\right) = \int_0^1 \frac{3}{2} \frac{y}{s} - \frac{1}{2} \left(\frac{y}{s}\right)^3 - \underbrace{\left(\frac{3}{2} \frac{y}{s} - \frac{1}{2} \left(\frac{y}{s}\right)^3 \right)^2}_{\frac{9}{4} \left(\frac{y}{s}\right)^2 - \frac{6}{4} \left(\frac{y}{s}\right)^4 + \frac{1}{4} \left(\frac{y}{s}\right)^6} d\left(\frac{y}{s}\right)$$

$$= \frac{3}{4} \left(\frac{y}{s}\right)^2 - \frac{9}{12} \left(\frac{y}{s}\right)^3 - \frac{1}{8} \left(\frac{y}{s}\right)^4 + \frac{6}{20} \left(\frac{y}{s}\right)^5 - \frac{1}{28} \left(\frac{y}{s}\right)^7 \Big|_0^1$$

$$= \frac{3}{4} - \frac{3}{4} - \frac{1}{8} + \frac{6}{20} - \frac{1}{28} = 0.1393$$

3. b. (cont.)

$$\text{Cubic: } C_F \cdot R_{eq} = 2 \frac{\partial(\frac{y_0}{s})}{\partial(\frac{y}{s})} \Big|_{y_s=0} = 2 \frac{\frac{1}{2}}{\partial(\frac{y}{s})} \left[\frac{3}{2} \left(\frac{y}{s} \right) - \frac{1}{2} \left(\frac{y}{s} \right)^3 \right]_{y_s=0}$$

$$= 2 \left[\frac{3}{2} - \frac{3}{2} \left(\frac{y}{s} \right)^2 \right]_{y_s=0} = 3$$

$$\text{Quartic: } \frac{\theta}{s} = \int_0^1 (1 - \frac{y_0}{s}) d(\frac{y}{s}) = \int_0^1 1 - 2 \frac{y}{s} + 2 \left(\frac{y}{s} \right)^3 - \left(\frac{y}{s} \right)^4 d(\frac{y}{s})$$

$$= \frac{y}{s} - \left(\frac{y}{s} \right)^2 + \frac{2}{4} \left(\frac{y}{s} \right)^4 - \frac{1}{5} \left(\frac{y}{s} \right)^5 \Big|_0^1 = 1 - 1 + \frac{1}{2} - \frac{1}{5} = \frac{3}{10} = 0.3$$

$$\frac{\theta}{s} = \int_0^1 \frac{4}{5} \left(1 - \frac{y}{s} \right) d(\frac{y}{s}) = \int_0^1 2 \frac{y}{s} - 2 \left(\frac{y}{s} \right)^3 + \left(\frac{y}{s} \right)^4 - \underbrace{\left[2 \frac{y}{s} - 2 \left(\frac{y}{s} \right)^3 + \left(\frac{y}{s} \right)^4 \right]}_{4 \left(\frac{y}{s} \right)^2 - 8 \left(\frac{y}{s} \right)^4 + 4 \left(\frac{y}{s} \right)^6 - 4 \left(\frac{y}{s} \right)^8 + \left(\frac{y}{s} \right)^{10}} d(\frac{y}{s})$$

$$= \left(\frac{y}{s} \right)^2 - \frac{4}{3} \left(\frac{y}{s} \right)^3 + \frac{2}{4} \left(\frac{y}{s} \right)^4 + \frac{9}{5} \left(\frac{y}{s} \right)^5 - \frac{4}{6} \left(\frac{y}{s} \right)^6 - \frac{4}{7} \left(\frac{y}{s} \right)^7 + \frac{4}{8} \left(\frac{y}{s} \right)^8 - \frac{1}{9} \left(\frac{y}{s} \right)^9 \Big|_0^1$$

$$= 1 - \frac{y}{s} - \frac{1}{2} + \frac{9}{5} - \frac{4}{6} - \frac{4}{7} + \frac{y}{s} - \frac{1}{9} = 0.1175$$

$$C_F \cdot R_{eq} = 2 \frac{\partial(\frac{y_0}{s})}{\partial(\frac{y}{s})} \Big|_{y_s=0} = 2 \frac{\frac{1}{2}}{\partial(\frac{y}{s})} \left[2 \left(\frac{y}{s} \right) - 2 \left(\frac{y}{s} \right)^3 + \left(\frac{y}{s} \right)^4 \right]_{y_s=0}$$

$$= 2 \left[2 - 6 \left(\frac{y}{s} \right)^2 + 4 \left(\frac{y}{s} \right)^3 \right]_{y_s=0} = 4$$

$$\text{Sinusoid: } \frac{\theta}{s} = \int_0^1 (1 - \frac{y_0}{s}) d(\frac{y}{s}) = \int_0^1 1 - \sin\left(\frac{\pi}{2} \frac{y}{s}\right) d(\frac{y}{s}) = \frac{y}{s} + \frac{2}{\pi} \cos\left(\frac{\pi}{2} \frac{y}{s}\right) \Big|_0^1$$

$$= 1 + \frac{2}{\pi} \left(\cos\left(\frac{\pi}{2} \frac{y}{s}\right) - \cos(0) \right) = 1 + \frac{2}{\pi} (0 - 1) = 1 - \frac{2}{\pi} = 0.3634$$

$$\frac{\theta}{s} = \int_0^1 \frac{y_0}{s} (1 - \frac{y_0}{s}) d(\frac{y}{s}) = \int_0^1 \sin\left(\frac{\pi}{2} \frac{y}{s}\right) \Big|_0^1 - \sin^2\left(\frac{\pi}{2} \frac{y}{s}\right) d(\frac{y}{s})$$

$$= -\frac{2}{\pi} \cos\left(\frac{\pi}{2} \frac{y}{s}\right) - \frac{1}{2} \left(\frac{y}{s} \right) + \frac{1}{2\pi} \sin\left(\frac{\pi}{2} \frac{y}{s}\right) \Big|_0^1 = -\frac{2}{\pi} (0 - 1) - \frac{1}{2} + 0 = \frac{2}{\pi} - \frac{1}{2} = 0.1366$$

$$C_F \cdot R_{eq} = 2 \frac{\partial(\frac{y_0}{s})}{\partial(\frac{y}{s})} \Big|_{y_s=0} = 2 \frac{\frac{1}{2}}{\partial(\frac{y}{s})} \left[\sin\left(\frac{\pi}{2} \frac{y}{s}\right) \right] = 2 \frac{\pi}{2} \cos\left(\frac{\pi}{2} \frac{y}{s}\right) \Big|_{y_s=0} = \pi = 3.14$$

3. b. (cont.)

In order to calculate the percent error relative to the Blasius solution, we use the following formula

$$\% \text{ error} = \left| \frac{\text{approximation} - \text{Blasius}}{\text{Blasius}} \right| * 100$$

The results are tabulated below

Profile	$\frac{\delta^*}{\delta}$ (%err)	$\frac{\theta}{\delta}$ (%err)	$C_{f, \text{Res}}$ (%err)
Blasius	0.337	0.125	3.3
linear	0.5 (48.4%)	0.167 (33.6%)	2 (39.4%)
quadratic	0.333 (1.2%)	0.133 (6.4%)	4 (21.2%)
cubic	0.375 (11.3%)	0.139 (11.2%)	3 (9.1%)
quartic	0.3 (11.0%)	0.118 (5.6%)	4 (21.2%)
sinusoid	0.363 (7.7%)	0.137 (9.6%)	3.14 (4.8%)

c. Which of the approximate profiles provides the best alternative for engineering purposes? why?

ANSWER

This is a difficult question because there is not one profile that performs the best across the board. Errors in $\frac{\delta^*}{\delta}$ have implications for accurately estimating the mass flow rate; while errors in $\frac{\theta}{\delta}$ have implications for accurately estimating the momentum flux which is important in terms of extracting usable work from the flow; and, finally $C_{f, \text{Res}}$ has implications for accurately estimating drag. So the choice of profile you use should depend on your application. The profiles giving the least error for $\frac{\delta^*}{\delta}$, $\frac{\theta}{\delta}$, and $C_{f, \text{Res}}$ are outlined in black. The profile giving the least average error is the sinusoid.