Fluid Mechanics (ME EN 5700/6700)
Equations Sheet Final Exam, Fall 2025

$$\vec{u} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\,\hat{i} + \frac{\partial}{\partial y}\,\hat{j} + \frac{\partial}{\partial z}\,\hat{k}$$

$$e_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)$$

$$\varepsilon_{ijk}\,\varepsilon_{klm} = \delta_{il}\delta_j m - \delta_{im}\delta_{jl}$$

$$\vec{\omega} = \vec{\nabla} \times \vec{u},$$

$$\omega_i = \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j}.$$

$$\Gamma = \int_{\mathcal{S}} \vec{\omega} \cdot d\vec{\mathcal{S}},$$

$$r_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\Big),$$

$$r_{ij} = -\frac{1}{2}\varepsilon_{ijk}\,w_k.$$

$$u \equiv \frac{\partial \psi}{\partial y}, \qquad v \equiv -\frac{\partial \psi}{\partial x}$$

$$\frac{D}{Dt}\int_{V(t)} F(\vec{x},t)\,dV = \int_{V(t)} \frac{\partial F}{\partial t}\,dV + \int_{A(t)} F\,(\vec{u}\cdot d\vec{A})$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho\,u_i)}{\partial x_i} = 0$$

$$\frac{D\,(m\vec{u})}{Dt} = \vec{F}$$

$$\rho\frac{\partial u_i}{\partial t} + \rho\,u_j\frac{\partial u_i}{\partial x_j} = \rho\,g_i + \frac{\partial \tau_{ij}}{\partial x_j}.$$

$$\tau_{ij} = -p\delta_{ij} + \sigma_{ij}$$

$$\tau_{ij} = -p\delta_{ij} + \lambda e_{mm}\delta_{ij} + 2\mu e_{ij}.$$

$$\tau_{ij} = -p\delta_{ij} + \mu(2e_{ij} - \frac{2}{3}e_{mm}\delta_{ij}).$$

$$\rho\frac{Du_i}{Dt} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\mu}{3}\frac{\partial}{\partial x_i}\Big(\frac{\partial u_m}{\partial x_m}\Big)$$

$$\rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

$$\rho\Big[C_p\frac{DT}{Dt}+\frac{T\beta}{\rho}\frac{Dp}{Dt}+p\Big(\frac{\partial u_i}{\partial x_i}\Big)\Big]=-\rho\frac{\partial u_i}{\partial x_i}+\phi-\frac{\partial q_i}{\partial x_i}.$$

$$\rho\, C_p\, u\frac{\partial T}{\partial x}+\rho\, C_p\, v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial x^2}+k\frac{\partial^2 T}{\partial y^2}+\mu\phi$$

$$q_i=-k\,\frac{\partial T}{\partial x_i},$$

$$\phi=2\mu\, e_{ij}\, e_{ij}=2\mu\Big[\Big(\frac{\partial u}{\partial x}\Big)^2+\Big(\frac{\partial v}{\partial y}\Big)^2\Big]+\mu\Big(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\Big)^2.$$

$$S_t=\frac{L}{tU}\quad E_u=\frac{\Delta p}{\rho U^2}\quad F_r=\frac{U^2}{gL}\quad R_e=\rho\frac{UL}{\mu}\tag{1}$$

$$M=\frac{U}{c}\quad C_D=\frac{F_D}{1/2\rho AU^2}\quad C_f=\frac{\tau_w}{1/2\rho U_\infty^2}\tag{2}$$

$$\frac{\delta}{L}\sim R_e^{-1/2}\tag{3}$$

$$\delta^*=\int_0^\infty\Big(1-\frac{u}{U_\infty}\Big)dy\qquad\theta=\int_0^\infty\frac{u}{U_\infty}\Big(1-\frac{u}{U_\infty}\Big)dy\tag{4}$$

$$-\frac{1}{2}ff''=f'''\quad f'(\infty)=0\quad f(0)=f'(0)=0\tag{5}$$

$$\delta_{99}=4.9\sqrt{\frac{\nu x}{U_\infty}}\quad\delta^*=1.72\sqrt{\frac{\nu x}{U_\infty}}\quad\theta=0.664\sqrt{\frac{\nu x}{U_\infty}}\tag{6}$$

$$\tau_w=\nu\frac{\partial u}{\partial y}\Big|_{y=0}\quad\tau_w=\frac{0.332\rho U_\infty^2}{\sqrt{Re_x}}\quad C_f=\frac{0.664}{\sqrt{Re_x}}\tag{7}$$

$$f'''+\frac{m+1}{2}ff''+m(1-f'^2)=0\quad f(\eta)=0\quad f'(\eta=0)=0\quad f'(\eta=\infty)=0\tag{8}$$

$$(U\delta^*)\frac{dU}{dx}+\frac{d}{dx}(U^2\theta)-Uv_w=\frac{\tau_w}{\rho}\tag{9}$$

$$(U\delta^*)\frac{dU}{dx}+\frac{d}{dx}(U^2\theta)=\frac{\tau_w}{\rho}\tag{10}$$

$$(U\delta^*)\frac{dU}{dx}+\frac{d}{dx}(U^2\theta)=\frac{\tau_w}{\rho}\tag{11}$$

$$\frac{\partial\vec{\omega}}{\partial t}+\Big(\vec{u}\cdot\vec{\nabla}\Big)\vec{\omega}=\Big(\vec{\omega}\cdot\vec{\nabla}\Big)\vec{u}+\frac{1}{\rho^2}\vec{\nabla}\rho\times\vec{\nabla}p+\nu\vec{\nabla}^2\vec{\omega}\tag{12}$$