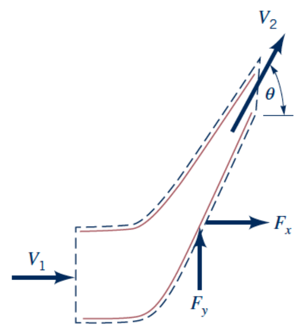
ME 3710
Homework 6
Due Tuesday February 29 at 11:59pm – upload to Canvas
[5 problems – 15 pts]

Solution 5.37

GIVEN: 10 °C liquid water, $A_1 = 1.0\,\text{m}^2, A_2 = 0.25\,\text{m}^2, V_1 = 20\,\dfrac{\text{m}}{\text{s}}$

$p_2 = p_{atm}$ and $p_1 = p_{atm} + 30\,\text{kPa}$. Neglect gravity.

FIND: F_x and F_y



SOLUTION: Apply the linear momentum equation in the x-direction to the control volume shown on the ring.

$$\dot{M}_{x,out} - \dot{M}_{x,in} = \sum F_x$$

where

$$\dot{M}_{x,out} = \dot{m}V_2\cos\theta_2 = \rho A_1 V_1 V_2 \cos\theta,$$

$$\dot{M}_{x,in} = \dot{m}V_1 = \rho A_1 V_1^2,$$

and

$$\sum F_x = F_x + (p_1 - p_{atm})A_1$$

where p_1 is an absolute pressure.

The x-direction linear momentum equation is

$$F_x = \rho A_1 V_1 (V_2\cos\theta - V_1) - (p_1 - p_{atm})A_1$$

Assuming constant fluid density, the continuity equation gives

$$\rho A_1 V_1 = \rho A_2 V_2 \cos\theta, \quad V_2 = \frac{A_1 V_1}{A_2 \cos\theta}$$

or

$$V_2 = \frac{(1.0\,\text{m}^2)\left(20\,\frac{\text{m}}{\text{s}}\right)}{(0.25\,\text{m}^2)\cos 45^\circ} = 113\,\frac{\text{m}}{\text{s}}.$$

where $\rho = 1000\,\frac{\text{kg}}{\text{m}^3}$ and

$$F_x = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)(1.0\,\text{m}^2)\left[\left(20\,\frac{\text{m}}{\text{s}}\right)\left(113\,\frac{\text{m}}{\text{s}}\right)\cos 45^\circ - 20\,\frac{\text{m}}{\text{s}}\right]\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right) - \left(30\times10^3\,\frac{\text{N}}{\text{m}^2}\right)(1.0\,\text{m}^2)$$

$$\boxed{F_x = 1.17\times10^6\,\text{N} = 1170\,\text{kN}}$$

We now apply the linear momentum equation in the y-direction to the control volume.

$$\dot{M}_{y,out} - \dot{M}_{y,in} = \sum F_y$$

where

$$\dot{M}_{y,out} = \dot{m} V_2 \sin\theta = \rho A_1 V_1 V_2 \sin\theta,$$

$$\dot{M}_{y,in} = \dot{m}(0) = 0,$$

and

$$\sum F_y = F_y$$

The y-direction linear momentum equation is

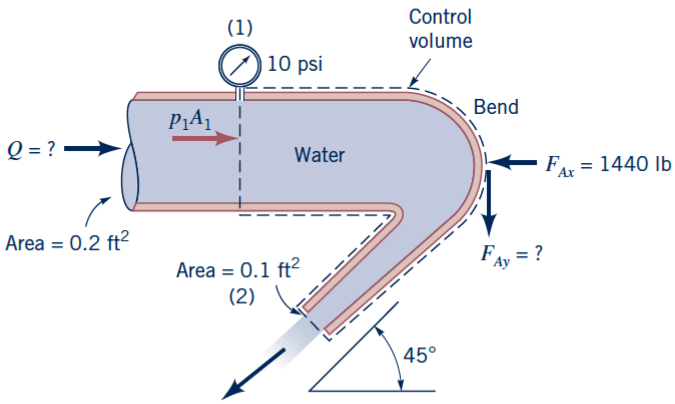
$$F_y = \rho A_1 V_1 V_2 \sin\theta.$$

The numerical values give

$$F_y = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)(1.0\,\text{m}^2)\left(20\,\frac{\text{m}}{\text{s}}\right)\left(113\,\frac{\text{m}}{\text{s}}\right)\sin 45^\circ\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$\boxed{F_y = 1.60\times10^6\,\text{N} = 1600\,\text{kN}}$$

Solution 5.38



A control volume that contains the bend and the water within the bend between sections (1) and (2) as shown in the sketch above is used. Application of the x-direction component of the linear momentum equation yields

$$-u_1\rho Q - V_2\cos 45^\circ \rho Q = p_1A_1 - F_{Ax} + p_2^{0\ \text{gage}} A_2\cos 45^\circ \tag{1}$$

with

$$u_1 = \frac{Q}{A_1} \quad \text{and} \quad V_2 = \frac{Q}{A_2}$$

Equation (1) becomes

$$-\frac{Q^2\rho}{A_1} - \frac{Q^2\rho\cos 45^\circ}{A_2} = p_1A_1 - F_{Ax}$$

or for part (a)

$$Q = \sqrt{\frac{-p_1 A_1 + F_{Ax}}{\rho\left(\frac{\cos 45^\circ}{A_2} + \frac{1}{A_1}\right)}}$$

$$Q = \sqrt{\frac{-\left(10\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)\left(0.2\,\text{ft}^2\right) + 1440\,\text{lb}}{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)\left(\frac{\cos 45^\circ}{0.1\,\text{ft}^2} + \frac{1}{0.2\,\text{ft}^2}\right)}}$$

$$Q = \underline{\underline{7.01}}\,\frac{\text{ft}^3}{\text{s}}$$

For part (b), we use the y-direction component of the linear momentum equation to get

$$F_{Ay} = V_2 \sin 45^\circ \rho Q = \frac{Q}{A_2} \sin 45^\circ \rho Q$$

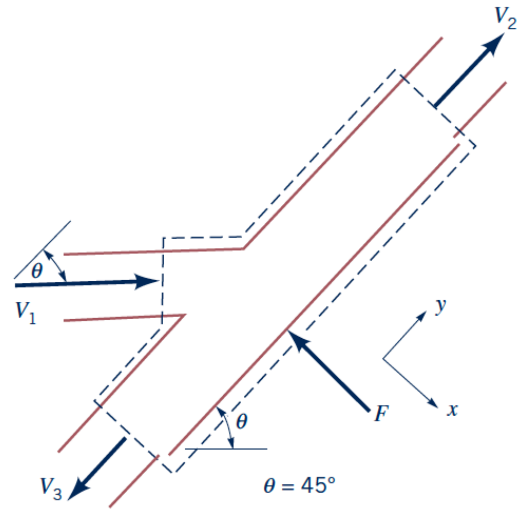
or

$$F_{Ay} = \frac{Q^2}{A_2} \sin 45^\circ \rho$$

and

$$F_{Ay} = \frac{\left(7.01\,\frac{\text{ft}^2}{\text{s}}\right)}{\left(0.01\,\text{ft}^2\right)} \sin 45^\circ \left(1.94\,\frac{\text{slugs}}{\text{ft}^2}\right)\left(1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right) = \underline{\underline{674}}\,\text{lb}$$

Solution 5.39



GIVEN: Plate hit with water in the figure in the problem. Neglect gravity. Frictionless plate.

FIND: Force F to hold plate stationary.

SOLUTION: Assume steady flow and apply the linear momentum equation in the x-direction to the control volume shown. F is the force on the plate.

$$\dot{M}_{x,out} - \dot{M}_{x,in} = \sum F_x$$

where

$$\dot{M}_{x,out} = 0$$

$$\dot{M}_{x,in} = \rho A_1 V_1 (V_1 \sin\theta),$$

and

$$\sum F_x = -F.$$

The x-direction linear momentum equation is then

$$0 - \rho A_1 V_1^2 \sin\theta = -F \quad \text{or} \quad F = \rho A_1 V_1^2 \sin\theta = \dot{m} V_1 \sin\theta$$

The numerical values give

$$F = \left(5\,\frac{\text{kg}}{\text{s}}\right)\left(30\,\frac{\text{m}}{\text{s}}\right)\sin 45^\circ \left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$\boxed{F = 106\,\text{N}}$$

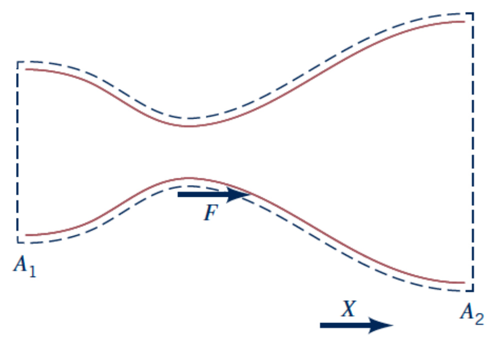
Solution 5.43

GIVEN: $T_1 = 300\,\mathrm{K}$, $p_1 = 303\,\mathrm{kPa}$, $V_1 = 0.5\dfrac{\mathrm{m}}{\mathrm{s}}$, $T_2 = 220\,\mathrm{K}$, and $p_2 = 101\,\mathrm{kPa}$, $A_1 = 0.6\,\mathrm{m}^2$ and $A_2 = 1.0\,\mathrm{m}^2$.

FIND: Horizontal force to hold ventori stationary.

SOLUTION:

Apply the linear momentum equation in the x-direction to the control volume enclosing the ventori.



$$\dot{M}_{x,out} - \dot{M}_{x,in} = \sum F_x$$

Now

$$\dot{M}_{x,out} = \dot{m}V_2 = \rho_1 A_1 V_1 V_2,$$

$$\dot{M}_{x,in} = \dot{m}V_1 = \rho_1 A_1 V_1^2,$$

and

$$\sum F_x = F + p_1 A_1 + p_{atm}(A_2 - A_1) - p_2 A_2$$
$$= F + (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2.$$

The linear momentum equation is

$$F + (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 = \rho_1 A_1 V_1 (V_2 - V_1)$$

or

$$F = (p_2 - p_{atm})A_2 - (p_1 - p_{atm})A_1 + \rho_1 A_1 V_1 (V_2 - V_1)$$

Assuming the air is an ideal gas

$$\rho_1 = \frac{p_1}{RT_1} = \frac{\left(303000\,\frac{\text{N}}{\text{m}^2}\right)}{\left(287\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\,\text{K})} = 3.52\,\frac{\text{kg}}{\text{m}^3}$$

V_2 is fluid from the continuity equation,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

or

$$V_2 = V_1\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{A_1}{A_2}\right) = V_1\left(\frac{p_1}{RT_1}\right)\left(\frac{RT_2}{p_2}\right)\left(\frac{A_1}{A_2}\right) = V_1\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{A_1}{A_2}\right)$$

$$= \left(0.5\,\frac{\text{m}}{\text{s}}\right)\left(\frac{303\,\text{kPa}}{101\,\text{kPa}}\right)\left(\frac{220\,\text{K}}{300\,\text{K}}\right)\left(\frac{0.6\,\text{m}^2}{1.0\,\text{m}^2}\right) = 0.66\,\frac{\text{m}}{\text{s}}$$

Then,

$$F = \left[(101-101)\frac{\text{kN}}{\text{m}^2}(1.0\,\text{m}^2) - (303-101)\frac{\text{kN}}{\text{m}^2}(0.6\,\text{m}^2)\right]\frac{10^3\,\text{N}}{\text{kN}}$$

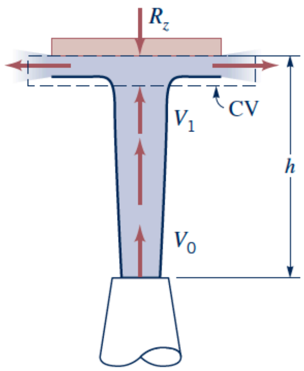
$$+\left(3.52\,\frac{\text{kg}}{\text{m}^3}\right)(0.6\,\text{m}^2)\left(0.50\,\frac{\text{m}}{\text{s}}\right)(0.66-0.5)\frac{\text{m}}{\text{s}}\,\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$= -121200\,\text{N}$$

or

$$\boxed{F = 121.2\,\text{kN, acting to left on venturi.}}$$

Solution 5.50



To determine the vertical distance h, we apply the vertical direction component of the linear momentum equation $\frac{\partial}{\partial t}\int_{CV} \mathbf{V}\rho d\mathcal{V} + \int_{CS} \mathbf{v}\rho\mathbf{v}\cdot\widehat{\mathbf{n}dA} = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$ to the water in the control volume shown in the sketch above. Thus,

$$-R_z - \rho g \mathcal{V}_{\text{water}} = -V_1 \rho A_1 V_1 = -\rho V_1^2 \frac{\pi}{4} D_1^2 \qquad (1)$$

The vertical reaction force of the plate on the water is equal in magnitude to the weight of the plate, or

$$R_z = g m_{\text{plate}} = \left(9.81 \frac{\text{m}}{\text{s}}\right)(1.5\,\text{kg}) = 14.7\,\text{N}$$

Also, the weight of the water within the control volume, $\rho g \mathcal{V}_{\text{water}}$, is negligible, and the mass flowrate is

$$\dot{m} = \rho A_1 V_1 = \rho A_0 V_0 = \left(999 \frac{\text{kg}}{\text{m}^3}\right)\frac{\pi}{4}(0.02\,\text{m})^2\left(10 \frac{\text{m}}{\text{s}}\right) = 3.13 \frac{\text{kg}}{\text{s}}$$

Thus, Eq. (1) becomes

$$-14.7\,\text{N} = -V_1\,\dot{m} \quad \text{or} \quad V_1 = \frac{14.7\,\text{N}}{3.13\,\frac{\text{kg}}{\text{s}}} = 4.70\,\frac{\text{m}}{\text{s}}$$

From the Bernoulli equation, $p + \frac{1}{2}\rho V^2 + \gamma z =$ constant along streamline, we have

$p_0 + \frac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1$, where $p_0 = p_1 = 0, z_0 = 0, z_1 = h$

Thus, $\frac{1}{2}\rho V_0^2 = \frac{1}{2}\rho V_1^2 h$

or since $\gamma = \rho g$

$$h = \frac{1}{2g}(V_0^2 - V_1^2) = \frac{1}{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)}()10^2 - 4.70^2\,\frac{\text{m}^2}{\text{s}^2} = \underline{\underline{3.97\,\text{m}}}$$