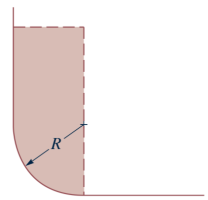
ME 3710 – Spring 2024
Homework 4
Due February 8 at 11:59pm – upload to files to Gradescope
21 points

Solution 2.114

In both cases, the magnitude of the vertical force is the weight of shaded section shown on the right. In addition, the location of the vertical force is the same (the centroid of the shaded section.) Therefore:



Alike: magnitude and location of vertical forces same.

However, the two vertical forces are different in that the force in (a) is acting upward and the force in (b) is acting downward. Therefore:

Different: direction of vertical forces opposite.

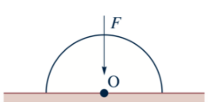
Solution 2.115

Due to symmetry, there is no net horizontal force on the roof. The vertical force is equal to the weight of fluid above the tunnel. This vertical force acts through the centroid of the fluid volume. Then for a tunnel length ℓ,

$$F = \gamma \mathcal{V} = \gamma \ell \left(2Rh - \frac{\pi}{2} R^2 \right)$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3} \right) (4\,\text{mi}) \left(5280 \frac{\text{ft}}{\text{mi}} \right) \left[2(20\,\text{ft})(70\,\text{ft}) - \frac{\pi}{2} (20\,\text{ft})^2 \right]$$

$\boxed{F = 2.86 \times 10^9\ \text{lb}}$.

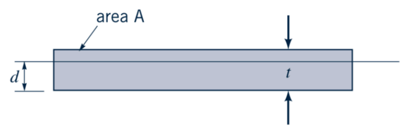


This force

acts downward through the point "O".

Solution 2.134

Without the polar bear on the ice, the submerged depth d of the ice is found by equating the weight of the ice and the buoyant force. Denoting the pure water specific weight by γ and the ice area by A gives



$$F_B = W_{\text{ice}}$$

or

$$W_{\text{ice}} = \gamma S A d.$$

The ice sinks an additional depth d' with the bear in the center of the ice. Equating the new buoyant force to the weight of the ice plus bear gives

$$F_B = W_{\text{ice}} + W_{\text{bear}},$$

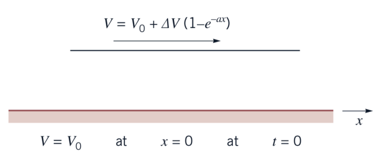
$$\gamma S A(d + d') = \gamma S A d + W_{\text{bear}},$$

or

$$A = \frac{W_{\text{bear}}}{\gamma S d'} = \frac{500\,\text{lb}}{\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)(1.03)\left(\dfrac{1}{12}\,\text{ft}\right)} \quad \text{or} \quad \boxed{A = 93.4\,\text{ft}^2}$$

Solution 4.2

GIVEN: See sketch.



FIND: $V(V_0, t)$ since we want the velocity of a fluid particle (Lagrangian description).

SOLUTION: The position x is found from

$$\frac{dx}{dt} = V = V_0 + \Delta V(1 - e^{-ax}).$$

Separating the variables and integrating gives

$$\int_0^x \frac{dx}{(V_0 + \Delta V) - \Delta V e^{-ax}} = \int_0^t dt$$

or

$$\left[\frac{x}{V_0 + \Delta V} - \frac{1}{(V_0 + \Delta V)(-a)} \ln(V_0 + \Delta V - \Delta V e^{-ax})\right]_0^x = t$$

Substituting the limits gives

$$\frac{x}{V_0+\Delta V}+\frac{1}{a(V_0+\Delta V)}\ln\left(\frac{V_0+\Delta V-\Delta V\mathrm{e}^{-ax}}{V_0}\right)=t$$

or

$$x+\frac{1}{a}\ln\left(1+\frac{\Delta V(1-e^{-ax})}{V_0}\right)=(V_0+\Delta V)t\,. \qquad (1)$$

Using the original equation for V gives

$$\frac{V-V_0}{\Delta V}=1-e^{-ax},\ e^{-ax}=1-\frac{V-V_0}{\Delta V},$$

and

$$x=-\frac{1}{a}\ln\left(1-\frac{V-V_0}{\Delta V}\right).$$

Substituting these expressions for e^{-ax} and x into Eq. (1) gives

$$-\frac{1}{a}\ln\left(1-\frac{V-V_0}{\Delta V}\right)+\frac{1}{a}\ln\left(\frac{\Delta V+V-V_0}{V_0}\right)=(V_0+\Delta V)t,$$

$$\frac{1}{a}\ln\left(\frac{V}{V_0}\frac{\Delta V}{\Delta V-V+V_0}\right)=(V_0+\Delta V)t,$$

$$\frac{V}{V_0}\frac{\Delta V}{\Delta V-V+V_0}=\mathrm{e}^{a(V_0+\Delta V)t},$$

$$V\Delta V=V_0(V_0+\Delta V-V)e^{a(V_0+\Delta V)t},$$

$$V(\Delta V+V_0e^{a(V_0+\Delta V)t})=V_0(V_0+\Delta V)e^{a(V_0+\Delta V)t}$$

or

$$\boxed{V=\frac{V_0(V_0+\Delta V)}{V_0+\Delta V_0e^{-a(V_0+\Delta V)t}}}$$

Solution 4.4

$u=1+y$ and $v=1$ so the streamlines are given by

$$\frac{dy}{dx}=\frac{v}{u}=\frac{1}{1+y}$$

Thus

$\int(1+y)dy=\int dx$ or

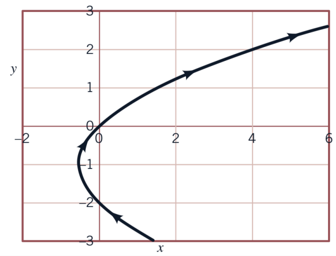
$y+\frac{1}{2}y^2=x+c$, where c is a constant.

For the streamline that goes through $x=y=0$, $c=0$.

Hence,

$$\underline{\underline{x=y+\frac{1}{2}y^2}}$$

This streamline is plotted below. Note that since $v = 1 > 0$, the direction of flow is as shown.



Solution 4.8

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(x+y)^2 + (xy^3 + 16)^2} = 0$$

or

$u = x + y = 0$ so that $x = -y$

and

$v = xy^3 + 16 = 0$ so that $xy^3 = -16$

Hence, $(-y)y^3 = -16$ or $y = 2$

Therefore, $V = 0$ at $\underline{\underline{x = -2,\ y = 2}}$

Solution 4.9

GIVEN: Two-dimensional, unsteady flow with

$$u = 5x(1+t) = \frac{dx}{dt} \qquad v = 5y(-1+t) = \frac{dy}{dt}$$

FIND: $x(t)$ and $y(t)$ if $x = x_0$ and $y = y_0$ at $t = 0$.

SOLUTION: Start with

$$\frac{dx}{dt} = 5x(1+t) \qquad \frac{dy}{dt} = 5y(-1+t).$$

Separating variables and integrating gives

$$\int_{x_0}^{x} \frac{dx}{x} = \int_0^t 5(1+t)dt, \qquad \int_{y_0}^{y} \frac{dy}{y} = \int_0^t 5(-1+t)dt,$$

$$\ln\left(\frac{x}{x_0}\right) = \frac{5}{2}[(1+t)^2]_0^t, \qquad \ln\left(\frac{y}{y_0}\right) = \frac{5}{2}[(-1+t)^2]_0^t$$

$$= \frac{5}{2}[(1+t)^2 - 1] \qquad = \frac{5}{2}[(-1+t)^2 - 1]$$

$$\boxed{x = x_0 e^{\frac{5(2t+t^2)}{2}}} \quad \text{and} \quad \boxed{y = y_0 e^{\frac{5(-2t+t^2)}{2}}}$$

The final point x(t) and y(t) are a Lagrangian description, you are following the point x and y in space only as a function of time. However, the original velocity field u and v is an Eulerian description;