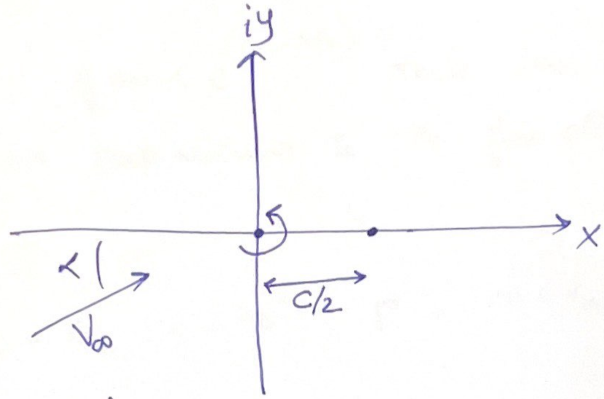
1) a)



$$W(z) = V_\infty e^{-i\alpha} - \frac{i\Gamma}{2\pi z}$$

$$W(c/2) = V_\infty e^{-i\alpha} - \frac{i\Gamma}{\cancel{2}\pi(c/\cancel{2})} = u - iv$$

$$v = V_\infty \sin\alpha + \frac{\Gamma}{\pi c} = 0 \Rightarrow \Gamma = -\pi c V_\infty \sin\alpha$$

so $$W(z) = V_\infty e^{-i\alpha} + \frac{i c V_\infty \sin\alpha}{2z}$$

$$F(z) = \int W dz = V_\infty z e^{-i\alpha} + \frac{i c V_\infty \sin\alpha}{2} \log_e(z)$$

b) $$C_l = \frac{l/s}{\frac{1}{2}\rho V_\infty^2 c} = \frac{-\rho V_\infty \Gamma}{\frac{1}{2}\rho V_\infty^2 c} = \frac{\cancel{-}\cancel{\rho}\cancel{V_\infty}(\cancel{-}\pi \cancel{c} \cancel{V_\infty} \sin\alpha)}{\frac{1}{2}\cancel{\rho}\cancel{V_\infty^2}\cancel{c}} = 2\pi\sin\alpha$$

so $\underline{C_l = 2\pi\sin\alpha}$

c) At stagnation point: $W(z) = 0 = V_\infty e^{-i\alpha} + \frac{i c V_\infty \sin\alpha}{2z}$

$$\Rightarrow z = -\frac{i c \cancel{V_\infty} \sin\alpha}{2\cancel{V_\infty} e^{-i\alpha}} = -i\sin\alpha \cdot e^{+i\alpha}\frac{c}{2}.$$

$$= + e^{-i\pi/2} \cdot e^{+i\alpha} \cdot \frac{c}{2}\sin\alpha$$

$\therefore z_{stag} = \frac{c}{2} \sin\alpha \, e^{i(\alpha-\pi/2)}$ Thus one stagnation point located $\frac{c}{2}\sin\alpha$ perpendicular to the free stream, beneath the vortex.

Streamlines → $\psi = Im(F(z)) = const.$

$$= Im\left(V_\infty z e^{-i\alpha} + \frac{icV_\infty sin\alpha}{2}\log(z)\right) = const.$$

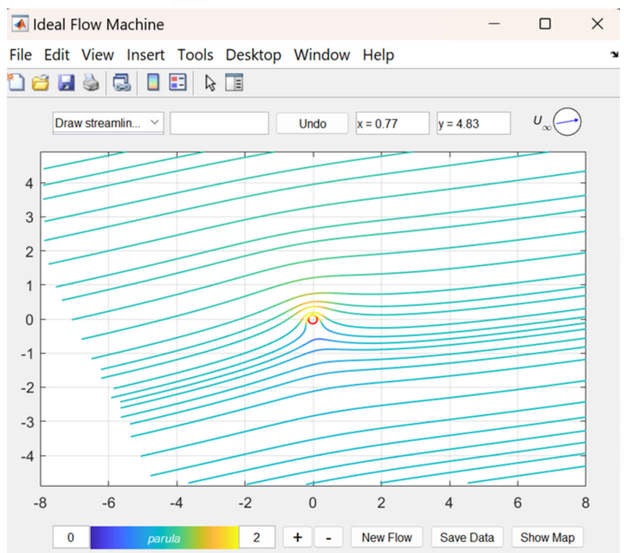
$$= Im\left(V_\infty r e^{i\theta} e^{-i\alpha} + \frac{icV_\infty sin\alpha}{2}\log(re^{i\theta})\right) = const$$

$$= Im\left(V_\infty r e^{i(\theta-\alpha)} + \frac{icV_\infty sin\alpha}{2}(\log(r) + i\theta)\right) = const$$

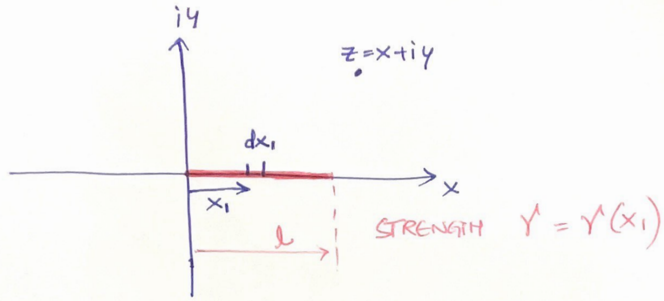
$$= Im\left(V_\infty r e^{i(\theta-\alpha)} + \frac{icV_\infty sin\alpha}{2}\log(r) - \frac{c\theta V_\infty sin\alpha}{2}\right) = const$$

$$= V_\infty r sin(\theta - \alpha) + \frac{cV_\infty sin\alpha}{2}\log(r) = const$$

$d)$ $V_\infty = 1, c = 4, \alpha = 4\ degrees,$
$\Gamma = -\pi c V_\infty \sin(\alpha) = -2.182$



2 a)



Consider the complex-velocity due to element dx_1:

$$dW(z) = \frac{-i\,\gamma\,dx_1}{2\pi(z - x_1)}$$

The velocity (dW) due to each element of the panel can be added to determine the velocity due to the entire panel.

$$W(z) = \int dW(z) = \int_0^l \frac{-i\,\gamma\,dx_1}{2\pi(z-x_1)}$$

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b) strength γ varies linearly from γ_1 at $x=0$ to γ_2 at $x_1 = l$.

$$\gamma(x_1) = \gamma_1 + (\gamma_2 - \gamma_1)\left(\frac{x_1}{l}\right)$$

$$W(z) = \int_0^l \frac{-i\left(\gamma_1 + (\gamma_2-\gamma_1)\frac{x_1}{l}\right)}{2\pi(z-x_1)}\,dx_1$$

$$= \int_0^l -\frac{i}{2\pi}\left(\frac{\gamma_1}{z-x_1}\right)dx_1 \mp i\int_0^l \frac{(\gamma_2-\gamma_1)}{2\pi l}\,\frac{x_1}{z-x_1}\,dx_1$$

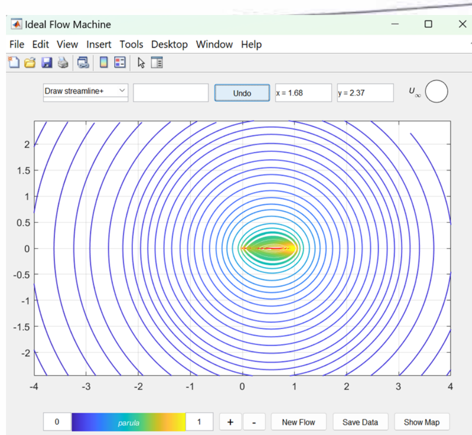
$$= \frac{-i\gamma_1}{2\pi}\int_0^l \frac{1}{z-x_1}\,dx_1 \mp \frac{i(\gamma_2-\gamma_1)}{2\pi l}\int_0^l \frac{x_1}{z-x_1}\,dx_1$$

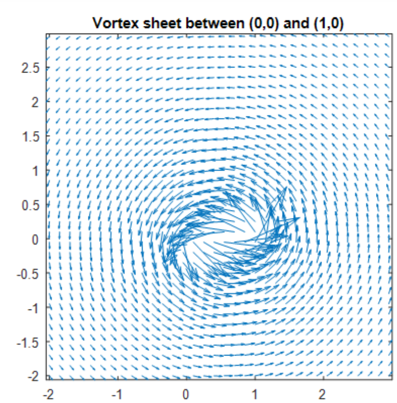
$$= \frac{-i\gamma_1}{2\pi}\left(\log(z-x_1)(-1)\right)\Big|_0^{l} + \frac{-i(\gamma_2-\gamma_1)}{2\pi l}\left[-z\log(x_1-z) - x_1\right]\Big|_0^{l}$$

$$= \frac{-i\gamma_1}{2\pi}\left[-\log(z-l)+\log(z)\right] - \frac{i}{2\pi}\,\frac{\gamma_2-\gamma_1}{l}\left[\begin{array}{l}-z\log(l-z)-l \\ \quad + z\log(0-z)\end{array}\right]$$

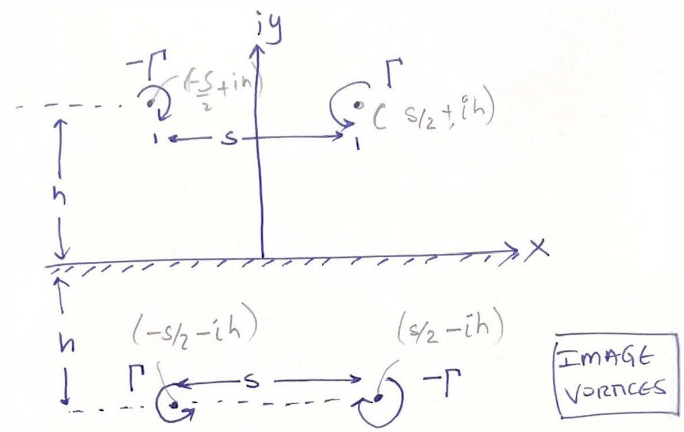
$$= \frac{-i\gamma_1}{2\pi}\log\frac{z}{(z-l)} - \frac{i}{2\pi}\,\frac{\gamma_2-\gamma_1}{l}\left[z\log\left(\frac{-z}{l-z}\right)-l\right]$$

$$W(z) = \frac{i\gamma_1}{2\pi}\log_e\left(\frac{z-l}{z}\right) + \frac{i}{2\pi}(\gamma_2-\gamma_1) + \frac{i(\gamma_2-\gamma_1)}{2\pi}\,\frac{z}{l}\log\left(\frac{z-l}{z}\right)$$





3)



The ground is simulating using the method of images.

a) $$W(z) = \frac{-i\Gamma}{2\pi(z-s/2-ih)} + \frac{i\Gamma}{2\pi(z+\frac{s}{2}-ih)} - \frac{i\Gamma}{2\pi(z+s/2+ih)} + \frac{i\Gamma}{2\pi(z-s/2+ih)}$$

$$F(z) = \frac{-i\Gamma}{2\pi}\log_e(z-s/2-ih) + \frac{i\Gamma}{2\pi}\log_e\left(z+\frac{s}{2}-ih\right)$$
$$-\frac{i\Gamma}{2\pi}\log_e(z+s/2+ih) + \frac{i\Gamma}{2\pi}\log_e(z-s/2+ih)$$