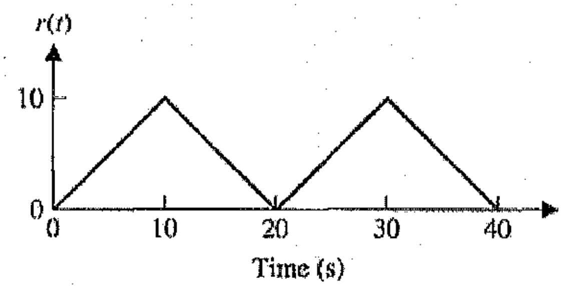
ME 5200/6200 and ECE 5615/6615 Classical Control

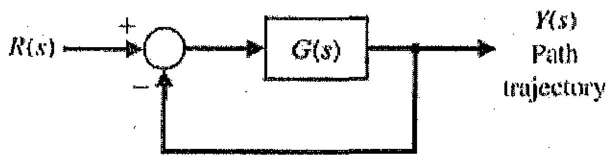
Homework 06 Solutions

Problem 1

Consider the following reference input and block diagram below:



(a)



(b)

Let the transfer function for the system be:

$$G(s) = \frac{75(s+1)}{s(s+5)(s+25)}$$

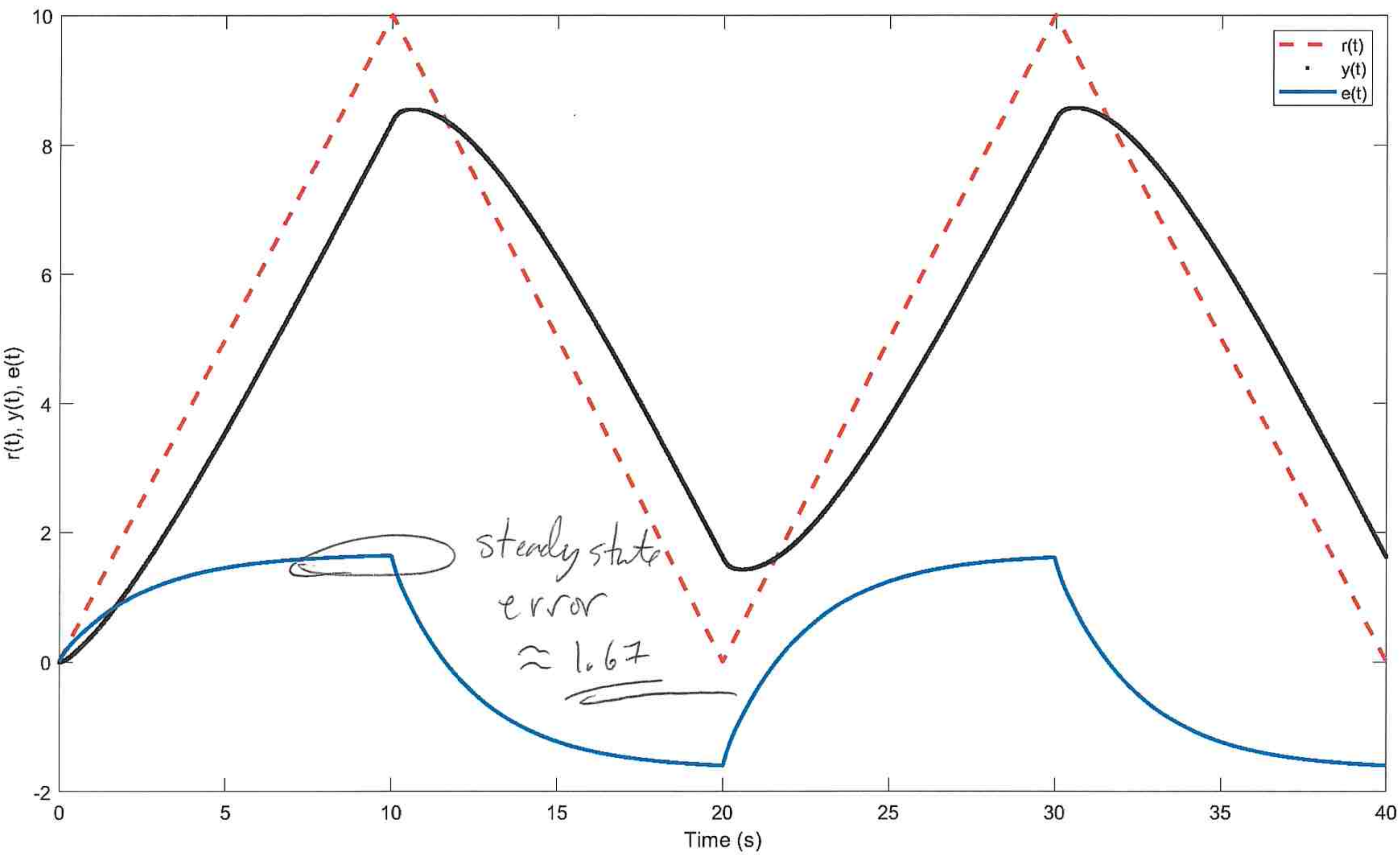
(a) Use Matlab to plot the time response of the output of the closed-loop system.
(b) Use Matlab to make a plot of the tracking error as a function of time.
(c) From the error plot, what is the steady state error when the input r(t) is shown above.

Make sure to submit a PDF print out of your Matlab code (m-file or Simulink diagram).

Can solve by hand as well, we have a ramp from $0 \to 10\,s$ and same from $10 \to 20\,s$, thus:

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} s\left(\frac{75(s+1)}{s(s+5)(s+25)}\right)$$

$$= \frac{75}{125} = 0.6 \Rightarrow e_{ss} = \frac{1}{K_v} = 1.67$$



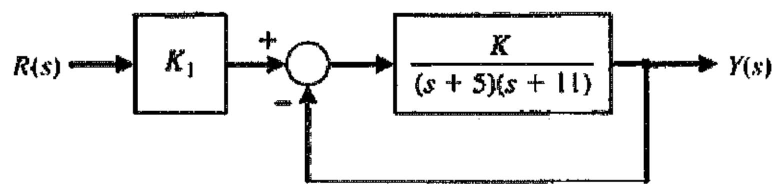
r(t)
y(t)
e(t)
10
8
6
4
2
0
-2
r(t), y(t), e(t)
0
5
10
15
20
25
30
35
40
Time (s)
steady state
error
≈ 1.67

Problem 2

For the closed-loop system below,

(a) Determine the steady-state error for a unit step input in terms of K and K_1, where $E(s) = R(s) - Y(s)$.

(b) Select K_1 so that the steady-state error is zero.



(a) Steady-state error is:

$$e_{ss} = \lim_{s \to 0} \frac{(s+5)(s+11) + K(1-K_1)}{(s+5)(s+11)+K}$$

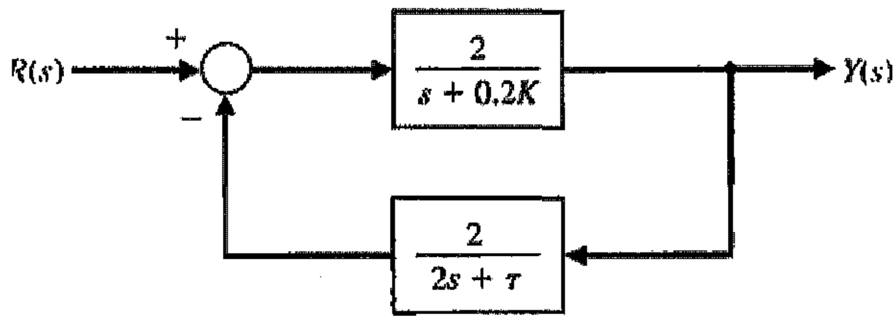
$$e_{ss} = \frac{55 + K(1-K_1)}{55+K}$$

(b) To achieve zero s.s., pick K_1 as:

$$K_1 = 1 + \frac{55}{K}$$

Problem 3

Consider the system below:



$$G_c(s)G(s) = \frac{2}{s + 0.2K} \quad \text{and} \quad H(s) = \frac{2}{2s + \tau}.$$

If $\tau = 2.43$, determine the value of K such that the steady-state error of the closed-loop system response to a unit step input, $R(s) = 1/s$, is zero.

Closed-loop T.F. is :

$$T(s) = \frac{2(2s+\tau)}{(s+0.2K)(2s+\tau)+4}$$

If $R(s) = 1/s$, then output is

$$Y(s) = \frac{2(2s+\tau)}{(s+0.2K)(2s+\tau)+4} * \frac{1}{s}$$

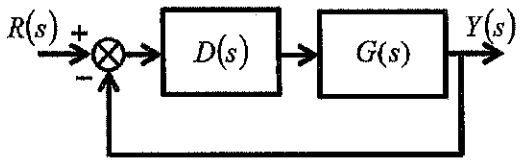
and $y_{ss} = \lim_{s\to 0} sY(s) = \frac{2\tau}{0.2K+4}$, and $e_{ss} = 1 - y_{ss}$

and we want $y_{ss} = 1$, so we pick K as

$K = 10 - 20/\tau \Rightarrow e_{ss} = 0$

Problem 4

Consider the closed-loop block diagram shown below. Let $G(s) = \frac{1}{(s+3)(s+4)}$. For each controller given below, find the steady-state error for: (i) a unit step input, (ii) a unit ramp input, and (iii) a unit parabolic input. State the system type and calculate the appropriate error constant for each of the five controllers. Assume, in each case, that the closed-loop system is stable.



(a) $D(s) = 2$ (a P controller)

(b) $D(s) = \frac{2(s+5)}{s}$ (a PI controller)

(c) $D(s) = 2(s+5)$ (a PD controller)

(d) $D(s) = \frac{2(s+5)(s+1)}{s(\tau s+1)}$, $\tau = \frac{1}{30}$ (a PID controller with a low-pass filter)

(e) $D(s) = \frac{2(s+5)(s+1)}{s^2}$ (a PID controller with a second integrator)

Problem 4

$\boxed{4}$ (a) $T(s) = \dfrac{2}{(s+3)(s+4)}$

No integrators
Type 0 (1 pt)
$K_p = \lim_{s\to 0} T(s) = \boxed{\dfrac{1}{6}}$ (1 pt)

(i) $e_{ss,step} = \dfrac{1}{1+K_p} = \dfrac{1}{1+\frac{1}{6}}$

$\boxed{e_{ss,step} = \dfrac{6}{7}}$ ← (1 pt)

(ii) $e_{ss,ramp} = \infty$
(iii) $e_{ss,para} = \infty$

Wrong Type for finite error!

(b) $T(s) = \dfrac{2(s+5)}{(s+3)(s+4)s}$

1 Integrator
Type 1 (1 pt)
$K_v = \lim_{s\to 0} sT(s) = \dfrac{10}{12} = \boxed{\dfrac{5}{6}}$ (1 pt)

(i) $e_{ss,step} = 0$
(iii) $e_{ss,para} = \infty$

From Type (1 pt)

(ii) $e_{ss,ramp} = \dfrac{1}{K_v}$

$\boxed{e_{ss,ramp} = \dfrac{6}{5}}$

(c) $T(s) = \dfrac{2(s+5)}{(s+3)(s+4)}$

No integrators
Type 0 (1 pt)
$K_p = \lim_{s\to 0} T(s) = \dfrac{10}{12} = \boxed{\dfrac{5}{6}}$ (1 pt)

(i) $e_{ss,step} = \dfrac{1}{1+K_p} = \dfrac{1}{1+\frac{5}{6}}$

$\boxed{e_{ss,step} = \dfrac{6}{11}}$ ← (1 pt)

(ii) $e_{ss,ramp} = \infty$
(iii) $e_{ss,para} = \infty$

(d) $T(s) = \dfrac{2(s+5)(s+1)}{s(\tau s+1)(s+3)(s+4)}$

1 integrator
Type 1 (1pt)

$K_v = \lim_{s \to 0} sT(s) = \frac{10}{12}$

$\boxed{K_v = \frac{5}{6}}$ (1 pt)

(i) $e_{ss,step} = 0$
(ii) $e_{ss,ramp} = \frac{1}{K_v} = \boxed{\frac{6}{5}}$
(iii) $e_{ss,para} = \infty$

(1pt)

(e) $T(s) = \dfrac{2(s+5)(s+1)}{s^2(s+3)(s+4)}$

2 integrators
Type 2 (1 pt)

$K_a = \lim_{s \to 0} s^2 T(s) = \frac{10}{12}$

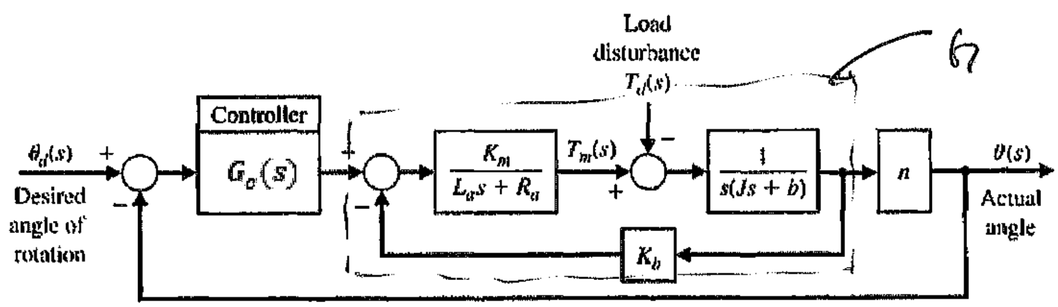
$\boxed{K_a = \frac{5}{6}}$ (1pt)

(i) $e_{ss,step} = 0$
(ii) $e_{ss,ramp} = 0$
(iii) $e_{ss,para} = \frac{1}{K_a} = \boxed{\frac{6}{5}}$

(1pt)

Problem 5

A robot arm has a shoulder joint where the model of the joint is given by the block diagram shown below. The joint uses a DC motor with armature control and has gears on the output shaft, so n is a constant. In the block diagram, there is a load disturbance $T_d(s)$ which represents the effect of the load. The entire closed-loop system shown below has two inputs, one for the desired angle of rotation and the other is the load disturbance. For simplicity, assume that K_m, L_a, R_a, J, b, n, and K_b are all equal to 1.



(a) Determine the transfer function that relates the desired angle of rotation $\theta_d(s)$ to the actual angle $\theta(s)$. What is the system type for this case?
(b) Determine the transfer function that relates the load disturbance $T_d(s)$ to the actual angle $\theta(s)$. What is the system type for this case?
(c) Determine the steady-state error when the desired angle $\theta_d(s)$ is a ramp input with slope B and the controller is $G_c(s) = 1/s$ (integral control). You can assume the disturbance is zero.
(d) Suppose the desired angle is zero and the load disturbance is a step of magnitude D. Determine the steady-state error if $G_c(s) = K$ (proportional control).

(a) T.F. from θ_d to θ:

$$\frac{\theta(s)}{\theta_d(s)} = \frac{G_c(s) G\, n}{1 + G_c(s) G\, n}$$

$$G = \frac{\left(\frac{K_m}{L_a s + R_a}\right)\left(\frac{1}{s(Js+b)}\right)}{1 + \left(\frac{K_m}{L_a s + R_a}\right)\left(\frac{1}{s(Js+b)}\right) K_b}$$

$$= \frac{K_m}{(L_a s + R_a) s (Js+b) + K_m K_b}$$

Hence:

$$\frac{\theta(s)}{\theta_d(s)} = \frac{G_c(s)\left(\frac{K_m}{(L_a s + R_a)(s)(Js+b) + K_m K_b}\right) n}{1 + G_c(s)\left(\frac{K_m}{(L_a s + R_a) s (Js+b) + K_m K_b}\right) n}$$

Looking at $G_c(s) G(s)\, n$, $G(s)$ is type 0, n is a constant, so system type will be determined by $\underline{G_c(s)}$!

(b)

$$\frac{\Theta(s)}{T_d(s)} = \frac{\dfrac{n}{s(sJ+b)}}{1 + G_c(s)\left(\dfrac{K_m}{(L_a s+R_a)(s)(Js+b)+K_m K_b}\right)n}$$

(c) $\Theta_d(s) = \frac{B}{s^2}$ $\quad G_c(s) = \frac{1}{s}$

$$e_{ss} = \frac{1}{K_v} \quad \text{and} \quad K_v = \lim_{s\to 0} s\,G_c(s)G(s)\,n$$

$$K_v = \lim_{s\to 0} \cancel{s}\left(\frac{1}{\cancel{s}}\right)\left(\frac{K_m}{(L_a s+R_a)s(Js+b)+K_m K_b}\right)n$$

$$K_v = \frac{\cancel{K_m}\, n}{\cancel{K_m} K_b} = \frac{n}{K_b} = \frac{1}{1} = 1$$

$$\Rightarrow \boxed{e_{ss} = 1}$$

(d) $G_c(s) = K$

$$\Theta_{ss} = \lim_{s\to 0} s\,\Theta(s) = \lim_{s\to 0} \cancel{s}\left(\frac{D}{\cancel{s}}\right)\left(\frac{\dfrac{K \cancel{K_m}\, n}{\cancel{K_m} K_b}}{1+K\left(\dfrac{K_m}{K_m K_b}\right)n}\right)$$

$$\Theta_{ss} = \frac{DK}{2}$$

$$\Rightarrow e_{ss} = \Theta_d - \Theta_{ss} = 0 - \frac{DK}{2} \Rightarrow \boxed{e_{ss} = -\frac{DK}{2}}$$