Metamaterial Analysis Through Finding γ

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1 Abstract

In this experiment I am looking to see if a soda can can act as a Helmholtz resonator that can alter the waveform of acoustic waves. To do so I see if I can find the adiabatic constant by varying the volume within the can and repeating the result at different frequencies. I find that a single soda can has the ability to increase the outgoing frequency of an acoustic wave but is not a strong enough Helmholtz resonator to find the adiabatic constant.

2 Introduction

A Helmholtz resonator is a type of metamaterial that can alter the waves that are sent through it by taking energy from the sound waves in the air. For such an object it is found that the resonance frequency ω is related to the volume of air inside the object V by the equation

$$\omega^2 = \frac{A^2 \gamma P}{mV}$$

where A is the area of the opening of the resonator, P is the ambient pressure of the environment, m is the effective mass of the air, and γ being the adiabatic constant that has an accepted value of ≈ 1.4. By varying the volume in the resonator and by knowing what frequency is being sent into the resonator, one could find the adiabatic constant if the object is a Helmholtz resonator. In this experiment the objective is to see if an ordinary soda can has the properties of a Helmholtz resonator.

3 Methods

In this experiment I used a waveform generator connected to a speaker on a breadboard to send sound waves through a can which then would be picked up by a microphone. Both the waveform generator and microphone are connected to an oscilloscope so that data can be recorded.The first test I conducted involved measuring the difference between waves that traveled through the can versus waves that had no can in their path. I did this at 50 Hz, 70 Hz, 90Hz, 110Hz, 130Hz all at an amplitude of 5Vpp. I initially used triangle waves and then used pulses so that any phase shifts in time were easy to see on the oscilloscope. [insert week 2 photos here] This offers an initial look to see if the can has any effect on the waveform and if the results are what is expected. I made a plot of the data to compare results.

The next experiment was to vary the volume of air within the can. To do so I measured the mass of the can and then filled the can with some small amount of water. I then placed the can in position of the speaker and microphone and recorded the data at a given frequency and then measured the mass of the can with the water. These steps were repeated until the can was full. The volume of air in the can is found by first calculating the volume of water in the can. Dividing the mass by the density of water which is approximately, $1g/cm^3$ for tap water, provides a reasonable estimate of the volume of water in the can, you can then find the volume of air remaining by subtracting from the total volume of the can. The total volume in the can should be measured using this method, fill the can with water and then find the mass of the water to get the volume of the can. It is also important to record the ambient pressure and to find the effective mass of the air. The mass of the air is heavily affected by the amount of CO2 in the room so I recorded the CO2 ppm (453 ppm) using the measuring device present in the lab room. The area of the opening of the can must also be measured, I decided to break off the top of the can so that the opening was approximately circular, and then I measured the radius of the opening (approximately 1cm) to find the area, which I found to be $3.14cm^2$. When taking this data I recorded the voltage vs time tables for 9 different volumes. In this case, I had the waveform generator create a pulse with a frequency of 90 Hz and a peak to peak voltage of 5 V. Once the data was collected I created a python program that takes a Voltage vs Time plot and produces a frequency spectrum that plots the weights of each frequency. In order to create such a plot you must preform a fast Fourier transformation which is demonstrated in the images at the end of this section. The starting point for the Fourier transform had to be changed to 0.001 to account for the time delay caused by the sound wave travelling to the microphone. The oscilloscope also had a limited precision which caused some complications. The time can only be shown with 4 digits which when transferred to a csv file, leads to results that appear to have multiple voltages at one given time. This cannot be allowed under Fourier transformation so in order to account for this issue I filtered the values and dropped duplicates, which increases the error in the measurement. The mean of the voltage values was calculated to account for DC offset as well. The peak frequency from the frequency spectrum plot is used as the natural frequency at the given volume. Using the equation

$$\gamma = \frac{\omega^2 m V}{A^2 P}$$

You can find the adiabatic constant at each frequency and take the average of all the values. The error is given by the devices used, the waveform generator, the oscilloscope, and the microphone. The uncertainty caused by the microphone is difficult to ascertain. The uncertainty for the frequency of the waveform generator is $\pm 1\%$. The uncertainty attributed to the oscilloscope is also more difficult to account for once the Fourier transformation is conducted. From the manual I believe an uncertainty of $\pm 5\%$ is reasonable.

4 Results Part 1

For experiment 1 at every frequency there was an observed decrease in peak to peak voltage when the can was in the path of the acoustic waves. This result is largely unremarkable as it is just a result of the sound being muffled by an object in its path. More interestingly, there was a change in the location of the peak when the can was present. As seen in the data below, in each case with

```python
#Fast Fourier Transform for frequency
# Convert columns to the appropriate data types if necessary
df['x-axis'] = pd.to_numeric(df['x-axis'], errors='coerce')
#df['voltage1'] = pd.to_numeric(df['voltage1'], errors='coerce')
df['2'] = pd.to_numeric(df['2'], errors='coerce')

# Drop rows with NaN values if necessary
df.dropna(inplace=True)

# Ensure time values are unique and sorted
df = df.sort_values(by='x-axis').drop_duplicates(subset='x-axis')

# Filter data to start from time >= 0.0010 seconds
df_filtered = df[df['x-axis'] >= 0.0010]

# Extract the filtered data into variables
time_filtered = df_filtered['x-axis'].values
voltage2_filtered = df_filtered['2'].values

# Check the sampling interval
if len(time_filtered) > 1:
    T = np.mean(np.diff(time_filtered))  # Average sampling interval
else:
    raise ValueError("Insufficient data points for FFT")
# Remove DC offset
voltage2_filtered -= np.mean(voltage2_filtered)

# Zero-padding to increase FFT resolution
N = len(time_filtered)
N_fft = 2**int(np.ceil(np.log2(N)))  # Next power of 2 for FFT
```

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# Plotting the time-domain data starting from 0.0010 seconds
plt.figure(figsize=(10, 6))
plt.plot(time_filtered, voltage2_filtered, label='Voltage From Mic (V)', color='r')
plt.xlabel('Time (s)')
plt.ylabel('Voltage (V)')
plt.title('Voltage vs. Time (Starting from 0.0010 s)')
plt.legend()
plt.xlim(left=0.0010)  # Ensure the x-axis starts at 0.0010 seconds
plt.grid()
plt.show()

# Perform FFT on the filtered data

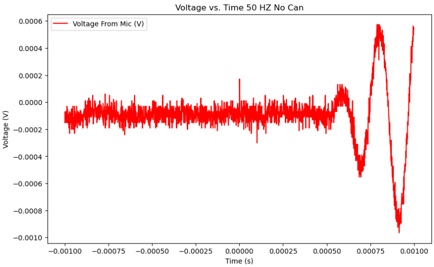
yf = fft(voltage2_filtered)
xf = fftfreq(N_fft, T)[:N_fft//2]  # Only take the positive frequencies

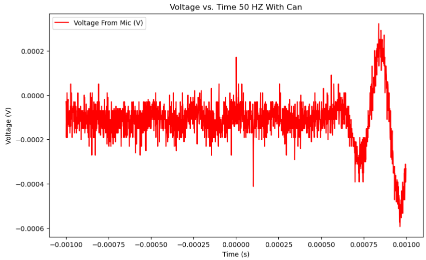
# Plot the frequency spectrum
plt.figure(figsize=(12, 6))
plt.plot(xf, 2.0/N * np.abs(yf[:N_fft//2]))
plt.title('Frequency Spectrum')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Amplitude')
plt.grid()
plt.show()

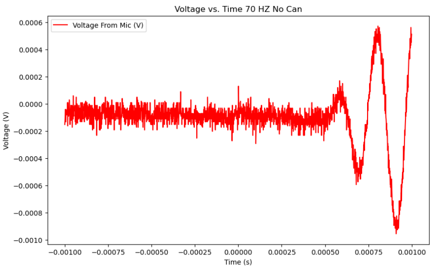
# Find the peak frequency
idx_peak = np.argmax(2.0/N * np.abs(yf[:N_fft//2]))
peak_frequency = xf[idx_peak]
print(f"The dominant frequency with 2.2 mL of air is {peak_frequency} Hz")
```

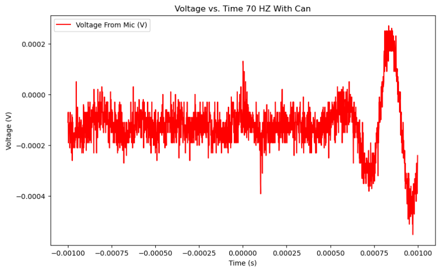
Figure 1: Fig. 1, Shows python code used to calculate a Fourier transformation from Voltage vs Time to Frequency.

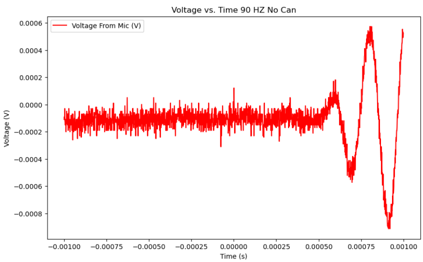
no can, approximately half of the next period can be seen in the data without the can which suggest that there is a phase shift caused by the can.

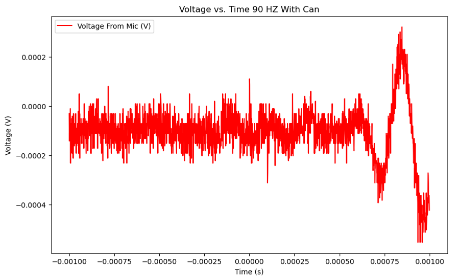

Voltage vs. Time 50 HZ No Can
Voltage From Mic (V)
Voltage (V)
Time (s)


Voltage vs. Time 50 HZ With Can
Voltage From Mic (V)
Voltage (V)
Time (s)


Voltage vs. Time 70 HZ No Can
Voltage From Mic (V)
Voltage (V)
Time (s)


Voltage vs. Time 70 HZ With Can
Voltage From Mic (V)
Voltage (V)
Time (s)


Voltage vs. Time 90 HZ No Can
Voltage From Mic (V)
Voltage (V)
Time (s)


Voltage vs. Time 90 HZ With Can
Voltage From Mic (V)
Voltage (V)
Time (s)

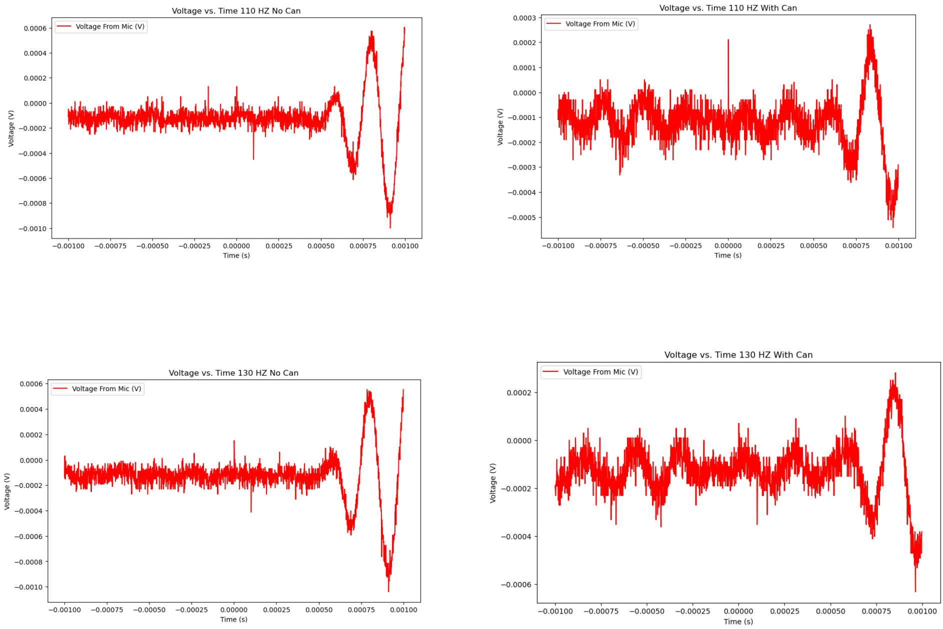


Figure 2: Fig. 2, Shows comparison between Voltage vs Time plots with and without the can for 50, 70, 90, 110, 130 Hz .

5 Results Part 2

The dominant frequency with 370 mL of air is 3906.25 ± 234.375, 314.8 mL of air has $3906.25 \pm 234.375Hz$, 284.8mL has $3515.625Hz \pm 210.9375$, 240.8mL has $2734.375 \pm 165.0625Hz$, 187.8 mL has $3906.25 \pm 234.375Hz$, 71.8 mL has $2734.375 \pm 164.0625Hz$, 73.8mL has $2734.375 \pm 165.0625Hz$, 2.2 mL has $2734.375 \pm 164.0625Hz$ The large uncertainties and the repeated values of frequency at different volumes signify that there are some systematic error in the experiment. This could be dude to a calculation error in the Fourier transformation but it is also possible that the size of the can being too small is a problem for an experiment that has such large uncertainty.For the adiabatic constant equation I use the values of $A = 0.000314m^2$ which I calculated by approximating the radius of the opening of being 1cm, $P = 1.013x10^5 N/m^2$ which is the average ambient pressure of a room at sea level. Of course the effective mass of air and frequency change with the volume of air in the can. To calculate the effective mass in the air I took the average density of air $1.293Kg/m^3$ and multiplied it by the volume of air such that

$$m = 1.293Kg/m^3 * V$$

We can construct a new equation

$$\frac{1.293V^2\omega^2}{PA^2} = \gamma$$

Now we calculate gamma for each volume. The value for the adiabatic constant with a volume of 370mL is 84914 which is very off from the expected value of 1.4. This signifies a very large systematic issue with my measurement. There are several components that can lead to small errors such as sounds picked up other than the waves generated from the speaker or water on the outside of the can increasing the weight but not contributing to the decrease of air volume in the can, but this level of error is most likely a computational mistake in the resonance frequency.

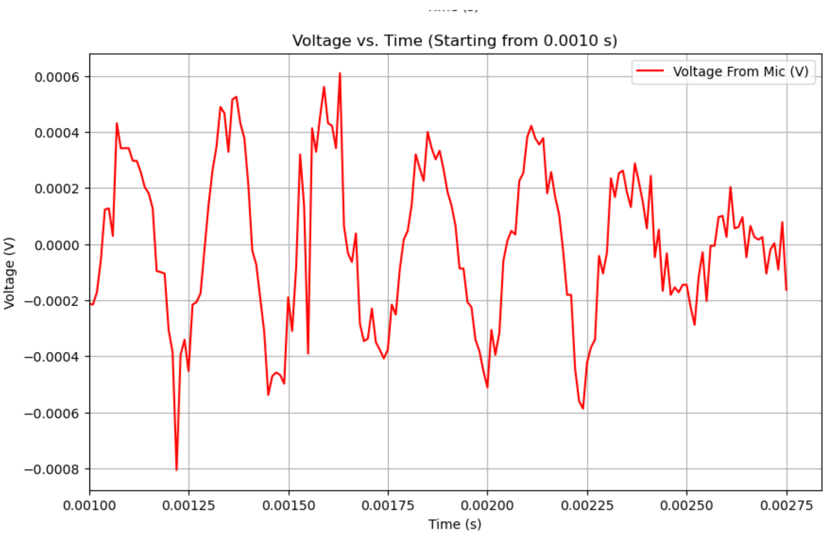


Figure 3: Shows Voltage vs Time plot .

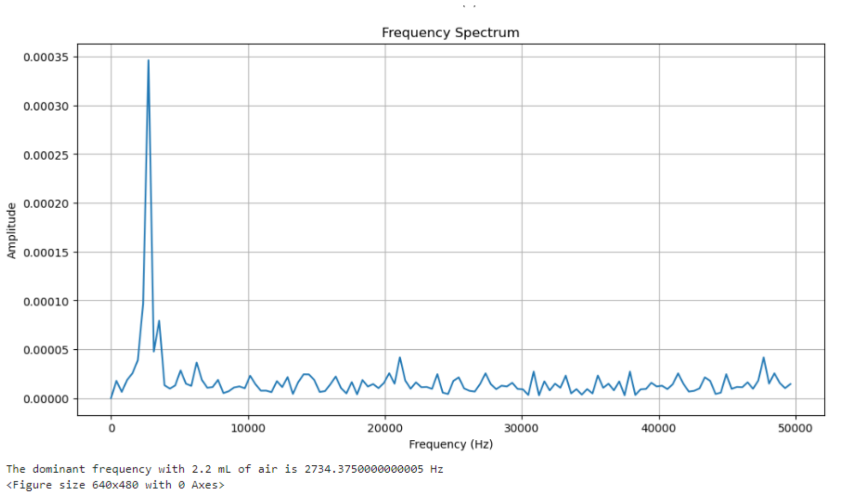


Figure 4: Shows Frequency Spectrum formed from Fourier transformation.