Modeling Meteorological Phenomena With Fourier Transforms

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In this paper I analyze how Fourier Transforms can be used to track and model weather systems. Mathematical derivation of these models along with graphical representations of the model is included and a comparison to other methods is made. The methods and materials necessary to provide meteorological information using Fourier Transforms offer a cost-effective alternative to more expensive methods that require technology that may not be affordable.

INTRODUCTION

Fourier transforms can be used to analyze complex periodic functions that comprise multiple periodic components. The Fourier transform itself deconstructs the complex periodic function such that you can isolate each component. Each Fourier transform is accompanied by a Fourier coefficient. The Fourier coefficient relates the frequency and time domains of the periodic function one is looking to analyze. By observing data sets that cover multiple years for a variety of weather systems such as temperature and wind speed, it is apparent that they are periodic in nature (2). Solar radiation, air temperature, and wind speed are dependent on a variety of variables that make the construction of a model nontrivial. Fourier transforms provide a way to create a model for these weather systems in an affordable manner with minimal equipment by deconstructing these complex periodic functions. This method must be applied to specific regions and environments in order to remain reasonably accurate. For example, if a region has low wind speeds and is a tropic environment, Fourier transforms can be used to obtain an equation that relates the two parameters (1). Even though this correlation is well known it is not easy to deduce one parameter from the other as there are a variety of other variables involved such as moisture in the air, cloud cover, etc. (4). Fourier transforms allow one to uncover the value of one of the chosen parameters. This relationship is possible because the data set taken from the area reveals that the temperature and net radiation can be modeled with periodic functions when taken to be diurnal as seen below.

Table I

Monthly minimum and maximum values of temperature (°C) and net radiation (W m^{-2}) at Osu station in 1995. Also included are the values of the average solar radiative input (W m^{-2} day^{-1}) and cloudiness (Oktas). The last column indicates the dates (not included in the means) selected for the case studies

Months	T (min.)	T (max.)	R (min.)	R (max.)	Average solar radiative input[1]	Av. Cloudiness[2]	Case studies
January	17.15	31.73	−34.89	436.83	4130.73	1	2nd
February	21.15	33.28	−33.93	479.91	4821.77	2	20th
March	23.20	31.18	−22.96	491.78	4952.57	2	10th
April	23.15	30.08	−15.77	438.96	4914.70	2	7th
May	22.03	29.33	−13.57	446.91	4843.48	4	19th
June	21.64	26.99	−22.28	347.82	4430.68	5	11th
July	21.74	25.48	−17.58	306.58	3192.70	5	27th
August	21.66	25.70	−16.15	318.02	3185.78	6	12th
September	21.49	26.78	−19.57	393.35	3820.13	4	23rd
October	21.43	27.66	−16.16	417.10	4480.83	3	6th
November	19.95	30.19	−22.40	519.60	4929.99	2	22nd
December	23.03	30.60	−20.82	463.89	4858.35	1	6th
Means	21.47	29.08	−21.34	421.73	4380.14	3	

[1] For Ile-Ife, Nigeria (7.29° N, 4.34° E). Period: 1992–94.
[2] Data from the Nigerian Meteorological Dept., Lagos. Period: 1971–84.

(Figure 1)

By using the methods described in the next section, it is possible to produce a model that can predict the net radiation from the temperature or vice versa. Therefore only one parameter needs to be tracked to find the other.

I. MATERIALS AND METHODS

Net radiation, air temperature, and or wind speed data that spans some substantial amount of time (days, months, years) is necessary. Random hours from the data set are chosen so that there is statistical independence between the Fourier transform coefficients and the test data (1).

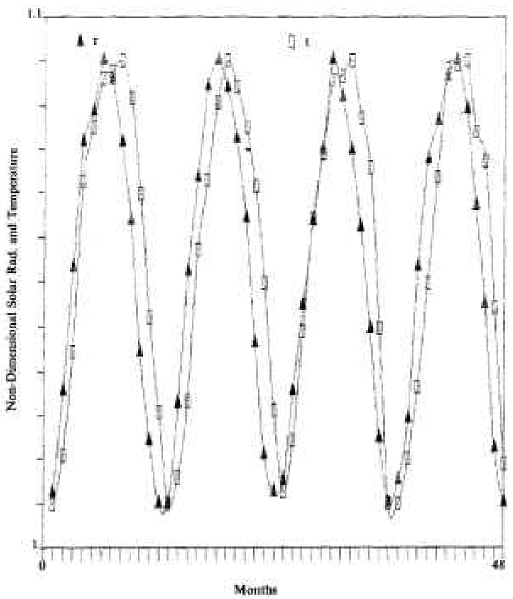


Fig. 1. The relationship between months of the year and non-dimensional solar radiation and temperature for Cairo city during the period (1989–1992)

(Figure 2)

Net radiation, air temperature, and wind speeds are all found to be periodic over a significant enough period of time with a time lag that is approximately constant. Solar radiation, air temperature, and wind speed are all expanded into the Fourier series by first finding the dimensionless equivalent (1). For example, radiation would be written as

$$r(k) = \frac{R(k) - R_{\min}}{R_{\max} - R_{\min}}$$

with min being the minimum radiation that year and max being the maximum radiation that year. We want to find the function R(k) which is possible using Fourier transforms. The trends in the data allow for a Fourier series to be constructed.

$$r_N = \frac{A_{or}}{2} + \sum_{j=1}^{\infty}\left[A_{rj}\cos\frac{\pi jN}{6} + B_{rj}\sin\frac{\pi jN}{6}\right]$$
$$t_N = \frac{A_{ot}}{2} + \sum_{j=1}^{\infty}\left[A_{tj}\cos\frac{\pi jN}{6} + B_{tj}\sin\frac{\pi jN}{6}\right]$$

One Fourier series for net radiation and temperature respectively. With N being the number of the associated with the time period you set, which in this example is the month. With A and B being the Fourier coefficients. Subtracting the radiation from the temperature series gives you the series

$$= \frac{A_o}{2} + \sum_{j=1}^{\infty}\left[A_j\cos\frac{\pi jN}{6} + B_j\sin\frac{\pi jN}{6}\right]$$

From here you can find the Fourier coefficients.

$$A_j = A_{rj} - A_{tj} = \frac{2}{12}\sum_{i=1}^{12}(r_i - t_i)\cos\frac{\pi ji}{6}$$

and

$$B_j = B_{rj} - B_{tj} = \frac{2}{12}\sum_{i=1}^{12}(r_i - t_i)\sin\frac{\pi ji}{6} \qquad (4)$$

The series goes from 1 to 12 because we are going by months. The difference between the radiation and temperature coefficients provides a singular coefficient for both that has an infinite amount of values, signified by the subscript j. Through comparisons of the model to real data it was found that the coefficients A and B for N/2 harmonics approximate the difference between radiation and temperature with the least root mean square error. With that in mind, we can construct a model for either temperature or solar radiation.

$$R = R_{\min} + (R_{\max} - R_{\min})\left(t_N + \frac{A_0}{2} + \sum_{j=1}^{6}\left[A_j \cos\frac{\pi j N}{6} + B_j \sin\frac{\pi j N}{6} \right]\right)$$

This method can also be applied to the temperature of the region. In doing so two Fourier transforms can be obtained.

$$F_r(n) = \frac{1}{N}\sum_{k=0}^{N-1} r(k)\cos\left(\frac{2\pi n k}{N}\right) - \frac{i}{N}\sum_{k=0}^{N-1} r(k)\sin\left(\frac{2\pi n k}{N}\right),$$

and

$$F_t(n) = \frac{1}{N}\sum_{k=0}^{N-1} t(k)\cos\left(\frac{2\pi n k}{N}\right) - \frac{i}{N}\sum_{k=0}^{N-1} t(k)\sin\left(\frac{2\pi n k}{N}\right),$$

By subtracting the two equations above one series can be obtained.

$$F_R(n) = \frac{1}{N}\sum_{k=0}^{N-1}[r(k) - t(k)]\cos\left(\frac{2\pi n k}{N}\right)$$

$$-\frac{i}{N}\sum_{k=0}^{N-1}[r(k) - t(k)]\sin\left(\frac{2\pi n k}{N}\right).$$

Now an inverse transform of the series can be computed so that it goes from being a function of frequencies (n = 0, 1, …, N -1) to a function of the hour of the day (time) (1).

$$A(k) \equiv [r(k) - t(k)] = 2\sum_{n=0}^{N/2}[F_{\text{real}}(n)]\cos\left(\frac{2\pi n k}{N}\right) - 2\sum_{n=0}^{N/2}[F_{\text{imag}}(n)]\sin\left(\frac{2\pi n k}{N}\right),$$

where,

$$F_{\text{real}}(n) = \frac{1}{N}\sum_{k=0}^{N-1}[r(k) - t(k)]\cos\left(\frac{2\pi n k}{N}\right),$$

and

$$F_{\text{imag}}(n) = \frac{1}{N}\sum_{k=0}^{N-1}[r(k) - t(k)]\sin\left(\frac{2\pi n k}{N}\right)$$

It is now possible to rewrite

$$r(k) = \frac{R(k) - R_{\min}}{R_{\max} - R_{\min}}$$

As

$$R(k) = R_{\min} + (R_{\max} - R_{\min})[t(k) + A(k)].$$

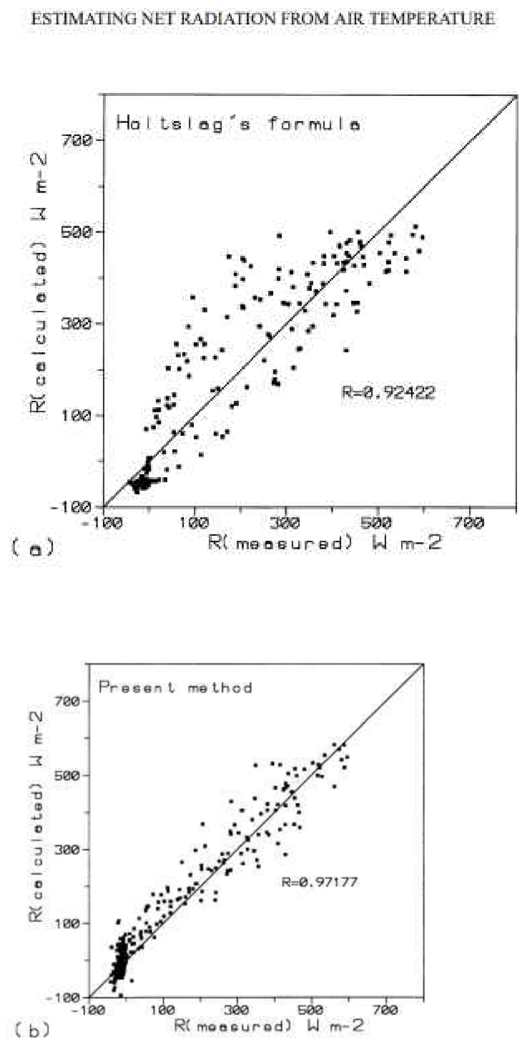
A model for net radiation dependent on time of day. The equation that is obtained depends on variables that are known from the data collected.

II. Results and Implications

Another way to create a model for net radiation would be to parametrize the relationship between temperature and radiation. This is known as the Holtslag formula

$$R = \frac{(1 - r)(a_1 \sin\phi + a_2)(1 - b_1 N^{b_2}) + c_1 T^6 - \sigma T^4 + c_2 N}{1 + c_3},$$

where there are several constants and variables that come from the environmental conditions (5). From a practical standpoint, the Fourier transform method may be a more desirable choice for situations in which variables in the equation such as solar elevation angle are difficult to obtain. Even when comparing the accuracy of the models the Fourier transform method has been found to be more effective within this specific data set (1). This could of course change depending on the location and environmental conditions that the model is based on.



(Figures 3 and 4)

Here we have a comparison of the Fourier method vs the Holtslag formula which uses parameterization to create a model for solar radiation. As you can see, the Fourier transform method gives you a more accurate model (4). This specific method relies on the accumulation of numerical data but it can be applied to image-based data such as hurricanes and other large weather systems (3). The main focus in that endeavor would be to use Fourier decomposition to analyze the boundaries to find a model for the path.

IV Discussion

These methods can provide a cost-effective way of predicting meteorological phenomena for specific areas and regions. While it requires data to be collected over a significant period of time and must be individually adapted to each location, it can be used further into the future and does not require the maintenance that other technology necessitates. This model is also reliant on the functions you are analyzing being periodic in nature, so in scales or time periods that are not in an appropriate scope, it may fail to produce a reasonably accurate result. Fourier transforms are a powerful tool for many other systems and weather patterns. Similarly with temperature and radiation, an accurate model of wind speed variability can be created by utilizing historical data and double Fourier transforms. A model such as this could be useful for the energy industry as a prediction of the energy that may be generated each month can inform decisions as to when the wind turbines should or should not be operating due to concerns of excess energy accumulation that may be too taxing on the power grid (2). Fourier transforms are vitally important in meteorological sciences.

V Acknowledgements

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VI References

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