Using the Michelson interferometer to measure the refractive index of air

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Abstract

We used the Michelson interferometer to measure the refractive index of air as a function of the pressure. We verified that the refractive index n is linearly related to the pressure P, and found that $n-1=(3.00\pm0.10)\times10^{-4}\cdot P/P_0$, where P_0 is the standard atmospheric pressure. Consequently, the atmospheric refractive index is $n=1+(3.00\pm0.10)\times10^{-4}$, which is larger than the expected value $1+2.8\times10^{-4}$. This discrepancy may be due to the overestimation of the wavelength of the laser emitters or the difference in the condition of the air in the gas cell from the standard condition. This experiment demonstrates the capability of the Michelson interferometer to measure small change in refractive index.

1 Introduction

When two coherent beams of light interfere, they typically produce interference patterns, where the bright regions are where the optical path length difference is an integer multiple of the wavelength (in vacuum) of the light, and the dark regions are where the optical path length difference is an odd multiple of half the wavelength. The optical path length of light traveling through such a material with refractive index n is nl, where l is the geometric path length. [1]

Therefore, we can change the optical path difference between two beams of light by changing the refractive index of the medium of one of the beams. For light traveling in the air, one way of changing the refractive index is by changing the pressure because the refractive index of air changes linearly in its pressure [2]. In this way, we can measure the refractive index of air as a function of the pressure.

This can be done with the Michelson interferometer, which consists of a beam splitter that splits on beam of light into two coherent beams. The two beams are reflected by mirrors and recombine at the beam splitter to interfere, creating an interference pattern on the viewing screen (if the source is an extended source) [3]. An illustration of the Michelson interferometer is shown in Figure 1.

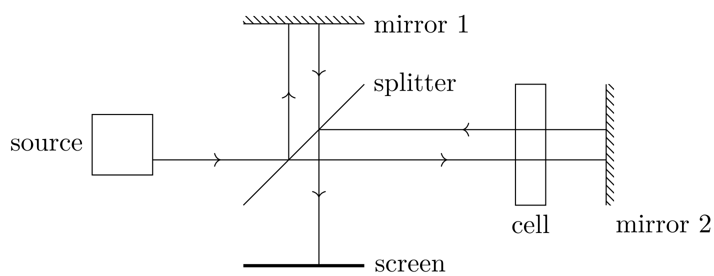


Figure 1: Illustration of the Michelson interferometer. The beam splitter splits the beam from source into two beams, which are reflected by mirror 1 and mirror 2 respectively to recombine at the splitter. If the source is an extended source (in this experiment, created by a laser with a convex lens in front) rather than a point source, an interference pattern can be seen on the screen because the two beams separated by the splitter have different optical path lengths. The gas cell is connected to a vacuum pump and a pressure gauge.

With this experiment, we can verify the linear relationship between the refractive index of air and the pressure, as well as showing the capability of the Michelson interferometer to measure small change in the refractive index, which is otherwise difficult to measure using usual methods of measuring the refractive index, such as using the refractometer.

2 Experimental methods

This experiment uses the PASCO™ Precision Interferometer kit [4] that provides all the necessary equipment with leveling screws. We start by setting up the laser and placing the components of the Michelson interferometer in place. In this experiment, for generality and accuracy, we used two different He–Ne lasers (called JDSU and Lumentum respectively) and interferometers and recorded data from both, the experimental setup is the same in both (both look like Figure 1). In the end, we average the results from the two lasers to reduce the uncertainties. The laser is placed behind a convex lens with a focal length of 18 mm to create an extended source. Mirror 1 can be moved to adjust its distance to the beam splitter, and the position of mirror 1 can be read from the micrometer attached to it.

Before any measurement, the interferometer needs calibrating. Before adding the splitter and the gas cell, mirror 2 is placed perpendicular to the table, and the laser is adjust so that it hits the center of mirror 2 and that the reflected beam hits exactly back to the aperture. The splitter is then placed at 45° according to the marks on the table, and mirror 1 is placed and adjusted so that the beam hits the center of mirror 1 and that the light spot from mirror 1 and mirror 2 overlap on the screen. After that, place the convex lens, and the interference pattern is seen.

Then, we can measure the wavelength of the laser. Move mirror 1 until a certain number M of rings on the screen appears or disappears. The distance d that mirror 1 moved is recorded, and the wavelength is

$$\lambda_0 = \frac{2d}{M}. \tag{1}$$

The factor of 2 is because the change in the path length is twice the distance traveled by mirror 1. Because the refractive index of air is close to 1, this wavelength is approximated as the wavelength in vacuum.

To measure the relationship between the pressure and the refractive index of air, we place a gas cell with thickness $t = 3\,\mathrm{cm}$ in the path of one of the beams. The gas cell is connected to a vacuum pump and a pressure gauge, which measures the pressure P in the gas cell relative to the atmospheric pressure. When the pressure changes by ΔP, the pattern on the screen changes by ΔN rings, They are linearly related as $|\Delta P/\Delta N| = k$, where k is a constant. The relationship between the refractive index n and pressure P is then given by

$$n = 1 + \frac{\lambda_0 P}{2kt}. \tag{2}$$

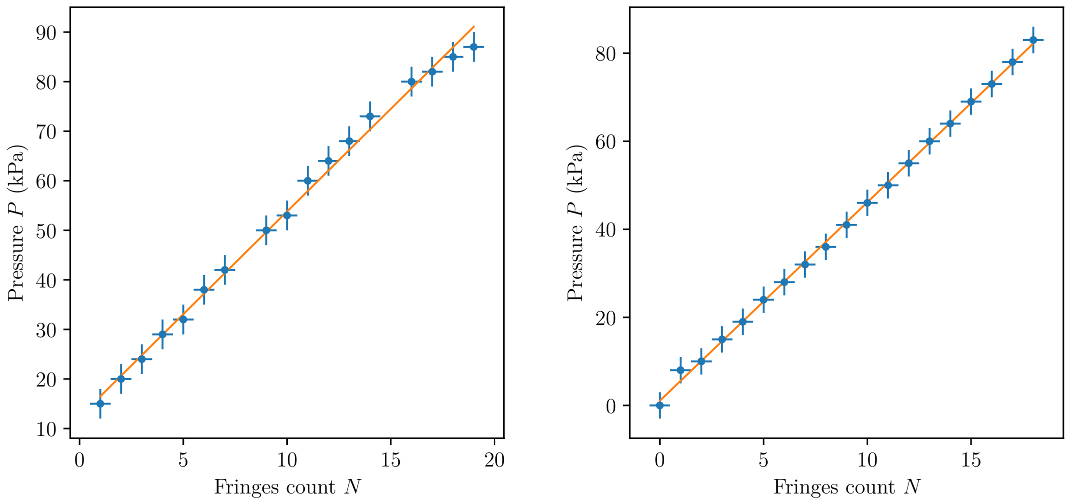
The derivation of this formula is given in Appendix A. Substitute P with the atmospheric pressure, and we can get the refractive index of air.

3 Results and analysis

For each wavelength measurement, we measure the distance d (standard uncertainty $0.5\,\mathrm{\mu m}$ set by the smallest scale of the micrometer) travelled by mirror 1 when the pattern changes by $M = 20.0 \pm 0.5$ rings (there is uncertainty because the pattern is not totally clear). For JDSU, 5 measurements are done; for Lumentum, 6 measurements are done. The data are substituted into Equation 1 to get the wavelength. Raw data are available in Appendix B. The results for JDSU is $\lambda_0 = 0.74 \pm 0.02\,\mathrm{\mu m}$; the results for Lumentum is $\lambda_0 = 0.80 \pm 0.02\,\mathrm{\mu m}$.

Then, the we change the pressure in the gas cell. Each time the pattern changes by one ring, the pressure is recorded (relative to the atmospheric pressure). The data are plotted in Figure 2. The ring counting has uncertainty of ± 0.5, and the pressure gauge reading has uncertainty $\pm 3\,\mathrm{kPa}$ set by the smallest scale of the pressure gauge. The raw data are available in Appendix B.

The data is then fitted to a linear function $P = kN + b$ using orthogonal distance regression (see Appendix C for the details) (with the tool NumPy [5] and SciPy [6]), where P is the pressure



(a) JDSU (b) Lumentum

Figure 2: The pressure P in the gas cell relative to the atmospheric pressure is plotted against the number N of rings. The blue points are the data points and error bars. The data is fitted to a linear function $P = kN + b$ using orthogonal distance regression. The orange line is the best fit.

reading on the gauge, and N is the number of rings. The result is

$$\begin{aligned} \text{JDSU:} \quad & b = 12.4 \pm 1.8\,\text{kPa}, \quad k = 4.14 \pm 0.16\,\text{kPa}, \\ \text{Lumentum:} \quad & b = 1.1 \pm 1.7\,\text{kPa}, \quad k = 4.51 \pm 0.16\,\text{kPa}. \end{aligned}$$

Substitute the results into Equation 2, and we have

$$\begin{aligned} \text{JDSU:} \quad & n = 1 + (3.02 \pm 0.15) \times 10^{-4}\,\frac{P}{P_0}, \\ \text{Lumentum:} \quad & n = 1 + (2.99 \pm 0.13) \times 10^{-4}\,\frac{P}{P_0}, \end{aligned}$$

where $P_0 = 101.325\,\text{kPa}$ is the standard atmospheric pressure [7]. The uncertainty is calculated by the standard error propagation formula, handled by the `uncertainties` Python package [8]. Average the results for $n-1$ from JDSU and Lumentum, and we have $n - 1 = (3.00 \pm 0.10) \times 10^{-4} \cdot$

P/P_0. This gives the atmospheric refractive index $n = 1 + (3.00 \pm 0.10) \times 10^{-4}$.

4 Discussion

The expected value of the refractive index of air for the wavelengths used in this experiment is $n = 1 + 2.8 \times 10^{-4}$ [9]. The result of $n = 1 + (3.00 \pm 0.10) \times 10^{-4}$ is larger than the expected value by about 2σ.

One possible reason for the overestimation is that the we overestimated the wavelength of the laser emitters. Both JDSU and Lumentum are He–Ne lasers, so their wavelengths should be $0.633\,\mu\mathrm{m}$ [10], which is smaller than the measured values. Another possible reason for the discrepancy is that the air in the gas cell is not in the same condition as the condition stated in [9], which is in temperature $15\,^\circ\mathrm{C}$ and $450\,\mathrm{ppm}$ CO_2 content.

A Formula of the refractive index

In this appendix, the derivation of Equation 2 is presented. Because the refractive index n is linearly related to the pressure P [2], we have $n = 1 + \alpha P$ (and thus $\Delta n = \alpha \Delta P$) for some constant α. The optical path length of the light in the gas cell is $2nt$, where t is the thickness of the gas cell. The factor of 2 is because the light travels through the gas cell twice. Therefore, when the pressure changes by ΔP, the optical path length changes by

$$\Delta l = 2t\,\Delta n = 2t\alpha\,\Delta P.$$

On the other hand, when the pattern changes by ΔN rings, the optical path length changes by $\Delta l = \lambda_0\,\Delta N$, where λ_0 is the wavelength of the light in the vacuum. Equate the two expressions for Δl, and we have

$$\alpha = \frac{\lambda_0}{2t}\,\frac{\Delta N}{\Delta P}.$$

Therefore, $\Delta P/\Delta N$ should be a constant $k = \lambda_0/2t\alpha$, and

$$n = 1 + \frac{\lambda_0 P}{2kt}.$$

This recovers Equation 2.

B Raw data and program

One can download the raw data and the Python program used in this experiment from `https://gist.github.com/UlyssesZh/4b445dd7a1d551cdb5526396023c5b5b`.

Here is the instruction to run the program. First, install Python 3.8 or later from `https://python.org`. Install required packages using `pip install -r requirements.txt`. Then, run the program using `python main.py`. It will output the numerical results with uncertainties. The plots will be saved as `jdsu.pdf` and `lumentum.pdf`.

C Orthogonal distance regression (ODR)

A brief introduction to orthogonal distance regression (ODR) is given here. For more details, see [11, 6]. The letters used in this section are not the same as those used previously in the report.

Let $(X_i \in \mathbb{R}^m, Y_i \in \mathbb{R})$ be a set of observed random variables with underlying true values, where $i = 1, \ldots, n$. Suppose the true values are related by a smooth function $f\colon \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}, \quad (x, \beta) \longmapsto y$, where β is a set of parameters. Then, the ODR refers to the regression to find the best fit β so that the sum of squared orthogonal distances from (X_i, Y_i) to the graph of $y = f(x, \beta)$ being minimized:

$$\left(\hat{\beta}, \left\{\hat{\delta}_i\right\}\right) = \mathop{\arg\min}_{\beta, \{\delta_i\}} \sum_{i=1}^{n} w_i^2 \left(\left(f(X_i + \delta_i, \beta) - Y_i\right)^2 + d_i^2 \delta_i^2\right)^2,$$

where $w_i \geq 0$ and $d_i > 0$ are some weights, and $\delta_i \in \mathbb{R}^m$ are the random error associated with X_i.

For the case in which we are interested, the weights are given by

$$w_i \coloneqq \frac{1}{\sigma_{\varepsilon_i}}, \quad d_i \coloneqq \frac{\sigma_{\varepsilon_i}}{\sigma_{\delta_i}},$$

where $\varepsilon_i \in \mathbb{R}$ are the random error associated with Y_i.

To do this regression, define the function $\mathbf{g}\colon \mathbb{R}^p \times (\mathbb{R}^m)^n \to \mathbb{R}^{2n}$ by

$$\begin{aligned} g_i(\beta, \{\delta_j\}) &\coloneqq w_i \left(f(X_i + \delta_i, \beta) - Y_i\right), & i &= 1, \ldots, n, \\ g_{n+i}(\beta, \{\delta_j\}) &\coloneqq w_i d_i \delta_i, & i &= 1, \ldots, n, \end{aligned}$$

and then the ODR problem is equivalent to the least squares problem

$$\hat{\theta} = \operatorname*{arg\,min}_{\theta \in \mathbb{R}^{p+mn}} \left\| \mathbf{g}(\theta) \right\|^2,$$

where $\|\cdot\|$ denotes the 2-norm on $\mathbb{R}^{2n}$, and $\theta \coloneqq (\beta, \{\delta_i\})$. Since it is now a least squares problem, we can use the Gauss–Newton algorithm to get the best fit $\hat{\theta}$ for θ, and for small error variances we can use the linearization method to estimate the covariance matrix of $\hat{\theta}$. The covariance matrix of $\hat{\beta}$ is then the upper-left $p \times p$ submatrix of the covariance matrix of $\hat{\theta}$, from which we can get the confidence intervals and thus the uncertainties in the parameters β.

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