

Theorem: Every odd natural number is of one of the forms $4n+1$ or $4n+3$, where n is an integer.

Proof: By induction.

Define an odd number as $2m + 1$

Assume $n = 0$, then:

$$4 \cdot 0 + 1 = 1$$

$$4 \cdot 0 + 3 = 3$$

These are the first two natural numbers, so we can assume n , and deduce $n+1$ by induction:

$$4 \cdot (n+1) + 1 = 4n + 4 + 1 = 4n + 5$$

$$4n + 5 = (4n + 3) + 2$$

5 is the next natural odd number after 3, since if $m = 1$, then using the definition of an odd number:

$$3 = 2 \cdot 1 + 1$$

and if $m = 2$, then:

$$5 = 2 \cdot 2 + 1$$

Hence, by induction, the theorem is proved.