Theorem:

If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as n$\rightarrow\infty$, then for any fixed number M>0, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML or $(\forall M \in \mathbb{N})(|Ma_n - ML| < \varepsilon)$.

Proof:

By contradiction.

Given ε > 0.

Assume the negation is correct:

$(\exists M \in \mathbb{N})(|Ma_n - ML| \geq \varepsilon)$

Rearranged:

$|a_n - L| \geq \frac{\varepsilon}{M}$

But we know that $|a_n - L| < \varepsilon$, therefore we can find N large enough so that $|a_N - L| < \frac{\varepsilon}{M}$. This is a contradiction, hence the theorem is proved.