Theorem: The sum of any five consecutive integers is divisible by 5 (without remainder) or:

$$(\exists p \in \mathbb{N}) f(n) = \sum_{i=n}^{n+4} i = 5 \cdot p$$

Proof: By induction.

Assume n = 0:

$$f(n) = \sum_{i=n}^{n+4} i = 0+1+2+3+4 = 10 = 5 \cdot 2$$

So the identity is valid for f(0), where p = 2.

Then we assume f(n) and we deduce f(n+1):
First we extract the last iteration, where the first digit, n, will be removed from the beginning of the sequence, and another digit will be added in at the end (n+5), since there must be 5 consecutive digits:

$$f(n+1) = \sum_{i=n+1}^{n+5} i = \sum_{i=n}^{n+4} i + (n+5) - n$$

Using the induction hypothesis, we know the previous sum is a multiple of 5 some number p:

$$f(n+1) = \sum_{i=n+1}^{n+5} i = 5p + (n+5) - n$$

Simplified:

$$f(m+1, n+1) = \sum_{i=m+1}^{n+1} i = 5p + 5 = 5(p+1)$$

This is the identity for n+1. Hence, by induction, the theorem is proved.