

Theorem: $A_n = (-\infty + n, \infty - n)$ or $\{x \in \mathbb{R} | -\infty + n < x < \infty - n\}$ has the properties $A_{n+1} \subset A_n$ for all n and their intersection $= \emptyset$.

Proof:

Start with $A_0 = (-\infty, \infty)$ which include all points on the real line.

Pick an arbitrary n . A_n will include $(-\infty + n + 1)$ since:

$$(-\infty + n) < (-\infty + n + 1) < (\infty - (n + 1)) < (\infty - n)$$

Now the next interval in the family is $(-\infty + (n + 1), \infty - (n + 1))$. Since

$\neg((-\infty + n + 1) < (-\infty + n + 1) < (\infty - (n + 1)) < (\infty - (n + 1)))$ is true, then A_{n+1} will not contain the point $(-\infty + n + 1)$.

Therefore a A_{n+1} is a subset of A_n .

Since the interval $A_\infty = (-\infty + \infty, \infty - \infty)$, which is the same as $(0, 0)$, there are no points within the set since for $(0 < x < 0)$, x cannot exist). Since there are no points within the set, it cannot intercept any other interval. This means the intersection for all intervals will be \emptyset .

This proves the theorem.