Theorem: $A_n = (n > 0)[-\infty + n, \infty - n]$ or $\{x \in \mathbb{R} | -\infty + n \leq x \leq \infty - n\}$ has the properties $A_{n+1} \subset A_n$ for all n and their intersection is a single real number.

Proof:

Pick an arbitrary n. $A_n$ will include $(-\infty + n + 1/2)$ since:

$$(-\infty + n) \leq (-\infty + n + \frac{1}{2}) < (\infty - (n + \frac{1}{2})) \leq (\infty - n)$$

Now the next interval in the family is is $(-\infty + (n + 1), \infty - (n + 1))$. Since $\neg((-\infty + n + 1) < (-\infty + n + \frac{1}{2}) < (\infty - (n + \frac{1}{2})) < (\infty - (n + 1)))$ is true, then $A_{n+1}$ will not contain the point $(-\infty + n + 1/2)$.

Therefore a $A_{n+1}$ is a subset of $A_n$.

Since the interval $A\infty = [-\infty + \infty, \infty - \infty]$, which is the same as [0, 0], there is one real number (0) within the set since for $(0 \leq x \leq 0)$, the only number x can be is 0 $(0 = x = 0)$. Since $A_{n+1} \subset A_n$ this means that every other interval will contain 0. Therefore the intersection for all intervals will be 0.

This proves the theorem.