Theorem: Prove that for any natural number n, $2+2^2+2^3+\ldots+2^n=2^{n+1}-2$.

Proof:

By induction.

First we prove where n = 1: $2=2^2-2=2$ Which is true.

Then we assume n and deduce n+1. First we extract that last iteration:

$$\sum_{i=1}^{n+1} 2^i = \sum_{i=1}^{n} 2^i + 2^{n+1} = 2^{n+1+1} - 2$$

Using the induction hypothesis we replace the sum up to n with $2^{n+1}-2$:

$$=\left(2^{n+1}-2\right)+2^{n+1}$$

Simplified:

$$=2\cdot 2^{n+1}-2$$

$$=2^{n+2}-2$$

Which is the same as $2^{n+1+1}-2$. Hence, by induction the theorem is proved.