

Theorem: $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

Proof: There are three cases:

Case 1: Suppose $n = 1$. Then: $3 \cdot 1 + 5m = 12$

Simplify: $5m = 12 - 3 = 9$

9 is not a multiple of 5 in the Natural numbers, so this cannot be correct.

Case 2: Suppose $m = 1$. Then: $3n + 5 \cdot 1 = 12$

Simplify: $3n = 12 - 5 = 7$

7 is not a multiple of 5 within the Natural numbers, so this also cannot be correct.

Case 3: Suppose $n = 2$ and $m = 2$. Then: $3 \cdot 2 + 5 \cdot 2 = 12$

Simplify: $6 + 10 = 16 = 12$

Which is also not correct, and more so, $16 > 12$, therefore for any m or n greater than 2, the result would be greater than 12. Therefore this theorem is false.