Theorem: For any integer n, the number n^2+n+1 is odd.

Proof:

Define odd as $(m \in \mathbb{N})[2m+1]$ and even as $(m \in \mathbb{N})[2m]$.
Either n is even or n is odd.
Case 1: If n is odd, then n = 2m+1:

$$n^2+n+1=(2m+1)^2+2m+1+1$$
$$expanded: \; m^2+4m+1+2m+1+1$$
$$simplified: \; 4m^2+6m+3$$
$$factor\,out: \; 2(2m^2+3m+1)+1$$

Which is the definition of an odd number.

Case 2: if n is even, then n = 2m:

$$n^2+n+1=(2m)^2+2m+1$$
$$expanded: \; 4m^2+2m+1$$
$$factor\,out: \; 2(2m^2+m)+1$$

which is the definition of an odd number.

Hence, whether n is odd or even, the result of n^2+n+1 is odd and the theorem is proved.