1 Acoustic Contrast Control

Acoustic contrast control was first proposed by Choi and Kim [1] as a constrained optimization cost function that maximizing the ratio between the mean square pressure in a bright (listening) and dark (quite) zone, denoted as the subscripts b and d. Acoustic contrast (AC) is defined as

$$AC = 10log_{10}\left(\frac{L_d \mathbf{p}_b^H \mathbf{p}_b}{L_b \mathbf{p}_d^H \mathbf{p}_d}\right) = 10log_{10}\left(\frac{L_d \mathbf{q}^H \mathbf{G}_b^H \mathbf{G}_b \mathbf{q}}{L_b \mathbf{q}^H \mathbf{G}_d^H \mathbf{G}_d \mathbf{q}}\right), \tag{1}$$

where $\mathbf{p}$ is an L x 1 column vector of pressure, $\mathbf{q}$ is an M x 1 column vector of complex source strengths, $\mathbf{G}$ is an L x M matrix of acoustic transfer functions between each M control sources and L control points, and H denotes the Hermitian operator. The AC is maximized by solving a constrained cost function where $\mathbf{p}_b^H \mathbf{p}_b$ is maximized under the constraint that $\mathbf{p}_d^H \mathbf{p}_d$ is kept at a constant real value D. With the use of the Lagrangian multiplier method, the cost function to be maximized is given by

$$J(\mathbf{q}, \lambda) = \mathbf{q}^H \mathbf{G}_b^H \mathbf{G}_b \mathbf{q} - \lambda(\mathbf{q}^H \mathbf{G}_d^H \mathbf{G}_d \mathbf{q} - D), \tag{2}$$

where λ is the Lagrangian multiplier. Taking the partial derivative with respect to $\mathbf{q}$ and λ and equating the results to 0, yields the stationary points as

$$\lambda \mathbf{q} = [\mathbf{G}_d^H \mathbf{G}_d]^{-1}[\mathbf{G}_b^H \mathbf{G}_b]\mathbf{q}, \tag{3}$$

$$\mathbf{q}^H \mathbf{G}_d^H \mathbf{G}_d \mathbf{q} = D. \tag{4}$$

It was shown by Choi and Kim [1] that the optimal source strengths $\mathbf{q}$ that maximizes the AC is proportional to the eigenvector that corresponds to the max eigenvalue of $[\mathbf{G}_d^H \mathbf{G}_d]^{-1}[\mathbf{G}_b^H \mathbf{G}_b]$. λ is then chosen manually so that Equation 4 is met.

References

[1] J.-W. Choi and Y.-H. Kim, "Generation of an acoustically bright zone with an illuminated region using multiple sources," *The Journal of the Acoustical Society of America*, vol. 111, no. 4, pp. 1695–1700, 2002.