# An Exploration of Large Numbers

## A UTM Math Club Presentation

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# Introduction

What is the largest number that you use on a daily basis?

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In everyday life, the average person is not going to use numbers larger than a few thousand. If they go to a bank that number might increase to the tens or hundreds of thousands depending on what is being discussed.

For people working in economics, they might regularly use numbers in the millions, billions, or even the trillions, and there are very few people who would use numbers larger than that in their daily life.

# Introduction

However, there are plenty of fields of math, both pure and applied, that produce much larger numbers; numbers that are so big that quantifying them, representing them, or even just comprehending them, become impossible to do. The study of

In this presentation, we will walk through some of the most famous large numbers in mathematics, in order of size, while also discussing the mathematics that produced them.

# The Rules

Before we start, let's outline some basic rules:

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2. Every number must be explicitly defined. Something like "the largest number that a human could ever think of, plus 1," or "the largest number that can be described in 14 or fewer English words," is not allowed.
3. Each number must not be a "naive extension" of the previous ones; we cannot just add one to the previous entry.

# The Fathomable

We begin with a look at some big, yet relatively easy to comprehend large numbers, as well as what they represent in mathematics.

# A "Monster" Outlier

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## Classification of Finite Simple Groups

Every finite simple group is isomorphic to one of the following groups:

1. A cyclic group of prime order
2. An alternating group of at least degree 5
3. A group of Lie type
4. The Tits group
5. One of 26 "Sporadic" groups

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5. One of 26 "Sporadic" groups

These "Sporadic" groups have some interesting properties, mainly their size relative to the rest of the groups.

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This group would go on to be called “The Monster Group,” denoted by $M$. It represents the symmetries of an object with 196,883 spacial dimensions. It has order

808,017,424,794,512,875,886,459,904,961,710,757,005,754,368,000,000,000

Which dwarfs the size of any other group we have come across.

# A Googol

This would not be a presentation about large numbers without mentioning a Googol.

A Googol is the number $10^{100}$, and is the first number with 100 digits.

The name Googol would go on to inspire the name of the world's most famous search engine "Google."

# Mersenne Primes

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**Definition**

A Mersenne Prime is any prime number of the form

$$2^n - 1$$

where $n$ is a positive integer. We denote this number by $M_n$.

The first few Mersenne primes are $M_2, M_3, M_5, M_7, M_{13}, M_{17}, \ldots$

# The Largest Known Prime Number

Since 1992, every new record holder for largest prime number has been a Mersenne Prime, and since 1996, each new largest Mersenne Prime has been found by GIMPS, the Great Internet Mersenne Prime Search, a collection of hundreds of volunteers running computations on their computers worldwide.

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As of today, the largest known prime number is $M_{82,589,933} = 2^{82,589,993} - 1$, found by Patrick Laroche in December 2018.

It is 24,862,048 digits long.

# The Largest Number We Could Theoretically Write

Obviously, we cannot write every single number down in the physcial universe. There is simply too little space. What is the largest number we can write down?

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The smallest thing we could write on is a single subatomic particle (things like protons, neutrons, quarks, etc.). Given that there are roughly $10^{80}$ subatomic particles in the universe, then the largest number we could write is

$$10^{10^{80}}$$

a number with $10^{80}$ digits.

# An Honourable Mention

**Question**

What is the significance of the number

$$10^{10^{30.8408}}$$

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What is the significance of the number

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Answer: It is the number of years it would take for a full can of beer at room temperature to spontaneously fall over as a result of quantum tunneling...

# The Really Hard to Fathom

We now move on to numbers that are still somewhat easy to understand, but fully comprehending their size is practically impossible.

# Counting Primes

## Definition

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$\pi(100) = 25$

$\pi(1000000) = 664579$

The algorithm for determining $\pi(x)$ for any $x$ is quite complex, and so mathematicians have sought to find simple approximations for it.

# The Logarithmic Integral

One of the best known approximations of the prime counting function is the **logarithmic integral**.

## Definition

The Logarithmic Integral is the function $\mathrm{Li} : \mathbb{R} \setminus \{1\} \to \mathbb{R}$ defined by

$$\mathrm{Li}(x) := \int_0^x \frac{1}{\ln(t)}\, dt$$

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## The Prime Number Theorem

$$\lim_{x \to \infty} \frac{\pi(x)}{\text{Li}(x)} = 1$$

# The Logarithmic Integral

## Examples

$\mathrm{Li}(10) = 6$

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# Constant Overestimating...

Notice that the logarithmic integral always overestimates the number of primes, though the margin of error does shrink as $x$ gets larger. A common question at this point is whether or not this is always true; that is, for all $x > 1$, it it the case that

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$$\pi(x) < \mathrm{Li}(x)$$

**Theorem (Littlewood, 1914)**

There are infinitely many values of $x > 1$ for which the value

$$\pi(x) - \mathrm{Li}(x)$$

changes signs.

# Skewes's Number

While Littlewood proved that the inequality did flip infinitely many times, he could not calculate the first time it happened.

It was his student, Stanley Skewes, who found an upper bound.

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**Theorem (Skewes, 1955)**

There exists a value $x$ such that

$$x < e^{e^{e^{e^{7.705}}}} \approx 10^{10^{10^{964}}}$$

where $\mathrm{li}(x) < \pi(x)$.

The value $10^{10^{10^{964}}}$ is now referred to as **Skewes's Number**.

# The Poincaré Recurrence Theorem

Take a deck of cards, shuffle them, and then deal the deck back to yourself. If you repeat this process, eventually you will deal out a sorted deck.

Similarly, if you place some gas in the corner of a room and let it disperse, after some time, the gas will eventually go back to the corner.

This concept is known as the Poincaré Recurrence Theorem, and the time it takes for the system to repeat is its Poincaré Recurrence Time.

# The Poincaré Recurrence Time of a Universe

In 2012, Don Page calculated what is believed to be the largest finite time ever calculated by a physicist.

He showed that, given a box containing a black hole with the mass of our universe, and assuming an inflationary model with inflation of $10^{-6}$ Plank masses, then the Poincaré Recurrence Time of that box would be

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He showed that, given a box containing a black hole with the mass of our universe, and assuming an inflationary model with inflation of $10^{-6}$ Plank masses, then the Poincaré Recurrence Time of that box would be

$$10^{10^{10^{10^{10^{1.1}}}}}$$

In short, assuming that a universe has the properties described above, then said universe will repeat itself arbitrarily many times, and this is the time it would take to repeat ONCE!

# Defining Some Notation

Before we venture any further, we must define some new notation to help display our next few numbers, otherwise they would not be possible to display on a screen in any meaningful fashion.

# Knuth's Up Arrow Notation

We will define $n \uparrow n$ as repeated multiplication, or just exponentiation.

$$n^n$$

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We will define $n \uparrow\uparrow n$ to be repeating $n \uparrow n$, $n$ times. This is called tetration.

$$\underbrace{n^{n^{n^{\cdots^{n}}}}}_{n \text{ times}}$$

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To simplify things, we also write $n$ arrows in a row as $\uparrow^n$. In general, we define $n \uparrow^k n$ to be repeating $n \uparrow^{k-1} n$, $k-1$ times. This produces pentation, hexation, etc.

# Example of Up Arrow Notation

## Examples

$3 \uparrow 3 = 3^3$

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# The Busy Beaver Game

Imagine we have an infinitely long hallway, with an infinite amount of rooms, each of which has the light off inside of it. We will send in a robot that we know will eventually halt. Said robot will have $n$ unique "states," and will do things like go to different rooms (called a shift), turn lights on and off, etc.

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What is the most amount of shifts an $n$-state robot can do before it halts?

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What is the most amount of shifts an $n$-state robot can do before it halts?

This question is at the heart of the Busy Beaver Game. A program with $n$ states which has the most shifts before halting is said to have won the $n$-th Busy Beaver Game. The number of shifts done by this program is referred to as the **$n$-th Busy Beaver Number**, or just $BB(n)$.

# BB(6)

As of today, only the first 5 Busy Beaver Numbers are known exactly: 1, 6, 21, 107, and 47176870, the latter of which was found this past summer.

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The sixth Busy Beaver Number, $BB(6)$, does not yet have an explicit value, but is known to be greater than

$$10 \uparrow\uparrow 15$$

a number represented as a power tower of 15 10's.

# mega

We conclude this section with a simple example of the power of up arrow notation.

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**mega**, is the value

$$(256 \uparrow\uparrow 256)^{257}$$

and was first defined by Hugo Steinhaus in the 1960s as part of his work on hyperoperations.

# The Unfathomable

We have finally reached the end. These numbers are so big, that any attempt at comprehending their size, digits, or anything related to their numerical value, becomes physically impossible, and it is impossible to represent them using our standard mathematical conventions.

# Colouring Cubes

Consider an $n$-dimensional hypercube (a 2D square, 3D cube, 4D tesseract, etc.), and draw all possible edges between vertices. Colour these edges either red or blue. Can you colour the edges such that there does not exist 4 co-planar vertices where the edges between them are all red or all blue?

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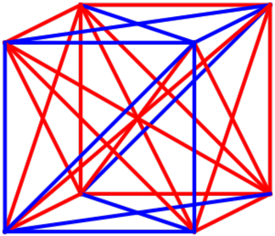
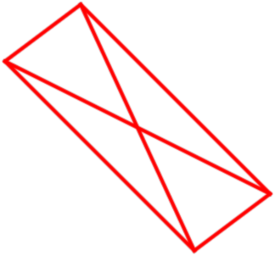
Clearly this can be done in 2, 3, and even 4 dimensions. Is this possible in all dimensions?





# Colouring Cubes

In 1971, Ronald Graham and Bruce Lee Rothschild proved that there exists a dimension at which this construction is no longer possible. While the exact dimension was not known, they did find an upper bound:

$$N = F(F(F(F(F(F(F(12)))))))$$

where $F(12) = 2 \uparrow^{12} 3$.

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where $F(12) = 2 \uparrow^{12} 3$.

In 1977, Martin Gardner found that Ronald Graham had an even large upper bound which was never published, and published the number and its meaning in Scientific American, flaunting it as "the largest number ever used in a serious mathematical proof."

This number would go on to be known as **Graham's Number**.

# Graham's Number

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## Definition

Define $g_n$ as follows:

$$g_n = \begin{cases} 3 \uparrow\uparrow\uparrow\uparrow 3 & \text{if } n = 1 \\ 3 \uparrow^{g_{n-1}} 3 & \text{if } n > 1 \end{cases}$$

Then Graham's Number, denoted by $G$, is equal to $g_{64}$.

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Then Graham's Number, denoted by $G$, is equal to $g_{64}$.

Surprisingly, the last 20 digits of $G$ are known:

$$\ldots 04575627262464195387$$

# Building Forests

In graph theory, a **tree** is a connected graph that contains no cycles within it, and a **forest** is a graph that can be described as a disjoint union of trees.

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Let's play a game. We're going to draw a forest of trees, drawing one tree at a time. We must abide by the following rules: Denote by $T_n$ the $n$-th tree in our forest, then

1. The $n$-th tree must contain at most $n$ vertices.
2. $T_m \not\leq T_n$ for any $m < n$; No tree can be contained within a larger one.

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1. The $n$-th tree must contain at most $n$ vertices.
2. $T_m \not\leq T_n$ for any $m < n$; No tree can be contained within a larger one.

Now assume that we colour each vertex one of $k$ colours. Using the rules defined above, how big of a forest can we draw?

# Building Forests

**Theorem (Kruskal's Tree Theorem)**

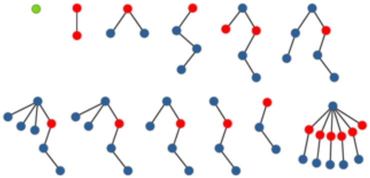
Given any infinite sequence $T_1, T_2, \ldots$ of trees labelled with $k$ colours, there is some $i < j$ such that $T_i \leq T_j$.

# Building Forests

**Theorem (Kruskal's Tree Theorem)**

Given any infinite sequence $T_1, T_2, \ldots$ of trees labelled with $k$ colours, there is some $i < j$ such that $T_i \leq T_j$.

This means that we can create a well-defined function, TREE, that takes in an input $k$, and outputs the largest value of $p$ for which $T_p$ must contain a smaller tree, given the vertices are labelled with $k$ colours.



# TREE(3)

**Examples**

$\mathrm{TREE}(1) =$

# TREE(3)

**Examples**

$\text{TREE}(1) = 1$

# TREE(3)

## Examples

TREE(1) $= 1$
TREE(2) $=$

# TREE(3)

### Examples

$\text{TREE}(1) = 1$
$\text{TREE}(2) = 6$

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**Examples**

TREE(1) $= 1$
TREE(2) $= 6$
TREE(3) $=$ I'm not even gonna bother typing this

TREE(3) is astoundingly larger than any number defined previously. To put this into perspective, a weak lower bound on the size of TREE(3) can be described using the recursive function defining Graham's Number:

$$g_{3\uparrow^{187196}3} \ll TREE(3)$$

# SCG(13) and SSCG(3)

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A **sub-cubic graph** is any graph (connected or not) where each vertex has at most degree 3. For two such graphs $T_n, T_m$, we say that $T_n \leq T_m$ if $T_n$ can be constructed from $T_m$ by removing or "contracting" edges from it. The resulting process is called is called SCG($n$).

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If we add the condition that our sub-cubic graph must be simple (no loops or multiple edges), we get a **simple sub-cubic graph**, and we get the function SSCG($n$).

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If we add the condition that our sub-cubic graph must be simple (no loops or multiple edges), we get a **simple sub-cubic graph**, and we get the function SSCG($n$).

These functions combine to produce the following inequality:

$$TREE(3) \ll SCG(13) \ll SSCG(3)$$

# The Big Number Duel

On January 26, 2007, MIT philosophy professors Augustin Rayo and Adam Elga met in what is now called the “Big Number Duel” to determine who could come up with the largest finite number.

Little did either of them know, this fun event would produce what many consider to be the largest named number ever conceived.

# A Hard Fought Battle

Rayo and Elga went back and forth for a while, producing some seriously big numbers:

$$111111111111111111111111111111$$

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$$BB(10^{100})$$

$$BB_1(10^{100})$$

$$BB_\omega(10^{100})$$

Eventually, Rayo would produce the winning answer.

# Rayo's Number

Rayo's Number can be described as the following number:

The smallest number bigger than every finite number that can be written in the language of first-order set-theory using less than $10^{100}$ symbols.

also denoted as Rayo($10^{100}$). This is a very simplified description. Rayo avoided ambiguities by defining a second-order formula and using a lot of logic stuff (I would go into more depth but I am not remotely qualified to do so).

# Can We Get Any Bigger?

Rayo's Number is defined using some of the fundamental building blocks of mathematics, including logic and set theory. Because of this, it is extremely difficult to come up with a larger number that cannot be expressed as a naive extension of the process that Rayo did. Many have tried, and almost all have failed.

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But "almost" does not mean all...

What is so special about first-order set-theory? Who's to say that we cannot modify this characteristic of Rayo's Number to produce an even larger number?

# A Mysterious Competitor

In October 2013, an anonymous googologist who goes by the name "Fish," constructed a series of numbers by "extending" some famous fast-growing functions, such as the Ackermann Function.

The largest of these numbers used an extension of the Rayo function defined previously, producing a much larger number than Rayo's Number.

# Oracle Formulas and the Rayo Hierarchy

Fish added an additional formula into the language used by Rayo by adding an oracle formula using some function $f$. The formula "$f(a) = b$", means that the $a$th and $b$th members of the sequence are such that $f(a) = b$.

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$R_1(n)$ is just the Rayo function described previously.

$R_2(n)$ is the Rayo function but the language used includes the oracle formula using $R_1(n)$.

$R_3(n)$ is the Rayo function with the oracle formula using $R_2(n)$ included, and so on.

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While this is number is significantly larger than Rayo($10^{100}$), there is some debate about whether or not it is a naive extension of it.