ECBS6001: Advanced Macroeconomics

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Lecture 18: A model of new products

Outline

- Today we apply continuous-time dynamic programming to a model of endogenous growth.
- We will use
  - dynamic programming with a jump process
  - differential equations
  - steady-state properties
- The application is also heavy in general equilibrium.
  - aggregation of individual decisions
  - resource constraints
  - endogenous prices

Outline

- Today we will study a model where growth occurs through an increase in the number of products.
- Product innovation is a source of technical progress.
  - Most of income differences across countries are due to differences in productivity.
- We will make use of the Dixit–Stiglitz model of monopolistic competition.

Innovation

- Innovation is a conscious economic activity.
  - In contrast to exogenous technical progress.
  - Responds to profit incentives of innovators.
- Process innovation reduces production costs of existing products. Product innovation entails coming up with new products.
  - We focus on product innovation.
- Horizontal innovation leads to products with new functions. Vertical innovation serves similar function at higher quality.
- Innovation ("idea") may or may not be replicable.
  - We begin with a setup with fully private benefits.
  - Then discuss the case of knowledge spillovers.

Product innovation

- Firms spend on R&D to come up with blueprints of products.
- Only products for which blueprints exist can be produced.
- The holder of a blueprint obtains a monopoly over producing that product.
  - Patent protection
  - Any small cost of imitation prevents it in equilibrium.
  - Later we will study imitation more generally.
- Firms do two things:
  1. develop blueprints
  2. produce from existing blueprints

Static equilibrium

Structure of the economy

- Firms produce goods based on existing blueprints.
  - using labor
- They also employ researchers to develop new blueprints
  - same type of labor
  - demand for funds
- Because of IPR protection, firms make profits.
- Workers earn wages and hold a portfolio of all firms (stock market).
- They decide how much to consume and how much to save.
  - supply of funds

Static equilibrium

- We begin by characterizing the equilibrium at a given point in time.
  - production of existing products
- We then move on to dynamic decisions
  - development of new products

Consumers

Love of variety

Love of variety

Firms

Market structure

Choice of numeraire

- We normalize aggregate expenditure to $E \equiv 1$.
- This is a weird choice of numeraire but will prove convenient later.
  - We are free to fix any one price in each time period to whatever value we want.
  - The price need not be 1, need not even be constant.
  - So we pick $P = 1/D$ so that $E = PD = 1$.

Firm profits

Determining wages

Checklist

So far we have determined

- how consumers value variety
- how firms price their products
- what is the total labor demand in production

Dynamic decisions

Supply of funds

Consumption smoothing

Deriving the Euler equation

Nominal vs real interest rate?

- All this referred to an economy in which there is no inflation.
- If there is inflation,
$$\dot{a} = ra + y - pc.$$
- And the FOC becomes
$$\frac{1}{c} = pV_a(a, y, p).$$

Exercise

- Derive the Euler equation in this economy.
- What is the optimal rate of growth for consumption?
- What do you need to know about $p(t)$?

The Euler equation

Demand for funds

The decision to innovate

- Take a firm with n products.
  - n is a firm-level state variable.
- The firm takes aggregates, N and L_m as given.
- The firm can raise capital at rate $r = \rho$.
- We next analyze the R&D decision.

Research and development

- R&D is costly and random.
  - Successes arrive with a Poisson process.
  - The arrival rate depends on R&D expenditure.
- So that a new product arrives with rate λ, the firm has to hire $a\lambda$ workers.
  - Again, λ is an instantaneous arrival rate.
  - R&D expenditure $a\lambda$ is a flflow.
  - Past expenditure and past success do not matter.
- Let us write down the Bellman equation for the value of the firm.

The value of a firm

Solution

Innovation and growth

- Note that the FOC did not pin down λ.
  - This because of the linearity of both the benefit and the cost of innovation.
  - Would change with convex costs of innovation.
- Suppose firm i innovates with rate λ_i, using $a\lambda_i$ R&D workers.
- This leads to a new product with arrival rate λ_i.

Aggregate innovation

Dynamic equilibrium

Long-run growth

Steady state

The steady-state N (and hence steady-state productivity) is

- increasing in country size L
- increasing in profit share $(1-\alpha)$
- decreasing in R&D cost a
- decreasing in discount rate ρ

$$\frac{D}{L_m} = N^{(1-\alpha)/\alpha} \rightarrow \left(\frac{(1-\alpha)L}{\alpha a \rho}\right)^{(1-\alpha)/\alpha}$$

Phase diagram

Recipe

1. Constant interest rate: $r = \rho$
2. Symmetric profits: $\pi = (1 - \alpha)/N$
3. Wage equation: $w = \alpha/L_m$
4. Firm valuation: $\rho v = \pi + \dot{v}$
5. Optimal R&D: $v \leq aw$
6. Resource constraint: $L_m + a\dot{N} = L$

Dynamic equilibrium

Long-run growth

- We first show that there is no long-run growth in this economy.
- This is because the incentive to innovate disappears as N grows large.
- Suppose

$$N > \bar{N} \equiv \frac{(1-\alpha)L}{\alpha a \rho}$$

  and there is no R&D.
- Then both N and L_m are constant, so is $v(N, L_m)$.
- From the Bellman equation,

$$v(N, L) = \frac{1-\alpha}{\rho N}.$$

- But because $N > \bar{N}$, this is indeed smaller than the cost of innovation $\alpha a / L$.
- So no innovation is a unique equilibrium.

Characterizing the dynamics

We collapse the six equations of the “recipe” into two.

Labor market clearing + wage equation + optimal R&D

$$\dot{N} = \max\left\{0, \frac{L}{a} - \frac{\alpha}{v}\right\}$$

Bellman equation

$$\dot{v} = \rho v - \frac{1-\alpha}{N}$$

Steady state

- The steady state is such that both $\dot{N}$ and $\dot{v}$ are zero.
- Steady-state N is
$$N_{ss} = \frac{(1-\alpha)L}{\alpha\rho a}.$$
- Steady-state productivity (output per worker) is
$$\frac{D}{L_m} = N^{(1-\alpha)/\alpha} \to \left(\frac{(1-\alpha)L}{\alpha a \rho}\right)^{(1-\alpha)/\alpha}.$$
    - (This only includes manufacturing, not R&D.)
- Both are
    - increasing in country size L
    - increasing in profit share $(1-\alpha)$
    - decreasing in R&D cost a
    - decreasing in discount rate ρ

Knowledge spillovers

Knowledge spillovers

- Now suppose that R&D has external benefits to other researchers.
- In particular, let the cost of R&D decrease with the number of existing products, N, a/N.
- This changes
  1. the incentive to innovate
  2. the resource requirements of innovation

The new Bellman

The new resource constraint

Balanced growth

Solution

Other ways to generate growth

Other ways to generate growth

- Knowledge spillovers reduce the cost of innovation so that profit *per cost* do not vanish.
- We have other ways to generate growth:
  - In a different demand system / competition, profits may not vanish. (See quality competition later.)
  - If innovation costs are in the final good rather than in labor units, they "mechanically" get lower with development:

$$P = N^{(\alpha-1)/\alpha}.$$

  - (This is also an external benefit of R&D but it is *pecuniary*.)
  - If population grows, firms keep doing R&D. This may even lead to growth in output *per capita*. (See later.)

Policy and welfare

Policies

- We want to see if policy has an effect on growth.
- We consider two policies:
    1. an R&D subsidy
    2. a production subsidy

R&D subsidy

- The government pays a fraction ϕ of research expenses.
- This is financed by a lump-sum tax.
- This changes the incentive to innovate,

$$\frac{\alpha a(1-\phi)}{NL_m} = v,$$

- and the Bellman equation

$$\rho = \frac{(1-\alpha)L_m}{\alpha a(1-\phi)} - g.$$

Solution

Production subsidy

- Manufacturers receive an ad valorem subsidy of ϕ_x.
- Their aggregate revenue is hence $1 + \phi_x$,
- profit per variety is
$$(1 + \phi_x)(1 - \alpha)/N.$$
- This seems to raise the profitability of R&D.

Production subsidy

Welfare

- Is the equilibrium growth rate g *optimal*?
- We see that R&D subsidies can increase the growth rate – should they be employed?

Welfare

- Is the equilibrium growth rate g *optimal*?
- We see that R&D subsidies can increase the growth rate – should they be employed?
- We answer that by solving the benevolent social planners problem.
- The social planner maximizes discounted utility subject to technology contraints.
  - (Prices and markets do not matter.)
- We begin with the case without knowledge spillovers.

Static optimum

- First note that the static equilibrium is optimal – despite imperfect competition.
- Given L_m workers and N existing varieties, the social planner would like to allocate L_m/N workers to each – just as in equilibrium.
- Because markups are symmetric, they do not involve any distrotion – relative prices across firms are unchanged.

Dynamic optimum

Bellman equation

Solution

Solution

The case with knowledge spillovers

Solution

Optimal growth

Discussion

- We have endogenized *technology*: companies invest in new technology just as they invested in physical capital in the Solow/Ramsey model.
- But it has proven difficult to endogenize *growth*: R&D can also be subject to decreasing returns to scale.
- We had to assume spillovers: the social returns to R&D are higher than the private returns.
- This model is not necessarily about *endogenous growth*, but certainly about *endogenous innovation* and technology.
- Innovation (and potentially growth) responds to taste and policy parameters and, notably, *country size*.
- Equilibrium growth is lower that optimal, there is room for policy.

Appendix

CES review

- Take the following CES utility function:

$$u(x_1, x_2) = [x_1^\alpha + x_2^\alpha]^{1/\alpha},$$

  and define $\varepsilon = 1/(1-\alpha)$, $\alpha = 1 - 1/\varepsilon$
- Maximize utility subject to prices p_1 and p_2:

$$p_1 x_1 + p_2 x_2 = E$$

- What is the relative demand for x_1 and x_2?

Utility maximization

- The marginal rate of substitution

$$\frac{u_1}{u_2} = \frac{x_1^{\alpha-1}}{x_2^{\alpha-1}} = \left(\frac{x_1}{x_2}\right)^{-1/\varepsilon}$$

- In the optimum, this equals the relative price, p_1/p_2:

$$\frac{x_1}{x_2} = \left(\frac{p_1}{p_2}\right)^{-\varepsilon}$$

- The relative demand is loglinear in relative prices.
    - The elasticity of substitution is constant at ε.

Cost minimization

- In parallel, we can solve the cost minimization problem.
- Minimize $E = p_1 x_2 + p_2 x_2$ subject to $u(x_1, x_2) = u_0$.
  - FOC:
$$p_i = \lambda x_i^{\alpha - 1}$$

$$E = u_0 \left[p_1^{1-\varepsilon} + p_2^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$$

- The term
$$P \equiv \left[p_1^{1-\varepsilon} + p_2^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$$

  is the *ideal price index*.

Markup pricing

- Take a demand function $D(p)$ and a cost function $C(Q)$.
- Maximize profit
$$pD(p) - C[D(p)]$$
- First-order condition
$$D(p) + pD'(p) - C'[D(p)]D'(p) = 0$$
- Divide by pD' and rearrange
$$\frac{p - C'[D(p)]}{p} = \frac{D(p)}{-pD'(p)} \equiv \frac{1}{\varepsilon}.$$
- Price–cost markup
$$\frac{p}{C'[D(p)]} = \frac{\varepsilon}{\varepsilon - 1}.$$