

# ECBS6001: Advanced Macroeconomics

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Fall 2019

# A model of firm and macro volatility

# Consumers

$$u_t = \left[ \int_{j=0}^1 y_{jt}^{1-1/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)}$$

## Production function

$$y_{jt} = \left[ \sum_{i=1}^{n_{jt}} x_{ijt}^{1-1/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}$$

Same  $\varepsilon$ . Linear technology subject to shocks:

$$x_{ijt} = \chi_{it} l_{ijt}$$

## Technology shocks

$$\chi_{i0} = 1$$

Technology fails after Poisson arrival  $\gamma$ :

$$d\chi_{it} = -dJ_i(\gamma t)\chi_{it}$$

# Symmetry

$$y_{jt} = \tilde{n}_{jt}^{\varepsilon/(\varepsilon-1)} l_{ijt} = \tilde{n}_{jt}^{1/(\varepsilon-1)} l_{jt}$$

with

$$l_{jt} = \sum_i l_{ijt}$$

## Pricing and demand

$$p_{jt} = \frac{\varepsilon}{\varepsilon - 1} w_t \tilde{n}_{jt}^{1/(1-\varepsilon)}$$

$$r_{jt} = \frac{R_t}{P_t^{1-\varepsilon}} p_{jt}^{1-\varepsilon} = R_t \frac{\tilde{n}_{jt}}{N_t}$$

with

$$N_t \equiv \int_{j=0}^1 \tilde{n}_{jt} dj$$

# Aggregation

$$l_{jt} = L \frac{\tilde{n}_{jt}}{N_t}$$

$$y_{jt} = \frac{L}{N_t} \tilde{n}_{jt}^{\varepsilon/(\varepsilon-1)}$$

$$Y_t = \frac{L}{N_t} N_t^{\varepsilon/(\varepsilon-1)} = L N_t^{1/(\varepsilon-1)}$$

assume  $\varepsilon = 2$  so that returns to new varieties do not diminish.



# Dynamic decisions

## Incentive to innovate

$$\pi_{jt} = (1 - \alpha)L\tilde{n}_{jt}$$

Firm-level state variable:  $\tilde{n}_{jt}$ . Do not care about competition,  $N_t$ .  
Cost of innovating with speed  $\lambda\tilde{n}_{jt}$  (as in Klette-Kortum):

$$\tilde{n}_{jt}c(\lambda)$$

# Bellman

$$\rho V(n) = \max_{\lambda} (1-\alpha)Ln - c(\lambda)n + \lambda n[V(n+1) - V(n)] + \gamma n[V(n-1) - v(n)]$$

Guess  $V(n) \equiv vn$ ,

$$\rho vn = \max_{\lambda} (1-\alpha)Ln - c(\lambda)n + \lambda vn - \gamma vn$$

## Firm-level solution

$$c'(\lambda^*) = v$$

$$v = \frac{(1 - \alpha)L - c(\lambda^*)}{\rho - \lambda^* + \gamma}$$