ECBS6001: Advanced Macroeconomics

Miklós Koren

Fall 2019

A model of firm and macro volatility

Consumers

$$u_t = \left[\int_{j=0}^{1} y_{jt}^{1-1/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)}$$

Production function

$$y_{jt} = \left[\sum_{i=1}^{n_{jt}} x_{ijt}^{1-1/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}$$

Same ε. Linear technology subject to shocks:

$$x_{ijt} = \chi_{it} l_{ijt}$$

Technology shocks

$$\chi_{i0} = 1$$

Technology fails after Poisson arrival γ:

$$d\chi_{it} = -dJ_i(\gamma t)\chi_{it}$$

Symmetry

$$y_{jt} = \tilde{n}_{jt}^{\varepsilon/(\varepsilon-1)} l_{ijt} = \tilde{n}_{jt}^{1/(\varepsilon-1)} l_{jt}$$

with

$$l_{jt} = \sum_i l_{ijt}$$

Pricing and demand

$$p_{jt} = \frac{\varepsilon}{\varepsilon - 1} w_t \tilde{n}_{jt}^{1/(1-\varepsilon)}$$

$$r_{jt} = \frac{R_t}{P_t^{1-\varepsilon}} p_{jt}^{1-\varepsilon} = R_t \frac{\tilde{n}_{jt}}{N_t}$$

with

$$N_t \equiv \int_{j=0}^{1} \tilde{n}_{jt} dj$$

Aggregation

$$l_{jt} = L\frac{\tilde{n}_{jt}}{N_t}$$

$$y_{jt} = \frac{L}{N_t}\tilde{n}_{jt}^{\varepsilon/(\varepsilon-1)}$$

$$Y_t = \frac{L}{N_t}N_t^{\varepsilon/(\varepsilon-1)} = LN_t^{1/(\varepsilon-1)}$$

assume $\varepsilon = 2$ so that returns to new varieties do not diminish.

Dynamic decisions

Incentive to innovate

$$\pi_{jt} = (1-\alpha)L\tilde{n}_{jt}$$

Firm-level state variable: $\tilde{n}_{jt}$. Do not care about competition, N_t.
Cost of innovating with speed $\lambda\tilde{n}_{jt}$ (as in Klette-Kortum):

$$\tilde{n}_{jt}c(\lambda)$$

Bellman

$$\rho V(n) = \max_{\lambda} (1-\alpha)Ln - c(\lambda)n + \lambda n[V(n+1) - V(n)] + \gamma n[V(n-1) - v(n)]$$

Guess $V(n) \equiv vn$,

$$\rho vn = \max_{\lambda} (1-\alpha)Ln - c(\lambda)n + \lambda vn - \gamma vn$$

Firm-level solution

$$c'(\lambda^*) = v$$

$$v = \frac{(1-\alpha)L - c(\lambda^*)}{\rho - \lambda^* + \gamma}$$

Firm dynamics

$$d\tilde{n}_{jt} = \tilde{n}_{jt} dJ_j(\lambda t) - \tilde{n}_{jt} dJ_i(\gamma t)$$

$$\mathsf{E}(d\tilde{n}_{jt}) = (\lambda - \gamma) n dt$$

$$\mathsf{Var}(d\tilde{n}_{jt}) = (\lambda + \gamma) n dt$$