Problem Set 1

Due Date 2020.10.21

1. Matlab basic operations: you should attach your Matlab codes and associated results in the answer.

   (a) Construct a vector from 0 to 100. The difference between two subsequent elements in the vector is 0.2. Count how many elements there are.

   (b) Construct an 5×5 identity matrix. Construct a 4×1 vector with all elements 1. Find the Kronecker product of $I_4 \otimes \mathbf{1}$.

   (c) Use for loop and while loop to find the sum of natural numbers from 1 to 100.

   (d) Write a function to find the sum of natural numbers from 1 to "N". (Basically, the argument of this function "sumN" is "n". Whenever you use this function, e.g. the sum from 1 to 20, just type "sumN(20)" and then you can get the sum.)

   (e) Use help in Matlab to learn the "find" function. What can this function do?

2. In this problem you will derive the HP filter and create your own code to implement it. Suppose you have a sequence of data, y_t, with $t = 1, \cdots, T$. Our objective is to find a trend $\{\tau\}_{t=1}^{T}$ to minimize the following objective function:

$$\min_{\tau_t} \quad \sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1}[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2, \quad \lambda \geq 0$$

   (a) Provide a verbal interpretation of this objective function and what exactly one is trying to get at when choosing a trend, $\{\tau\}_{t=1}^{T}$.

   (b) Prove that, if $\lambda = 0$, then the solution is $\tau_t = y_t, \forall t$. In the other words, the trend and the actual series would be identical.

   (c) Prove that, as $\lambda \to \infty$, then the solution is for the trend to be a linear time trend, i.e. for $\tau_t = \beta + \alpha t$ for some α and β.

   (d) For the more general case in which $0 < \lambda < \infty$, derive analytical solutions for the trend. (Hint: notice that this a unconstrained optimization problem. When the objective is convex in τ_t, you should find the optimizer by making every derivative with respect to τ_t zero and then you can find the solution.)

(e) Write a Matlab code to find the trend taking the actual data series y_t. To do this, express the first order conditions in (d) in matrix form as follows:

$$\begin{aligned} \Lambda\Gamma &= Y \\ \Gamma &= \Lambda^{-1}Y \end{aligned}$$

where Λ is a $T \times T$ matrix whose elements are functions of λ; Y is a $T \times 1$ vector of actual data. Γ is the vector of τ_t. Download quarterly and seasonly adjusted US real GDP data from 1947:Q1 to 2020:Q2 (FRED: https://fred.stlouisfed.org/). Take natural logs of the data and then use your code to compute the HP trend using $\lambda = 1600$. Show a plot of both the real GDP and the trend. Show a plot of the cyclical component. What is the standard deviation of the HP detrended GDP? How should you interpret this number?

3. Consider a more simplified one-period model without government. The consumer's problem is:

$$\begin{aligned} \max_{C,l} \quad & \ln C + \theta \ln l \\ s.t. \quad & C + wl = wh + \pi \end{aligned}$$

The firm is endowed with linear technology $y = zN^d$ instead of the Cobb-Douglous technology in the lecture notes.

$$\max_{N^d} \quad zN^d - wN^d$$

(a) Solve the consumer's problem. What is the marginal substitution rate of the leisure for consumption?

(b) Solve the firm's problem.

(c) Define the competitive equilibrium.

(d) Define the Pareto optimum of this problem. Write down the PPF. What is the marginal rate of transformation? Explain verbally why the PPF is not concave but linear or why the MRT is constant in the problem.

(e) Compare the PO and CE. Show that CE is a PO in this problem. Check that $MRS = MRT$ in this case.

(f) Apply the implicit function theorem to find the comparative statics of C^*, l^* with respect to z. Disentangle the substitution effect and income effect.

4. Consider the economy with a frictional wage income tax t. Other setups are the same as the one-period model in our lecture notes. Basically, the representative consumer solves the problem:

$$\begin{aligned} \max_{C,l} \quad & \ln C + \theta \ln l \\ s.t. \quad & C + w(1-t)l = w(1-t)h + \pi - T \end{aligned}$$

The representative firm maximizes its profit:

$$\max_{N^d} \quad zK^{\alpha}(N^d)^{1-\alpha} - wN^d$$

The government runs a balanced budget.

$$G = T + twN^d$$

(a) Solve the consumer's problem. Describe verbally the tradeoff between consumption and leisure of the consumer by the equilibrium condition. What is the marginal substitution rate of the leisure for consumption?

(b) Solve the firm's problem.

(c) Define the competitive equilibrium.

(d) Define the Pareto optimum of this problem. Write down the PPF. What is the marginal rate of transformation?

(e) Compare the PO and CE. Show that CE is not a PO in this problem. You can see that why the first theorem of welfare economics does not hold when there is frictional tax? Check that $MRS \neq MRT$ in this case.