WORLDQUANT UNIVERSITY

MScFE 640: PORTFOLIO THEORY AND ASSET PRICING

Group Members

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GROUP ASSIGNMENT 2

Over View

The main objective of the Assignment is to measure the performance of the large-Cap US equity market. Where 500 Companies are classified in 11 sectors such as Energy, Material, Industrials, Consumer discretionary, Consumer staples, Health Care, Financials, Information Technology, Communication Services, Utilities, and Real Estates. The performance of the S&P 500 index in a period will be equal to the weighted-average of each sectors. Before we begin the Group Two Assignment, we need to understand what Efficient Frontier is.

i) Efficient Frontier

Efficient Frontier refers to the set of optimal portfolio that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. The Efficient Frontier rate portfolios (investment) on a scale of return (y-axis) against risk (x-axis), where compound annual growth rate is commonly used as the return component while annualized standard deviation depicts the risk component.

The eleven (11) portfolios were constructed using the two risky assets of XLI and XLE.

- The expected return of each portfolio is a weighted average of its individual assets' expected returns, given by the mathematical expression,

  $E(R_P) = w_1 E(R_1) + w_2 E(R_2)$. Where w_1, w_2 are the respective given weights of the two assets, and $E(R_1), E(R_2)$ are the respective expected returns of each asset.

  For this particular assignment, the expected return for the two assets was taken from the CAPM calculations from the previous work which gave a return of 0.0947(9.47%) and 0.0941(9.41%) for XLE and XLI respectively.

  The expected return of each of the eleven (11) portfolio was constructed using the above mathematical expression where the weight and the expected return of both XLE and XLI were stated. The attached excel spreadsheet shows the calculations of the expected returns for the portfolios.

- The variance of the portfolio is just not the weighted average of the variance of individual assets but it also depends on the covariance and correlation of the two assets.

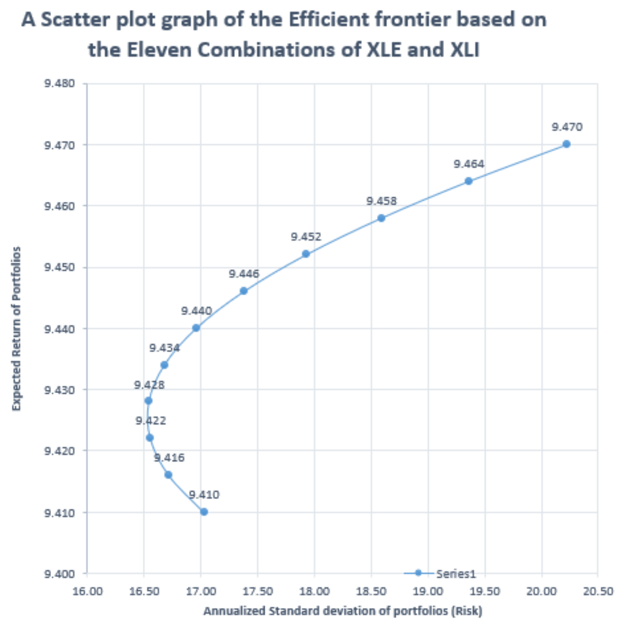
  The levels of variance translate directly with the levels of risk. The higher the variance, the higher the levels of risk and vice versa.

  The variance of each portfolio was calculated using the following expression,

$Var(R_P) = w_1^2 Var(R_1) + w_2^2 Var(R_2) + 2w_1 w_2 Cov(R_1, R_2)$, where $Cov(R_1, R_2)$ is the covariance of the two assets.

The standard deviation of each portfolio was then computed by taking the positive square root of the variance. The calculations for the variance and standard deviations can again be found in the attached excel spreadsheet which was done by excel functions.

- The diagram below shows a scatter plot graph of the Efficient Frontier based on the eleven combinations of XLE and XLI.



The various calculations regarding the above graph can be found in the attached excel file.

ii) **Selecting a Portfolio that Satisfying the given Constraint**

The portfolio that satisfies the given constraint that, the return is greater than 9.43% and the volatility is not greater than 16.8% is **portfolio seven (7)** that has an expected return of 9.434% which is slightly greater than the given constraint and a volatility of 16.68%.

- With regarding returns, portfolio seven (7) would generate a return of 9.434% as against an average return of 0.00911% for S&P 500, for the same period of 27/11/2017 to 23/11/2018. The significant difference in returns indicate that, portfolio seven (7) is superior in that regard.

The return for the S&P 500, this was also calculated in excel spreadsheet by using the following formula,

$$\frac{(P_t - P_{t-1})}{P_{t-1}} \times 100$$

Where P_t is the current daily price and P_{t-1} is the previous daily price.

By the same excel spreadsheet, the average return was computed using the average function.

- Reference was also made to the annualized standard deviation which is a good metric for the measure of risk in this particular case. Per the calculations made in the excel spreadsheet, the annualized volatility for portfolio Seven (7) calculated to 16.68% and that of S&P 500 was 14.79%. This result indicates that portfolio seven (7) is more risky than S&P 500.

iii) **Portfolio's Performance Relative to the S&P500**

An average return of 0.00911% was obtained for the S&P500 whilst an average return of -0.00741% was obtain for the portfolio. Hence, the S&P500 yielded a higher return.

An annualized volatility of 16.66% was obtained for the Portfolio which is higher in contract with the 14.76% obtained for the S&P 500. This makes our portfolio investment riskier.

Risk-adjusted return defines an investment's return by measuring how much risk is involved in producing that return, which is generally expressed as a number or rating. Some common risk measures include alpha, beta, R-squared, standard deviation and the Sharpe ratio.

For the purpose of this work, we employed the use of the Sharpe ratio as that is what is required. The Sharpe ratio is a measure of an investment's excess return, above the risk-free rate, per unit of standard deviation. It is calculated by taking the return of the investment, subtracting the risk-free rate, and dividing this result by the investment's standard deviation. This is mathematically expressed as,

$$\text{Sharpe-ratio} = \frac{R_P - R_f}{\sigma_P}$$

Where,

R_P = return of portfolio

R_f =Risk-free rate

σ_P =Standard deviation of the portfolio's excess return

For portfolio 7,

R_7 =9.434%, R_f =2.25% and σ_7 =16.68%

$$\text{Sharpe-ratio} = \frac{9.434\% - 2.25\%}{16.68\%} = 0.4307$$

For S&P 500,

$R_{S\&P}$ =0.00911%, R_f =2.25% and $\sigma_{S\&P}$ =14.79%

Sharpe-ratio = $\frac{0.00911\% - 2.25\%}{14.78\%}$ =-0.1516

The Sharpe ratio for portfolio 7 is greater than that of S&P 500 and it means that, it gained more per unit of total risk than S&P 500.

iv) <u>S&P Correlation with Portfolio</u>

Let's consider the following quantities and their mathematical expressions since they have been employed in various computations here to tell if the benchmark is appropriate.

- Active returns: This is simply the difference between return of portfolio and that of the benchmark return. It has the mathematical expression, $R_{Active} = R_P - R_B$ where R_P and R_B, are the portfolio return and benchmark return respectively.
  According to the above formula was used in the calculation of the Active return in excel spreadsheet.
- Tracking Error: This is simply the standard deviation of the active returns. Tracking error is a measure of the risk in an investment portfolio that is due to active management decisions made by the portfolio manager; it indicates how closely a portfolio follows the index to which it is benchmarked.
  The tracking error for this exercise was computed in excel by applying the standard deviation function on the active returns that had been generated.
  Manually this can be computed by using the formula, Tracking Error=
  $$= \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}(R_i - \bar{R})^2}$$
  Where, R_i is the active return, $\bar{R}$ is the mean active return and T is the time period.
  The tracking error computed was 0.515% which indicates a relatively stable active return. This also means the portfolio is closely following the benchmark.

- The mean adjusted tracking error (MATE): This metric uses both the squared active return and the squared tracking error. Therefore, if a benchmark has a large squared active return or a large squared tracking error, then it will have a large mean-adjusted tracking error. The mathematical expression is given by, MATE= $= \sqrt{\frac{1}{T}\sum_{i=1}^{T}(R_{Active})^2}$.
  The mean adjusted tracking error was computed using the above formula per the work at hand and the result was 0.513% which is a good indication that the portfolio follows the benchmark closely.

References

1. Lecture notes 4
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4. Chen, J. Readjusted Return. Taken from (https://www.investopedia.com/terms/r/riskadjustedreturn.asp) on 26/6/2020