



Tecnológico de Estudios Superiores  
de Ecatepec  
División de Ingeniería en Sistemas Computacionales



# Evaluación 3º parcial

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Grupo: 5501

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Materia: Métodos numéricos

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## Ejercicios Métodos numéricos

- 4 iteraciones Jacobi;
- 4 iteraciones Gauss - Siedel

$$1) \begin{array}{l} 9x + 2y - z = -2 \\ 7x + 8y + 5z = 3 \\ 3x + 4y - 10z = 6 \end{array}$$

Jacobi:

$$x = \frac{-2 - 2y + 1z}{9} = -0.22 - 0.22y + 0.11z$$

$$y = \frac{3 - 7x - 5z}{8} = 0.375 - 0.875x - 0.625z$$

$$z = \frac{-6 + 3x + 4y}{10} = -0.6 + 0.3x + 0.4y$$

$$x = -0.22 - 0.22y + 0.11z$$

$$y = 0.375 - 0.875x - 0.625z$$

$$z = -0.6 + 0.3x + 0.4y$$

$$x_0 = 0$$

$$y_0 = 0$$

$$z_0 = 0$$

Calcular iteración ①

$$x_1 = -0.22 - \cancel{0.22y} + \cancel{0.11z} = -0.22$$

$$y_1 = 0.375 - \cancel{0.875x} - \cancel{0.625z} = 0.375$$

$$z_1 = -0.6 + \cancel{0.3x} + \cancel{0.4y} = -0.6$$

$$x_1 = -0.22$$

$$y_1 = 0.375$$

$$z_1 = -0.6$$

Calcular iteración 2

$$x_2 = -0.22 - 0.22y_1 + 0.11z_1 = -0.22 - 0.22(0.375) + 0.11(-0.6)$$
$$= -0.3685$$

$$y_2 = 0.375 - 0.875x_1 - 0.625z_1 = 0.375 - 0.875(-0.22) - 0.625(-0.6) = 0.9425$$

$$z_2 = -0.6 + 0.3x_1 + 0.4y_1 = -0.6 + 0.3(-0.22) + 0.4(0.375)$$
$$= -0.516$$

$$x_2 = -0.3685$$

$$y_2 = 0.9425$$

$$z_2 = -0.516$$

Calcular iteración 3

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$$x_3 = -0.22 - 0.22y_2 + 0.11z_2 = -0.22 - 0.22(0.9425) \\ + 0.11(-0.516) = \underline{-0.48411}$$

$$y_3 = 0.375 - 0.875x_2 - 0.625z_2 = 0.375 - 0.875(-0.3685) \\ - 0.625(-0.516) = \underline{1.0199375}$$

$$z_3 = -0.6 + 0.3x_2 + 0.4y_2 = -0.6 + 0.3(-0.3685) \\ + 0.4(0.9425) = \underline{-0.33355}$$

$$x_3 = -0.48411$$

$$y_3 = 1.0199375$$

$$z_3 = -0.33355$$

Calcular iteración ④

$$x_4 = -0.22 - 0.22y_3 + 0.11z_3 = -0.22 - 0.22(1.0199375) \\ + 0.11(-0.33355) = \underline{-0.48107675}$$

$$y_4 = 0.375 - 0.875x_3 - 0.625z_3 = 0.375 - 0.875(-0.48411) \\ - 0.625(-0.33355) = \underline{1.00828375}$$

$$z_4 = -0.6 + 0.3x_3 + 0.4y_3 = -0.6 + 0.3(-0.48411) \\ + 0.4(1.0199375) = \underline{-0.337258}$$

$$x_4 = -0.48107675$$

$$y_4 = 1.00828375$$

$$z_4 = -0.337258$$

# Gauss-Seidel

$$\begin{aligned} 1) \quad & 9x + 2y - z = -2 \\ & 7x + 8y + 5z = 3 \\ & 3x + 4y - 10z = 6 \end{aligned}$$

$$\begin{aligned} x &= -0.22 - 0.22y + 0.11z \\ y &= 0.375 - 0.875x - 0.625z \\ z &= -0.6 + 0.3x + 0.4y \end{aligned}$$

$$\begin{aligned} x^0 &= 0 \\ y^0 &= 0 \\ z^0 &= 0 \end{aligned}$$

Calcular iteración ①

$$x' = -0.22 - 0.22y^0 + 0.11z^0 = -0.22$$

$$y' = 0.375 - 0.875x' - 0.625z^0 = 0.375 - 0.875(-0.22)$$

$$y' = 0.5675$$

$$\begin{aligned} z' &= -0.6 + 0.3x' + 0.4y' = -0.6 + 0.3(-0.2) + \\ &0.4(0.5675) = -0.433 \end{aligned}$$

$$x' = -0.22$$

$$y' = 0.5675$$

$$z' = -0.433$$

Calculando iteración ②

$$\begin{aligned} x^2 &= -0.22 - 0.22y' + 0.11z' = -0.22 - 0.22(0.5675) \\ &+ 0.11(-0.433) = -0.39248 \end{aligned}$$

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$$y^2: 0.375 - 0.875x^2 - 0.625z^2 = 0.375 - 0.875(-0.39248)$$
$$- 0.625(-0.433) = \underline{0.989045}$$

$$z^2: -0.6 + 0.3x^2 + 0.4y^2 = -0.6 + 0.3(0.39248)$$
$$+ 0.4(0.989045) = \underline{-0.322126}$$

$$x^2 = -0.39248$$

$$y^2 = 0.989045$$

$$z^2 = -0.322126$$

Calculando iteración ③

$$x^3 = -0.22 - 0.22y^2 + 0.11z^2 = -0.22 - 0.22(0.989045)$$
$$+ 0.11(-0.322126) = \underline{-0.47380718}$$

$$y^3 = 0.375 - 0.875x^3 - 0.625z^2 = 0.375 - 0.875(-0.47380718)$$
$$- 0.625(-0.322126) = \underline{0.98995190749}$$

$$z^3 = -0.6 + 0.3x^3 + 0.4y^3 = -0.6 + 0.3(-0.47380718)$$
$$+ 0.4(0.98995190749) = \underline{-0.346161391}$$

$$x^3 = -0.47380718$$

$$y^3 = 0.98995190749$$

$$z^3 = -0.346161391$$

Calculando iteración ④

$$x^4 = -0.22 - 0.22y^3 + 0.11z^3 = -0.22 - 0.22(0.9899518) \\ + 0.11(-0.346161391) = \underline{-0.4758671726}$$

$$y^4 = 0.375 - 0.875x^4 - 0.625z^3 = 0.375 - 0.875(-0.4758671726) \\ - 0.625(-0.346161391) = \underline{1.007734645}$$

$$z^4 = -0.6 + 0.3x^4 + 0.4y^4 = -0.6 + 0.3(-0.4758671726) \\ + 0.4(1.007734645) = \underline{-0.3396662936}$$

$$\boxed{\begin{aligned} x^4 &= -0.4758671726 \\ y^4 &= 1.007734645 \\ z^4 &= -0.3396662936 \end{aligned}}$$

$$2) \begin{aligned} 5x_1 + 2x_2 &= 12 \\ -x_1 + 10x_2 &= 8 \end{aligned}$$

Jacobi

$$x_1 = \frac{12 - 2x_2}{5} = 2.4 - 0.4x_2$$

$$x_2 = \frac{8 + 1x_1}{10} = 0.8 + 0.1x_1$$

$$x_1 = 2.4 - 0.4x_2$$

$$x_1^0 = 0$$

$$x_2 = 0.8 + 0.1x_1$$

$$x_2^0 = \emptyset$$

Calculamos iteración ①

$$x_1^1 = 2.4 - 0.4x_2^0 = \underline{2.4}$$

$$x_2^1 = 0.8 + 0.1x_1^1 = \underline{0.8}$$

$$\boxed{\begin{aligned} x_1^1 &= 2.4 \\ x_2^1 &= 0.8 \end{aligned}}$$

Calculamos iteración ②

$$x_1^2 = 2.4 - 0.4x_2^1 = 2.4 - 0.4(0.8) = \underline{2.08}$$

$$x_2^2 = 0.8 + 0.1x_1^1 = 0.8 + 0.1(2.4) = \underline{1.04}$$

$$\boxed{\begin{aligned} x_1^2 &= 2.08 \\ x_2^2 &= 1.04 \end{aligned}}$$

Calculamos iteración ③

$$x_1^3 = 2.4 - 0.4x_2^2 = 2.4 - 0.4(1.04) = \underline{1.984}$$

$$x_2^3 = 0.8 + 0.1x_1^2 = 0.8 + 0.1(2.08) = \underline{1.008}$$

$$\boxed{\begin{aligned} x_1^3 &= 1.984 \\ x_2^3 &= 1.008 \end{aligned}}$$

Calculamos iteración ④

$$x_1^4 = 2.4 - 0.4x_2^3 = 2.4 - 0.4(1.008) = \underline{1.9968}$$

$$x_2^4 = 0.8 + 0.1x_1^3 = 0.8 + 0.1(1.984) = \underline{0.9984}$$

$$\boxed{\begin{aligned} x_1^4 &= 1.9968 \\ x_2^4 &= 0.9984 \end{aligned}}$$

# Gauss-Seidel

$$2) \begin{aligned} 5x_1 + 2x_2 &= 12 \\ -x_1 + 10x_2 &= 8 \end{aligned}$$

$$\begin{aligned} x_1 &= 2.4 - 0.4x_2 \\ x_2 &= 0.8 + 0.1x_1 \end{aligned}$$

$$\begin{aligned} x_1^0 &= 0 \\ x_2^0 &= 0 \end{aligned}$$

Calculamos iteración ①

$$x_1^1 = 2.4 - 0.4x_2^0 = 2.4$$

$$x_2^1 = 0.8 + 0.1x_1^1 = 0.8 + 0.1(2.4) = 1.08$$

$$\boxed{\begin{aligned} x_1^1 &= 2.4 \\ x_2^1 &= 1.08 \end{aligned}}$$

Calculamos iteración ②

$$x_1^2 = 2.4 - 0.4x_2^1 = 2.4 - 0.4(1.08) = 1.968$$

$$x_2^2 = 0.8 + 0.1x_1^2 = 0.8 + 0.1(1.968) = 0.9968$$

$$\boxed{\begin{aligned} x_1^2 &= 1.968 \\ x_2^2 &= 0.9968 \end{aligned}}$$

Calculamos iteración ③

$$x_1^3 = 2.4 - 0.4x_2^2 = 2.4 - 0.4(0.9968) = \underline{2.00128}$$

$$x_2^3 = 0.8 + 0.1x_1^3 = 0.8 + 0.1(2.00128) = \underline{1.000128}$$

$$\boxed{\begin{aligned} x_1^3 &= 2.00128 \\ x_2^3 &= 1.000128 \end{aligned}}$$

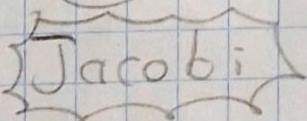
Calculamos iteración ④

$$x_1^4 = 2.4 - 0.4x_2^3 = 2.4 - 0.4(1.000128) = \underline{1.9999488}$$

$$x_2^4 = 0.8 + 0.1x_1^4 = 0.8 + 0.1(1.9999488) = \underline{0.99999488}$$

$$\boxed{\begin{aligned} x_1^4 &= 1.9999488 \\ x_2^4 &= 0.99999488 \end{aligned}}$$

3)  $\begin{cases} 8x_1 + x_2 = 4 \\ 2x_1 + 5x_2 = 3 \\ x_1 + 4x_3 = 3 \end{cases}$

 Jacobi

$$x_1 = \frac{4}{8} - \frac{1}{8}x_2 = 0.5 - 0.125x_2$$

$$x_2 = \frac{3}{5} - \frac{2}{5}x_1 = 0.6 - 0.4x_1$$

$$x_3 = \frac{3}{4} - \frac{1}{4}x_1 = 0.75 - 0.25x_1$$

$$\begin{aligned} x_1^0 &= 0 \\ x_2^0 &= 0 \\ x_3^0 &= 0 \end{aligned}$$

Calculamos iteración ①

$$x_1^1 = 0.5 - 0.125 \cancel{x_2^0} = 0.5$$

$$x_2^1 = 0.6 - 0.4 \cancel{x_1^0} = 0.6$$

$$x_3^1 = 0.75 - 0.25 \cancel{x_1^0} = 0.75$$

$$\boxed{\begin{array}{l} x_1^1 = 0.5 \\ x_2^1 = 0.6 \\ x_3^1 = 0.75 \end{array}}$$

Calculamos iteración ②

$$x_1^2 = 0.5 - 0.125 x_2^1 = 0.5 - 0.125(0.6) = 0.425$$

$$x_2^2 = 0.6 - 0.4 x_1^1 = 0.6 - 0.4(0.5) = 0.4$$

$$x_3^2 = 0.75 - 0.25 x_1^1 = 0.75 - 0.25(0.5) = 0.625$$

$$\boxed{\begin{array}{l} x_1^2 = 0.425 \\ x_2^2 = 0.4 \\ x_3^2 = 0.625 \end{array}}$$

Calculamos iteración ③

$$x_1^3 = 0.5 - 0.125 x_2^2 = 0.5 - 0.125(0.4) = 0.45$$

$$x_2^3 = 0.6 - 0.4 x_1^2 = 0.6 - 0.4(0.425) = 0.43$$

$$x_3^3 = 0.75 - 0.25 x_1^2 = 0.75 - 0.25(0.425) = 0.64375$$

$$x_1^3 = 0.45$$

$$x_2^3 = 0.43$$

$$x_3^3 = 0.6375$$

Calculamos iteración ④

$$x_1^4 = 0.5 - 0.125 x_2^3 = 0.5 - 0.125(0.43) = 0.44625$$

$$x_2^4 = 0.6 - 0.4 x_1^3 = 0.6 - 0.4(0.45) = 0.42$$

$$x_3^4 = 0.75 - 0.25 x_1^3 = 0.75 - 0.25(0.45) = 0.6375$$

$$x_1^4 = 0.44625$$

$$x_2^4 = 0.42$$

$$x_3^4 = 0.6375$$

### GAUSS-SEIDEL

$$③ 8x_1 + x_2 = 4$$

$$2x_1 + 5x_2 = 3$$

$$x_1 + 4x_3 = 3$$

$$x_1 = 0.5 - 0.125 x_2$$

$$x_2 = 0.6 - 0.4 x_1$$

$$x_3 = 0.75 - 0.25 x_1$$

$$x_1^0 = 0$$

$$x_2^0 = 0$$

$$x_3^0 = 0$$

Calcular iteración ①

$$x_1^1 = 0.5 - 0.125 \cancel{x_2^0} = \underline{0.5}$$

$$x_2^1 = 0.6 - 0.4 x_1^1 = 0.6 - 0.4(0.5) = \underline{0.4}$$

$$x_3^1 = 0.75 - 0.25 x_1^1 = 0.75 - 0.25(0.5) = \underline{0.625}$$

$$x_1^1 = 0.5$$

$$x_2^1 = 0.4$$

$$x_3^1 = 0.625$$

Calcular iteración ②

$$x_1^2 = 0.5 - 0.125 x_2^1 = 0.5 - 0.125(0.4) = \underline{0.45}$$

$$x_2^2 = 0.6 - 0.4 x_1^2 = 0.6 - 0.4(0.45) = \underline{0.42}$$

$$x_3^2 = 0.75 - 0.25 x_1^2 = 0.75 - 0.25(0.45) = \underline{0.6375}$$

$$x_1^2 = 0.45$$

$$x_2^2 = 0.42$$

$$x_3^2 = 0.6375$$

Calcular iteración ③

$$x_1^3 = 0.5 - 0.125 x_2^2 = 0.5 - 0.125(0.42) = \underline{0.4475}$$

$$x_2^3 = 0.6 - 0.4 x_1^3 = 0.6 - 0.4(0.4475) = \underline{0.4211}$$

$$x_3^3 = 0.75 - 0.25 x_1^3 = 0.75 - 0.25(0.4475) = \underline{0.638125}$$

$$\boxed{\begin{aligned}x_1^3 &= 0.4475 \\x_2^3 &= 0.421 \\x_3^3 &= 0.638125\end{aligned}}$$

Calcular iteración ④

$$x_1^4 = 0.5 - 0.125 x_2^3 = 0.5 - 0.125(0.421) = 0.447375$$

$$x_2^4 = 0.6 - 0.4 x_1^4 = 0.6 - 0.4(0.447375) = 0.42105$$

$$x_3^4 = 0.75 - 0.25 x_1^4 = 0.75 - 0.25(0.447375) = 0.63815625$$

4)  $6x_1 + 2x_2 + x_3 = 22$

$$-x_1 + 8x_2 + 2x_3 = 20$$

$$x_1 - x_2 + 6x_3 = 23$$

Jacob

$$x_1 = \frac{22}{6} - \frac{2x_2}{6} - \frac{x_3}{6} = 3.66 - 0.33x_2 - 0.166x_3$$

$$x_2 = \frac{20}{8} + \frac{1x_1}{8} - \frac{2x_3}{8} = 2.5 + 0.125x_1 - 0.25x_3$$

$$x_3 = \frac{23}{6} - \frac{1x_1}{6} + \frac{1x_2}{6} = 3.833 - 0.166x_1 + 0.166x_2$$

$$x_1 = 3.66 - 0.33x_2 - 0.166x_3$$

$$x_2 = 2.5 + 0.125x_1 - 0.25x_3$$

$$x_3 = 3.833 - 0.166x_1 + 0.166x_2$$

$$x_1^0 = 0$$

$$x_2^0 = 0$$

$$x_3^0 = 0$$

Calcular iteración ①

$$X_1^1 = 3.66 - 0.33X_2^0 - 0.166X_3^0 = 3.66$$

$$X_2^1 = 2.5 + 0.125X_1^0 - 0.25X_3^0 = 2.5$$

$$X_3^1 = 3.833 - 0.166X_1^0 + 0.166X_2^0 = 3.833$$

$$X_1^1 = 3.66$$

$$X_2^1 = 2.5$$

$$X_3^1 = 3.833$$

Calcular iteración ②

$$X_1^2 = 3.66 - 0.33X_2^1 - 0.166X_3^1 = 3.66 - 0.33(2.5) \\ - 0.166(3.833) = 2.198722$$

$$X_2^2 = 2.5 + 0.125X_1^1 - 0.25X_3^1 = 2.5 + 0.125(3.66) \\ - 0.25(3.833) = 1.99925$$

$$X_3^2 = 3.833 - 0.166X_1^1 + 0.166X_2^1 = 3.833 - 0.166(3.66) \\ + 0.166(2.5) = 3.64044$$

$$X_1^2 = 2.198722$$

$$X_2^2 = 1.99925$$

$$X_3^2 = 3.64044$$

Calcular iteración ③

$$X_1^3 = 3.66 - 0.33 X_2^2 - 0.166 X_3^2 = 3.66 - 0.33(1.99925) \\ - 0.166(3.64044) = \underline{2.39593446}$$

$$X_2^3 = 2.5 + 0.125 X_1^2 - 0.25 X_3^2 = 2.5 + 0.125(2.198722) \\ - 0.25(3.64044) = \underline{1.86473025}$$

$$X_3^3 = 3.833 - 0.166 X_1^2 + 0.166 X_2^2 = 3.833 - 0.166(2.198722) \\ + 0.166(1.99925) = \underline{3.7999887648}$$

$$\boxed{\begin{aligned} X_1^3 &= 2.39593446 \\ X_2^3 &= 1.86473025 \\ X_3^3 &= 3.7999887648 \end{aligned}}$$

Calcular iteración ④

$$X_1^4 = 3.66 - 0.33 X_2^3 - 0.166 X_3^3 = 3.66 - 0.33(1.86473025) \\ - 0.166(3.7999887648) = \underline{2.413840883}$$

$$X_2^4 = 2.5 + 0.125 X_1^3 - 0.25 X_3^3 = 2.5 + 0.125(2.39593446) \\ - 0.25(3.7999887648) = \underline{1.849494617}$$

$$X_3^4 = 3.833 - 0.166 X_1^3 + 0.166 X_2^3 = 3.833 - 0.166(2.39593446) \\ + 0.166(1.86473025) = \underline{3.744820101}$$

$$\boxed{X_1^4 = 2.413840883}$$

$$\boxed{X_2^4 = 1.849494617}$$

$$\boxed{X_3^4 = 3.744820101}$$

# GAUSS - SEIDEL

$$4) \quad 6x_1 + 2x_2 + x_3 = 22$$

$$-x_1 + 8x_2 + 2x_3 = 20$$

$$x_1 - x_2 + 6x_3 = 23$$

$$x_1 = 3.66 - 0.33x_2 - 0.166x_3$$

$$x_2 = 2.5 + 0.125x_1 - 0.25x_3$$

$$x_3 = 3.833 - 0.166x_1 + 0.166x_2$$

$$\boxed{\begin{aligned} x_1^0 &= 0 \\ x_2^0 &= 0 \\ x_3^0 &= 0 \end{aligned}}$$

Calculamos iteración ①

$$x_1^1 = 3.66 - 0.33x_2^0 - 0.166x_3^0 = 3.66$$

$$x_2^1 = 2.5 + 0.125x_1^1 - 0.25x_3^0 = 2.5 + 0.125(3.66) = 2.9575$$

$$x_3^1 = 3.833 - 0.166x_1^1 + 0.166x_2^1 = 3.833 - 0.166(3.66) + 0.166(2.9575) = 3.716385$$

$$\boxed{x_1^1 = 3.66}$$

$$\boxed{x_2^1 = 2.9575}$$

$$\boxed{x_3^1 = 3.716385}$$

## Calculamos Iteración ②

$$X_1^2 = 3.66 - 0.33 X_2^2 - 0.166 X_3^2 = 3.66 - 0.33(2.9575) \\ - 0.166(3.716385) = \underline{2.06710509}$$

$$X_2^2 = 2.5 + 0.125 X_1^2 - 0.25 X_3^2 = 2.5 + 0.125(2.06710509) \\ - 0.25(3.716385) = \underline{1.829291886}$$

$$X_3^2 = 3.833 - 0.166 X_1^2 + 0.166 X_2^2 = 3.833 - 0.166(2.06710509) \\ + 0.166(1.829291886) = \underline{3.793523008}$$

$$X_1^2 = 2.06710509$$

$$X_2^2 = 1.829291886$$

$$X_3^2 = 3.793523008$$

## Calculamos iteración ③

$$X_1^3 = 3.66 - 0.33 X_2^2 - 0.166 X_3^2 = 3.66 - 0.33(1.829291886) \\ - 0.166(3.793523008) = \underline{2.426608858}$$

$$X_2^3 = 2.5 + 0.125 X_1^2 - 0.25 X_3^2 = 2.5 + 0.125(2.426608858) \\ - 0.25(3.793523008) = \underline{1.854945355}$$

$$X_3^3 = 3.833 - 0.166 X_1^2 + 0.166 X_2^2 = 3.833 - 0.166(2.426608858) \\ + 0.166(1.854945355) = \underline{3.738103859}$$

$$X_1^3 = 2.426608859$$

$$X_2^3 = 1.854945355$$

$$X_3^3 = 3.738103859$$

## Calculamos iteración ④

$$x_1^4 = 3.66 - 0.33x_2^3 - 0.166x_3^3 = 3.66 - 0.33(1.854945) \\ - 0.166(3.738103859) = \underline{2.427342909}$$

$$x_2^4 = 2.5 + 0.125x_1^3 - 0.25x_3^3 = 2.5 + 0.125(2.427342909) \\ - 0.25(3.738103859) = \underline{1.868891899}$$

$$x_3^4 = 3.833 - 0.166x_1^3 + 0.166x_2^3 = 3.833 - 0.166(2.427342909) \\ + 0.166(1.868891899) = \underline{3.740297132}$$

$$x_1^4 = 2.427342909$$

$$x_2^4 = 1.868891899$$

$$x_3^4 = 3.740297132$$