Villagrand Aparicio Brandon Jair

1) Sea el sig. polinomio

$$f(x) = -0.5x^2 + 2.5x + 4.5$$

formula general

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2.5) \pm \sqrt{(2.5)^2 - 4(-0.5)(4.5)}}{2(-0.5)}$$

$$x = \frac{-2.5 \pm \sqrt{6.25 - (-9)}}{-1}$$

$$x = \frac{-2.5 \pm \sqrt{6.25 + 9}}{-1}$$

$$x = \frac{-2.5 \pm \sqrt{15.25}}{-1}$$

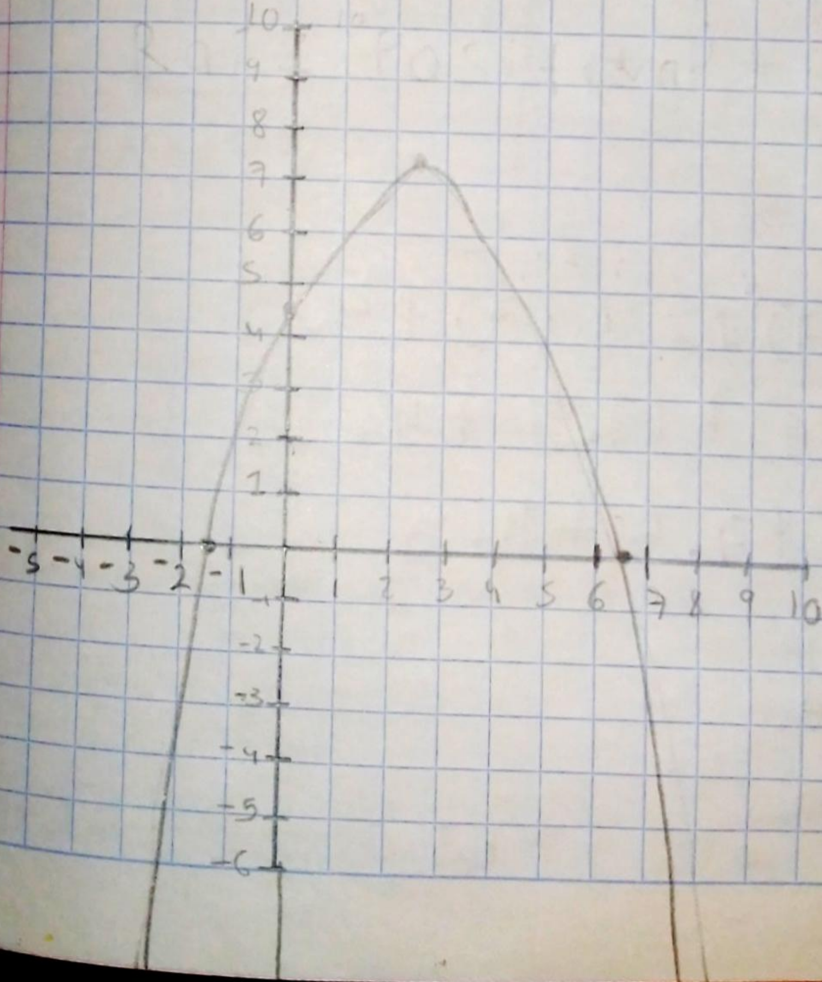
$$x = \frac{-2.5 \pm 3.9}{-1}$$

$$x_1 = \frac{-2.5 + 3.9}{-1}$$

$$x_1 = \frac{1.4}{-1} = \boxed{-1.4}$$ Raíz negativa

$$x_2 = \frac{-2.5 - 3.9}{-1}$$

$$x_2 = \frac{-6.4}{-1} = \boxed{6.4}$$ Raíz positiva



Villagrana Aparicio Brandon Jair

Bisección (raíz positiva)

$f(x) = -0.5x^2 + 2.5x + 4.5$

intervalo $[6, 7]$

$x_u = 7$

$x_l = 6$

Sust. "x_u" en $f(x)$

$f(x_l) = -0.5(6)^2 + 2.5(6) + 4.5$

$= -0.5(36) + 15 + 4.5$

$= -18 + 15 + 4.5$

$\underline{f(x_l) = 1.5}$

$f(x_u) = -0.5(7)^2 + 2.5(7) + 4.5$

$= -0.5(49) + 17.5 + 4.5$

$= -24.5 + 17.5 + 4.5$

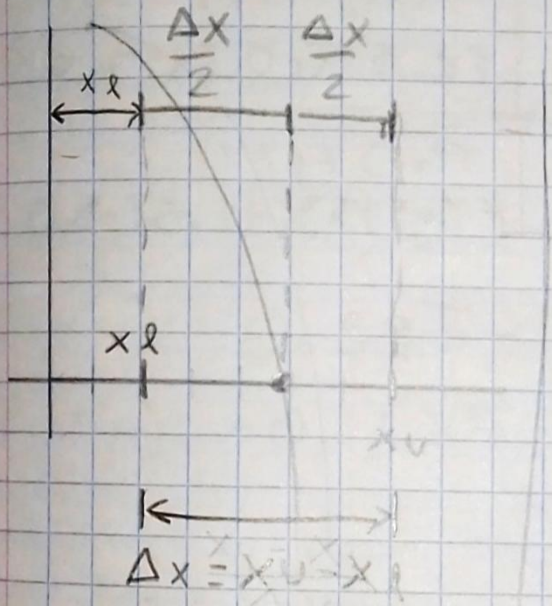
$\underline{f(x_u) = -2.5}$

Villagrana Aparicio Brandon Jair

$= (1.5)(-2.5) = -3.75 < 0$

$F(x_u)F(x_l) < 0$

Si hay raíz en el intervalo



$$X_r = X_l + \frac{\Delta x}{2}$$

$$= \frac{2}{2}x_l + \frac{x_u - x_l}{2}$$

$$= \frac{2x_l + x_u - x_l}{2}$$

$$X_r = \frac{x_u + x_l}{2}$$

Sustituir

$$X_r = \frac{7+6}{2} = \frac{13}{2} = 6.5$$

Sustituir

$$F(x_r) = -0.5(6.5)^2 + 2.5(6.5) + 4.5$$

$f(x_r) = -0.5(42.25) + 16.25 + 4.5$

$f(x_r) = -21.125 + 16.25 + 4.5$

$f(x_r) = -0.375$

$f(x_l) = +1.5$

$f(x_r) = -0.375$

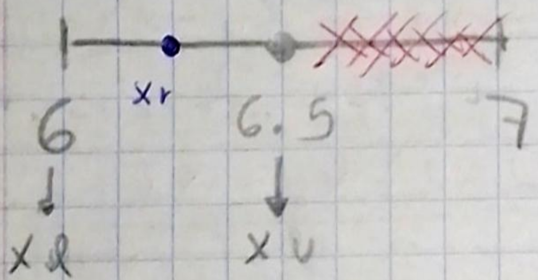
$f(x_u) = -2.5$

$f(x_l)\,f(x_r)$

$(1.5)(-0.375) < 0$ ✓

$f(x_r)\,f(x_u)$

$(-0.375)(-2.5) > 0$ ✗



Primera iteración

$x'_l = 6$

$x'_r = ?$

$x'_u = 6.5$

$$x'_r = \frac{x'_l + x'_u}{2} = \frac{6 + 6.5}{2} = 6.25$$

sustituir

$$f(x'_r) = -0.5(6.25)^2 + 2.5(6.25) + 4.5$$

$$f(x'_r) = -0.5(39.0625) + 15.625 + 4.5$$

$$= -19.53125 + 15.625 + 4.5$$

$$f(x'_r) = \underline{0.59375}$$

$f(x'_l) = f(1) = 1.5$

$f(x'_r) = 0.59375$

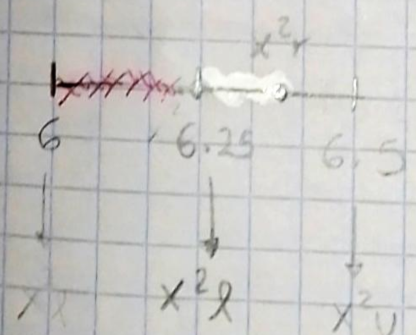
$f(x'_u) = -0.375$

$f(x'_l)\,f(x'_r)$

$(1.5)(0.59375) > 0$ X

$f(x'_r)\,f(x'_u)$

$(0.59375)(-0.375) < 0$ ✓



$x^2_l = 6.25$

$x^2_r = ?$

$x^2_u = 6.5$

Segunda iteración

$$x^2_r = \frac{x^2_l + x^2_u}{2} = \frac{6.25 + 6.5}{2} = \underline{6.375}$$

sustituir

$$f(x^2_r) = -0.5(6.375)^2 + 2.5(6.375) + 4.5$$

$F(x^2{}_r) = -0.5(40.640625) + 15.9375 + 4.5$

$F(x^2{}_r) = -20.3203125 + 15.9375 + 4.5$

$F(x^2{}_r) = \underline{0.1171875}$

$F(x^2{}_l) = 0.59375$

$F(x^2{}_r) = 0.1171875$

$F(x^2{}_u) = -0.375$

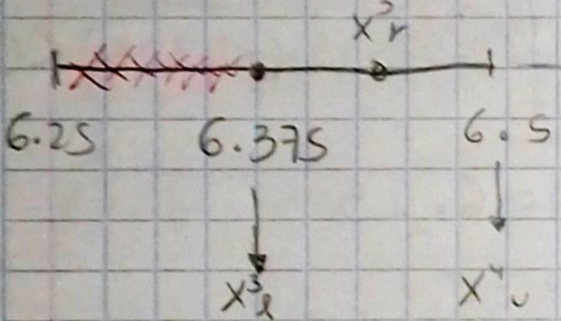
$F(x^2{}_l)\,F(x^2{}_r)$ -

$(0.59375)(0.1171875) > 0$

$F(x^2{}_r)\,F(x^2{}_u)$

$(0.1171875)(-0.375) < 0$ ✓

Tercera Iteración



$x^3{}_l = 6.375$

$x^3{}_r = ?$

$x^3{}_u = 6.5$

$$x^3{}_r = \frac{x^3{}_l + x^3{}_u}{2} = \frac{6.375 + 6.5}{2} = \underline{6.4375}$$

$f(x^3 r) = -0.5(6.4375)^2 + 2.5(6.4375) + 4.5$

$= -0.5(41.44140625) + 16.09375 + 4.5$

$f(x^3 r) = -20.72070313 + 16.09375 + 4.5$

$f(x^3 r) = -0.12695313$

$f(x^3 l) = 0.1171875$

$f(x^3 r) = -0.12695313$

$f(x^3 u) = -0.375$

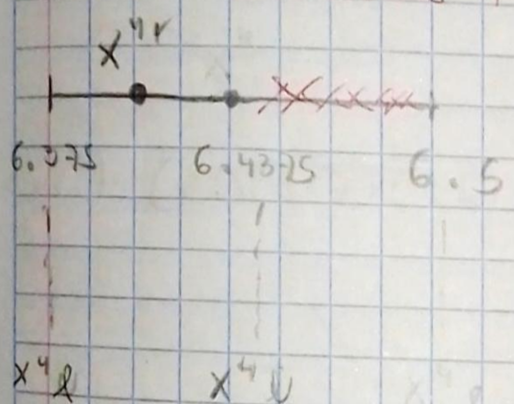
$f(x^3 l)\, f(x^3 r)$

$(0.1171875)(-0.12695313) < 0$ ✓

$f(x^3 l)\, f(x^3 r)$

$(-0.12695313)(-0.375) > 0$ ✗

Cuarta iteración



$x^4_r = \dfrac{6.375 + 6.437}{2} = \underline{6.406}$

$f(x^4 r) = -0.5(6.406)^2 + 2.5(6.406) + 4.5$

$f(x^4 r) = -0.5(41.036836) + 2.5(6.406) + 4.5$

$$f(x^4 r) = -0.003418$$

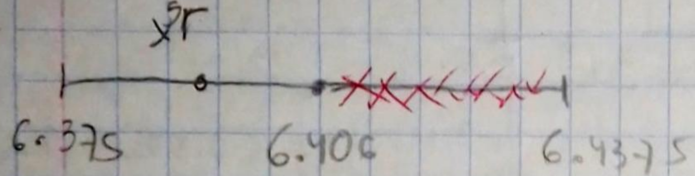
$$f(x^4 l) = 0.1171875$$
$$f(x^4 r) = -0.003418$$
$$f(x^4 u) = -0.12695313$$

$f(x^4 l)\, f(x^4 r)$ ✓

$(0.1171875)(-0.003418) < 0$

X

Quinta iteración



$$x^5 r = \frac{6.375 + 6.406}{2} = 6.3905$$

$$f(x^5 r) = -0.5(6.3905)^2 + 2.5(6.3905) + 4.5$$
$$f(x^5 r) = 0.057004875$$

$$f(x^5 l) = 0.1171875$$
$$f(x^5 r) = 0.057004875$$
$$f(x^5 u) = -0.12695313$$

X

$f(x^5 r)\, f(x^5 u)$

$(0.057004875)(-0.12695313)$
< 0

$$\varepsilon_a = \left| \frac{x_r^{actual} - x_r^{anterior}}{x_r^{actual}} \right|$$

$$\boxed{\varepsilon_a < 0.001}$$ ← Criterio paro

$x_r^{act} = 6.3905 \qquad x_r^{ant} = 6.406$

$$\varepsilon_a = \left| \frac{6.3905 - 6.406}{6.3905} \right| = -0.002425475$$

$-0.002425475 < 0.001$ ✓

Raíz negativa
(Falsa posición)

$f(x) = -0.5x^2 + 2.5x + 4.5$

intervalo $[-2, -1]$

$x_u = -1$

$x_l = -2$

sust. "x_u" en $f(x)$

$f(x_u) = -0.5(-1)^2 + 2.5(-1) + 4.5$

$= -0.5(1) - 2.5 + 4.5$

$f(x_u) = 1.5$

$f(x_l) = -0.5(-2)^2 + 2.5(-2) + 4.5$

$= -0.5(4) - 5 + 4.5$

$= -2 - 5 + 4.5$

$= -2.5$

$f(x_u)\,f(x_l) < 0$

$(1.5)(-2.5) < 0 \rightarrow$ si hay raíz en el intervalo formula

$$X_r = X_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

$$Xr = -1 - \frac{1.5(-2-(-1))}{-2.5 - 1.5}$$

$$Xr = -1 - \frac{1.5(-2+1)}{-4}$$

$$Xr = -1 - \frac{1.5(-1)}{-4}$$

$$Xr = -1 + \frac{1.5}{-4}$$

$$Xr = -1 - 0.375$$

$$Xr = \underline{-1.375}$$

$$f(xr) = -0.5(-1.375)^2 + 2.5(-1.375) + 4.5$$

$$= \underline{0.1171875}$$

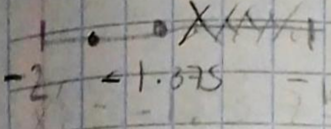
$f(x_l) = -2.5$

$f(x_r) = 0.1171875$

$f(x_u) = 1.5$

$(-2.5)(0.1171875) < 0$ ✓

X



$x'_l = -2$

$x'_r = ?$

$x'_u = -1.375$

$f(x'_u) = 0.1171875$

$f(x'_l) = -2.5$

$$x'_r = -1.375 - \frac{0.1171875(-2-(-1.375))}{-2.5 - 0.1171875}$$

$$x'_r = -1.375 - \frac{0.1171875(-2+1.375)}{-2.6171875}$$

$$x'_r = -1.375 - \left(\frac{0.1171875(-0.625)}{-2.6171875}\right)$$

$$x'_r = -1.347014925$$

$f(x^1 r) = -0.5(-1.347014925)^2 + 2.5(-1.347014925)$
$+4.5$

$f(x^1 r) = 0.225238083$

$f(x^1 l) = -2.5$

$f(x^1 r) = 0.225238083$

$f(x^1 u) = 0.1171875$

$(-2.5)(0.225238083) < 0$ ✓

X

-2 $-1.3470..$ -1.375

$x^2 l = -2$

$x^2 r = ?$

$x^2 u = -1.3470$

$f(x^2 u) = 0.225238083$

$f(x^2 l) = -2.5$

$$x^2 r = -1.3470 - \frac{(0.225238083)(-2-(-1.3470))}{-2.5-0.225238083}$$

$$x^2 r = -1.3470 - \frac{(0.225238083)(-0.653)}{-2.725238083}$$

$x^2 r = -1.400969768$

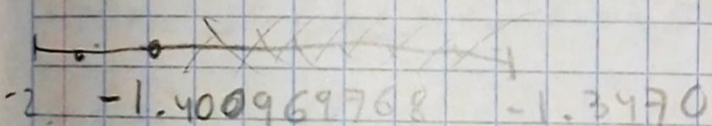
$f(x^2_r) = -0.5(-1.400969768)^2 + 2.5(-1.400969768)$
$+ 4.5$

$f(x^2 r) = 0.016217434$

$f(x^2 l) = -2.5$

$f(x^2 r) = 0.016217434$ I $(-2.5)(0.016217434) < 0$

$f(x^2 u) = 0.225238083$ I X



$x^3 l = -2$ $f(x^3 l) = -2.5$

$x^3 r = ?$ $f(x^3 u) = 0.016217434$

$x^3 u = -1.400969768$

$$x^3 r = -1.400969768 - \frac{0.016217434(-2-(-1.4009697))}{-2.5 - 0.016217434}$$

$$x^3 r = -1.400969768 - \frac{0.016217434(-0.599030232)}{-2.516217434}$$

$x^3 r = -1.404830616$

$F(x^3 r) = -0.5(-1.404830616)^2 + 2.5(-1.404830616)$
$+ 4.5$

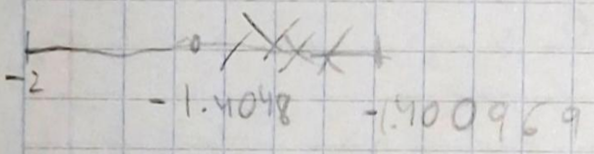
$F(x^3 r) = 0.00114893$

$F(x^3 l) = -2.5$

$F(x^3 r) = 0.00114893$

$F(x^3 u) = 0.001621734$

$(-2.5)(0.00114893) < 0$



$x^4 l = -2$

$x^4 r = ?$

$x^4 u = -1.404830616$

$F(x^4 u) = 0.00114893$

$F(x^4 l) = -2.5$

$$x^4 r = -1.404830616 - \frac{0.00114893(-2-(-1.404830616))}{-2.5 - 0.00114893}$$

$$x^4 r = -1.404830616 - \frac{0.00114893(-0.595169384)}{-2.50114893}$$

$$x^4 r = -1.405104014$$

$$f(x^4 r) = -0.5(-1.405104014)^2 + 2.5(-1.405104014)$$
$$+4.5$$

$$f(x^4) = 0.000081319$$

$$f(x^4 l) = -2.5$$
$$f(x^4 r) = 0.000081319$$
$$f(x^4 u) = 0.00114893$$

$(-2.5)(0.000081319) < 0$

$$x^5 l = -2$$
$$x^5 r = ?$$
$$x^5 u = -1.405104014$$

$$f(x^5 u) = 0.000081319$$
$$f(x^5 l) = -2.5$$

$$x^5 r = -1.405104014 - \frac{0.000081319(-2-(-1.405104014))}{-2.5 - 0.000081319}$$

$$x^s r = -1.405104014 - \frac{0.000081319(-0.594445986)}{-2.500081319}$$

$$x^s r = -1.405123364$$

$$f(x^s r) = -0.5(-1.405123364)^2 + 2.5(-1.405123364) + 4.5$$

$$f(x^s r) = 0.000000575 5$$

$$f(x^s l) = -2.5$$

$$f(x^s r) = 0.0000005755$$

$$f(x^s u) = 0.000081319$$

$(-2.5)(0.0000005755) < 0$ ✓

$$\varepsilon a = \left| \frac{xr^{act} - xr^{ant}}{xr^{act}} \right|$$

$$\boxed{\varepsilon a < 0.001}$$ ← criterio paro

$xr^{act} = 1.405123364 \quad x_r^{ant} = 1.405104014$

$$\varepsilon a = \left| \frac{-1.405123364 - (-1.405104014)}{1.405123364} \right|$$

$\varepsilon_a = -0.000013771$

$-0.000013771 < 0$

Bisección

Intervalo $[0, 1]$

$x_l = 0$

$x_u = 1$

$$x_r = \frac{0 + 1}{2} = 0.5$$

$f(x_r) = 5(0.5)^3 - 5(0.5)^2 + 6(0.5) - 2$

$f(x_r) = 0.375$

$f(x_l) = \cancel{5(0)^3} - \cancel{5(0)^2} + \cancel{6(0)} - 2$

$f(x_l) = -2$

$f(x_u) = \cancel{5(1)^3 - 5(1)^2} + 6(1) - 2$

$= 4$

$f(x_l) = -2$

$f(x_r) = 0.375$ I < 0

$f(x_u) = 4$

3) D Sea la función

$\ln(x^2) = 0.7$

$x^{2} = e^{0.7}$

$x_1 = \pm\sqrt{e^{0.7}}$

$x_1 = 1.41406754 9$

$x_2 = -1.41406754 9$

Determinar las primeras 3 iteraciones a mano utilizando o bisección en un intervalo $x \in [0,5, 2]$

$f(x) = \ln(x^2 = 0.7)$

intervalos

$[0.5, 2]$

$f(x_l) = \ln((0.5)^2) - 0.7 = -2.086294361$

$f(x_u) = \ln((2)^2) - 0.7 = 0.6862943611$

$f(x_u)\, f(x_l) < 0$

$(-2.086294361)(0.6862943611) < 0$

$= -1.431812059 < 0$ si hay raíz en el intervalo

$$x_r = \frac{x_l + x_u}{2} = \frac{0.5+2}{2} = 1.25$$

$$f(x_r) = \ln((1.25)^2) - 0.7$$
$$= -0.2537128974$$

$f(x_l) = -2.086294361$

$f(x_r) = -0.2537128974$

$f(x_u) = 0.6862943611$

$(-0.2537128974)(0.6862943611) < 0$

Primera Iteración

$x'_l = 1.25$

$x'_r = ?$

$x'_u = 2$

$$x'_r = \frac{1.25+2}{2} = \frac{3.25}{2} = 1.625$$

$$f(x'_r) = \ln((1.625)^2) - 0.7 = 0.2710156316$$

$f(x'_l) = -0.2537128974$

$f(x'_r) = 0.2710156316$

$f(x'_u) = 0.6862943611$

$(-0.2537128974)(0.2710156316) < 0$

Segunda iteración

$x^2 l = 1.25$

$x^2 r = ?$

$x^2 u = 1.625$

$$x^2 r = \frac{1.25 + 1.625}{2} = 1.4375$$

$f(x^2 r) = \ln((1.4375)^2) - 0.7 = 0.0258109873 8$

$f(x^2 l) = -0.2537128974$

$f(x^2 r) = 0.025810987 38$

$f(x^2 u) = 0.686294361$ 1

$(-0.2537128974)(0.025810987 38) < 0$

Tercera iteración

$x^3 l = 1.4375 \quad x^3 r = ? \quad x^3 u = 1.625$

$$x^3 r = \frac{1.4375 + 1.625}{2} = 1.53125$$

$f(x^3 r) = \ln((1.53125)^2) - 0.7 =$

$f(x^3 r = 0.1521687906$

$F(x^3 l) = -0.2537128974$ $(-0.253712897$

$F(x^3 r) = 0.025810987738$ $(0.025810987738) < 0$

$F(x^3 u) = 0.2710156316$

Falsa posición

$F(x) = \ln(x^2) = 0.7$

$x_l = 0.5 \quad x_u = 2$

$F(x_l) = -2.08629436 1$

$F(x_u) = 0.686294361 1$

$F(x_l)\,F(x_u) < 0$

$= -1.4318122056 < 0$ Si hay raíz en el intervalo

$$X_r = \frac{(-2.08629436)(2) - (0.6862943611)(0.5)}{-2.086294361 - 0.6862943611}$$

$$X_r = 1.628707448$$

$$f(x_r) = \ln((1.628707448)^2) - 0.7$$

$$f(x_r) = 0.2755734471$$

$$f(x_l) = -2.086294361 \quad \mathrm{I} \quad <0$$
$$f(x_r) = 0.2755734472 \quad \mathrm{I}$$
$$f(x_u) = 0.6862943611 \quad \mathrm{I} \quad \times$$

Primera iteración

$$x'_l = 0.5$$
$$x'_r = ?$$
$$x'_u = 1.628707448$$

$$x_r' = \frac{(-2.086294361)(1.628707448) - (0.275573)(0.5)}{-2.086294361 - 0.2755734471}$$

$$x'_r = 1.49703$$

$f(x'r) = \ln((1.49703)^2) - 0.7$

$f(x'r) = 0.10696$

$f(xl) = -2.08629$ I ✓ < 0

$f(xr) = 0.10696$

$f(xu) = 0.27557$ I X

$x^2 l = 0.5$

$x^2 r = ?$

$x^2 u = 1.49703$

$$x^2 r = \frac{(-2.08629)(1.49703) - (0.80696)(0.5)}{-2.08629 - 0.80696}$$

$x^2 r = 1.21894$

$f(x^2 r) = \ln((1.21894)^2) - 0.7$

$f(x^2 r) = -0.30403$

$f(x^2 l) = -2.08629$ I ✓ < 0

$f(x^2 r) = 1.21894$

$f(x^2 u) = 0.10696$ I X

Tercera Iteración

$x^3 l = 1.21894$

$x^3 r = ?$

$x^3 u = 1.49703$

$$x^3 r = \frac{(-0.30403)(1.49703) - (0.10696)(1.21894)}{-0.30403 - 0.10646}$$

$x^3 r = 1.42465$

$F(x^3 r) = \ln((1.42465)) - 0.7$

$f(x^3 r) = 0.078523339035$

$f(x^3 l) = -2.08629$

$F(x^3 r) = 0.0785239035$ ✓ < 0

$f(x^3 u) = 1.21894$

4) Determine la raiz bajo los siguientes casos

$$x^{3.5} = 80$$

• De forma analítica

$$x^{3.5} = 80$$

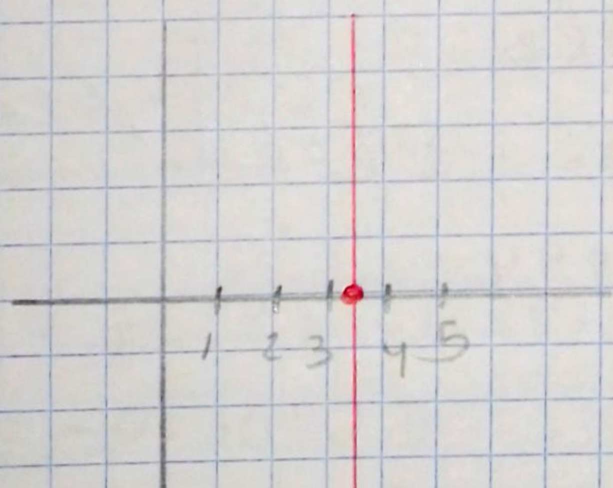
$$x^{7/2} = 80$$

$$\boxed{x = 8^{2/7}}$$

$$\boxed{x = \sqrt[7]{8^{2}}}$$

De forma gráfica

$$f(x) = x^{3.5} - 80$$



Utilizando bisección (3 iteraciones a mano) y el resto por programa

$f(x) = x^{3.5} - 80$

Intervalos $[1, 3]$

$x_l = 1$
$x_u = 3$

$f(x_l) = (1)^{3.5} - 80 = -79$

$f(3)^{3.5} - 80 = -33.2346282$

$(-79)(-33.2346282) > 0$

No hay raíz en intervalo

Utilizando la falsa posición (3 iteraciones a mano) el resto por programa

$f(x) = x^{3.5} - 80 = 0$

$x_l = 1$
$x_u = 3$

$f(x_l) = (1)^{3.5} - 80 = -79$
$f(x_u) = (3)^{3.5} - 80 = -33.2346282$

$(-79)(-33.2346282) > 0$ No existe raíz en intervalo